# CHAPTER 3

## SIGNALS & SYSTEMS

**YEAR 2012** **ONE MARK**

**MCQ 3.1** If $x[n] = (1/3)^{|n|} - (1/2)^n u[n]$, then the region of convergence (ROC) of its $z$-transform in the $z$-plane will be

(A) $\frac{1}{3} < |z| < 3$ (B) $\frac{1}{3} < |z| < \frac{1}{2}$

(C) $\frac{1}{2} < |z| < 3$ (D) $\frac{1}{3} < |z|$

**MCQ 3.2** The unilateral Laplace transform of $f(t)$ is $\frac{1}{s^2 + s + 1}$. The unilateral Laplace transform of $tf(t)$ is

(A) $-\frac{s}{(s^2 + s + 1)^2}$ (B) $-\frac{2s+1}{(s^2 + s + 1)^2}$

(C) $\frac{s}{(s^2 + s + 1)^2}$ (D) $\frac{2s+1}{(s^2 + s + 1)^2}$



**YEAR 2012** **TWO MARKS**

**MCQ 3.3** Let $y[n]$ denote the convolution of $h[n]$ and $g[n]$, where $h[n] = (1/2)^n u[n]$ and $g[n]$ is a causal sequence. If $y[0] = 1$ and $y[1] = 1/2$, then $g[1]$ equals

(A) 0 (B) 1/2

(C) 1 (D) 3/2

**MCQ 3.4** The Fourier transform of a signal $h(t)$ is $H(j\omega) = (2\cos\omega)(\sin 2\omega)/\omega$. The value of $h(0)$ is

(A) 1/4 (B) 1/2

(C) 1 (D) 2

**MCQ 3.5** The input $x(t)$ and output $y(t)$ of a system are related as $y(t) = \int_{-\infty}^{t} x(\tau)\cos(3\tau)\,d\tau$. The system is

(A) time-invariant and stable

(B) stable and not time-invariant

(C) time-invariant and not stable

(D) not time-invariant and not stable

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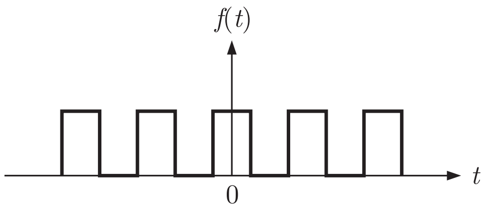
## YEAR 2011 ONE MARK

**MCQ 3.6** The Fourier series expansion $f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t$ of the periodic signal shown below will contain the following nonzero terms




(A) $a_0$ and $b_n, n = 1,3,5,...\infty$

(B) $a_0$ and $a_n, n = 1,2,3,...\infty$

(C) $a_0\, a_n$ and $b_n, n = 1,2,3,...\infty$

(D) $a_0$ and $a_n\; n = 1,3,5,...\infty$

**MCQ 3.7** Given two continuous time signals $x(t) = e^{-t}$ and $y(t) = e^{-2t}$ which exist for $t > 0$, the convolution $z(t) = x(t)^{*} y(t)$ is

(A) $e^{-t} - e^{-2t}$

(B) $e^{-3t}$

(C) $e^{+t}$

(D) $e^{-t} + e^{-2t}$

## YEAR 2011 TWO MARKS

**MCQ 3.8** Let the Laplace transform of a function $f(t)$ which exists for $t > 0$ be $F_1(s)$ and the Laplace transform of its delayed version $f(t-\tau)$ be $F_2(s)$. Let $F_1^*(s)$ be the complex conjugate of $F_1(s)$ with the Laplace variable set $s = \sigma + j\omega$. If $G(s) = \frac{F_2(s)\,F_1^*(s)}{|F_1(s)|^2}$, then the inverse Laplace transform of $G(s)$ is an ideal

(A) impulse $\delta(t)$

(B) delayed impulse $\delta(t-\tau)$

(C) step function $u(t)$

(D) delayed step function $u(t-\tau)$

**MCQ 3.9** The response $h(t)$ of a linear time invariant system to an impulse $\delta(t)$, under initially relaxed condition is $h(t) = e^{-t} + e^{-2t}$. The response of this system for a unit step input $u(t)$ is

(A) $u(t) + e^{-t} + e^{-2t}$

(B) $(e^{-t} + e^{-2t})\,u(t)$

(C) $(1.5 - e^{-t} - 0.5e^{-2t})\,u(t)$

(D) $e^{-t}\delta(t) + e^{-2t}u(t)$

## YEAR 2010 ONE MARK

**MCQ 3.10** For the system $2/(s+1)$, the approximate time taken for a step response to reach 98% of the final value is

(A) 1 s

(B) 2 s

(C) 4 s

(D) 8 s

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**MCQ 3.11** The period of the signal $x(t) = 8\sin\left(0.8\pi t + \frac{\pi}{4}\right)$ is

(A) $0.4\pi$ s (B) $0.8\pi$ s

(C) 1.25 s (D) 2.5 s

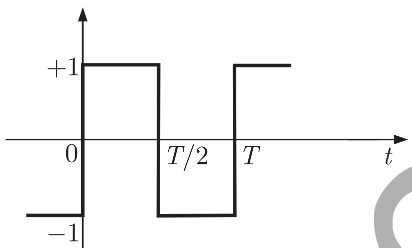
**MCQ 3.12** The system represented by the input-output relationship

$$y(t) = \int_{-\infty}^{5t} x(\tau)\,d\tau,\ t > 0$$

(A) Linear and causal (B) Linear but not causal

(C) Causal but not linear (D) Neither liner nor causal

**MCQ 3.13** The second harmonic component of the periodic waveform given in the figure has an amplitude of

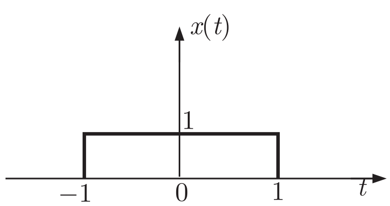



(A) 0 (B) 1

(C) $2/\pi$ (D) $\sqrt{5}$

**YEAR 2010 TWO MARKS**

**MCQ 3.14** $x(t)$ is a positive rectangular pulse from $t = -1$ to $t = +1$ with unit height as shown in the figure. The value of $\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$ {where $X(\omega)$ is the Fourier transform of $x(t)$} is.




(A) 2 (B) $2\pi$

(C) 4 (D) $4\pi$

**MCQ 3.15** Given the finite length input $x[n]$ and the corresponding finite length output $y[n]$ of an LTI system as shown below, the impulse response $h[n]$ of the system is

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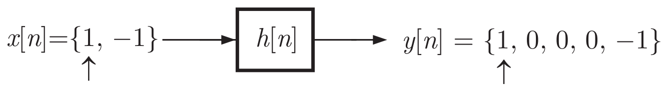
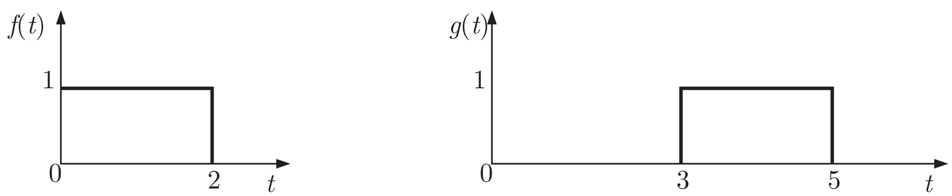



(A) $h[n]=\{\underset{\uparrow}{1},0,0,1\}$
(B) $h[n]=\{\underset{\uparrow}{1},0,1\}$
(C) $h[n]=\{\underset{\uparrow}{1},1,1,1\}$
(D) $h[n]=\{\underset{\uparrow}{1},1,1\}$

**Common Data Questions Q.6-7.**

Given $f(t)$ and $g(t)$ as show below




**MCQ 3.16** $g(t)$ can be expressed as

(A) $g(t)=f(2t-3)$
(B) $g(t)=f\left(\frac{t}{2}-3\right)$
(C) $g(t)=f\left(2t-\frac{3}{2}\right)$
(D) $g(t)=f\left(\frac{t}{2}-\frac{3}{2}\right)$

**MCQ 3.17** The Laplace transform of $g(t)$ is

(A) $\frac{1}{s}(e^{3s}-e^{5s})$
(B) $\frac{1}{s}(e^{-5s}-e^{-3s})$
(C) $\frac{e^{-3s}}{s}(1-e^{-2s})$
(D) $\frac{1}{s}(e^{5s}-e^{3s})$

**YEAR 2009 ONE MARK**

**MCQ 3.18** A Linear Time Invariant system with an impulse response $h(t)$ produces output $y(t)$ when input $x(t)$ is applied. When the input $x(t-\tau)$ is applied to a system with impulse response $h(t-\tau)$, the output will be

(A) $y(\tau)$
(B) $y(2(t-\tau))$
(C) $y(t-\tau)$
(D) $y(t-2\tau)$

**YEAR 2009 TWO MARKS**

**MCQ 3.19** A cascade of three Linear Time Invariant systems is causal and unstable. From this, we conclude that

(A) each system in the cascade is individually causal and unstable
(B) at least on system is unstable and at least one system is causal
(C) at least one system is causal and all systems are unstable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(D) the majority are unstable and the majority are causal

**MCQ 3.20** The Fourier Series coefficients of a periodic signal $x(t)$ expressed as $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T}$ are given by $a_{-2} = 2 - j1$, $a_{-1} = 0.5 + j0.2$, $a_0 = j2$, $a_1 = 0.5 - j0.2$, $a_2 = 2 + j1$ and $a_k = 0$ for $|k| > 2$
Which of the following is true ?

(A) $x(t)$ has finite energy because only finitely many coefficients are non-zero

(B) $x(t)$ has zero average value because it is periodic

(C) The imaginary part of $x(t)$ is constant

(D) The real part of $x(t)$ is even

**MCQ 3.21** The z-transform of a signal $x[n]$ is given by $4z^{-3} + 3z^{-1} + 2 - 6z^2 + 2z^3$
It is applied to a system, with a transfer function $H(z) = 3z^{-1} - 2$
Let the output be $y[n]$. Which of the following is true ?

(A) $y[n]$ is non causal with finite support

(B) $y[n]$ is causal with infinite support

(C) $y[n] = 0; |n| > 3$

(D) $\mathrm{Re}[Y(z)]_{z=e^{j\theta}} = -\mathrm{Re}[Y(z)]_{z=e^{-j\theta}}$
$\mathrm{Im}[Y(z)]_{z=e^{j\theta}} = \mathrm{Im}[Y(z)]_{z=e^{-j\theta}}; \; -\pi \le \theta < \pi$



## YEAR 2008 ONE MARK

**MCQ 3.22** The impulse response of a causal linear time-invariant system is given as $h(t)$. Now consider the following two statements :
**Statement (I)**: Principle of superposition holds
**Statement (II):** $h(t) = 0$ for $t < 0$
Which one of the following statements is correct ?

