It's the Network

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Abstract

Very High Frequency (VHF) repeaters can be used to create a relatively low-maintenance radio network. The goal of this paper to determine the minimum number of repeaters needed to allow 1,000 or 10,000 people to communicate using VHF radios on a region with a 40 mile radius. To tailor our solution to different topographic terrains, we design a method for dealing with such a situation on a mountainous region. We look at strategies to allocate bandwidth to users, maximizing the usefulness of the network. In order to boost bandwidth efficiency, we propose using Single-sideband modulation (SSB) to maximize the number of available channels. Using a greedy heuristic, we can allocate users to channels 3 times as efficiently as random placement. In addition, we design a spectrum isolation scheme that eliminates interference between adjacent repeaters. Using this method and 40 foot repeater towers, we propose a network of 32 repeaters to facilitate communication with interrupted communication expected 6 percent of the time for 1,000 people. However, it does not scale as well to 10,000 people. Schemes for wireless repeater networks are created that would allow our system to be implemented in areas with minimal infrastructure. To handle situations with mountainous terrain, we developed two models, the partition method and the local maximum method. While both models placed repeaters on certain peaks, we found that the local maximum method was over four times better than the partition method.

Contents

1 Introduction

Repeaters are used in VHF transmissions to propagate signals over long distances. Several characteristics of VHF signals make repeater networks an attractive option for connecting users within range of the antenna network. First, the line-of-sight transmission and reception of VHF signals means that coverage range is a function of antenna height and topography. Secondly, technology such as CTCSS and in-band signaling allows us to coordinate repeaters in such a way that any user within range of one repeater can access any user within range of another repeater. The wavelength of this portion of the spectrum is approximately 2 meters, hence it is referred to as the 2-meter band.

The frequency spectrum we are working with is 145 MHz to 148 MHz. This portion of the 2-meter band is primarily used by amateur radio operators, who in most areas use the radio for recreational or emergency communication purposes. Hence, amateur radio, or "ham", communities provide us with established protocols that we will take into consideration; however, in the interest of pursuing an optimal repeater configuration, we will liberally break from convention when it is logical to do so.

1.1 Assumptions

To simplify our model we generally assume that our radio system works in a mathematically perfect way. Our specific assumptions are:

- Radio signals will go on forever unless they hit something (without considering power)
- The Earth is a perfect sphere and only its curvature blocks signals (we will relax this later)
- VHF waves do not reflect off surfaces
- All radio users are 5 feet tall (ie their antennae have a height of 5 feet)

1.2 Definitions

- Simplex mode: parties in communication with one another transmit and receive on the same channel (and tone).
- Full-duplex mode: parties in communication with one another transmit on one channel (and tone) and receive on another channel, allowing for simultaneous transmission and receiving.
- Co-channel users: the set of all parties operating on the same channel.
- CTCSS: Continuous Tone-Coded Squelch System allows co-channel users to selectively receive audio that is transmitted with a certain subaudible tone.
- Range: the maximum distance between two parties in which communication is possible.

1.3 Problem Formulation

How can we implement a repeater network, route traffic, and allocate bandwidth to accommodate as many users as possible?

We try to provide the most consistent coverage to the population under the constraint that our network be reasonable in infrastructure. Rather than focus on hardware, repeaters, materials, and advanced radio spectrum issues, we approach this by first optimizing our usage of bandwidth for the communications. We then analyze techniques to distribute the traffic efficiently. Finally, we design techniques to adapt to hilly environments.

2 Range of the VHF Signal

Based on our assumptions, VHF signals travel in a straight line forever unless they come into contact with an object. Since we are modeling radio transmissions on a perfect sphere, the signal will be blocked by contact with the Earth because of the sphere's curvature. We want to determine the maximum distance that two people (this can also be the distance between objects) can be apart and still contact each other via VHF radio. Given the heights of the two people, say h_1 and h_2, and the radius of the Earth, R, we can determine the maximum distance they can be apart using the Pythagorean Theorem.

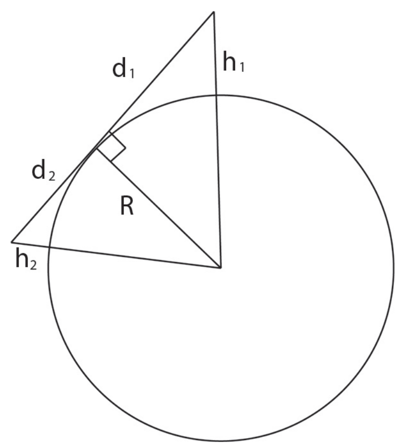


Figure 1: The segment that connects the two objects is tangent to the sphere

As we can see:

$$\mathsf{range}(h_1, h_2) = d_1 + d_2 = \sqrt{(R+h_1)^2 - R^2} + \sqrt{(R+h_2)^2 - R^2}$$

3 Approximating the Number of Repeaters Needed

3.1 Minimizing the Number of Repeaters: One Repeater Serves All

The maximum range of two users who are 5 feet tall is 5.48 miles. Their communication range can be vastly improved using a repeater.

