

**Edition 2020-21**

# **Signals & Systems**

PEN-Drive / G-Drive Course & LIVE Classroom Program

**Workbook**

Electronics & Telecommunication Engineering

Electrical Engineering

Electrical & Electronics Engineering

Instrumentation Engineering

**GATE / ESE / PSUs**



**GATE ACADEMY®**  
*steps to success...*

# **Signals & Systems**

PEN-Drive / G-Drive Course & LIVE Classroom Program

Workbook

ETC / EE / EEE / IN

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**Edition : 2020-21**

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## GATE Syllabus

**Electronics and Communications (EC)** : Continuous-time signals: Fourier series and Fourier transform representations, sampling theorem and applications; Discrete-time signals: discrete-time Fourier transform (DTFT), DFT, FFT, Z-transform, interpolation of discrete-time signals; LTI systems: definition and properties, causality, stability, impulse response, convolution, poles and zeros, parallel and cascade structure, frequency response, group delay, phase delay, digital filter design techniques.

**Electrical Engineering (EE)** : Representation of continuous and discrete-time signals, Shifting and scaling operations, Linear Time Invariant and Causal systems, Fourier series representation of continuous periodic signals, Sampling theorem, Applications of Fourier Transform, Laplace Transform and z-Transform.

**Instrumentation Engineering (IN)** : Periodic, aperiodic and impulse signals; Laplace, Fourier and z-transforms; transfer function, frequency response of first and second order linear time invariant systems, impulse response of systems; convolution, correlation. Discrete time system: impulse response, frequency response, pulse transfer function; DFT and FFT; basics of IIR and FIR filters.

## ESE Syllabus

**Electrical Engineering (EE)** : Representation of continuous and discrete-time signals, shifting and scaling operations, linear, time-invariant and causal systems, Fourier series representation of continuous periodic signals, sampling theorem, Fourier and Laplace transforms, Z transforms, Discrete Fourier transform, FFT, linear convolution, discrete cosine transform, FIR filter, IIR filter, bilinear transformation.

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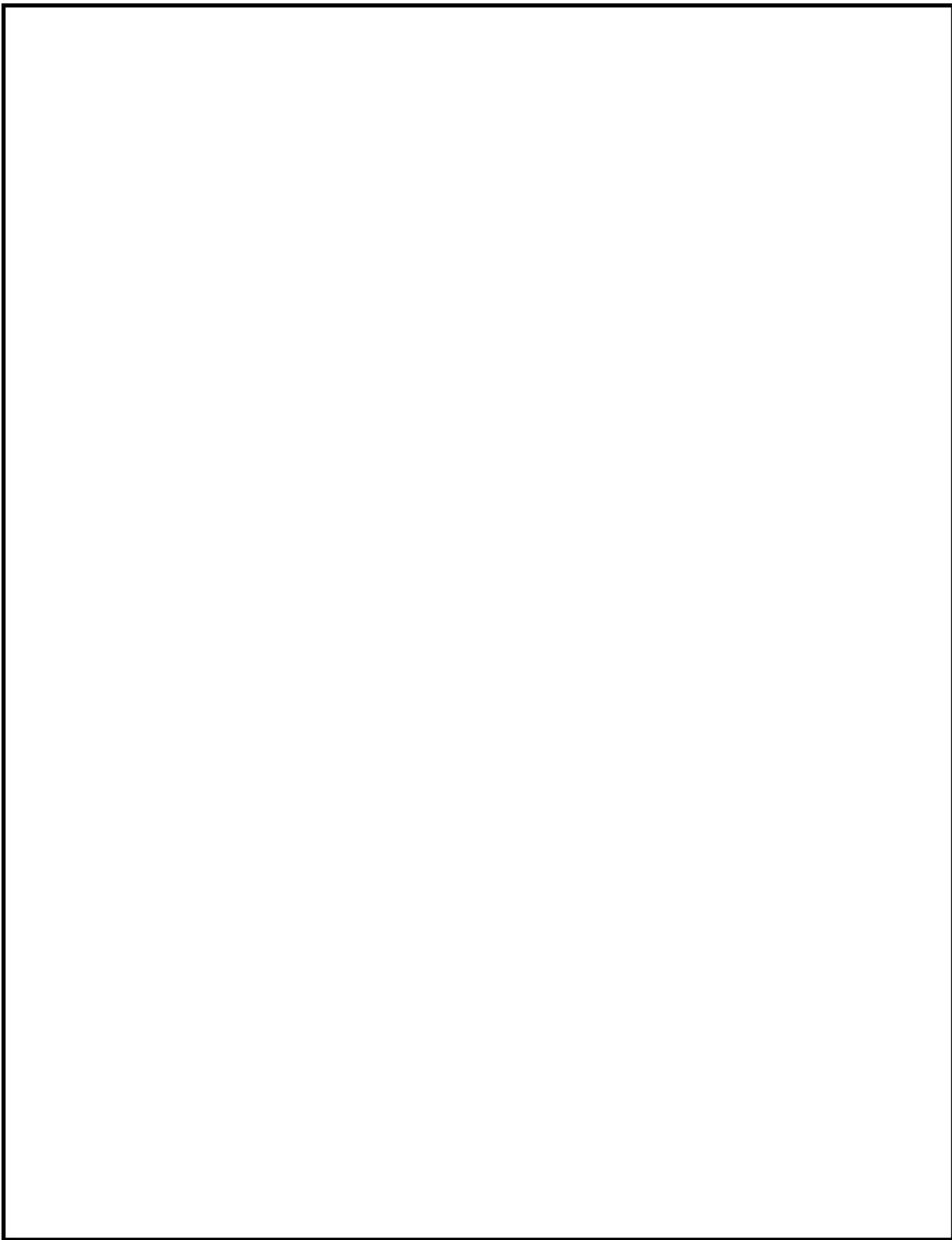
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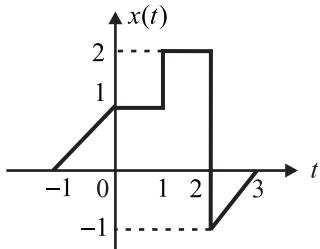


# 1

# Continuous & Discrete Time Signals

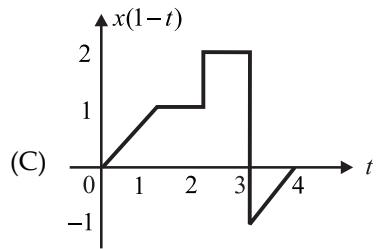
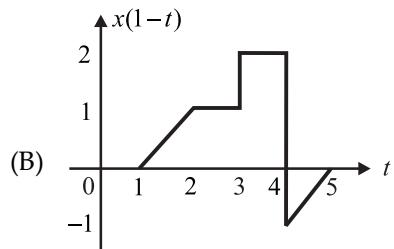
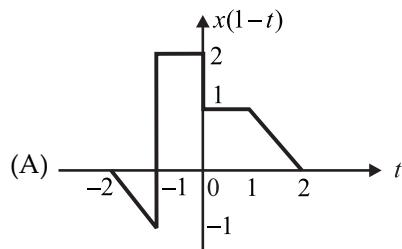
## Objective & Numerical Ans Type Questions :

- Q.1** If a plot of a signal  $x(t)$  is as shown in below figure.



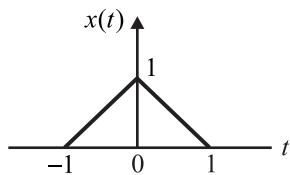
Then the plot of the signal  $x(1-t)$  will be

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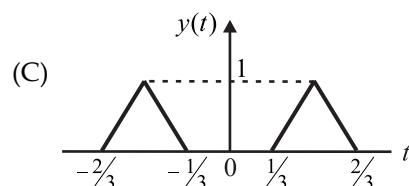
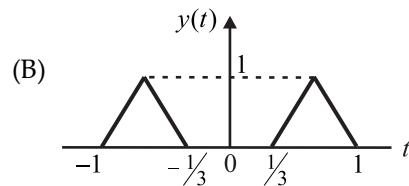
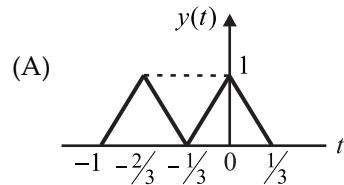


(D) None of the above

- Q.2** For the continuous-time signal  $x(t)$  shown in below figure.

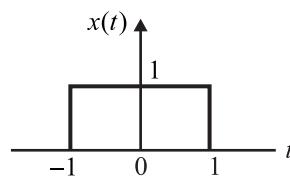


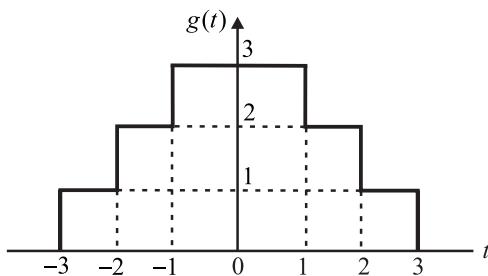
The signal  $y(t) = x(3t) + x(3t + 2)$  is



(D) None of the above

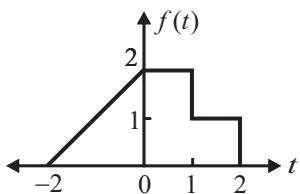
- Q.3** If  $x(t)$  is shown in below figure then  $g(t)$  can be represented in terms of  $x(t)$  as



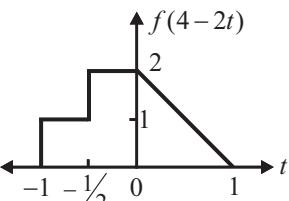
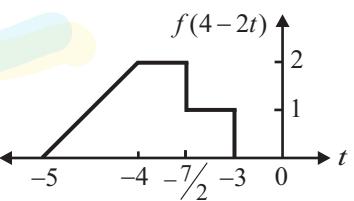
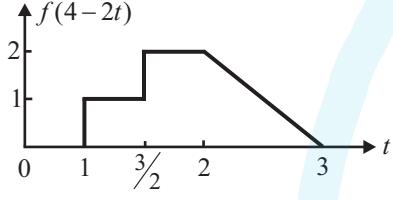
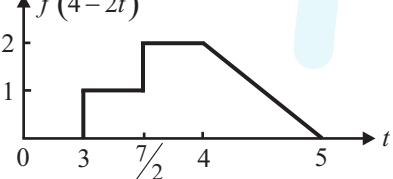


- (A)  $x(t) + x(t/2) + x(t/3)$   
 (B)  $x(t) + x(2t) + x(3t)$   
 (C)  $x(t) + 2x(t/2) + 3x(t/3)$   
 (D)  $x(t) + 2x(2t) + 3x(3t)$

**Q.4** Consider a signal  $f(t)$  as shown in figure

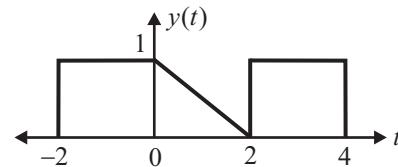


The plot of signal  $f(4-2t)$  is

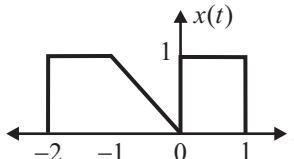
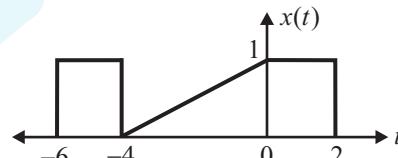
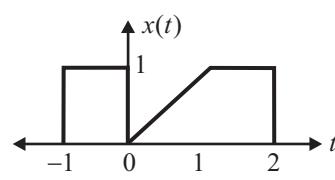
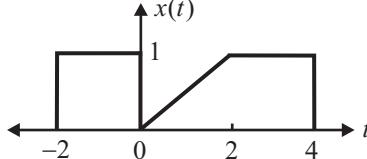
- (A)   $f(4-2t)$
- (B)   $f(4-2t)$
- (C)   $f(4-2t)$
- (D)   $f(4-2t)$

**Q.5** A signal  $x(t)$  is transformed into another signal

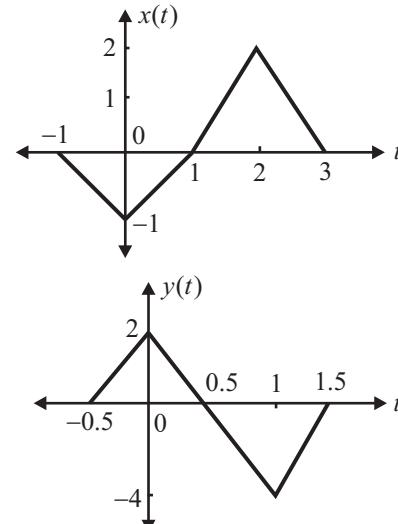
$$y(t) \text{ given as } y(t) = x\left(1 - \frac{t}{2}\right)$$



The waveform of the original signal  $x(t)$  is

- (A)   $x(t)$
- (B)   $x(t)$
- (C)   $x(t)$
- (D)   $x(t)$

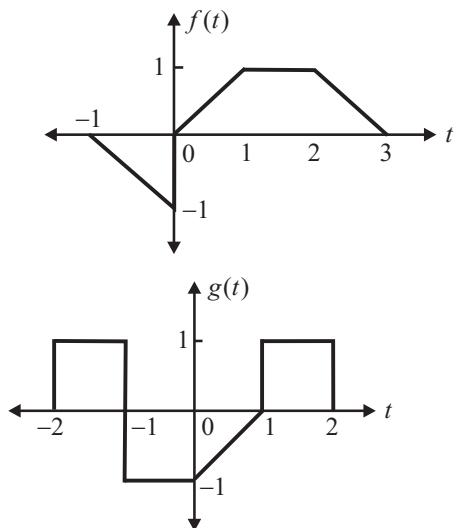
**Q.6** Consider two signals  $x(t)$  and  $y(t)$  shown in figure below.



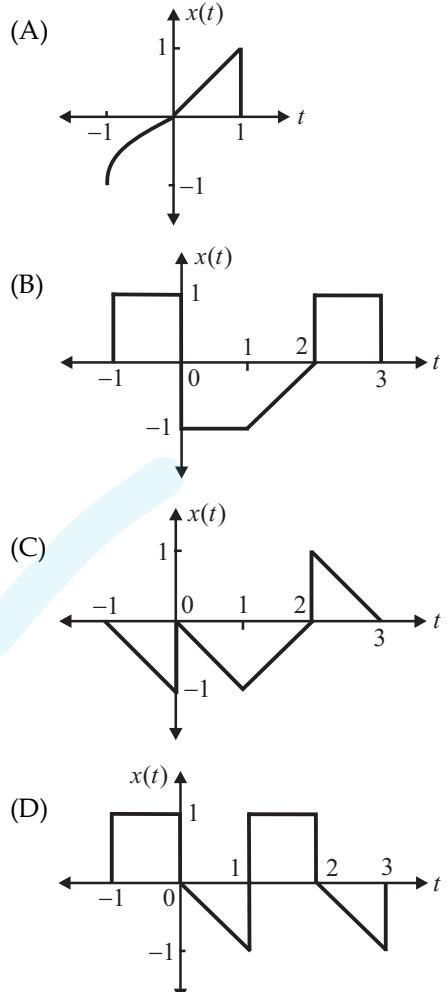
If  $y(t) = Ax\left(\frac{t-t_0}{W}\right)$  then, the value of  $A$ ,  $t_0$  and  $W$  are respectively

- (A)  $-2, 0, 2$       (B)  $-2, 1, \frac{1}{2}$   
 (C)  $-2, 0, \frac{1}{2}$       (D)  $2, 1, 2$

- Q.7** Two CT signals  $f(t)$  and  $g(t)$  are shown in following figure :



The plot for a signal  $x(t) = f(t)g(t-1)$  will be



- Q.8** Given a sequence  $x[n]$ , to generate the sequence  $y[n] = x[3-4n]$ , which one of the following procedures would be correct?

[GATE EE 2008-Bangalore]

(A) First delay  $x[n]$  by 3 samples to generate  $z_1[n]$ , then pick every 4<sup>th</sup> sample of  $z_1[n]$  to generate  $z_2[n]$ , and then finally time reverse  $z_2[n]$  to obtain  $y[n]$

(B) First advance  $x[n]$  by 3 samples to generate  $z_1[n]$ , then pick every 4<sup>th</sup> sample of  $z_1[n]$  to generate  $z_2[n]$ , and then finally time reverse  $z_2[n]$  to obtain  $y[n]$

(C) First pick every fourth sample of  $x[n]$  to generate  $v_1[n]$ , time-reverse  $v_1[n]$  to obtain  $v_2[n]$ , and finally advance  $v_2[n]$  by 3 sample to obtain  $y[n]$

(D) First pick every fourth sample of  $x[n]$  to generate  $v_1[n]$ , time-reverse  $v_1[n]$  to obtain  $v_2[n]$ , and finally delay  $v_2[n]$  by 3 samples to obtain  $y[n]$

- Q.9**  $x[n]$  is defined as [ESE EC 2006]

$$x[n] = \begin{cases} 0 & \text{for } n < -2 \text{ and } n > 4 \\ 1 & \text{otherwise} \end{cases}$$

Determine the value of  $n$  for which  $x[-n-2]$  is guaranteed to be zero.

- (A)  $n < 1$  and  $n > 7$       (B)  $n < -4$  and  $n > 2$   
 (C)  $n < -6$  and  $n > 0$       (D)  $n < -2$  and  $n > 4$

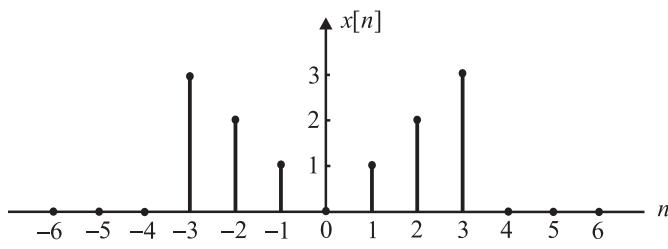
- Q.10** Let  $x[n] = \begin{cases} n & n = \text{multiple of 3} \\ 0 & \text{otherwise} \end{cases}$

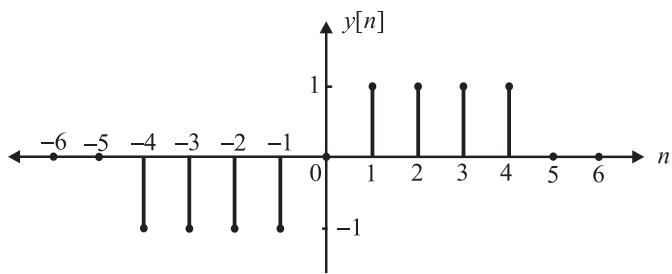
Then  $y[n] = x[3n]$  is :

- (A)  $y[n] = n$  for multiple of 3  
 (B)  $y[n] = 3n$  for all  $n$   
 (C)  $y[n] = 9n$  for multiple of 3  
 (D)  $y[n] = 3n$  for multiple of 3

**Common Data for Questions 11, 12 and 13**

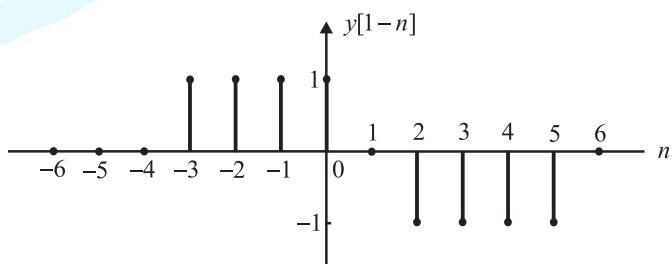
Consider the signal  $x[n]$  and  $y[n]$  given below in figures.



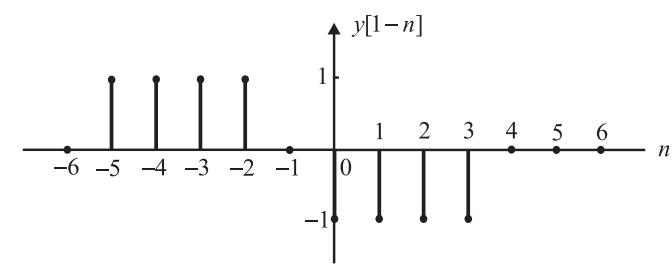


Q.11 The signal  $y[1-n]$  is

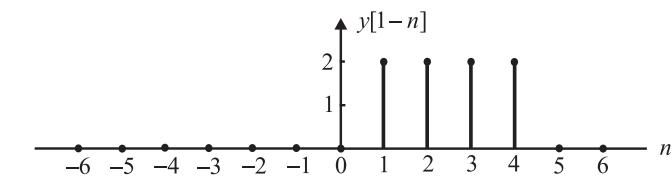
(A)



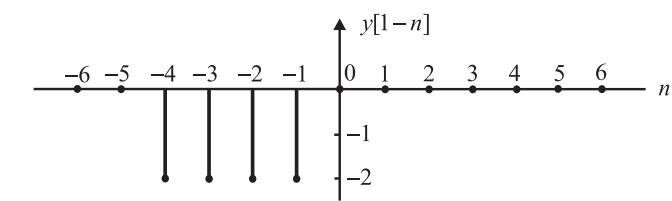
(B)



(C)

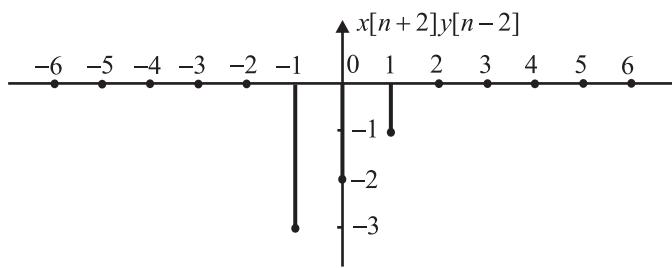


(D)

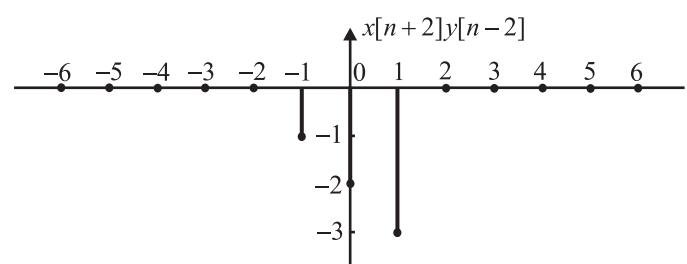


Q.12 The signal  $x[n+2]y[n-2]$  is

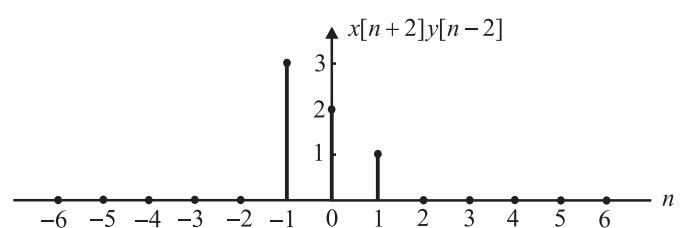
(A)



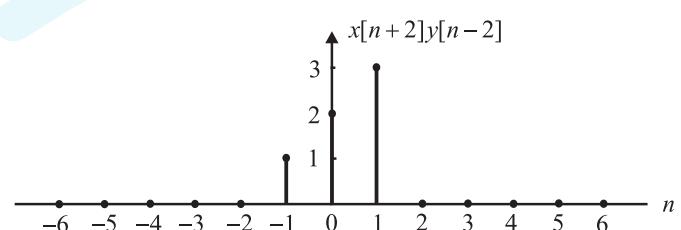
(B)