(A) Statements (I) is correct and statement (II) is wrong

(B) Statements (II) is correct and statement (I) is wrong

(C) Both Statement (I) and Statement (II) are wrong

(D) Both Statement (I) and Statement (II) are correct

**MCQ 3.23** A signal $e^{-\alpha t}\sin(\omega t)$ is the input to a real Linear Time Invariant system. Given $K$ and $\phi$ are constants, the output of the system will be of the form $Ke^{-\beta t}\sin(vt + \phi)$ where

(A) $\beta$ need not be equal to $\alpha$ but $v$ equal to $\omega$

(B) $v$ need not be equal to $\omega$ but $\beta$ equal to $\alpha$

(C) $\beta$ equal to $\alpha$ and $v$ equal to $\omega$

(D) $\beta$ need not be equal to $\alpha$ and $v$ need not be equal to $\omega$

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**YEAR 2008** **TWO MARKS**

**MCQ 3.24** A system with $x(t)$ and output $y(t)$ is defined by the input-output relation :

$$y(t) = \int_{-\infty}^{-2t} x(t)\, d\tau$$

The system will be

(A) Casual, time-invariant and unstable

(B) Casual, time-invariant and stable

(C) non-casual, time-invariant and unstable

(D) non-casual, time-variant and unstable

**MCQ 3.25** A signal $x(t) = \mathrm{sinc}(\alpha t)$ where $\alpha$ is a real constant $(\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x})$ is the input to a Linear Time Invariant system whose impulse response $h(t) = \mathrm{sinc}(\beta t)$, where $\beta$ is a real constant. If min $(\alpha, \beta)$ denotes the minimum of $\alpha$ and $\beta$ and similarly, max $(\alpha,\beta)$ denotes the maximum of $\alpha$ and $\beta$, and K is a constant, which one of the following statements is true about the output of the system ?

(A) It will be of the form $K\mathrm{sinc}(\gamma t)$ where $\gamma = \min(\alpha, \beta)$

(B) It will be of the form $K\mathrm{sinc}(\gamma t)$ where $\gamma = \max(\alpha, \beta)$

(C) It will be of the form $K\mathrm{sinc}(\alpha t)$

(D) It can not be a sinc type of signal

**MCQ 3.26** Let $x(t)$ be a periodic signal with time period $T$, Let $y(t) = x(t - t_0) + x(t + t_0)$ for some $t_0$. The Fourier Series coefficients of $y(t)$ are denoted by $b_k$. If $b_k = 0$ for all odd $k$, then $t_0$ can be equal to

(A) $T/8$ (B) $T/4$

(C) $T/2$ (D) $2T$

**MCQ 3.27** $H(z)$ is a transfer function of a real system. When a signal $x[n] = (1 + j)^n$ is the input to such a system, the output is zero. Further, the Region of convergence (ROC) of $(1 - \frac{1}{2}z^{-1})$ H(z) is the entire Z-plane (except $z = 0$). It can then be inferred that $H(z)$ can have a minimum of

(A) one pole and one zero

(B) one pole and two zeros

(C) two poles and one zero

D) two poles and two zeros

**MCQ 3.28** Given $X(z) = \frac{z}{(z - a)^2}$ with $|z| > a$, the residue of $X(z)z^{n-1}$ at $z = a$ for $n \geq 0$ will be

(A) $a^{n-1}$ (B) $a^n$

(C) $na^n$ (D) $na^{n-1}$

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**MCQ 3.29** Let $x(t) = \text{rect}(t - \frac{1}{2})$ (where $rect(x) = 1$ for $-\frac{1}{2} \leq x \leq \frac{1}{2}$ and zero otherwise. If $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$, then the FTof $x(t) + x(-t)$ will be given by

(A) $\text{sinc}\left(\frac{\omega}{2\pi}\right)$

(B) $2\text{sinc}\left(\frac{\omega}{2\pi}\right)$

(C) $2\text{sinc}\left(\frac{\omega}{2\pi}\right)\cos\left(\frac{\omega}{2}\right)$

(D) $\text{sinc}\left(\frac{\omega}{2\pi}\right)\sin\left(\frac{\omega}{2}\right)$

**MCQ 3.30** Given a sequence $x[n]$, to generate the sequence $y[n] = x[3 - 4n]$, which one of the following procedures would be correct ?

(A) First delay $x(n)$ by 3 samples to generate $z_1[n]$, then pick every $4^{th}$ sample of $z_1[n]$ to generate $z_2[n]$, and than finally time reverse $z_2[n]$ to obtain $y[n]$.

(B) First advance $x[n]$ by 3 samples to generate $z_1[n]$, then pick every $4^{\text{th}}$ sample of $z_1[n]$ to generate $z_2[n]$, and then finally time reverse $z_2[n]$ to obtain $y[n]$

(C) First pick every fourth sample of $x[n]$ to generate $v_1[n]$, time-reverse $v_1[n]$ to obtain $v_2[n]$, and finally advance $v_2[n]$ by 3 samples to obtain $y[n]$

(D) First pick every fourth sample of $x[n]$ to generate $v_1[n]$, time-reverse $v_1[n]$ to obtain $v_2[n]$, and finally delay $v_2[n]$ by 3 samples to obtain $y[n]$

## YEAR 2007 ONE MARK

**MCQ 3.31** Let a signal $a_1 \sin(\omega_1 t + \phi)$ be applied to a stable linear time variant system. Let the corresponding steady state output be represented as $a_2 F(\omega_2 t + \phi_2)$. Then which of the following statement is true?

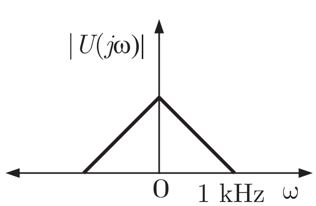
(A) $F$ is not necessarily a "Sine" or "Cosine" function but must be periodic with $\omega_1 = \omega_2$.

(B) $F$ must be a "Sine" or "Cosine" function with $a_1 = a_2$

(C) $F$ must be a "Sine" function with $\omega_1 = \omega_2$ and $\phi_1 = \phi_2$

(D) $F$ must be a "Sine" or "Cosine" function with $\omega_1 = \omega_2$

**MCQ 3.32** The frequency spectrum of a signal is shown in the figure. If this is ideally sampled at intervals of 1 ms, then the frequency spectrum of the sampled signal will be

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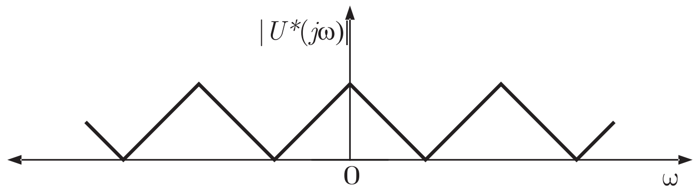
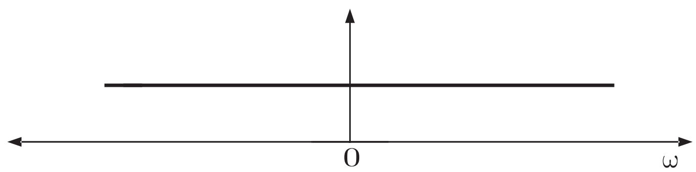
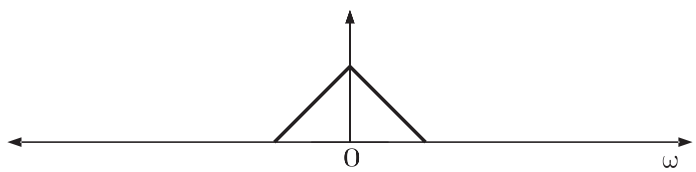
(A)



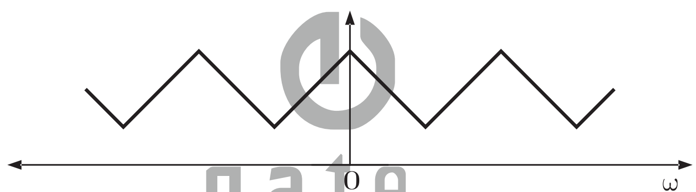

(B)




(C)




(D)




**YEAR 2007** **TWO MARKS**

**MCQ 3.33** A signal $x(t)$ is given by

$$x(t) = \begin{cases} 1, - T/4 < t \leq 3T/4 \\ -1, 3T/4 < t \leq 7T/4 \\ -x(t+T) \end{cases}$$

Which among the following gives the fundamental fourier term of $x(t)$ ?

(A) $\frac{4}{\pi}\cos\left(\frac{\pi t}{T} - \frac{\pi}{4}\right)$ (B) $\frac{\pi}{4}\cos\left(\frac{\pi t}{2T} + \frac{\pi}{4}\right)$

(C) $\frac{4}{\pi}\sin\left(\frac{\pi t}{T} - \frac{\pi}{4}\right)$ (D) $\frac{\pi}{4}\sin\left(\frac{\pi t}{2T} + \frac{\pi}{4}\right)$

**Statement for Linked Answer Question 34 and 35 :**

**MCQ 3.34** A signal is processed by a causal filter with transfer function $G(s)$
For a distortion free output signal wave form, $G(s)$ must
(A) provides zero phase shift for all frequency
(B) provides constant phase shift for all frequency

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(C) provides linear phase shift that is proportional to frequency

(D) provides a phase shift that is inversely proportional to frequency

**MCQ 3.35** $G(z) = \alpha z^{-1} + \beta z^{-3}$ is a low pass digital filter with a phase characteristics same as that of the above question if

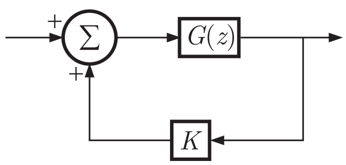
(A) $\alpha = \beta$

(B) $\alpha = -\beta$

(C) $\alpha = \beta^{(1/3)}$

(D) $\alpha = \beta^{(-1/3)}$

**MCQ 3.36** Consider the discrete-time system shown in the figure where the impulse response of $G(z)$ is $g(0) = 0, g(1) = g(2) = 1, g(3) = g(4) = \cdots = 0$




This system is stable for range of values of $K$

(A) $[-1, \frac{1}{2}]$

(B) $[-1, 1]$

(C) $[-\frac{1}{2}, 1]$

(D) $[-\frac{1}{2}, 2]$

**MCQ 3.37** If $u(t), r(t)$ denote the unit step and unit ramp functions respectively and $u(t) * r(t)$ their convolution, then the function $u(t+1) * r(t-2)$ is given by

