Structural Analysis

The team utilised the Canoe Analysis Program (CAP), an in-house software developed in MATLAB® (MathWorks 2019), to analyse the effects of bending, gravity load from canoe, and paddlers with paddler loading factors for over 300 two dimensional traverse cross sections. The team determined the main contributors to shear and moment as the longitudinal and transverse bending from the load of paddlers, punching shear from paddlers pushing on the toe-block and leaning against the canoe body, as well as the hydrostatic forces of water on the submerged portion of the canoe. The wall thickness had the greatest influence on maximum stresses predicted by CAP, which were compared with experimental concrete strength results to determine the canoe's structural integrity.

After obtaining the stress profile and principal stresses of the canoe from CAP, the team incorporated them into the Mohr-Coulomb failure criterion to determine the region of no crack formation under any combination of stresses. This process was conducted to prevent seepage and damage from cracks that can form prior to concrete reaching its ultimate tensile strength, particularly under biaxial stresses from longitudinal and transverse bending.

In view of using an innovative approach to evaluate the formation of cracks at a microscopic level, the team utilised the biaxial failure contour proposed by Como-Luciano. It is established on the assumption that significant failure in the form of cracking occurs when the local maximum principal stresses around pores reach the tensile strength of the hardened cement paste, thus causing destruction in the concrete binder.

Structural Analysis Results

After running 270 loading cases with 2 and 4 paddlers weighing between 75-200 kg, CAP anticipated a maximum principal stress and minimum principal stress of 0.9369 MPa and 0.9199 MPa respectively. The stress envelope in Figure 9 concatenates the maximum stresses of the 270 loading cases to summarize the most extreme stress values at each location along the canoe. With an experimental tensile strength of 1.68 MPa that yields a factor of safety of 1.79 in addition to the paddler loading factor, the team has deemed the concrete mix sufficient to resist all loads on the concrete structure along with sufficient margin for some inconsistent quality control during concrete casting and sanding.

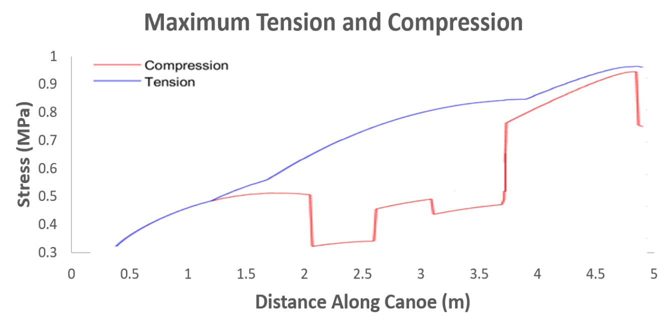

Figure 9: Tension and Compression Stress Envelopes of All Load Cases

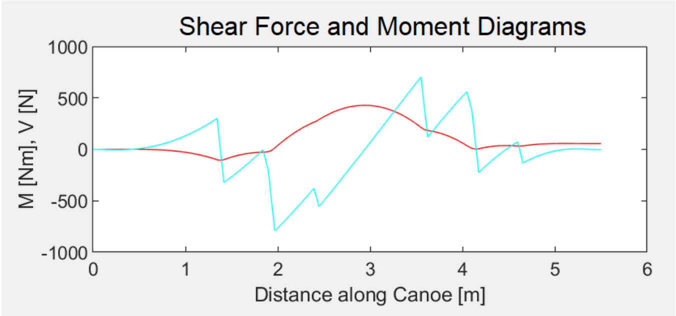

Figure 10: Shear Force and Moment Diagrams of Most Critical Load Case

In addition to stress calculations, primary reinforcement calculation was conducted under the assumption that the glass fibre mesh will be in a state of tension on the bottom of the C channel. The usage of short PVC fibres for secondary reinforcement allows the concrete to achieve a tensile strength of 1.68 MPa, and this empirical result is used to calculate the modulus of rupture. The calculated cracking moment of 2812 Nm and an ultimate bending capacity of 2233 Nm are well above an anticipated maximum moment of 400 Nm predicted by CAP (Figure 10). Since the capacity is more than one third greater than factored demand, the minimum reinforcement criterion may be waived (CSA A23.3 Clause 10.5.1.3). A large safety margin in tensile and bending capacity ensures that the team's structural strength is sufficient to resist all loading demands.

