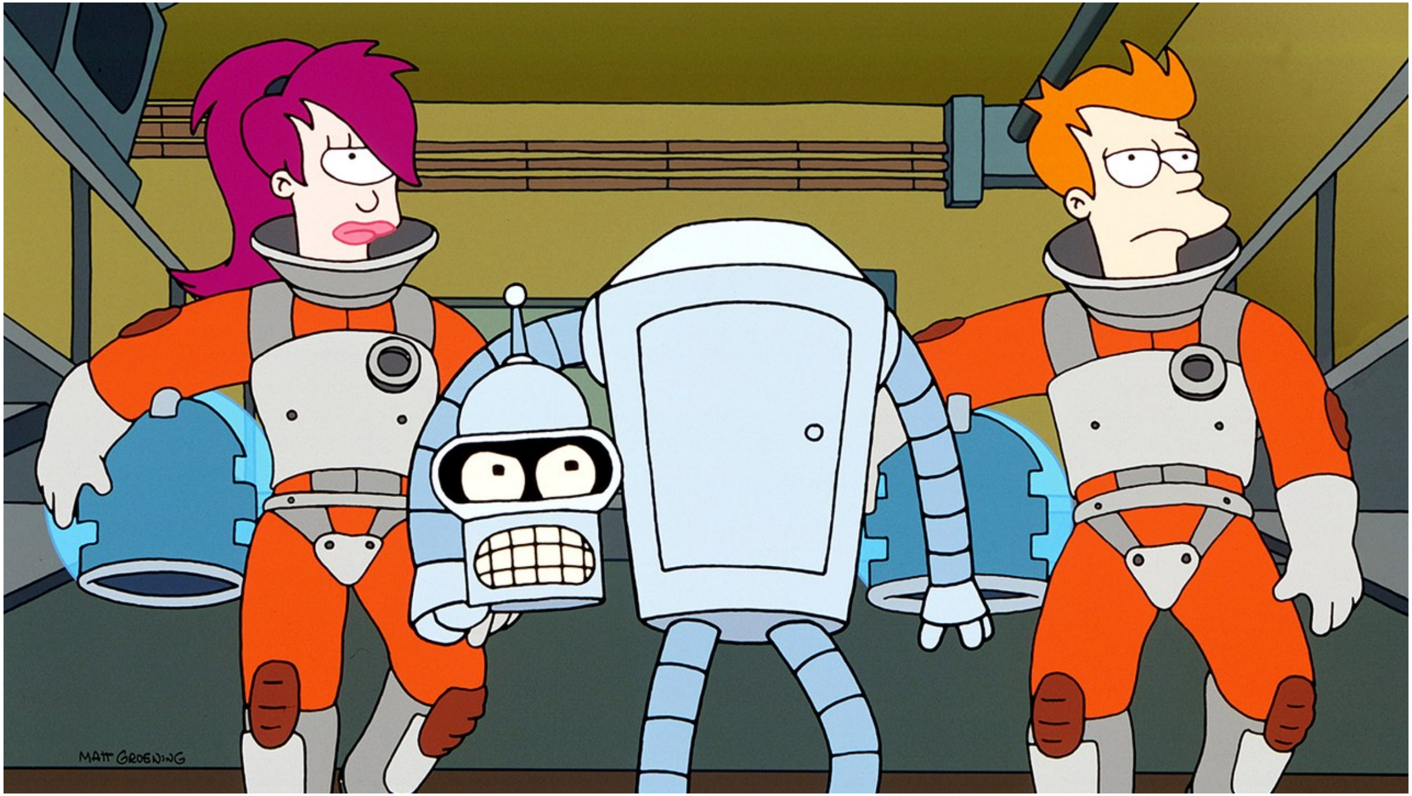
Math Camp: Lesson 4

Statistics and Probability

UW–Madison Political Science

August 20 & 21, 2020

Hang in there



Why do we mess with statistics?

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- Models that describe our data have *unknown variables* ("parameters")
- How do we estimate those parameters?

Let's flip a coin

If we flip a *fair* coin, what is the probability that it lands heads up?

Let's say we got Heads

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- The coin flip (data) is influenced by an underlying *probability of Heads*
- There is a *systematic* component and a *random* component
- Statistical modeling is (in part) distinguishing systematic forces from random forces

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- We collect data
- We *make inferences* about the processes that influence the data
- Are those inferences consistent with our theories?

Statistics and probability

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- We can calculate the *probability of the data* under each model to pick the best model
- And then evaluate how certain (or rather, uncertain) we are about our findings

But before we can do any of that

We have to learn some basic math of probability

Agenda

- Counting
- Set theory
- Probability
- Independence, joint probability
- Bayes' Theorem
- Looking ahead

Helpful vocabulary

A **random variable** is a realization of a process that is at least partially random (i.e. unpredictable)

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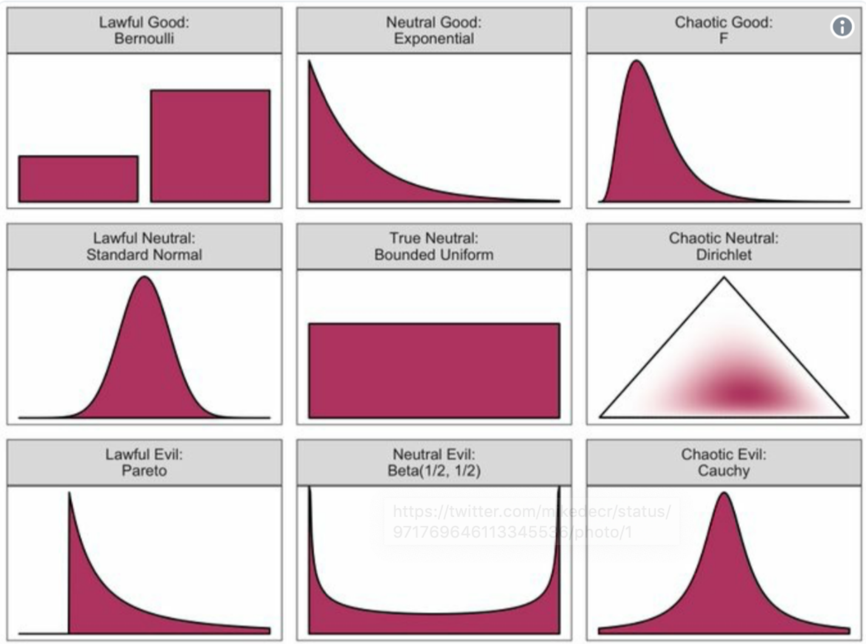
If we wanted to describe the probability of each potential outcome, we would do so with a *probability distribution.*

- A probability distribution is a *function* that maps potential outcomes to the probability of those outcomes
- x = potential outcome
- $f(x) =$ probability of x
- These also matter for formal (non-statistical) models (e.g. utility shocks)

Some distributions...



Mike DeCrescenzo
@mikedecr

Probability distribution alignments

10:28 AM - Mar 8, 2018

15 See Mike DeCrescenzo's other Tweets

More about probability distributions

Probability distributions can describe discrete outcomes (coin flips) or continuous outcomes (height, vote margin)

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Probability distributions are the basis for statistical inference

- z-scores, p-values
- Prior and posterior beliefs

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$$\prod_{k=1}^{K} n_k$$

(multiply the n_k's)

I roll a 6-sided die 4 times. How many unique sets of 4 rolls can I obtain (assuming that different orderings of the same 4 numbers are different events)?

Complex counting considerations

Does the *order of selection* matter? (Is $\{1, 2\} = \{2, 1\}$?)

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Are selected objects *replaced* (able to be selected again) or *not replaced*?

Ordering with replacement

This is easiest because (a) no need to adjust for "double-counting" and (b) the number of possibilities is always constant.

The number of possible ways to select k elements from a larger pool of n is

$$n \times n \times n \times \ldots \times n = n^k$$

Intuition: in each draw, there are n possibilities. Each of n outcomes in one draw can be combined with the n outcomes in any (and all) other draws.

Example: rolling two dice several times

Order, no replacement

Also called **permutation**.

The number of ways to select k objects from a pool of n possible objects, where order matters but replacement does not occur.

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For example: number of possible ways to deal a card game, winning lottery numbers

Unordered, no replacement

Also called combinations: The number of possible ways to select k objects from a pool of n possible objects, where *order does not matter* and *replacement does not occur*

Intuition: we have fewer possibilities than before, substantively identical elements (A and then B; B and then A) are not double counted

$$\frac{n!}{k!(n-k)!} = \binom{n}{k}$$

For example: survey samples, raffles, possible groups of 2 in a classroom

Unordered, with replacement

The number of possible ways to select k elements from a larger pool of n possible elements, where order does not matter and replacement does occur

$$\frac{(n+k-1)!}{(n-1)!k!} = \binom{n+k-1}{k}$$

Example: the number of heads if you flip a coin n times

Exercises

Imagine we rank the 3 top swimmers in this room.

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Imagine we have 5 identical candies for students in this room. You can win more than 1 candy. How many different sets of winners can there be?

Imagine we have 2 identical bicycles for students in this room. You can only win 1 bicycle. How many sets of winners?

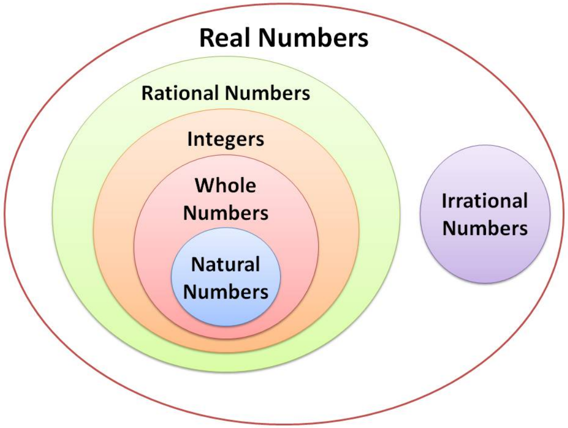
Set theory

Sets

Remember: a set is a collection of elements. Could be numbers, units, areas in space...

- $F = \{1, 2, 3, 4\}$
- $G = \{1, 3, 5\}$
- $H = [0, 1] \cup (2, 3)$

What are unions? Intersections? Disjoints?
Subsets? Supersets?



- $P = \{\text{Reagan}, \text{Bush41}, \text{Clinton}, \text{Bush43}, \text{Obama}, \text{Trump}\}$
- $D = \{\text{Carter}, \text{Mondale}, \text{Dukakis}, \text{Clinton}, \text{Gore}, \text{Kerry}, \text{Obama}, \text{HRC}\}$
- $R = \{\text{Reagan}, \text{Bush41}, \text{Dole}, \text{Bush43}, \text{McCain}, \text{Romney}, \text{Trump}\}$
- $I = \{\text{Perot}, \text{Nader}\}$

The sample space

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Not the same as the set that contains *everything*. Only the relevant things for what we're currently talking about.

Complementary sets

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What is Ω^C?

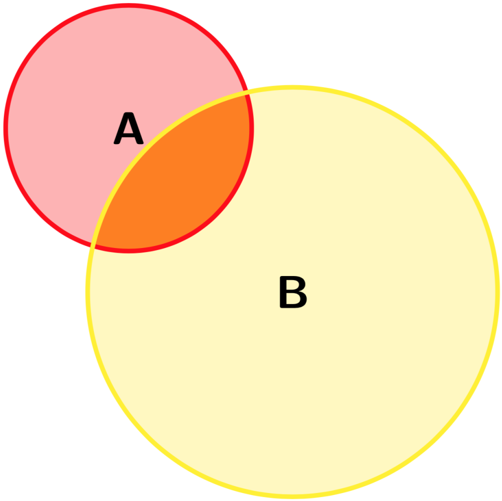
- $\emptyset$

Making sense?