(A) $\frac{1}{2}(t-1)\,u(t-1)$

(B) $\frac{1}{2}(t-1)\,u(t-2)$

(C) $\frac{1}{2}(t-1)^2 u(t-1)$

(D) None of the above

**MCQ 3.38** $X(z) = 1 - 3z^{-1}$, $Y(z) = 1 + 2z^{-2}$ are Z transforms of two signals $x[n], y[n]$ respectively. A linear time invariant system has the impulse response $h[n]$ defined by these two signals as $h[n] = x[n-1] * y[n]$ where * denotes discrete time convolution. Then the output of the system for the input $\delta[n-1]$

(A) has Z-transform $z^{-1}X(z)\,Y(z)$

(B) equals $\delta[n-2] - 3\delta[n-3] + 2\delta[n-4] - 6\delta[n-5]$

(C) has Z-transform $1 - 3z^{-1} + 2z^{-2} - 6z^{-3}$

(D) does not satisfy any of the above three

**YEAR 2006** **ONE MARK**

**MCQ 3.39** The following is true

(A) A finite signal is always bounded

(B) A bounded signal always possesses finite energy

(C) A bounded signal is always zero outside the interval $[-t_0, t_0]$ for some $t_0$

(D) A bounded signal is always finite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**MCQ 3.40** $x(t)$ is a real valued function of a real variable with period $T$. Its trigonometric Fourier Series expansion contains no terms of frequency $\omega = 2\pi(2k)/T; k = 1, 2\cdots$ Also, no sine terms are present. Then $x(t)$ satisfies the equation

(A) $x(t) = -x(t-T)$

(B) $x(t) = x(T-t) = -x(-t)$

(C) $x(t) = x(T-t) = -x(t-T/2)$

(D) $x(t) = x(t-T) = x(t-T/2)$

**MCQ 3.41** A discrete real all pass system has a pole at $z = 2\angle 30^\circ$: it, therefore

(A) also has a pole at $\frac{1}{2}\angle 30^\circ$

(B) has a constant phase response over the $z$-plane: $\arg|H(z)| =$ constant constant

(C) is stable only if it is anti-causal

(D) has a constant phase response over the unit circle: $\arg|H(e^{i\Omega})| =$ constant

**YEAR 2006 TWO MARKS**

**MCQ 3.42** $x[n] = 0; n < -1, n > 0, x[-1] = -1, x[0] = 2$ is the input and $y[n] = 0; n < -1, n > 2, y[-1] = -1 = y[1], y[0] = 3, y[2] = -2$ is the output of a discrete-time LTI system. The system impulse response $h[n]$ will be

(A) $h[n] = 0; n < 0, n > 2, h[0] = 1, h[1] = h[2] = -1$

(B) $h[n] = 0; n < -1, n > 1, h[-1] = 1, h[0] = h[1] = 2$

(C) $h[n] = 0; n < 0, n > 3, h[0] = -1, h[1] = 2, h[2] = 1$

(D) $h[n] = 0; n < -2, n > 1, h[-2] = h[1] = h[-1] = -h[0] = 3$

**MCQ 3.43** The discrete-time signal $x[n] \longleftrightarrow X(z) = \sum_{n=0}^{\infty} \frac{3^n}{2+n} z^{2n}$, where $\longleftrightarrow$ denotes a transform-pair relationship, is orthogonal to the signal

(A) $y_1[n] \leftrightarrow Y_1(z) = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n z^{-n}$

(B) $y_2[n] \leftrightarrow Y_2(z) = \sum_{n=0}^{\infty} (5^n - n) z^{-(2n+1)}$

(C) $y_3[n] \leftrightarrow Y_3(z) = \sum_{n=-\infty}^{\infty} 2^{-|n|} z^{-n}$

(D) $y_4[n] \leftrightarrow Y_4(z) = 2z^{-4} + 3z^{-2} + 1$

**MCQ 3.44** A continuous-time system is described by $y(t) = e^{-|x(t)|}$, where $y(t)$ is the output and $x(t)$ is the input. $y(t)$ is bounded

(A) only when $x(t)$ is bounded

(B) only when $x(t)$ is non-negative

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(C) only for $t \le 0$ if $x(t)$ is bounded for $t \ge 0$

(D) even when $x(t)$ is not bounded

**MCQ 3.45** The running integration, given by $y(t) = \int_{-\infty}^{t} x(t')\,dt'$

(A) has no finite singularities in its double sided Laplace Transform $Y(s)$

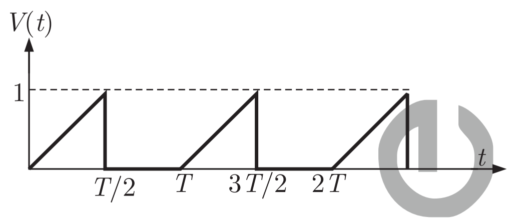
(B) produces a bounded output for every causal bounded input

(C) produces a bounded output for every anticausal bounded input

(D) has no finite zeroes in its double sided Laplace Transform $Y(s)$

**YEAR 2005 TWO MARKS**

**MCQ 3.46** For the triangular wave from shown in the figure, the RMS value of the voltage is equal to




(A) $\sqrt{\frac{1}{6}}$

(B) $\sqrt{\frac{1}{3}}$

(C) $\frac{1}{3}$

(D) $\sqrt{\frac{2}{3}}$

**MCQ 3.47** The Laplace transform of a function $f(t)$ is $F(s) = \frac{5s^2 + 23s + 6}{s(s^2 + 2s + 2)}$ as $t \to \infty$, $f(t)$ approaches

(A) 3

(B) 5

(C) $\frac{17}{2}$

(D) $\infty$

**MCQ 3.48** The Fourier series for the function $f(x) = \sin^2 x$ is

(A) $\sin x + \sin 2x$

(B) $1 - \cos 2x$

(C) $\sin 2x + \cos 2x$

(D) $0.5 - 0.5\cos 2x$

**MCQ 3.49** If $u(t)$ is the unit step and $\delta(t)$ is the unit impulse function, the inverse $z$ -transform of $F(z) = \frac{1}{z+1}$ for $k > 0$ is

(A) $(-1)^k \delta(k)$

(B) $\delta(k) - (-1)^k$

(C) $(-1)^k u(k)$

(D) $u(k) - (-1)^k$

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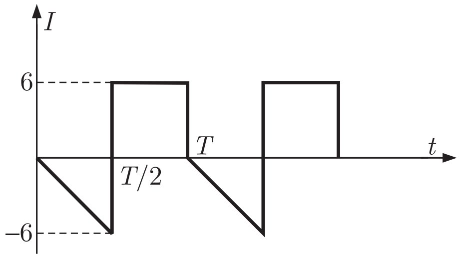
**YEAR 2004** **TWO MARKS**

**MCQ 3.50** The rms value of the periodic waveform given in figure is




(A) $2\sqrt{6}$ A (B) $6\sqrt{2}$ A

(C) $\sqrt{4/3}$ A (D) 1.5 A

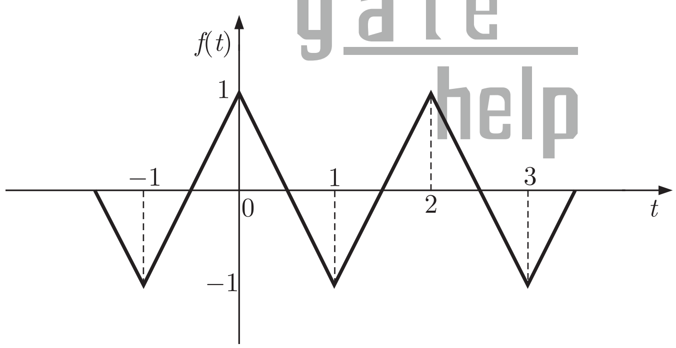
**MCQ 3.51** The rms value of the resultant current in a wire which carries a dc current of 10 A and a sinusoidal alternating current of peak value 20 is

(A) 14.1 A (B) 17.3 A

(C) 22.4 A (D) 30.0 A

**YEAR 2002** **ONE MARK**

**MCQ 3.52** Fourier Series for the waveform, $f(t)$ shown in Figure is




(A) $\frac{8}{\pi^2}\left[\sin(\pi t) + \frac{1}{9}\sin(3\pi t) + \frac{1}{25}\sin(5\pi t) + .....\right]$

(B) $\frac{8}{\pi^2}\left[\sin(\pi t) - \frac{1}{9}\cos(3\pi t) + \frac{1}{25}\sin(5\pi t) + .......\right]$

(C) $\frac{8}{\pi^2}\left[\cos(\pi t) + \frac{1}{9}\cos(3\pi t) + \frac{1}{25}\cos(5\pi t) + .....\right]$

(D) $\frac{8}{\pi^2}\left[\cos(\pi t) - \frac{1}{9}\sin(3\pi t) + \frac{1}{25}\sin(5\pi t) + .......\right]$

**MCQ 3.53** Let $s(t)$ be the step response of a linear system with zero initial conditions; then the response of this system to an an input $u(t)$ is

(A) $\int_0^t s(t-\tau)\,u(\tau)\,d\tau$ (B) $\frac{d}{dt}\left[\int_0^t s(t-\tau)\,u(\tau)\,d\tau\right]$

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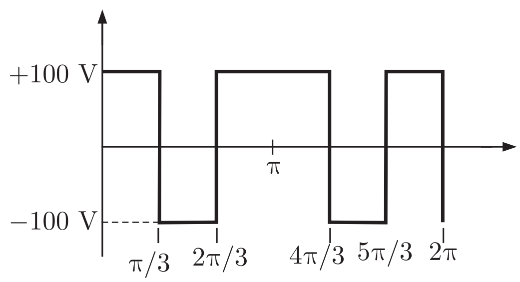
(C) $\int_0^t s(t-\tau)\left[\int_0^t u(\tau_1)\,d\tau_1\right]d\tau$ (D) $\int_0^1 [s(t-\tau)]^2 u(\tau)\,d\tau$

**MCQ 3.54** Let $Y(s)$ be the Laplace transformation of the function $y(t)$, then the final value of the function is

(A) $\underset{s\to 0}{\text{Lim}}\,Y(s)$ (B) $\underset{s\to\infty}{\text{Lim}}\,Y(s)$

(C) $\underset{s\to 0}{\text{Lim}}\,sY(s)$ (D) $\underset{s\to\infty}{\text{Lim}}\,sY(s)$

