Riccati Differential Equation

Idle Mode:

$$\dot{\Omega}(t) = A\,\Omega(t) + \Omega(t)A^T + Q$$

Scaler case: $a \leftarrow A,\ q \leftarrow Q,\ \Omega_0 \leftarrow \Omega(t_0)$

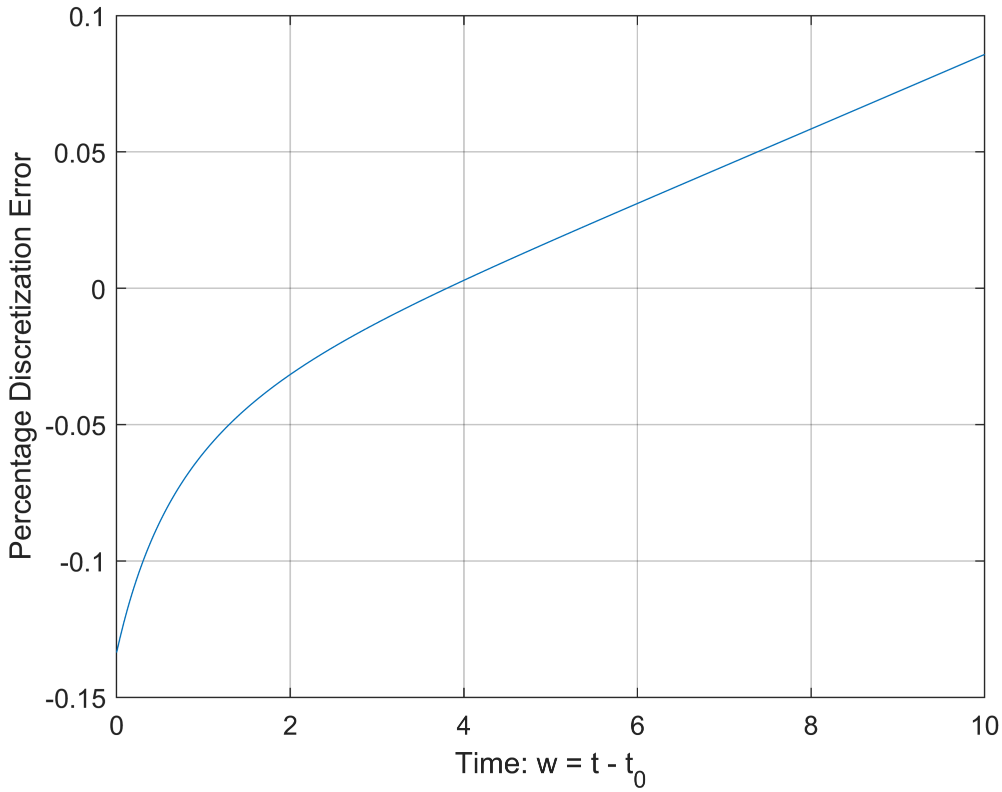
$$\Omega(t) = \left(\Omega_0 + \frac{q}{2a}\right)e^{2a(t-t_0)} - \frac{q}{2a}$$

$$J(t_0, t_0 + w) = \frac{1}{2a}\left(\Omega_0 + \frac{q}{2a}\right)\left(e^{2aw} - 1\right) - \frac{q}{2a}w$$

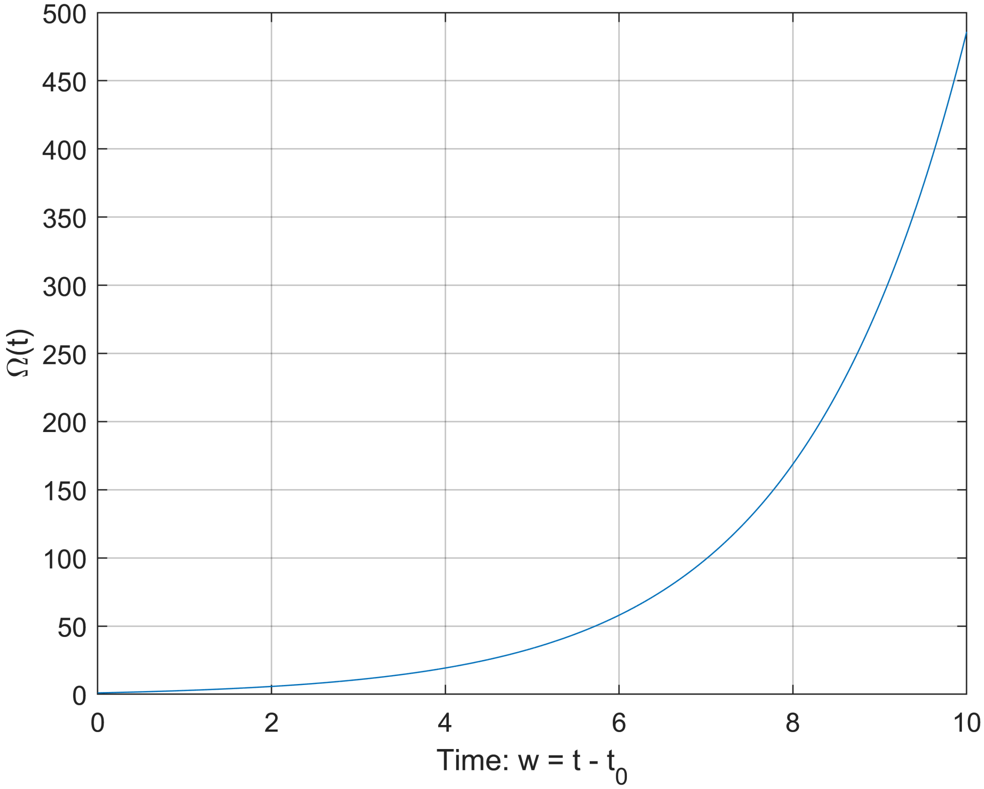
```
clear all; close all; clc;

a = 0.2628;
q = 0.8107;
t_0 = 5;
Omega_0 = 1
```

```
Omega_0 = 1
```