Probability (beginning with sets)

Probability as Sets

We can use sets to represent the probability of events. Total area represents total probability of all events (equal to 1).



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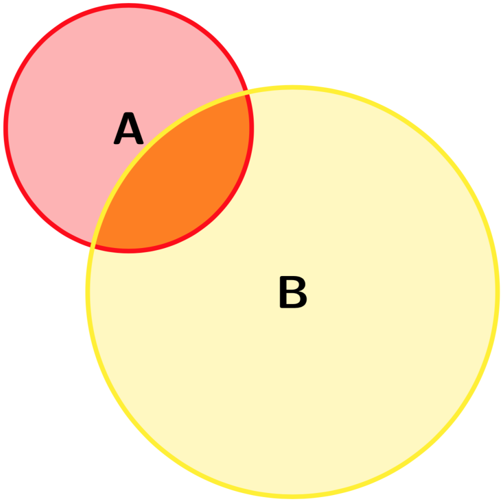

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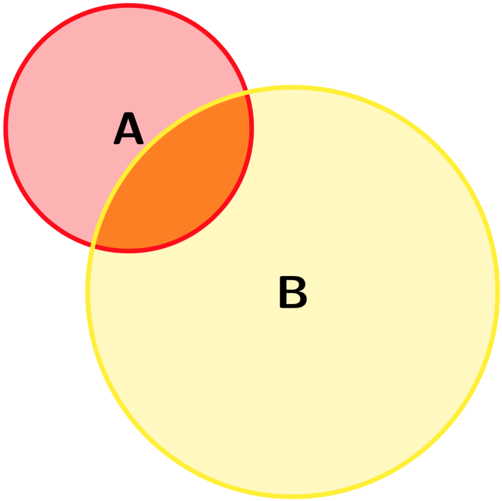

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$\Pr(A)$?

$\Pr(A^C)$?

Let's play cards

We have 4 suits (hearts, diamonds, spades, clubs) and 13 card values (Ace, 2, 3, ..., Jack, Queen, King). Suits and values can both be sets.

	A	2	3	4	5	6	7	8	9	10	J	Q	K
♡H													
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Total area = 1

Probability of an individual card: $\frac{1}{52}$

Properties of probabilities

Probabilities are strictly bounded on the closed interval $[0, 1]$

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If we had N many *collectively exhaustive* and *mutually exclusive* sets of potential outcomes, their probabilities sum to 1. Which is to say, *something must happen*.

$$\sum_{n=1}^{N} p(A_n) = 1$$

Probability of complements

If Ω contains the set of all potential outcomes, and A is an event that is a subset of the outcome space that occurs with $p(A)$

- What is $p(A^C)$?

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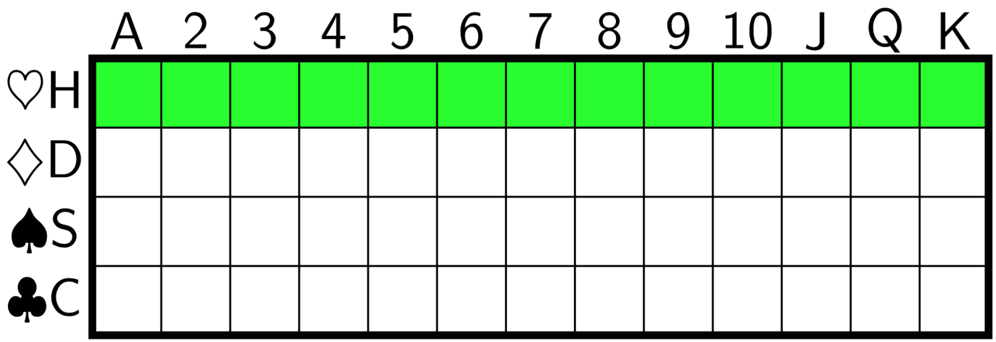
The intuition: *Something* must happen, either A or not A

Example of complements

Probability that a random card is a Heart? $p(H) = \frac{1}{4}$



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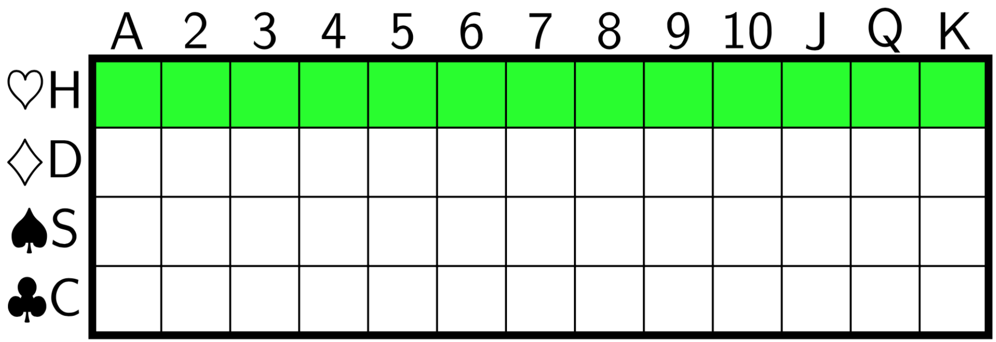
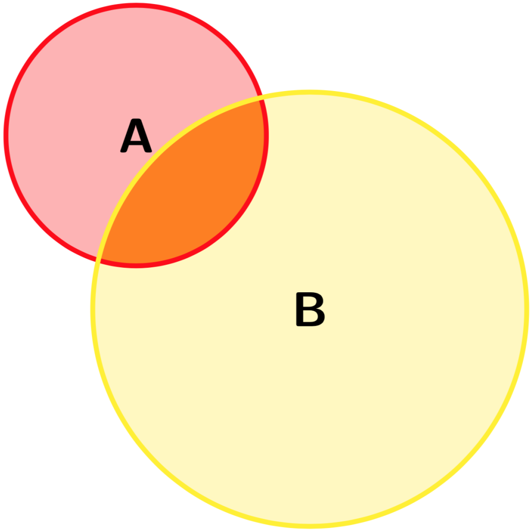


Probability that a card is not a Heart? $1 - p(H) = \frac{3}{4}$

Probability of unions

The probability of $A \cup B$

The probability that *either* A or B occurs



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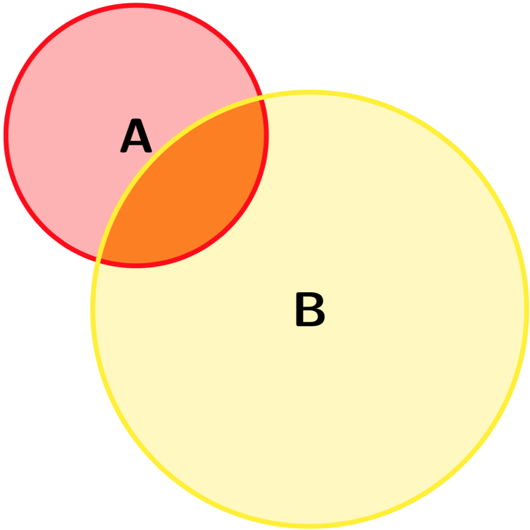

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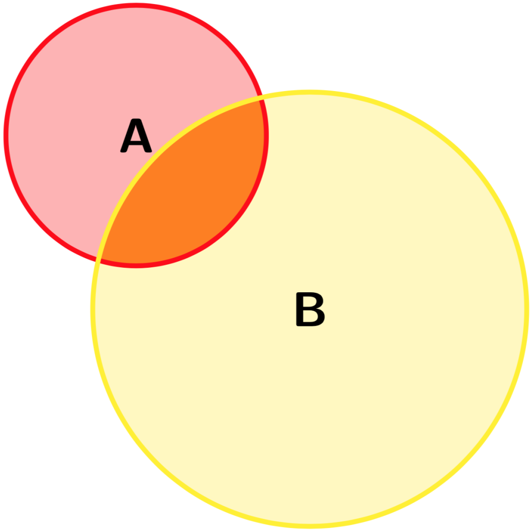
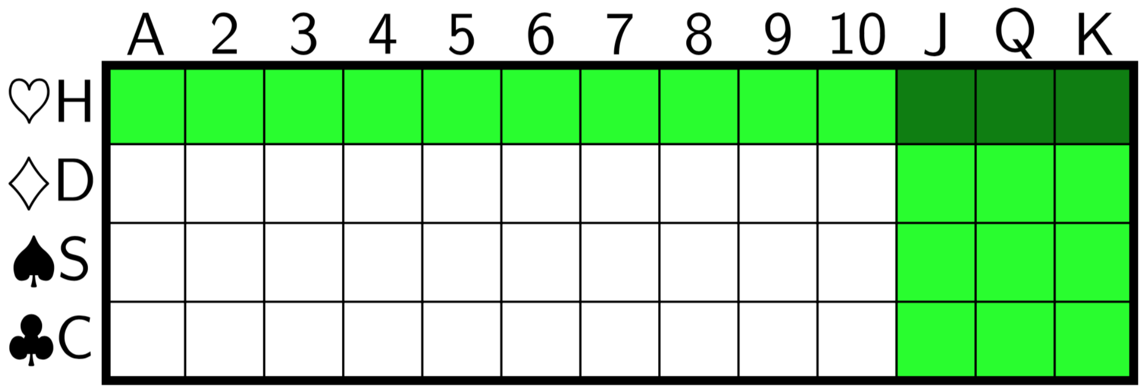


$$p(A \cup B) = p(A) + p(B) - p(A \cap B)$$

The intuition: the sum of A and B will double count $A \cap B$, so we need to subtract one instance of $A \cap B$

Probability of unions

What is the probability that we draw a card that is *either* a Heart *or* a face card?



