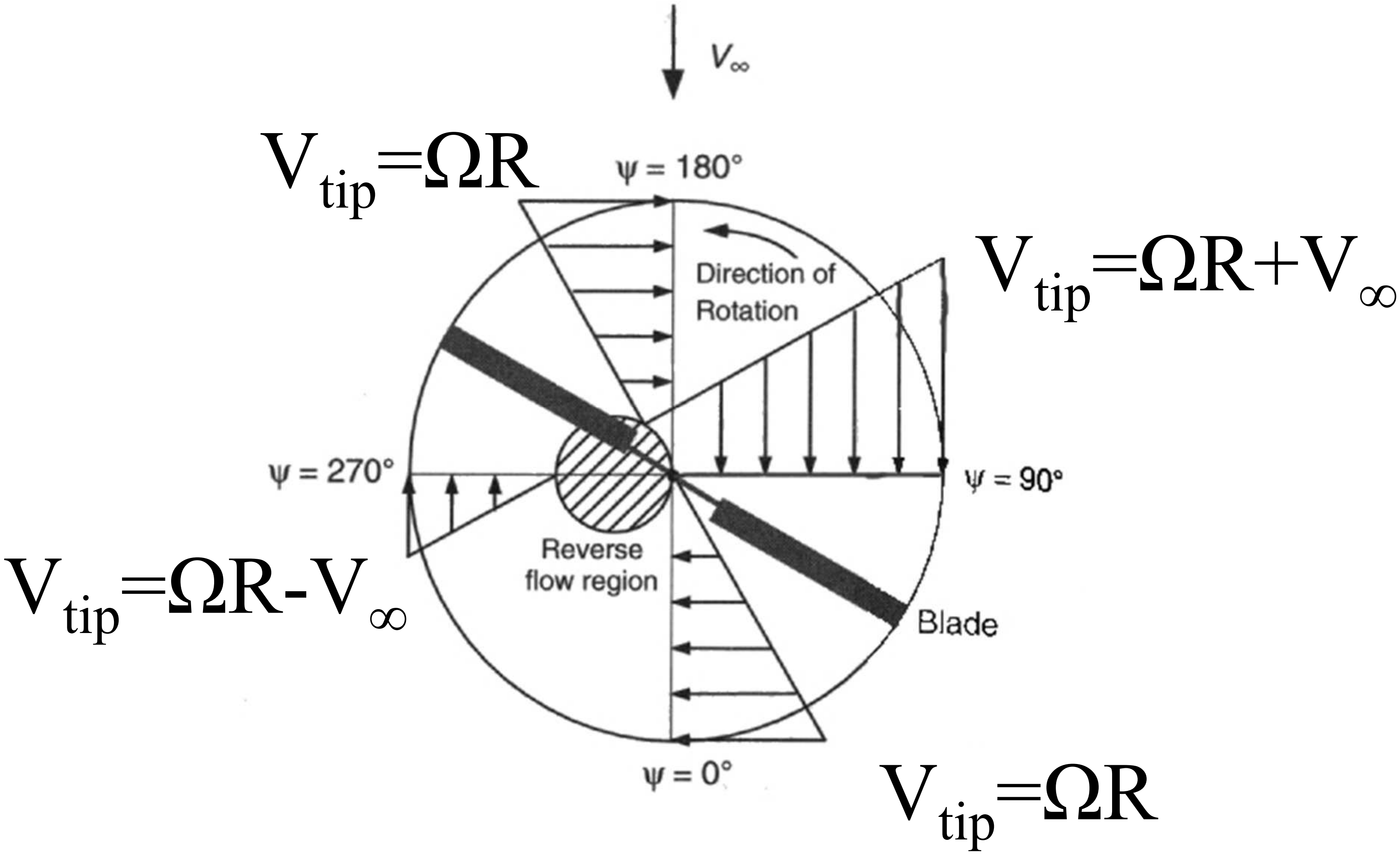
Momentum Theory in Forward Flight

•In helicopter forward flight the rotor moves through the air with an edgewise velocity component that is parallel to the rotor plane