What happens if we have one repeater antenna that services the 40 mile radius geographic area? Using the range formula, we find that the repeater needs to be mounted atop a 925 foot tall tower. This is not structurally impractical (Tokyo Tower stands at $1,091$ feet), and would make sense in temperate climates. Browsing Ham Radio clubs around the nation that operate on the 2 meter band, we see that it is common practice to avoid the boundary frequencies 145.000 and 148.000 (because signal fluctuations below and above the intended frequency can cause loss of information). following the convention of these amateur radio enthusiasts, we begin by allocating 145.45-145.75 MHz as follows:

Frequency Spectrum	Channel
145.15-145.45 MHz	Repeater Channel 1 (Output)
145.45-145.75 MHz	Repeater Channel 2 (Output)
145.75-146.05 MHz	Repeater Channel 1 (Input)
146.05-146.35 MHz	Repeater Channel 2 (Input)
146.35-146.65 MHz	Repeater Channel 3 (Output)
146.65-146.95 MHz	Repeater Channel 4 (Output)
146.95-147.25 MHz	Repeater Channel 3 (Input)
147.25- 147.55 MHz	Repeater Channel 4 (Input)
147.55-147.85 MHz	Simplex Channel

If a party wanted his or her message to be transmitted by the repeater at, say, frequency 145.60 MHz (Repeater Channel 2), then he or she would transmit 600 kHz higher at 146.20 MHz. Hence, there are effectively 5 channels that can be used simultaneously with no interference. Within each channel, users can filter for signals with one of 54 sub-audible squelch tones, allowing users 270 private-line channels to choose from.

This is not to say that we can accommodate 270 simultaneous users with zero interference. CTCSS gives users a way to selectively receive signals containing a certain tone, however, signals transmitted with a different tone at the same frequency will cause interference [7]. So the question of how many simultaneous users this system can support comes down to the level of interference tolerance, measured by *co-channel transmit-time* (defined above). Supposing n active users and uniform traffic throughout the region, the transmit-time is $\frac{60}{n}$ seconds per minute. For $n = 1000$, and $n = 10,000$, each user has an average of 0.06 and 0.006 seconds per minute to transmit their message. Clearly, this is not an optimal solution. And so begins our journey!

3.2 Simple Models for Determining an Upperbound and a Lowerbound

Using the equation for the range of the signal described above, we constructed a simple greedy algorithm to determine the number of repeaters we need to cover an area with a 40 mile radius so that everyone can communicate with each other using VHF radios. For this particular algorithm we assumed that the repeater towers were 40 feet tall and that people were 5 feet tall.

This algorithm takes two of the inputed n people (who have been placed randomly on the disk) and checks to see if there is a pathway for the VHF signal to get from one person to the other. We begin by choosing one person to be the starting point and the other to be the target and check if they are within range of each other. If they are within range of each other, no extra tower is needed. If the two people are not in range of each other we check to see if there are any repeater towers in the area that are closer to the person whom we are trying to reach (we choose the tower that is closest to the target). When there are no towers within range of the starting person's location that are closer to the target, we add a tower on the shortest line connecting the two people that is just within the range of the person making the call. After we have a tower that is closer to the target, we apply the algorithm again with the new tower as the starting point and the target remaining the same. Once we have run the algorithm on every combination of people, we have the number of repeater towers needed.

We ran this algorithm several times using 1000 people and 10,000 people. The average number of towers that this algorithm gave us was 137.6 towers with a maximum value of 154 towers and a minimum value of 120 towers. These estimates are clearly upper bounds on the number of towers that we need for several reasons. First, it is likely that people are taller than 5 feet. Second, the placement of towers depends on the placement of people and is therefore very unlikely to be optimal. The most important reason this algorithm gives us an upper bound is that because it is greedy, it rejects paths that do not give an immediate improvement even if they were optimal and would lead us to the target.

Now to get a lower bound on the number of repeater towers we do a simple area calculation. The area of the region is approximately 5026.5 miles^2 and the area that a 40 foot tower covers is approximately 188.6 miles^2. Dividing the region's area with the area covered by a tower we get 27 towers needed. This is clearly a lower bound because we cannot cover a larger circle with smaller circles and have no overlap.

Thus, using simple methods we can conclusively say that the number of repeaters we need is between 27 and 154.

4 Channel Allocation

There are many ways that communications can interfere with each other. If two simplex conversations share the same channel but have different CTCSS tones then, if they are in range of each other, whenever someone is talking in each conversation there is interference. If these simplex conversations also share a CTCSS tone then whenever someone talks in one conversation everyone in both conversations hears the message. These collisions are a problem for users that we must fix to run a practical repeater network.

Since we only have a finite number of channels and CTCSS tones an efficient channel allocation scheme is needed to prevent the users from being too interrupted. We assume that users will use simplex frequencies whenever possible and that users want to minimize interruptions in their conversation. Our metric of the allocation quality is the ratio of collisions to attempted calls.

4.1 Greedy Channel Allocation Strategy

Explicitly solving the channel allocation problem is more difficult than we could do over a weekend (we believe the problem is NP-Hard, but we have not proven that). So we propose a greedy heuristic for each conversation of choosing the least crowded channel and then the least crowded CTCSS tone on that channel. We have a central hub that stores the information about channel usage and that does the allocation of channels to individual users.