(C)

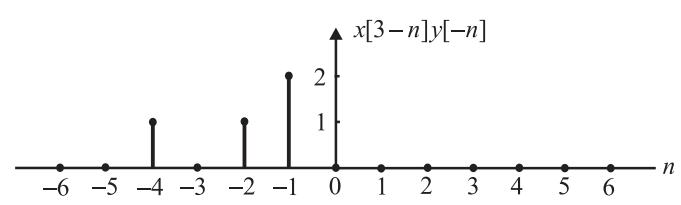


(D)

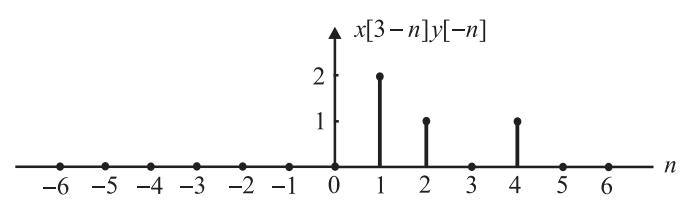


Q.13 The signal  $x[3-n]y[-n]$  is

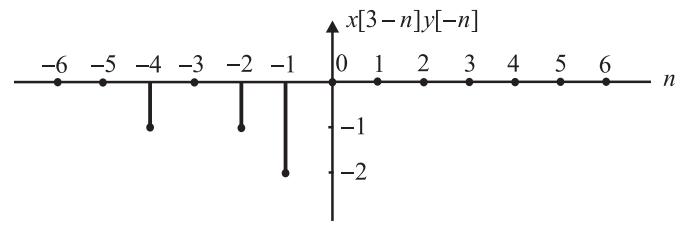
(A)



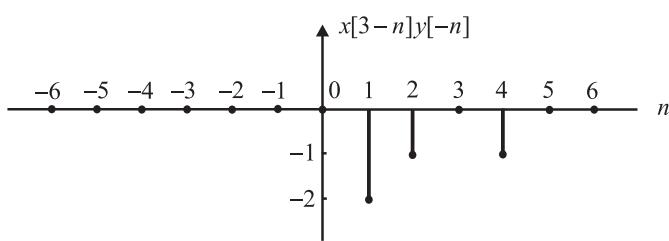
(B)



(C)



(D)



- Q.14** The value of  $\int_{-\infty}^{+\infty} e^{-t} \delta(2t-2) dt$ , where  $\delta(t)$  is the Dirac delta function, is

[GATE EE 2016-Bangalore]

- (A)  $\frac{1}{2e}$       (B)  $\frac{2}{e}$   
 (C)  $\frac{1}{e^2}$       (D)  $\frac{1}{2e^2}$

- Q.15** Consider signal  $x(t) = \begin{cases} 1, & |t| \leq 2 \\ 0, & |t| > 2 \end{cases}$ . Let  $\delta(t)$  denote the unit impulse (Dirac-delta) function. The value of the integral  $\int_0^5 2x(t-3)\delta(t-4)dt$  is

[GATE IN 2018-Guwahati]

- (A) 2      (B) 1  
 (C) 0      (D) 3

- Q.16** The integral [GATE IN 2011-Madras]

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} t^2 e^{-t^2/2} \delta(1-2t) dt$$

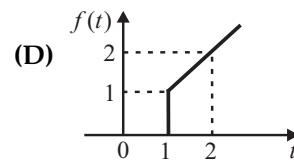
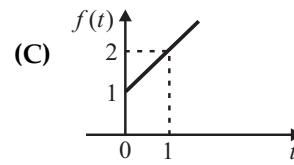
- is equal to  
 (A)  $\frac{1}{8\sqrt{2\pi}} e^{-1/8}$       (B)  $\frac{1}{4\sqrt{2\pi}} e^{-1/8}$   
 (C)  $\frac{1}{\sqrt{2\pi}} e^{-1/2}$       (D) 1

- Q.17** Match the waveforms on the left hand side with the correct mathematical description listed on the right hand side. [GATE EE 1994-Kharagpur]

Waveform

 $f(t)$ 

- (A)   
 (P)  $t.u(t-1)$
- (B)   
 (Q)  $(t+1).u(t-1)$



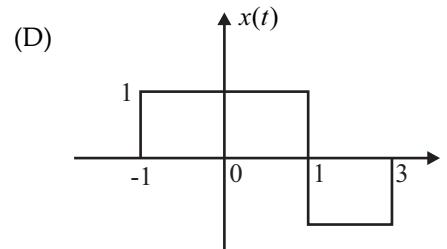
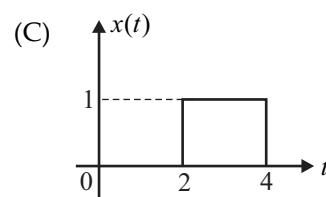
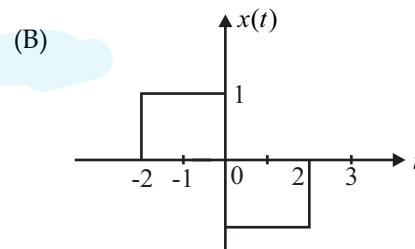
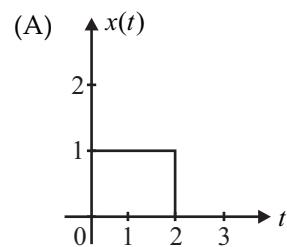
- (T)  $(t-1).u(t-1)$   
 (U)  $(t-1).u(t-1)$

Codes : A B C D

- (A) R U S P  
 (B) R P U S  
 (C) R Q P S  
 (D) R S U P

- Q.18** The signal  $x(t) = u(t+2) - 2u(t) + u(t-2)$  is represented by

[ESE EC 2016]



**Q.19** The ramp function can be obtained from the unit impulse at  $t = 0$  by [ESE EC 2013]

- (A) Differentiating unit impulse function once
- (B) Differentiating unit impulse function twice
- (C) Integrating unit impulse function once
- (D) Integrating unit impulse function twice

**Q.20** The value of the integral  $I = \int_1^2 (5t^2 + 1)\delta(t) dt$  is [ESE EC 2013]

- (A) 0
- (B) 1
- (C)  $\frac{42}{3}$
- (D)  $\frac{125}{3}$

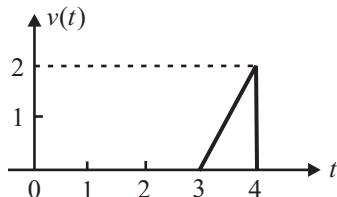
**Q.21** A signal  $f(t)$  is described as [ESE EC 2012]

$$\begin{aligned} f(t) &= [1 - |t|] && \text{when } |t| \leq 1 \\ &= 0 && \text{when } |t| > 1 \end{aligned}$$

This represents the unit

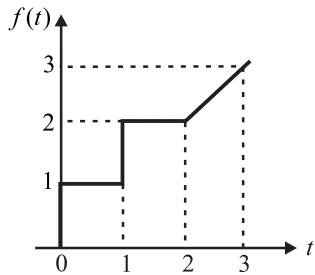
- (A) sinc function
- (B) triangular function
- (C) signum function
- (D) parabolic function

**Q.22** In the graph shown below, which one of the following is the expression for  $v(t)$ ?



- (A)  $(2t+6)[u(t-3)+2u(t-4)]$  [ESE EC 2005]
- (B)  $(-2t-6)[u(t-3)+u(t-4)]$
- (C)  $(-2t+6)[u(t-3)+u(t-4)]$
- (D)  $(2t-6)[u(t-3)-u(t-4)]$

**Q.23** Consider the following waveform. Which one of the following gives the correct description of the waveform shown in the below waveform?



- (A)  $u(t) + u(t-1)$
- (B)  $u(t) + (t-1)u(t-1)$

- (C)  $u(t) + u(t-1) + (t-2)u(t-2)$

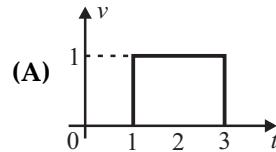
- (D)  $u(t) + (t-2)u(t-2)$

**Q.24** If a function  $f(t)u(t)$  is shifted to right side by  $t_0$ , then the function can be expressed as [ESE EC 2001]

- (A)  $f(t-t_0)u(t)$
- (B)  $f(t)u(t-t_0)$
- (C)  $f(t-t_0)u(t-t_0)$
- (D)  $f(t+t_0)u(t-t_0)$

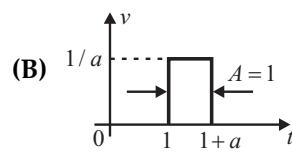
**Q.25** Match List-I with List-II and select the correct answer using the codes given below the Lists : [ESE EC 1997]

List-I

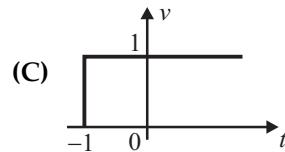


List-II

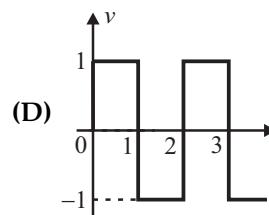
1.  $v(t) = u(t+1)$



2.  $v(t) = u(t) - 2u(t-1) + 2u(t-2) - 2u(t-3)$



3.  $v(t) = u(t-1) - u(t-3)$

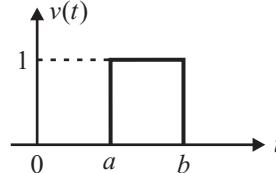


4.  $\lim_{\alpha \rightarrow 0} v(t) = \delta(t-1)$

Codes : A B C D

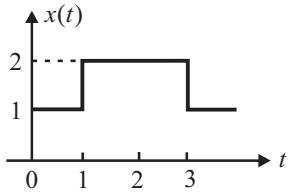
- (A) 1 2 3 4
- (B) 3 4 1 2
- (C) 4 3 2 1
- (D) 4 3 1 2

**Q.26** Consider the following functions for the rectangular voltage pulse shown in below figure



- (a)  $v(t) = u(a-t)u(t-b)$
- (b)  $v(t) = u(b-t)u(t-a)$
- (c)  $v(t) = u(t-a) - u(t-b)$
- (d)  $v(t) = u(b-t) - u(a-t)$

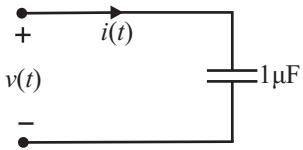
- The functions that describe the pulse are :
- (A) c & d                          (B) b, c & d  
 (C) a, c & d                          (D) Only c
- Q.27** Consider the following waveform. Which one of the following gives the correct description of the waveform shown in the below waveform?



- (A)  $u(t) + 2u(t-1) - 2u(t-3)$   
 (B)  $0.5[1 + \text{sgn}(t) + \text{sgn}(t-1) - \text{sgn}(t-3)]$   
 (C)  $0.5\text{sgn}(t) + 0.5\text{sgn}(t-1) - \text{sgn}(t-3)$   
 (D)  $u(t) - u(t-1) - u(t-3)$
- Q.28** The value of  $\delta(3t)$  and  $\delta(3n)$  are respectively
- (A)  $\frac{1}{3}\delta(t)$  and  $\delta(n)$                   (B)  $\frac{1}{3}\delta(t)$  and  $3\delta(n)$   
 (C)  $3\delta(t)$  and  $\frac{1}{3}\delta(n)$                   (D)  $\frac{1}{3}\delta(t)$  and  $\frac{1}{3}\delta(n)$

- Q.29** Which is not a true statement?
- (A)  $\delta(at) = \frac{1}{|a|}\delta(t)$   
 (B)  $\delta(-t) = \delta(t)$   
 (C)  $x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$   
 (D)  $\int_{-\infty}^{\infty} f(t)\delta(t-t_0)dt = f(t-t_0)$

- Q.30** In the given figure, if  $v(t) = \delta(t)$  then  $i(t)$  will be a unit



- (A) Step function                          (B) Impulse function  
 (C) Doublet function                          (D) Triplet function
- Q.31** The value of  $\int_{-2}^6 [\delta(t-1) + \delta(t+3) - \delta(t-5)]dt$  is
- (A) 3    (B) 2  
 (C) 1    (D) 0
- Q.32** The value of  $f(t) = \left( \frac{\sin[0.5\pi(t-2)]}{t^2 + 4} \right) \delta(t-1)$  is
- (A)  $-1/5$     (B)  $-2/5$   
 (C)  $1/5$     (D) None of these

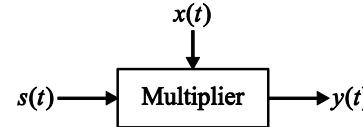
- Q.33** The value of  $\int_{-\infty}^{\infty} (t^3 + \sin \pi t)\delta'(t-1)dt$  is
- (A)  $-(3-\pi)$                                   (B)  $-(3+\pi)$   
 (C) 1    (D) -1

- Q.34** The value of  $\left( \frac{\sin kt}{t} \right) \delta(t)$  is
- (A) 0    (B)  $\infty$   
 (C) Indeterminate                                  (D)  $k\delta(t)$

- Q.35** A signal  $x(t) = \sin 5\pi t + 0.5 \cos 10\pi t$  is applied to a multiplier as shown. The output is of the form

$$y(t) = \sum_{n=-\infty}^{\infty} A_n \delta(t - 0.2n)$$

$$\text{where } s(t) = 2 \sum_{n=-\infty}^{\infty} \delta(t - 0.2n)$$



The magnitude  $A_n$  for  $n = 2$  is :

- (A) 0    (B) 0.5  
 (C) 1    (D) 1.5

- Q.36** The value of the integral  $\int_{-2}^2 \delta(t-1.5) \sin c(t) dt$  is
- (A) 0    (B)  $\frac{2}{3\pi}$   
 (C)  $-\frac{2}{3\pi}$     (D) 1

- Q.37** A signal is defined as  $x(t) = 4\text{tri}(t)$ . The value of  $x(1/2)$  is

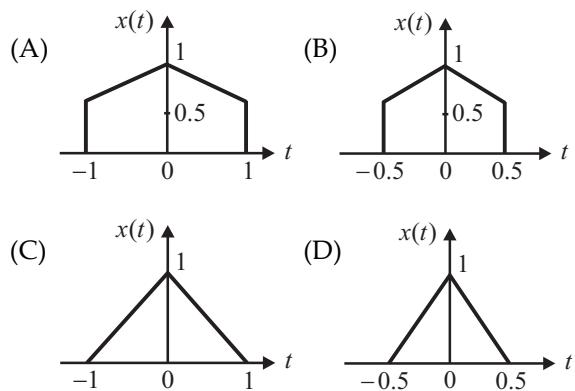
- (A)  $1/2$     (B) 1  
 (C) 2    (D) 0

- Q.38** Value of  $\int_0^{20} \delta(t-8) \text{tri}\left(\frac{t}{32}\right) dt$  is
- (A)  $1/4$     (B)  $1/2$   
 (C)  $3/4$     (D) 1

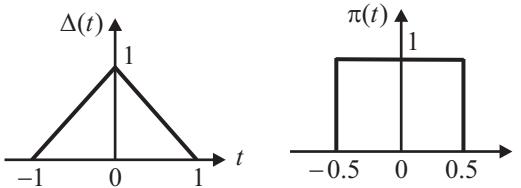
- Q.39** The derivative of the function  $g(t) = 2\text{tri}\left(\frac{t}{2}\right) - 1$  is
- (A)  $u(t+2) + u(t-2)$   
 (B)  $\text{sgn}(t+2) - \text{sgn}(t-2)$   
 (C)  $u(t+2) - u(t-2)$   
 (D)  $u(t+2) - 2u(t) + u(t-2)$

- Q.40** A signal is defined as  $x(t) = 2\text{tri}[2(t-1)] + 6\text{rect}\left(\frac{t}{4}\right)$ . The value of  $x(3/2)$  is  
 (A) 4                        (B) 5  
 (C) 6                        (D) 7

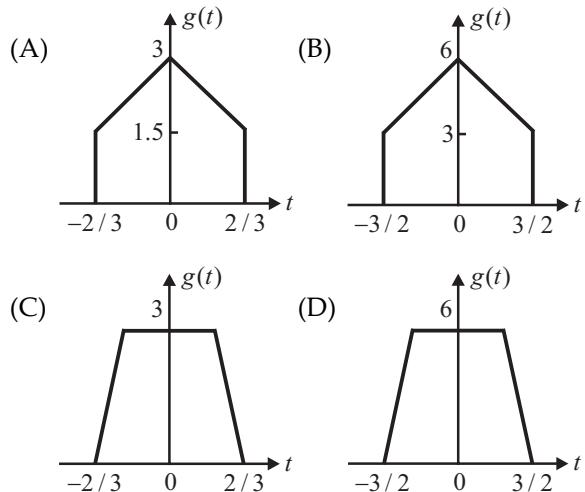
- Q.41** Consider the signal  $x(t) = \text{rect}(t)\text{tri}(t)$ . The graph of  $x(t)$  is



- Q.42** Consider a unit triangular function  $\Delta(t)$  and a unit rectangular function  $\pi(t)$  as shown in figure



Which of the following waveforms is correct for  $g(t) = 3\Delta(2t/3) + 3\pi(t/3)$ ?



- Q.43** The value of accumulation  $\sum_{n=0}^{10} \text{ramp}[n]$  is  
 (A) 50                        (B) 10  
 (C) 55                        (D) 11

**Statement for Linked Questions 44 & 45**

A discrete time signal  $g[n]$  is given as  

$$g[n] = u[n+3] - u[n-5]$$

- Q.44** Sum of all values of  $g[n]$  is given as  
 (A) 2                        (B) -2  
 (C) 4                        (D) 8

- Q.45** Sum of all values of  $g[3n]$  is  
 (A) 1                        (B) 3  
 (C) 2                        (D) 4

- Q.46** Evaluate the value of  $x[n] = \sum_{n=0}^{\infty} n \cdot \left(\frac{1}{3}\right)^n$   
 (A) 9/4                        (B) 3/4  
 (C) 1/3                        (D) 2/3

- Q.47** A discrete time signal  $x[n]$  is defined in terms of unit impulse function as follow

$$x[n] = 1 - \sum_{r=3}^{\infty} \delta[n-1-r]$$

If  $x[n]$  is expressed in terms of unit step function as  $x[n] = u[an-b]$  then the values of a and b are

- (A)  $a=1, b=3$                         (B)  $a=-1, b=-3$   
 (C)  $a=-1, b=-4$                         (D)  $a=1, b=-3$

- Q.48** A discrete time sequence is described as  

$$x[n] = (-1)^{n+1} \frac{1}{n} u[n-1]$$
. The value of  $\sum_{n=-\infty}^{\infty} |x[n]|$  is  
 (A) 2                        (B) 1/2  
 (C) 0                        (D) None of these

- Q.49** Let  $S = \sum_{n=0}^{\infty} n \alpha^n$  where  $|\alpha| < 1$ . The value of  $\alpha$  in the range  $0 < \alpha < 1$ , such that  $S = 2\alpha$  is \_\_\_\_\_.

[GATE EE 2016-Bangalore]

- Q.50** Consider the discrete-time signal  $x(n) = \left(\frac{1}{3}\right)^n u(n)$ , where  $u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$ . Define the signal  $y(n)$  as  $y(n) = x(-n), -\infty < n < \infty$ . Then  $\sum_{n=-\infty}^{\infty} y(n)$  equals

[GATE IN 2007-Kanpur]

- (A)  $-\frac{2}{3}$                         (B)  $\frac{2}{3}$   
 (C)  $\frac{3}{2}$                         (D) 3

**Q.51** The value of the function  $\int_{-\infty}^{\infty} \delta(at-b) \sin^2(t-4) dt$

where  $a > 0$ , is

(A) 1

$$(B) \frac{1}{b} \sin^2\left(\frac{a}{b} - 4\right)$$

(C) 0

$$(D) \frac{1}{a} \sin^2\left(\frac{b}{a} - 4\right)$$

**Q.52** Consider discrete time signal

$$x[n] = 0.2x_1[n-n_0] - 2$$

A discrete transformed signal of  $x[n]$  is given as

$$x_1[n] = 5x[n+n_0] + K. \text{ The value of constant } K \text{ is}$$

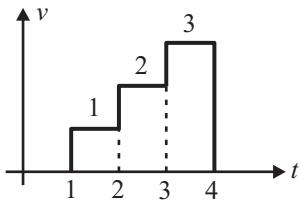
(A) -2

(B) 10

(C) -5

(D) -10

**Q.53** The expression for the waveform in terms of step function is given by [ESE EC 1991]



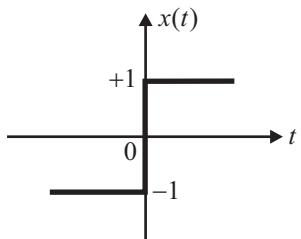
(A)  $v = u(t-1) - u(t-2) + u(t-3)$

(B)  $v = u(t-1) + u(t-2) + u(t-3)$

(C)  $v = u(t-1) + u(t-2) - u(t-3)$

(D)  $v = u(t-1) + u(t-2) + u(t-3) - 3u(t-4)$

**Q.54** The function  $x(t)$  is shown in the below figure. Even and odd parts of a unit step function  $u(t)$  are respectively [GATE EC 2005-Bombay]



(A)  $\frac{1}{2}, \frac{x(t)}{2}$

(B)  $-\frac{1}{2}, \frac{x(t)}{2}$

(C)  $\frac{1}{2}, -\frac{x(t)}{2}$

(D)  $-\frac{1}{2}, -\frac{x(t)}{2}$

**Q.55** Consider the sequence

$$x[n] = [-4-j5 \quad 1+j2 \quad 4]$$

The conjugate anti-symmetric part of the sequence is [GATE EC 2004-Delhi]

(A)  $[-4-j2.5 \quad j2 \quad 4-j2.5]$

(B)  $[-j2.5 \quad 1 \quad j2.5]$

(C)  $[-j2.5 \quad j2 \quad 0]$

(D)  $[-4 \quad 1 \quad 4]$

**Q.56** If from the function  $f(t)$  one forms the function,  $\psi(t) = f(t) + f(-t)$ , then  $\psi(t)$  is [ESE EC 1991]

(A) Even

(B) Odd

(C) Neither even nor odd

(D) Both even and odd

**Q.57** The even part of the signal  $x(t) = 1 + t + 3t^2 + 5t^3 + 9t^4$  at  $t = 0$  is

(A) 1

(B) 13

(C) 8

(D) 0

**Q.58** Odd component of the signal  $x(t) = \sin t \cdot \cos t$  is

(A)  $\sin t$

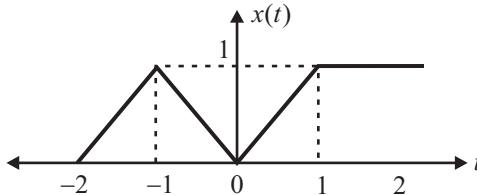
(B)  $\sin t \cdot \cos t$

(C)  $\cos t$

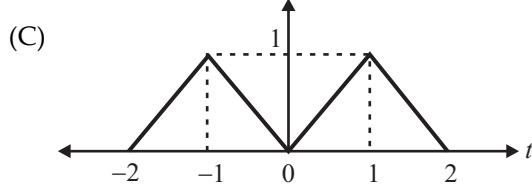
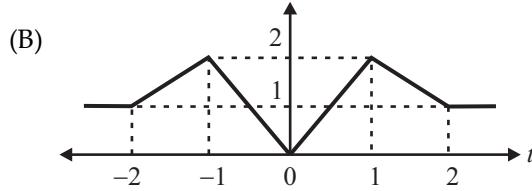
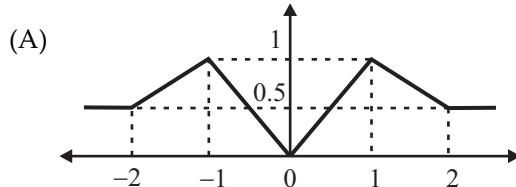
(D) None of these

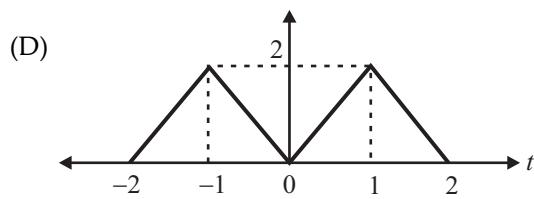
#### Common Data for Questions 59 & 60

Consider the signal shown in below figure.

