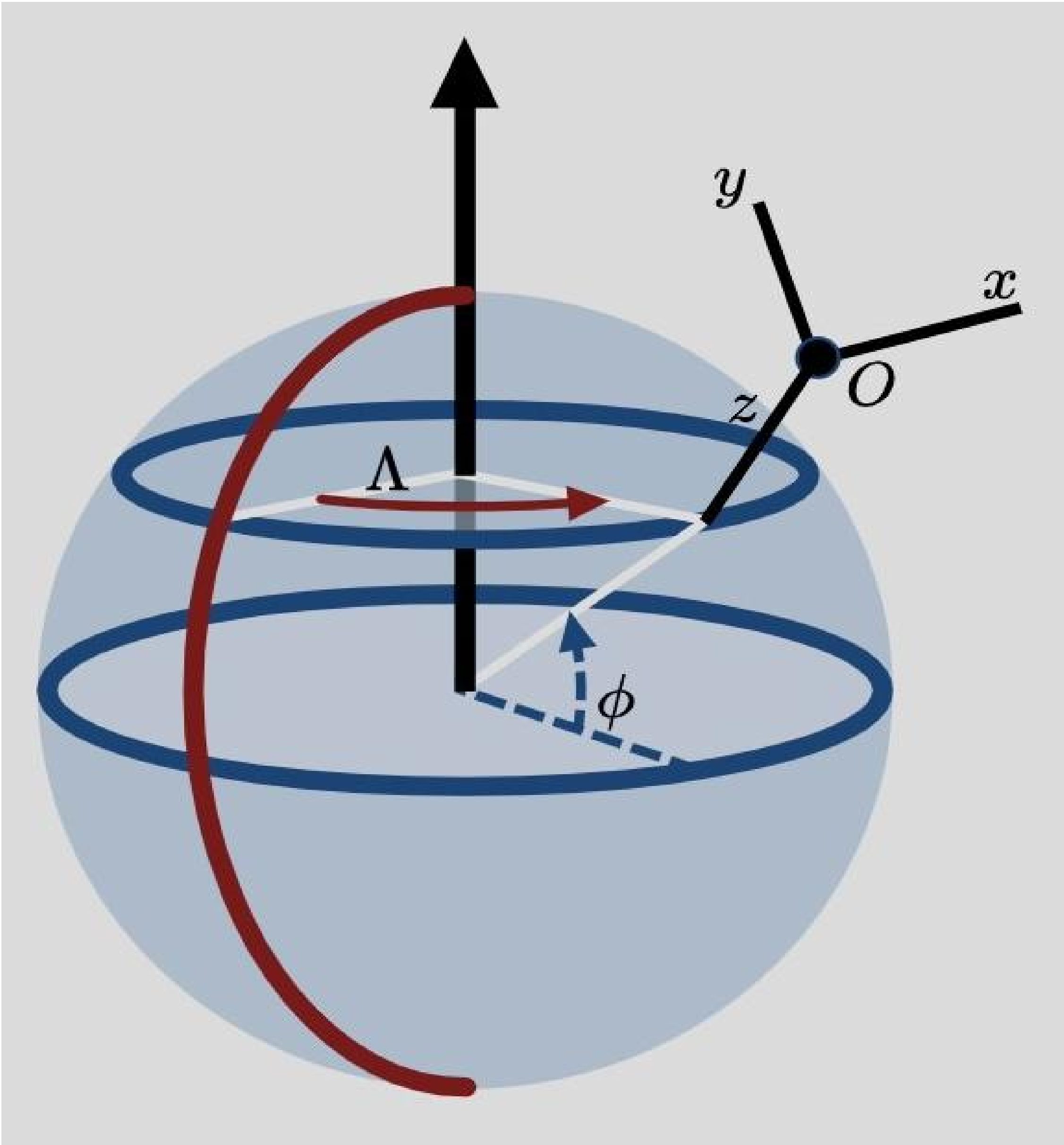
TOPOCENTRIC COORDINET SYSTEM



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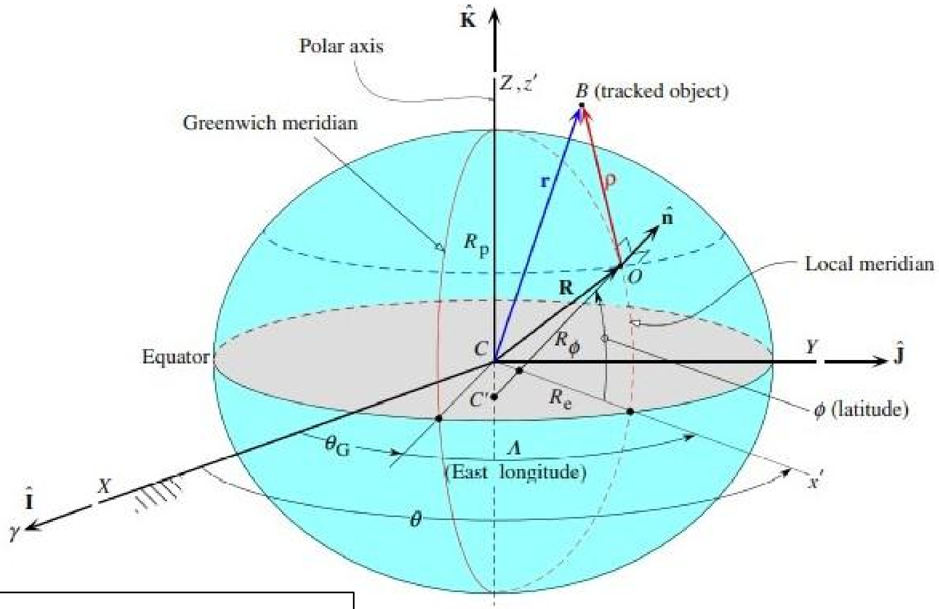
A topocentric coordinat system is one that is centered at the observers location on the surface of the earth.



r=R+ρ (2.1)

Where :

- *r* : the position of the body relative to the center of attraction 'c'
- *R* : the position vector of the observer relative to the center of attraction 'c'
- ρ : the position vector of the body relative to the observer.

The location of the observer site (o) is determined by specifying

1. It's east longitude(**Λ)** : measured positive eastward from the Greanwich meridian through o.
2. It's latitude (ϕ) : the angle between the equator and the normal to the earth surface at o.