In the real world this process could be implemented in a server that people could check into before using the radio (probably by an automated process in the radio). Then, when they leave the channel, they check out. This server would have to track the current state of each channel and tone so that the allocation could reflect the current state of the airwaves. Since each allocation only depends on the previous state this could be done in an online system without any major difficulties.

4.1.1 Greedy Channel Allocation Flaws

Our main disadvantage is that if you have two simplex conversations that are far enough apart not to interact, there is no reason not to give them the same channel, but our algorithm could give them different channels. This could cause unnecessary collisions in practice if many non-interfering simplex conversations were allocated first.

4.1.2 Channel Allocation Testing

To test the effectiveness of this allocation strategy we created a numerical simulation that chose pairs of points to be communicating uniformly over a circular region of space that we assumed was served by a single tower. We then fixed our variable parameters (call frequency, number of channels, number of CTCSS tones, number of conversations, etc). The difference between our two simulations was that one used our greedy allocation algorithm and the other chose channels randomly. We then compared the net number of collisions per try for each setup run 100 times (so the placement of people is not a major factor). Our algorithm had a ratio of collisions per try of 0.2267, while random allocation had a ratio of 0.6569. We chose another set of parameters and got 0.1587 for our algorithm and 0.5211 for random allocation. This suggests that our algorithm does a reasonable job of allocating channels to users.

5 Tower Capacity

Before we can analyze how many towers we need to support 1000 people we must know how many people a single tower can support. We look at how well one tower would provide service to the residents of the total region. We can generalize this to many towers by assuming all of our towers have no spatial overlap in range and are connected by a wired network. Then whenever any tower receives a signal on some channel to be repeated we assume all of them repeat the message on the desired output channel.

When considering the parameters that could change the number of people served, a handful come to mind as possibly important. The number of channels used by repeaters and as simplex channels should both be related to the number of collisions because with more channels there is more space not to have a conflict. Similarly it would make sense for there to be fewer collisions when there are more squelch tones. We can also imagine that adjusting the amount of conflicts the users will tolerate would allow more users to be accommodated. Additionally, some sources recommend not using CTCSS on simplex frequencies so we should see how that changes the network. Finally, reducing the number of calls per unit time would also allow more people to share the network.

5.1 Capacity Simulation

To analyze these potential factors in the number of people we can accommodate we created a numerical simulation that took these parameters and a number of people. This program simulated our network with a certain number of people by first placing them randomly, then used our greedy allocation scheme to assign channels and squelch tones. Finally, it simulated this network and its traffic for 1000 time steps (they are arbitrary units but we can handle using real numbers too) and outputted the number of collisions and the number of connection tries in the network. We then used this simulation (run 50 times for numerical stability) to estimate whether we could add more people safely or not. This procedure was used to refine the people estimate until it was sufficiently close to the maximum population. In this way we worked back from the parameters to get the maximum tolerable number of people.

5.2 Channel Variations

We will analyze the number of people our network supports for varying choices of number of channels for repeaters and number of simplifies. These simulations were over a 40 mile radius circle, each user had a fixed chance of making a call, and a fixed annoyance threshold (ie the ratio of the number of collisions to the number of tried connections) was used. The reason we note the 40 mile radius is because that affects the portion of connections that are simplex. The rest of the conditions should not be as significant a factor in the rough behavior so the specific values are unimportant.

		Number of simplex channels				
		1	2	3	4	5
Number of repeater channels	1	45	45	44	44	44
	2	88	88	90	87	86
	3	130	132	132	130	131
	4	174	172	173	172	174
	5	215	217	213	216	214
	6	259	260	258	260	259
	7	301	304	302	299	299
	8	340	341	345	344	343
	9	386	386	386	388	386
	10	426	426	433	427	427
	11	478	473	471	473	472
	12	514	516	516	515	515
	13	560	558	554	555	560
	14	604	600	598	598	603
	15	648	645	642	648	644

The most interesting detail of this table is that adding new simplex channels does not help as much as adding new repeater channels. This numerically validates the approach of greedily allocating as many repeater channels as possible at the expense of more simplex channels. The repeater channels are directly related. This makes sense because every user could hear the output of a repeater channel but many users could not overhear a simplex conversation.

5.3 CTCSS Codes

CTCSS allows for users to ignore conversations using different tones. So, many conversations could be on the same channel but as long as two people don't try to talk at the same time the users will never hear each other. So we would expect as we added squelch tones that we could accommodate more people. We tested increments of 5 squelch tones on a network with 7 repeater channels and 3 simplex channels as well as on a network of 4 repeater channels and 1 simplex channel.

	Number of channels	7, 3	4, 1
	1	16	10
	6	92	54
	11	162	94
	16	231	132
	21	296	170
Number of CTCSS tones	26	304	178
	31	304	174
	36	299	174
	41	300	173
	46	300	172
	51	301	172
	56	302	175

Here the results more closely match the prediction of more being better. The data suggests that there is a optimum around 26. It could be the case that those numbers cooperate a little more with the allocation algorithm so the result is nicer, but it could just be a fluke of the numerical simulation. Either way the peak level is very close to the steady state after that, and we can assume more is better.