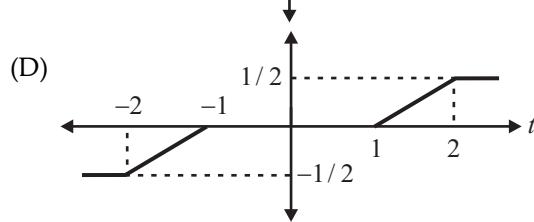
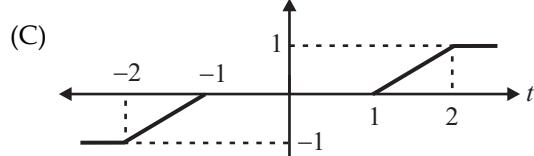
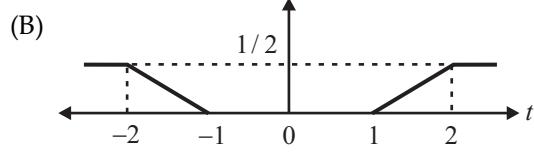
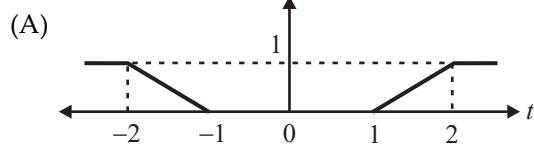


**Q.59** Even part of the signal  $x(t)$  is



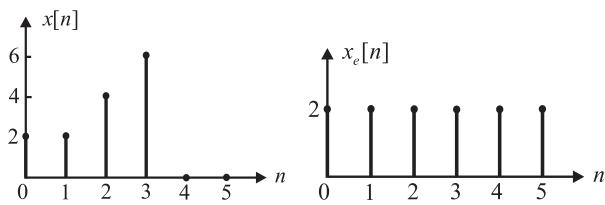


Q.60 Odd part of the signal is

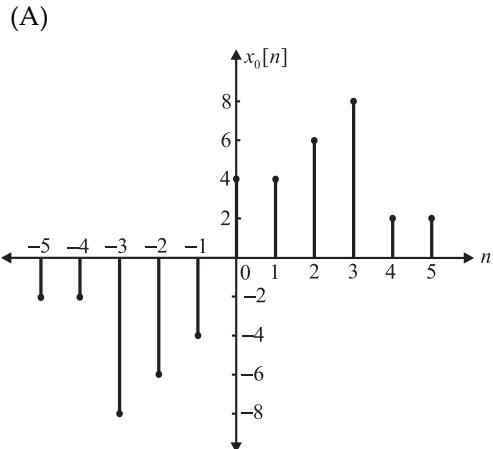


**Common Data for Questions 61 & 62**

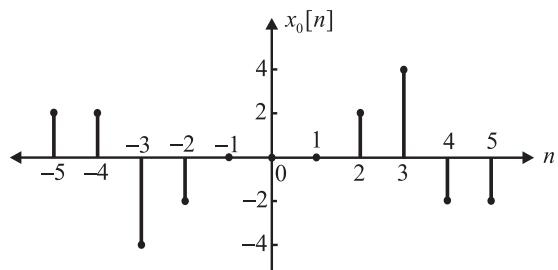
The figure below shows a signal  $x[n]$  and its even part  $x_e[n]$  for  $n \geq 0$  only, that is for  $n < 0$ ,  $x[n]$  and  $x_e[n]$  are not given.



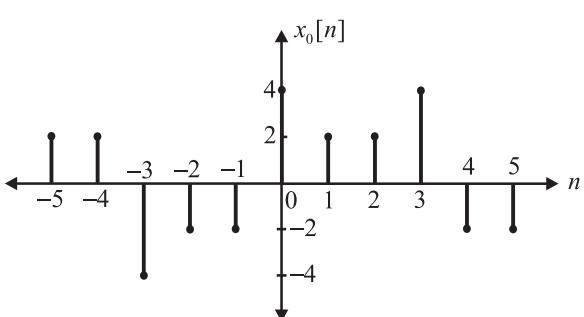
Q.61 The complete odd part of the signal  $x_0[n]$  is



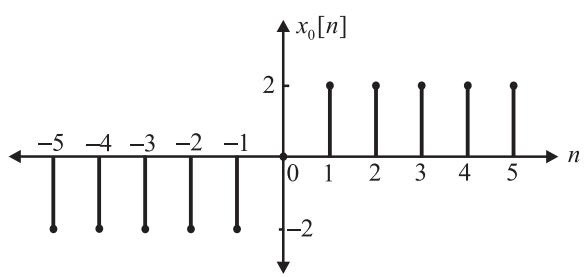
(B)



(C)

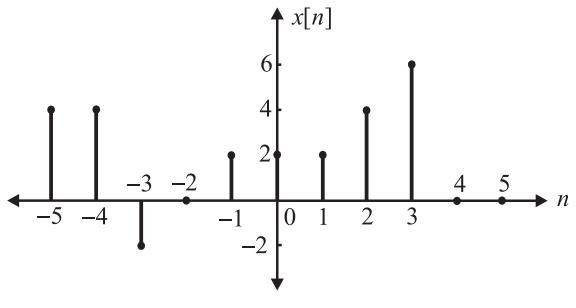


(D)

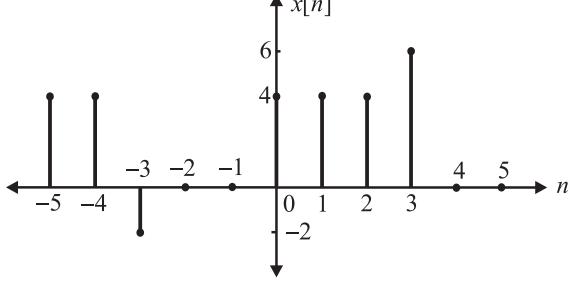


Q.62 The complete plot for signal  $x[n]$  is

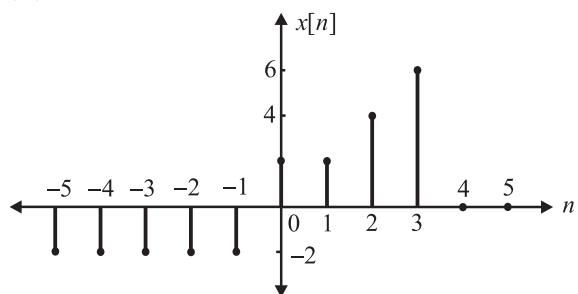
(A)



(B)



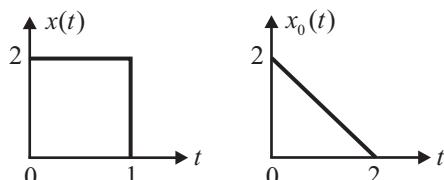
(C)



- (D) Complete  $x[n]$  cannot be determined from the given information.

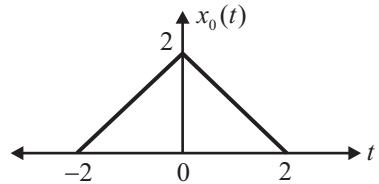
**Common Data for Questions 63 & 64**

The figure shows parts of a signal  $x(t)$  and its odd part  $x_0(t)$ , for  $t \geq 0$  only, that is  $x(t)$  and  $x_0(t)$  are not given for  $t < 0$ .

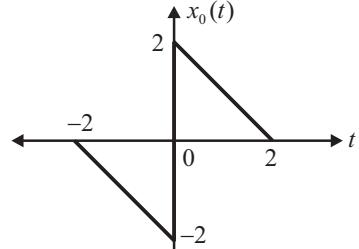


- Q.63** The complete odd part  $x_0(t)$  of the signal will be

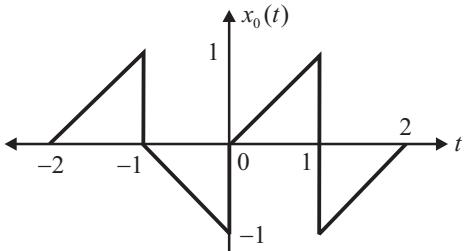
(A)



(B)



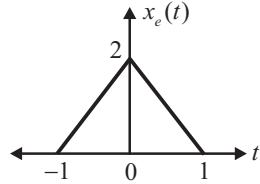
(C)



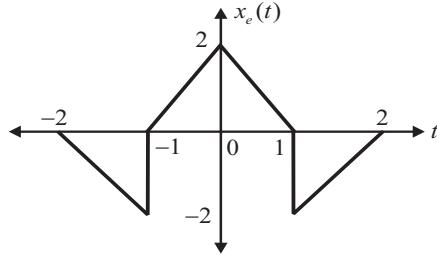
- (D) Cannot be determined

- Q.64** The complete even part  $x_e(t)$  of the signal  $x(t)$  is

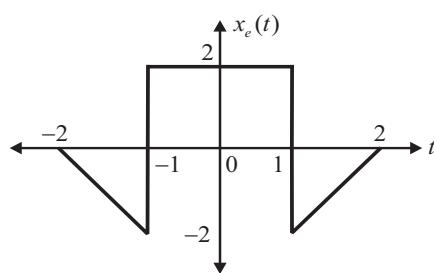
(A)



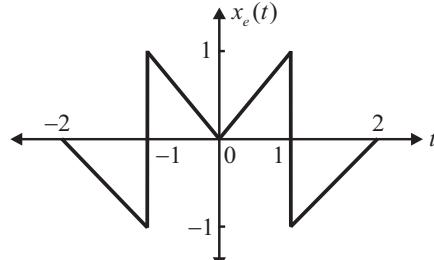
(B)



(C)



(D)



- Q.65** Two periodic signals  $x(t)$  and  $y(t)$  have the same fundamental period of 3 seconds. Consider the signal  $z(t) = x(-t) + y(2t+1)$ . The fundamental period of  $z(t)$  in seconds is

[GATE IN 2018-Guwahati]

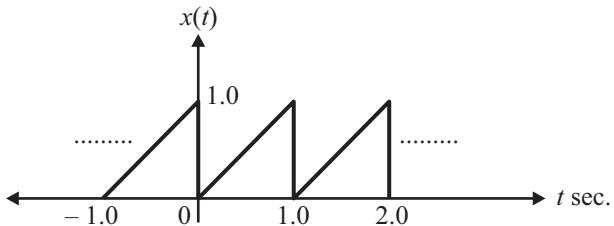
(A) 1

(B) 1.5

(C) 2

(D) 3

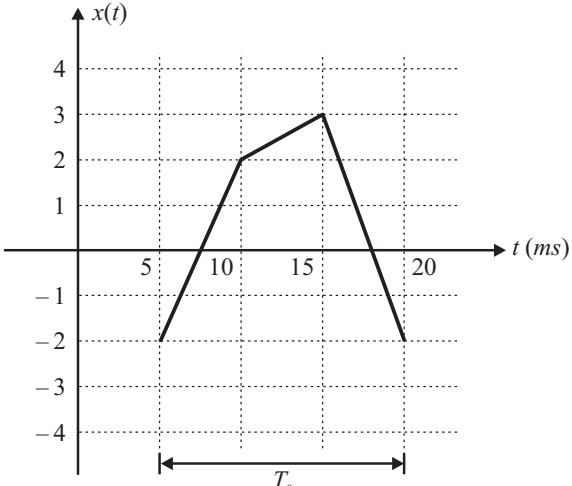
- Q.66** A periodic signal  $x(t)$  is shown in the figure. The fundamental frequency of the signal  $x(t)$  in Hz is \_\_\_\_\_. [GATE IN 2017-Roorkee]



- Q.67** The fundamental period of the signal  $x(t) = 2 \cos\left(\frac{2\pi}{3}t\right) + \cos(\pi t)$ , in seconds, is \_\_\_\_\_ s. [GATE IN 2015-Kharagpur]

- Q.68** For a periodic signal  $v(t) = 30 \sin 100t + 10 \cos 300t + 6 \sin(500t + \pi/4)$ . The fundamental frequency in rad/s is [GATE EC, EE, IN 2013-Bombay]

- (A) 100                      (B) 300  
 (C) 500                      (D) 1500
- Q.69** A function is given by  $f(t) = \sin^2 t + \cos 2t$ . Which of the following is true?
- [GATE EC 2009-Roorkee]
- (A)  $f$  has frequency components at 0 and  $1/2\pi$  Hz  
 (B)  $f$  has frequency components at 0 and  $1/\pi$  Hz  
 (C)  $f$  has frequency components at  $1/2\pi$  and  $1/\pi$  Hz  
 (D)  $f$  has frequency components at 0,  $1/2\pi$  and  $1/\pi$  Hz
- Q.70** Consider the periodic signal,  
 $x(t) = (1 + 0.5 \cos 40\pi t) \cos 200\pi t$
- where  $t$  is in seconds. Its fundamental frequency, in Hz, is [GATE IN 2007-Kanpur]
- (A) 20                      (B) 40  
 (C) 100                      (D) 200
- Q.71** Which of the following signal is not periodic?  
 [GATE EC 1992-Delhi]
- (A)  $S(t) = \cos 2t + \cos 3t + \cos 5t$   
 (B)  $S(t) = \exp(j8\pi t)$   
 (C)  $S(t) = \exp(-7t) \sin 10\pi t$   
 (D)  $S(t) = \cos 2t \cos 4t$
- Q.72** Consider two signals  $x_1(t) = e^{j20t}$  and  $x_2(t) = e^{(-2+j)t}$ . Which one of the following statements is correct? [ESE EC 2007]
- (A) Both  $x_1(t)$  and  $x_2(t)$  are periodic  
 (B)  $x_1(t)$  is periodic but  $x_2(t)$  is not periodic  
 (C)  $x_2(t)$  is periodic but  $x_1(t)$  is not periodic  
 (D) Neither  $x_1(t)$  nor  $x_2(t)$  is periodic
- Q.73** For the given signals  $x_1(t)$ ,  $x_2(t)$  and  $x_3(t)$  the periods are respectively 1.08, 3.6 and 2.025 sec, then  $x(t) = x_1(t) + x_2(t) + x_3(t)$  is :
- (A) Non periodic  
 (B) Periodic with fundamental period 30  
 (C) Periodic with fundamental period 32.4  
 (D) Periodic with fundamental period 5

- Q.74** The signal  $x(t) = \sum_{k=-\infty}^{\infty} [\delta(t - 4k) - \delta(t - 4k - 1)]$  is
- (A) Non periodic  
 (B) Periodic with fundamental period 3  
 (C) Periodic with fundamental period 4  
 (D) Periodic with fundamental period 5
- Q.75** One period of a periodic signal  $x(t)$  with period  $T_0$  is graphed in figure. Assuming  $x(t)$  has a period  $T_0$ , what is the value of  $x(t)$  at time  $t = 220$  ms?
- 
- (A) -2                      (B) 2  
 (C) 3                           (D) 1
- Q.76** If a continuous time signal  $x(t) = \cos(2\pi t)$  is sampled at 4 Hz, the value of the discrete time sequence  $x(n)$  at  $n = 5$  is
- [GATE IN 2017-Roorkee]
- (A) -0.707                      (B) -1  
 (C) 0                              (D) 1
- Q.77** A continuous-time function  $x(t)$  is periodic with period  $T$ . The function is sampled uniformly with a sampling period  $T_s$ . In which one of the following cases is the sampled signal periodic?
- [GATE EC 2016-Bangalore]
- (A)  $T = \sqrt{2} T_s$                       (B)  $T = 1.2 T_s$   
 (C) Always                              (D) Never
- Q.78** The fundamental period  $N_0$  of the discrete-time sinusoid  $x[n] = \sin\left(\frac{301}{4}\pi n\right)$  is \_\_\_\_\_
- [GATE IN 2016-Bangalore]

**Q.79** The continuous signal  $x(t) = \sin \omega_0 t$  is a periodic signal. However, for its discrete-time counterpart  $x(n) = \sin \omega_0 n$  to be periodic, the necessary condition is [GATE IN 2011-Madras]

- (A)  $0 < \omega_0 < 2\pi$
- (B)  $2\pi / \omega_0$  to be an integer
- (C)  $2\pi / \omega_0$  to be a ratio of integers
- (D) None of the above

**Q.80** The fundamental period of the sequence  $x[n] = 3\sin(1.3\pi n + 0.5\pi) + 5\sin(1.2\pi n)$  is

[GATE IN 2005-Bombay]

- (A) 20
- (B)  $\frac{2\pi}{1.3\pi}$
- (C)  $\frac{2\pi}{1.2\pi}$
- (D) 10

**Q.81** A signal  $x(t) = 5\cos(150\pi t - 60)$  is sampled at 200 Hz. The fundamental period of the sampled sequence  $x(n)$  is, [GATE IN 2004-Delhi]

- (A)  $\frac{1}{200}$
- (B)  $\frac{2}{200}$
- (C) 4
- (D) 8

**Q.82** Consider a complex exponential sequence  $e^{j\omega_0 n}$  with frequency  $\omega_0$ . Suppose  $\omega_0 = 1$ , then

[ESE EC 2016, 2015]

- (A) Such a sequence is periodic.
- (B) Such a sequence is not periodic at all.
- (C) Periodic for some value of period 'N'.
- (D) Some definite range  $N_0 < n < N_1$  exists for a periodic sequence.

**Q.83** Which of the following statement is correct for

$$x(n) = \cos\left(\frac{n\pi}{4}\right) + \sin\left(\frac{n\pi}{8}\right) - 2\cos\left(\frac{n\pi}{2}\right)$$

- (A) Periodic,  $N = 8$
- (B) Non periodic
- (C) Periodic,  $N = 16$
- (D) Periodic,  $N = 64$

**Q.84** If  $x_1(t) = \begin{cases} \sin t, & t \geq 0 \\ \cos t, & t < 0 \end{cases}$  and

$$x_2(n) = u(n) + u(-n-1), \text{ then}$$

- (A)  $x_1$  and  $x_2$  both are periodic
- (B)  $x_1$  and  $x_2$  both are non periodic
- (C)  $x_1$  is periodic but  $x_2$  is non periodic
- (D)  $x_1$  is non periodic, but  $x_2$  is periodic

**Q.85** A periodic time signal is given by

$$x(n) = \cos(3\pi n) + \sin(7\pi n) + \cos(2.5\pi n)$$

The term  $\sin(7\pi n)$  in  $x(n)$  corresponds to

- (A) 14<sup>th</sup> harmonic
- (B) 7<sup>th</sup> harmonic
- (C) 6<sup>th</sup> harmonic
- (D) 5<sup>th</sup> harmonic

**Q.86** A signal  $x(t) = 2\cos(40\pi t) + \sin(60\pi t)$  is sampled at 75 Hz. Then period of discrete time signal  $x(n)$  is

- (A) 5
- (B) 10
- (C) 15
- (D) Non periodic

**Q.87** For a periodic signal  $x[n]$  with period  $N$  it is given that  $y[n] = x[2n]$ . Which of the following is true?

- (A)  $y[n]$  is periodic with period  $N_0 = N/2$  if  $N$  is even and with period  $N_0 = N$  if  $N$  is odd.
- (B)  $y[n]$  is periodic with period  $N_0 = N$ , whether  $N$  is even or odd.
- (C)  $y[n]$  is periodic with period  $N_0 = N/2$ , whether  $N$  is even or odd.
- (D)  $y[n]$  is not necessarily periodic.

**Q.88** For a discrete time signal  $x[n]$ , it is given that

$$y[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

Consider the following statements :

$S_1$  : If  $x[n]$  is periodic, then  $y[n]$  is periodic

$S_2$  : If  $y[n]$  is periodic then  $x[n]$  is periodic

Which of above statements is/are true?

- (A)  $S_1$  only
- (B)  $S_2$  only
- (C) Both  $S_1$  and  $S_2$
- (D) None of these

**Q.89** The signal energy of the continuous time signal

$$x(t) = [(t-1)u(t-1)] - [(t-2)u(t-2)]$$

$$-[(t-3)u(t-3)] + [(t-4)u(t-4)]$$

[GATE EE 2018-Guwahati]

$$(A) \frac{11}{3} \quad (B) \frac{7}{3}$$

$$(C) \frac{1}{3} \quad (D) \frac{5}{3}$$

**Q.90** Consider the two continuous-time signals defined below : [GATE EE 2018-Guwahati]

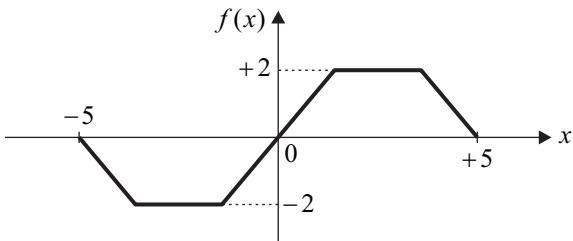
$$x_1(t) = \begin{cases} |t|, & -1 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

$$x_2(t) = \begin{cases} 1-|t|, & -1 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

These signals are sampled with a sampling period of  $T = 0.25$  seconds to obtain discrete-time signals  $x_1[n]$  and  $x_2[n]$ , respectively. Which one of the following statements is true?

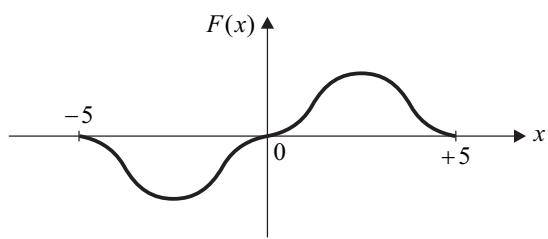
- (A) The energy of  $x_1[n]$  is greater than the energy of  $x_2[n]$ .
- (B) The energy of  $x_2[n]$  is greater than the energy of  $x_1[n]$ .
- (C)  $x_1[n]$  and  $x_2[n]$  have equal energies.
- (D) Neither  $x_1[n]$  nor  $x_2[n]$  is a finite-energy signal.

**Q.91** Consider the plot of  $f(x)$  versus  $x$  as shown below. [GATE EC 2016-Bangalore]

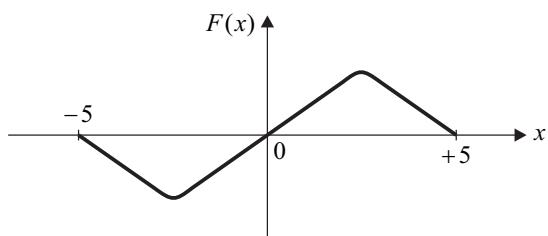


Suppose  $F(x) = \int_{-5}^x f(y) dy$ . Which one of the following is a graph of  $F(x)$ ?