Since the earth is not a perfect sphere ,then the oblateness or flattening f is defined as,

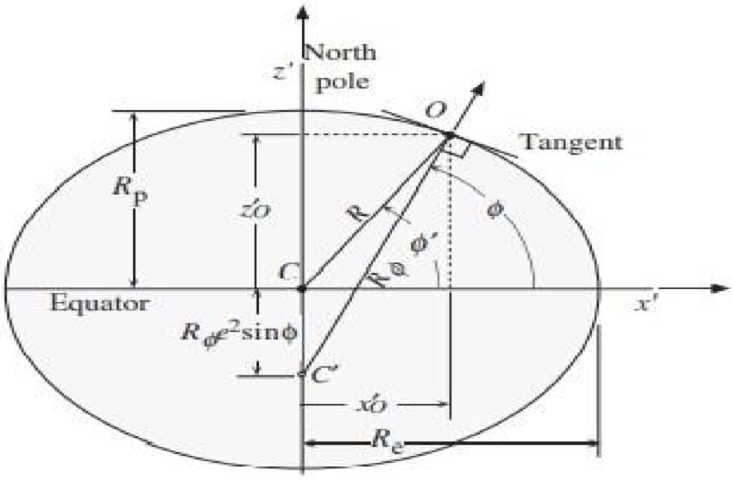
$$f= \frac{Re - Rp}{Re} \qquad (2.2)$$

Where:

- *Re* : the equatorial radius
- *Rp* : the polar radius

Note: (For the earth f=0.000335)

The relationship between geocentric latitude Ǿ and geodetic latitude ϕ.



$$Rp = Re\sqrt{1-e^2} = Re(1-f) \qquad (2.3)$$

Where :

- *Re* : semi major axis
- *Rp* : semi minor axis

$$e = \sqrt{2f - f^2} \qquad (2.4)$$

$$F = 1 - \sqrt{1-e^2} \qquad (2.5)$$

$$R = \left[\frac{Re}{\sqrt{1-(2f-f^2)\sin^2\phi}} + H\right]\cos Ø\,(\cos\theta\ I + \sin\theta\ J) + \left[\frac{Re(1-f)^2}{\sqrt{1-(2f-f^2)\sin^2\phi}} + H\right]\sin\phi\ K \qquad (2.6)$$