Appendix B – Structural Calculations

Load Calculations

Assumptions & Predefined Variables

The team assumes the buoyancy force acting on the canoe submerged in water has a rectangular projection at the bottom. The team also assumes the self-weight dead-load of the canoe can be represented as a uniform distributed load across the whole canoe length. Since both the dead-load and buoyancy acts across the whole canoe length in opposite directions, the resultant force is regarded as the buoyancy force to support the paddlers and cargo. Therefore, the buoyancy force is treated as a uniform distributed load with its total force equal the sum of two paddlers of 90.7 kg at 15% and 85% span of the canoe, and a cargo with 1168 Nm^{-1} distributed load of 1.52 m centered at the midspan.

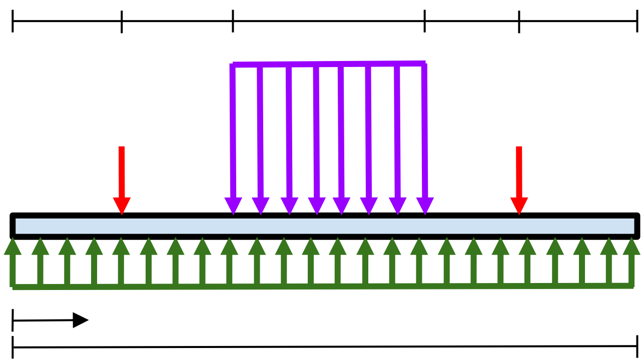

Figure B-1: Free body diagram of the buoyancy, paddler and cargo load on the canoe

<u>Buoyancy force:</u>

$$F_b = \frac{(90.7 \times 9.81 \times 2 \ + 1168 \times 1.52)}{5.5} = 646.344\ Nm^{-1}$$

<u>Equations for shear force and bending moment across the canoe and maximum moment:</u>

The length of the canoe is divided into 5 segments: $0 < x < 0.825$, $0.825 < x < 1.99$, $1.99 < x < 3.51$, $3.51 < x < 4.675$, $4.675 < x < 5.5$. This is because the shear force diagram is formed by 5 different functions. The equation for load $w(x)$ is integrated to obtain the shear force equation $V(x)$, and is integrated again to obtain the moment equation $M(x)$.

$w(x) = F_b$ for $0 < x < 1.99$ *and* $3.51 < x < 5.5$
$w(x) = F_b - 1168$ for $1.99 < x < 3.51$