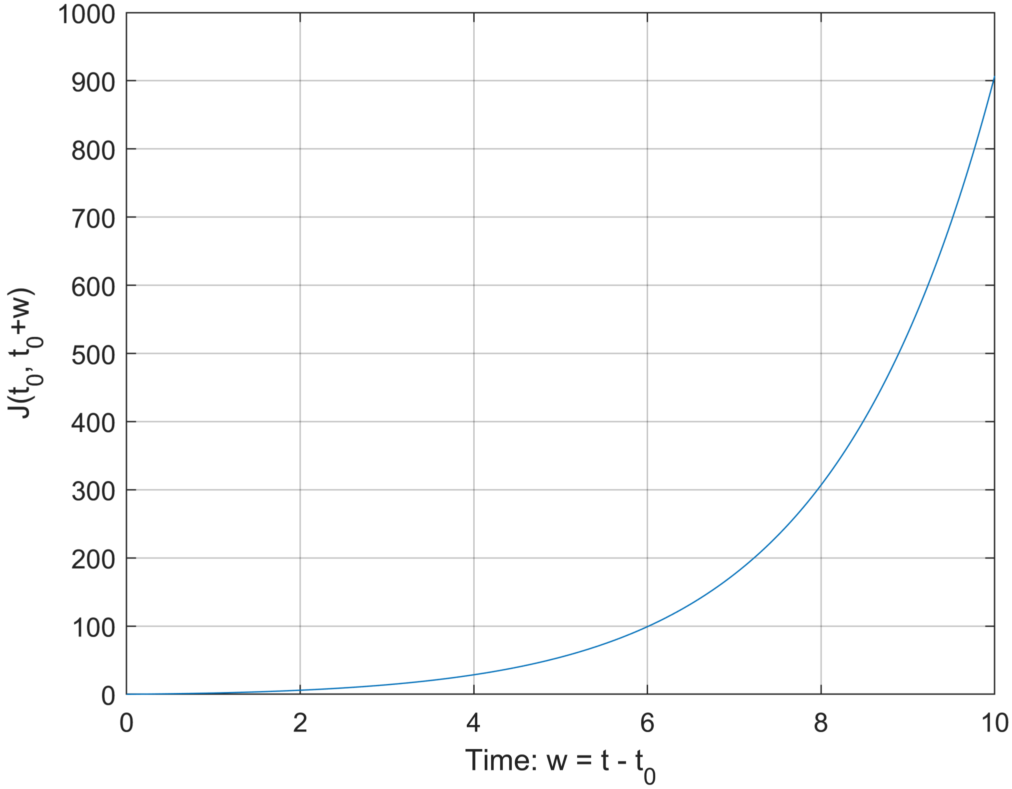
```

deltaT = 0.001;
periodT = 10;
Omega = Omega_0;
JArray = [];
errorFrac = [];
OmegaArray = [];
for w = 0:deltaT:periodT % w = t-t_0
    Omega = Omega + deltaT*(2*a*Omega + q);
    OmegaAct = (Omega_0 + (q/(2*a)))*exp(2*a*w)-(q/(2*a));
    OmegaArray = [OmegaArray, OmegaAct];
    errorFrac = [errorFrac, (OmegaAct-Omega)*100/OmegaAct];

    % cost upto w
    J_w = (1/(2*a))*(Omega_0 + (q/(2*a)))*(exp(2*a*w)-1)-(q/(2*a))*w;
    JArray = [JArray, J_w];
end
figure
plot(0:deltaT:periodT,errorFrac)
ylabel('Percentage Discretization Error')
xlabel('Time: w = t - t_0')
grid on
```




```
% Percentage discretization error grows linearly but is less than 1%

figure
plot(0:deltaT:periodT,OmegaArray)
ylabel('\Omega(t)')
xlabel('Time: w = t - t_0')
grid on
```




```
figure
plot(0:deltaT:periodT,JArray)
ylabel('J(t_0, t_0+w)')
xlabel('Time: w = t - t_0')
grid on
```




```
JArray1 = JArray;
```

Dwell Mode

$\dot{\Omega}(t) = A\,\Omega(t) + \Omega(t)A^T + Q - \Omega(t)\,G\,\Omega(t)$ with $G = H^T R^{-1} H$

Scaler case: $a \leftarrow A,\ q \leftarrow Q,\ \ g \leftarrow G,\ \ \Omega_0 \leftarrow \Omega(t_0)$

$v_1 = \frac{1}{q}\left(-a + \sqrt{a^2 + qg}\right)$ and $v_2 = \frac{1}{q}\left(-a - \sqrt{a^2 + qg}\right)$ and $\lambda = 2\sqrt{a^2 + qg}$

$$\Omega(t) = \frac{(v_2\,\Omega_0 - 1) + (-v_1\,\Omega_0 + 1)e^{-\lambda(t-t_0)}}{v_1(v_2\,\Omega_0 - 1) + v_2(-v_1\,\Omega_0 + 1)e^{-\lambda(t-t_0)}}$$

$$J(t_0, t_0 + w) = \frac{1}{g}\ln\left(v_1(v_2\,\Omega_0 - 1) + v_2(-v_1\,\Omega_0 + 1)e^{-\lambda w}\right) + \frac{w}{v_1} - \frac{1}{g}\ln(v_2 - v_1)$$

```
% clear all; close all; clc;

a = 0.2628;
q = 0.8107;
r = 5.7044;
h = 1;
g = h'*r^(-1)*h;
```

```
v_1 = (-a+sqrt(a^2+q*g))/q
```

```
v_1 = 0.2427
```

```
v_2 = (-a-sqrt(a^2+q*g))/q
```

```
v_2 = -0.8910
```

```
lambda = 2*sqrt(a^2+q*g)
```

```
lambda = 0.9191
```

```
Omega_ss = 1/v_1
```

```
Omega_ss = 4.1205
```

```
t_0 = 5;
Omega_0 = 100
```

```
Omega_0 = 100
```

```
c_1 = v_2*Omega_0 - 1
```

```
c_1 = -90.1015
```

```
c_2 = -v_1*Omega_0 + 1
```

```
c_2 = -23.2686
```

```
c_3 = v_1*c_1
```