**MCQ 3.55** What is the rms value of the voltage waveform shown in Figure ?




(A) $(200/\pi)$ V (B) $(100/\pi)$ V

(C) 200 V (D) 100 V

**YEAR 2001 ONE MARK**

**MCQ 3.56** Given the relationship between the input $u(t)$ and the output $y(t)$ to be

$$y(t) = \int_0^t (2+t-\tau)\,e^{-3(t-\tau)}u(\tau)\,d\tau,$$

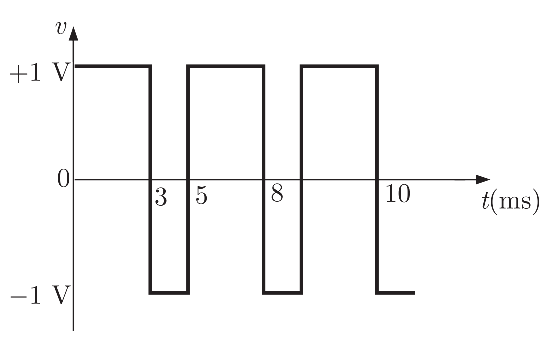
The transfer function $Y(s)/U(s)$ is

(A) $\frac{2e^{-2s}}{s+3}$ (B) $\frac{s+2}{(s+3)^2}$

(C) $\frac{2s+5}{s+3}$ (D) $\frac{2s+7}{(s+3)^2}$

### Common data Questions Q.57-58*

Consider the voltage waveform $v$ as shown in figure

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**MCQ 3.57** The DC component of $v$ is

(A) 0.4 (B) 0.2

(C) 0.8 (D) 0.1

**MCQ 3.58** The amplitude of fundamental component of $v$ is

(A) 1.20 V (B) 2.40 V

(C) 2 V (D) 1 V

***********

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# SOLUTION

**SOL 3.1** Option (C) is correct.

$$x[n] = \left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{2}\right)^n u[n]$$
$$= \left(\frac{1}{3}\right)^n u[n] + \left(\frac{1}{3}\right)^{-n} u[-n-1] - \left(\frac{1}{2}\right)^n u(n)$$

Taking $z$-transform

$$X[z] = \sum_{n=-\infty}^{\infty}\left(\frac{1}{3}\right)^n z^{-n} u[n] + \sum_{n=-\infty}^{\infty}\left(\frac{1}{3}\right)^{-n} z^{-n} u[-n-1]$$
$$- \sum_{n=-\infty}^{\infty}\left(\frac{1}{2}\right)^n z^{-n} u[n] = \sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n z^{-n} + \sum_{n=-\infty}^{-1}\left(\frac{1}{3}\right)^{-n} z^{-n} - \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n z^{-n}$$
$$= \underbrace{\sum_{n=0}^{\infty}\left(\frac{1}{3z}\right)^n}_{\text{I}} + \underbrace{\sum_{m=1}^{\infty}\left(\frac{1}{3}z\right)^m}_{\text{II}} - \underbrace{\sum_{n=0}^{\infty}\left(\frac{1}{2z}\right)^n}_{\text{III}} \qquad \text{Taking } m = -n$$

Series I converges if $\left|\frac{1}{3z}\right| < 1$ or $|z| > \frac{1}{3}$

Series II converges if $\left|\frac{1}{3}z\right| < 1$ or $|z| < 3$

Series III converges if $\left|\frac{1}{2z}\right| < 1$ or $|z| > \frac{1}{2}$

Region of convergence of $X(z)$ will be intersection of above three

So, ROC : $\frac{1}{2} < |z| < 3$

**SOL 3.2** Option (D) is correct.

Using $s$-domain differentiation property of Laplace transform.

If $$f(t) \overset{\mathcal{L}}{\longleftrightarrow} F(s)$$

$$tf(t) \overset{\mathcal{L}}{\longleftrightarrow} -\frac{dF(s)}{ds}$$

So, $$\mathcal{L}[tf(t)] = \frac{-d}{ds}\left[\frac{1}{s^2+s+1}\right] = \frac{2s+1}{(s^2+s+1)^2}$$

**SOL 3.3** Option (A) is correct.

Convolution sum is defined as

$$y[n] = h[n] * g[n] = \sum_{k=-\infty}^{\infty} h[n]\, g[n-k]$$

For causal sequence, $$y[n] = \sum_{k=0}^{\infty} h[n]\, g[n-k]$$

$$y[n] = h[n]\, g[n] + h[n]\, g[n-1] + h[n]\, g[n-2] + \ldots..$$

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For $n = 0$, $$y[0] = h[0]\,g[0] + h[1]\,g[-1] + ..........$$
$$= h[0]\,g[0] \qquad g[-1] = g[-2] = ....0$$
$$= h[0]\,g[0] \qquad ...(i)$$

For $n = 1$, $$y[1] = h[1]\,g[1] + h[1]\,g[0] + h[1]\,g[-1] + ....$$
$$= h[1]\,g[1] + h[1]\,g[0]$$
$$\frac{1}{2} = \frac{1}{2}g[1] + \frac{1}{2}g[0] \qquad h[1] = \left(\frac{1}{2}\right)^1 = \frac{1}{2}$$
$$1 = g[1] + g[0]$$
$$g[1] = 1 - g[0]$$

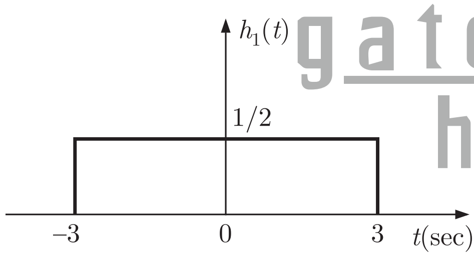
From equation (i), $$g[0] = \frac{y[0]}{h[0]} = \frac{1}{1} = 1$$

So, $$g[1] = 1 - 1 = 0$$

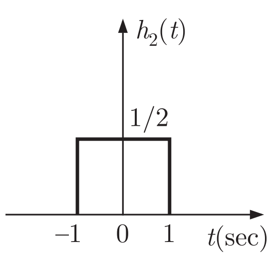
**SOL 3.4** Option (C) is correct.

$$H(j\omega) = \frac{(2\cos\omega)(\sin 2\omega)}{\omega} \qquad = \frac{\sin 3\omega}{\omega} + \frac{\sin\omega}{\omega}$$

We know that inverse Fourier transform of $\sin c$ function is a rectangular function.




$$\overset{\mathcal{F}}{\longleftrightarrow} \quad \frac{\sin 3\omega}{\omega}$$




$$\overset{\mathcal{F}}{\longleftrightarrow} \quad \frac{\sin\omega}{\omega}$$

So, inverse Fourier transform of $H(j\omega)$

$$h(t) = h_1(t) + h_2(t)$$
$$h(0) = h_1(0) + h_2(0) = \frac{1}{2} + \frac{1}{2} = 1$$

**SOL 3.5** Option (D) is correct.

$$y(t) = \int_{-\infty}^{t} x(\tau)\cos(3\tau)\,d\tau$$

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**Time invariance :**

Let, $x(t) = \delta(t)$

$$y(t) = \int_{-\infty}^{t} \delta(t)\cos(3\tau)\,d\tau = u(t)\cos(0) = u(t)$$

For a delayed input $(t - t_0)$ output is

$$y(t, t_0) = \int_{-\infty}^{t} \delta(t - t_0)\cos(3\tau)\,d\tau = u(t)\cos(3t_0)$$

Delayed output

$$y(t - t_0) = u(t - t_0)$$

$$y(t, t_0) \neq y(t - t_0)$$

System is not time invariant.

**Stability :**

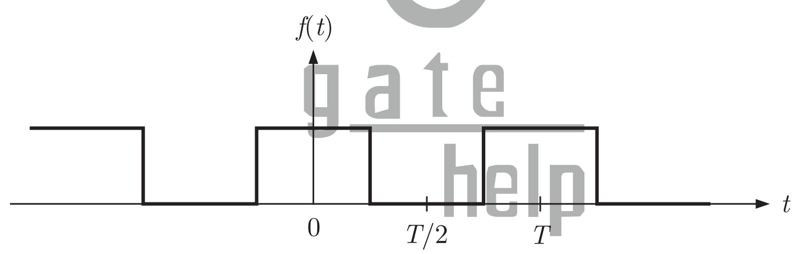
Consider a bounded input $x(t) = \cos 3t$

$$y(t) = \int_{-\infty}^{t} \cos^2 3t = \int_{-\infty}^{t} \frac{1 - \cos 6t}{2} = \frac{1}{2}\int_{-\infty}^{t} 1\,dt - \frac{1}{2}\int_{-\infty}^{t} \cos 6t\,dt$$

As $t \to \infty$, $y(t) \to \infty$ (unbounded)

System is not stable.

**SOL 3.6** Option (D) is correct.




$$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos\omega t + b_n \sin n\omega t)$$

- The given function $f(t)$ is an even function, therefore $b_n = 0$
- $f(t)$ is a non zero average value function, so it will have a non-zero value of $a_0$

$$a_0 = \frac{1}{(T/2)}\int_0^{T/2} f(t)\,dt \text{ (average value of } f(t))$$

- $a_n$ is zero for all even values of $n$ and non zero for odd $n$

$$a_n = \frac{2}{T}\int_0^{T} f(t)\cos(n\omega t)\,d(\omega t)$$

So, Fourier expansion of $f(t)$ will have $a_0$ and $a_n$, $n = 1, 3, 5 \dots \infty$

**SOL 3.7** Option (A) is correct.

$$x(t) = e^{-t}$$

Laplace transformation

$$X(s) = \frac{1}{s+1}$$

$$y(t) = e^{-2t}$$

$$Y(s) = \frac{1}{s+2}$$

Convolution in time domain is equivalent to multiplication in frequency domain.

$$z(t) = x(t) * y(t)$$

$$Z(s) = X(s)\,Y(s) = \left(\frac{1}{s+1}\right)\left(\frac{1}{s+2}\right)$$

By partial fraction and taking inverse Laplace transformation, we get

$$Z(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

$$z(t) = e^{-t} - e^{-2t}$$

**SOL 3.8** Option (D) is correct.

$$f(t) \overset{\mathcal{L}}{\longleftrightarrow} F_1(s)$$

$$f(t-\tau) \overset{\mathcal{L}}{\longleftrightarrow} e^{-s\tau} F_1(s) = F_2(s)$$

$$G(s) = \frac{F_2(s)\,F_1^*(s)}{|F_1(s)|^2} = \frac{e^{-s\tau} F_1(s)\,F_1^*(s)}{|F_1(s)|^2}$$

$$= \frac{e^{-sE}|F_1(s)|^2}{|F_1(s)|^2} \qquad \{\because\ F_1(s)\,F_1^*(s) = |F_1(s)|^2$$

$$= e^{-s\tau}$$

Taking inverse Laplace transform

$$g(t) = \mathcal{L}^{-1}[e^{-s\tau}] = \delta(t-\tau)$$

**SOL 3.9** Option (C) is correct.

$$h(t) = e^{-t} + e^{-2t}$$

Laplace transform of $h(t)$ i.e. the transfer function

$$H(s) = \frac{1}{s+1} + \frac{1}{s+2}$$

For unit step input

$$r(t) = \mu(t)$$

or $$R(s) = \frac{1}{s}$$

Output, $$Y(s) = R(s)\,H(s) = \frac{1}{s}\left[\frac{1}{s+1} + \frac{1}{s+2}\right]$$