5.4 Threshold Variation

Another intuitive factor is the level of problems that is tolerable. That is the maximum ratio of collisions to attempted communication allowed. We would assume that the more collisions we are allowed the more people can be in the network. We ran the simulation on a network with 8 repeater channels and 2 simplex channels and a network with 4 repeater channels and 1 simplex channel.

		8, 2	4, 1
	0.01	34	18
	0.05	100	50
	0.10	182	91
	0.15	260	132
Acceptable threshold	0.20	344	173
	0.25	430	212
	0.30	509	258
	0.35	588	294
	0.40	672	338
	0.45	748	376
	0.50	834	422

As one would expect allowing more collisions makes the network accommodate more users. Just ask AT&T. However, doubling the acceptable threshold does not double the possible population, so this is not a sustainable way to increase users. Also, in practice you would rather do reduction techniques other than this.

5.5 Simplex CTCSS

Some sources recommend not using CTCSS on simplex channels because it can cause unintentional collisions [10]. So we disabled simplex CTCSS and did an analysis of number of repeater channels vs number of simplex channels.

Number of repeater channels \ Number of simplex channels	1	2	3	4	5
2	86	86	87	87	90
4	172	171	175	174	173
6	259	261	258	258	259
8	344	342	343	344	344
10	428	428	426	426	427
12	514	511	511	514	514

Even with the lack of CTCSS on the simplex frequencies there appears to be no added population by adding simplex channels. Also, the table closely aligns to the previous table relating number of repeater channels and number of simplex channels to the max population. Therefore, it does not matter whether we allow CTCSS on simplex channels.

5.6 Talking Ratio

In practice we would not have control of the probability at any instance that an individual would be talking. However, we want to analyze this because it shows an important constraint on our network. So we test on a network with 4 repeater channels and 1 simplex channel as well as on a network with 8 repeater channels and 2 simplex channels.

		4, 1	8, 2
	0.000001	514	1003
	0.00001	483	970
	0.0001	482	963
	0.001	466	935
	0.002	454	908
	0.003	442	883
Talking ratio	0.004	418	839
	0.005	334	672
	0.006	283	560
	0.007	243	484
	0.008	213	425
	0.009	195	383
	0.010	173	345
	0.011	159	315
	0.012	147	292
	0.013	134	269
	0.014	127	254
	0.015	118	236
	0.016	111	223
	0.017	106	208
	0.018	101	195
	0.019	95	190
	0.020	90	181

This shows that a network where people talk less would allow a larger population. It also shows us that there is a limit to how much a difference the talking ratio can make. So depending on people to not be talkative helps, but very small talking ratios do not allow us to fit a very large number of users.

5.7 Real World Values

5.7.1 Realistic Talk Ratio

Using data from Nielson [5] and The New York Times [13], we determined the probability of there being a person making a call at any given minute.

$$\frac{204 \text{ calls/month}}{30 \text{ days/month}} * 1.81 \text{ minutes/call} = 12.31 \text{ minutes on phone/day}$$

Dividing this number by the minutes in a day, 1440, we get $\approx .0085$ as the probability that a person is making a call at any given minute of the day. This probability is clearly not completely accurate because people are more apt to make calls at certain times of the day, but this is the most reasonable probability we can get. We will use this probability later.

5.7.2 FM and SSB

In the 2-meter band there are many different encoding strategies, two prominent ones are FM (Frequency Modulation) and SSB (Single-sideband Modulation) [11]. Using FM encoding, we know that the needed bandwidth for an individual signal (ie a channel) is conservatively 20 kHz [12]. We will allocate channels by breaking the 3 MHz band into 150 channels of 20 kHz width. Then we allocate a channel and the channel 600 kHz above it to the repeater channels if both are valid channels and neither has been allocated to a repeater channel. The remainder become simplex channels. So for FM we get 60 channels for repeaters and 30 simplex frequencies.

Using SSB the bandwidth needed for the encoding is at most 6 kHz [6]. Using the same procedure as we used for FM we get that SSB supports 200 repeater channels and 100 simplex channels. So we get more channels by using a scheme like SSB.

5.7.3 Real Numbers

Using the values we predict in the real world we can run computational models to predict how many people our network can handle. Our only value that is not fixed by this is the acceptable threshold of collisions. So we tabulate max population given our values subject to different thresholds. So we fix the talk ratio to be 0.0085 as predicted and we try both FM and SSB for our number of channels. We will allow CTCSS tones on simplex conversations and all 54 CTCSS codes.

Threshold	FM	SSB
0.01	259	872
0.02	407	1351
0.03	560	1848
0.04	701	2331
0.05	842	2834
0.10	1566	5272
0.15	2290	7682
0.20	3010	10064

So given our numbers we can support 1000 users very reasonably with a collision threshold of 0.02 and SSB. We could also use the more unreasonable threshold of 0.20 and support 10,000. Alternatively, we could use smaller SSB bandwidth, for instance 4 kHz which is still a reasonable SSB bandwidth and would have a higher population [6].

6 Wireless Networking

How is a wireless network of repeater towers an improvement over a single centralized repeater tower? Besides the advantage of decreased elevation (and therefore, decreased susceptibility to weather disasters), it turns out that a network of repeaters allows trans-repeater communication to be routed selectively to boost the network's traffic threshold.