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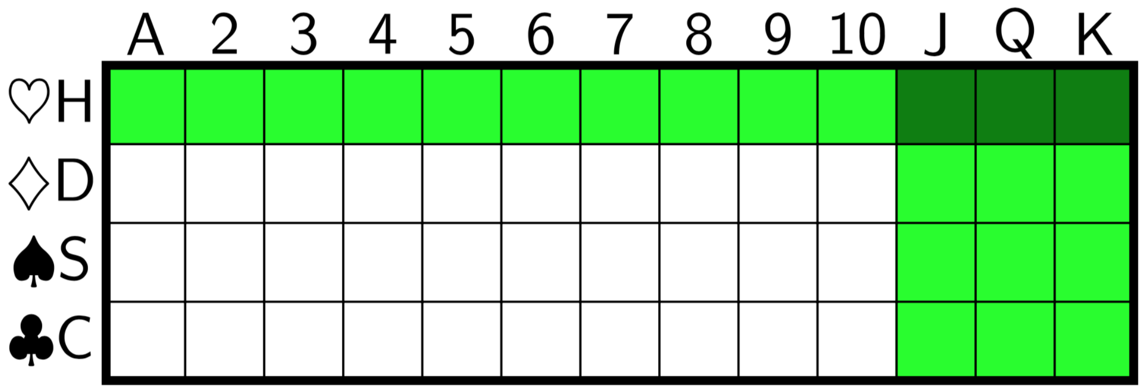

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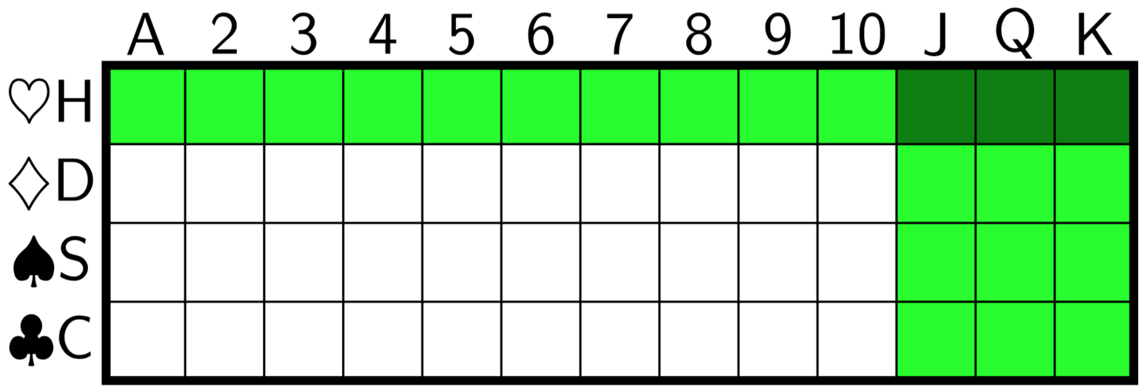

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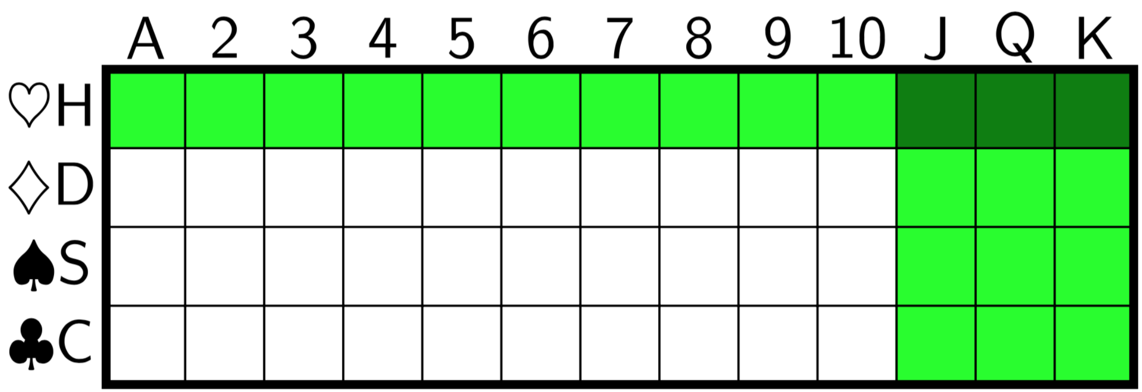

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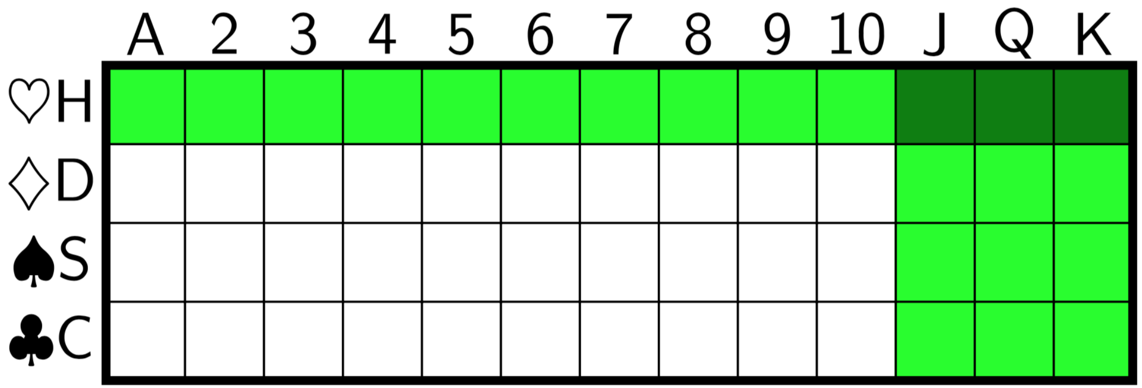


$p(H) = ?$

$p(F) = ?$

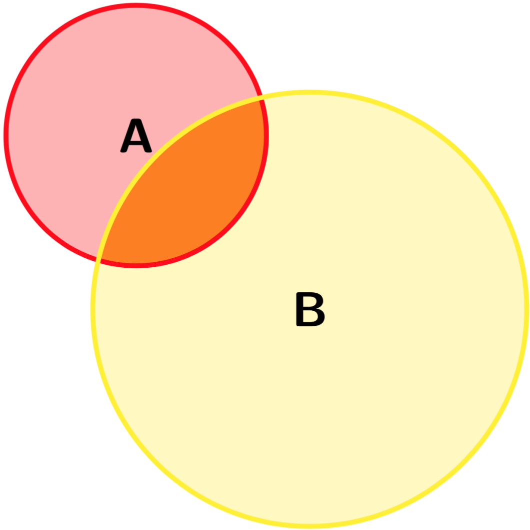
$p(H \cap F) = ?$

$p(H \cup F) = \frac{1}{4} + \frac{12}{52} - \frac{3}{52} = \frac{22}{52}$

Probability of intersections

The probability of $A \cap B$

The probability that *both* A and B occur



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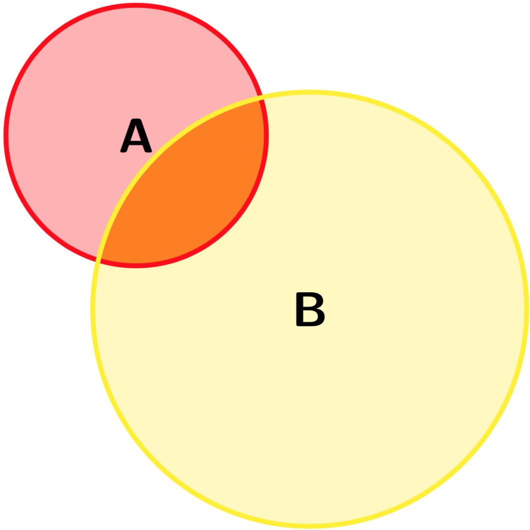
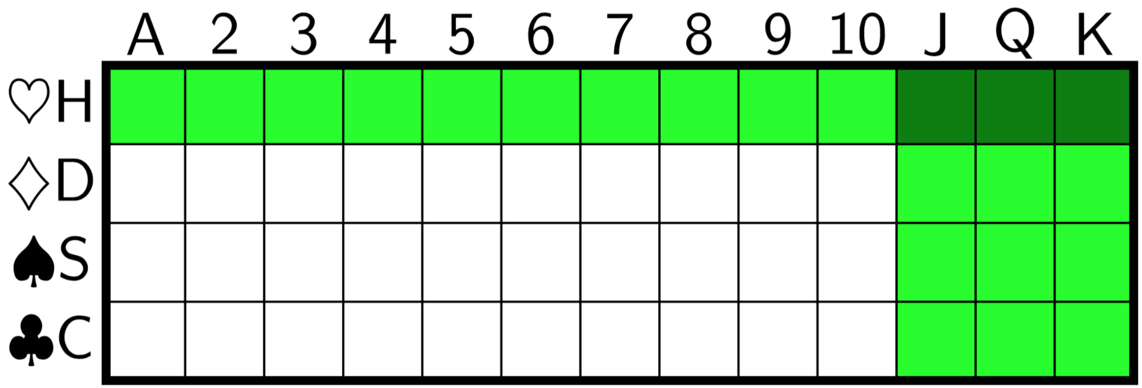


$$p(A \cap B) = p(A) + p(B) - p(A \cup B)$$

The intuition: We care only about the component that we double counted

Probability of intersections

What is the probability that we draw a card that is *both* a heart *and* a face card?



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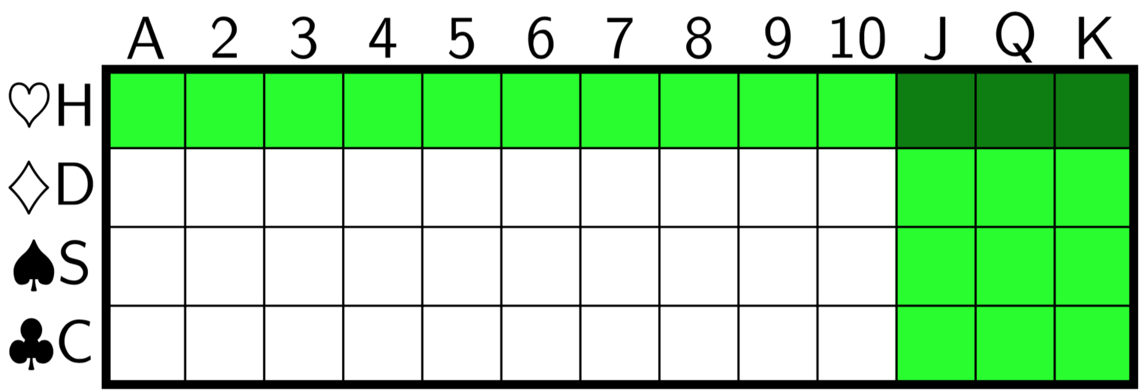

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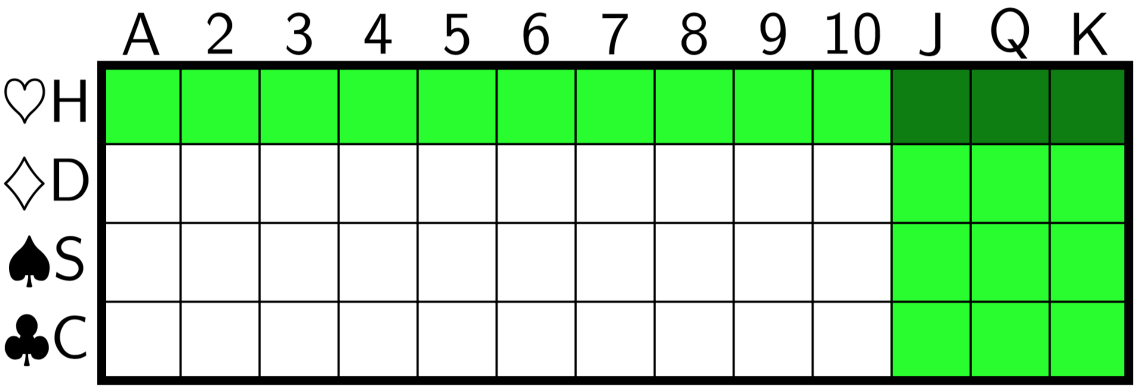