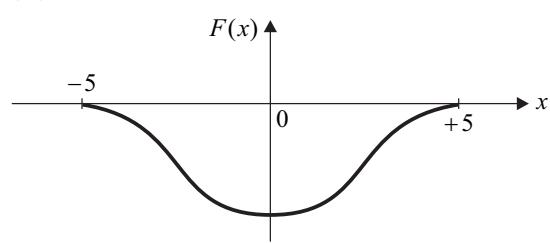
(A)



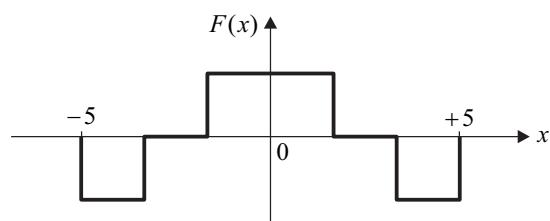
(B)



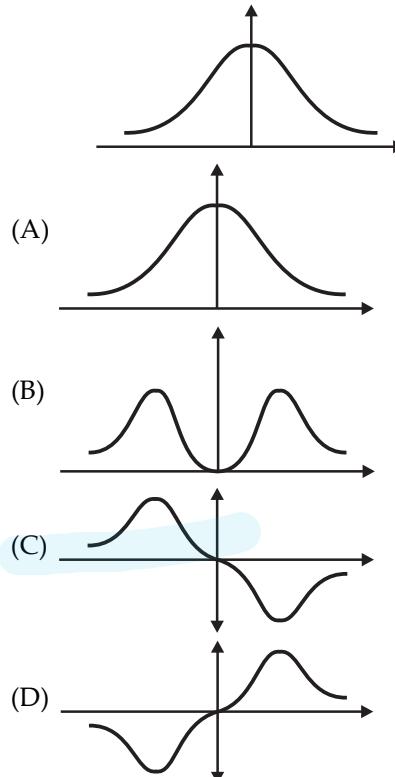
(C)



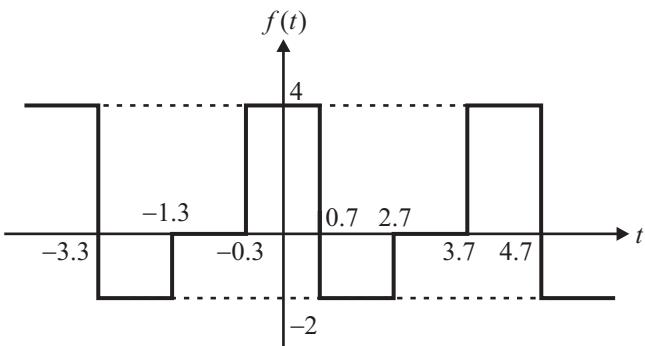
(D)



**Q.92** The derivative of the symmetric function drawn in figure will look like [GATE EC 2005-Bombay]



**Q.93** The mean square value of the given periodic waveform  $f(t)$  is [GATE EE 2017-Roorkee]

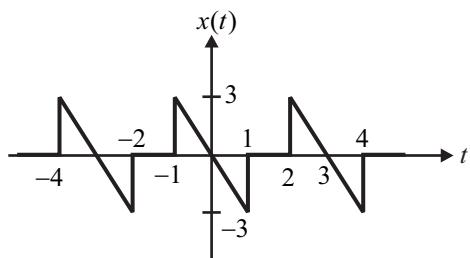


**Q.94** The energy of the signal  $x(t) = \frac{\sin(4\pi t)}{4\pi t}$  is \_\_\_\_\_.

[GATE EC 2016-Bangalore]

**Q.95** The waveform of aperiodic signal  $x(t)$  is shown in the figure.

[GATE EC 2015-Kharagpur]



A signal  $g(t)$  is defined by  $g(t) = x\left(\frac{t-1}{2}\right)$ . The average power of  $g(t)$  is \_\_\_\_\_.

**Q.96** The value of the integral  $\int_{-\infty}^{\infty} \sin c^2(5t) dt$  is \_\_\_\_\_.

[GATE EC 2014-Kharagpur]

**Q.97** The power of the signal

$$s(t) = 8 \cos\left(20\pi t - \frac{\pi}{2}\right) + 4 \sin(15\pi t)$$

[GATE EC 2005-Bombay]

- (A) 40                                     (B) 41  
 (C) 42                                     (D) 82

**Q.98** Match List-I with List-II and select the correct answer using the codes given below the Lists :

**List-I**                                   **List-II** [ESE EC 2012]

- |                    |   |
|--------------------|---|
| A. Even signal     | 1. $x(n) = \left(\frac{1}{4}\right)^n u(n)$ |
| B. Causal signal   | 2. $x(-n) = x(n)$                           |
| C. Periodic signal | 3. $x(n)u(n)$                               |
| D. Energy signal   | 4. $x(n) = x(n+N)$                          |

**Codes :**    A    B    C    D

- (A) 2    3    4    1  
 (B) 1    3    4    2  
 (C) 2    4    3    1  
 (D) 1    4    3    2

**Q.99** If the energy of the signal  $x(t)$  is  $E_x$  then what will be the energy for a signal  $x(at-b)$ ?

- (A)  $\frac{E_x}{a}$                                      (B)  $\left(\frac{b}{a}\right)E_x$   
 (C)  $\frac{1}{a}E_x + b$                              (D)  $\left(\frac{1}{a}+b\right)E_x$

**Q.100** The order of the energy of given below signals are

- |                     |                     |
|---------------------|---------------------|
| (a) $x(t)$          | (b) $x(0.5t)$       |
| (c) $x(4t-3)$       | (d) $x(-0.25t)$     |
| (A) $b > a > c > d$ | (B) $a > b > c > d$ |
| (C) $d > b > a > c$ | (D) $c > a > d > b$ |

**Q.101** Energy and power of the signal  $x(t) = e^{-3t}$  is

- |                   |                   |
|-------------------|-------------------|
| (A) $1/6, \infty$ | (B) $1/3, \infty$ |
| (C) $\infty, 1/6$ | (D) None of these |

**Q.102** The energy of the signal  $x[n]$  is

$$x[n] = \begin{cases} n & 0 \leq n \leq 5 \\ 10-n & 6 \leq n \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

- |        |         |
|--------|---------|
| (A) 75 | (B) 85  |
| (C) 95 | (D) 110 |

**Q.103** A discrete variable is given by

$$x(n) = (-1)^n [u(n) - u(n-3)]$$

The energy of the signal is given by

- |       |       |
|-------|-------|
| (A) 1 | (B) 2 |
| (C) 3 | (D) 6 |

**Q.104** The energy of the signal  $x(t) = u(t) - u(10-t)$  is

- |        |              |
|--------|--------------|
| (A) 0  | (B) 5        |
| (C) 10 | (D) $\infty$ |

**Q.105** The energy of signal  $A\delta(n)$  is

- |             |             |
|-------------|-------------|
| (A) $A^2$   | (B) $A^2/2$ |
| (C) $A^2/4$ | (D) 0       |

**Q.106** The raised cosine pulse  $x(t)$  is defined as

$$x(t) = \begin{cases} \frac{1}{2}(\cos \omega t + 1), & -\frac{\pi}{\omega} \leq t \leq \frac{\pi}{\omega} \\ 0, & \text{otherwise} \end{cases}$$

The total energy of  $x(t)$  is

- |                            |                            |
|----------------------------|----------------------------|
| (A) $\frac{3\pi}{4\omega}$ | (B) $\frac{3\pi}{8\omega}$ |
| (C) $\frac{3\pi}{\omega}$  | (D) $\frac{3\pi}{2\omega}$ |

**Q.107** Energy of  $x(t) = 10 \operatorname{rect}\left(\frac{t}{20}\right) \cos \pi t$  is

- |          |           |
|----------|-----------|
| (A) 2000 | (B) 20000 |
| (C) 1000 | (D) 10000 |

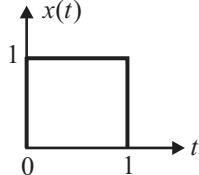
**Q.108** Energy of  $x(t) = 3 \operatorname{tri}\left(\frac{t}{4}\right)$  is

- (A) 6                      (B) 12  
 (C) 18                      (D) 24

**Q.109** Ratio of energies of  $x_1(t) = e^{-t^2}$  and  $x_2(t) = e^{-t^2/2}$  is

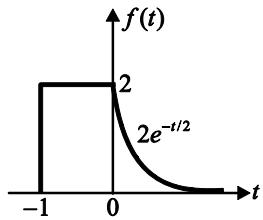
- (A) 1.414                (B) 0.707  
 (C) 2                      (D) None of these

**Q.110** The energy of even and odd parts of given signal are respectively



- (A) 0.5, 0                (B) 0.5, 0.5  
 (C) 0.25, 0              (D) 0.25, 0.25

**Q.111** Determine the energy of given below figure.



- (A) 4                      (B) 8  
 (C) 12                    (D) 16

**Q.112** Consider the signal  $x(t) = \delta(t+2) - \delta(t-2)$ . The value of energy for the signal  $y(t) = \int_{-\infty}^t x(\tau) d\tau$  is

- (A) 4                      (B) 2  
 (C) 1                      (D)  $\infty$

**Q.113** The energy and power of the signal  $x(t) = 2 \cos(3t + 90^\circ) + 4 \cos(3t + 30^\circ)$  is

- (A)  $\infty, 20$             (B)  $0, 10$   
 (C)  $\infty, 14$             (D) None of these

**Q.114** Energy of a signal  $x(t) = A \operatorname{rect}(t) + B \operatorname{rect}(t-0.5)$  is

- (A)  $A^2 + B^2$             (B)  $A^2 - B^2$   
 (C)  $A^2 + B^2 + AB$     (D)  $A^2 + B^2 + 2AB$

**Q.115** The energy of the signal  $e^{-3t} \sin 6t \cdot u(t)$  is

- (A)  $1/12$                 (B)  $1/15$   
 (C)  $1/60$                 (D)  $1/10$

**Q.116** Consider the following statements regarding a DT signal  $x[n] = (-1)^n u[n]$

1.  $x[n]$  is periodic with period  $N = 2$

2. Power of  $x[n]$  is 0.5.

3. The signal has infinite energy.

Which of the above statement is/are true?

- (A) 1 and 2                (B) 2 and 3  
 (C) 1, 2 and 3            (D) None of these

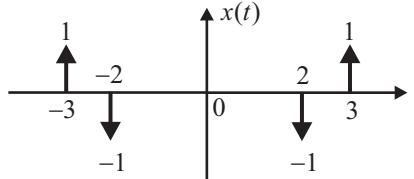
**Q.117** The DT signal  $g[n] = \frac{1}{\sqrt{n}} u[n-1]$  is

- (A) An energy signal  
 (B) A power signal  
 (C) Neither energy nor power signal  
 (D) None of the above

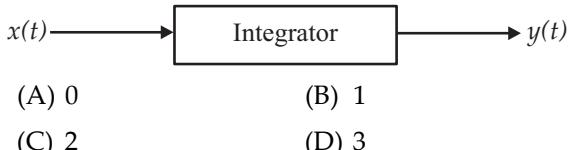
**Q.118** Energy of the signal  $x(n) = (0.9)^{|n|} \sin(\pi n / 2)$  is

- (A) 1.71                   (B) 2.71  
 (C) 3.71                   (D) 4.71

**Q.119** If the signal shown in below figure



is passed through an integrator then the energy of  $y(t)$  is

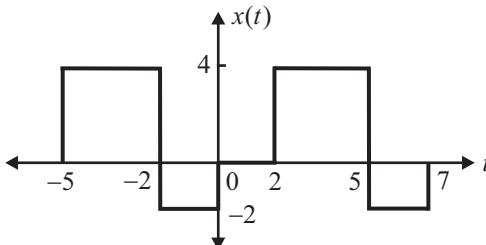


- (A) 0                      (B) 1  
 (C) 2                      (D) 3

**Q.120** The energy of a signal is 5, and the even component of the signal is  $(0.5)^{|n|}$ . Find the energy of its odd part.

- (A)  $5/3$                    (B)  $7/3$   
 (C)  $8/3$                     (D)  $10/3$

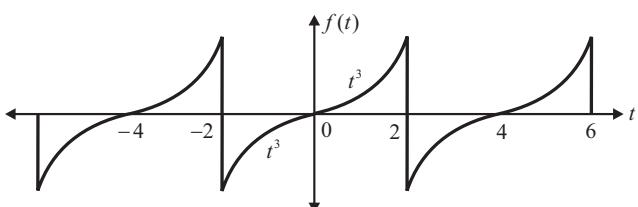
**Q.121** The power of a periodic signal shown in figure is



- (A) 56 unit      (B) 8 unit  
 (C) 11.2 unit      (D) 32 unit

**Q.122** The periodic signal  $f(t)$  shown in below figure.

The power of signal  $x(t) = -f(t)$  is



- (A)  $\frac{64}{7}$       (B)  $\frac{32}{7}$   
 (C)  $\frac{256}{7}$       (D)  $\frac{-32}{7}$

**Q.123** A signal  $x(t)$  is periodic with fundamental period  $T_0 = 6$ .

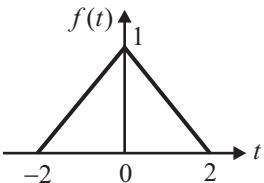
$$x(t) = \text{rect}\left[\frac{t-2}{3}\right] - 4\text{rect}\left[\frac{t-4}{2}\right]$$

The signal power is

- (A)  $\frac{35}{6}$       (B)  $\frac{31}{6}$   
 (C)  $\frac{1}{2}$       (D)  $\frac{16}{3}$

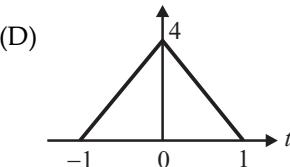
### Practice (objective & Num Ans) Questions :

**Q.1** A signal  $f(t)$  is given as :      [ESE EC 2004]

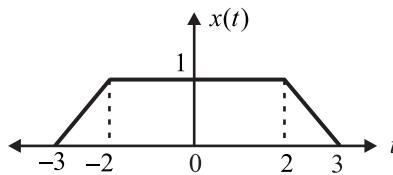


Then,  $f(2t)$  is

- (A)   
 (B)   
 (C)



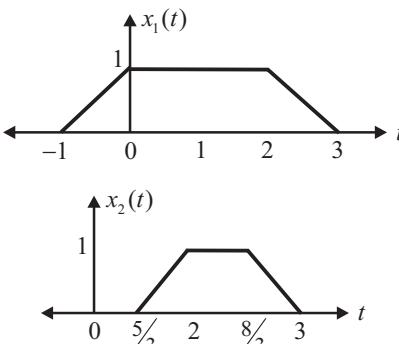
**Q.2** Consider a signal  $x(t)$  as shown in figure :



Now sketch of  $y(t) = x(6t - 3)$

- (A)   
 (B)   
 (C)   
 (D) None of the above

**Q.3** Consider two signals  $x_1(t)$  and  $x_2(t)$  as shown below



Which of the following procedure is correct to obtain  $x_2(t)$  from  $x_1(t)$  ?

- (A) First compress  $x_1(t)$  by a factor of 3, then shift to the right by 6 time units.  
 (B) First expand  $x_1(t)$  by a factor of 6, then shift to the right by 3 time units.

- (C) First compress  $x_1(t)$  by a factor of 3, then shift to the right by 2 time units.  
(D) First shift  $x_1(t)$  to the right by 2 time units then expand by a factor of 3.

**Q.4** A signal  $v[n]$  is defined by [ESE EC 2007, 2005]

$$v[n] = \begin{cases} 1 & n=1 \\ -1 & n=-1 \\ 0 & n=0 \text{ and } |n| > 1 \end{cases}$$

The value of the composite signal defined as  $v[n] + v[-n]$  is :

- (A) 0 for all integer values of  $n$   
(B) 2 for all integer values of  $n$   
(C) 1 for all integer values of  $n$   
(D) -1 for all integer values of  $n$

**Q.5** The discrete-time signal  $x[n]$  is given as

$$x[n] = \begin{cases} 1, & n=1, 2 \\ -1, & n=-1, -2 \\ 0, & n=0 \text{ and } |n| > 2 \end{cases}$$

Which one of the following is the time-shifted signal  $y[n] = x[n+3]$  [ESE EC 2006]

- (A)  $y[n] = \begin{cases} 1, & n=-1, -2 \\ -1, & n=-4, -5 \\ 0, & n=-3, n < -5 \text{ and } n > -1 \end{cases}$   
(B)  $y[n] = \begin{cases} 0, & n=-1, -2 \\ -1, & n=-4, -5 \\ 1, & n=-3, n < -5 \text{ and } n > -1 \end{cases}$   
(C)  $y[n] = \begin{cases} 1, & n=-1, -2 \\ 0, & n=-4, -5 \\ 1, & n=-3, n < -5 \text{ and } n > -1 \end{cases}$   
(D)  $y[n] = \begin{cases} -1, & n=1, 2 \\ 1, & n=4, 5 \\ 0, & n=3, n > 5 \text{ and } n < 1 \end{cases}$

**Q.6** Let  $x[n]$  be a signal with  $x[n] = 0$  for  $n < -2$  and  $n > 4$ . Find the value of  $n$  for which the signal  $x[n+4]$  is guaranteed to be zero :

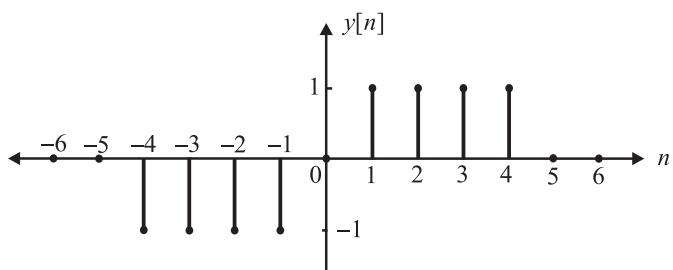
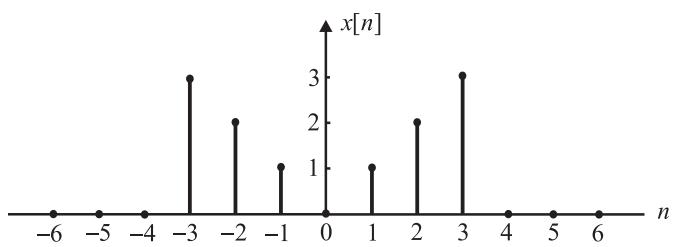
- (A)  $n < -6$  and  $n > 0$     (B)  $n < 1$  and  $n > 7$   
(C)  $n < -4$  and  $n > 2$     (D)  $n < -2$  and  $n > 4$

**Q.7** Tick the false statement.

- (A)  $t\delta(t) = 0$   
(B)  $\cos t \delta(t-\pi) = -\delta(t-\pi)$   
(C)  $\delta(t) = \int u(t) dt$   
(D)  $t\delta'(t) = -\delta(t)$

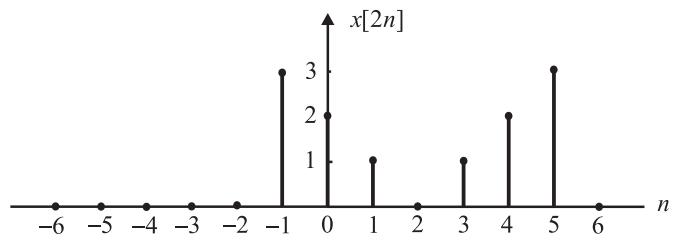
**Common Data for Questions 8, 9 and 10**

Consider the signal  $x[n]$  and  $y[n]$  given below in figures.

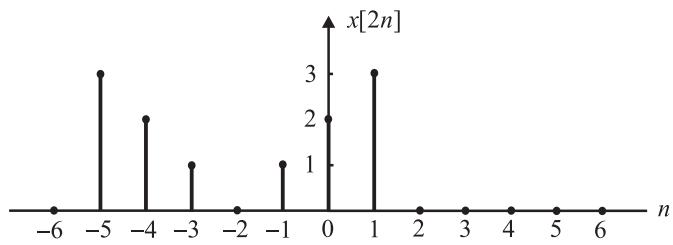


**Q.8** The signal  $x[2n]$  is

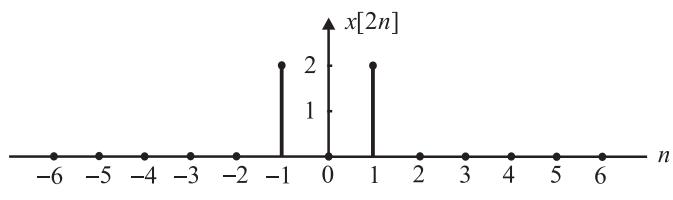
(A)



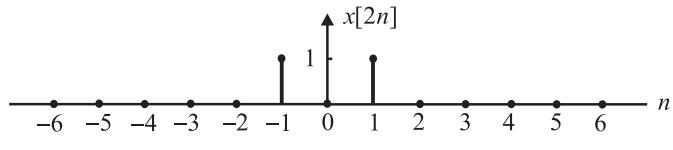
(B)



(C)

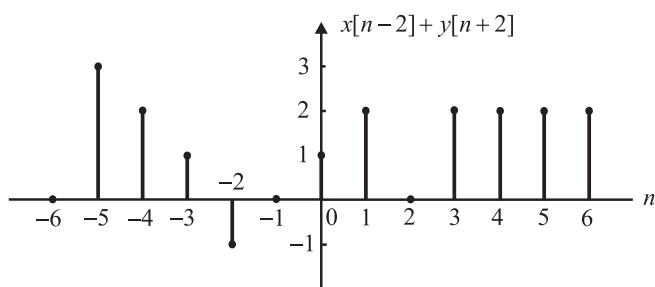


(D)

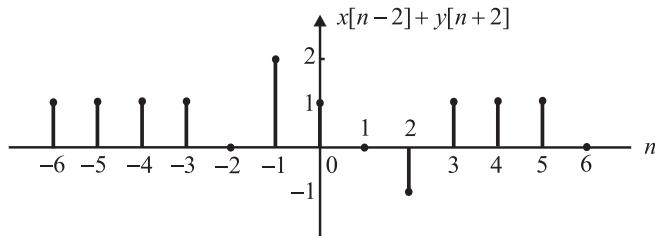


**Q.9** The signal  $x[n-2] + y[n+2]$  is

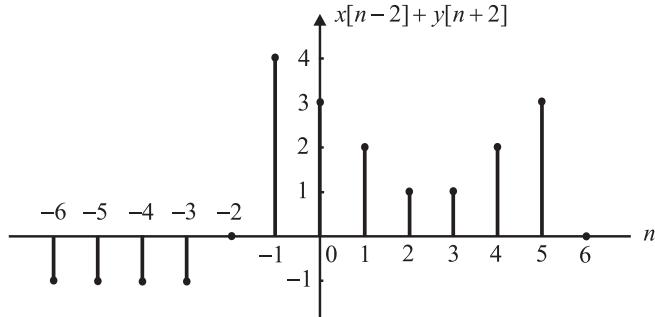
(A)



