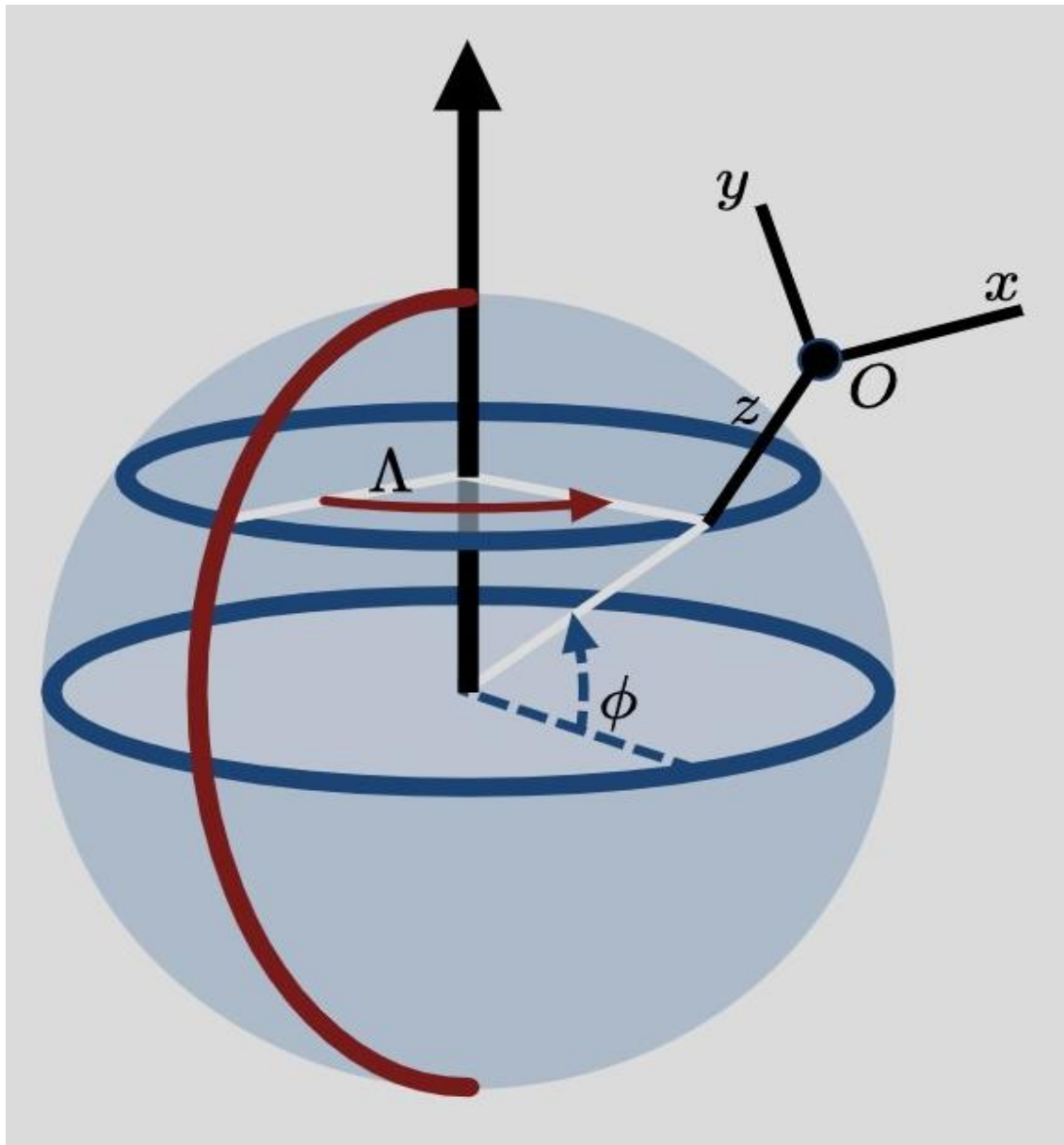


TOPOCENTRIC COORDINATE SYSTEM

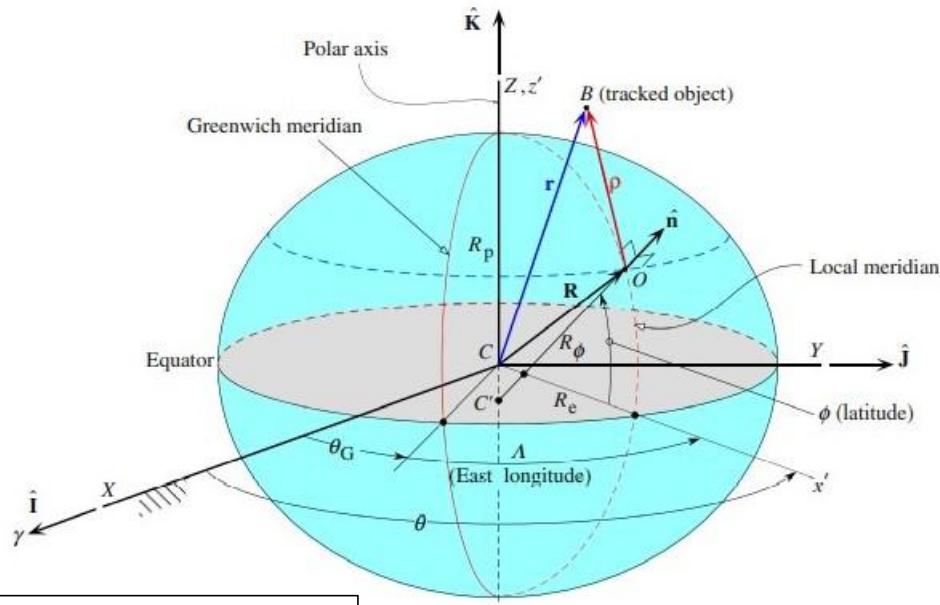


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A topocentric coordinat system is one that is centered at the observers location on the surface of the earth.



$$r=R+\rho \quad (2.1)$$

Where :

- r : the position of the body relative to the center of attraction 'c'
- R : the position vector of the observer relative to the center of attraction 'c'
- ρ : the position vector of the body relative to the observer.