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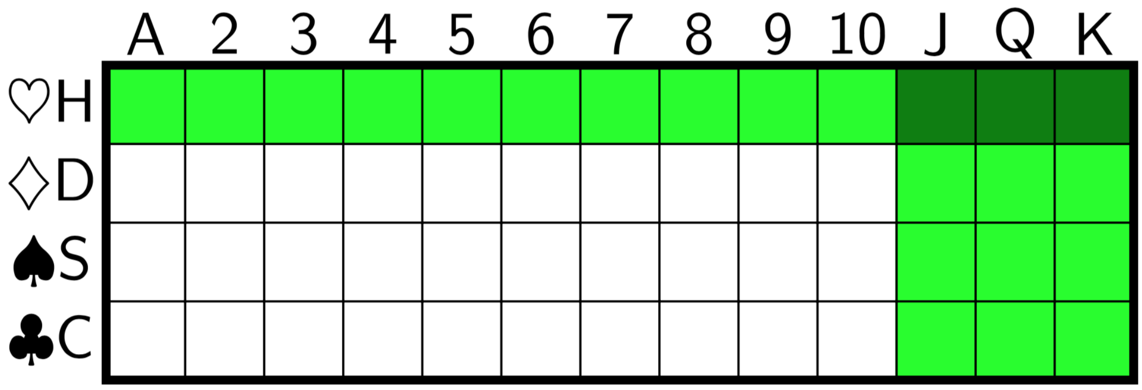

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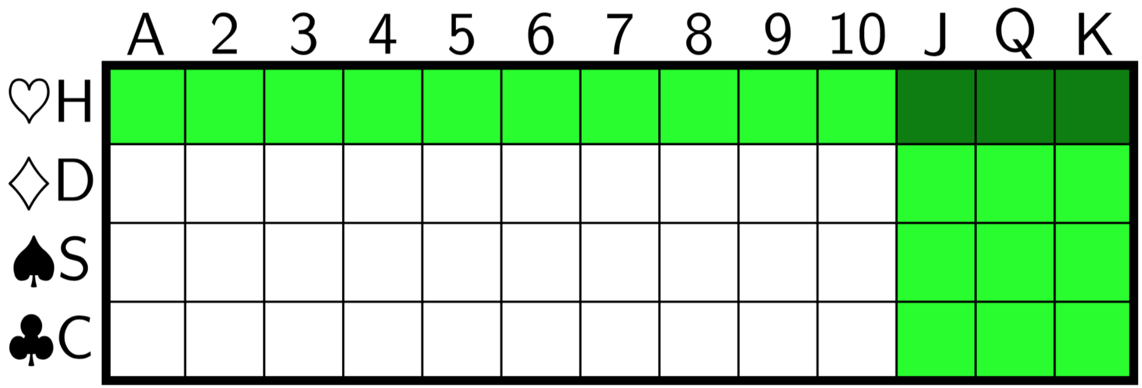


$p(H) =?$

$p(F) =?$

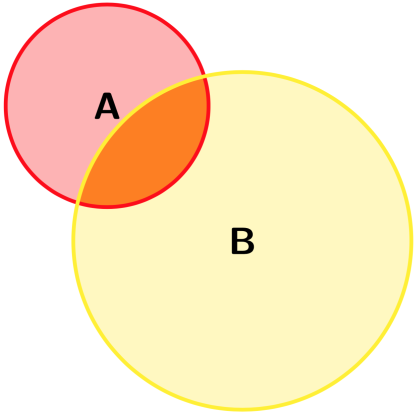
$p(H \cup F) =?$

$p(H \cap F) = \frac{1}{4} + \frac{12}{52} - \frac{22}{52} = \frac{3}{52}$

Conditional probability

The probability of A, given B, is expressed as $p(A \mid B)$

What is the probability of A, given that B also occurs?



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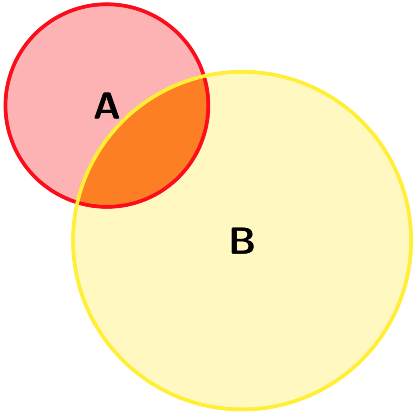

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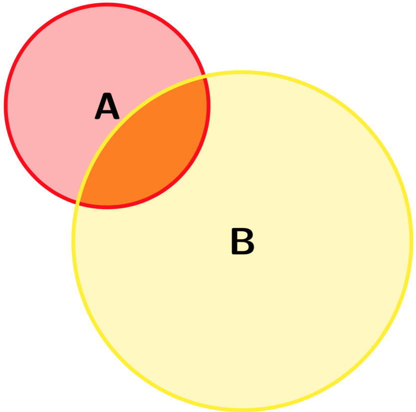

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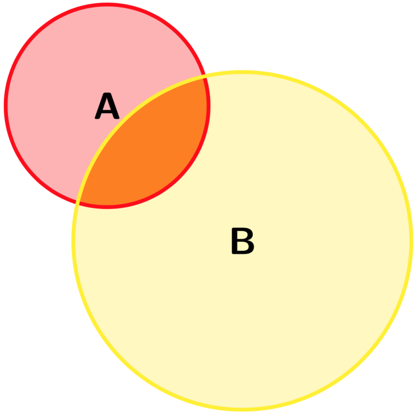

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- If we *know* that B happened, we only care about the space within B
- the probability that both A and B happen, divided by the probability of B
- $p(\text{intersection}) \ / \ p(\text{conditioning event})$

Conditional probability

	A	2	3	4	5	6	7	8	9	10	J	Q	K
♡H													
♢D													
♠S													
♣C													

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Conditional probability

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- $p(\text{Ace}) = \frac{4}{52}$
- $p(\text{Ace of Diamonds} \mid \text{Ace}) = \frac{1/52}{4/52} = \frac{1}{4}$

What's the probability?

	A	2	3	4	5	6	7	8	9	10	J	Q	K
♡H													
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$p(\{8, 9, 10\})$

$p(\{5, 6\} \cup \{6, 10\})$

$p(A \mid H^C)$

The notion of *independence*

Two events are **independent** if knowing the outcome of one event does not change the probability of the other

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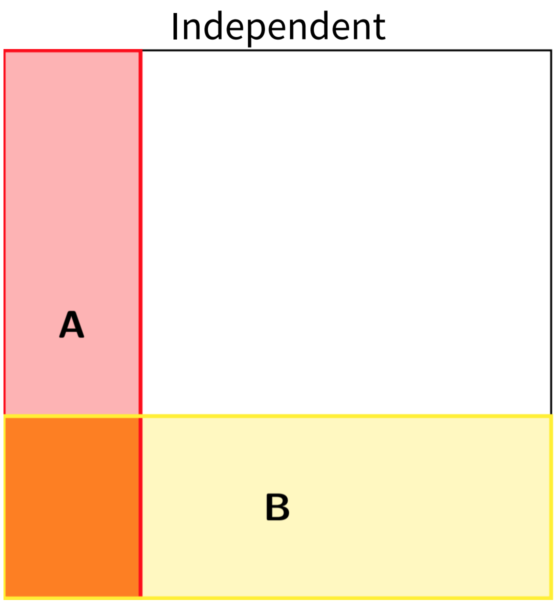

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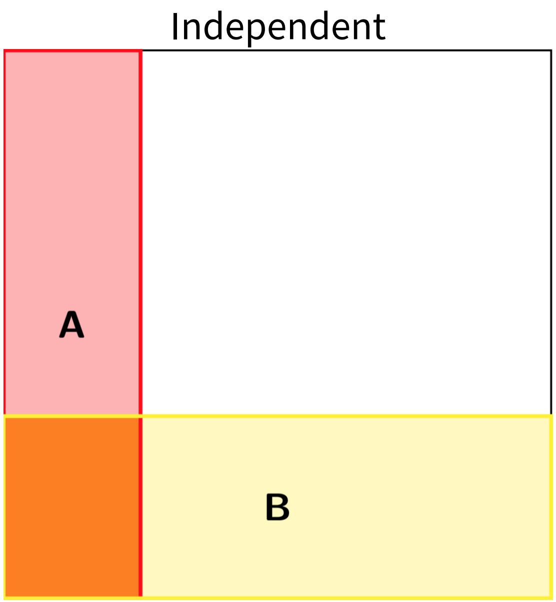

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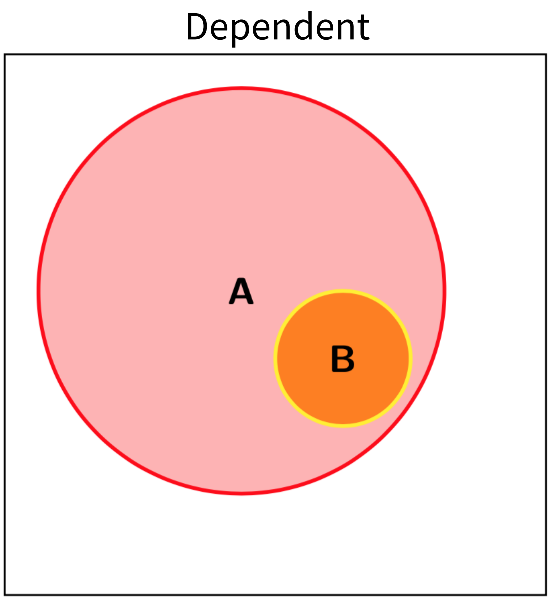
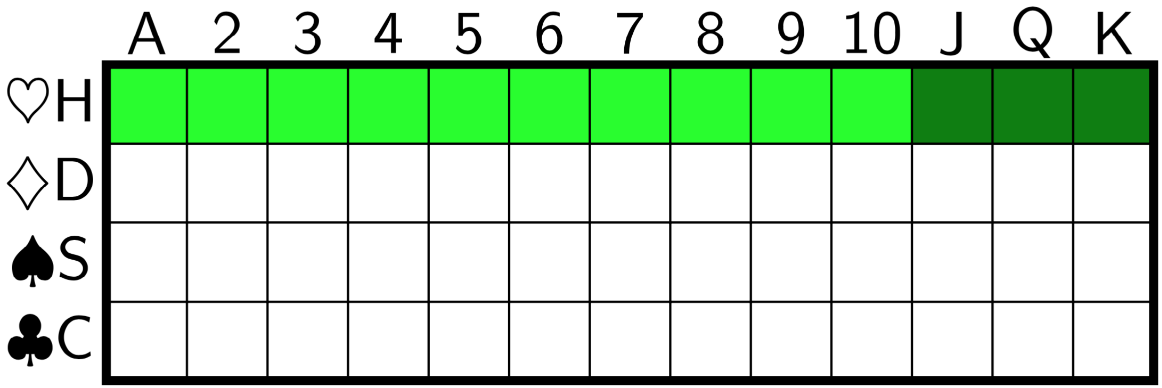


$$p(B) \neq p(B \mid A)$$

Independence of Events

Is drawing a face card independent of drawing a Hearts card?