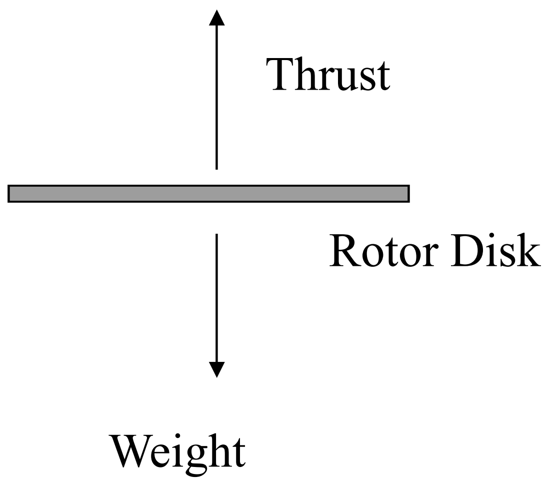
•Since the helicopter rotor has to produced both the lifting force and the propulsion force.

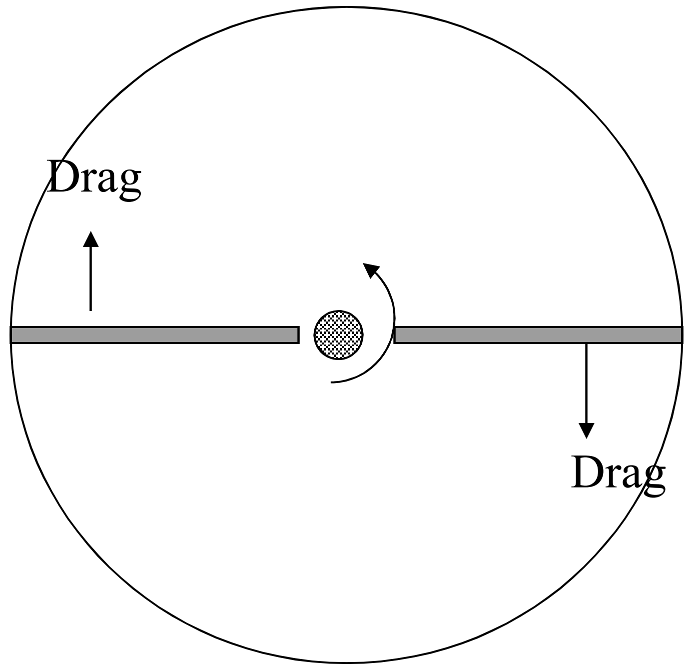
•The rotor must be tilted forward at an AOA relatively to the oncoming flow.