Then the flattening and eccentricit are related to each other.
Since,

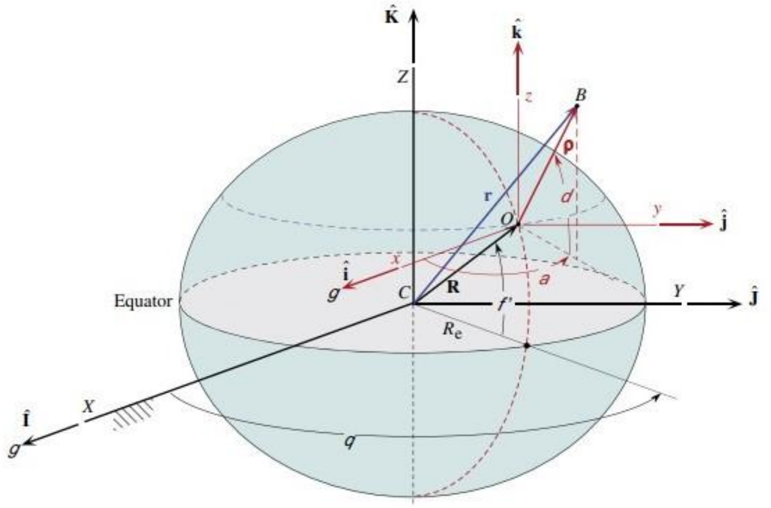
- geodetic latitude ϕ : the angle between the normal and the equator.
- Egocentric latitude Ǿ: the angle between the equator plane and line joining o to the center of the earth.
- XO and ZO : are the meridian coordinate of O.

Then:

$$\tan Ǿ = (1-f)^2 \tan\phi \qquad (2.7)$$

Thus at sea level , the geodetic latitude is related to geocentric latitude.

Topocentric equatorial coordinate system



$\hat{\boldsymbol{\rho}} = \cos\delta\cos\alpha\ \mathbf{I} + \cos\delta\sin\alpha\ \mathbf{Y} + \sin\delta\ \mathbf{K}$ (2.8)

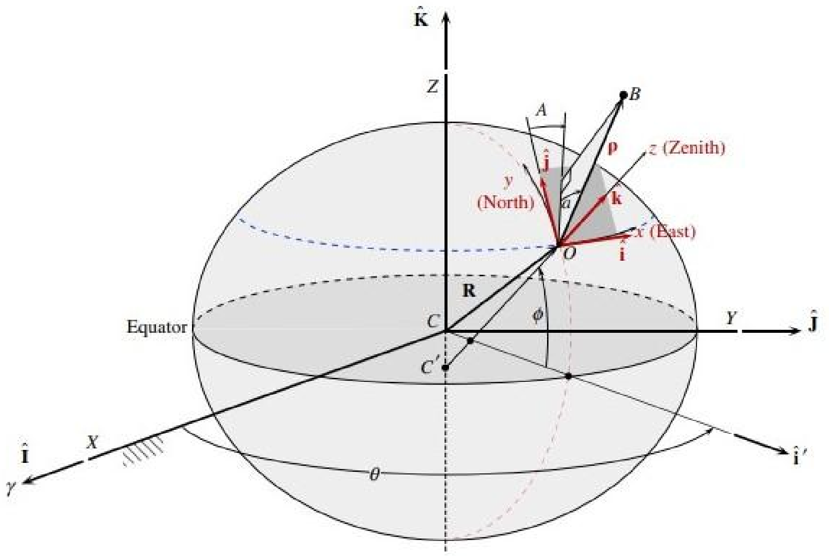
Where:

- α: right asccension.
- δ :declination.
- ρ :slant range.
-

XYZ axes through O coincides with XYZ axes of the egocentric equatorial frame.

*Note: (In topocentric coordinat system **i=I , y= Y , k=K**)*

Topocentric horizon coordinate system



It is centered at the observation point o whose position vector is **R.**

- *Xy plane* **:** local horizon, which is the plane tangent to the ellipsaid at point o.
- *X axis* : is determined eastward.
- *Y axis*: is determined to the north.
- *Z axis*: normal to xy plane directed outward the zenith.