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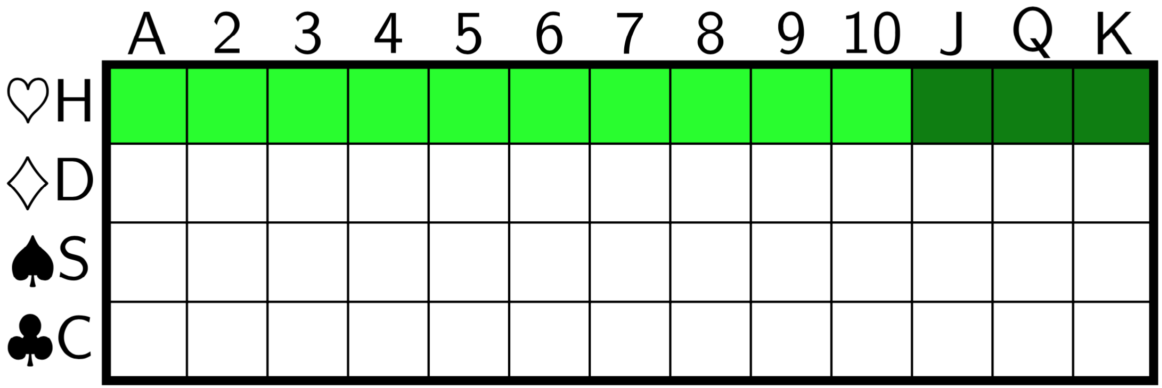

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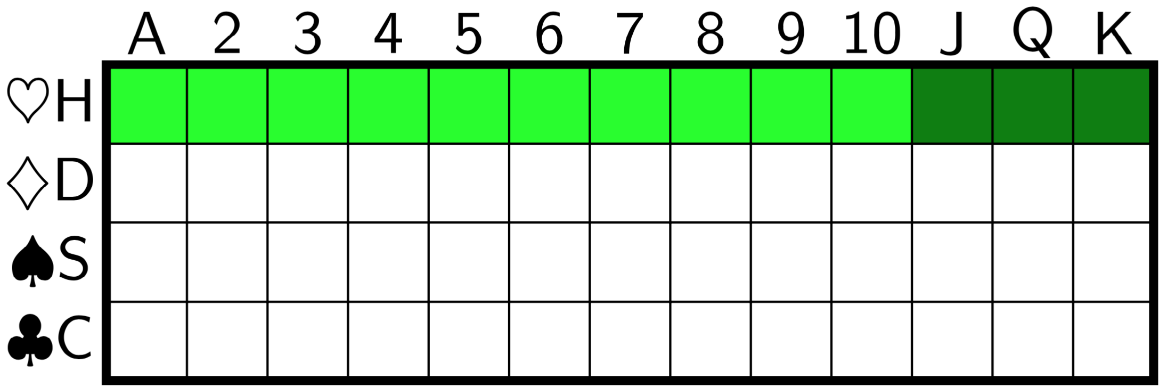

$p(F \mid H) = \frac{3}{13}$

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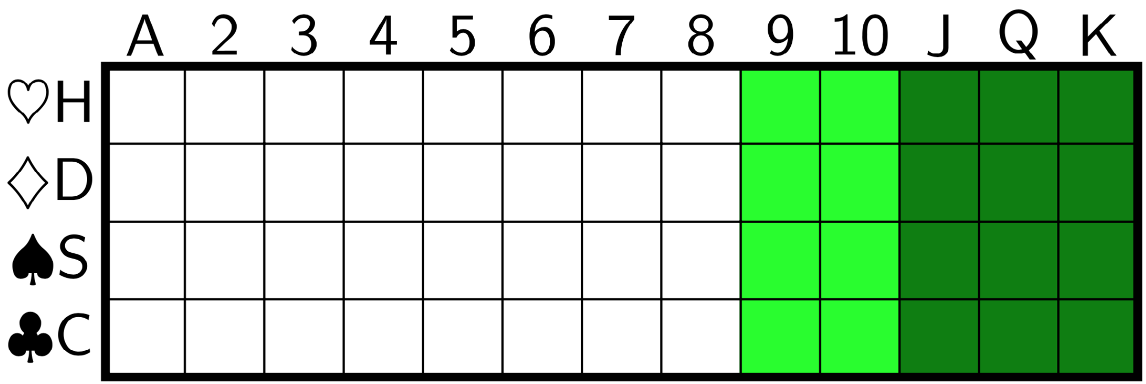
Independence of Events

What about drawing a face card independent of drawing a card greater than 8?



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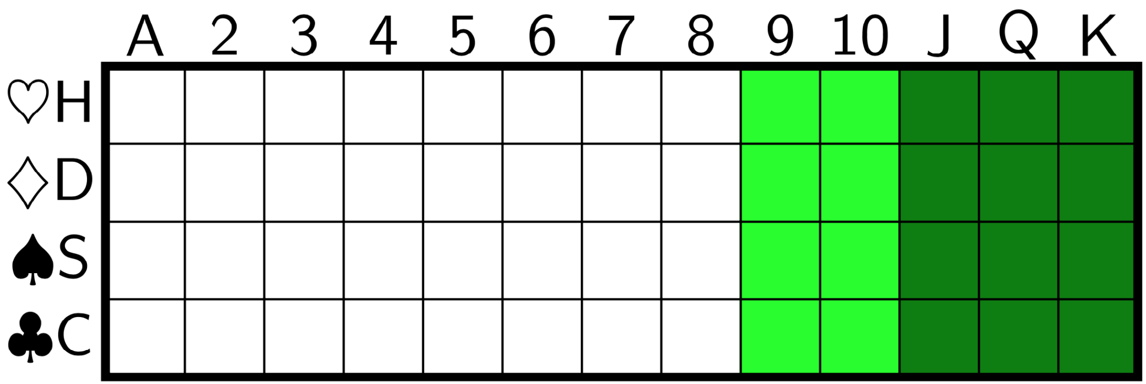

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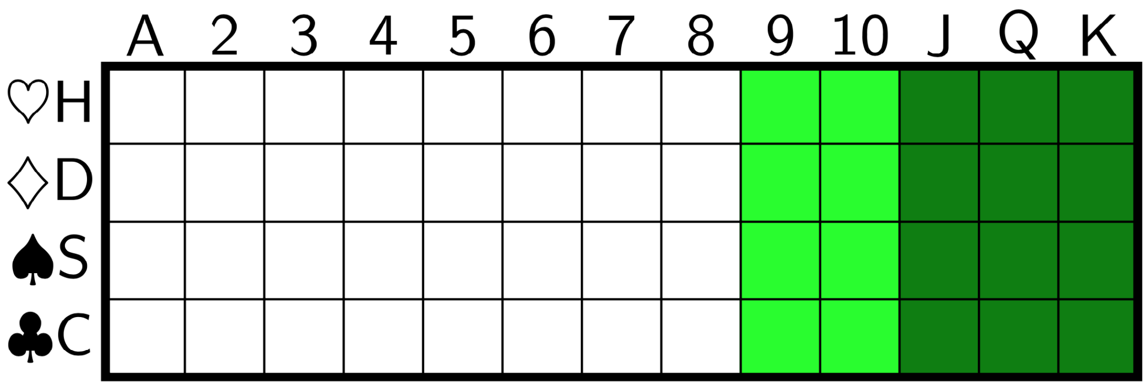

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Joint probability

What we're doing here is considering the probability of *multiple events*

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The exact equation for the joint probability depends on whether the events are *independent*

Joint probability of independent events

If multiple events are independent of one another, the joint probability of all events is the *product* of the individual probabilities.

Example: we flip three coins independently of one another. What's the probability of the sequence $\{H, H, H\}$?

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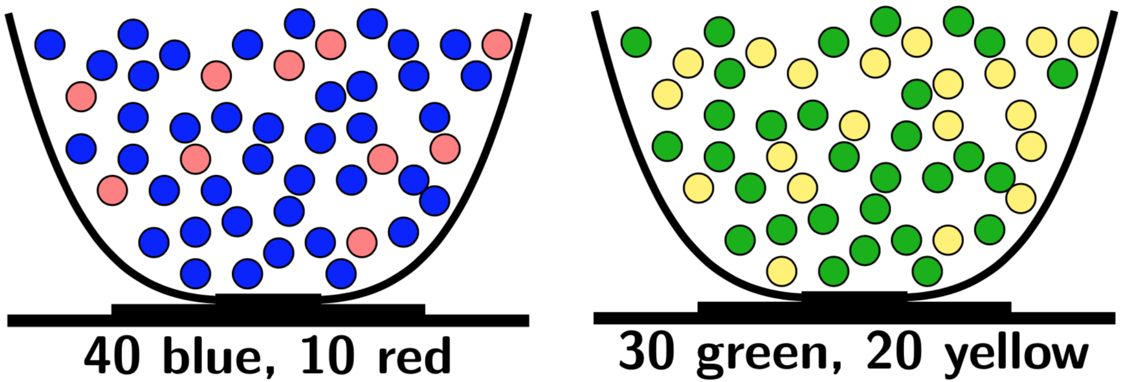
$$\begin{aligned} p(H) \times p(H) \times p(H) &= .5 \times .5 \times .5 \\ &= 0.125 \end{aligned}$$

So we've got two bowls



If we draw a ball from each bowl, what is the joint probability of...

- $p(\text{blue}, \text{green}) = ?$
- $p(\text{blue}, \text{yellow}) = ?$
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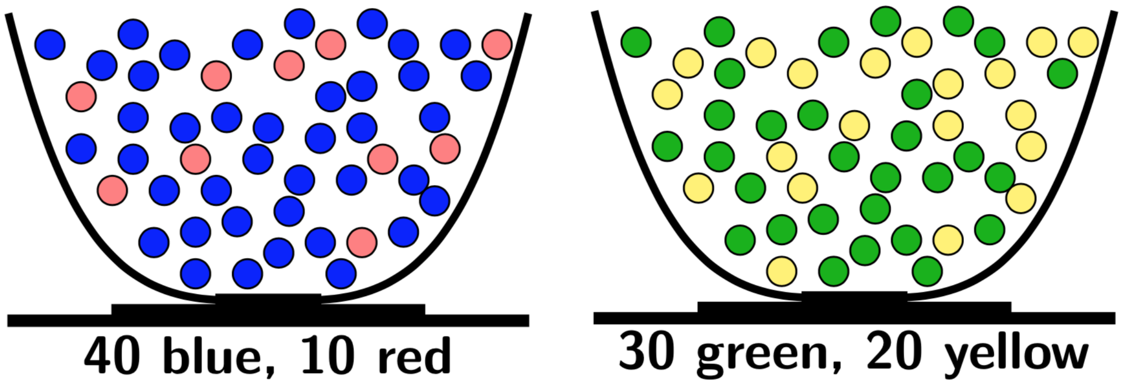

If we draw a ball from each bowl, what is the joint probability of...

