Q1

a) $\mathbb{E}[\mathcal{L}(y = \text{Keep}, t)] = 0.1 \times 1 = 0.1$

$\mathbb{E}[\mathcal{L}(y = \text{Remove}, t)] = 0.9 \times 100 = 90$

b) We want to minimize expected loss given conditional probability, let $p = Pr(t = \text{Spam}|x)$, so compute $\min \mathbb{E}[\mathcal{L}(y = \text{Keep}, t)] \cdot Pr(t = \text{Spam}|x) = p$ and $\min \mathbb{E}[\mathcal{L}(y = \text{Remove}, t)] \cdot Pr(t = \text{Not Spam}|x) = 100(1-p)$.

Set $p = 100(1-p)$, calculate to get $p = \frac{100}{101}$, so if $Pr(t = \text{Spam}|x) \leq \frac{100}{101}$, keep the email, if $Pr(t = \text{Spam}|x) > \frac{100}{101}$, remove the email.

c) We can use conditional probability formula to compute $Pr(Spam|x)$, i.e. $Pr(Spam|x) = \frac{Pr(x|Spam)Pr(Spam)}{Pr(x)}$.

$Pr(Spam|x = (0,0)) = \frac{0.4 \times 0.1}{0.4 \times 0.1 + 0.998 \times 0.9} = 0.043$

$Pr(Spam|x = (0,1)) = \frac{0.3 \times 0.1}{0.3 \times 0.1 + 0.001 \times 0.9} = 0.971$

$Pr(Spam|x = (1,0)) = \frac{0.1 \times 0.1}{0.1 \times 0.1 + 0 \times 0.9} = 0.957$

$Pr(Spam|x = (1,1)) = \frac{0.1 \times 0.1}{0.1 \times 0.1 + 0 \times 0.9} = 1$

From what we have in b), if $x = (1,1)$, remove the email, otherwise if $x = (0,0), (0,1)$ *or* $(1,0)$, keep the email

Q2

a) The three points are on the same line, so they can't be in two different half-space at the same time, so this dataset is not linearly separable.

b)

$\psi_1(x)$	$\psi_2(x)$	t
-1	1	1
1	1	0
3	9	1

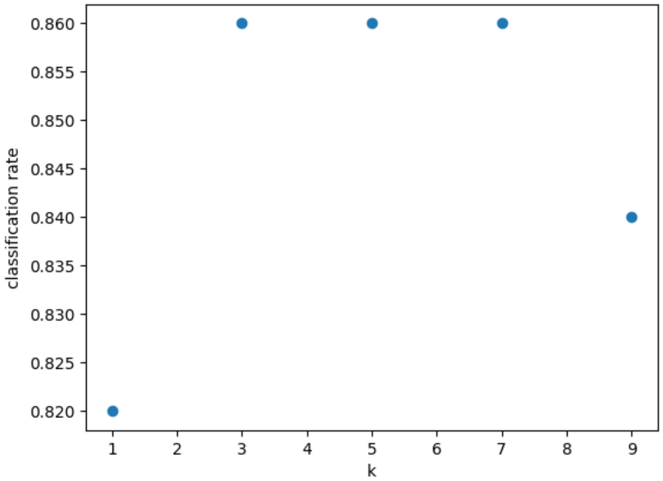
We want $w^\top x \geq 0$ when $t = 1$, $w^\top x < 0$ when $t = 0$, so

$$-w_1 + w_2 \geq 0 \quad w_1 + w_2 < 0 \quad 3w_1 + 9w_2 \geq 0$$

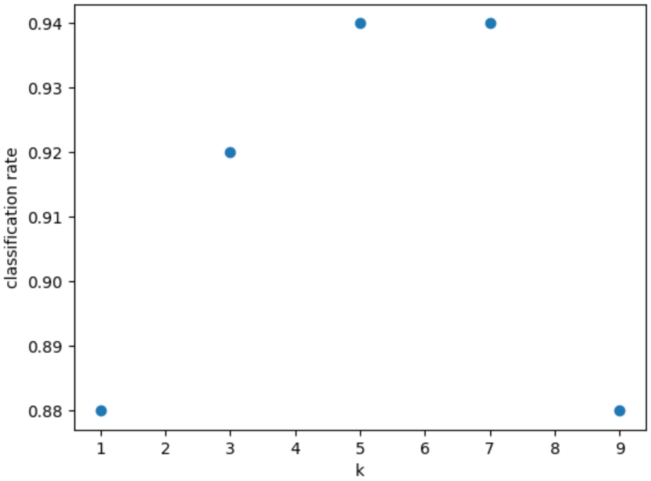
From the inequalities above, $(-2, 1)$ correctly classify all the examples

Q3

3.1

a) 

b) Choose $k = 5$ because when $k = 5$ the highest accuracy of 86% is achieved. The accuracy also have a trend to first increase then decrease, so choose 5 instead of 3 and 7 because 5 is in the middle, and more likely to have high accuracy with different dataset.