The Dynamic Pressure varies Radially and Azimuthally



Force Balance in Hover





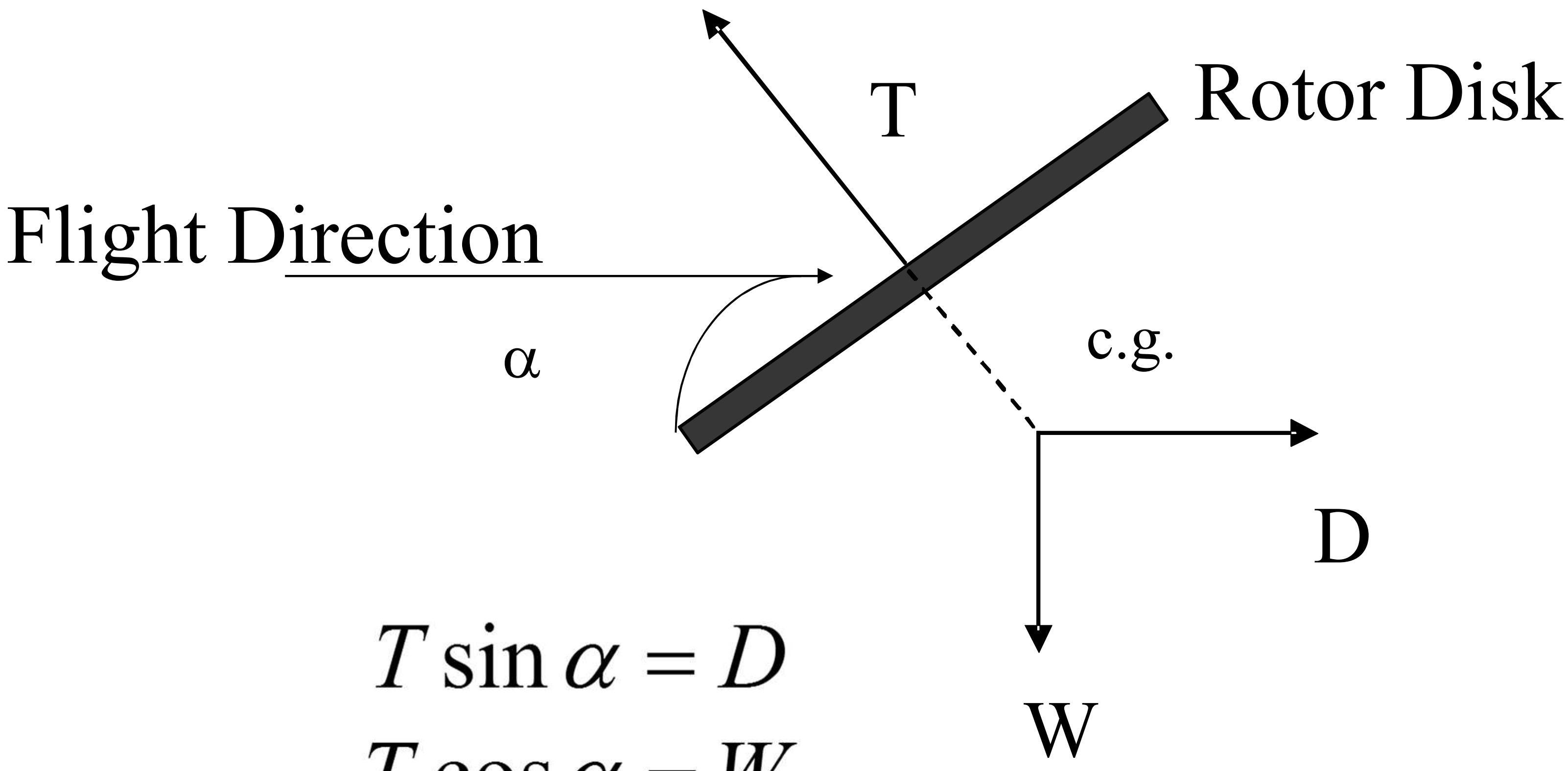
- In hover, T= W
- The drag forces on the individual blades cancel each other out, when summed up.

Glauert's flow model

- To start this effort, we will need a very simple inflow model.
- A model proposed by Glauert (1926) is used.
- This model is phenomenological, not mathematically well founded.
- It gives reasonable estimates of inflow velocity at the rotor disk, and is a good starting point.
- It also gives the correct results for an elliptically loaded wing.