The position vector (ρ) of a body relative to the topocentric horizon is ,

$$\boldsymbol{\rho} = \rho \cos a \sin A\ \mathbf{I} + \rho \cos a \cos A\ \mathbf{Y} + \rho \sin a\ \mathbf{K} \qquad (2.9)$$

Where:

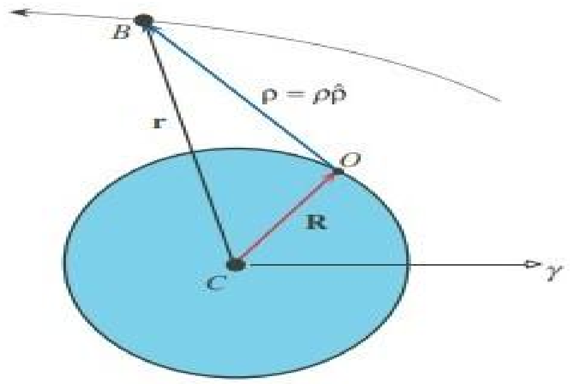
- *A*: the azimuth measured positive clockwise from due North (0 $\leq$ A $\leq$ 360°)
- *a*: the elevation or attitude measured from the horizon to the line of sight of the body B (0 $\leq$ a $\leq$ 90°).

The transformation between the geocentric equatorial and topocentric horizon systems.

$$\mathbf{I} = -\sin\theta\ \mathbf{I} + \cos\theta\ \mathbf{J}$$
$$\mathbf{J} = -\sin\theta\cos\theta\ \mathbf{I} - \sin\varphi\sin\theta\ \mathbf{J} + \cos\varphi\ \mathbf{K} \qquad (2.10)$$
$$\mathbf{K} = \cos\varphi\cos\theta\ \mathbf{I} + \cos\varphi\sin\theta\ \mathbf{Y} + \sin\varphi\ \mathbf{K}$$

Orbit determination from angle and range measurements.

The Orbit around the earth is determined once the state vectors r and v in the inertial geocentric frame.



$$\mathbf{r} = \mathbf{R} + \rho\hat{\boldsymbol{\rho}} \quad (2.11)$$

By differentiating this equation with respect to time,

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{\mathbf{R}} + \rho\dot{\boldsymbol{\rho}} + \dot{\rho}\boldsymbol{\rho} \quad (2.12)$$

$$\mathbf{a} = \ddot{\mathbf{r}} = \ddot{\mathbf{R}} + \ddot{\rho}\hat{\boldsymbol{\rho}} + 2\dot{\rho}\dot{\hat{\boldsymbol{\rho}}} + \rho\ddot{\hat{\boldsymbol{\rho}}} \quad (2.13)$$

Note: (the vector in these equations must all be expressed in the common basis IJK of the intertial (non rotating) geocentric equatorial frame)

To summarize, given the topocentric azimuth A and altitude a of the target together with sidereal time θ and latitude φ of the tracking station,we can compute the topocentric declination δ and the right ascension α as follows,

1.

$$\delta = \sin^{-1}(\cos\phi \cos A \cos a + \sin\phi \sin a) \qquad (2.14)$$

2.

$$h = \begin{cases} 360^\circ - \cos^{-1}\left(\dfrac{\cos\phi \sin a - \sin\phi \cos A \cos a}{\cos\delta}\right) & 0^\circ < A < 180^\circ \\ \cos^{-1}\left(\dfrac{\cos\phi \sin a - \sin\phi \cos A \cos a}{\cos\delta}\right) & 180^\circ \le A \le 360^\circ \end{cases} \qquad (2.15)$$

3.

$$\alpha = \theta - h \qquad (2.16)$$

Where :

- h : the angular distance between the orbit and local meridian.
  - If h is positive : the object is west of the meridian.
  - If h is negative :the object is east of the meridian.