Classification rate for $k^* = 5$ is 94%, $k^* - 2 = 3$ have accuracy 92%, $k^* + 2 = 7$ have accuracy 94%, the accuracy of test data is higher than that of validation data for all values of k.

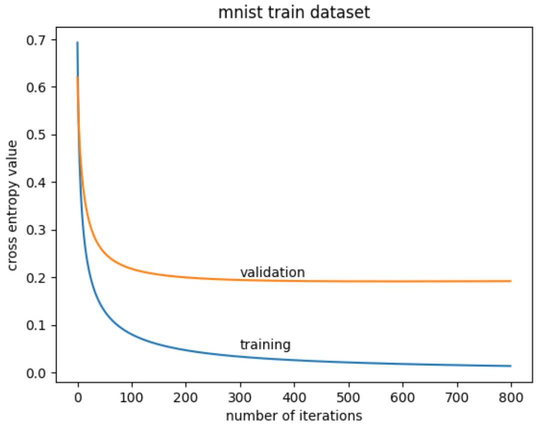
3.2

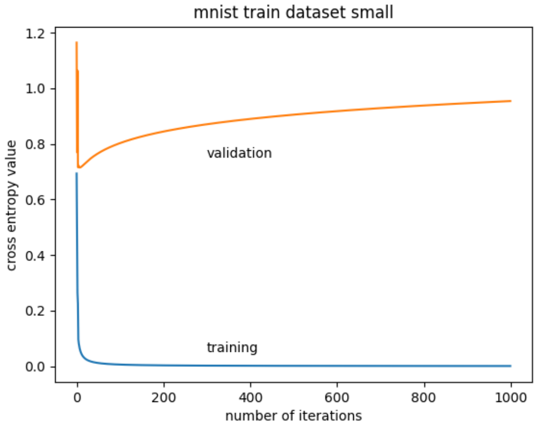
a) see python file

b) For mnist_train, use learning rate 0.2 and iterations 800 as hyperparameters, these are the parameters that yield a small entropy and high classification; for training data, final cross entropy is 0.014 and classification rate is 100%, for validation data, final cross entropy is 0.192, the classification is 88%, for test data, final cross entropy is 0.227, the classification is 92%.

For mnist_train_small, use learning rate 0.4 and iterations 1000 as hyperparameters, they generate relatively low entropy and high classification rate; for training data, final cross entropy is 0.0006 and classification rate is 100%, for validation data, final cross entropy is 0.954, the classification is 74%, for test data, final cross entropy is 0.966, the classification is 78%.

c) The plot are as followed. The results change due to change in initial weights and bias when using random weights. In this case, try several times and take an average over a set of entropy values of classification rates, then decide the best hyperparameters choice.





Q4

a) Note the function is sum of squares, so it's sum of convex functions, so it's convex and the local minimum (obtained when gradient to 0) is the global minimum. The design matrix X is the matrix with $x(i)$ on row i, and let y be the vector with $y(i)$ on row i, so this problem can be re-write as:

$$\begin{aligned} f(w) &= \frac{1}{2}(y - Xw)^T A(y - Xw) + \frac{\lambda}{2}||w||^2 \\ &= \frac{1}{2}(y^T - w^T X^T)A(y - Xw) + \frac{\lambda}{2}||w||^2 \\ &= \frac{1}{2}(y^T Ay - y^T AXw - w^T X^T Ay + w^T X^T AXw) + \frac{\lambda}{2}||w||^2 I \end{aligned}$$

Differentiate with respect to w:

$$\nabla f = \frac{1}{2}(-y^T AX - y^T AX + 2X^T AXw) + \lambda wI$$

Set to 0 to find minimum point:

$$\begin{aligned} &\frac{1}{2}(-y^T AX - y^T AX + 2X^T AXw) + \lambda wI = 0 \\ &X^T AXw + \lambda wI = y^T AX \\ &w = (X^T AX + \lambda I)^{-1} X^T Ay \end{aligned}$$

as desired.

b) Sorry I can't figure out how to use the l2 function