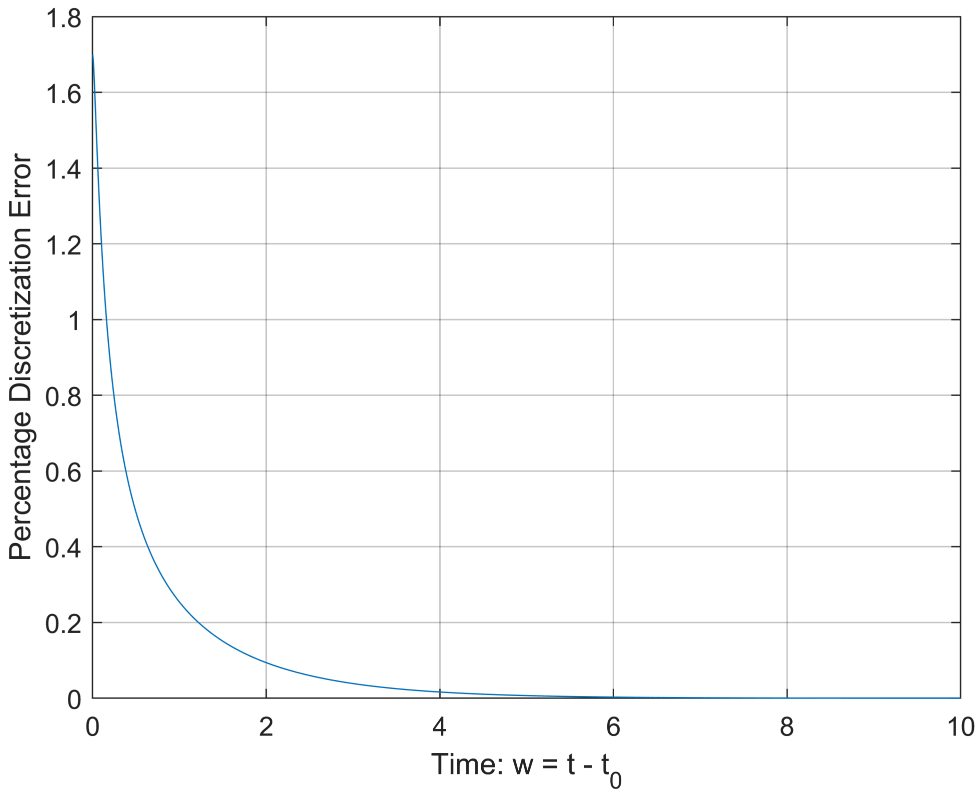
```
c_3 = -21.8664
```

```
c_4 = v_2*c_2
```

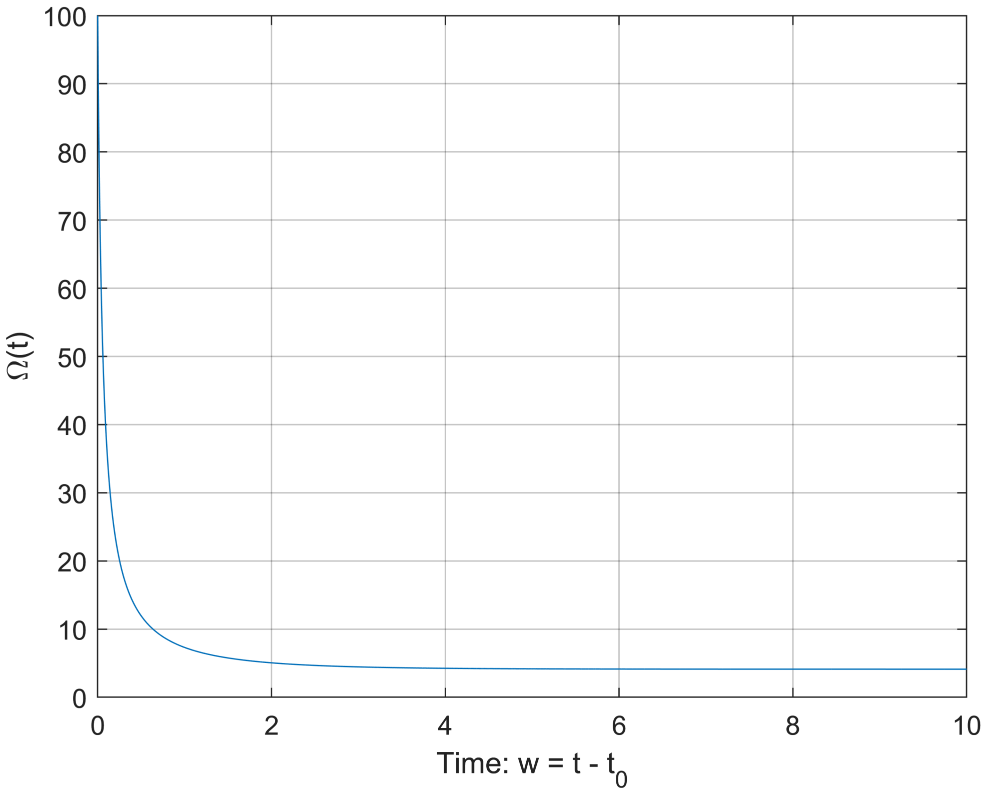
```
c_4 = 20.7327
```

```
deltaT = 0.001;
periodT = 10;
Omega = Omega_0;
JArray = [];
errorFrac = [];
OmegaArray = [];
for w = 0:deltaT:periodT % w = t-t_0
    Omega = Omega + deltaT*(2*Omega*a + q - Omega^2*g);
    OmegaAct = (c_1 + c_2*exp(-lambda*w))/(c_3 + c_4*exp(-lambda*w));
    OmegaArray = [OmegaArray, OmegaAct];
    errorFrac = [errorFrac, (OmegaAct-Omega)*100/OmegaAct];

%     % cost upto w
    J_w = (1/g)*log(c_3+c_4*exp(-lambda*w)) + (1/v_1)*w - (1/g)*log(v_2-v_1);
    JArray = [JArray, J_w];
end
figure
```