- $p(\text{blue}, \text{green}) = \left(\frac{40}{50}\right)\left(\frac{30}{50}\right) = (.8)(.6) = .48$
- $p(\text{blue}, \text{yellow}) = \left(\frac{40}{50}\right)\left(\frac{20}{50}\right) = (.8)(.4) = .32$
- $p(\text{red}, \text{green}) = \left(\frac{10}{50}\right)\left(\frac{30}{50}\right) = (.2)(.6) = .12$
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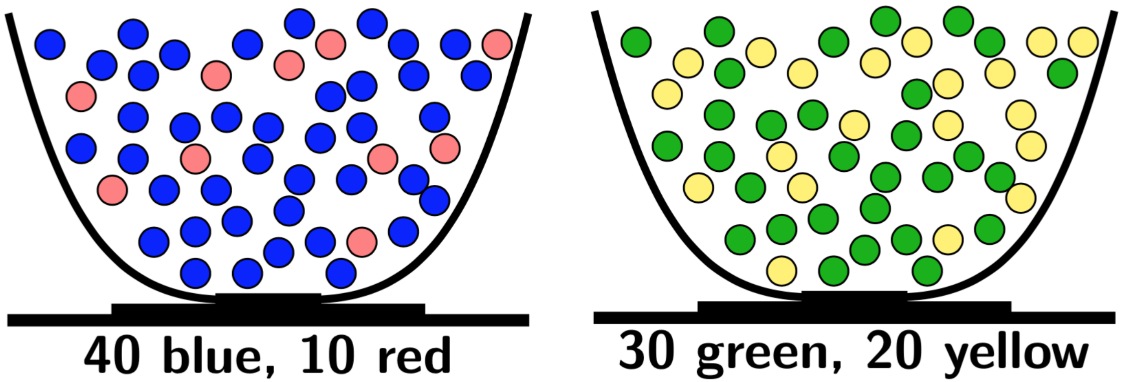

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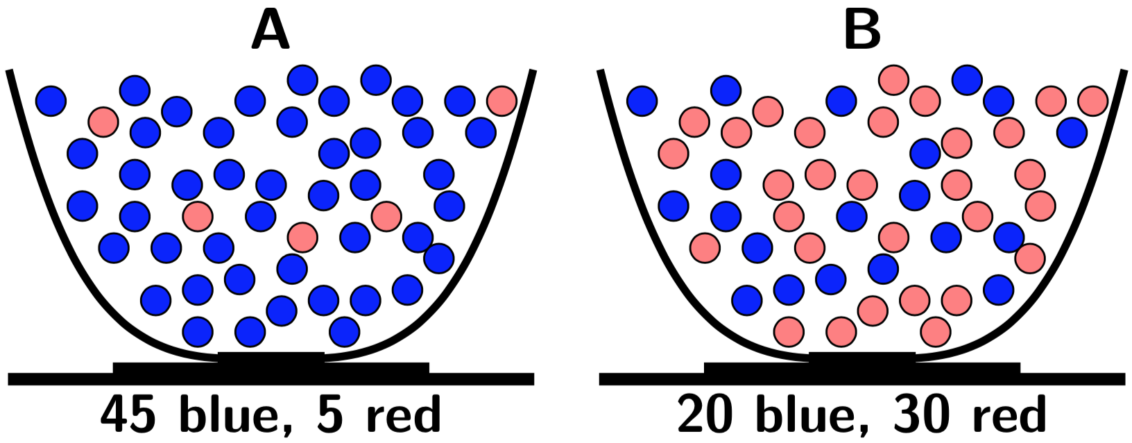
Because these are mutually exclusive and exhaustive events, probabilities sum to 1

Imagine we flip a coin. If heads, we draw a ball from the left urn. If tails, we draw from the right.



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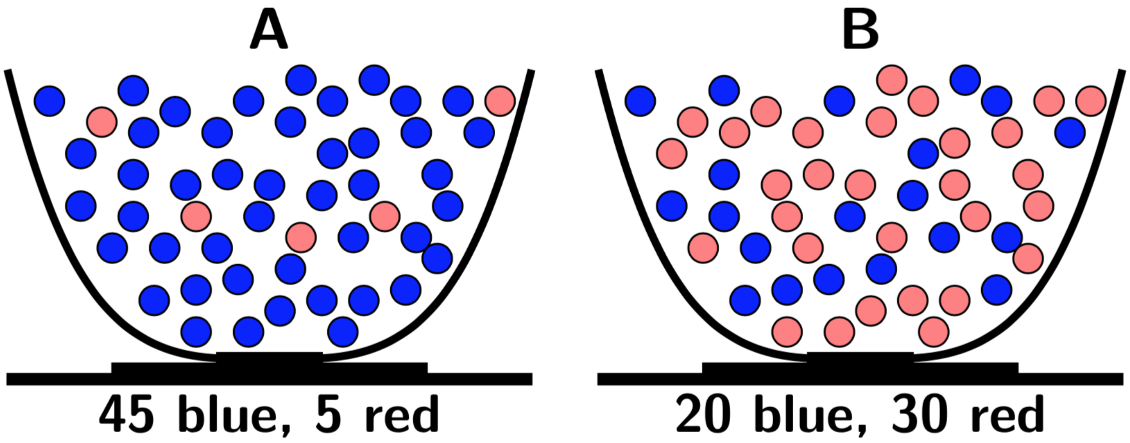

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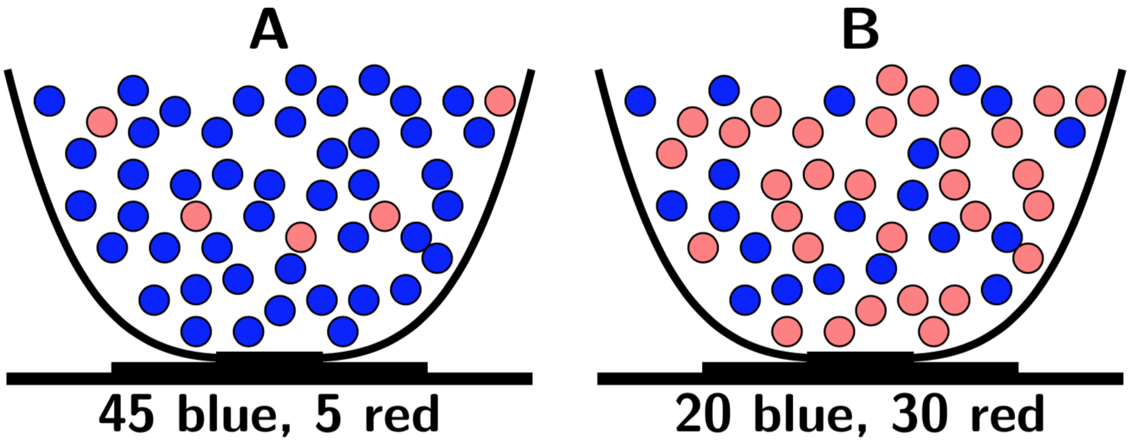

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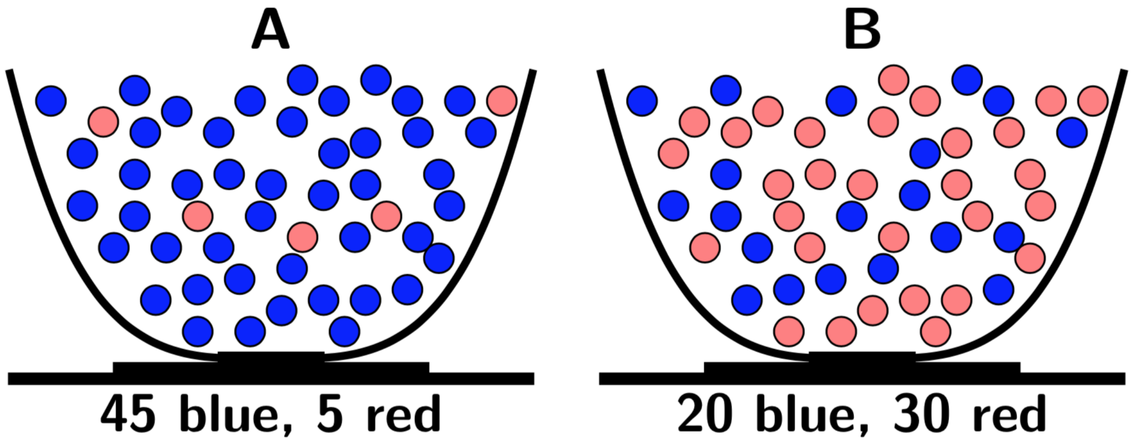

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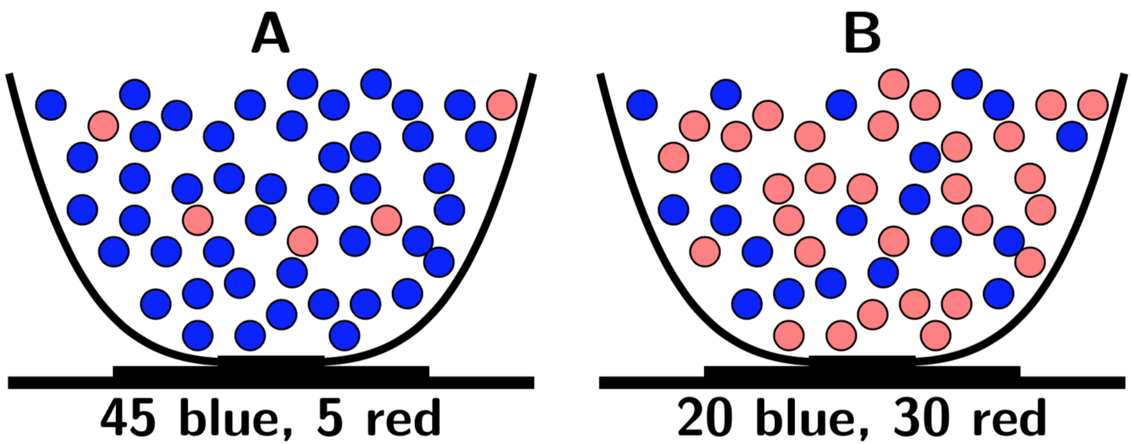

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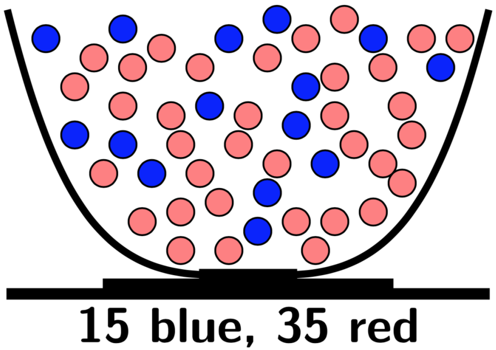
$$\begin{aligned} p(\text{blue}) &= p(\text{blue} \mid A) * p(A) + p(\text{blue} \mid B) * p(B) \\ &= p(\text{blue} \mid A) * p(A) + p(\text{blue} \mid A^C) * p(A^C) \end{aligned}$$

Thinking about order and replacement

We draw 5 balls from one urn, replacing each time. We get the following sequence:



$\{\text{blue, red, blue, blue, red}\}$

The probability of *this specific sequence* is $.3 * .7 * .3 * .3 * .7 = 0.01323$,

or if we simplify: $0.3^3 0.7^2$

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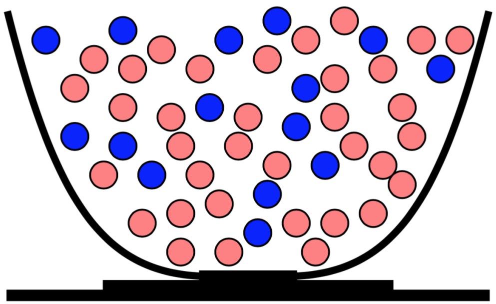

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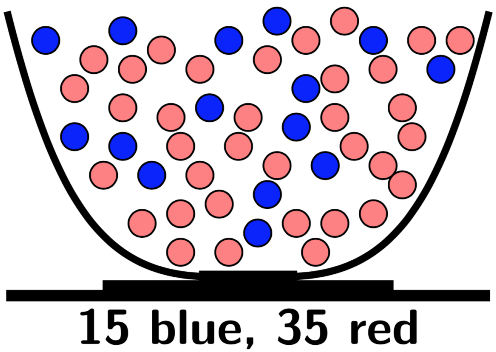

