Level: Bachelor　　　　Semester: Fall　　　　Year　　: 2018
Programme: BE　　　　　　　　　　　　Full Marks: 100
Course: Engineering Mathematics II　　　Pass Marks: 45
　　　　　　　　　　　　　　　　　　　Time　　: 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Find the image of the point (1,2,3) in the plane 2x-y+z+3=0.　　8

    OR

    Find the shortest distance between the lines
    ax + by + cz +d = 0 = a_1x +b_1y + c_1z + d, and z-axis.

   b) Find the equation of sphere, its centre and radius which has the circle　　7
   $x^2 + y^2 + z^2 = 9$, x − 2y + 2z = 5 as a great circle.

2. a) Write down the criteria for a function f(x, y) of two variables x and y　　8
   to have maximum or minimum values at a point.
   If the sum of the dimension of a rectangular swimming pool is given.
   Prove that the amount of water in the pool is maximum when it is cube.

   b) State and prove Euler's theorem for homogeneous function of two　　7
   variables. If $v = \log\left(\dfrac{x^2 + y^2}{x + y}\right)$ Prove that $x\dfrac{\partial v}{\partial x} + y\dfrac{\partial v}{\partial y} = 1.$

3.
   a) Evaluate $\displaystyle\int_0^4\int_y^4 \dfrac{xdxdy}{x^2 + y^2}$ by changing the order of integration.　　8

   b) Find the volume of the solid whose base is the region in the xy-plane　　7
   that is bounded by the parabola $y = 4-x^2$ and line y=3x while the top of
   the solid is bounded by the plane z = x+4.

4. a) Solve $\dfrac{dy}{dx} + \dfrac{1}{x}\sin 2y = x^3 \cos^2 y$　　7

   b) Solve. y"+9y=sec3x; (method of variation of parameter)　　8

5. a) Solve the differential equation : $(1+x^2)y''+xy'-y=0$, by using　　7
   power series methods.

    OR

Define Bessel Equation and Bessel function of order n. Also show that
$\dfrac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x).$

   b) Solve the initial value problem: $y''-4y'+3y=10e^{-2x}$ where y(0)= 1　　8
   and $y'(0)=3$.

6. a) Define convolution theorem for inverse Laplace Transform and use it　　8
   to find $L^{-1}\left(\dfrac{s}{(s^2 + w^2)^2}\right)$

   b) Using Laplace Transform solve the initial value problem　　7
   $y'' - 4y' +3y = e^{-t}$　$y(0) = y'(0) = 1$

7. Write short notes :　　　　　　　　　　　　　　　　2.5×4
   a) Find the equation of the plane through (−1, 1,−1) and (6, 2, 1) and
   normal to the plane 2x+y+z = 5.
   b) Solve e^{x-y} dx+e^{y-x}dy=0.
   c) If f(x,y,z) $= \dfrac{x}{y}+\dfrac{y}{z}+\dfrac{z}{x}$, then show that $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y} + z\dfrac{\partial f}{\partial z} = 0.$
   d) Find f(t) if $F(s)=\dfrac{1}{s^2 +36}$