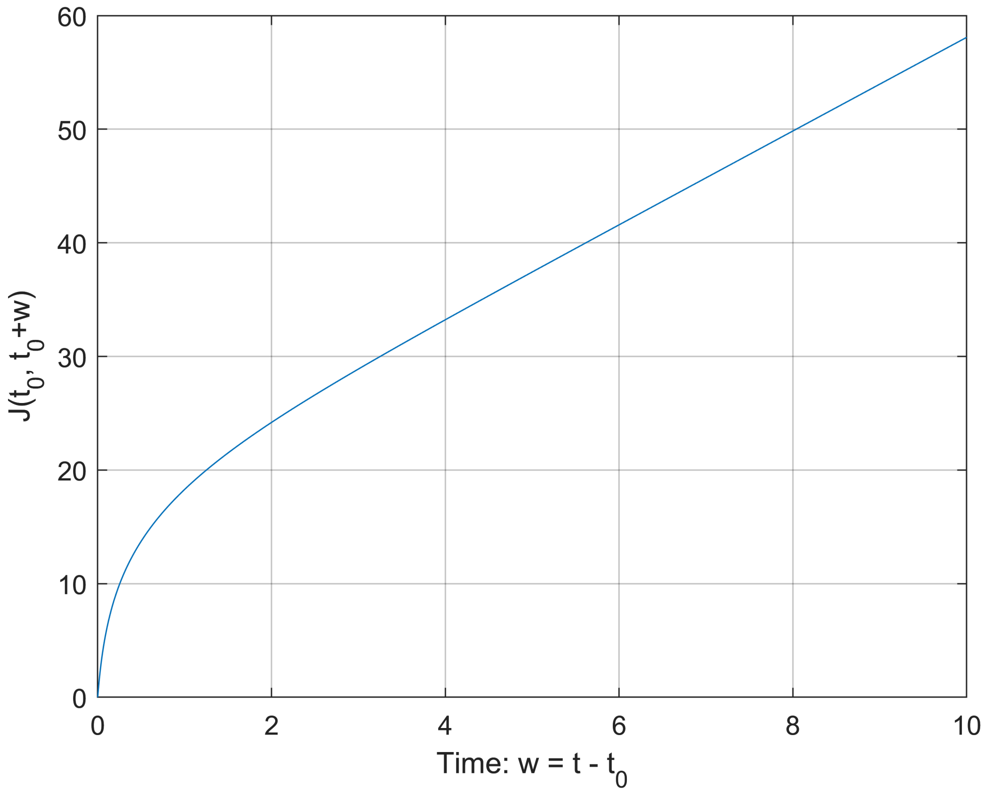
```
plot(0:deltaT:periodT,errorFrac)
ylabel('Percentage Discretization Error')
xlabel('Time: w = t - t_0')
grid on
```




```
% Percentage discretization error grows linearly but is less than 1%


figure
plot(0:deltaT:periodT,OmegaArray)
ylabel('\Omega(t)')
xlabel('Time: w = t - t_0')
grid on
```



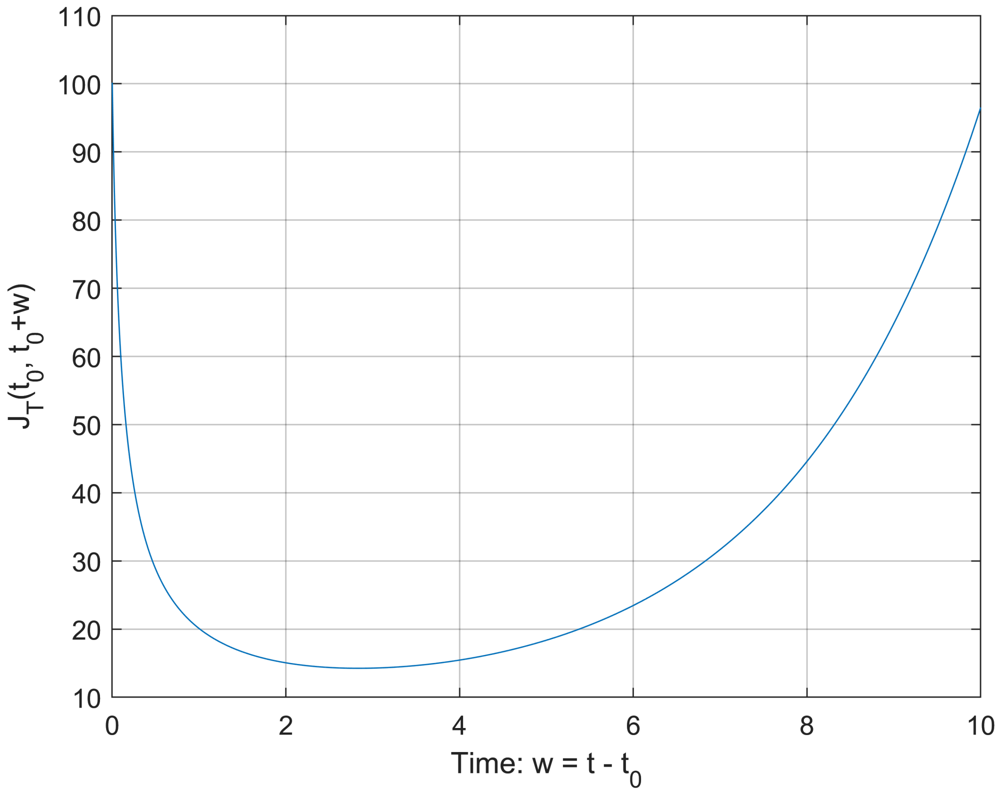

```
figure
plot(0:deltaT:periodT,JArray)
ylabel('J(t_0, t_0+w)')
xlabel('Time: w = t - t_0')
grid on
```




```
JArray2 = JArray;
```

Case 2:

```
figure
plot(0:deltaT:periodT,(JArray1+JArray2)./(0:deltaT:periodT))
% plot(0:deltaT:periodT,(JArray1+JArray2))
ylabel('J_T(t_0, t_0+w)')
xlabel('Time: w = t - t_0')
grid on
```



Next target solution: RHCP2

```
syms u positive
syms lambda_1 lambda_2 positive
% with no external neighbors
% Agent 1 finished dwelling at t = 0.539 at taregt 1
% No Solution! i=1, j=2, cost=2.6809
H = 0.2318*u + 5.6831*exp(0.5256*u + 0.19392) + 5.2993*log(0.26685*exp(-0.63111*u) + 0.93361) -
```