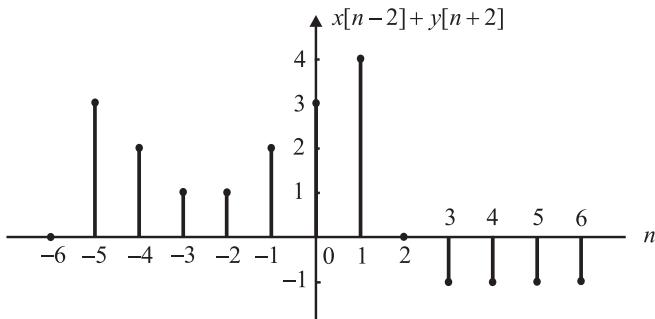
(B)



(C)

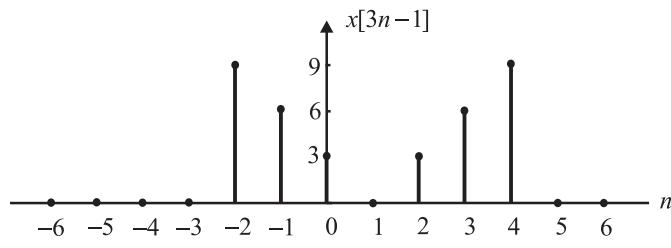


(D)

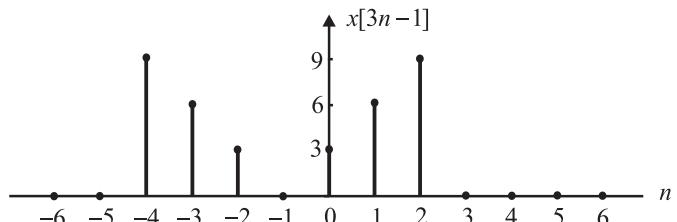


**Q.10** The signal  $x[3n-1]$  is

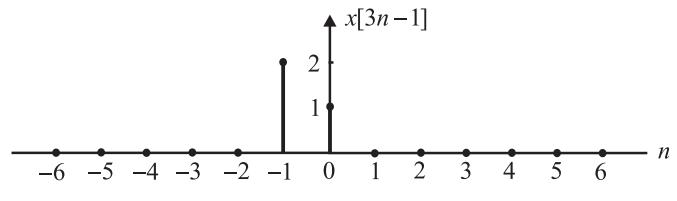
(A)



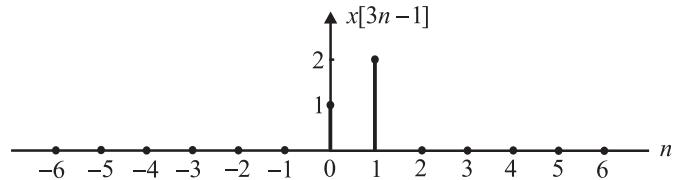
(B)



(C)



(D)



**Q.11** The value of  $\sum_{n=0}^{\infty} n \left(\frac{1}{2}\right)^n$  is \_\_\_\_\_.

[GATE EC 2015-Kanpur]

**Q.12** The integral  $\int_{-\infty}^{\infty} \delta(t - \pi/6) 6 \sin(t) dt$  evaluates to

[GATE IN 2010-Guwahati]

(A) 6 (B) 3

(C) 1.5 (D) 0

**Q.13** The Dirac delta function is defined as

[GATE EC 2006-Kharagpur]

(A)  $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$

(B)  $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{otherwise} \end{cases}$

(C)  $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$  and  $\int_{-\infty}^{\infty} \delta(t) dt = 1$

(D)  $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{otherwise} \end{cases}$  and  $\int_{-\infty}^{\infty} \delta(t) dt = 1$

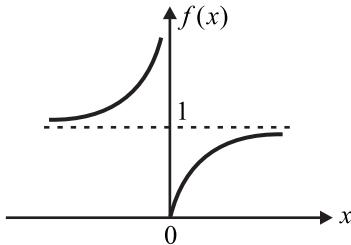
**Q.14** As  $x$  is increased from  $-\infty$  to  $\infty$ , the function

$$f(x) = \frac{e^x}{1+e^x} \quad [\text{GATE EC 2006-Kharagpur}]$$

(A) monotonically increases

(B) monotonically decreases

- (C) increases to a maximum value and then decreases  
 (D) decreases to a minimum value and then increases
- Q.15** The plot of a function  $f(x)$  is shown in the following figure : [GATE IN 2006-Kharagpur]



A possible expression for the function  $f(x)$  is :

- (A)  $\exp(|x|)$  (B)  $\exp\left(\frac{-1}{x}\right)$   
 (C)  $\exp(-x)$  (D)  $\exp\left(\frac{1}{x}\right)$
- Q.16** The value of the integral  $I = \frac{1}{\sqrt{2\pi}} \int_0^{\infty} \exp\left(-\frac{x^2}{8}\right) dx$  is [GATE EC 2005-Bombay]  
 (A) 1 (B)  $\pi$   
 (C) 2 (D)  $2\pi$
- Q.17** Let  $\delta(t)$  denote the delta function. The value of the integral  $\int_{-\infty}^{\infty} \delta(t) \cos\left(\frac{3t}{2}\right) dt$  is [GATE EC 2001-Kanpur]  
 (A) 1 (B) -1  
 (C) 0 (D)  $\pi/2$
- Q.18** The value of the integral  $\int_{-5}^6 e^{-2t} \delta(t-1) dt$  is equal to [GATE EE 1994-Kharagpur]  
 (A) 1 (B)  $e^{-2}$   
 (C) 0 (D)  $e^2$
- Q.19** The system described by the difference equation  $y(n) - 2y(n-1) + y(n-2) = x(n) - x(n-1)$  has  $y(n) = 0$  for  $n < 0$ . [ESE EC 1997]  
 If  $x(n) = \delta(n)$ , then  $y(2)$  will be  
 (A) 2 (B) 1  
 (C) Zero (D) -1
- Q.20** Area of the signal  $x(t) = 1 - |t|$  is  
 (A) 1 (B) 1/2  
 (C) 0 (D) 2

- Q.21** If  $\int_{-\infty}^{\infty} \phi'(t) \delta(t) dt = \phi'(0)$  then  $\int_{-\infty}^{\infty} \phi(t) \delta'(t) dt$  is  
 (A)  $\phi'(0)$  (B)  $-\phi'(0)$   
 (C) 0 (D) None of these
- Q.21** The value of  $\int_{-4}^6 (5t^2 + 2) \delta(t - 5/2) dt$  is  
 (A)  $129/2$  (B)  $133/4$   
 (C)  $131/4$  (D)  $131/2$
- Q.22** If  $g(t) = \sin c[2(t+1)]$ , then the value of  $10 g(t/10)$  at  $t = 4$  is  
 (A) 0 (B) 0.334  
 (C) 0.668 (D) None of these
- Q.23** The value of the integral  $\int_0^{20} \delta(t-5) \text{rect}\left(\frac{t}{16}\right) dt$  is  
 (A) 0 (B)  $1/2$   
 (C) 1 (D) 2
- Q.24** The value of CT signal  $x(t) = 8 \text{tri}\left(\frac{t}{4}\right) \otimes \delta(t-2)$  at  $t = 1$  sec is  
 (A) 8 (B) 4  
 (C) 6 (D) 0
- Q.25** Even and odd parts of the signal  $x[n] = \delta[n]$  are respectively  
 (A)  $\delta[n], \delta[n]$  (B) 0,  $\delta[n]$   
 (C)  $\delta[n], 0$  (D) 0, 0
- Q.26** Odd and even parts of  $e^{jt}$  are respectively  
 (A)  $\cos t, j \sin t$  (B)  $j \sin t, \cos t$   
 (C)  $\sin t, j \cos t$  (D)  $j \cos t, \sin t$
- Q.27** The period of the signal  $x(t) = 8 \sin\left(0.8\pi t + \frac{\pi}{4}\right)$  is [GATE EE 2010-Guwahati]  
 (A)  $0.4\pi$  sec (B)  $0.8\pi$  sec  
 (C) 1.25 sec (D) 2.5 sec
- Q.28** The fundamental period of  $x(t) = 2 \sin 2\pi t + 3 \sin 3\pi t$  with  $t$  expressed in seconds, is [GATE IN 2009-Roorkee]  
 (A) 1 sec (B) 0.67 sec  
 (C) 2 sec (D) 3 sec
- Q.29** The Fourier series for a periodic signal is given as  

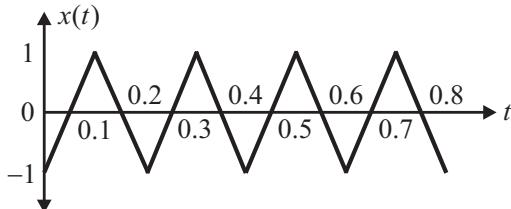
$$x(t) = \cos(1.2\pi t) + \cos(2\pi t) + \cos(2.8\pi t)$$
 The fundamental frequency of the signal is : [GATE IN 2006-Kharagpur]

- (A) 0.2 Hz                          (B) 0.6 Hz  
 (C) 1.0 Hz                           (D) 1.4 Hz

**Q.30** Identify non-periodic signal

- (A)  $x(t) = \cos^2 t$                           (B)  $x(t) = \cos 2\pi t u(t)$   
 (C)  $x(t) = \sin \frac{2\pi}{3} t$                           (D)  $x(t) = \sin^2 t$

**Q.31** What is the fundamental frequency in rad/sec for a given below figure?



- (A) 5                                      (B) 10  
 (C)  $10\pi$                                     (D)  $5\pi$

**Q.32** A discrete-time signal  $x[n] = \sin(\pi^2 n)$ ,  $n$  being an integer, is                                         [GATE EC 2014-Kharagpur]

- (A) Periodic with period  $\pi$   
 (B) Periodic with period  $\pi^2$   
 (C) Periodic with period  $\pi/2$   
 (D) Not periodic

**Q.33** The fundamental period of the discrete-time signal  $x[n] = e^{j(\frac{5\pi}{6})n}$  is

[GATE IN 2008-Bangalore]

- (A)  $\frac{6}{5\pi}$                                       (B)  $\frac{12}{5}$   
 (C) 6    (D) 12

**Q.34** A signal  $x[n] = 5 \cos\left[\frac{4\pi n}{41}\right]$  is

- (A) Periodic with  $N_0 = 41/2$   
 (B) Periodic with  $N_0 = 41$   
 (C) Periodic with  $N_0 = 2/41$   
 (D) Periodic with  $N_0 = 1/41$

**Q.35** Period of the signal

$x[n] = \cos[0.5\pi n] + \sin[0.125\pi n] + 3 \cos[0.25\pi n + \pi/3]$  would be

- (A) 16    (B) 8  
 (C) 4    (D)  $x[n]$  is aperiodic

**Q.36** Two sequences  $x_1[n]$  and  $x_2[n]$  have the same energy. Suppose  $x_1[n] = \alpha 0.5^n u[n]$ , where  $\alpha$  is a positive real number and  $u[n]$  is the unit step

sequence. Assume  $x_2[n] = \begin{cases} \sqrt{1.5} & \text{for } n = 0, 1 \\ 0 & \text{otherwise} \end{cases}$ .

Then the value of  $\alpha$  is \_\_\_\_\_.

[GATE EC 2015-Kharagpur]

**Q.37** If a signal  $f(t)$  has energy  $E$ , then energy of the signal  $f(2t)$  is equal to

[GATE EC 2001-Kanpur]

- (A)  $E$     (B)  $E/2$   
 (C)  $2E$     (D)  $4E$

**Q.38** Which one of the following sequences is NOT a power signal?                                         [GATE IN 2001-Kanpur]

- (A) Unit step sequence  
 (B)  $e^{j\omega_0 n}$   
 (C) A periodic sequence  
 (D) Unit ramp sequence

**Q.39** Statement (I) : The total energy of an energy signal falls between the limits 0 and  $\infty$ .

Statement (II) : The average power of an energy signal is zero.                                         [ESE EC 2015]

(A) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).

(B) Both Statement (I) and Statement (II) are individually true and Statement (II) is the NOT the correct explanation of Statement (I).

(C) Statement (I) is true but Statement (II) is false.

(D) Statement (I) is false but Statement (II) is true.

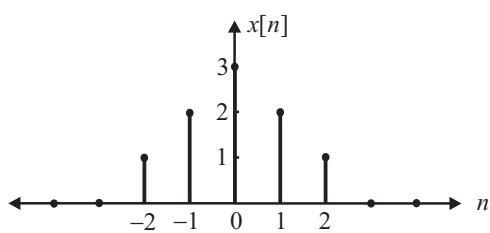
**Q.40** Which one of the following is the mathematical representation for the average power of the signal  $x(t)$ ?     [ESE EC 2007]

- (A)  $\frac{1}{T} \int_0^T x(t) dt$                                       (B)  $\frac{1}{T} \int_0^T x^2(t) dt$   
 (C)  $\frac{1}{T} \int_{-T/2}^{T/2} x(t) dt$                                       (D)  $\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) dt$

**Q.41** Power of  $8e^{j2\pi t} u(t)$  is

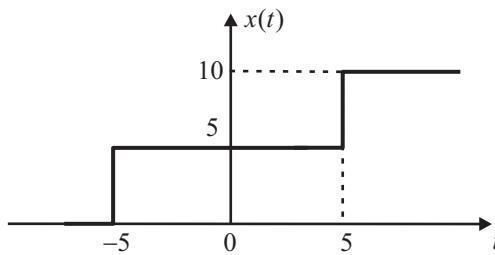
- (A) 64    (B) 32  
 (C) 128    (D) 16

**Q.42** The energy of a discrete time signal  $x[n]$  which is shown in figure below equal to



- (A) Zero      (B) 36 unit  
 (C) 19 unit      (D) 72 unit

**Q.43** Calculate the power of the signal shown below



- (A) 125 W      (B) Infinity  
 (C) 0 W      (D) 50 W

**Q.44** The energy of the function  $x(t) = \frac{d}{dt}[\text{rect}(t)]$  is  
 (A) 0      (B) 1  
 (C) 2      (D)  $\infty$

**Q.45** Energy of  $\sin c(4t)$  is

- (A) 4      (B) 0.25  
 (C) 1      (D) 0.5

**Q.46** The signal energy of  $x(t) = \text{sinc}(5t) \otimes \delta(2t)$  is  
 (A) 0.1      (B) 0.5  
 (C) 0.05      (D) 0.2

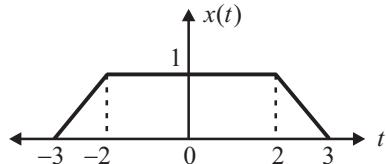
**Q.47** A DT signal is given as

$$g[n] = \cos\left(\frac{\pi n}{3}\right)[u(n) - u(n-6)].$$

The energy of the signal is

- (A) 2 unit      (B) 3 unit  
 (C) 4 unit      (D) Infinite

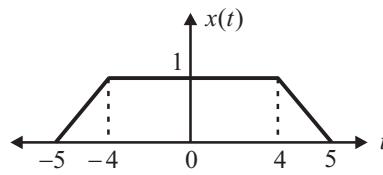
**Q.48** Energy of the signal  $x(t)$  is



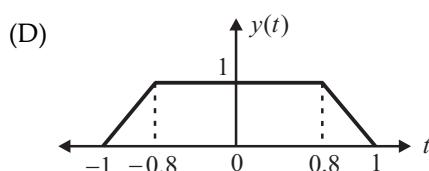
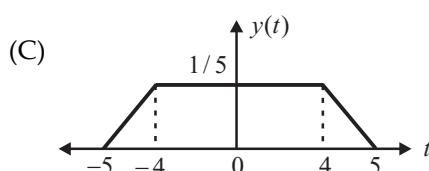
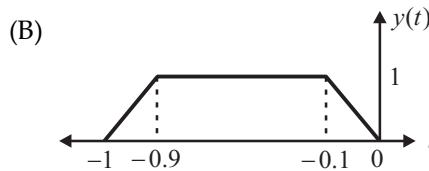
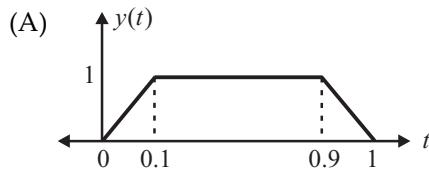
- (A) 1      (B)  $1/3$   
 (C)  $2/3$       (D)  $14/3$

**Common Data for Questions 49 to 53**

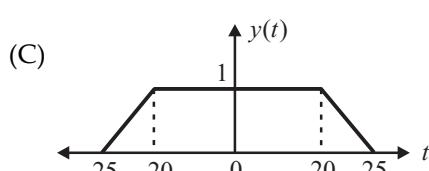
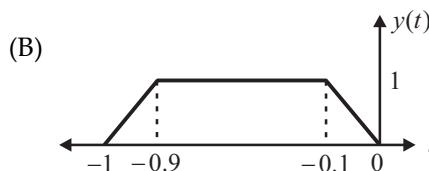
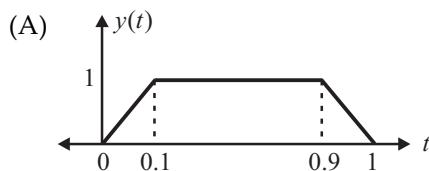
The signal  $x(t)$  is depicted in below figure

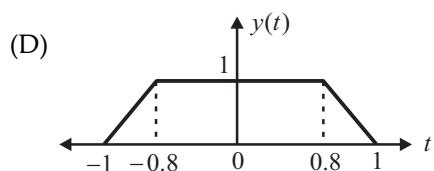


**Q.49** The trapezoidal pulse  $y(t)$  is related to the  $x(t)$  as  $y(t) = x(10t - 5)$ . The sketch of  $y(t)$  is

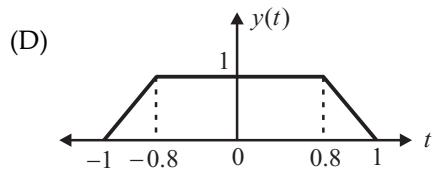
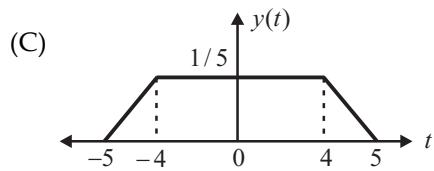
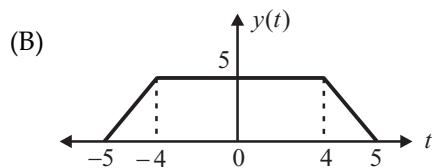
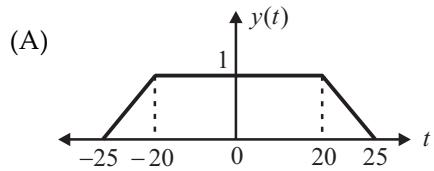


**Q.50** The trapezoidal pulse  $x(t)$  is time scaled producing  $y(t) = x(5t)$ . The sketch of  $y(t)$  is





- Q.51** The trapezoidal pulse  $x(t)$  is time scaled producing  $y(t) = x(t/5)$ . The sketch of  $y(t)$  is



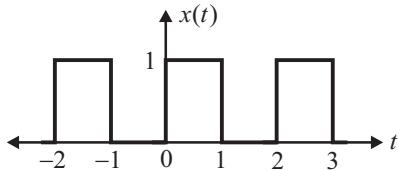
- Q.52** The trapezoidal pulse  $x(t)$  is applied to a differentiator, defined by  $y(t) = dx(t)/dt$ . The total energy of  $y(t)$  is

- (A) 0 J                          (B) 1 J  
 (C) 2 J                          (D) 3 J

- Q.53** The total energy of  $x(t)$  is

- (A) 0 J                          (B) 13 J  
 (C)  $\frac{13}{3}$  J                      (D)  $\frac{26}{3}$  J

- Q.54** Consider a periodic signal  $x(t)$  as shown in figure. The power in this signal is



- (A) 0.376                          (B) 0.5  
 (C) 0.576                          (D) 1

**Answer Keys**

Objective & Numerical Answer Type Questions									
1.	A	2.	A	3.	A	4.	C	5.	C
6.	C	7.	C	8.	B	9.	C	10.	B
11.	A	12.	B	13.	D	14.	A	15.	A
16.	A	17.	A	18.	B	19.	D	20.	A
21.	B	22.	D	23.	C	24.	C	25.	B
26.	B	27.	B	28.	A	29.	D	30.	C
31.	D	32.	D	33.	A	34.	D	35.	C
36.	C	37.	C	38.	C	39.	D	40.	C
41.	B	42.	B	43.	C	44.	D	45.	B
46.	B	47.	B	48.	D	49.	0.2928	50.	C
51.	D	52.	B	53.	D	54.	A	55.	A
56.	A	57.	A	58.	B	59.	A	60.	D
61.	B	62.	A	63.	B	64.	D	65.	D
66.	1	67.	6	68.	A	69.	B	70.	A
71.	C	72.	B	73.	C	74.	C	75.	B
76.	C	77.	B	78.	8	79.	C	80.	A
81.	D	82.	B	83.	C	84.	D	85.	A
86.	C	87.	A	88.	C	89.	D	90.	A
91.	C	92.	C	93.	6	94.	0.25	95.	2
96.	0.2	97.	A	98.	A	99.	A	100.	C
101.	D	102.	B	103.	C	104.	D	105.	A
106.	A	107.	C	108.	D	109.	B	110.	B
111.	B	112.	A	113.	C	114.	C	115.	B
116.	B	117.	C	118.	D	119.	C	120.	D
121.	B	122.	A	123.	B				

**Practice (Objective & Numerical Answer) Questions**

1.	A	2.	B	3.	C	4.	A	5.	A
6.	A	7.	C	8.	C	9.	C	10.	D
11.	2	12.	B	13.	D	14.	A	15.	B
16.	A	17.	A	18.	B	19.	B	20.	A
21.	B	22.	C	23.	C	24.	C	25.	C
26.	B	27.	D	28.	C	29.	A	30.	B
31.	C	32.	D	33.	D	34.	B	35.	A
36.	1.5	37.	B	38.	D	39.	B	40.	D
41.	B	42.	C	43.	D	44.	D	45.	B
46.	C	47.	B	48.	D	49.	A	50.	D
51.	A	52.	C	53.	D	54.	B		

# 2

# Properties of Continuous & Discrete Time Systems

## Objective & Numerical Ans Type Questions :

- Q.1** Consider a continuous-time system with input  $x(t)$  and output  $y(t)$  given by

$$y(t) = x(t) \cos(t)$$

This system is [GATE EE 2016-Bangalore]

- (A) linear and time-invariant
- (B) non-linear and time-invariant
- (C) linear and time-varying
- (D) non-linear and time-varying

- Q.2** The system represented by the input output

$$\text{relationship } y(t) = \int_{-\infty}^{5t} x(\tau) d\tau, t > 0 \text{ is}$$

[GATE EE 2010-Guwahati]

- (A) Linear and causal
- (B) Linear but not causal
- (C) Causal but not linear
- (D) Neither linear nor causal

- Q.3** For input  $x(t)$ , an ideal impulse sampling system produces the output

$$y(t) = \sum_{k=-\infty}^{\infty} x(kT) \delta(t - kT) \text{ where } \delta(t) \text{ is the Dirac}$$

delta function.

The system is [GATE IN 2009-Roorkee]

- (A) Non-linear and time-invariant
- (B) Non-linear and time-varying
- (C) Linear and time-invariant
- (D) Linear and time-varying

- Q.4** The input and output of a continuous time system are respectively denoted by  $x(t)$  and

$y(t)$  Which of the following descriptions corresponds to a causal system?