By partial fraction

$$Y(s) = \frac{3}{2s} - \frac{1}{s+1} - \left(\frac{1}{s+2}\right)\frac{1}{2}$$

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Taking inverse Laplace

$$\begin{aligned} y(t) &= \frac{3}{2}u(t) - e^{-t}u(t) - \frac{e^{-2t}u(t)}{2} \\ &= u(t)[1.5 - e^{-t} - 0.5e^{-2t}] \end{aligned}$$

**SOL 3.10** Option (C) is correct.

System is given as

$$H(s) = \frac{2}{(s+1)}$$

Step input $\quad R(s) = \frac{1}{s}$

Output $\quad Y(s) = H(s)\,R(s) = \frac{2}{(s+1)}\left(\frac{1}{s}\right) = \frac{2}{s} - \frac{2}{(s+1)}$

Taking inverse Laplace transform

$$y(t) = (2 - 2e^{-t})\,u(t)$$

Final value of $y(t)$,

$$y_{ss}(t) = \lim_{t \to \infty} y(t) = 2$$

Let time taken for step response to reach 98% of its final value is $t_s$.

So,

$$\begin{aligned} 2 - 2e^{-t_s} &= 2 \times 0.98 \\ 0.02 &= e^{-t_s} \\ t_s &= \ln 50 = 3.91 \text{ sec.} \end{aligned}$$

**SOL 3.11** Option (D) is correct.

Period of $x(t)$,

$$T = \frac{2\pi}{\omega} = \frac{2\,\pi}{0.8\,\pi} = 2.5 \text{ sec}$$

**SOL 3.12** Option (B) is correct.

Input output relationship

$$y(t) = \int_{-\infty}^{5t} x(\tau)\,d\tau, \qquad t > 0$$

**Causality :**

$y(t)$ depends on $x(5t), t > 0$ system is non-causal.

For example $t = 2$

$y(2)$ depends on $x(10)$ (future value of input)

**Linearity :**

Output is integration of input which is a linear function, so system is linear.

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**SOL 3.13** Option (A) is correct.

Fourier series of given function

$$x(t) = A_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$\because x(t) = -x(t) \text{ odd function}$$

So, $A_0 = 0$

$$a_n = 0$$

$$b_n = \frac{2}{T}\int_0^T x(t)\sin n\omega_0 t\,dt$$

$$= \frac{2}{T}\left[\int_0^{T/2}(1)\sin n\omega_0 t\,dt + \int_{T/2}^{T}(-1)\sin n\omega_0 t\,dt\right]$$

$$= \frac{2}{T}\left[\left(\frac{\cos n\omega_0 t}{-n\omega_0}\right)_0^{T/2} - \left(\frac{\cos n\omega_0 t}{-n\omega_0}\right)_{T/2}^{T}\right]$$

$$= \frac{2}{n\omega_0 T}[(1-\cos n\pi) + (\cos 2n\pi - \cos n\pi)]$$

$$= \frac{2}{n\pi}[1-(-1)^n]$$

$$b_n = \begin{cases} \frac{4}{n\pi}, & n \text{ odd} \\ 0 \quad, & n \text{ even} \end{cases}$$

So only odd harmonic will be present in $x(t)$

For second harmonic component $(n = 2)$ amplitude is zero.

**SOL 3.14** Option (D) is correct.

By parsval's theorem

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2 d\omega = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$\int_{-\infty}^{\infty}|X(\omega)|^2 d\omega = 2\pi \times 2 = 4\pi$$

**SOL 3.15** Option (C) is correct.

Given sequences $x[n] = \{\underset{\uparrow}{1}, -1\}, \ 0 \le n \le 1$

$$y[n] = \{\underset{\uparrow}{1},0,0,0,-1\}, \ 0 \le n \le 4$$

If impulse response is $h[n]$ then

$$y[n] = h[n] * x[n]$$

Length of convolution $(y[n])$ is 0 to 4, $x[n]$ is of length 0 to 1 so length of $h[n]$ will be 0 to 3.

Let $h[n] = \{\underset{\uparrow}{a}, b, c, d\}$

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Convolution

| | $a$ | $b$ | $c$ | $c$ |
|---|---|---|---|---|
| 1 | $a$ | $b$ | $c$ | $d$ |
| $-1$ | $-a$ | $-b$ | $-c$ | $-d$ |

$$y[n] = \{\underset{\uparrow}{a}, -a+b, -b+c, -c+d, -d\}$$

By comparing

$$\begin{aligned} a &= 1 \\ -a+b &= 0 \quad \Rightarrow b = a = 1 \\ -b+c &= 0 \quad \Rightarrow c = b = 1 \\ -c+d &= 0 \quad \Rightarrow d = c = 1 \end{aligned}$$

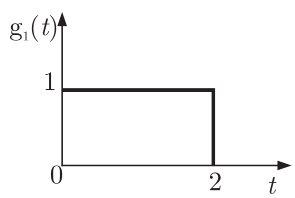
So, $$h[n] = \{\underset{\uparrow}{1}, 1, 1, 1\}$$

**SOL 3.16** Option (D) is correct.

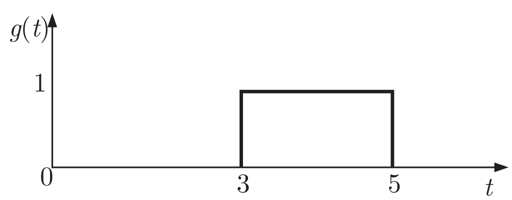
We can observe that if we scale $f(t)$ by a factor of $\frac{1}{2}$ and then shift, we will get $g(t)$.

First scale $f(t)$ by a factor of $\frac{1}{2}$

$$g_1(t) = f(t/2)$$




Shift $g_1(t)$ by 3, $$g(t) = g_1(t-3) = f\left(\frac{t-3}{2}\right)$$




$$g(t) = f\left(\frac{t}{2} - \frac{3}{2}\right)$$

**SOL 3.17** Option (C) is correct.

$g(t)$ can be expressed as

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$$g(t) = u(t-3) - u(t-5)$$

By shifting property we can write Laplace transform of $g(t)$

$$G(s) = \frac{1}{s}e^{-3s} - \frac{1}{s}e^{-5s} = \frac{e^{-3s}}{s}(1-e^{-2s})$$

**SOL 3.18** Option (D) is correct.

Let $x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s)$

$y(t) \overset{\mathcal{L}}{\longleftrightarrow} Y(s)$

$h(t) \overset{\mathcal{L}}{\longleftrightarrow} H(s)$

So output of the system is given as

$$Y(s) = X(s)H(s)$$

Now for input $x(t-\tau) \overset{\mathcal{L}}{\longleftrightarrow} e^{-s\tau}X(s)$ (shifting property)

$h(t-\tau) \overset{\mathcal{L}}{\longleftrightarrow} e^{-s\tau}H(s)$

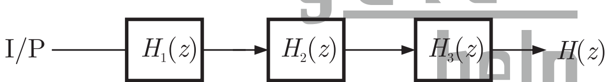
So now output is $Y'(s) = e^{-s\tau}X(s)\cdot e^{-\tau s}H(s)$

$$= e^{-2s\tau}X(s)H(s) = e^{-2s\tau}Y(s)$$

$$y'(t) = y(t-2\tau)$$

**SOL 3.19** Option (B) is correct.

Let three LTI systems having response $H_1(z), H_2(z)$ and $H_3(z)$ are Cascaded as showing below




Assume $H_1(z) = z^2 + z^1 + 1$ (non-causal)

$H_2(z) = z^3 + z^2 + 1$ (non-causal)

Overall response of the system

$$H(z) = H_1(z)H_2(z)H_3(z)$$

$$H(z) = (z^2+z^1+1)(z^3+z^2+1)H_3(z)$$

To make $H(z)$ causal we have to take $H_3(z)$ also causal.

Let $H_3(z) = z^{-6} + z^{-4} + 1$

$$= (z^2+z^1+1)(z^3+z^2+1)(z^{-6}+z^{-4}+1)$$

$H(z) \to$ causal

Similarly to make $H(z)$ unstable atleast one of the system should be unstable.

**SOL 3.20** Option (C) is correct.

Given signal

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T}$$

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Let $\omega_0$ is the fundamental frequency of signal $x(t)$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \qquad \because \frac{2\pi}{T} = \omega_0$$

$$\begin{aligned} x(t) &= a_{-2} e^{-j2\omega_0 t} + a_{-1} e^{-j\omega_0 t} + a_0 + a_1 e^{j\omega_0 t} + a_2 e^{j2\omega_0 t} \\ &= (2-j)\, e^{-2j\omega_0 t} + (0.5+0.2j)\, e^{-j\omega_0 t} + 2j + \\ &\qquad + (0.5-0.2)\, e^{j\omega_0 t} + (2+j)\, e^{j2\omega_0 t} \\ &= 2[e^{-j2\omega_0 t} + e^{j2\omega_0 t}] + j[e^{j2\omega_0 t} - e^{-j2\omega_0 t}] + \\ &\qquad 0.5[e^{j\omega_0 t} + e^{-j\omega_0 t}] - 0.2j[e^{+j\omega_0 t} - e^{-j\omega_0 t}] + 2j \\ &= 2(2\cos 2\omega_0 t) + j(2j\sin 2\omega_0 t) + 0.5(2\cos\omega_0 t) - \\ &\qquad 0.2j(2j\sin\omega_0 t) + 2j \\ &= [4\cos 2\omega_0 t - 2\sin 2\omega_0 t + \cos\omega_0 t + 0.4\sin\omega_0 t] + 2j \\ \mathrm{Im}[x(t)] &= 2 \quad \text{(constant)} \end{aligned}$$

**SOL 3.21** Option (A) is correct.

Z-transform of $x[n]$ is

$$X(z) = 4z^{-3} + 3z^{-1} + 2 - 6z^2 + 2z^3$$

Transfer function of the system

$$H(z) = 3z^{-1} - 2$$

Output

$$\begin{aligned} Y(z) &= H(z)X(z) \\ Y(z) &= (3z^{-1} - 2)(4z^{-3} + 3z^{-1} + 2 - 6z^2 + 2z^3) \\ &= 12z^{-4} + 9z^{-2} + 6z^{-1} - 18z + 6z^2 - 8z^{-3} - 6z^{-1} - 4 + 12z^2 - 4z^3 \\ &= 12z^{-4} - 8z^{-3} + 9z^{-2} - 4 - 18z + 18z^2 - 4z^3 \end{aligned}$$

Or sequence $y[n]$ is

$$\begin{aligned} y[n] &= 12\delta[n-4] - 8\delta[n-3] + 9\delta[n-2] - 4\delta[n] - \\ &\qquad 18\delta[n+1] + 18\delta[n+2] - 4\delta[n+3] \end{aligned}$$

$$y[n] \neq 0,\ n < 0$$

So $y[n]$ is non-causal with finite support.