Let's set up a network of 40 foot towers in our geographic region. Based on the radio range between two towers in a smooth region, towers must be no more than 15.4078 miles apart. So two contiguous repeater regions would look like this:

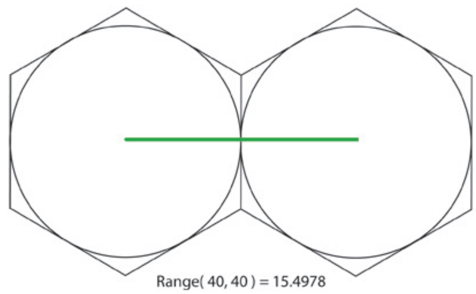


Figure 2: Of the three regular polygons that tessellate, the hexagon most closely approximates a circle. Note: It may appear in this diagram that some areas have no reception. However, remember that users located outside the circle and within the hexagon have height at least 5 feet, hence they will be within range of the repeater tower.

Tessellating a smooth region or radius 40 miles, we need 32 40 foot tall towers to provide coverage to this region.

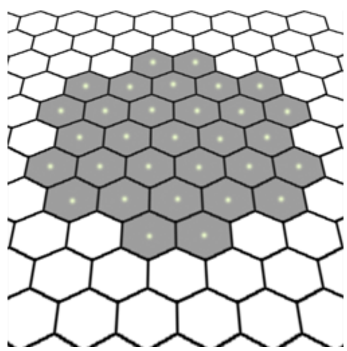


Figure 3: 32 hexagons tile to provide coverage for this region.

When a user within the range of repeater A intends to contact another user in the range of repeater B, he or she transmits at the input frequency of A, which retransmits an amplified version of the message at its output frequency. Meanwhile, repeater B is listening at A's output frequency, so it takes the message and retransmits it at its own output frequency, allowing users within range of B to pick up the signal.

6.1 Selectively Routing Out-of-range Calls

If each repeater were programmed to indiscriminately transmit the calls of its neighbors, then one signal would be propagated throughout the entire network. In terms of traffic alleviation, implementing this setup would be no better than mounting one repeater atop a 925 foot tower. In order to exploit the network's structure, we take advantage of the tessellated regions as follows: we notice that transmission between two non-contiguous repeaters can be routed in a multiple of ways.

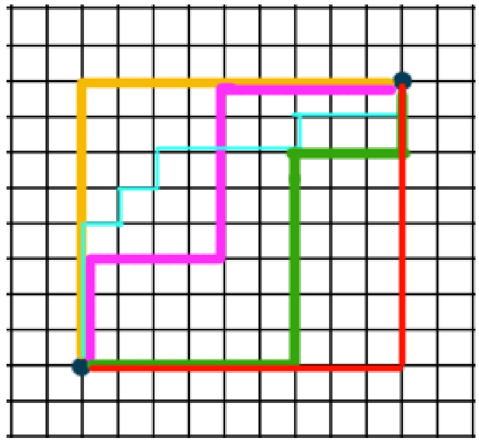


Figure 4: Each node represents a repeater, and edges between nodes represent wireless lines of communication. The number of minimum paths between two nodes increases with the distance between the nodes.

Hence, we can *concentrate* the signal load distribution by propagating signals along single paths rather than outward waves.

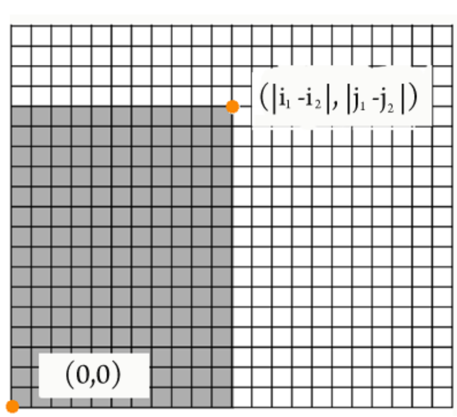
6.2 Probabilistic Propagation

In wireless communications, we assume that x percent of total users want to make targeted calls rather than network-wide broadcasts. These users know the appropriate call number that will route their call to their intended receiver parties. These calls are *smart* in that the $Caller's$ calls are encrypted using an in-band tag identifying the $Intended's$ local repeater tower.

We will now construct a wireless, randomly-selective signal propagation scheme.

6.2.1 Random-path Propagation Along a Grid

Consider a grid network connected wirelessly. Suppose a user at grid location (i_1, j_1) wants to contact a user at (i_2, j_2). The set of routes between (i_1, j_1) and (i_2, j_2) is an affine transformation of the set of routes between $(0, 0)$ and $(|i_1 - i_2|, |j_1 - j_2|)$, so we will consider:



Consider the signal propagation method by which the repeater disregards the set of neighboring repeaters (hereafter called neighbors) that are geographically farther away from the destination repeater. At each step, the repeater is either left with 1 or 2 neighbors to distribute the signal to. If it has two possible neighbors, it randomly selects one with equal probability and propagates the signal overlaid with the appropriate squelch tone. As long as moving up (and increasing j_k) and moving right (and increasing i_k) makes progress towards the destination, then choosing one path rather than indiscriminately propagating the signal will decrease the probability that repeaters that are k steps from the origin will be employed.

