

Level: Bachelor	Semester:Spring	Year : 2019
Programme:BE		Full Marks: 100
Course: Engineering Mathematics II		Pass Marks: 45
		Time : 3hrs.

*The figures in the margin indicate full marks.*

*Attempt all the questions.*

1. a) Find the equation of the plane through the line  $2x+3y-5z-4=0=3x-4y+5z-6$ , parallel to the z-axis.  
b) Find the equation of the sphere through the circle  $x^2 + y^2 + z^2 = 4$ ,  $z = 0$  and is cut by the plane  $x + 2y + 3z = 0$  in a circle of radius 3.
2. a) State and prove Euler's Theorem on homogenous function of two independent variable of degree n. if  $u = \sin^{-1} \left( \frac{x^3+y^3+z^3}{ax+by+cz} \right)$  prove that  $x \frac{du}{dx} + y \frac{du}{dy} + z \frac{du}{dz} = 2 \tan u$   
b) A rectangular box open at the top, is to have a volume of 32C.C. Find the dimensions of the box requiring least material for its construction.
3. a) Evaluate the integral  $\int_0^{4a} \int_{y^2/4a}^y \frac{x^2 - y^2}{x^2 + y^2} dx dy$  changing into polar co-ordinates.  
b) Find the volume of the solid cut from the first octant by the surface  $Z = 4-x^2-y$ .
4. a) Solve the equation  $\frac{dy}{dx} + \frac{y \log y}{x} = \frac{y(\log y)^2}{x^2}$   
b) Solve the initial value problem.  $y'' + y' - 2y = 14 + 2x - 2x^2$ ,  $y(0) = 0$ ,  $y'(0) = 0$ .
5. a) Solve  $y'' - 4xy' + (4x^2 - 2)y = 0$  by using power series method.

OR

Define Bessel equation and Bessel function of order  $n$  Also show that

$$\frac{d}{dx}[X^{-n}j_n(X)] = -X^{-n}j_{n+1}(X)$$

- b) Find the general solution of  
 $y'' - 4y' + 5y = e^{2x} \operatorname{cosec} x$  by using method of variation of parameter.
6. a) Define Laplace transform. Evaluate  
 i)  $L(t^2 \sin wt)$  ii)  $L^{-1}\left(\frac{1}{s^2(s^2 + w^2)}\right)$
- b) Using Laplace Transform solve the initial value problem  
 $y'' - 3y' + 2y = 4t + e^{3t}$        $y(0) = y'(0) = -1$ .
7. Attempt all the questions.
- a) Find the equation of plane which through (1,1,1) and parallel to the plane  $3x - 4y + 5z = 0$ .
- b) Find Laplace transform of  $e^{-2t} \cos t$ .
- c) Solve :  $\frac{dy}{dx} + y \cot x = e^{\cos x}$ ,
- d) If  $V = \sqrt{x^2 + y^2 + z^2}$  Show that  $V_{xx} + V_{yy} + V_{zz} = \frac{2}{V}$