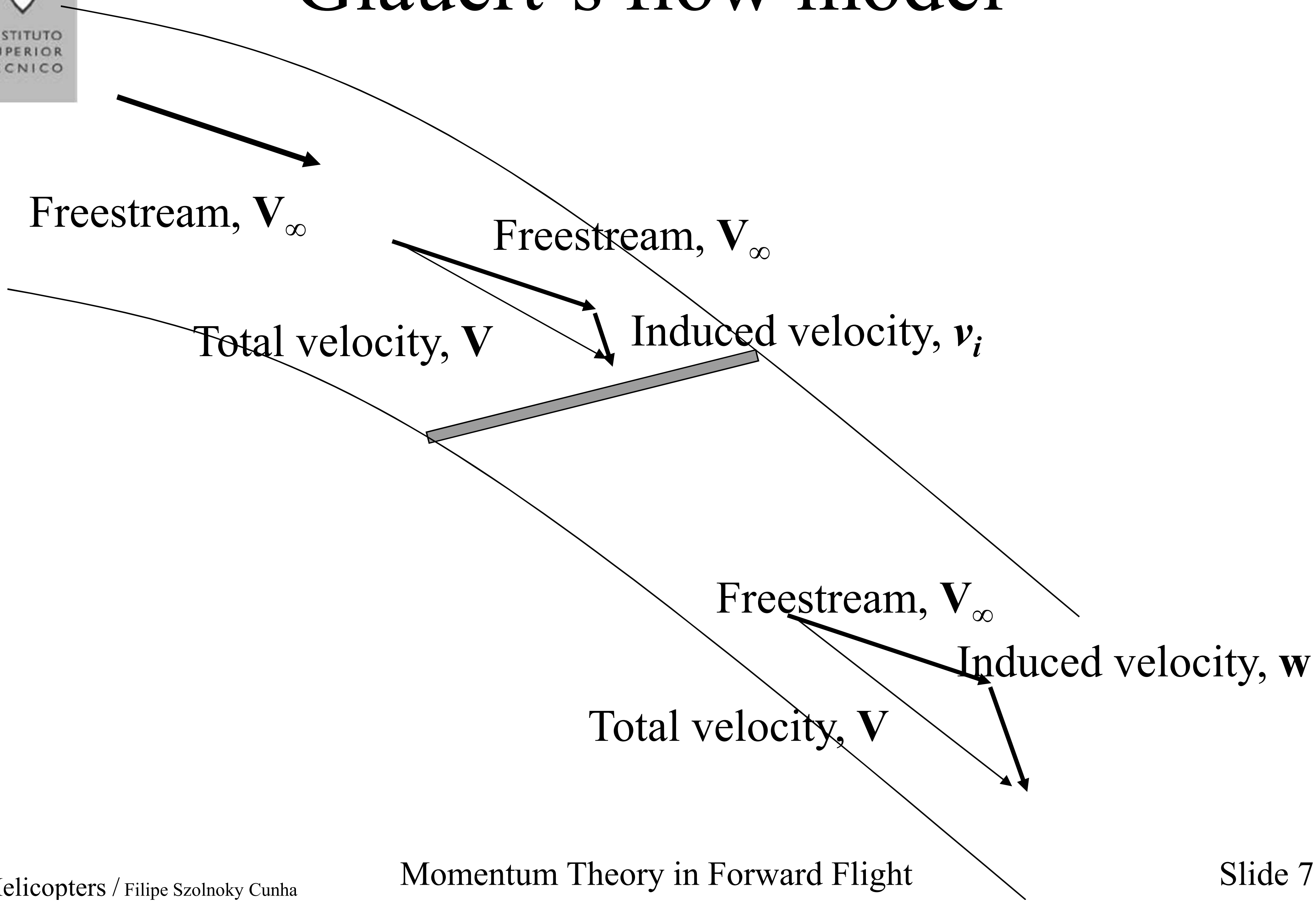
Force Balance in Forward Flight

Force Balance in Forward Flight

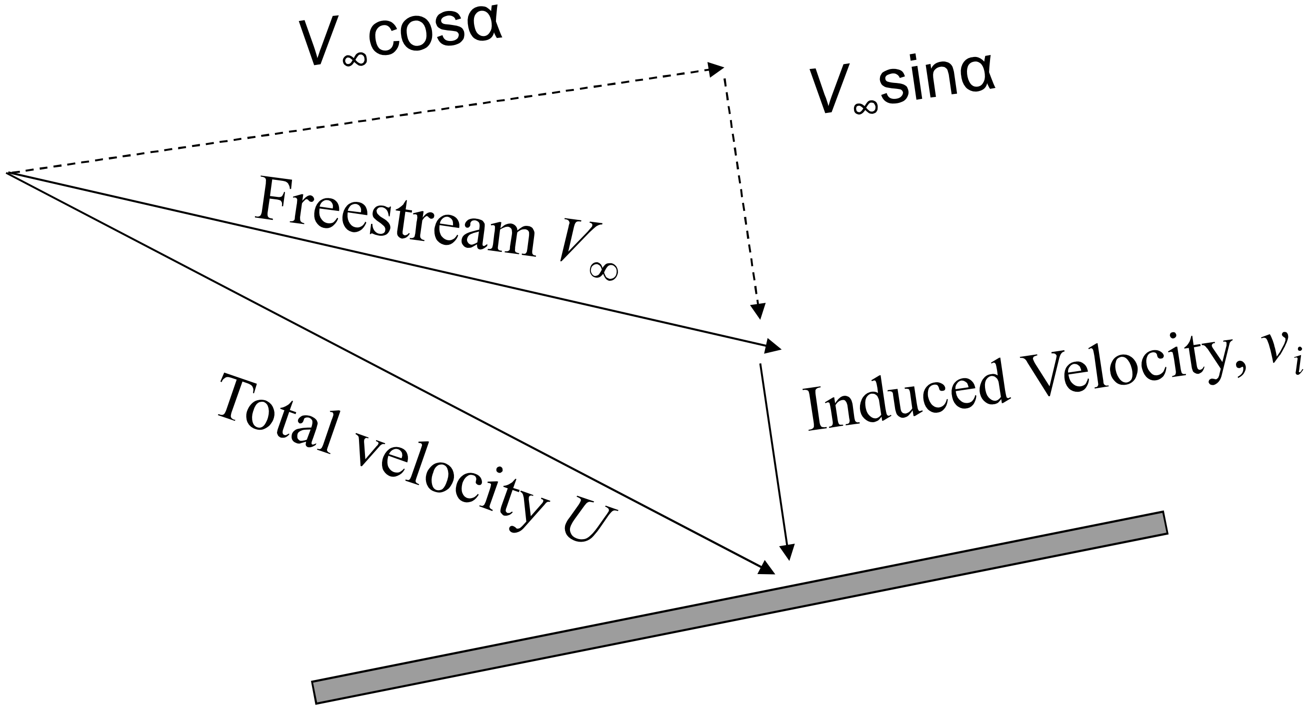


$$T\sin\alpha = D$$
$$T\cos\alpha = W$$

Glauert’s flow model



Total Velocity at the Rotor Disk



$$U = \sqrt{\left(V_\infty \cos\alpha\right)^2 + \left(V_\infty \sin\alpha + v_i\right)^2}$$

Conservation laws

- Conservation of momentum in the direction normal to the disk:

$$T = \dot{m}\left(V_\infty \sin\alpha + w\right) - \dot{m}\left(V_\infty \sin\alpha\right) = \dot{m}w$$

- Conservation of energy in the same direction

$$P = T\left(V_\infty \sin\alpha + v_i\right) =$$

$$= \tfrac{1}{2}\dot{m}\left(V_\infty \sin\alpha + w\right)^2 - \tfrac{1}{2}\dot{m}\left(V_\infty \sin\alpha\right)^2 =$$

$$= \tfrac{1}{2}\dot{m}\left(2V_\infty w \sin\alpha + w^2\right)$$

Conservation laws

- From the two previous equations we can write:

$$2wv_i + 2V_\infty w \sin\alpha = 2V_\infty w \sin\alpha + w^2$$

- And reach the conclusion that $w=2v_i$, the same result was in the previous cases.
- Knowing that the mass flow at the disk is ρAU:

$$T = 2\dot{m}v_i = 2\rho A v_i \sqrt{V_\infty^2 + 2V_\infty v_i \sin\alpha + v_i^2}$$

- In high speed forward flight $V_\infty >> v_i$ so:

$$T = 2\rho A v_i V_\infty$$

Induce velocity

- We know for the hover case that:

$$v_h^2 = \frac{T}{2\rho A}$$

- Then from the previous equation:

$$v_i = \frac{v_h^{\ 2}}{\sqrt{\left(V_\infty \cos\alpha\right)^2 + \left(V_\infty \sin\alpha + v_i\right)^2}}$$

Non dimensional forms

- The non-dimensional form using the tip speed ΩR:

$$\begin{cases} \mu = \dfrac{V_\infty \cos\alpha}{\Omega R} \\ \lambda = \dfrac{V_\infty \sin\alpha + v_i}{\Omega R} = \dfrac{V_\infty \sin\alpha}{\Omega R} + \dfrac{v_i}{\Omega R} = \mu \tan\alpha + \lambda_i \end{cases}$$

- So that the non-dimensional induced velocity equation can be written as:

$$\lambda_i = \frac{\lambda_h^2}{\sqrt{\mu^2 + \lambda^2}}$$

Non dimensional forms

- Since we already know that: $\lambda_h = \sqrt{\dfrac{C_T}{2}}$
- We can write:

$$\lambda_i = \frac{\lambda_h^2}{\sqrt{\mu^2 + \lambda^2}} = \frac{C_T}{2\sqrt{\mu^2 + \lambda^2}} \Rightarrow$$