The location of the observer site (o) is determined by specifying

1. It's east longitude(Λ) : measured positive eastward from the Greenwih meridian through o.
2. It's latitude (ϕ) : the angle between the equator and the normal to the earth surface at o.

Since the earth is not a perfect sphere ,then the oblateness or flattening f is defined as,

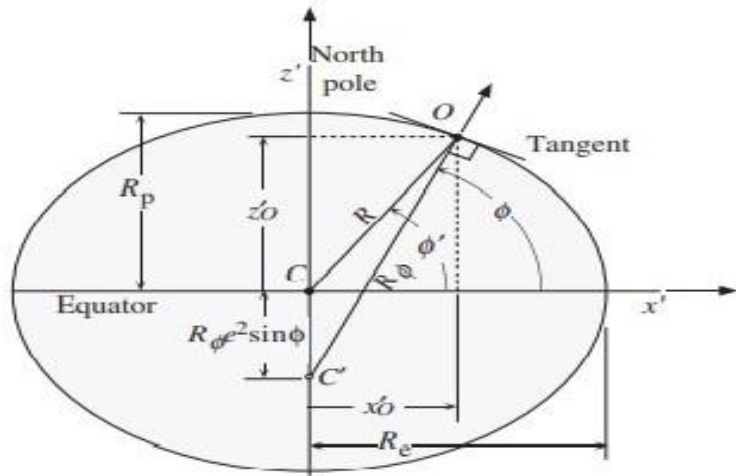
$$f = \frac{R_e - R_p}{R_e} \quad (2.2)$$

Where:

- R_e : the equatorial radius
- R_p : the polar radius

Note: (For the earth $f=0.000335$)

