POKHARA UNIVERSITY

Level: Bachelor   Semester: Spring   Year : 2017
Programme: BE   Full Marks: 100
Course: Engineering Mathematics II   Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1\. a) Prove that the lines $\frac{x+1}{1}=\frac{y+1}{2}=\frac{z+1}{3}$; $x+2y+3z-8=0=2x+3y+4z-11$ are coplanar and find the point of contact and equation of plane containing them. 8

b) Prove that the circles 7

$x^2+y^2+z^2-2x+3y+4z-5=0$, $5y+6z+1=0$ and

$x^2+y+z^2-3x-4y+5z-6=0$, $x+2y-7z=0$

lie on the same sphere and find its equation.

2\. a) State and prove Eulers theorem for homogeneous function of two variables in x and y of degree n. If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\tan u$. 7

b) What are the criteria for a function of two independent variables to have extreme values? Find the minimum value of $f=x^2+y^2+z^2$ such that $x+y+z=1$ and $xyz=1$. 8

3\. a) Draw the region of integration of $\int_0^\pi\int_x^\pi \frac{\sin y}{y}\,dy\,dx$, and find its value interchanging the order of integration. 7

OR

Find the volume of the solid in the first octant bounded by the coordinate planes, the cylinder $x^2+y^2=4$ and the plane $z+y=3$.J

b) Evaluate $\int_0^{\frac{a}{\sqrt{2}}}\int_y^{\sqrt{a^2-y^2}} \log(x^2+y^2)\,dx\,dy, (a>0)$ by changing into polar integral. 8

4\. a) Define Bernoulli's differential equation and solve 7

$$\frac{dy}{dx}-\frac{\tan y}{1+x}=(1+x)e^x\sec y$$

b) Solve $\frac{d^2y}{dx^2}-\frac{dy}{dx}+4y=\frac{e^x}{x}$ by method of variation of parameter. 8

5\. a) Solve by power series method 7

$(1-x)y^I=y$

OR

Define Bessel's differential equation and find the Bessel function of first kind.

b) Solve the following initial value problem. 8

$$y''+2y'+y=e^{-x},\ y(0)=-1,\ y'(0)=1$$

6\. a) Define Laplace transform of a function. Using Laplace transform prove the following: 8

i. $L(\sin at\ \cos at)=\frac{a(s^2-2a^2)}{s^4+4a^4}$

ii. $L^{-1}\left\{\frac{1}{s^2(s^2+w^2)}\right\}=\frac{1}{w^2}\left(t-\frac{\sin wt}{w}\right)$

b) Using Laplace Transform solve the initial value problem 7

$$y''+4y'+3y=e^{-t},\ y(0)=y'(0)=1$$

7\. Attempt all questions 4×2.5

a) Find the equation of the line through (1,3,5) and (2,3,4) perpendicular to the plane $3x-4y+5z=0$.

b) Verify Euler's theorem for $f(x, y)=x^3+y^3+z^3$

c) Solve the differential equation

$(1+x)y\,dx+(1+y)x\,dy=0$

d) Find the Laplace transform of te^{2t}