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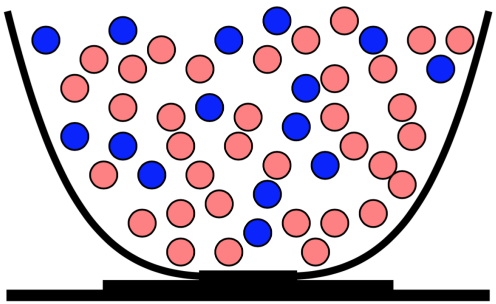

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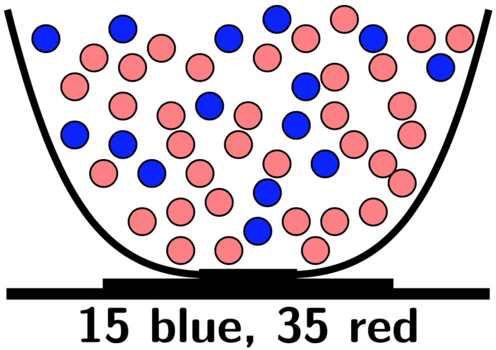

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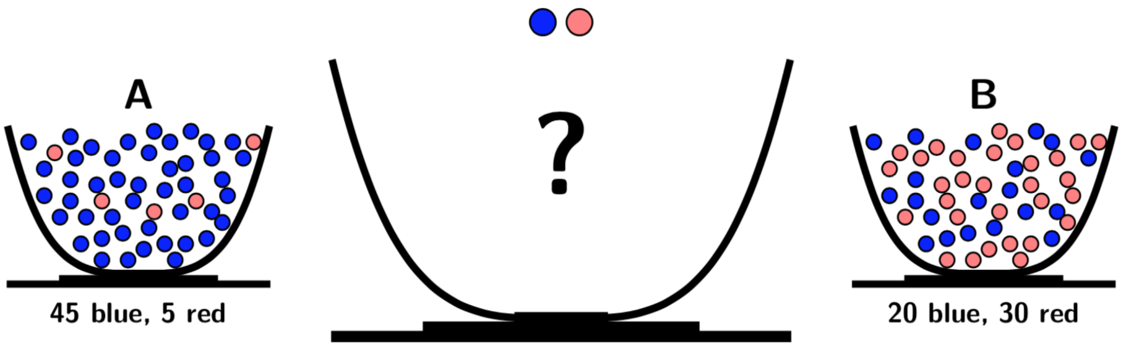
$$\left(\frac{5!}{3!(5-3)!}\right)(.3)^3(.7)^2 = \binom{5}{3}(.3)^3(.7)^2 = (10)(.01323) = .1323$$

Inverse conditional probability



Someone flips a coin to decide whether to draw a ball from bowl A or B (each with 50% probability), but the bowl is hidden from us.

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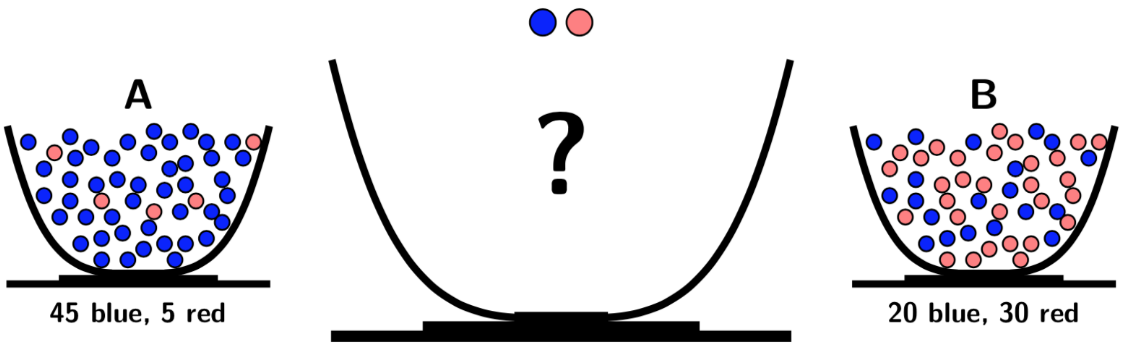

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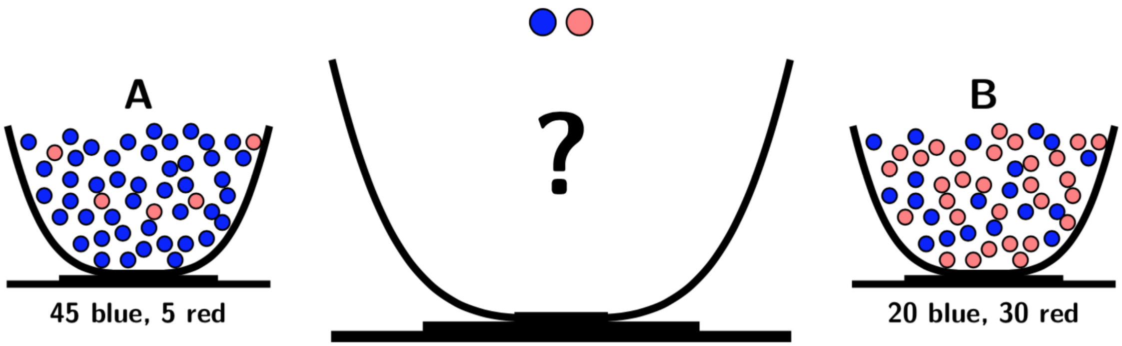

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- We've drawn a *blue* ball. What's the probability that we drew from A?

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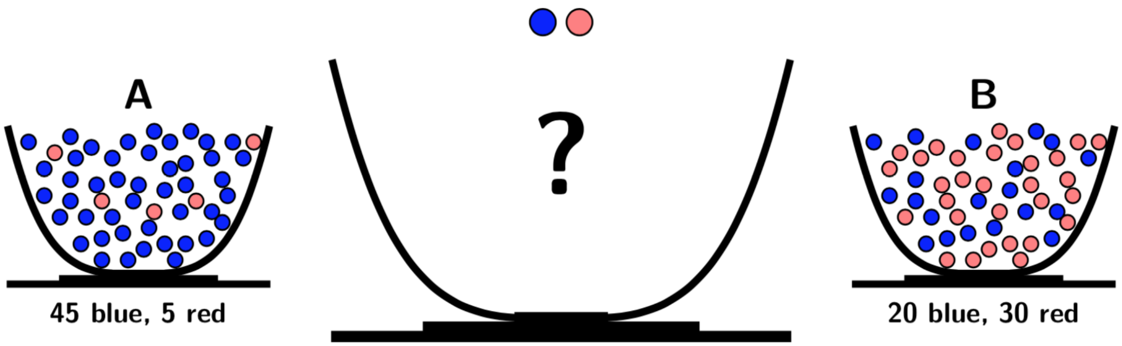


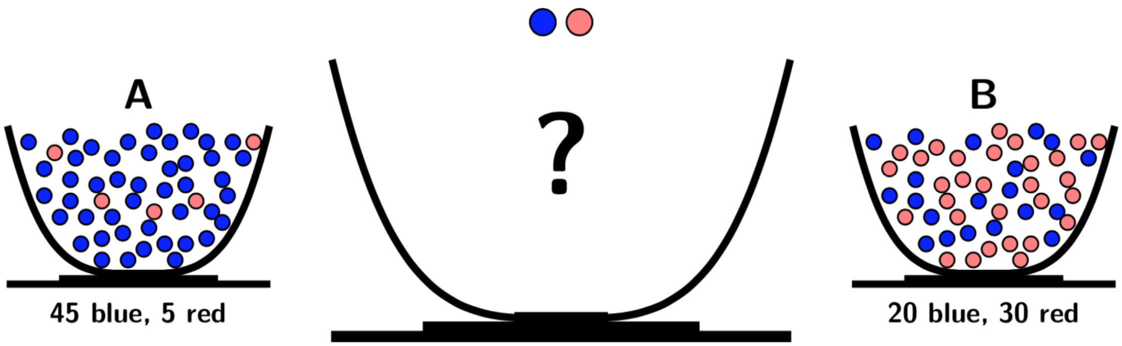
Someone flips a coin to decide whether to draw a ball from bowl A or B (each with 50% probability), but the bowl is hidden from us.

- What is the probability of drawing from bowl A?
- We've drawn a *blue* ball. What's the probability that we drew from A?

"Inverse" conditional probability problem:

- It's easy to find $p(\text{blue} \mid A)$,
- but how can we *invert* it to find $p(A \mid \text{blue})$?

Find $p(A \mid \text{blue})$



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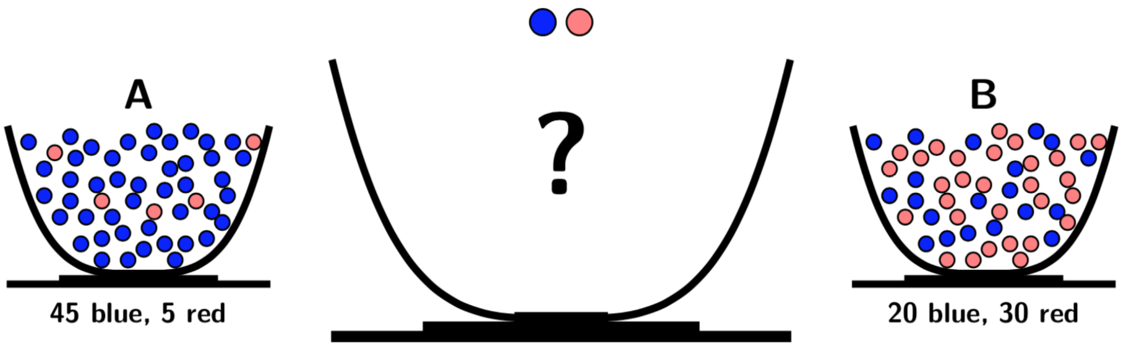

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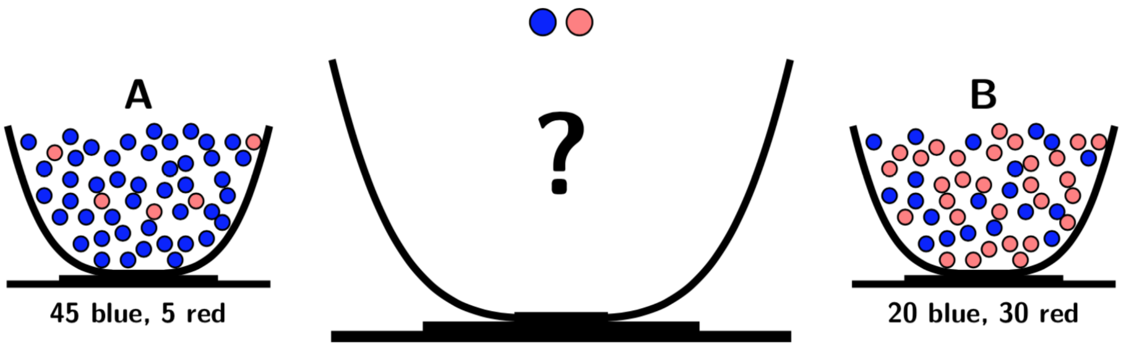