H =

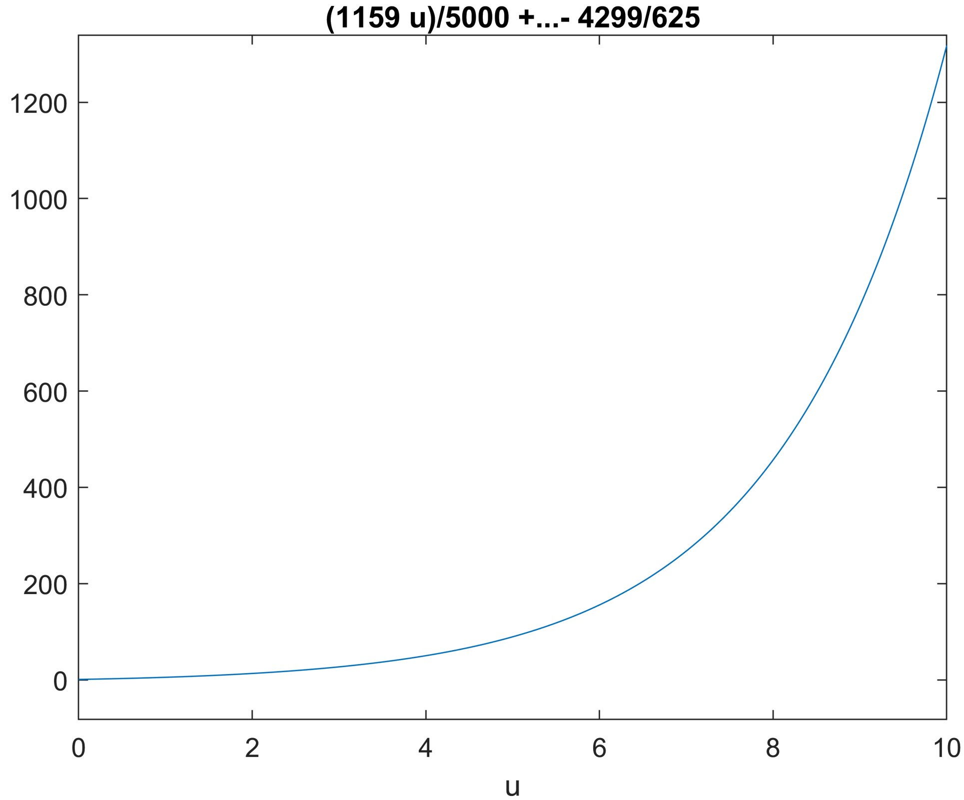
$$\frac{1159\,u}{5000} + \frac{52993\,\log\left(\frac{5337\,\mathrm{e}^{-\frac{63111\,u}{100000}}}{20000} + \frac{93361}{100000}\right)}{10000} + \frac{56831\,\mathrm{e}^{\frac{657\,u}{1250} + \frac{606}{3125}}}{10000} - \frac{4299}{625}$$

```
J = (0.2318*u + 5.6831*exp(0.5256*u + 0.19392) + 5.2993*log(0.26685*exp(-0.63111*u) + 0.93361)
```

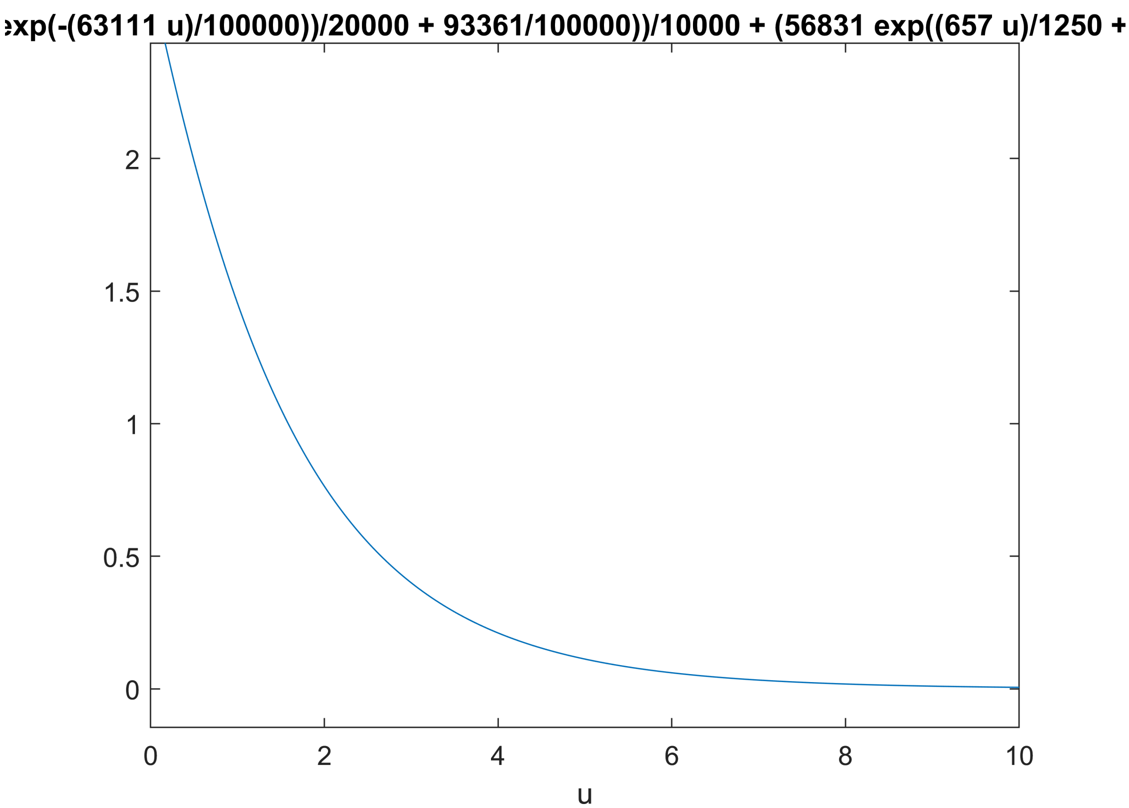
J =

$$\frac{\frac{1159\,u}{5000} + \frac{52993\,\log\left(\frac{5337\,\mathrm{e}^{-\frac{63111\,u}{100000}}}{20000} + \frac{93361}{100000}\right)}{10000} + \frac{56831\,\mathrm{e}^{\frac{657\,u}{1250} + \frac{606}{3125}}}{10000} - \frac{4299}{625}}{u + \frac{1153}{3125}}$$

```
figure
ezplot(H,[0,10])
```




```
figure
ezplot(J,[0,10])
```



```
Horizon = 10;
rho = 0.36896;
Hamil = J+lambda_1*(-u)+lambda_2*(u-(Horizon-rho));
eqn1 = diff(Hamil,u)==0
```