[GATE EC 2008-Bangalore]

- (A)  $y(t) = x(t-2) + x(t+4)$
- (B)  $y(t) = x(t-4) + x(t+1)$
- (C)  $y(t) = x(t-4) + x(t-1)$
- (D)  $y(t) = x(t+5) + x(t-5)$

- Q.5** Let  $x(t)$  be the input and  $y(t)$  be the output of a continuous time system. Match the system properties P1, P2 and P3 with system relations R1, R2, R3, R4. [GATE EC 2008-Bangalore]

**(Properties)**

P1 : Linear but not time-invariant

P2 : Time-invariant but not linear

P3 : Linear and time-invariant

**(Relations)**

$$R1 : y(t) = t^2 x(t)$$

$$R2 : y(t) = t |x(t)|$$

$$R3 : y(t) = |x(t)|$$

$$R4 : y(t) = x(t-5)$$

(A) (P1, R1), (P2, R3), (P3, R4)

(B) (P1, R2), (P2, R3), (P3, R4)

(C) (P1, R3), (P2, R1), (P3, R2)

(D) (P1, R1), (P2, R2), (P3, R3)

- Q.6** The input and output of a continuous time system are respectively denoted by  $x(t)$  and  $y(t)$  Which of the following descriptions corresponds to a causal system?

[GATE EE 2008-Bangalore]

- (A)  $y(t) = x(t-2) + x(t+4)$
- (B)  $y(t) = x(t-4).x(t+1)$

	(C) $y(t) = (t+4)x(t-1)$ (D) $y(t) = (t+5)x(t+5)$	
Q.7	Which one of the following discrete-time systems is time-invariant?  [GATE IN 2008-Bangalore]	
	(A) $y[n] = nx[n]$ (B) $y[n] = x[3n]$ (C) $y[n] = x[-n]$ (D) $y[n] = x[n-3]$	
Q.8	An excitation is applied to a system at $t = T$ and its response is zero for $-\infty < t < T$ . Such a system is  [GATE EC 1991-Madras]	
	(A) Non-causal system (B) Stable system (C) Causal system (D) Unstable system	
Q.9	Consider a discrete time system which satisfies the additivity property, i.e., if the output for $u_1[n]$ is $y_1[n]$ and that for $u_2[n]$ is $y_2[n]$ , then output for $u_1[n]+u_2[n]$ is $y_1[n]+y_2[n]$ . Such a system is  [ESE EC 2016]	
	(A) Linear (B) Sometimes linear (C) Non-linear (D) Sometimes non-linear	
Q.10	Consider a continuous time system A, modeled by the equation $y(t) = t x(t) + 4$ and a discrete time system modelled by the equation $y[n] = x^2[n]$ . These systems are [ESE EC 2015]  (A) A-time invariant and B-time invariant (B) A-time varying and B-time invariant (C) A-time invariant and B-time varying (D) A-time invariant and B-time varying	
Q.11	The response of a system to a complex input $x(t) = e^{j2t}$ is specified as $y(t) = t \cdot e^{j2t} + e^{-j2t}$ . The system  [ESE EC 2015]	
	(A) is definitely LTI (B) is definitely not LTI (C) may be LTI (D) information is insufficient	
Q.12	The discrete LTI System is represented by impulse response $h(n) = \left(\frac{1}{2}\right)^n u(n)$ , then the system is  [ESE EC 2015]	
	(A) Causal and stable (B) Causal and unstable (C) Non causal and stable (D) Non causal and unstable	
Q.13	The following equation describes a linear time-varying discrete time system [ESE EC 2012]  (A) $y(k+2) + ky(k+1) + y(k) = u(k)$ (B) $y(k+2) + ky^2(k+1) + y(k) = u(k)$ (C) $y(k+2) + 3y(k+1) + 2y(k) = u(k)$ (D) $y(k+2) + y^2(k+1) + ky(k) = u(k)$	
Q.14	With the following equations, the time-invariant systems are [ESE EC 2012]  1. $\frac{d^2 y(t)}{dt^2} + 2t \frac{dy(t)}{dt} + 5y(t) = x(t)$ 2. $y(t) = e^{-2x(t)}$ 3. $y(t) = \left[ \frac{d}{dt} x(t) \right]^2$ 4. $y(t) = \frac{d}{dt} \left[ e^{-2t} x(t) \right]$  (A) 1 and 2      (B) 1 and 4 (C) 2 and 3      (D) 3 and 4	
Q.15	<b>Assertion (A) :</b> The system described by $y^2(t) + 2y(t) = x^2(t) + x(t) + c$ is a linear and static system.  <b>Reason (R) :</b> The dynamic system is characterized by differential equation.  <b>Codes :</b> [ESE EC 2010] (A) Both A and R are individually true and R is the correct explanation of A (B) Both A and R are individually true but R is not the correct explanation of A (C) A is true but R is false (D) A is false but R is true	
Q.16	The outputs of two systems $S_1$ and $S_2$ for the same input $x[n] = e^{j\pi n}$ are 1 and $(-1)^n$ , respectively. Which one of the following statements is correct? [ESE EC 2007]  (A) Both $S_1$ and $S_2$ are LTI (B) $S_1$ is LTI but $S_2$ is not LTI (C) $S_1$ is not LTI but $S_2$ is LTI (D) Neither $S_1$ nor $S_2$ is LTI	

- Q.17** The system characterized by the equation  $y(t) = ax(t) + b$  is [ESE EC 2006]

- (A) Linear for any value of  $b$
- (B) Linear if  $b > 0$
- (C) Linear if  $b < 0$
- (D) Non-linear

- Q.18** Consider the following systems : [ESE EC 2004]

1.  $y[k] = x[k] + a_1x[k-1] - b_1y[k-1] - b_2y[k-2]$
2.  $y[k] = x[k] + a_1x[k-1] + a_2x[k-2]$
3.  $y[k] = x[k+1] + a_1x[k] + a_2x[k-1]$
4.  $y[k] = a_1x[k] + a_2x[k-1] - b_1y[k-2]$

Which of the systems given above represent recursive discrete systems?

- (A) 1 and 4
- (B) 1 and 2
- (C) 1, 2 and 3
- (D) 2, 3 and 4

- Q.19** Which one of the following systems is a causal system? [ESE EC 2000]

- (A)  $y(t) = \sin(u(t+3))$
- (B)  $y(t) = 5u(t) + 3u(t-1)$
- (C)  $y(t) = 5u(t) + 3u(t+1)$
- (D)  $y(t) = \sin(u(t-3)) + \sin(u(t+3))$

- Q.20** Which of the following system is additive but not homogeneous?

- (A)  $x^2(t)$
- (B)  $x(-t)$
- (C)  $t.x(t)$
- (D)  $x^*(t)$

- Q.21** A system is specified by its input output relationship as  $y(t) = \frac{f^2(t)}{(df/dt)}$ . Which of the following statement is correct?

- (A) System is additive but not homogeneous
- (B) System is homogeneous but not additive
- (C) System is linear
- (D) System is neither homogeneous nor additive

- Q.22** Identify the time invariant system

- (A)
- (B)
- (C)
- (D)

- Q.23** Consider two CT systems given by following input output relationship :

$$\text{System 1 : } y(t) = \sin[x(t)]$$

$$\text{System 2 : } y(t) = t \cdot \sin[x(t)]$$

Which of the above systems is/are time invariant?

- (A) 1 only
- (B) 2 only
- (C) Both 1 and 2
- (D) Neither 1 nor 2

- Q.24** The response of a system S to a complex input  $x(t) = e^{j5t}$  is specified as  $y(t) = te^{j5t}$ . The system

- (A) is definitely LTI
- (B) is definitely not LTI
- (C) may be LTI
- (D) information is insufficient

- Q.25** The response of a system S to a complex input  $x(t) = e^{j8t}$  is specified as  $y(t) = \cos 8t$ . The system

- (A) is definitely LTI
- (B) is definitely not LTI
- (C) may be LTI
- (D) information is insufficient

- Q.26** Consider a discrete time system S whose response to a complex exponential input  $e^{j\pi n/2}$  is specified as  $S : e^{j\pi n/2} \Rightarrow e^{j3\pi n/2}$ . The system is

- (A) definitely LTI
- (B) definitely not LTI
- (C) may be LTI
- (D) information is not sufficient

- Q.27** A continuous-time linear system with input  $x(t)$  and output  $y(t)$  yield the following input-output pairs :

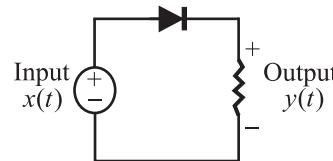
$$x(t) = e^{j2t} \Leftrightarrow y(t) = e^{j5t}$$

$$x(t) = e^{-j2t} \Leftrightarrow y(t) = e^{-j5t}$$

If  $x_1(t) = \cos(2t-1)$ , the corresponding  $y_1(t)$  is

- (A)  $\cos(5t-1)$
- (B)  $e^{-j} \cos(5t-1)$
- (C)  $\cos 5(t-1)$
- (D)  $e^j \cos(5t-1)$

- Q.28** Consider an ideal diode circuit shown in the figure



The system is

- (A) causal, linear and time-invariant
- (B) non-causal, non-linear and time-variant
- (C) causal, non-linear and time-variant
- (D) causal, non-linear and time-invariant

**Q.29** The following input output pairs have been observed during the operation of a linear system

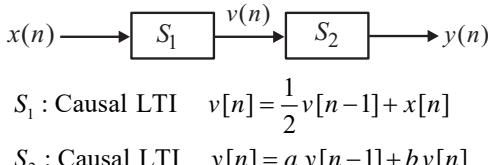
$$\begin{aligned}x_1[n] = \{-1, 2, 1\} &\xrightarrow{s} y_1[n] = \{1, 2, -1, 0, 1\} \\x_2[n] = \{1, -1, -1\} &\xrightarrow{s} y_2[n] = \{-1, 1, 0, 2\} \\x_3[n] = \{0, 1, 1\} &\xrightarrow{s} y_3[n] = \{1, 2, 1\}\end{aligned}$$

The conclusion regarding the time invariance of the system is

- (A) System is time invariant
- (B) System is time variant
- (C) One more observation is required
- (D) Conclusion cannot be drawn from observation

**Statement for Linked Questions 30 & 31**

Consider the cascade of the following two systems  $S_1$  and  $S_2$  as shown in below figure



The difference equation for cascaded system is :

$$y[n] = x[n] + \frac{3}{4}y[n-1] - \frac{1}{8}y[n-2]$$

**Q.30** The value of  $a$  is

- (A) 1/4
- (B) 1
- (C) 4
- (D) 2

**Q.31** The value of  $b$  is

- (A) 1/4
- (B) 1
- (C) 4
- (D) 2

**Statement for Linked Questions 32 & 33**

Consider a discrete time system whose input output relationship is given by following equation

$$y[n] = \frac{1}{2} \left( y[n-1] + \frac{x[n]}{y[n-1]} \right)$$

For an input  $x[n] = \alpha u[n]$ , with condition  $y[-1] = 1$ , the output converges to some constant  $k$  as  $n \rightarrow \infty$

**Q.32** The value of  $k$  is

- (A) 0
- (B)  $\alpha^1$
- (C)  $\alpha^{1/2}$
- (D)  $\alpha^2$

**Q.33** The system is

- (A) linear, time-variant
- (B) non-linear, time-variant
- (C) linear, time-invariant
- (D) non-linear, time-invariant

**Q.34**  $y(n) = \sum_{k=-\infty}^n x(k)$

[ESE EC 2004]

Which one of the following systems is inverse of the system given above?

- (A)  $x(n) = y(n) - y(n-1)$
- (B)  $x(n) = y(n)$
- (C)  $x(n) = y(n+4)$
- (D)  $x(n) = n y(n)$

**Q.35** Which of the following systems is non-invertible?

- (A)  $y(t) = x(t-2)$
- (B)  $y(t) = x^3(t)$
- (C)  $y(t) = x^2(t)$
- (D)  $y(t) = x(t) + 5$

**Q.36** From the given signals, determine which one of the following is not invertible?

- (A)  $y(t) = 2x(t)$
- (B)  $y(t) = \int_{-\infty}^t x(\tau) d\tau$
- (C)  $y[n] = \sum_{k=-\infty}^n x[k]$
- (D)  $y[n] = n \cdot x[n]$

**Q.37** Out of the following signals :

$$\begin{array}{ll} x_1 = \frac{d}{dt} x(t) & x_2 = x(t-4) \\ x_3 = \int_{-\infty}^t x(t) dt & x_4 = x^2(t) \\ x_5 = x^3(t) & \end{array}$$

(A)  $x_1, x_4$  are non invertible

(B)  $x_1, x_4, x_5$  are non invertible

(C)  $x_2, x_3, x_4$  are invertible

(D)  $x_1, x_2, x_3, x_5$  are non invertible

**Q.38** The inverse system of the equation

$$y(t) = \int_{-\infty}^{t/3} x(\lambda) d\lambda$$

(A) Not possible      (B)  $z(t) = y(3t)$

(C)  $z(t) = \frac{d}{dt} y\left(\frac{t}{3}\right)$       (D)  $z(t) = \frac{dy(3t)}{dt}$

**Q.39** Consider the two systems :

$$S_1: y[n] = \begin{cases} x[n+1], & n \geq 0 \\ x[n], & n \leq -1 \end{cases}$$

$$S_2: y[n] = \begin{cases} x\left[\frac{n}{2}\right], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

The invertible system is

- (A)  $S_1$                           (B)  $S_2$   
 (C) Both  $S_1$  &  $S_2$               (D) None of these

**Q.40** Consider two LTI systems

$$S_1: y(n) = x\left(\frac{n}{2}-1\right) \quad S_2: y(n) = x(2n-1)$$

Choose the correct option

- (A)  $S_1$  is invertible with inverse  $x(2n+1)$  and  
 $S_2$  is invertible with inverse  $x\left(\frac{n+1}{2}\right)$
- (B)  $S_1$  is non-invertible and  $S_2$  is invertible with  
 inverse  $x\left(\frac{n}{2}+1\right)$
- (C)  $S_1$  is invertible with inverse  $x(2n+2)$  and  
 $S_2$  is non-invertible
- (D) None of the above

**Q.41** The input  $x(t)$  and output  $y(t)$  of a system are

related as  $y(t) = \int_{-\infty}^t x(\tau) \cos(3\tau) d\tau$ . The system is

[GATE EC, EE, IN 2012-Delhi]

- (A) time-invariant and stable  
 (B) stable and not time-invariant  
 (C) time-invariant and not stable  
 (D) not time-invariant and not stable

**Q.42** Consider a system with input  $x(t)$  and output  $y(t)$  related as follows [GATE IN 2011-Madras]

$$y(t) = \frac{d}{dt} \{e^{-t} x(t)\}$$

Which one of the following statements is TRUE?

- (A) The system is non linear  
 (B) The system is time-invariant  
 (C) The system is stable  
 (D) The system has memory

**Q.43** A system with input  $x(t)$  and output  $y(t)$  is defined by the input-output relation

$$y(t) = \int_{-\infty}^{-2t} x(\tau) d\tau$$

The system will be : [GATE EE 2008-Bangalore]

- (A) Causal, time-invariant and unstable  
 (B) Causal, time-invariant and stable  
 (C) Non-causal, time-invariant and unstable  
 (D) Non-causal, time-variant and unstable

**Q.44** A system with input  $x[n]$  and output  $y[n]$  is

$$\text{given as } y[n] = \left( \sin \frac{5}{6} \pi n \right) x[n]$$

The system is : [GATE EC 2006-Kharagpur]

- (A) Linear, stable and invertible  
 (B) Non-linear, stable and non-invertible  
 (C) Linear, stable and non-invertible  
 (D) Linear, unstable and invertible

**Q.45** Which of the following is true?

[GATE EE 2006-Kharagpur]

- (A) A finite signal is always bounded.  
 (B) A bounded signal always possesses finite energy.  
 (C) A bounded signal is always zero outside the interval  $[-t_0, t_0]$  for some  $t_0$   
 (D) A bounded signal is always finite.

**Q.46** A continuous time system is described by  $y(t) = e^{-|x(t)|}$ , where  $y(t)$  is the output and  $x(t)$  is the input,  $y(t)$  is bounded

[GATE EE 2006-Kharagpur]

- (A) only when  $x(t)$  is bounded  
 (B) only when  $x(t)$  is non-negative  
 (C) only for  $t \geq 0$  if  $x(t)$  is bounded for  $t \geq 0$   
 (D) even when  $x(t)$  is not bounded

**Q.47** Let P be linearity, Q be time-invariance, R be causality and S be stability. A discrete time system has the input-output relationship,

$$y[n] = \begin{cases} x[n] & n \geq 1 \\ 0, & n = 0 \\ x[n+1] & n \leq -1 \end{cases}$$

where  $x[n]$  is the input and  $y[n]$  is the output.

The above system has the properties :

[GATE EC 2003-Madras]

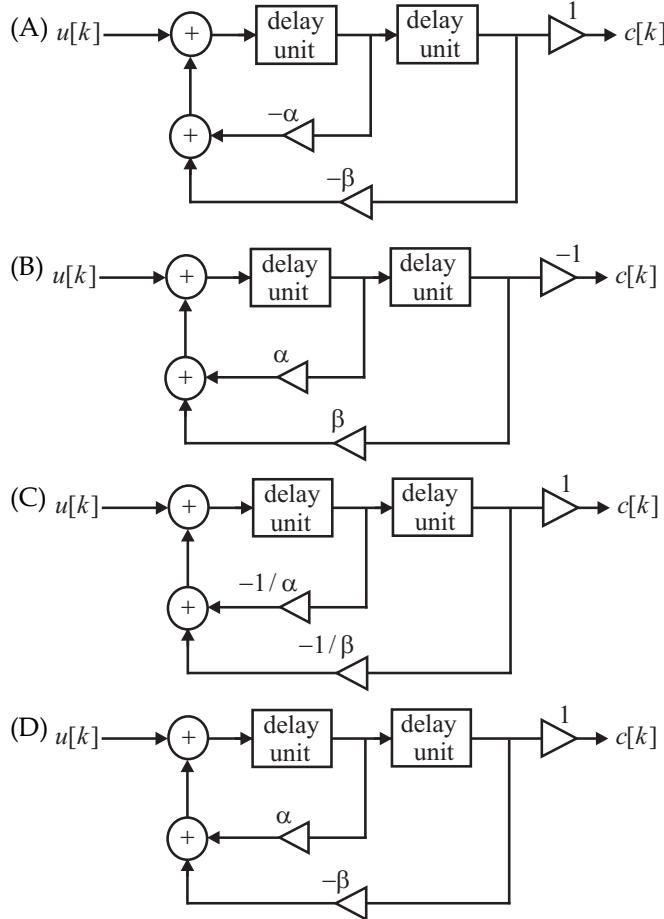
- (A) P, S but not Q, R    (B) P, Q, S but not R  
 (C) P, Q, R, S    (D) Q, R, S but not P

**Q.48** A discrete-time system has input  $x[\cdot]$  and output  $y[\cdot]$  satisfying  $y[m] = \sum_{j=-\infty}^m x[j]$ . The system is

- (A) linear and unstable    **[ESE EC 2014]**  
 (B) linear and stable  
 (C) non-linear and stable  
 (D) non-linear and unstable

**Q.49** Which one of the following is the correct state-specialization of a discrete system given by the difference equation

$$c[k+2] + \alpha c[k+1] + \beta c[k] = u[k] ? \quad \text{[ESE EC 2000]}$$



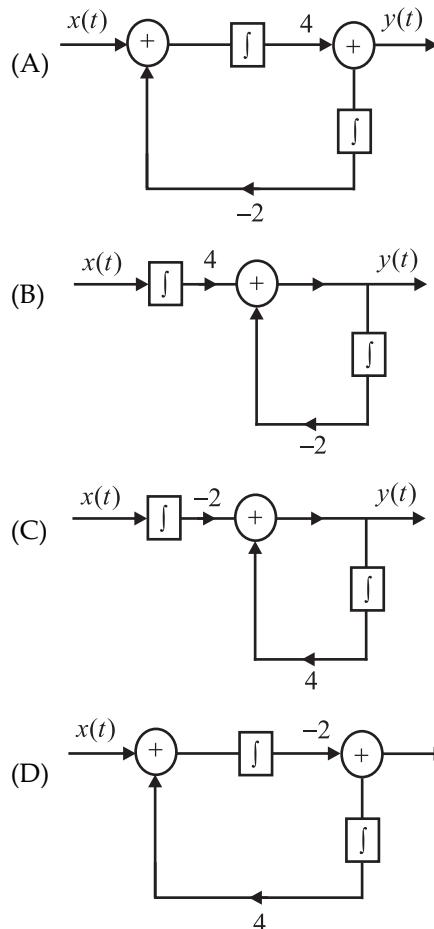
**Q.50** Which one of the following pairs are NOT correctly matched?    **[ESE EC 1999]**

- (A) Unstable system....  $\frac{dy(t)}{dt} - 0.1y(t) = x(t)$   
 (B) Non linear system ....  $\frac{dy(t)}{dt} + 2t^2 y(t) = x(t)$   
 (C) Non causal system ....  $y(t) = x(t+2)$   
 (D) Non dynamic system ....  $y(t) = 3x^2(t)$

**Q.51** Consider a causal LTI system described by following differential equation

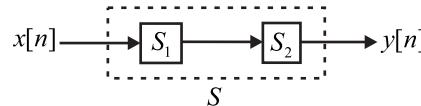
$$\frac{dy(t)}{dt} + 2y(t) = 4x(t)$$

The correct block diagram representation of the system is



**Common Data for Questions 52 to 54**

Two discrete time system  $S_1$  &  $S_2$  are connected in cascade to form a new system S as shown in figure below.



**Q.52** Consider the following statements :

- (a) If  $S_1$  &  $S_2$  are linear, then S is linear  
 (b) If  $S_1$  &  $S_2$  are nonlinear, then S is nonlinear  
 (c) if  $S_1$  &  $S_2$  are causal, then S is causal  
 (d) If  $S_1$  &  $S_2$  are time invariant, then S is time invariant

True statements are :

- (A) a, b, c    (B) b, c, d  
 (C) a, c, d    (D) All of these

**Q.53** Consider the following statements

- (a) If  $S_1$  &  $S_2$  are linear and time invariant, then interchanging their order does not change the system.
- (b) If  $S_1$  &  $S_2$  are linear and time varying, then interchanging their order does not change the system.

True statements are

- (A) Both a & b
- (B) Only a
- (C) Only b
- (D) None of these

**Q.54** Consider the following statements

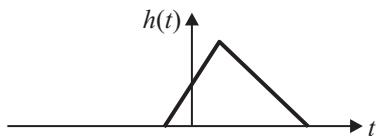
- (a) If  $S_1$  &/or  $S_2$  are non causal, then S is non causal.
- (b) If  $S_1$  &/or  $S_2$  are unstable, then S is unstable.