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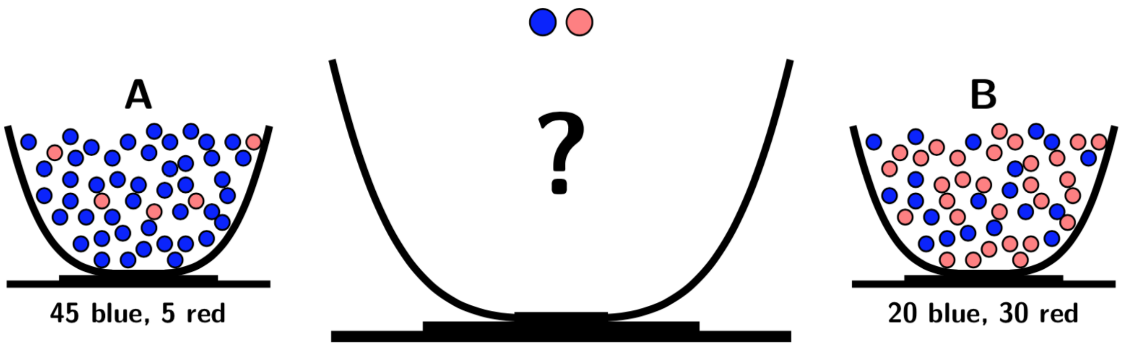

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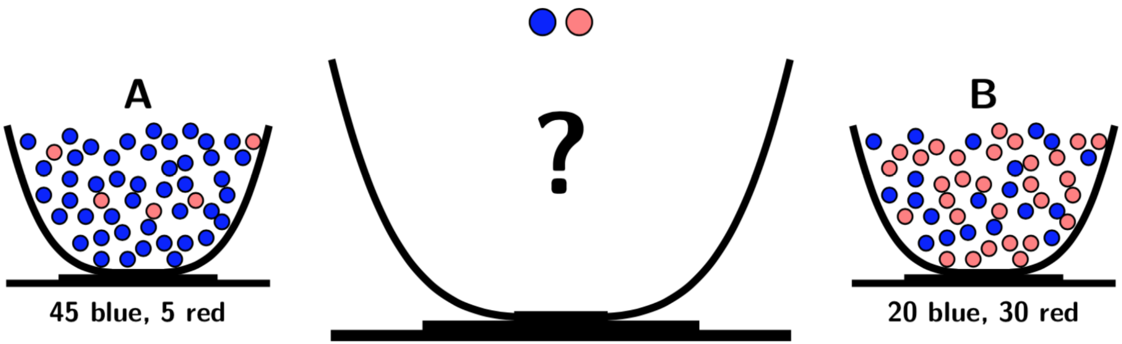

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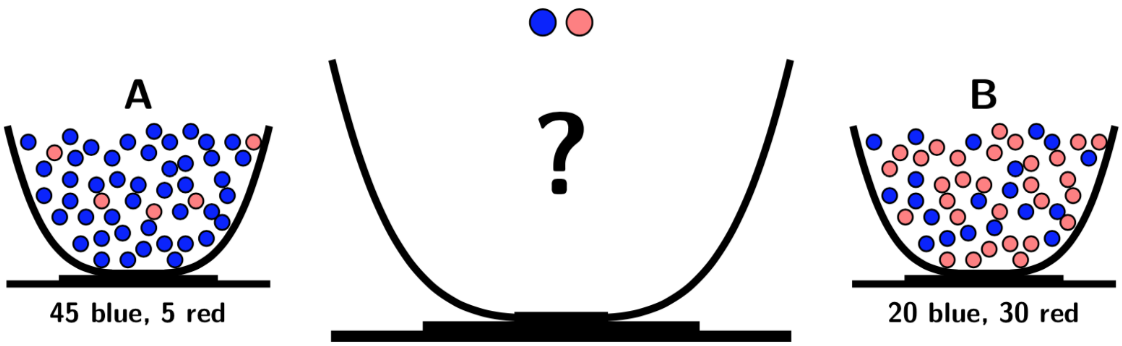

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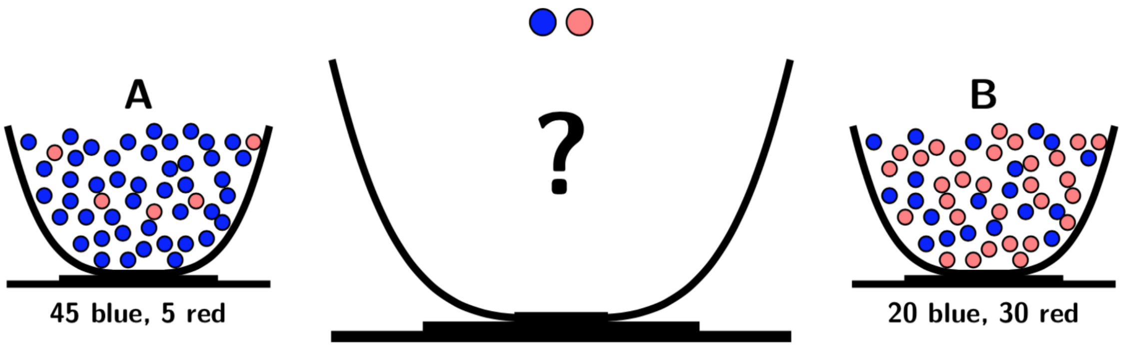

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- $(0.5)(0.9) = 0.45$
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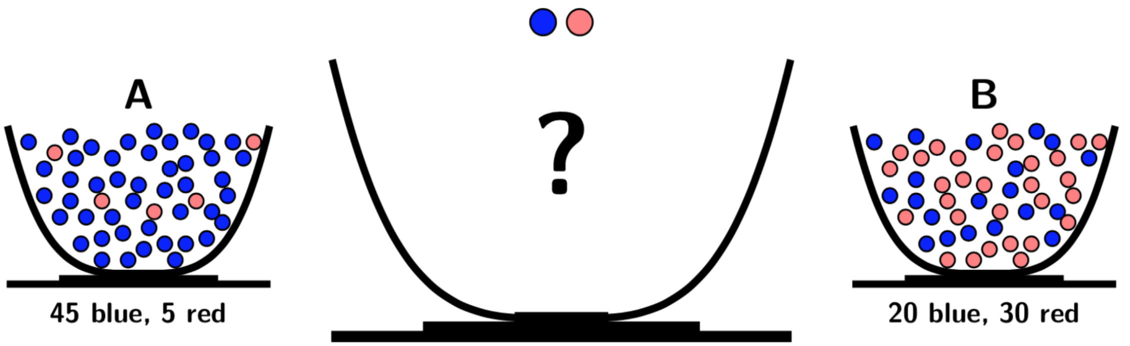

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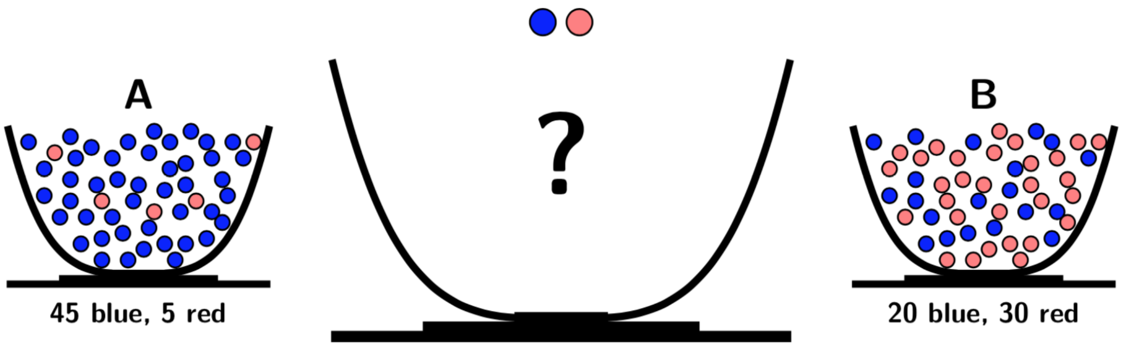

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- $p(A \cap \text{blue}) + p(B \cap \text{blue})$
- $(0.5)(0.9) + (0.5)(0.4) = 0.45 + 0.20 = 0.65$

Find $p(A \mid \text{blue})$

$$p(A \mid \text{blue}) = \frac{p(A \cap \text{blue})}{p(\text{blue})}$$

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This is **inverse conditional probability**: how we find $p(A \mid \text{blue})$ by starting with $p(\text{blue} \mid A)$.

We just did Bayes' theorem

Bayes' Theorem

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Or, more generally

$$p(x \mid y) = \frac{p(y \mid x)p(x)}{p(y \mid x)p(x) + p(y \mid x^c)p(x^c)}$$

A common Bayes example

A rare disease occurs in .01% of the population. We have a test for it, but it isn't perfect. 98% of individuals with the condition will test positive (2% false negative). 97% of those without the condition test negative (3% false positive).

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What's the probability that you have the disease?

- **Prior probability:** .01% you have the disease
- What is the **updated (posterior) probability** that you have the disease, *given that you test positive*

Applying Bayes

$$\text{Posterior probability} = \frac{p(\text{data} \mid \text{prior}) \times \text{prior}}{p(\text{data})}$$

$$p(\text{disease}|+) = \frac{p(+ \cap \text{ disease})}{p(+)}$$

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This is called *Bayesian updating*

Why Bayesian statistics is hard

Take a look at the denominator of Bayes' theorem

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Moreover, our results are *distributions*, not just single estimates.

A continuous example

We think that the probability of a "heads" on a coin is most likely 0.5, but we aren't certain about that. We flip the coin 12 times and find 10 heads. What is our revised belief?

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2. The parameter is unknown, and our information about it will always be imperfect. The information we obtain (*conditioning on* our model, on our data) can only approximate a *distribution* of possible values that are more or less plausible.

The two statistical genders

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"Frequentism"

- Statistical properties come from repeated sampling assumptions
- There exists a true parameter, which we estimate
- We can calculate probability that our data were created by different assumed parameter values
- Low probability of data can be used to reject parameter values
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"Bayesianism"

- Statistical properties come from *posterior distribution*
- Parameters are "random" (not fixed), only approximated with a distribution
- We have prior notions about plausible parameter values
- We can estimate the likelihood of data at different prior values
- Data updates our prior to form posterior beliefs
- Focus is on the probability of the *parameter*, updating a prior with data

Looking ahead

Methods courses

If you want to understand statistical work in political science, you should do:

- 812, 813, MLE
- Empirical methods (817)

Formal theory courses:

- 835 (game theory)
- Formal models of domestic (836) and international (837) politics

Advanced methods courses include

- Multilevel modeling, Time series, Panel data, Bayesian analysis, Experimental methods, Event history

Courses outside the department:

- Ag econ: applied regression, choice models
- Sociology: causal inference, survey methods
- Statistics: networks, machine learning

Methods pathways

Take the foundations courses no matter what

First field: "I want to study *how to study* politics". You still need a substantive interest

Second field: "I want to teach and research about/use new methods," not just, "I can do statistics okay"

Minor: 3 courses (see reqs)

Advice for methods courses

Take as many as you feasibly can.

Don't delay MLE.

Even if you a qualitative researcher, the *epistemological* lessons of large-N analysis are valuable.

If you're going to *read* empirical social science, you should take empirical social science courses.

Pick something you like and get good at it

- Time series, Bayes, text as data, causal inference, experiments

Do replication projects

Advice for methods in the *discipline*

Learn an unfamiliar method from a different field/subfield and apply it to your interests

Take the open science and the "replication crisis" seriously

Take math seriously (it helps you ride the learning curve)

Be a plain text social scientist (take your computer seriously)

Learn $\LaTeX$, learn R, learn `git`. Stata works but we may be way past the inflection point

If you might leave academia for data science, consider Python and machine learning