4.

$$\dot{\boldsymbol{\rho}} = \cos\delta \cos\alpha \ \mathbf{I} + \cos\delta \sin\alpha \ \mathbf{Y} + \sin\delta \ \mathbf{K} \qquad (2.17)$$

5.

$$\mathbf{R} = R_e \cos\phi' \cos\theta \hat{\mathbf{I}} + R_e \cos\phi' \sin\theta \hat{\mathbf{J}} + R_e \sin\phi' \hat{\mathbf{K}} \qquad (2.18)$$

$$\tan \acute{Ø} = (1 - f)^2 \tan\phi \qquad (2.19)$$

6.

$$\mathbf{r} = \mathbf{R} + \boldsymbol{\rho} \qquad (2.1)$$

7.

$$\dot{h} = -\frac{\dot{A}\cos A \cos a - \dot{a} \sin A \sin a + \dot{\delta} \sin A \cos a \tan\delta}{\cos\phi \sin a - \sin\phi \cos A \cos a} \qquad (2.20)$$

8.

$$\dot{\delta} = \frac{1}{\cos\delta}\left[-\dot{A}\cos\phi\sin A\cos a + \dot{a}(\sin\phi\cos a - \cos\phi\cos A\sin a)\right] \quad (2.21)$$

9.

$$\dot{h} = \dot{\theta} - \dot{\alpha} = \omega_E - \dot{\alpha} \quad (2.22)$$

10.

$$\dot{\alpha} = \omega_E + \frac{\dot{A}\cos A\cos a - \dot{a}\sin A\sin a + \dot{\delta}\sin A\cos a\tan\delta}{\cos\phi\sin a - \sin\phi\cos A\cos a} \quad (2.23)$$

11.

$$\dot{\mathbf{R}} = \mathbf{\Omega} \times \mathbf{R} \quad (2.24)$$

Where:

- *Ω = WE **K** (angular velocity)*

12.

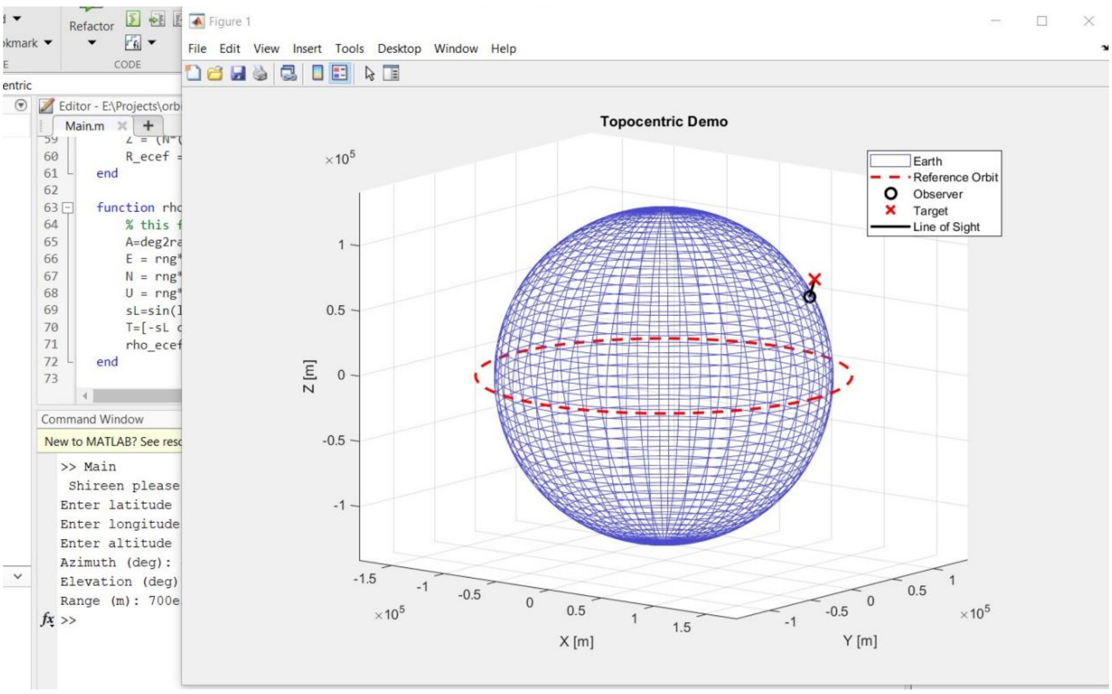
$$\mathbf{v} = \dot{\mathbf{r}} = \dot{\mathbf{R}} + \rho\dot{\boldsymbol{\rho}} + \dot{\rho}\boldsymbol{\rho} \quad (2.25)$$