**SOL 3.22** Option (D) is correct.

Since the given system is LTI, So principal of Superposition holds due to linearity.

For causal system $h(t) = 0,\ t < 0$

Both statement are correct.

**SOL 3.23** Option (C) is correct.

For an LTI system output is a constant multiplicative of input with same frequency.

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Here input $g(t) = e^{-\alpha t}\sin(\omega t)$

output $y(t) = Ke^{-\beta t}\sin(vt+\phi)$

Output will be in form of $Ke^{-\alpha t}\sin(\omega t+\phi)$

So $\alpha = \beta, v = \omega$

**SOL 3.24** Option (D) is correct.

Input-output relation

$$y(t) = \int_{-\infty}^{-2t} x(\tau)\, d\tau$$

**Causality :**

Since $y(t)$ depends on $x(-2t)$, So it is non-causal.

**Time-variance :**

$$y(t) = \int_{-\infty}^{-2t} x(\tau-\tau_0)\, d\tau \neq y(t-\tau_0)$$

So this is time-variant.

**Stability :**

Output $y(t)$ is unbounded for an bounded input.

For example

Let $x(\tau) = e^{-\tau}$ (bounded)

$$y(t) = \int_{-\infty}^{-2t} e^{-\tau} d\tau = \left[\frac{e^{-\tau}}{-1}\right]_{-\infty}^{-2t} \longrightarrow \text{Unbounded}$$

**SOL 3.25** Option (A) is correct.

Output $y(t)$ of the given system is

$$y(t) = x(t) * h(t)$$

Or $$Y(j\omega) = X(j\omega)\, H(j\omega)$$

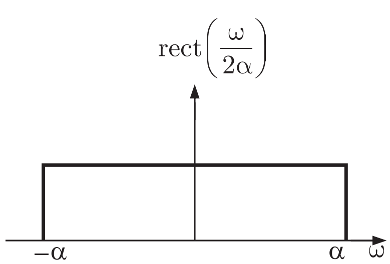
Given that, $x(t) = \text{sinc}(\alpha t)$ and $h(t) = \text{sinc}(\beta t)$

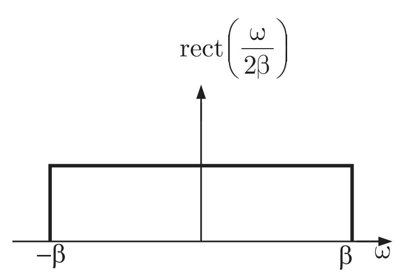
Fourier transform of $x(t)$ and $h(t)$ are

$$X(j\omega) = \mathcal{F}[x(t)] = \frac{\pi}{\alpha} rect\left(\frac{\omega}{2\alpha}\right), -\alpha < \omega < \alpha$$

$$H(j\omega) = \mathcal{F}[h(t)] = \frac{\pi}{\beta} rect\left(\frac{\omega}{2\beta}\right), -\beta < \omega < \beta$$

$$Y(j\omega) = \frac{\pi^2}{\alpha\beta}\text{rect}\left(\frac{\omega}{2\alpha}\right)\text{rect}\left(\frac{\omega}{2\beta}\right)$$

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So, $Y(j\omega) = \text{K rect}\left(\frac{\omega}{2\gamma}\right)$

Where $\gamma = \min(\alpha, \beta)$

And $y(t) = \text{K sinc}(\gamma t)$

**SOL 3.26** Option (B) is correct.

Let $a_k$ is the Fourier series coefficient of signal $x(t)$

Given $y(t) = x(t - t_0) + x(t + t_0)$

Fourier series coefficient of $y(t)$

$$b_k = e^{-jk\omega t_0} a_k + e^{jk\omega t_0} a_k$$

$$b_k = 2a_k \cos k\omega t_0$$

$$b_k = 0 \text{ (for all odd } k)$$

$$k\omega t_0 = \frac{\pi}{2}, \text{ k} \rightarrow \text{odd}$$

$$k\frac{2\pi}{T} t_0 = \frac{\pi}{2}$$

For $k = 1$, $t_0 = \frac{T}{4}$

**SOL 3.27** Option ( ) is correct.

**SOL 3.28** Option (D) is correct.

Given that $X(z) = \frac{z}{(z-a)^2}, \; |z| > a$

Residue of $X(z) z^{n-1}$ at $z = a$ is

$$= \frac{d}{dz}(z-a)^2 X(z) z^{n-1}\Big|_{z=a}$$

$$= \frac{d}{dz}(z-a)^2 \frac{z}{(z-a)^2} z^{n-1}\Big|_{z=a}$$

$$= \frac{d}{dz} z^n\Big|_{z=a} = nz^{n-1}\Big|_{z=a} = na^{n-1}$$

**SOL 3.29** Option (C) is correct.

Given signal

$$x(t) = \text{rect}\left(t - \frac{1}{2}\right)$$

So, $$x(t) = \begin{cases} 1, & -\frac{1}{2} \le t - \frac{1}{2} \le \frac{1}{2} \quad \text{or} \quad 0 \le t \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Similarly

$$x(-t) = \text{rect}\left(-t - \frac{1}{2}\right)$$

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$$x(-t) = \begin{cases} 1, & -\frac{1}{2} \le -t - \frac{1}{2} \le \frac{1}{2} \quad \text{or} \quad -1 \le t \le 0 \\ 0, & \text{elsewhere} \end{cases}$$

$$\mathcal{F}[x(t) + x(-t)] = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt + \int_{-\infty}^{\infty} x(-t)\, e^{-j\omega t}\, dt$$

$$= \int_0^1 (1)\, e^{-j\omega t}\, dt + \int_{-1}^{0} (1)\, e^{-j\omega t}\, dt$$

$$= \left[\frac{e^{-j\omega t}}{-j\omega}\right]_0^1 + \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-1}^0 = \frac{1}{j\omega}(1 - e^{-j\omega}) + \frac{1}{j\omega}(e^{j\omega} - 1)$$

$$= \frac{e^{-j\omega/2}}{j\omega}(e^{j\omega/2} - e^{-j\omega/2}) + \frac{e^{j\omega/2}}{j\omega}(e^{j\omega/2} - e^{-j\omega/2})$$

$$= \frac{(e^{j\omega/2} - e^{-j\omega/2})(e^{-j\omega/2} + e^{j\omega/2})}{j\omega}$$

$$= \frac{2}{\omega}\sin\left(\frac{\omega}{2}\right) \cdot 2\cos\left(\frac{\omega}{2}\right) = 2\cos\frac{\omega}{2}\,\mathrm{sinc}\left(\frac{\omega}{2\pi}\right)$$

**SOL 3.30** Option (B) is correct.

In option (A)

$$z_1[n] = x[n-3]$$
$$z_2[n] = z_1[4n] = x[4n-3]$$
$$y[n] = z_2[-n] = x[-4n-3] \ne x[3-4n]$$

In option (B)

$$z_1[n] = x[n+3]$$
$$z_2[n] = z_1[4n] = x[4n+3]$$
$$y[n] = z_2[-n] = x[-4n+3]$$

In option (C)

$$v_1[n] = x[4n]$$
$$v_2[n] = v_1[-n] = x[-4n]$$
$$y[n] = v_2[n+3] = x[-4(n+3)] \ne x[3-4n]$$

In option (D)

$$v_1[n] = x[4n]$$
$$v_2[n] = v_1[-n] = x[-4n]$$
$$y[n] = v_2[n-3] = x[-4(n-3)] \ne x[3-4n]$$

**SOL 3.31** Option ( ) is correct.

The spectrum of sampled signal $s(j\omega)$ contains replicas of $U(j\omega)$ at frequencies $\pm nf_s$.

Where $n = 0, 1, 2.......$

$$f_s = \frac{1}{T_s} = \frac{1}{1\,\text{m sec}} = 1\,\text{kHz}$$

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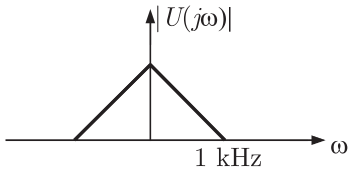
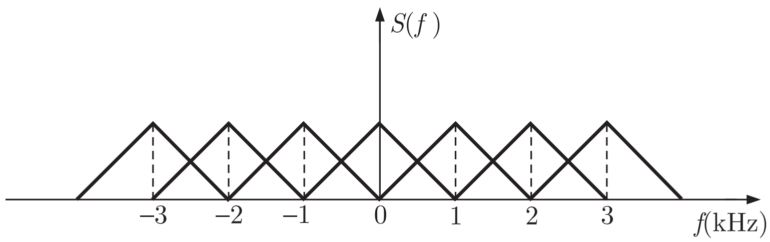
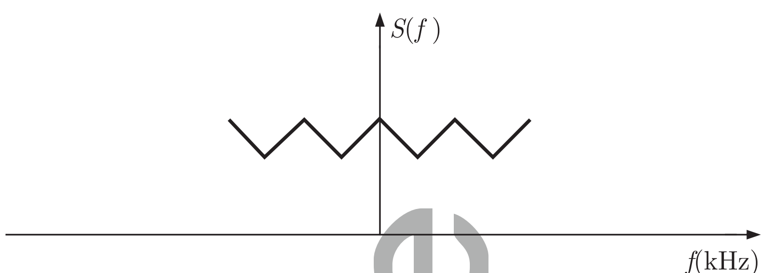


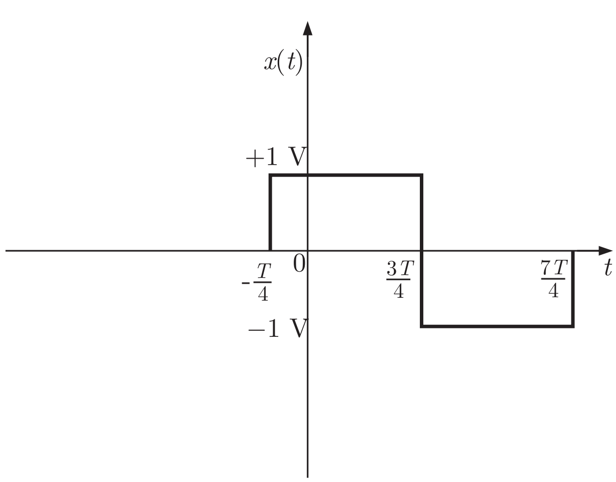

**SOL 3.32** Option (D) is correct.

For an LTI system input and output have identical wave shape (i.e. frequency of input-output is same) within a multiplicative constant (i.e. Amplitude response is constant)

So $F$ must be a sine or cosine wave with $\omega_1 = \omega_2$

**SOL 3.33** Option (C) is correct.

Given signal has the following wave-form

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Function x(t) is periodic with period $2T$ and given that

$$x(t) = -x(t+T) \text{ (Half-wave symmetric)}$$

So we can obtain the fourier series representation of given function.