eqn1 =

$$\lambda_2 - \lambda_1 + \frac{\frac{37337967\,\sigma_2}{12500000} - \frac{17849282807151\,e^{-\frac{63111\,u}{100000}}}{20000000000000\,\sigma_1} + \frac{1159}{5000}}{u + \frac{1153}{3125}} - \frac{\frac{1159\,u}{5000} + \frac{52993\log(\sigma_1)}{10000} + \frac{56831\,\sigma_2}{10000} - \frac{4299}{625}}{\left(u + \frac{1153}{3125}\right)^2} =$$

where

$$\sigma_1 = \frac{5337\,e^{-\frac{63111\,u}{100000}}}{20000} + \frac{93361}{100000}$$

$$\sigma_2 = e^{\frac{657\,u}{1250} + \frac{606}{3125}}$$

```
eqn2 = lambda_1*(-u)==0
```

eqn2 = $-\lambda_1\,u = 0$

```
eqn3 = lambda_2*(u-(Horizon-rho))==0
```

eqn3 =

$$\lambda_2\left(u - \frac{30097}{3125}\right) = 0$$

```
[uSol,L1,L2] = vpasolve([eqn1,eqn2,eqn3],[u,lambda_1,lambda_2],[0,Horizon;0 inf;0 inf])
```

uSol = 5.9245203964401789124598162777042e-61

L1 = 1.1759663799123572823624386103741

L2 = −1.2008066248393619905769695979013e-72

```
% DDJ = vpa(simplify(diff(diff(J,u),u)),5)
% ezplot(DDJ,[0,10])

% Agent 2 finished dwelling at t = 3.298 at taregt 2
% Solution Found: i=2, j=1, cost=14.3485, u=0.063446
H = 397.2*exp(0.038474*u + 0.014195) - 9.5439*u + 5.7044*log(3.2513 - 2.1175*exp(-0.91909*u)) -
```

H =

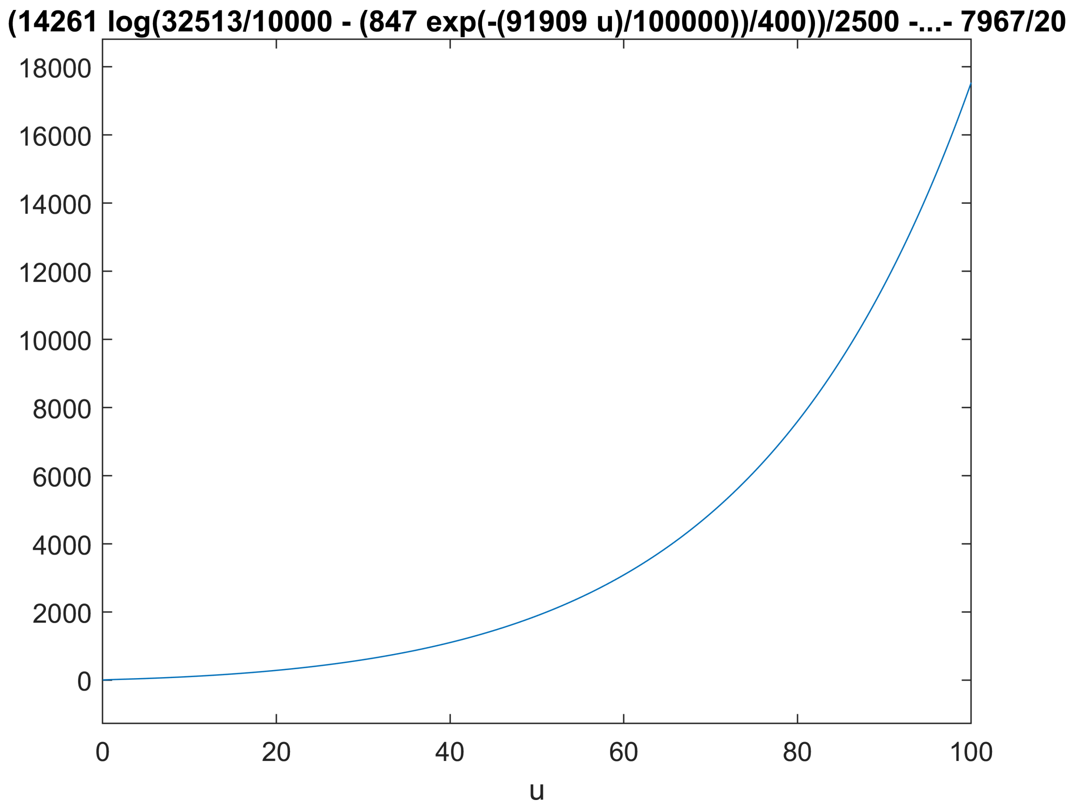
$$\frac{14261\log\left(\frac{32513}{10000} - \frac{847\,e^{-\frac{91909\,u}{100000}}}{400}\right)}{2500} - \frac{95439\,u}{10000} + \frac{1986\,e^{\frac{5544687746030479\,u}{144115188075855872} + \frac{1022857547368387}{72057594037927936}}}{5} - \frac{7967}{20}$$

```
J = -(1.0*(9.5439*u - 397.2*exp(0.038474*u + 0.014195) - 5.7044*log(3.2513 - 2.1175*exp(-0.919
```