$$\Rightarrow \lambda = \mu \tan\alpha + \frac{C_T}{2\sqrt{\mu^2 + \lambda^2}}$$

- Which requires a numerical solution

Approximate Form at High Speed Forward Flight

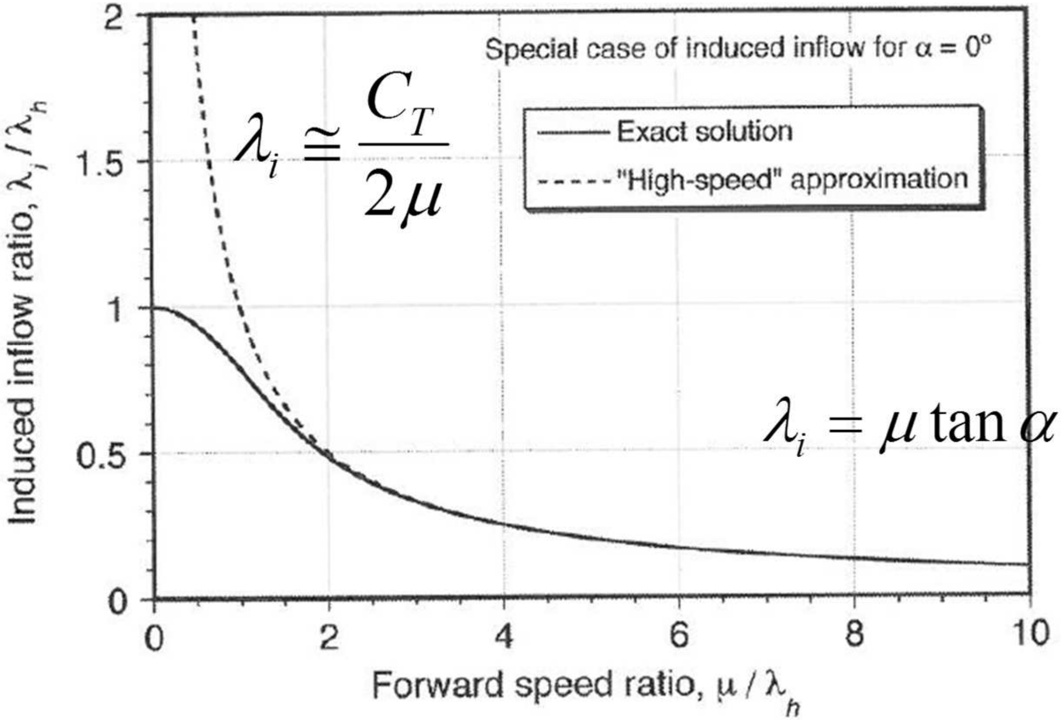
- If the advance ratio μ is higher than *0.2* and α is small, μ far exceeds the inflow ratio λ:

$$\lambda = \mu \tan\alpha + \frac{C_T}{2\sqrt{\mu^2 + \lambda^2}} \Rightarrow$$

$$\Rightarrow (\lambda - \mu \tan\alpha) 2\sqrt{\mu^2 + \lambda^2} = C_T \Rightarrow$$