**SOL 3.34** Option (C) is correct.

Output is said to be distortion less if the input and output have identical wave shapes within a multiplicative constant. A delayed output that retains input waveform is also considered distortion less.

Thus for distortion less output, input-output relationship is given as

$$y(t) = Kg(t - t_d)$$

Taking Fourier transform.

$$Y(\omega) = KG(\omega)\, e^{-j\omega t_d} = G(\omega) H(\omega)$$

$H(\omega) \Rightarrow$ transfer function of the system

So, $$H(\omega) = Ke^{-j\omega t_d}$$

Amplitude response $|H(\omega)| = K$

Phase response, $\theta_n(\omega) = -\omega t_d$

For distortion less output, phase response should be proportional to frequency.

**SOL 3.35** Option (A) is correct.

$$G(z)\big|_{z = e^{j\omega}} = \alpha e^{-j\omega} + \beta e^{-3j\omega}$$

for linear phase characteristic $\alpha = \beta$.



**SOL 3.36** Option (A) is correct.

System response is given as

$$H(z) = \frac{G(z)}{1 - KG(z)}$$

$$g[n] = \delta[n-1] + \delta[n-2]$$

$$G(z) = z^{-1} + z^{-2}$$

So $$H(z) = \frac{(z^{-1} + z^{-2})}{1 - K(z^{-1} + z^{-2})} = \frac{z+1}{z^2 - Kz - K}$$

For system to be stable poles should lie inside unit circle.

$$|z| \le 1$$

$$z = \frac{K \pm \sqrt{K^2 + 4K}}{2} \le 1 \quad K \pm \sqrt{K^2 + 4K} \le 2$$

$$\sqrt{K^2 + 4K} \le 2 - K$$

$$K^2 + 4K \le 4 - 4K + K^2$$

$$8K \le 4$$

$$K \le 1/2$$

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**SOL 3.37** Option (C) is correct.

Given Convolution is,

$$h(t) = u(t+1) * r(t-2)$$

Taking Laplace transform on both sides,

$$H(s) = \mathcal{L}[h(t)] = \mathcal{L}[u(t+1)] * \mathcal{L}[r(t-2)]$$

We know that, $\mathcal{L}[u(t)] = 1/s$

$$\mathcal{L}[u(t+1)] = e^{s}\left(\frac{1}{s^2}\right) \qquad \text{(Time-shifting property)}$$

and $$\mathcal{L}[r(t)] = 1/s^2$$

$$\mathcal{L}\, r(t-2) = e^{-2s}\left(\frac{1}{s^2}\right) \qquad \text{(Time-shifting property)}$$

So $$H(s) = \left[e^{s}\left(\frac{1}{s}\right)\right]\left[e^{-2s}\left(\frac{1}{s^2}\right)\right]$$

$$H(s) = e^{-s}\left(\frac{1}{s^3}\right)$$

Taking inverse Laplace transform

$$h(t) = \frac{1}{2}(t-1)^2 u(t-1)$$

**SOL 3.38** Option (C) is correct.

Impulse response of given LTI system.

$$h[n] = x[n-1] * y[n]$$

Taking $z$-transform on both sides.

$$H(z) = z^{-1}X(z)\,Y(z) \qquad \because\; x[n-1] \overset{\mathcal{Z}}{\longleftrightarrow} z^{-1}x(z)$$

We have $X(z) = 1 - 3z^{-1}$ and $Y(z) = 1 + 2z^{-2}$

So

$$H(z) = z^{-1}(1-3z^{-1})(1+2z^{-2})$$

Output of the system for input $u[n] = \delta[n-1]$ is,

$$y(z) = H(z)\,U(z) \qquad U[n] \overset{\mathcal{Z}}{\longleftrightarrow} U(z) = z^{-1}$$

So

$$Y(z) = z^{-1}(1-3z^{-1})(1+2z^{-2})\,z^{-1}$$

$$= z^{-2}(1-3z^{-1}+2z^{-2}-6z^{-3}) = z^{-2}-3z^{-3}+2z^{-4}-6z^{-5}$$

Taking inverse z-transform on both sides we have output.

$$y[n] = \delta[n-2] - 3\delta[n-3] + 2\delta[n-4] - 6\delta[n-5]$$

**SOL 3.39** Option (B) is correct.

A bounded signal always possesses some finite energy.

$$E = \int_{-t_0}^{t_0} |g(t)^2|\,dt < \infty$$

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**SOL 3.40** Option (C) is correct.

Trigonometric Fourier series is given as

$$x(t) = A_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

Since there are no sine terms, so $b_n = 0$

$$b_n = \frac{2}{T_0}\int_0^{T_0} x(t) \sin n\omega_0 t\, dt$$

$$= \frac{2}{T_0}\left[\int_0^{T_0/2} x(\tau)\sin n\omega_0\tau\; d\tau + \int_{T_0/2}^{T} x(t)\sin n\omega_0 t\, dt\right]$$

Where $\tau = T - t \;\; \Rightarrow d\tau = -\, dt$

$$= \frac{2}{T_0}\left[\int_{T_0}^{T_0/2} x(T-t)\sin n\omega_0(T-t)\,(-\,dt) + \int_{T_0/2}^{T} x(t)\sin n\omega_0 t\, dt\right]$$

$$= \frac{2}{T_0}\left[\int_{T_0/2}^{T_0} x(T-t)\sin n\Big(\frac{2\pi}{T}T - t\Big)dt + + \int_{T_0/2}^{T} x(t)\sin n\omega_0 t\, dt\right]$$

$$= \frac{2}{T_0}\left[\int_{T_0/2}^{T_0} x(T-t)\sin(2n\pi - n\omega_0)\, dt + \int_{T_0/2}^{T_0} x(t)\sin n\omega_0 t\, dt\right]$$

$$= \frac{2}{T_0}\left[-\int_{T_0/2}^{T_0} x(T-t)\sin(n\omega_0 t)\, dt + + \int_{T_0/2}^{T_0} x(t)\sin n\omega_0 t\, dt\right]$$

$b_n = 0$ if $\;x(t) = x(T-t)$

From half wave symmetry we know that if

$$x(t) = -\,x\Big(t \pm \frac{T}{2}\Big)$$

Then Fourier series of $x(t)$ contains only odd harmonics.

**SOL 3.41** Option (C) is correct.

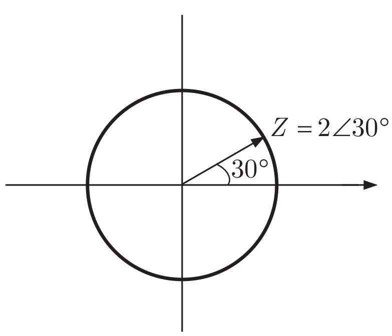
$Z$-transform of a discrete all pass system is given as

$$H(z) = \frac{z^{-1} - z_0^*}{1 - z_0 z^{-1}}$$

It has a pole at $z_0$ and a zero at $1/z_0^*$.

Given system has a pole at

$$z = 2\angle 30^\circ = 2\frac{(\sqrt{3} + j)}{2} = (\sqrt{3} + j)$$

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system is stable if $|z| < 1$ and for this it is anti-causal.

**SOL 3.42** Option (A) is correct.

According to given data input and output Sequences are

$$x[n] = \{-1, \underset{\uparrow}{2}\}, -1 \le n \le 0$$

$$y[n] = \{-1, \underset{\uparrow}{3}, -1, -2\}, -1 \le n \le 2$$

If impulse response of system is $h[n]$ then output

$$y[n] = h[n] * x[n]$$

Since length of convolution $(y[n])$ is $-1$ to 2, $x[n]$ is of length $-1$ to 0 so length of $h[n]$ is 0 to 2.

Let $h[n] = \{\underset{\uparrow}{a}, b, c\}$

Convolution

| | $a$ | $b$ | $c$ |
|---|---|---|---|
| $-1$ | $-a$ | $-b$ | $-c$ |
| 2 | $2a$ | $2b$ | $2c$ |

$$y[n] = \{-a, \underset{\uparrow}{2a - b}, 2b - c, 2c\}$$

$$y[n] = \{-1, \underset{\uparrow}{3}, -1, -2\}$$

So,

$a = 1$

$2a - b = 3 \Rightarrow b = -1$

$2a - c = -1 \Rightarrow c = -1$

Impulse response $h[n] = \{\underset{\uparrow}{1}, -1, -1\}$

**SOL 3.43** Option ( ) is correct.

**SOL 3.44** Option (D) is correct.

Output $y(t) = e^{-|x(t)|}$

If $x(t)$ is unbounded, $|x(t)| \rightarrow \infty$

$$y(t) = e^{-|x(t)|} \rightarrow 0 \quad \text{(bounded)}$$

So $y(t)$ is bounded even when $x(t)$ is not bounded.

**SOL 3.45** Option (B) is correct.

Given $y(t) = \int_{-\infty}^{t} x(t')\,dt'$

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Laplace transform of $y(t)$

$$Y(s) = \frac{X(s)}{s}, \text{ has a singularity at } s = 0$$

For a causal bounded input, $y(t) = \int_{-\infty}^{t} x(t')\,dt'$ is always bounded.