Great! Now, what is this probability? For a repeater located at (i, j), the number of paths leading to (i, j) is given by the $(i + 1)$th and $(j + 1)$th component in Pascal's Triangle:

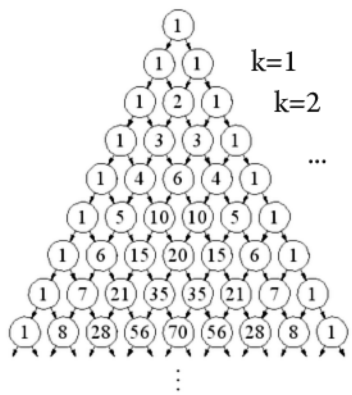


Figure 5: The value of each node is the number of paths leading to it from $(1, 1)$.

Running a simulation to calculate the probability that (i, j) will be hit under a random-path signal propagation scheme, we find that the the repeater nodes near the boundary of the grid enjoy significantly reduced traffic; the probability that a boundary node located k steps away from the origin repeater is 2^{-k}.

The repeaters in the interior also enjoy alleviated traffic. For instance, consider the random-path signal propagation from $(0, 0)$ to $(4, 5)$. The probability that each repeater node is employed in the signal propagation is given in the chart below.

1.0000	0.5000	0.2500	0.1250	0.0667
0.5000	0.5000	0.3750	0.2667	0.2000
0.2500	0.3750	0.4000	0.4000	0.4286
0.1250	0.2667	0.4000	0.5714	1.0000

6.2.2 Random-path Propagation Through a Hexagonal Tessellation

The results from the grid analysis generalizes nicely to our geographic placement of repeater nodes within hexagonal tessellations.

At step k of the propagation, the repeater node has a choice to propagate the signal "up" or to the

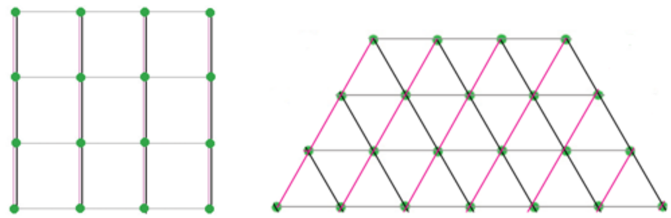


Figure 6: Signals more efficiently propagate along triangular “grids” than on square grids.

“right”. Hence, along the boundary of the set of minimum paths, the probability of traffic is again 2^{-k} after k steps.

7 Repeater Interference

Another issue is that if two repeaters work on the same frequency over some areas the signal from one could arrive before the other. In effect this is just as bad as a collision of two different signals because the message from the closer tower is interrupted by the message from the further tower. We will assume our network is wired, that is none of the radio channels are used for repeater to repeater communication.

We will choose to place the repeaters in the center of hexagonal regions; so a person uses a repeater if they are in the hexagon around that repeater. Observe that hexagonal tilings of the plane are three color able. Our strategy for avoiding interference is by taking a coloring of the hexagonal tiling to get a color for each tower. Since the colors came from a valid coloring we know that no repeaters whose hexagons share an edge have the same color. That means that for any tower all the towers that it could interfere with have a different color. Then we take the possible repeater channels and choose three distinct channels to be channel one for each of the colors. Then we remove those from the possible repeater channels and repeat the process to get the remaining channels one by one. If at the end there are 1 or 2 repeater channels we just allocate those remaining channels as simplex channels.

So when two people want to talk to each other we assume they know which channel and code to use (either through central allocation or allowing users choice). Then each chooses the color of the hexagon they are in as the channel color they will use. For example Alice and Bob are assigned channel 1. Alice knows she is in a red hexagon so she uses the repeater frequency channel 1-red. Bob knows he is in a blue hexagon, so he uses channel 1-blue. Whenever Alice says something on channel 1-red in each hexagon of color c will output it on channel 1-c.

While this removes the problem of interference between repeaters it also lessens the number of repeater channels available. So, using FM (20 kHz bandwidth) there are normally 60 repeater channels and 30 simplex; if this process is used there are only 20 repeater channels and 30 simplex. For SSB (6 kHz bandwidth) there are normally 200 repeater channels and 100 simplex; using this there are 66 repeater channels and 104 simplex. Taking these into account we get more accurate data on the number of people the network can support. We used the same numerical simulation as

