Designing Circular and Non-Circular Gears for FDM 3d-Printing

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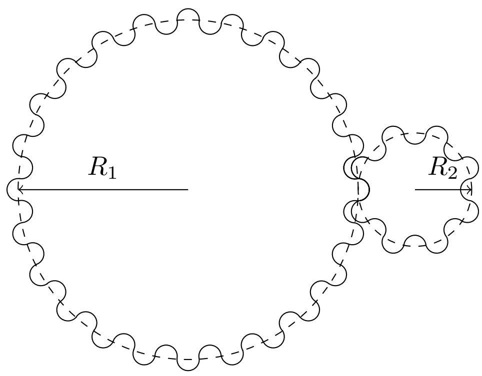

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1 Tooth Profile

Since FDM 3d printed gears are weak and therefore torque and transmission efficiency is not as important we are going to use a circular tooth profile. Please note that this profile is not optimal but it is easy to calculate non circular gears and it is sufficient for most 3d printed parts. If this is not suitable for your design take a look at involute gears.

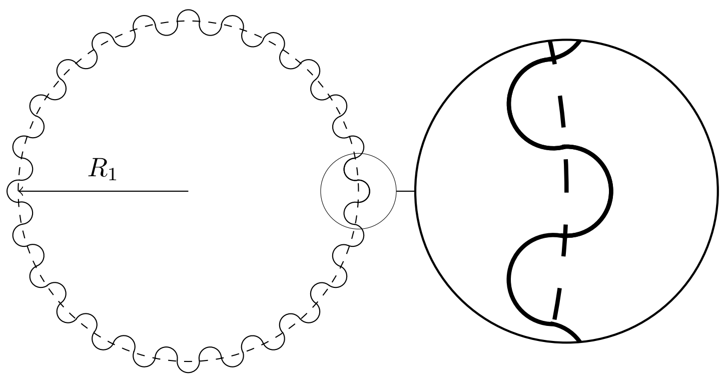


Figure 1: Gear profile construction using circles

2 Drawing a Gear

Start by drawing the general shape of your gear. As we will see in section 3.4 this can be any convex shape. As an example we will take a look at a circular gear. Then, using a compas, mark points with distance r on the outline of your shape and draw a circle with radius r on every second mark.

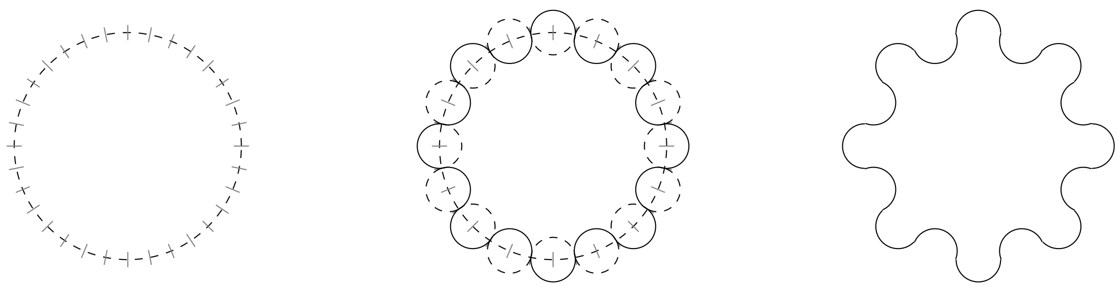


Figure 2: Drawing a circular gear

3 Calculating Gear and teeth Size

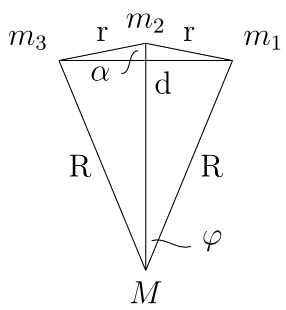
3.1 Circular Gears.

Designing a circular gear is straightforward. We start by drawing a circle K with radius R. This will be the size of our final gear. We then look for a sequence of smaller circles $(k_i)_{i\in I}$ with radius r such that their center is on K, they intersect each other on K, they cover K completely and the number of k_i must be even ($|I| = 2n : n \in \mathbb{N}$). This is important since the number of teeth has to be a positive integer.

Theorem 3.1
Given a circle K with radius R and $n \in \mathbb{N}$, it is possible to find a sequence of n circles $(k_i)_{i\in I}$ with radius r and center $(m_i)_{i\in I} \in K$ such that $|\{x \in K : d(x, m_i) > r, \forall i \in I\}| = 0$ and $|\{x \in K : \exists i, j \in I, d(x, m_i) \leq r \wedge d(x, m_j) \leq r\}| = n$. Where $d(x, y)$ is the distance between x and y. Let $\varphi = \frac{2Pi}{n}$ then

$$R = \frac{r \cdot \cos\left(\frac{\varphi}{4}\right)}{\sin(\frac{\varphi}{2})}. \tag{3.1}$$

Proof
Let K be a circle with radius R and center $(0, 0)$ and $n = 2k$, $k \in \mathbb{N}$ we can, using polar coordinates, construct a sequence $(m_i)_{i\in\mathbb{Z}_{2n}}$, $m_i = \left(R, i \cdot \frac{\pi}{n}\text{rad}\right)$ of $2n$ points on K. They are evenly spaced and $r \coloneqq d(m_0, m_1)$. We can now construct a sequence $(k_j)_{j\in\mathbb{Z}_n}$of n circles with radius r and center $m_j : j \equiv 0 \pmod 2$. Therefore $\{x \in K : \exists i, j \in \mathbb{Z}_{2n} : d(x, m_{2i}) \leq r \wedge d(x, m_{2j}) \leq r\} = \{m_i : i \equiv 1 \pmod 2, i \in \mathbb{Z}_{2n}\}$ and $|\{m_i : i \equiv 1 \pmod 2, i \in \mathbb{Z}_{2n}\}| = n$.