**SOL 3.46** Option (A) is correct.

RMS value is given by

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2(t)\,dt}$$

Where

$$V(t) = \begin{cases} \left(\frac{2}{T}\right)t, & 0 \le t \le \frac{T}{2} \\ 0, & \frac{T}{2} < t \le T \end{cases}$$

So $$\frac{1}{T}\int_0^T V^2(t)\,dt = \frac{1}{T}\left[\int_0^{T/2}\left(\frac{2t}{T}\right)^2 dt + \int_{T/2}^{T}(0)\,dt\right]$$

$$= \frac{1}{T}\cdot\frac{4}{T^2}\int_0^{T/2} t^2\,dt = \frac{4}{T^3}\left[\frac{t^3}{3}\right]_0^{T/2}$$

$$= \frac{4}{T^3}\times\frac{T^3}{24} = \frac{1}{6}$$

$$V_{rms} = \sqrt{\frac{1}{6}}\ \text{V}$$

**SOL 3.47** Option (A) is correct.

By final value theorem

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s\,F(s) = \lim_{s\to 0} s\frac{(5s^2+23s+6)}{s(s^2+2s+2)}$$

$$= \frac{6}{2} = 3$$

**SOL 3.48** Option (D) is correct.

$$f(x) = \sin^2 x = \frac{1-\cos 2x}{2}$$

$$= 0.5 - 0.5\ \cos 2x$$

$$f(x) = A_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 x + b_n \sin n\omega_0 x$$

$f(x) = \sin^2 x$ is an even function so $b_n = 0$

$$A_0 = 0.5$$

$$a_n = \begin{cases} -0.5, & n = 1 \\ 0 & , \text{otherwise} \end{cases}$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{T} = 2$$

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**SOL 3.49** Option (B) is correct.

Z-transform $$F(z) = \frac{1}{z+1} = 1 - \frac{z}{z+1} = 1 - \frac{1}{1+z^{-1}}$$

so, $$f(k) = \delta(k) - (-1)^k$$

Thus $$(-1)^k \xleftrightarrow{\mathcal{Z}} \frac{1}{1+z^{-1}}$$

**SOL 3.50** Option (A) is correct.

Root mean square value is given as

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I^2(t)\,dt}$$

From the graph, $$I(t) = \begin{cases} -\left(\frac{12}{T}\right)t, 0 \le t < \frac{T}{2} \\ 6, \qquad T/2 < t \le T \end{cases}$$

So $$\frac{1}{T}\int_0^T I^2\,dt = \frac{1}{T}\left[\int_0^{T/2}\left(\frac{-12t}{T}\right)^2 dt + \int_{T/2}^T (6)^2\,dt\right]$$

$$= \frac{1}{T}\left(\frac{144}{T^2}\left[\frac{t^3}{3}\right]_0^{T/2} + 36[t]_{T/2}^T\right)$$

$$= \frac{1}{T}\left[\frac{144}{T^2}\left(\frac{T^3}{24}\right) + 36\left(\frac{T}{2}\right)\right] = \frac{1}{T}[6T + 18T] = 24$$

$$I_{rms} = \sqrt{24} = 2\sqrt{6}\text{ A}$$

**SOL 3.51** Option (B) is correct.

Total current in wire

$$I = 10 + 20\sin\omega t$$

$$I_{rms} = \sqrt{(10)^2 + \frac{(20)^2}{2}} = 17.32\text{ A}$$

**SOL 3.52** Option (C) is correct.

Fourier series representation is given as

$$f(t) = A_0 + \sum_{n=1}^{\infty} a_n\cos n\omega_0 t + b_n \sin n\omega_0 t$$

From the wave form we can write fundamental period $T = 2$ sec

$$f(t) = \begin{cases} \left(\frac{4}{T}\right)t, \quad -\frac{T}{2} \le t \le 0 \\ -\left(\frac{4}{T}\right)t, 0 \le t \le \frac{T}{2} \end{cases}$$

$$f(t) = f(-t),\ f(t) \text{ is an even function}$$

So, $$b_n = 0$$

$$A_0 = \frac{1}{T}\int_T f(t)\,dt$$

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$$= \frac{1}{T}\left[\int_{-T/2}^{0}\left(\frac{4}{T}\right)t dt + \int_{0}^{T/2}\left(-\frac{4}{T}\right)t dt\right]$$

$$= \frac{1}{T}\left(\frac{4}{T}\left[\frac{t^2}{2}\right]_{-T/2}^{0} - \frac{4}{T}\left[\frac{t^2}{2}\right]_{0}^{T/2}\right)$$

$$= \frac{1}{T}\left[\frac{4}{T}\left(\frac{T^2}{8}\right) - \frac{4}{T}\left(\frac{T^2}{8}\right)\right] = 0$$

$$a_n = \frac{2}{T}\int_{T} f(t)\cos n\omega_0 t\, dt$$

$$= \frac{2}{T}\left[\int_{-T/2}^{0}\left(\frac{4}{T}\right)t\cos n\omega_0 t + \int_{0}^{T/2}\left(-\frac{4}{T}\right)t\cos n\omega_0 t dt\right]$$

By solving the integration

$$a_n = \begin{cases} \frac{8}{n^2\pi^2}, & n \text{ is odd} \\ 0, & n \text{ is even} \end{cases}$$

So,

$$f(t) = \frac{8}{\pi^2}\left[\cos \pi t + \frac{1}{9}\cos(3\pi t) + \frac{1}{25}\cos(5\pi t) + ....\right]$$

**SOL 3.53** Option (A) is correct.

Response for any input $u(t)$ is given as

$$y(t) = u(t) * h(t) \qquad h(t) \to \text{impulse response}$$

$$y(t) = \int_{-\infty}^{\infty} u(\tau)\, h(t-\tau)\, d\tau$$

Impulse response $h(t)$and step response $s(t)$ of a system is related as

$$h(t) = \frac{d}{dt}[s(t)]$$

So $$y(t) = \int_{-\infty}^{\infty} u(\tau)\frac{d}{dt}s[t-\tau]\, d\tau = \frac{d}{dt}\int_{-\infty}^{\infty} u(\tau)\, s(t-\tau)\, d\tau$$

**SOL 3.54** Option (B) is correct.

Final value theorem states that

$$\lim_{t \to \infty} y(t) \ \lim_{s \to \infty} Y(s)$$

**SOL 3.55** Option (D) is correct.

$$V_{rms} = \sqrt{\frac{1}{T_0}\int_{T_0} V^2(t)\, dt}$$

here $T_0 = \pi$

$$\frac{1}{T_0}\int_{T_0} V^2(t)\, dt = \frac{1}{\pi}\left[\int_{0}^{\pi/3}(100)^2 dt + \int_{\pi/3}^{2\pi/3}(-100)^2 dt + \int_{2\pi/3}^{\pi}(100)^2 dt\right]$$

$$= \frac{1}{\pi}\left[10^4\left(\frac{\pi}{3}\right) + 10^4\left(\frac{\pi}{3}\right) + 10^4\left(\frac{\pi}{3}\right)\right] = 10^4 \text{ V}$$

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$$V_{rms} = \sqrt{10^4} = 100 \text{ V}$$

**SOL 3.56** Option (D) is correct.

Let $h(t)$ is the impulse response of system

$$y(t) = u(t) * h(t)$$

$$y(t) = \int_0^t u(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_0^t (2 + t - \tau)\, e^{-3(t-\tau)} u(\tau)\, d\tau$$

So $\quad h(t) = (t+2)\, e^{-3t} u(t), t > 0$

Transfer function

$$H(s) = \frac{Y(s)}{U(s)} = \frac{1}{(s+3)^2} + \frac{2}{(s+3)}$$

$$= \frac{1 + 2s + 6}{(s+3)^2} = \frac{(2s+7)}{(s+3)^2}$$

**SOL 3.57** Option (B) is correct.

Fourier series representation is given as

$$v(t) = A_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

period of given wave form $T = 5$ ms

DC component of $v$ is

$$A_0 = \frac{1}{T} \int_T v(t)\, dt$$

$$= \frac{1}{5}\left[\int_0^3 1\, dt + \int_3^5 -1\, dt\right]$$

$$= \frac{1}{5}[3 - 5 + 3] = \frac{1}{5}$$

**SOL 3.58** Option (A) is correct.

Coefficient, $a_n = \dfrac{2}{T} \displaystyle\int_T v(t) \cos n\omega_0 t\, dt$

$$= \frac{2}{5}\left[\int_0^3 (1) \cos nwt\, dt + \int_3^5 (-1) \cos nwt\, dt\right]$$

$$= \frac{2}{5}\left(\left[\frac{\sin n\omega t}{n\omega}\right]_0^3 - \left[\frac{\sin n\omega t}{n\omega}\right]_3^5\right)$$

Put $\quad \omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{5}$

$$a_n = \frac{1}{n\pi}[\sin 3n\omega - \sin 5n\omega + \sin 3n\omega]$$

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$$= \frac{1}{n\pi}\left[2\sin\left(3n\frac{2\pi}{5}\right) - \sin\left(5n\frac{2\pi}{5}\right)\right]$$

$$= \frac{1}{n\pi}\left[2\sin\left(\frac{6\pi n}{5}\right) - \sin(2n\pi)\right]$$

$$= \frac{2}{n\pi}\sin\left(\frac{6\pi n}{5}\right)$$

Coefficient, $b_n = \frac{2}{T}\int_T v(t)\sin n\omega_0 t\,dt$

$$= \frac{2}{5}\left[\int_0^3 (1)\sin nwt\,dt + \int_3^5 (-1)\sin nwt\,dt\right]$$

$$= \frac{2}{5}\left(\left[-\frac{\cos n\omega t}{n\omega}\right]_0^3 - \left[-\frac{\cos n\omega t}{n\omega}\right]_3^5\right)$$

put $\omega = \frac{2\pi}{T} = \frac{2\pi}{5}$

$$b_n = \frac{1}{n\pi}[-\cos 3n\omega + 1 + \cos 5n\omega - \cos 3n\omega]$$

$$= \frac{1}{n\pi}[-2\cos 3n\omega + 1 + \cos 5n\omega]$$

$$= \frac{1}{n\pi}\left[-2\cos\left(3n\frac{2\pi}{5}\right) + 1 + \cos\left(5n\frac{2\pi}{5}\right)\right]$$

$$= \frac{1}{n\pi}\left[-2\cos\left(\frac{6\pi n}{5}\right) + 1 + 1\right]$$

$$= \frac{2}{n\pi}\left[1 - \cos\left(\frac{6\pi n}{5}\right)\right]$$

Amplitude of fundamental component of $v$ is

$$v_f = \sqrt{a_1^2 + b_1^2}$$

$$a_1 = \frac{2}{\pi}\sin\left(\frac{6\pi}{5}\right),\; b_1 = \frac{2}{\pi}\left(1 - \cos\frac{6\pi}{5}\right)$$

$$v_f = \frac{2}{\pi}\sqrt{\sin^2\frac{6\pi}{5} + \left(1 - \cos\frac{6\pi}{5}\right)^2}$$

$$= 1.20 \text{ Volt}$$

***********

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