True statements are :

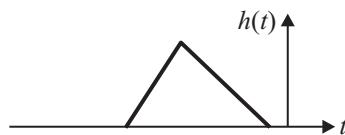
- (A) Both a & b
- (B) Only a
- (C) Only b
- (D) None of these

**Q.55** Which of the following can be impulse response of a causal system? [GATE EC 2005-Bombay]

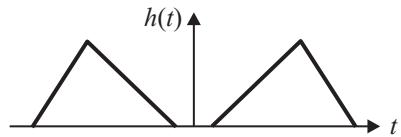
(A)



(B)



(C)



(D)



**Q.56** A discrete LTI system is non-causal if its impulse response is [ESE EC 2003]

- (A)  $a^n u(n-2)$
- (B)  $a^{n-2} u(n)$
- (C)  $a^{n+2} u(n)$
- (D)  $a^n u(n+2)$

**Q.57** Let  $h(t)$  be the response of a linear system to a unit impulse  $\delta(t)$ . [ESE EC 1998]

Consider the following statements in this regard

1. If the system is causal,  $h(t) = 0$  for  $t < 0$
2. If the system is time-variable, then the response of the system to an input of  $\delta(t-T)$  is  $h(t-T)$  for all values of the constant T.
3. If the system is non-dynamic, then  $h(t)$  is of the form  $A\delta(t)$ , where the constant A depends on the system.

Of these statements

- (A) 1 and 2 are correct
- (B) 1 and 3 are correct
- (C) 2 and 3 are correct
- (D) 1, 2 and 3 are correct

**Q.58** Let  $h(t)$  be the impulse response of a CT system. which of the following system is memory less?

- (A)  $h(t) = \delta(t) - \delta(t-4)$
- (B)  $h(t) = e^{-4t} u(t)$
- (C)  $h(t) = 4\delta(t)$
- (D)  $h(t) = (1+t^2)u(t)$

**Q.59** Consider the two system  $S_1$  &  $S_2$  having impulse response  $h_1[n] = (2)^n u[n-2]$  and  $h_2[n] = (2)^n u[n+2]$  respectively. Which of these systems are causal?

- (A)  $S_1$
- (B)  $S_2$
- (C) both  $S_1$  &  $S_2$
- (D) None of these

**Q.60** Which of the following system with an impulse response  $h(t)$  is non-invertible?

- (A)  $h(t) = \delta(t+5)$
- (B)  $h(t) = \delta(t) - \delta(t-2)$
- (C)  $h(t) = \delta(t-5)$
- (D) None of these

**Q.61** Which of the following LTI discrete system with an impulse response  $h[n]$  is non-invertible?

- (A)  $h[n] = \delta[n-2]$
- (B)  $h[n] = \delta[n+3]$
- (C)  $h[n] = \delta[n] - \delta[n-2]$
- (D) None of these

**Q.62** For linear time invariant systems, that are Bounded Input Bounded Output stable, which one of the following statements is TRUE?

[GATE EE 2015-Kharagpur]

- (A) The impulse response will be integrable, but may not be absolutely integrable.
- (B) The unit impulse response will have finite support.

- (C) The unit step response will be absolutely integrable.  
(D) The unit step response will be bounded.
- Q.63** Let  $h(t)$  denote the impulse response of a causal system with transfer function  $\frac{1}{s+1}$ . Consider the following three statements.

[GATE EC 2014-Kharagpur]

**S1 :** The system is stable

**S2 :**  $\frac{h(t+1)}{h(t)}$  is independent of  $t$  for  $t > 0$ .

**S3 :** A non-causal system with the same transfer function is stable.

For the above system,

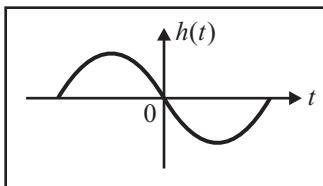
- (A) only S1 and S2 are true  
(B) only S2 and S3 are true  
(C) only S1 and S3 are true  
(D) S1 , S2 and S3 are true

- Q.64** Consider a system whose input  $x$  and output  $y$  are related by the equation

[GATE EC 2009-Roorkee]

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)h(2\tau)d\tau,$$

where  $h(t)$  is shown in the graph.



Which of the following four properties are possessed by the system?

BIBO : Bounded input gives a bounded output.

Causal : The system is causal.

LP : The system is low pass.

LTI : The system is linear and time-invariant.

- (A) Causal, LP      (B) BIBO, LTI  
(C) BIBO, Causal, LTI    (D) LP, LTI

- Q.65** A cascade of 3 Linear Time Invariant systems is causal and unstable. From this, we conclude that

[GATE EE 2009-Roorkee]

- (A) each system in the cascade is individually causal and unstable  
(B) at least one system is unstable and at least one system is causal

- (C) at least one system is causal and all systems are unstable

- (D) the majority are unstable and the majority are causal

- Q.66** The impulse response  $h(t)$  of a linear time-invariant continuous time system is described by  $h(t) = \exp(\alpha t)u(t) + \exp(\beta t)u(-t)$ , where  $u(t)$  denotes the unit step function, and  $\alpha$  and  $\beta$  are real constants. This system is stable if :

[GATE EC 2008-Bangalore]

- (A)  $\alpha$  is positive and  $\beta$  is positive  
(B)  $\alpha$  is negative and  $\beta$  is negative  
(C)  $\alpha$  is positive and  $\beta$  is negative  
(D)  $\alpha$  is negative and  $\beta$  is positive

- Q.67** The impulse response of a causal linear time invariant system is given as  $h(t)$ . Now consider the following two statements :

[GATE EE 2008-Bangalore]

**Statement (I) :** Principle of superposition holds

**Statement (II) :**  $h(t) = 0$  for  $t < 0$

Which one of the following statements is correct?

- (A) Statement (I) is correct and statement (II) is wrong.  
(B) Statement (II) is correct and statement (I) is wrong.  
(C) Both statement (I) and statement (II) are wrong.  
(D) Both statement (I) and statement (II) are correct.

- Q.68** The impulse response  $h[n]$  of a linear time-invariant system is given by

$$h[n] = u[n+3] + u[n-2] - 2u[n-7]$$

Where  $u[n]$  is the unit step sequence. The above system is

[GATE EC 2004-Delhi]

- (A) Stable but not causal  
(B) Stable and causal  
(C) Causal but unstable  
(D) Unstable and not causal

- Q.69** The impulse response functions of four linear systems  $S_1, S_2, S_3, S_4$  are given respectively by

$$h_1(t) = 1, \quad h_2(t) = u(t), \quad h_3(t) = \frac{u(t)}{t+1}, \quad h_4(t) = e^{-3t}u(t)$$

where  $u(t)$  is the unit step function. Which of these systems is time-invariant, causal, and stable?

[GATE EC 2002-Bangalore]

- (A)  $S_1$                           (B)  $S_2$   
 (C)  $S_3$                             (D)  $S_4$

**Q.70** A system has impulse response  $h[n] = \cos(n)u[n]$

The system is

[ESE EC 2016]

- (A) Causal and stable  
 (B) Non causal and stable  
 (C) Non causal and not stable  
 (D) Causal and not stable

**Q.71** **Statement (I)** : The standard definition of stability precludes  $\sin \omega_0 t$  term in impulse response.

**Statement (II)** :  $\sin \omega_0 t$  is a periodic function.

[ESE EC 2014]

- (A) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I)  
 (B) Both Statement (I) and Statement (II) are individually true but Statement (II) is **not** the correct explanation of Statement (I)  
 (C) Statement (I) is true but Statement (II) is false  
 (D) Statement (I) is false but Statement (II) is true

**Q.72** **Assertion (A)** : A linear system gives a bounded output if the input is bounded.

**Reason (R)** : The roots of the characteristic equation have all negative real parts and the response due to initial conditions decay to zero as time  $t$  tends to infinity.

**Codes :** [ESE EC 2010]

- (A) Both A and R are individually true and R is the correct explanation of A  
 (B) Both A and R are individually true but R is not the correct explanation of A  
 (C) A is true but R is false  
 (D) A is false but R is true

**Q.73** Consider the following statements about linear time invariant (LTI) continuous time systems :

1. The output signal in an LTI system with known input and known impulse response can always be determined.

2. A causal LTI system is always stable.

3. A stable LTI system has an impulse response,  $h(t)$  which has a finite value when integrated over whole of the time axis, i.e.  $\int_{-\infty}^{\infty} h(\lambda) d\lambda$  is finite.

Which of the statements given above are correct?

[ESE EC 2005]

- (A) 1 and 3                          (B) 1 and 2  
 (C) 2 and 3                           (D) 1, 2 and 3

**Q.74** The range of values of  $a$  and  $b$  for which the linear time invariant system with impulse response

$$\begin{aligned} h(n) &= a^n, n \geq 0 \\ &= b^n, n < 0 \end{aligned}$$

will be stable is

[ESE EC 2002]

- (A)  $|a| > 1, |b| > 1$                           (B)  $|a| < 1, |b| < 1$   
 (C)  $|a| < 1, |b| > 1$                             (D)  $|a| > 1, |b| < 1$

**Q.75** Which of the following system with impulse response  $h(t)$  is not stable?

- (A)  $h(t) = e^{-2t}u(t-1)$                           (B)  $h(t) = e^{2t}u(-1-t)$   
 (C)  $h(t) = e^{2t}u(t-1)$                             (D)  $h(t) = te^{-t}u(t)$

**Q.76** The impulse response of a continuous-time LTI system is

$$h(t) = \left[ 2e^{-t} - e^{\frac{(t-100)}{100}} \right] u(t)$$

The system is

- (A) Causal and stable  
 (B) Causal but not stable  
 (C) Stable but not causal  
 (D) Neither causal nor stable

**Q.77** The impulse response of four CT systems  $S_1, S_2, S_3$  and  $S_4$  are given respectively as

$$h_1(t) = \delta(t), \quad h_2(t) = \frac{1}{\tau} e^{-t/\tau} u(t)$$

$$h_3(t) = u(t) - u(t-2), \quad h_4(t) = \cos(100t)u(t)$$

Which of these systems is not BIBO stable?

- (A)  $S_1$                                   (B)  $S_2$   
 (C)  $S_3$                                     (D)  $S_4$

**Q.78** Consider the impulse response of two LTI system.

$$S_1 : h_1(t) = e^{-(1-2j)t}u(t) \quad S_2 : h_2(t) = e^{-t} \cos 2t u(t).$$

The stable system is

- (A)  $S_1$                           (B)  $S_2$   
 (C) Both  $S_1$  &  $S_2$               (D) None of these

**Common Data for Questions 79 to 81**

The impulse response of continuous-time LTI system is given. Determine whether the system is causal and/or stable and choose correct option.

**Q.79**  $h(t) = e^{-6|t|}$

- (A) causal and stable  
 (B) causal but not stable  
 (C) stable but not causal  
 (D) neither causal nor stable

**Q.80**  $h(t) = e^{-6t}u(3-t)$

- (A) causal and stable  
 (B) causal but not stable  
 (C) stable but not causal  
 (D) neither causal nor stable

**Q.81**  $h(t) = e^{-4t}u(t-2)$

- (A) causal and stable  
 (B) causal but not stable  
 (C) stable but not causal  
 (D) neither causal nor stable

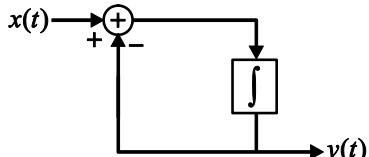
**Q.82** Given an LTI system :

1. The inverse of a causal LTI system is always causal.
2. If  $h(t)$  is the impulse response of an LTI system and  $h(t)$  is periodic and non-zero, the system is unstable.
3. If an LTI system is causal, it is stable.
4. The cascade of a non-causal LTI system with a causal one is necessarily non-causal.
5. If a discrete time LTI system has an impulse response  $h(n)$  of finite duration, the system is stable.

The correct statement are

- (A) 1, 2, 3                          (B) 1, 3, 5  
 (C) 2, 4                              (D) 2

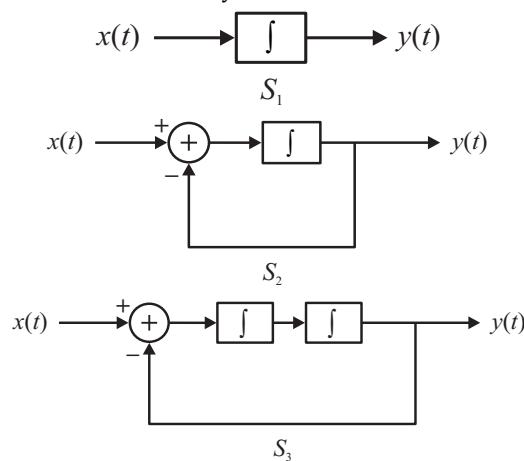
**Q.83** A system is shown in below figure :



The system is

- (A) Stable                          (B) Unstable  
 (C) Marginal stable              (D) Not able to determine

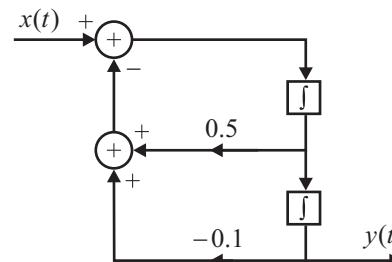
**Q.84** Consider three systems



Choose the correct option

- (A) Only  $S_2$  is stable  
 (B) Only  $S_3$  is stable  
 (C) Both  $S_2$  and  $S_3$  are stable  
 (D) All are unstable

**Q.85** The block diagram representation of CT system is shown in the figure below. The system is



- (A) BIBO Stable                    (B) BIBO Unstable  
 (C) Marginally Stable            (D) None of these

**Practice (objective & Num Ans) Questions :**

**Q.1** The input  $x(t)$  and the corresponding output  $y(t)$  of a system are related by  $y(t) = \int_{-\infty}^{5t} x(\tau) d\tau$ .

The system is                        [GATE IN 2010-Guwahati]

- (A) Time invariant and causal  
 (B) Time invariant and non-causal  
 (C) Time variant and non-causal  
 (D) Time variant and causal

**Q.2** A system with an input  $x(t)$  and output  $y(t)$  is described by the relation :  $y(t) = t x(t)$ . This system is                        [GATE EC 2000-Kharagpur]

- (A) Linear and time-invariant  
 (B) Linear and time-varying  
 (C) Non-linear and time-invariant  
 (D) Non-linear and time-varying
- Q.3** A system is characterized by the input-output relation  $y(t) = x(2t) + x(3t)$  for all  $t$ , where  $y(t)$  is the output and  $x(t)$  is the input. It is  
**[ESE EC 2014]**
- (A) linear and causal  
 (B) linear and non-causal  
 (C) non-linear and causal  
 (D) non-linear and non-causal
- Q.4** The discrete time system described by  $y(n) = x^2(n)$  is  
**[ESE EC 2012]**
- (A) causal and linear  
 (B) causal and non-linear  
 (C) non-causal and linear  
 (D) non-causal and non linear
- Q.5** The output  $y(t)$  of a continuous-time system S for the input  $x(t)$  is given by  
**[ESE EC 2009]**
- $$y(t) = \int_{-\infty}^t x(\lambda) d\lambda$$
- Which one of the following is correct?
- (A) S is linear and time-invariant  
 (B) S is linear and time-variant  
 (C) S is non-linear and time-invariant  
 (D) S is non-linear and time-variant
- Q.6** If  $v-i$  characteristic of a circuit is given by  $v(t) = t.i(t) + 2$ , the circuit is of which type?  
 (A) Linear and time invariant **[ESE EC 2008]**  
 (B) Linear and time variant  
 (C) Non-linear and time invariant  
 (D) Non-linear and time variant
- Q.7** Which one of the following systems described by the following input-output relations is non-linear?  
**[ESE EC 2007]**
- (A)  $y(n) = nx(2n)$       (B)  $y(n) = x(n^2)$   
 (C)  $y(n) = n^2x(n)$       (D)  $y(n) = x^2(n)$
- Q.8** Which one of the following is the correct statement? The continuous time system described by  $y(t) = x(t^2)$  is  
**[ESE EC 2006]**

- (A) Causal, linear and time-varying  
 (B) Causal, non-linear and time-varying  
 (C) Non-causal, non-linear and time invariant  
 (D) Non-causal, linear and time-variant
- Q.9** The governing differential equations connecting the output  $y(t)$  and the input  $x(t)$  of four continuous time systems are given in the List-I and List-II respectively. Match List-I (Equation) with List-II (System Category) and select the correct answer using the code give below the lists  
**[ESE EC 2005]**
- A.  $2t \frac{dy}{dt} + 4y = 2tx$   
 B.  $y \frac{dy}{dt} + 4y = 2x$   
 C.  $4 \frac{d^2y}{dt^2} + 2 \frac{dy}{dt} + y = 3 \frac{dx}{dt}$   
 D.  $\left( \frac{dy}{dt} \right)^2 + 2ty = 4 \frac{dx}{dt}$
- List-II (System Category)**
- Linear, time invariant and dynamic
  - Non-linear, time-invariant and dynamic
  - Linear, time-variant and dynamic
  - Non-linear, time-variant and dynamic
  - Non-linear, time-invariant and non dynamic
- Codes :** A B C D
- (A) 3 2 1 4  
 (B) 4 1 5 3  
 (C) 3 1 5 4  
 (D) 4 2 1 3
- Q.10** Match List-I (Equation connecting input  $x(n)$  and output  $y(n)$ ) of four discrete time systems with List-II (System category) and select the correct answer using the codes given below the lists :  
**[ESE EC 2004]**
- List-I**
- A.  $y(n+2) + y(n+1) + y(n) = 2x(n+1) + x(n)$   
 B.  $n^2 y^2(n) + y(n) = x^2(n)$   
 C.  $y(n+1) + ny(n) = 4nx(n)$   
 D.  $y(n+1)y(n) = 4x(n)$

**List-II (System category)**

1. Linear, time-variant, dynamic
2. Linear, time-invariant, dynamic
3. Non-linear, time-variant, dynamic
4. Non-linear, time-invariant, dynamic
5. Non-linear, time-variant, memoryless

**Codes :** A B C D

- (A) 3 5 2 1  
 (B) 3 2 5 4  
 (C) 2 3 5 1  
 (D) 2 5 1 4

**Q.11** If the response of a system to an input does not depend on the future values of the input. Then which one of the following is true for the system? [ESE EC 2004]

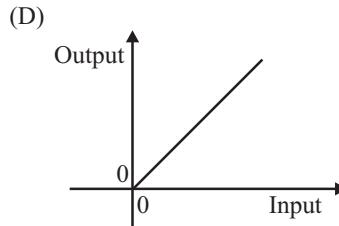
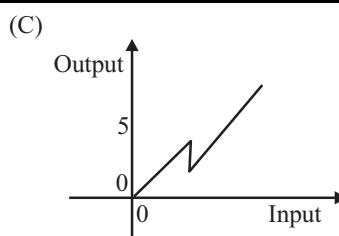
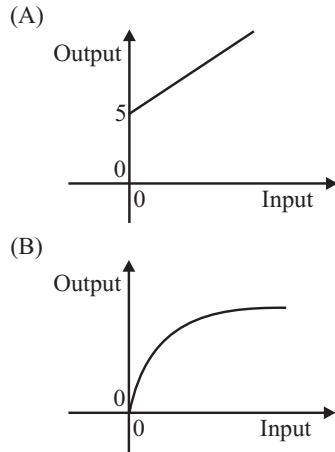
- (A) it is aperiodic      (B) it is causal  
 (C) it is anticipatory    (D) it is discrete

**Q.12** A continuous time system is governed by the equation  $3y^3(t) + 2y^2(t) + y(t) = x^2(t) + x(t)$  [  $y(t)$  and  $x(t)$  respectively are output and input]. The system is [ESE EC 2000]

- (A) linear and dynamic  
 (B) linear and non-dynamic  
 (C) non-linear and dynamic  
 (D) non-linear and non-dynamic

**Ans.** D

**Q.13** Which one of the following input-output relationship is that of a linear system?

[ESE EC 1999]

**Q.14** The discrete-time equation  $y(n+1) + 0.5n y(n) = 0.5x(n+1)$  is NOT attributable to a [ESE EC 1999]

- (A) memoryless system  
 (B) time-varying system  
 (C) linear system  
 (D) causal system

**Q.15** Which one of the following system is non-linear? [ESE EC 1998]

- (A)  $y(t) = 2x(t-1) - 3x(t-2) + x(t-3)$   
 (B)  $y(t) = 5x(t)$   
 (C)  $y(t) = 2x(t-1) - x(t-2) - x(t-4)$   
 (D)  $y(t) = 2x(t) + 3.6$

**Q.16** **Assertion (A) :** A memory less system is causal.

**Reason (R) :** A system is causal if the output at any time depends only on values of input at that time and in the past. [ESE EC 1994]

- (A) Both A and R are true and R is the correct explanation of A  
 (B) Both A and R are true and R is not a correct explanation of A  
 (C) A is true but R is false  
 (D) A is false but R is true

**Q.17** If an input  $x(n)$  is applied to an LTI system it yields an output  $y(n)$ . Which of the following statements is not true?

- (A) When input  $x(n-n_0)$  is applied, output will be  $y(n-n_0)$   
 (B) When input  $ax(n)$  is applied, output will be  $ay(n)$

- (C) When input  $x(2n)$  is applied, output will be  $y(2n)$   
 (D) None of the above

**Q.18** A system described by equation  $y(t) = \text{Odd}[x(t)]$  is

- (A) Linear, Static, Causal, Time invariant  
 (B) Linear, Dynamic, Non-Causal, Time variant  
 (C) Non-Linear, Dynamic, Non-Causal, Time variant  
 (D) Non-Linear, Static, Causal, Time invariant

**Statement for Linked Questions 19 & 20**

Consider a discrete time system whose input output relationship is given by equation

$$y[n] = y[n-1] + 3x[n], n \geq 0$$

**Q.19** Given that  $y[-1]=1$  and  $\sum_{k=0}^2 x[k] = 2$ , the value of  $y[2]$  is

- (A) 3                      (B) 2  
 (C) 7                      (D) 6

**Q.20** The given system is

- (A) linear and time-invariant only if  $y[-1]=0$   
 (B) linear and time-invariant only if  $y[-1] \neq 0$   
 (C) non-linear and time-variant only if  $y[-1]=0$   
 (D) linear only if  $y[-1]=0$  and time-invariant for any value of  $y[-1]$

**Q.21** For the system described by the equation below, with the input  $f(t)$  and output  $y(t)$ .

$$y(t) = f^n(t) \quad n, \text{ integer}$$

Which of the following statement is correct?

- (A) Invertible for even values of  $n$  and the inverse system equation is  $y(t) = [f(t)]^{1/n}$ .  
 (B) Invertible for odd values of  $n$  and the inverse system equation is  $y(t) = [f(t)]^{1/n}$ .  
 (C) Not invertible for any values of  $n$ .  
 (D) Invertible for any values of  $n$  and the inverse system equation is  $y(t) = [f(t)]^{1/n}$ .