Matlab script

```
% % % % By Shireen Fathy
% % % % This shows how to see a target from a point on Earth

function simple_topocentric_demo()
% % % User Inputs
fprintf(" Shireen please enter the data \n")
lat = input('Enter latitude (deg): '); %latitude on Earth
lon = input('Enter longitude (deg): '); % longitude on Earth
h = input('Enter altitude (m): '); % height from sea level
A = input('Azimuth (deg): '); % which direction is the target (0=N,90=E)
el = input('Elevation (deg): '); % how high is the target in the sky
rng = input('Range (m): '); % distance to the target
% % % Earth Parameters
Re = 6378137; % radius of Earth in meters
f = 1/298.257; % how squished is the Earth (flattening)
% % % Compute Positions
R_ecef = geodetic2ecef(lat, lon, h, Re, f); % my position in 3D space
rho_ecef = azelrange2ecef(A, el, rng, lat, lon); % target vector from me
r_ecef = R_ecef + rho_ecef; % target absolute position
% % % % Plotting
figure; hold on; grid on; axis equal;
scale = 1/50;
Re_vis = Re*scale; % shrink everything to see it better

% --- Plot Earth as a sphere ---
[Xe,Ye,Ze] = sphere(50);
mesh(Re_vis*Xe, Re_vis*Ye, Re_vis*Ze, ...
'EdgeColor',[0.3 0.3 0.8], 'FaceAlpha',0); % just the outline, looks like a
globe
% --- Plot a simple circular orbit for fun ---
theta = linspace(0,2*pi,200);
r_orbit = (Re+700e3)*scale; % 700 km above Earth
plot3(r_orbit*cos(theta), r_orbit*sin(theta), zeros(size(theta)),'r--
','LineWidth',2);

% --- Plot me and the target ---
plot3(R_ecef(1)*scale,R_ecef(2)*scale,R_ecef(3)*scale,'ko','MarkerSize',8,'
LineWidth',2); % me = black dot
plot3(r_ecef(1)*scale,r_ecef(2)*scale,r_ecef(3)*scale,'rx','MarkerSize',12,
'LineWidth',2); % target = red X

% --- Draw line from me to the target ---
plot3([R_ecef(1) r_ecef(1)]*scale, ...
[R_ecef(2) r_ecef(2)]*scale, ...
[R_ecef(3) r_ecef(3)]*scale,'k-','LineWidth',2); % my line of sight

% Labels and Title
xlabel('X [m]'); ylabel('Y [m]'); zlabel('Z [m]'); % axes labels
title(' Topocentric Demo '); % fun title
legend('Earth','Reference Orbit','Observer','Target','Line of
Sight','Location','best');

view(40,25); % nice angle to see everything
End
```

```
% % % % % % % Helper Functions
function R_ecef = geodetic2ecef(phi, lambda, H, Re, f)
% this function convert lat/lon/alt to X,Y,Z (ECEF)
phi = deg2rad(phi); lambda = deg2rad(lambda);
e2 = f*(2-f);
N = Re ./ sqrt(1 - e2*sin(phi).^2);
X = (N+H).*cos(phi).*cos(lambda);
Y = (N+H).*cos(phi).*sin(lambda);
Z = (N*(1-e2)+H).*sin(phi);
R_ecef = [X;Y;Z]; % return as a column vector
end

function rho_ecef = azelrange2ecef(A, el, rng, phi, lambda)
% this function convert azimuth/elevation/range to a 3D vector in ECEF
A=deg2rad(A); el=deg2rad(el); phi=deg2rad(phi); lambda=deg2rad(lambda);
E = rng*cos(el).*sin(A);
N = rng*cos(el).*cos(A);
U = rng*sin(el);
sL=sin(lambda); cL=cos(lambda); sP=sin(phi); cP=cos(phi);
T=[-sL cL 0; -sP*cL -sP*sL cP; cP*cL cP*sL sP];
rho_ecef=T*[E;N;U]; % final vector from observer to target
end
```

Visualization



Reference:

[1] Curtis, Howard D., "Orbital Mechanics for Engineering Students", 4th Edition, Elsevier, 2020.