$V(x) = \int w(x)\, dx + C$
$M(x) = -\int V(x)\, dx + C$

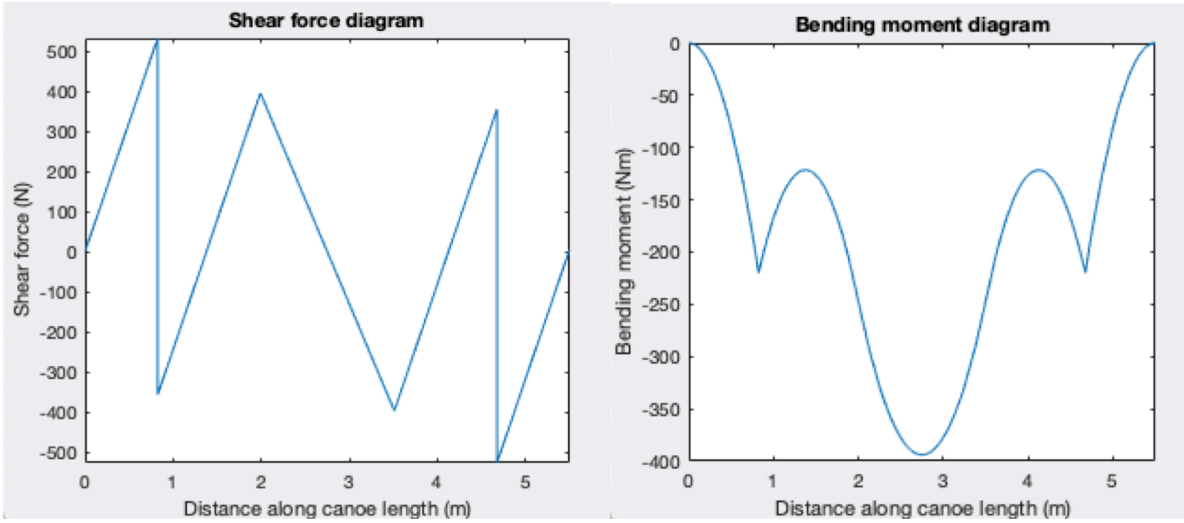

Figure B-2: Shear force and bending moment diagrams along canoe length

The maximum bending moment $M_{max} = -393.8698\ Nm$

The cross-section of the canoe at midspan, where the maximum moment occurs, as a C channel with dimensions as shown below:

Figure B-3: Cross-sectional C channel

Solving for neutral axisy_c:
$A_1 = t \times h$
$A_2 = (w - 2 \times t) \times t$
$y_c = (A_1 \times h \times \frac{1}{2} \times 2 + A_2 \times t \times \frac{1}{2}) \div (A_1 \times 2 + A_2) = 0.0957\ m$

Solving for moment of inertia$I$:
$I_1 = t \times h^3 \div 12 + A_1 \times (y_c - h \times \frac{1}{2})^2$
$I_2 = (w - 2 \times t) \times h^3 \div 12 + A_2 \times (y_c - t \times \frac{1}{2})^2$
$I = I_1 \times 2 + I_2 = 2.4691 \times 10^{-4}\ m^4$

<u>Tensile, compressive, and shear stress at maximum bending moment:</u>
$\sigma_{tension} = M_{max} \times y_c \div I = 1.5259 \times 10^5\ Pa$
$\sigma_{compression} = M_{max} \times (0.35 - y_c) \div I = 4.0573 \times 10^5\ Pa$
$\tau = V(x = 2.75) \div (2 \times A_1 + A_2) = \ \ 7.4652 \times 10^{-12}\ Pa$

<u>Bending moment at which cracking of concrete begins to occur:</u>

Tensile strength of concrete: 1.68MPa

$Section\ modulus\ S = \frac{I}{y_c} = \frac{2.4691 \times 10^{-4}\ m^4}{0.0957\ m} = 2.58 \times 10^3 m^3$

Resistance factor of concrete ϕ = 0.65

Modulus of rupture: $fr = (0.65)\ (1.68MPa) = 1.09MPa$

Cracking moment of concrete: $Mcr = frS = (1.09MPa)\ (0.00258m^3) = 2812\ Nm$

<u>Ultimate bending moment, including the effects of reinforcement</u>

Compressive strength of concrete = 5.44MPa

Yield force in reinforcement:$Ty = (556lb/ft)\ (3ft) = 7420N$

Parameter $\alpha = 0.85 - 0.0015f'c = 0.842$

Depth of stress block: $a = \frac{Ty}{\alpha\phi c f' cb} = \frac{7420N}{(0.842)\ (0.65)\ (5.44MPa)\ (30mm)} = 83\ mm$

Distance from extreme compression fibre to centroid of steel:

$d = 350mm - 7.5mm = 342.5mm$

Lever arm: $z = d - \frac{a}{2} = 342.5mm - 83mm/2 = 301\ mm$

Ultimate bending moment: $Mr = Tyz = (7420N)\ (301mm) = 2233\ Nm$

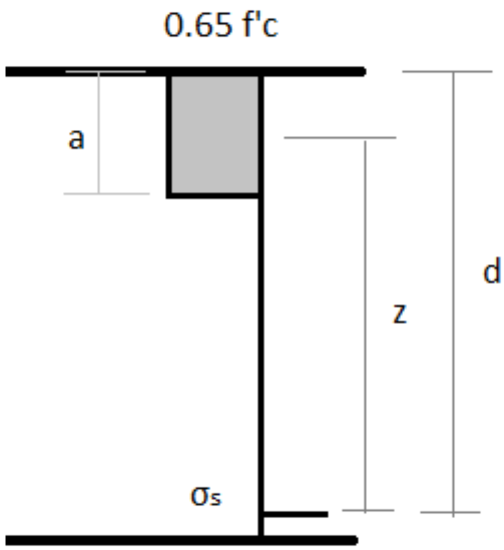

Figure B-4: Stress in a Reinforced Concrete Section