The relationship between geocentric latitude θ and geodetic latitude ϕ .



$$R_p = R_e \sqrt{1 - e^2} = R_e(1-f) \quad (2.3)$$

Where :

- R_e : semi major axis
- R_p : semi minor axis

$$e = \sqrt{2f - f^2} \quad (2.4)$$

$$F = 1 - \sqrt{1 - e^2} \quad (2.5)$$

$$R = \left[\frac{R_e}{\sqrt{1 - (2f - f^2) \sin^2 \phi}} + H \right] \cos \phi' (\cos \theta \mathbf{i} + \sin \theta \mathbf{j}) + \left[\frac{R_e(1-f)^2}{\sqrt{1 - (2f - f^2) \sin^2 \phi}} + H \right] \sin \phi \mathbf{k} \quad (2.6)$$

Then the flattening and eccentricity are related to each other.

Since,

- geodetic latitude ϕ : the angle between the normal and the equator.
- Geocentric latitude ϕ' : the angle between the equator plane and line joining O to the center of the earth.
- XO and ZO : are the meridian coordinate of O.

Then:

$$\tan \phi' = (1 - f)^2 \tan \phi \quad (2.7)$$

Thus at sea level , the geodetic latitude is related to geocentric latitude.

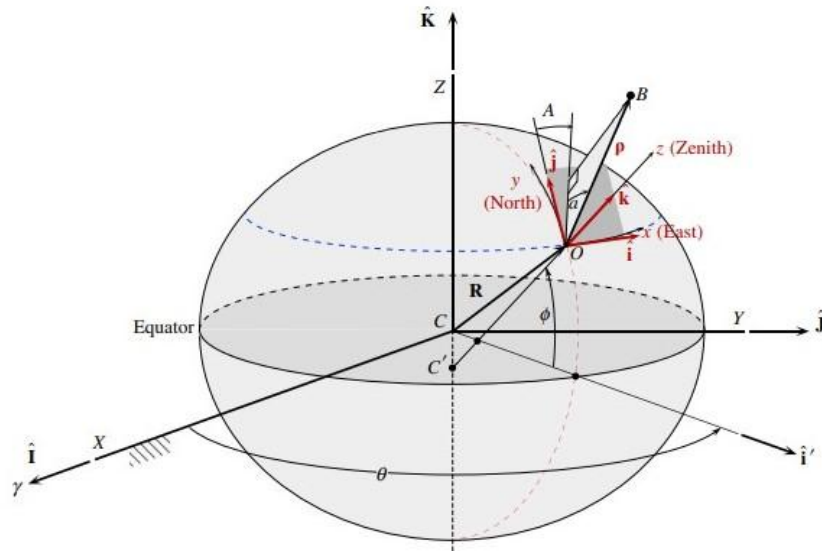
[illegible]

Where:

- α : right ascension.
- δ : declination.
- ρ : slant range.
-

Note: (In topocentric coordinat system $i=I$, $y=Y$, $k=K$)

Topocentric horizon coordinate system



It is centered at the observation point O whose position vector is \mathbf{R} .

- *Xy plane* : local horizon, which is the plane tangent to the ellipsoid at point O .
- *X axis* : is determined eastward.
- *Y axis*: is determined to the north.
- *Z axis*: normal to *xy plane* directed outward the zenith.

The position vector (ρ) of a body relative to the topocentric horizon is ,

$$\rho = \rho \cos a \sin A \mathbf{I} + \rho \cos a \cos A \mathbf{Y} + \rho \sin a \mathbf{K} \quad (2.9)$$

Where:

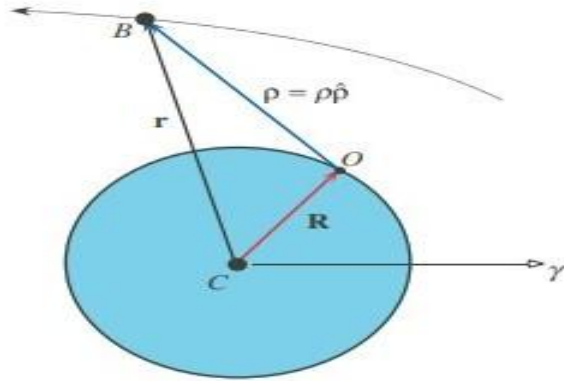
- A : the azimuth measured positive clockwise from due North ($0 \leq A \leq 360^\circ$)
- a : the elevation or attitude measured from the horizon to the line of sight of the body B ($0 \leq a \leq 90^\circ$).

The transformation between the geocentric equatorial and topocentric horizon systems.

$$\begin{aligned} \mathbf{I} &= -\sin \theta \mathbf{I} + \cos \theta \mathbf{J} \\ \mathbf{J} &= -\sin \theta \cos \theta \mathbf{I} - \sin \phi \sin \theta \mathbf{J} + \cos \phi \mathbf{K} \\ \mathbf{K} &= \cos \phi \cos \theta \mathbf{I} + \cos \phi \sin \theta \mathbf{Y} + \sin \phi \mathbf{K} \end{aligned} \quad (2.10)$$

Orbit determination from angle and range measurements.

The Orbit around the earth is determined once the state vectors \mathbf{r} and \mathbf{v} in the inertial geocentric frame.



$$\mathbf{r} = \mathbf{R} + \rho \hat{\rho} \quad (2.11)$$

By differentiating this equation with respect to time,

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{\mathbf{R}} + \rho \dot{\hat{\rho}} + \dot{\rho} \hat{\rho} \quad (2.12)$$

$$\mathbf{a} = \ddot{\mathbf{r}} = \ddot{\mathbf{R}} + \ddot{\rho} \hat{\rho} + 2\dot{\rho} \dot{\hat{\rho}} + \rho \ddot{\hat{\rho}} \quad (2.13)$$

Note: (the vector in these equations must all be expressed in the common basis IJK of the inertial (non rotating) geocentric equatorial frame)

To summarize, given the topocentric azimuth A and altitude a of the target together with sidereal time θ and latitude ϕ of the tracking station, we can compute the topocentric declination δ and the right ascension α as follows,

1.

$$\delta = \sin^{-1}(\cos \phi \cos A \cos a + \sin \phi \sin a) \quad (2.14)$$

2.

$$h = \begin{cases} 360^\circ - \cos^{-1}\left(\frac{\cos \phi \sin a - \sin \phi \cos A \cos a}{\cos \delta}\right) & 0^\circ < A < 180^\circ \\ \cos^{-1}\left(\frac{\cos \phi \sin a - \sin \phi \cos A \cos a}{\cos \delta}\right) & 180^\circ \leq A \leq 360^\circ \end{cases} \quad (2.15)$$

3.

$$\alpha = \theta - h \quad (2.16)$$

Where :

- h : the angular distance between the orbit and local meridian.
 - If h is positive : the object is west of the meridian.
 - If h is negative : the object is east of the meridian.

4.

$$\dot{\mathbf{p}} = \cos \delta \cos \alpha \mathbf{I} + \cos \delta \sin \alpha \mathbf{Y} + \sin \delta \mathbf{K} \quad (2.17)$$

5.

$$\mathbf{R} = R_e \cos \phi' \cos \theta \hat{\mathbf{I}} + R_e \cos \phi' \sin \theta \hat{\mathbf{J}} + R_e \sin \phi' \hat{\mathbf{K}} \quad (2.18)$$

$$\tan \emptyset = (1 - f)^2 \tan \phi \quad (2.19)$$

6.

$$\mathbf{r} = \mathbf{R} + \mathbf{p} \quad (2.1)$$

7.

$$\dot{h} = -\frac{\dot{A} \cos A \cos a - \dot{a} \sin A \sin a + \dot{\delta} \sin A \cos a \tan \delta}{\cos \phi \sin a - \sin \phi \cos A \cos a} \quad (2.20)$$