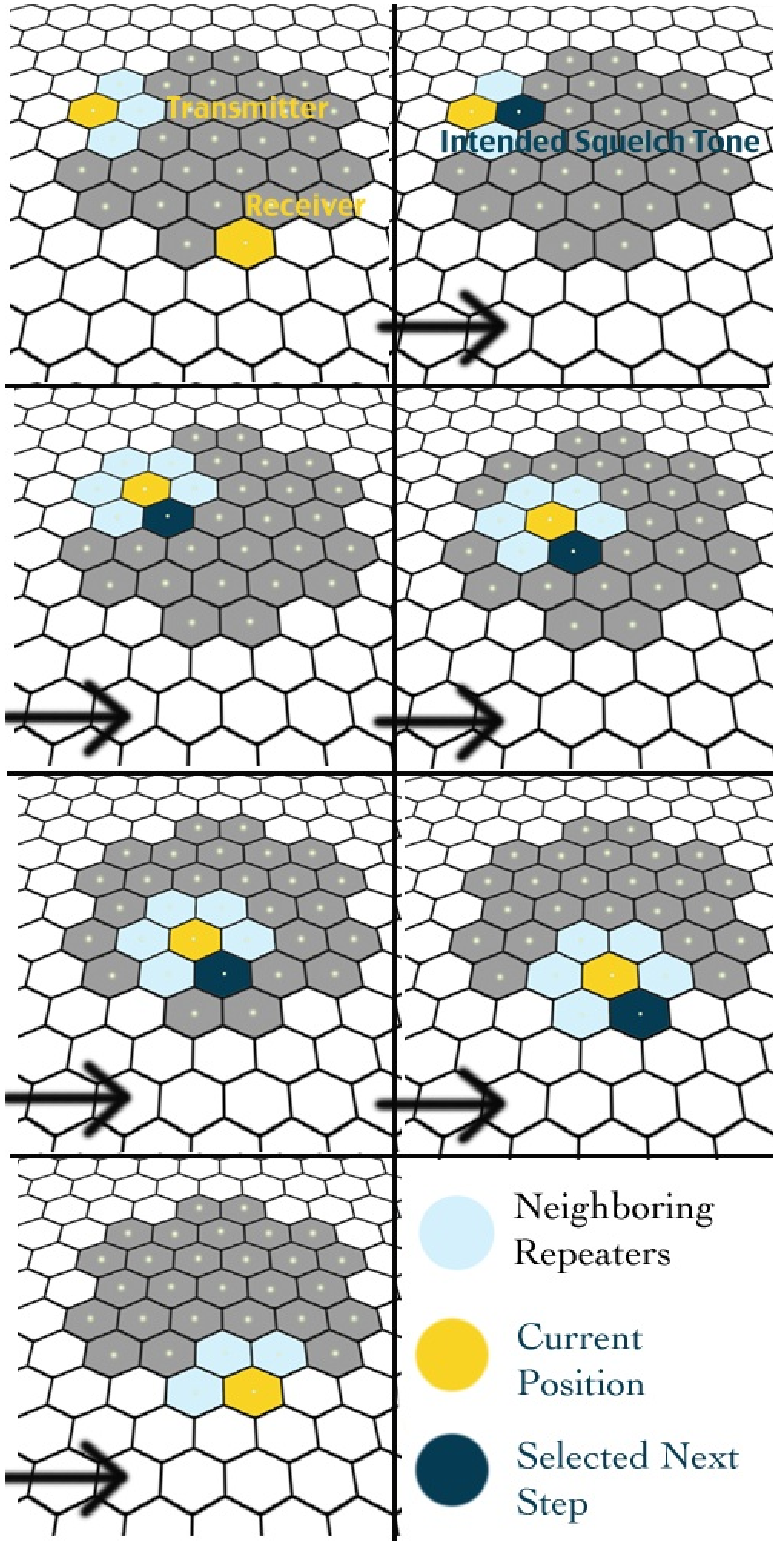


Figure 7: Using CTCSS, each repeater can propagate its signal to some, none, or all of contiguous repeaters.

our previous estimate but we used this adjusted number of channels.

Threshold	Number of People FM	Number of People SSB
0.01	92	291
0.02	136	451
0.03	181	602
0.04	234	768
0.05	280	928
0.06	330	1090
0.07	384	1242
0.08	424	1394
0.09	482	1570
0.10	527	1716

As we would expect doing this decreases the number of people our network can support. Unfortunately it is not computationally feasible to find what threshold would allow for 10,000 users, but we do know that, on SSB, a fairly reasonable threshold of 0.06 does allow us to support 1000 users. This strategy would be important for implementing a network of towers (which may be more cost effective than one large tower), and it could be important given the technical capabilities of the repeaters. So, despite the decreased capacity we could get important advantages using this system. Using FM we would need an unreasonably high threshold to get 1000 people, so we should use a lower bandwidth encoding like SSB.

8 Dealing With Changes in Elevation

Once the elevation of the region in question is no longer constant, the previous value that we determined as the number of repeaters needed to cover everyone is no longer valid. With changes in elevation, the repeaters' line of sight is severally diminished in a numerous areas. Our repeater placement can no longer be determined solely by the region covered on a perfect sphere and the placement of other repeaters, but must be placed in a way so as to minimize the number of obstructions in the way of a signal.

Other problems posed by the variations in elevation are that the VHF signals are reflected off objects and continue to spread. This deflection of the signal depends a lot on the composition and the curvature of the surface. Modeling this would be extremely complex and require actual data about the curvature of the surface for us to ascertain anything useful. Therefore, we assume that objects completely absorb VHF signals that reach them.

8.1 The Partition Approach

One of the models that we formed to deal with the changes in elevation was partitioning the region into equal areas. Within each partition, we placed the repeater on the apex. We then checked each square mile to see if it is covered or not; we took obstructions into account. After we had all the uncovered areas, additional towers would be placed to provide full coverage.

We applied this algorithm to a square region that was 80 miles wide and had varying elevations (the elevations were formed at random) and found that with 1600 towers, between 40% to 45% of the area was uncovered. This result was extremely poor and led us to conclude that there was little reason to add repeaters to cover all the territory.