Let $\alpha \coloneqq \sphericalangle m_3 m_2 m_1$, $\varphi \coloneqq \sphericalangle m_1 M m_3$ and $d \coloneqq \overline{m_3 m_1}$. Using the inscribed angle theorem(Weisstein 2020), we know that $\alpha = \frac{2\pi - \varphi}{2}$. By drawing a line from M to

m we get two right triangles. $\overline{Mm_2}$ also halves φ, α and d. Thus

$$\frac{d}{2} = r \cdot \sin\left(\frac{\alpha}{2}\right) \tag{3.2}$$

and

$$R = \frac{d}{2\sin\left(\frac{\varphi}{2}\right)}. \tag{3.3}$$

By substituting in α in eq. (3.2) and using $\cos(x) = \sin(\frac{\pi}{2} - x)$, we see that

$$R = \frac{r \cdot \cos\left(\frac{\varphi}{4}\right)}{\sin(\frac{\varphi}{2})}. \tag{3.4}$$

□

3.2 Rectangular Gears

3.3 Elliptical Gears

3.4 Other Non Circular Gears

3.5 Examples

Example 3.5.1 Circular gears with given gear ratio and centerdistance

Lets say we want a 3:1 gear reduction and the center points of each gear should be 30mm apart. We can now start to build a system of equations:

$$R_1 + R_2 = 30$$
$$n_1 = 3n_2$$

Where R_1 and R_2 are the radii of the two gears and n_1 and n_2 denote the number of teeth on each. Using eq. (3.1) we also know that:

$$\varphi_1 = \frac{2Pi}{n_1}$$
$$R_1 = \frac{r * \cos\left(\frac{\varphi_1}{4}\right)}{\sin(\frac{\varphi}{2})}$$
$$\varphi_2 = \frac{2Pi}{n_2}$$
$$R_2 = \frac{r * \cos\left(\frac{\varphi_2}{4}\right)}{\sin(\frac{\varphi}{2})}$$

If we specify the number of teeth on either one of them lets say

$$n_2 = 16.$$

we can solve the system of equations using wolframalpha, maple or another computerprogram to find a unique solution. In this example we get:

$$r = 1.471832758$$
$$R_1 = 22.49196236$$
$$R_2 = 7.508037642$$
$$n_1 = 48$$
$$n_2 = 16$$
$$\varphi_1 = 0.1308996939$$
$$\varphi_2 = 0.3926990818$$

If r is too small/large change the number of teeth you specified.

I: $R_1 + R_2 = 30$

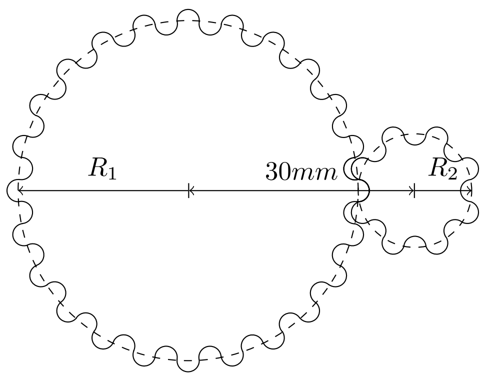


Figure 3: Resulting Gears

4 Rack and Pinion

4.1 Constant rotational Speed

4.2 Non-constant rotational Speed

4.2.1 Continuous functions

4.2.2 Discontinuous functions

4.3 Non-constant shaft position

4.4 Examples

Example 4.4.1 Shaft following a Sinus(t) curve

We want a gear such that the shaft is following a $\sin(t)$ curve. We know that $\sin(t)$ is a 2π periodic function with $\sin(t) > 0, t \in (0, \pi)$ and $\sin(t) < 0, t \in (\pi, 2\pi)$. As seen in section 4 this will result in a non convex gear shape. To use the method described in section 3.4 we need a new function $g(t) := sin(t) + |min(sin[0, 2\pi])|$. Thus $g(t) \geq 0, t \in (0, 2\pi)$ resulting in a convex gear shape. However the shaft will now follow $g(t)$ and not $\sin(t)$. Therefore we need to offset both the rack and pinion by $|min(sin[0, 2\pi])|$.
We can now start by choosing a minimum radius $R := 5$ for the gear.

Example 4.4.2 cos(t)

References

Weisstein, Eric W. (2020). *Inscribed Angle.* URL: `https://mathworld.wolfram.com/InscribedAngle.html` (visited on 05/05/2020) (cit. on p. 4).

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