8.

$$\dot{\delta} = \frac{1}{\cos \delta} \left[-\dot{A} \cos \phi \sin A \cos a + \dot{a} (\sin \phi \cos a - \cos \phi \cos A \sin a) \right] \quad (2.21)$$

9.

$$\dot{h} = \dot{\theta} - \dot{\alpha} = \omega_E - \dot{\alpha} \quad (2.22)$$

10.

$$\dot{\alpha} = \omega_E + \frac{\dot{A} \cos A \cos a - \dot{a} \sin A \sin a + \dot{\delta} \sin A \cos a \tan \delta}{\cos \phi \sin a - \sin \phi \cos A \cos a} \quad (2.23)$$

11.

$$\dot{\mathbf{R}} = \boldsymbol{\Omega} \times \mathbf{R} \quad (2.24)$$

Where:

- $\boldsymbol{\Omega} = \omega_E \mathbf{K}$ (angular velocity)

12.

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{\mathbf{R}} + \rho \dot{\mathbf{p}} + \dot{\rho} \mathbf{p} \quad (2.25)$$

Matlab script

```
%% %% By Shireen Fathy
%% %% This shows how to see a target from a point on Earth

function simple_topocentric_demo()
%% %% User Inputs
fprintf(" Shireen please enter the data \n")
lat = input('Enter latitude (deg): '); %latitude on Earth
lon = input('Enter longitude (deg): '); % longitude on Earth
h = input('Enter altitude (m): '); % height from sea level
A = input('Azimuth (deg): '); % which direction is the target (0=N,90=E)
el = input('Elevation (deg): '); % how high is the target in the sky
rng = input('Range (m): '); % distance to the target
%% %% Earth Parameters
Re = 6378137; % radius of Earth in meters
f = 1/298.257; % how squished is the Earth (flattening)
%% %% Compute Positions
R_ecef = geodetic2ecef(lat, lon, h, Re, f); % my position in 3D space
rho_ecef = azelrange2ecef(A, el, rng, lat, lon); % target vector from me
r_ecef = R_ecef + rho_ecef; % target absolute position
%% %% Plotting
figure; hold on; grid on; axis equal;
scale = 1/50;
Re_vis = Re*scale; % shrink everything to see it better

% --- Plot Earth as a sphere ---
[Xe,Ye,Ze] = sphere(50);
mesh(Re_vis*Xe, Re_vis*Ye, Re_vis*Ze, ...
'EdgeColor',[0.3 0.3 0.8], 'FaceAlpha',0); % just the outline, looks like a
globe
% --- Plot a simple circular orbit for fun ---
theta = linspace(0,2*pi,200);
r_orbit = (Re+700e3)*scale; % 700 km above Earth
plot3(r_orbit*cos(theta), r_orbit*sin(theta), zeros(size(theta)), 'r--',
'LineWidth',2);

% --- Plot me and the target ---
plot3(R_ecef(1)*scale,R_ecef(2)*scale,R_ecef(3)*scale,'ko','MarkerSize',8,'
LineWidth',2); % me = black dot
plot3(r_ecef(1)*scale,r_ecef(2)*scale,r_ecef(3)*scale,'rx','MarkerSize',12,
'LineWidth',2); % target = red X

% --- Draw line from me to the target ---
plot3([R_ecef(1) r_ecef(1)]*scale, ...
[R_ecef(2) r_ecef(2)]*scale, ...
[R_ecef(3) r_ecef(3)]*scale,'k-','LineWidth',2); % my line of sight

% Labels and Title
xlabel('X [m]'); ylabel('Y [m]'); zlabel('Z [m]'); % axes labels
title(' Topocentric Demo '); % fun title
legend('Earth','Reference Orbit','Observer','Target','Line of
Sight','Location','best');

view(40,25); % nice angle to see everything
End
```