$$C_T = 2\lambda\mu$$

Variation of Non-Dimensional Inflow with Advance Ratio

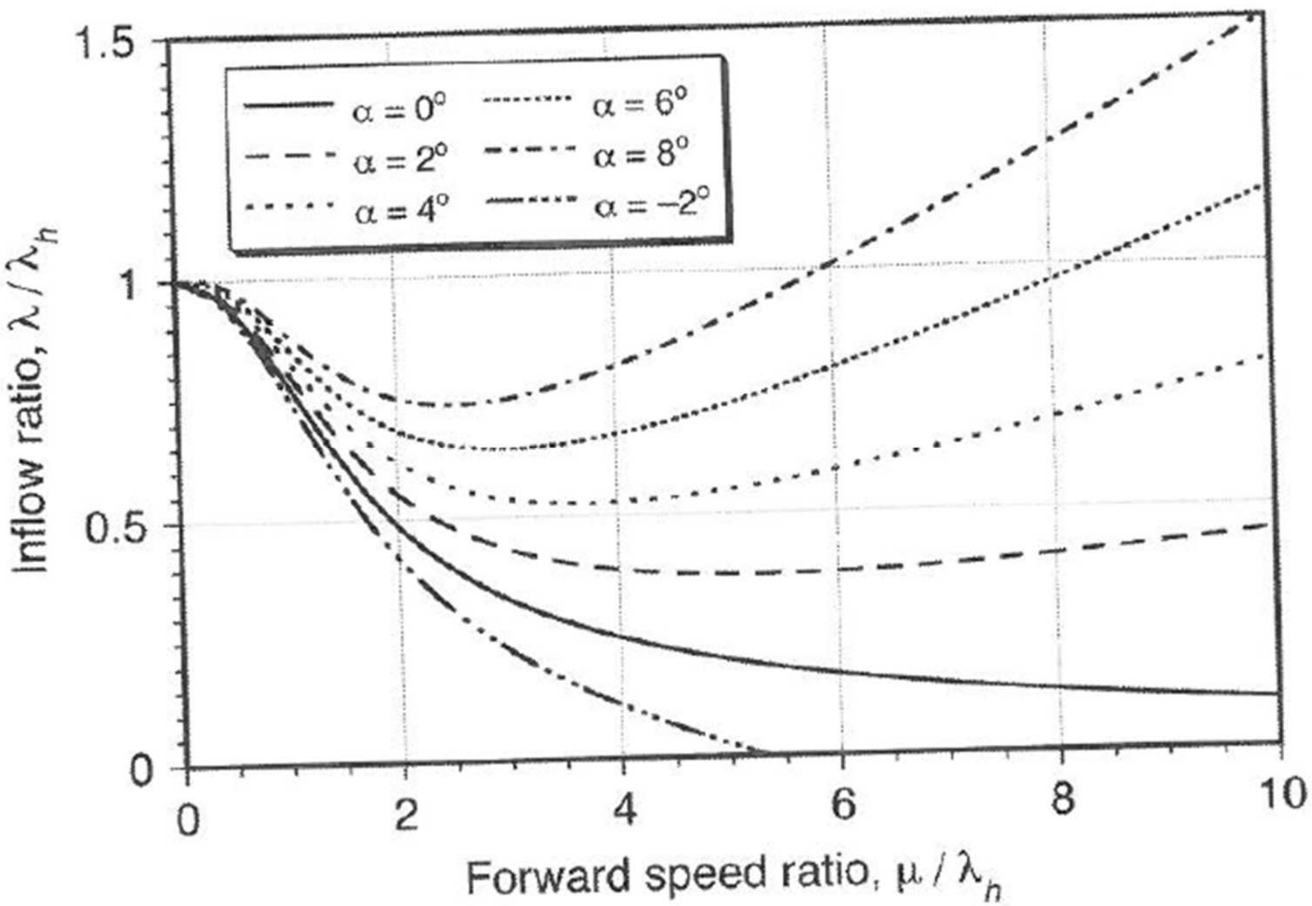


$$\lambda_i \cong \frac{C_T}{2\mu}$$

$$\lambda_i = \mu \tan\alpha + \frac{C_T}{2\sqrt{\mu^2 + \lambda_i^2}}$$

- Notice that inflow velocity rapidly decreases with advance ratio

Variation of Non-Dimensional Inflow with Advance Ratio



Power Consumption in Forward Flight

- The ideal power from Glauert's theory is

$$P_{ideal} = T\left(V_\infty \sin\alpha + v_i\right)$$

- For the actual power we have to take into account the blade profile power

$$P = T\left(V_\infty \sin\alpha + v_i\right) + P_0$$

- From the equilibrium of forces *Tsinα=D* so:

$$P = Tv_i + DV_\infty + P_0$$

- Where Tv_i is the induce power and DV_∞ is the Parasitic power

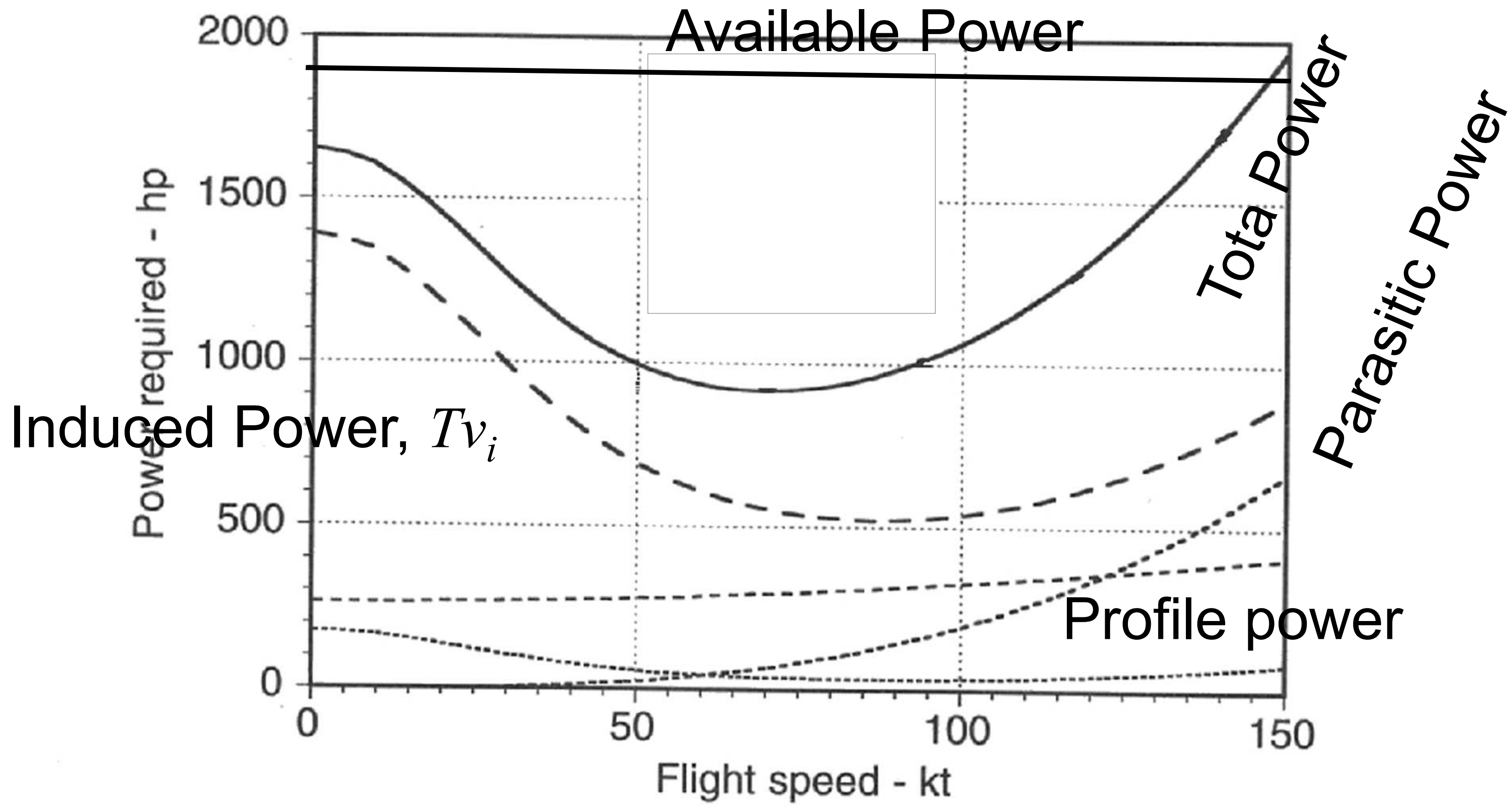
Power Consumption in Forward Flight

- The induce power decrease with the advance ratio μ
- The Parasite power can be calculated:

$$DV_\infty = \left[\tfrac{1}{2}\rho V_\infty^2 C_D S\right] V_\infty = \tfrac{1}{2}\rho V_\infty^3 C_D S$$

- The Parasite power increases with the cube of the forward velocity (or advance ratio μ)

Power in Forward Flight



Power Coefficient

$$C_{P_{\text{Uncorrected}}} = C_T\lambda_i + \frac{1}{2}\frac{f}{A}\mu^3 + \frac{\sigma C_{d0}}{8}\left[1+3\mu^2\right]$$

Induced power

Parasite Power

Profile Power

$$C_{P_{\text{Corrected}}} = \kappa C_T\lambda_i + \frac{1}{2}\frac{f}{A}\mu^3 + \frac{\sigma C_{d0}}{8}\left[1+4.6\mu^2\right]$$

1.15

C_D is the vehicle parasite drag coefficient and S the reference area. Because there is no agreement on a common reference area it is customary to supply the product $C_D S=f$ equivalent flat plate area