8.2 The Local Maximum Approach

The alternate model we constructed to handle mountainous regions was finding all of the local maximums and placing repeaters on all of them. This method is guaranteed to work because every possible obstruction has a repeater on it that will cover the surrounding area.

This algorithm was run 320 times on different elevation maps and clearly agrees with the theory. In every simulation the difference between the apex and the nadir was at least 2000 feet, and no areas were out of range of a repeater. The average number of repeaters needed was approximately 418 with a standard deviation of 16.15. This data indicates that this model is over 4 times better than the partition approach because it covers all the areas with a fourth of the repeaters. We can also confidently say that the actual number of towers is fewer than 418 because a square with side lengths of 80 has a greater area than a circle with a radius of 40.

Unfortunately, this model does not tell us the number of repeaters needed to handle the user load, but it gives us highly useful information about how to achieve total coverage of the region with varying elevations.

9 Conclusion

Comparison of a wired and wireless implementation directs our focus to their trade-offs. As we have shown, a wired network is advantageous because it allows for dynamic bandwidth allocation by a centralized hub. One can imagine how this dynamic system would be an attractive option in an urban setting. On the other hand, a wireless network can diffuse the flow of traffic by selectively propagating signals. In this case, one obvious weakness is the assumption that users are making calls targeted across the network. Our analysis has shown that the wired versus wireless implementations would perform very differently given different parameters (traffic density, geographic terrain, resource limitations). As such, our final recommendation requires that we know physical and demographic stats of the site. Our solution is two-fold: we recommend a network of repeaters linked by wires as our base system. In some conditions where infrastructure is either hard to come by or difficult to maintain we would propose the use of wireless repeater links.

Assuming the area is a perfect circle on the Earth we would say in either case (wired or wireless) use a network of 32 repeaters each 40 feet tall. In both cases there would be a 0.06 ratio of collisions to attempted communications. In the wired case we have the advantage of no extra data required. However, for wireless communication we need to add data packets to each message to ensure it does not cause a feedback loop. Additionally, in a wired network we can use our channel allocation scheme to give us better results, while that would be more difficult to implement in an wireless system.

If we needed to construct a VHF repeater network on mountainous terrain, we have made significant advances in solving this problem. By placing the repeaters on every local maximum, we will succeed in providing coverage to all areas. Applying this process on terrain with great differences in elevation indicated that approximately 420 repeaters would be needed to achieve the coverage goal. Of course, the actual number would depend on the topography of a specific area.

9.1 Limitations

- Our model has no good mechanism to allow for calls between two users without propagating the message through the entire network. Especially our wired system should have some sort of better scheme for efficient network usage.
- Our model cannot handle extremely large numbers of people without decreasing the quality of communications.
- Our scheme for covering overlapped coverage costs us repeater channels, which decreases our capacity.
- Our current model does not do anything in response to failures of repeaters.
- Our current model to deal with elevation does not address how many repeaters would be needed to handle a given number of people.

References

[1] Brian, Marshall, Jeff Tyson, and Julia Layton. How Cell Phones Work. n.d. http://www.howstuffworks.com/cell-phone.htm/printable (accessed February 12, 2011).

[2] Compute Support Group, Inc. VHF Marine Radio Service Frequency Table. n.d. http://www.csgnetwork.com/marinefreqtable.html (accessed February 2011).

[3] Kozlowski, Michael. CB Radio Etiquette. n.d. http://www.ehow.com/way_5290843_cb-radio-etiquette.html (accessed February 2011).

[4] Murray State University. MSUARC - Echolink. n.d. http://campus.murraystate.edu/org /msuarc/echolink.htm (accessed February 2011).

[5] Nielsen Wire. In U.S., SMS Text Messaging Tops Mobile Phone Calling. September 22, 2008. http://blog.nielsen.com/nielsenwire/online_mobile/in-us-text-messaging-tops-mobile-phone-calling/ (accessed February 2011).

[6] QRZ Forums. SSB Bandwidth and ”Good Practice”. May 2008. http://forums.qrz.com/archive/index.php/t-162069.html (accessed February 2011).

[7] Suffern, Maurice Grayle. Basic Radio Principles. New York: McGraw-Hill Book Company, Inc., 1943.

[8] The Economist. Difference Engine: Bigger and Better Than WiFi. December 9, 2010. http://www.economist.com/node/17647517?story_id=17647517 (accessed February 11, 2011).

[9] The Utah VHF Society. Frequency Coordinator FAQ. n.d. http://utahvhfs.org/freqfaq1.shtml#pl_simplex (accessed February 2011).

[10] -. Simplex Frequency Assignment. n.d. http://utahvhfs.org/simpfreq1.html (accessed February 2011).

[11] Wikipedia. 2-meter band. February 13, 2011. http://en.wikipedia.org/wiki/2-meter_band (accessed February 14, 2011).

[12] Witte, Bob. 2M Frequencies. September 4, 2006. http://www.k0nr.com/rwitte/2m_frequencies.html (accessed February 2011).

[13] Wortham, Jenna. Cellphones Now Used More for Data Than for Calls. May 2010, 2010. http://www.nytimes.com/2010/05/14/technology/personaltech/14talk.html?_r=1 (accessed February 13, 2011).