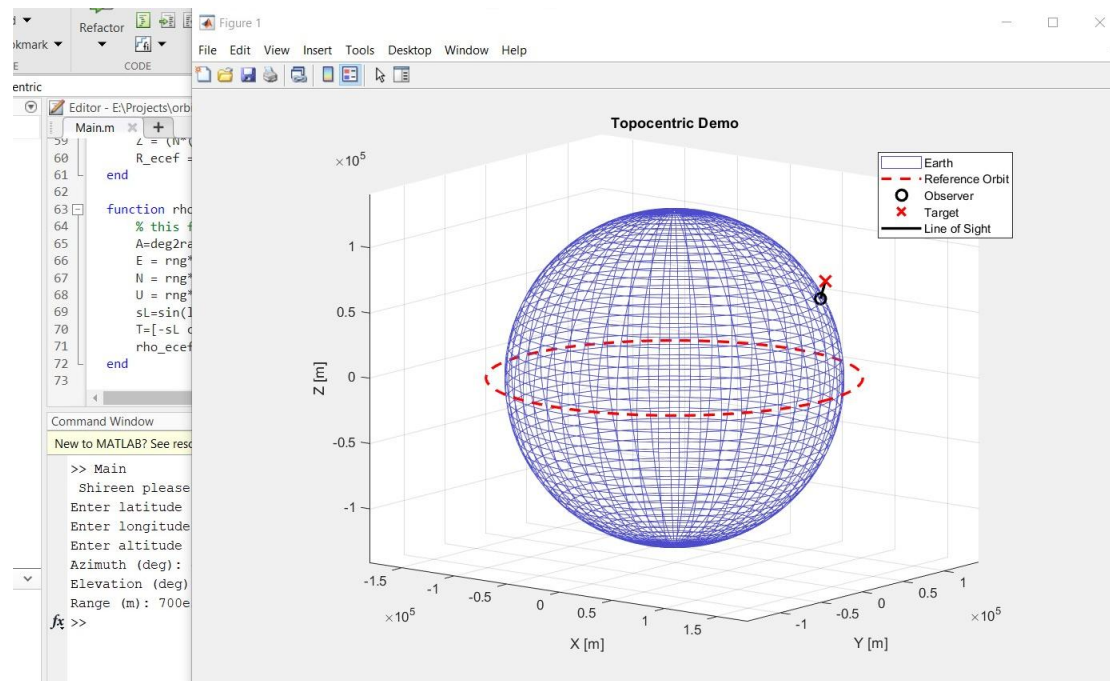
```

% % % % % % Helper Functions
function R_ecef = geodetic2ecef(phi, lambda, H, Re, f)
% this function convert lat/lon/alt to X,Y,Z (ECEF)
phi = deg2rad(phi); lambda = deg2rad(lambda);
e2 = f*(2-f);
N = Re ./ sqrt(1 - e2*sin(phi).^2);
X = (N+H).*cos(phi).*cos(lambda);
Y = (N+H).*cos(phi).*sin(lambda);
Z = (N*(1-e2)+H).*sin(phi);
R_ecef = [X;Y;Z]; % return as a column vector
end

function rho_ecef = azelrange2ecef(A, el, rng, phi, lambda)
% this function convert azimuth/elevation/range to a 3D vector in ECEF
A=deg2rad(A); el=deg2rad(el); phi=deg2rad(phi); lambda=deg2rad(lambda);
E = rng*cos(el).*sin(A);
N = rng*cos(el).*cos(A);
U = rng*sin(el);
sL=sin(lambda); cL=cos(lambda); sP=sin(phi); cP=cos(phi);
T=[-sL cL 0; -sP*cL -sP*sL cP; cP*cL cP*sL sP];
rho_ecef=T*[E;N;U]; % final vector from observer to target
end

```

Visualization



Reference:

[1] Curtis, Howard D., "Orbital Mechanics for Engineering Students", 4th Edition, Elsevier, 2020.