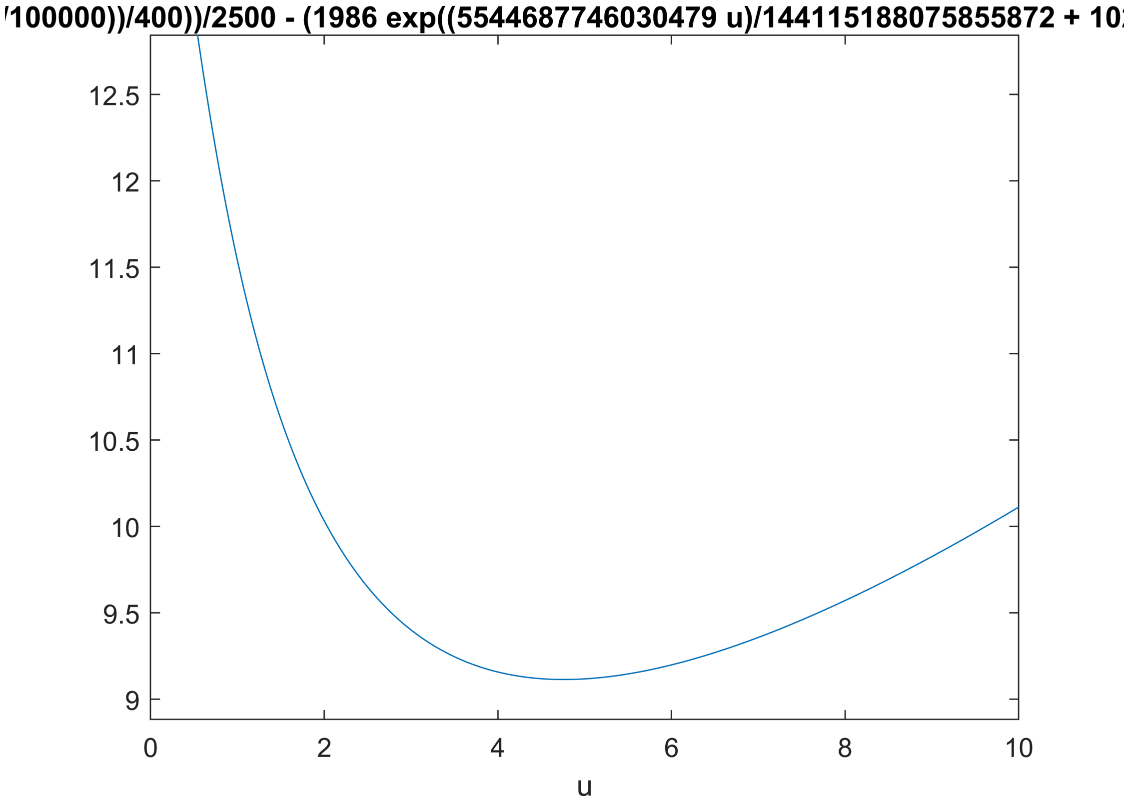
J =

$$-\frac{\frac{95439\,u}{10000}-\frac{14261\log\left(\frac{32513}{10000}-\frac{847\,e^{-\frac{91909\,u}{100000}}}{400}\right)}{2500}-\frac{1986\,e^{\frac{5544687746030479\,u}{144115188075855872}+\frac{1022857547368387}{72057594037927936}}}{5}+\frac{7967}{20}}{u+\frac{1153}{3125}}$$

```
figure
ezplot(H,[0,100])
```




```
figure
ezplot(J,[0,10])
```




```
Hamil = J+lambda_1*(-u)+lambda_2*(u-(Horizon-rho));
eqn1 = diff(Hamil,u)==0
```

eqn1 =

$$\lambda_2 - \lambda_1 + \frac{\frac{95439\,u}{10000} - \frac{14261\log\left(\frac{32513}{10000}-\sigma_1\right)}{2500} - \frac{1986\,\sigma_2}{5} + \frac{7967}{20}}{\left(u+\frac{1153}{3125}\right)^2} - \frac{\frac{1110174968903\,\mathrm{e}^{-\frac{91909\,u}{100000}}}{100000000000\,\left(\sigma_1 - \frac{32513}{10000}\right)} - \frac{550587\ldots}{36028\ldots}}{u+\frac{1153}{3125}}$$

where

$$\sigma_1 = \frac{847\,\mathrm{e}^{-\frac{91909\,u}{100000}}}{400}$$

$$\sigma_2 = \mathrm{e}^{\frac{5544687746030479\,u}{144115188075855872} + \frac{1022857547368387}{72057594037927936}}$$

```
eqn2 = lambda_1*(-u)==0
```

eqn2 = $-\lambda_1\,u = 0$

```
eqn3 = lambda_2*(u-(Horizon-rho))==0
```

eqn3 =

$$\lambda_2 \left(u - \frac{30097}{3125}\right) = 0$$

```
[uSol,L1,L2] = vpasolve([eqn1,eqn2,eqn3],[u,lambda_1,lambda_2],[0,Horizon;0 inf;0 inf])
```

uSol = $4.7592734325196821176210956401858$

L1 = $1.0277349931374323230374538424263\text{e-}52$

L2 = $5.6211940652754005355400679458419\text{e-}52$

```
% with all the neighbors
% Agent 2 finished dwelling at t = 3.378 at taregt 3
%
H = 33.301*exp(0.57729*u + 0.19643) - 35.155*u + 420.02*exp(0.038474*u + 0.013091) + 44.83*exp(
```

H =

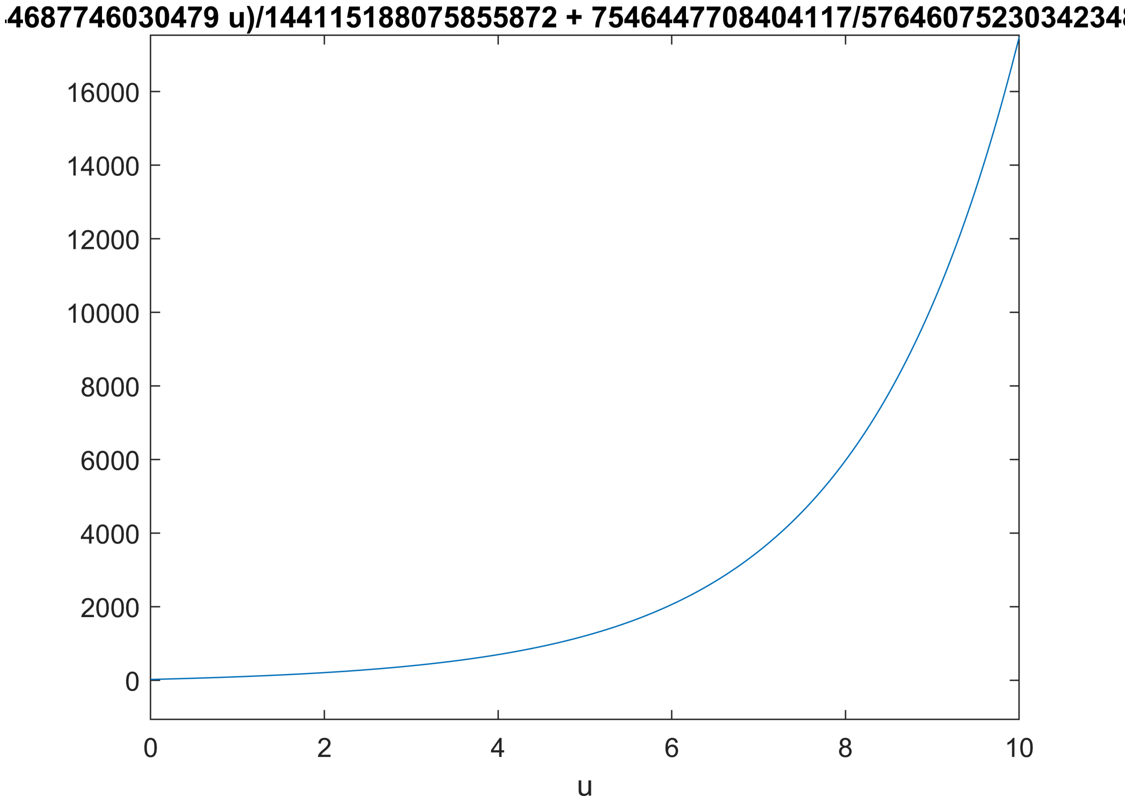
$$\frac{21001\, e^{\frac{5544687746030479\, u}{144115188075855872}+\frac{7546447708404117}{576460752303423488}}}{50} - \frac{7031\, u}{200} + \frac{18243\, e^{\frac{8104173486257679\, u}{288230376151711744}+\frac{2757471185605811}{288230376151711744}}}{50} + \frac{14261 \log\left(\frac{1}{\cdot}\right.}{}$$

```
J = (33.301*exp(0.57729*u + 0.19643) - 35.155*u + 420.02*exp(0.038474*u + 0.013091) + 44.83*exp
```

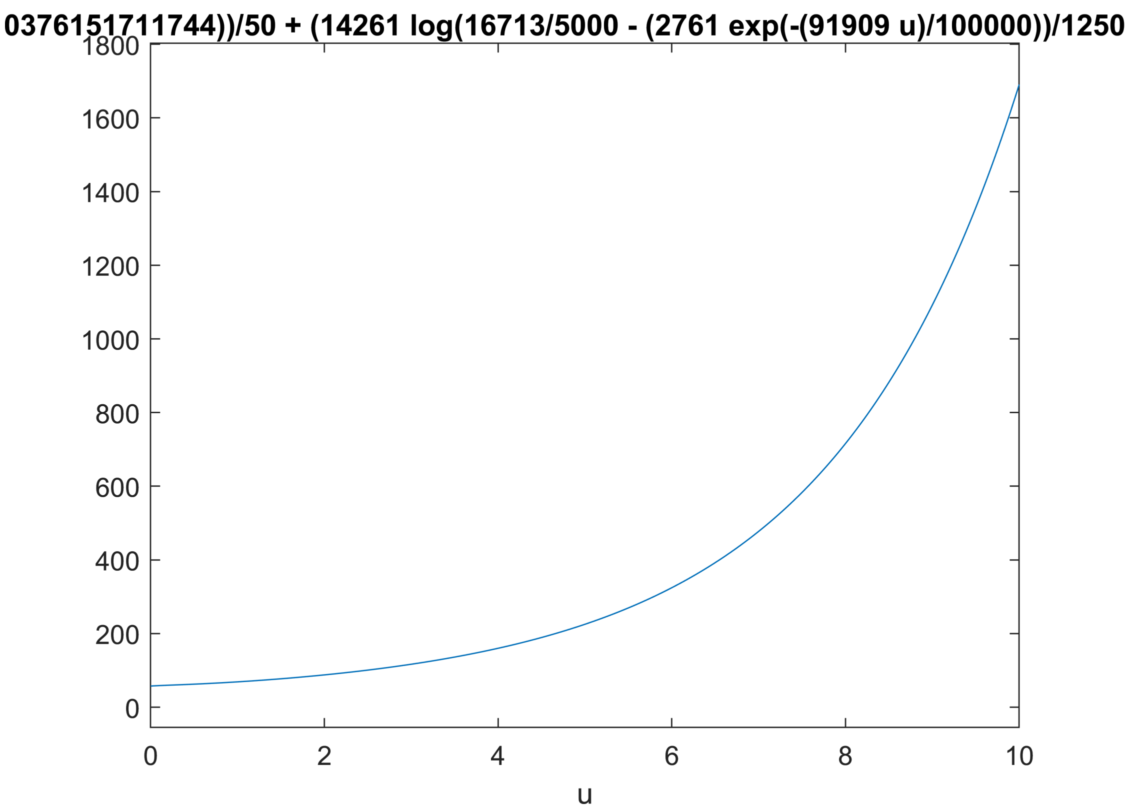
J =

$$\frac{\frac{21001\, e^{\frac{5544687746030479\, u}{144115188075855872}+\frac{7546447708404117}{576460752303423488}}}{50} - \frac{7031\, u}{200} + \frac{18243\, e^{\frac{8104173486257679\, u}{288230376151711744}+\frac{2757471185605811}{288230376151711744}}}{50} + \frac{14261 \log\left(\frac{1}{\cdot}\right.}{}}{u + \frac{17\cdot}{50\cdot}}$$

```
figure
ezplot(H,[0,10])
```




```
figure
ezplot(J,[0,10])
```



```
syms u positive
%  Agent 1 finished dwelling at t = 0.001 at taregt 1
J = (4.41361e+32*exp(0.5256*u) - 2.29096e+33*u + 9.66704e+32*exp(0.440604*u) + 2.72006e+34*exp(
```

J =

$$\frac{441360999999999967720076325421056\, e^{\frac{657\,u}{1250}} - 229096000000000001094471698402508\,8\, u + 89144099}{1439389999}$$

```
figure
ezplot(J,[0,10])
```