**Q.22** A DT system has following input-output relationship

$$y(n) = \begin{cases} 3x(n), & n < 0 \\ 0, & n \geq 0 \end{cases}$$

Consider the following properties

$P_1$  : System is linear

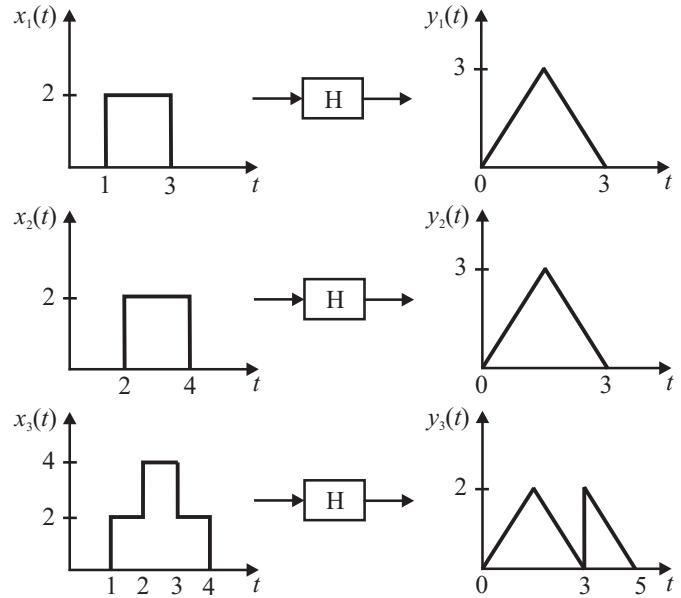
$P_2$  : System is causal

$P_3$  : System is invertible

Which of the above properties are possessed by the system?

- (A)  $P_1, P_2$                       (B) Only  $P_1$   
 (C)  $P_1, P_3$                       (D) Only  $P_2$

**Q.23** Three signals  $x_1(t), x_2(t), x_3(t)$  are sent to the system and the corresponding output signals  $y_1(t), y_2(t), y_3(t)$  are obtained as shown in figure below :

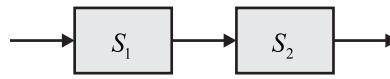


The system is

- (A) Non-linear, time variant, non-invertible, non-causal  
 (B) Linear, time invariant, causal, non-invertible  
 (C) Non-linear, non-invertible, causal, time-invariant  
 (D) Non-linear, time invariant, invertible, causal

**Q.24** Two linear time invariant discrete time systems  $S_1$  and  $S_2$  are cascaded as shown in the figure. Each system is modeled by a second order difference equation. The difference equation of the overall cascaded system can be of the order of

[ESE EC 2000]



- (A) 0, 1, 2, 3 or 4                      (B) either 2 or 4  
 (C) 2                                      (D) 4

**Q.25** Which of the following system is not stable with input  $x(n)$  and output  $y(n)$ ?

(A)  $y(n) = 20 \sin\{x(n)\} + 10$  (B)  $y(n) = e^{x(n)}$

(C)  $y(n) = \sum_{k=-\infty}^n x(k)$  (D)  $y(n) = \sum_{k=-2}^2 x(n-k)$

**Q.26** A discrete time system with an input  $x(n)$  and output  $y(n)$  is governed by following difference equation  $y(n) + 2y(n-1) = x(n) + x^2(n)$ . The system is

- (A) non-linear but stable
- (B) linear but unstable
- (C) non-linear and unstable
- (D) linear and stable

**Common Data for Questions 27 to 33**

Let P be linearity, Q be time-invariance, R be causality and S be stability. In questions input  $x(t)$  and output  $y(t)$  relationship has been given. In options properties of system has been given. Choose the option which matches the properties for the system given.

**Q.27**  $y(t) = u\{x(t)\}$

- (A) P, Q, R, S
- (B) Q, R, S
- (C) R, S
- (D) S

**Q.28**  $y(t) = x(t-5) - x(3-t)$

- (A) P, Q, R, S
- (B) Q, R, S
- (C) P, R
- (D) P, S

**Q.29**  $y(t) = x\left(\frac{t}{2}\right)$

- (A) P, Q, R, S
- (B) P, Q
- (C) P, R
- (D) P, S

**Q.30**  $y(t) = \cos(2\pi t).x(t)$

- (A) P, Q, R, S
- (B) Q, R, S
- (C) P, R, S
- (D) P, Q, S

**Q.31**  $y(t) = |x(t)|$

- (A) P, Q, R, S
- (B) P, Q, R
- (C) Q, R, S
- (D) R, S, P

**Q.32**  $t \frac{d}{dt} y(t) - 8y(t) = x(t)$

- (A) P, R
- (B) P, Q, R
- (C) P, Q, R, S
- (D) P, S

**Q.33**  $y(t) = \int_{-\infty}^{t+3} x(\lambda) d\lambda$

- (A) P, Q, R
- (B) P, Q
- (C) Q, R
- (D) P, S

**Common Data for Questions 34 to 38**

Let P be linearity, Q be time invariance, R be causality and S be stability.

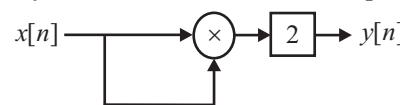
**Q.34** The system  $y[n] = \text{rect}(x[n])$  has the properties

- (A) P, Q, R
- (B) Q, R, S
- (C) R, S, P
- (D) S, P, Q

**Q.35** The system  $y[n] = \sum_{m=-\infty}^{n+1} x[m]$  has the properties

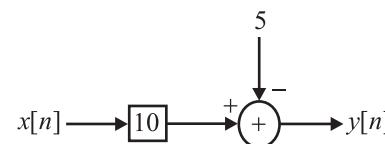
- (A) P, Q, R, S
- (B) R, S
- (C) P, Q
- (D) Q, R

**Q.36** The system shown below has the properties



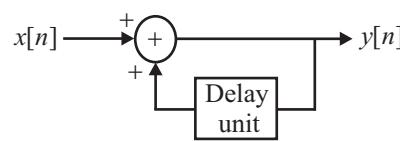
- (A) P, Q, R, S
- (B) Q, R, S
- (C) P, Q
- (D) R, S

**Q.37** The system shown below has the properties



- (A) P, Q, R, S
- (B) P, Q, S
- (C) P, R, S
- (D) Q, R, S

**Q.38** The system shown below has the properties



- (A) P, Q, R, S
- (B) P, Q, R
- (C) P, Q
- (D) Q, R, S

**Q.39** Which one of the following is correct? A system can be completely described by a transfer function if it is : [ESE EC 2007]

- (A) Non-linear and continuous
- (B) Linear and time-varying
- (C) Non-linear and time-invariant
- (D) Linear and time invariant

**Q.40** Let  $h(t)$  be the impulse response of CT system.

Which of the following system is causal?

- (A)  $h(t) = e^{-2t}u(t+2)$
- (B)  $h(t) = 2t^2u(-t) + e^{-t}u(t)$
- (C)  $h(t) = e^{-4t}u(t-2)$
- (D)  $h(t) = u(t) + u(t+2)$

**Q.41** A system is defined by its impulse response  $h(n) = 2^n u(n-2)$ . The system is

- (A) stable and causal [GATE EC 2011-Madras]
- (B) causal but not stable
- (C) stable but not causal
- (D) unstable and non causal

**Q.42** Match each of the items 1, 2 on the left with the most appropriate item A, B, C or D on the right. In the case of a linear time invariant system

[GATE EC 1997-Madras]

- (1) Poles in the right half plane implies
- (2) Impulse response zero for  $t \leq 0$  implies
- (A) Exponential decay of output
- (B) System is causal
- (C) No stored energy in the system
- (D) System is unstable

**Q.43** The impulse response  $h[n]$  of an LTI system is

$$h[n] = u[n+3] + u[n-2] - 2u[n-7]$$

Then the system is : [ESE EC 2010]

- |             |               |
|-------------|---------------|
| 1. Stable   | 2. Causal     |
| 3. Unstable | 4. Not causal |

Which of these are correct?

- (A) 1 and 2 only
- (B) 2 and 3 only
- (C) 3 and 4 only
- (D) 1 and 4 only

**Q.44** The impulse response of a system  $h(n) = a^n u(n)$ .

What is the condition for the system to be BIBO stable? [ESE EC 2006]

- (A)  $a$  is real and positive
- (B)  $a$  is real and negative
- (C)  $|a| > 1$
- (D)  $|a| < 1$

**Q.45** A discrete time system has impulse response  $h(n) = a^n u(n+2)$ ,  $|a| < 1$ . Which one of the following statement is correct? The system is

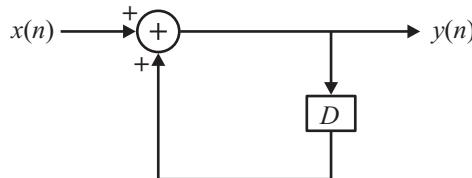
[ESE EC 2004]

- (A) Stable, causal and memory less
- (B) Unstable, causal and has memory
- (C) Stable, non-causal and has memory
- (D) Unstable, non-causal and memoryless

**Q.46** The impulse response of discrete time LTI system is given by  $h(n) = (0.99)^n u(n+3)$

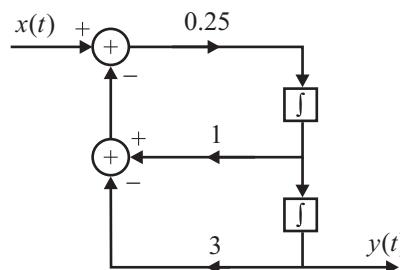
- (A)  $h(n)$  is unstable and causal
- (B)  $h(n)$  is unstable and non-causal
- (C)  $h(n)$  is stable and causal
- (D)  $h(n)$  is stable and non-causal

**Q.47** The system described by the figure is



- (A) Linear, Time invariant, Unstable, Dynamic
- (B) Linear, Time variant, Stable, Dynamic
- (C) Non-Linear, Time variant, Unstable, Dynamic
- (D) None of the above

**Q.48** A system is described by the block diagram



This system is

- (A) Linear, Static, Non-invertible and Stable
- (B) Non Linear, Dynamic, Invertible and Stable
- (C) Linear, Dynamic, Non Invertible and Unstable
- (D) None of the above



**Answer Keys**

Objective & Numerical Answer Type Questions									
1.	C	2.	B	3.	D	4.	C	5.	A
6.	C	7.	D	8.	C	9.	B, D	10.	B
11.	B	12.	A	13.	A	14.	C	15.	D
16.	C	17.	D	18.	A	19.	B	20.	D
21.	B	22.	B	23.	A	24.	B	25.	B
26.	B	27.	A	28.	D	29.	B	30.	A
31.	B	32.	C	33.	D	34.	A	35.	C
36.	D	37.	A	38.	D	39.	B	40.	C
41.	D	42.	D	43.	D	44.	C	45.	D
46.	D	47.	A	48.	A	49.	A	50.	B
51.	B	52.	C	53.	B	54.	D	55.	D
56.	D	57.	B	58.	C	59.	A	60.	D
61.	D	62.	D	63.	A	64.	B	65.	B
66.	D	67.	D	68.	A	69.	D	70.	D
71.	B	72.	D	73.	A	74.	C	75.	C
76.	B	77.	D	78.	C	79.	C	80.	D
81.	A	82.	D	83.	A	84.	A	85.	B

Practice (Objective & Numerical Answer) Questions									
1.	C	2.	B	3.	B	4.	B	5.	A
6.	D	7.	D	8.	D	9.	A	10.	D
11.	B	12.	D	13.	D	14.	A	15.	D
16.	A	17.	C	18.	B	19.	C	20.	D
21.	B	22.	A	23.	A	24.	D	25.	C
26.	C	27.	B	28.	D	29.	D	30.	C
31.	C	32.	A	33.	B	34.	B	35.	C
36.	B	37.	D	38.	B	39.	D	40.	C
41.	B	42.	1-D, 2-B	43.	D	44.	D	45.	C
46.	D	47.	A	48.	D				

# 3

# Continuous Time Fourier Series

## Objective & Numerical Ans Type Questions :

- Q.1** Let  $f(x)$  be a real, periodic function satisfying  $f(-x) = -f(x)$ . The general of its Fourier series representation would be

[GATE EE 2016-Bangalore]

(A)  $f(x) = a_0 + \sum_{k=1}^{\infty} a_k \cos(kx)$

(B)  $f(x) = \sum_{k=1}^{\infty} b_k \sin(kx)$

(C)  $f(x) = a_0 + \sum_{k=1}^{\infty} b_{2k} \cos(kx)$

(D)  $f(x) = \sum_{k=0}^{\infty} a_{2k+1} \sin(2k+1)x$

- Q.2** The signum function is given by

$$\text{sgn}(x) = \begin{cases} \frac{x}{|x|}; & x \neq 0 \\ 0; & x = 0 \end{cases}$$

The Fourier series expansion of  $\text{sgn}(\cos(t))$  has

[GATE EE 2015-Kanpur]

- (A) Only sine terms with all harmonics  
 (B) Only cosine terms with all harmonics  
 (C) Only sine terms with even numbered harmonics  
 (D) Only cosine terms with odd numbered harmonics

- Q.3** For a periodic square wave, which one of the following statements is TRUE?

[GATE EE 2014-Kharagpur]

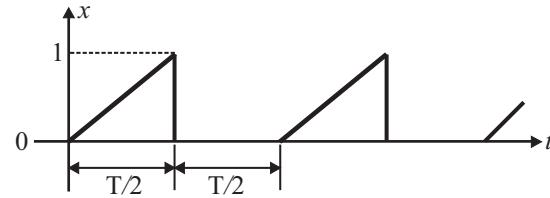
- (A) The Fourier series coefficients do not exist.  
 (B) The Fourier series coefficients exist but the reconstruction converges at no point.

(C) The Fourier series coefficients exist and the reconstruction converges at most points.

(D) The Fourier series coefficients exist and the reconstruction converges at every point.

- Q.4** A periodic variable  $x$  is shown in the figure as a function of time. The root-mean-square (rms) value of  $x$  is \_\_\_\_\_.

[GATE EC 2014-Kharagpur]

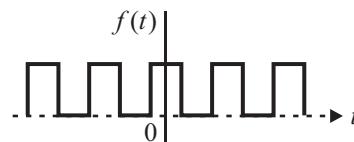


- Q.5** The Fourier series expansion

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t$$

of the periodic signal shown below will contain the following nonzero terms

[GATE EE 2011-Madras]



(A)  $a_0$  and  $b_n$ ,  $n = 1, 3, 5, \dots, \infty$

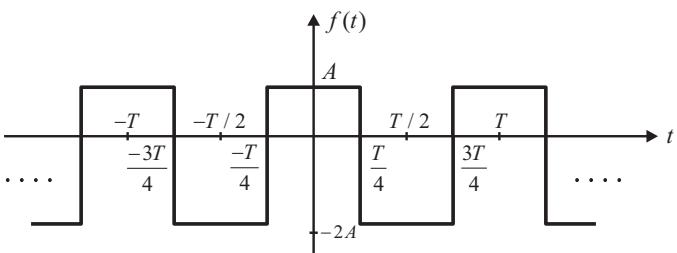
(B)  $a_0$  and  $a_n$ ,  $n = 1, 2, 3, \dots, \infty$

(C)  $a_0$ ,  $a_n$  and  $b_n$ ,  $n = 1, 2, 3, \dots, \infty$

(D)  $a_0$  and  $a_n$ ,  $n = 1, 3, \dots, \infty$

- Q.6** The trigonometric Fourier series for the waveform  $f(t)$  shown below contains

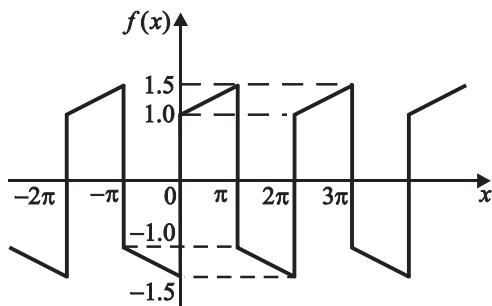
[GATE EC 2010-Guwahati]



- (A) only cosine terms and zero value for the dc component.  
 (B) only cosine terms and a positive value for the dc component.  
 (C) only cosine terms and a negative value for the dc component.  
 (D) only sine terms and a negative value for the dc component.

**Q.7**  $f(x)$ , shown in the adjoining figure is represented by  $f(x) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(nx) + b_n \sin(nx)]$ . The value of  $a_0$  is

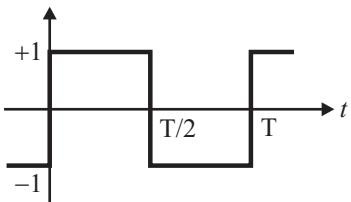
[GATE IN 2010-Guwahati]



- (A) 0                          (B)  $\pi/2$   
 (C)  $\pi$                           (D)  $2\pi$

**Q.8** The second harmonic component of the periodic waveform given the figure has an amplitude of

[GATE EE 2010-Guwahati]



- (A) 0                                  (B) 1  
 (C)  $2/\pi$                                   (D)  $\sqrt{5}$

**Q.9** The root mean squared value of  $x(t) = 3 + 2 \sin(t) \cos(2t)$  [GATE IN 2009-Roorkee]

- (A)  $\sqrt{3}$                                   (B)  $\sqrt{8}$   
 (C)  $\sqrt{10}$                                   (D)  $\sqrt{11}$

**Q.10**  $x(t)$  is a real valued function of a real variable with period  $T$ . Its trigonometric Fourier series expansion contains no terms of frequency  $\omega = 2\pi(2k)/T$ ;  $k = 1, 2, \dots$ . Also, no sine terms are present. Then  $x(t)$  satisfies the equation

[GATE EE 2006-Kharagpur]

- (A)  $x(t) = -x(t-T)$   
 (B)  $x(t) = x(T-t) = -x(-t)$   
 (C)  $x(t) = x(T-t) = -x(t-T/2)$   
 (D)  $x(t) = x(t-T) = x(t-T/2)$

**Q.11** Choose the function  $f(t)$ ,  $-\infty < t < \infty$  for which a Fourier series cannot be defined

- (A)  $3\sin(25t)$                           [GATE EC 2005-Bombay]  
 (B)  $4\cos(20t+3) + 2\sin(710t)$   
 (C)  $e^{-t}\sin(25t)$   
 (D) 1

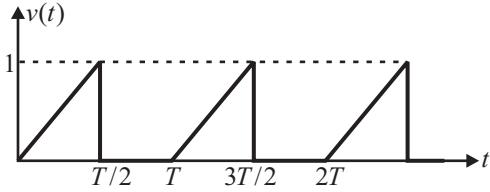
**Q.12** The Fourier series for function  $f(x) = \sin^2 x$  is

[GATE EE 2005-Bombay]

- (A)  $\sin x + \sin 2x$                           (B)  $1 - \cos 2x$   
 (C)  $\sin 2x + \cos 2x$                           (D)  $0.5 - 0.5 \cos 2x$

**Q.13** For the triangular wave form shown in the figure, the RMS value of the voltage is equal to

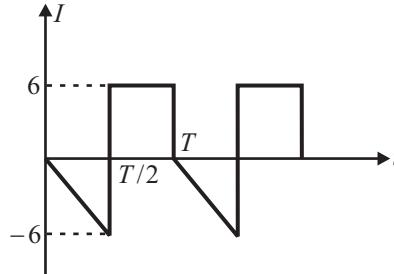
[GATE EE 2005-Bombay]



- (A)  $\frac{1}{\sqrt{6}}$                                   (B)  $\frac{1}{\sqrt{3}}$   
 (C)  $\frac{1}{3}$     (D)  $\frac{\sqrt{2}}{3}$

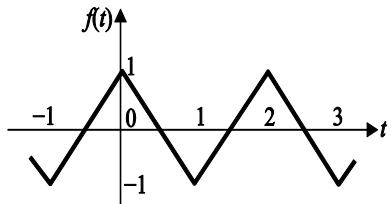
**Q.14** The rms value of the periodic waveform given in figure is

[GATE EE 2004-Delhi]



- (A)  $2\sqrt{6}$  A                                  (B)  $6\sqrt{2}$  A  
 (C)  $\sqrt{4/3}$  A    (D) 1.5 A

- Q.15** Fourier series for the waveform,  $f(t)$  shown in figure. [GATE EE 2002-Bangalore]



- (A)  $\frac{8}{\pi^2} \left[ \sin(\pi t) + \frac{1}{9} \sin(3\pi t) + \frac{1}{25} \sin(5\pi t) + \dots \right]$
- (B)  $\frac{8}{\pi^2} \left[ \sin(\pi t) - \frac{1}{9} \cos(3\pi t) + \frac{1}{25} \sin(5\pi t) + \dots \right]$
- (C)  $\frac{8}{\pi^2} \left[ \cos(\pi t) + \frac{1}{9} \cos(3\pi t) + \frac{1}{25} \cos(5\pi t) + \dots \right]$
- (D)  $\frac{8}{\pi^2} \left[ \cos(\pi t) - \frac{1}{9} \sin(3\pi t) + \frac{1}{25} \sin(5\pi t) + \dots \right]$

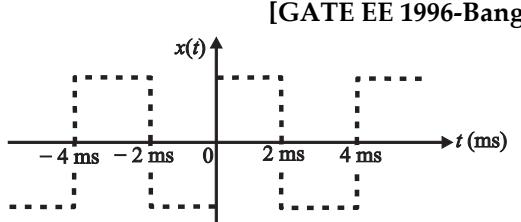
- Q.16** A periodic signal  $x(t)$  of period  $T_0$  is given by [GATE EC 1998-Delhi]

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & T_1 < |t| < \frac{T_0}{2} \end{cases}$$

The dc component of  $x(t)$  is

- (A)  $\frac{T_1}{T_0}$  (B)  $\frac{T_1}{(2T_0)}$   
 (C)  $\frac{2T_1}{T_0}$  (D)  $\frac{T_0}{T_1}$

- Q.17** A periodic rectangular signal,  $x(t)$  has the wave form shown in figure. [GATE EE 1996-Bangalore]



Frequency of the fifth harmonic of its spectrum is

- (A) 40 Hz (B) 200 Hz  
 (C) 250 Hz (D) 1250 Hz

- Q.18** The RMS value of a rectangular wave of period  $T$ , having a value of  $+V$  for a duration  $T_1 (< T)$  and  $-V$  for the duration,  $T - T_1 = T_2$  equals

[GATE EC 1995-Kanpur]

- (A)  $V$  (B)  $\frac{T_1 - T_2}{T} V$   
 (C)  $\frac{V}{\sqrt{2}}$  (D)  $\frac{T_1}{T_2} V$

- Q.19**  $x(t) = \frac{1}{T_0} + \sum_{k=1}^N \frac{2}{T_0} \cos k\omega_0 t$ , is the combined trigonometric form of Fourier series for : [ESE EC 2016]

- (A) Half rectified wave  
 (B) Saw-tooth wave  
 (C) Rectangular wave  
 (D) Impulse train

- Q.20** Statement (I) : Dirichlet's conditions restrict the periodic signal  $x(t)$ , to be represented by Fourier series, to have only finite number of maxima and minima.

Statement (II) :  $x(t)$  should possess only a finite number of discontinuities. [ESE EC 2015]

- (A) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).  
 (B) Both Statement (I) and Statement (II) are individually true and Statement (II) is the NOT the correct explanation of Statement (I).  
 (C) Statement (I) is true but Statement (II) is false.  
 (D) Statement (I) is false but Statement (II) is true.

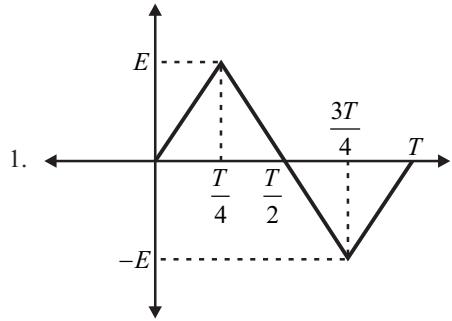
- Q.21** A square wave is defined by [ESE EC 2004]

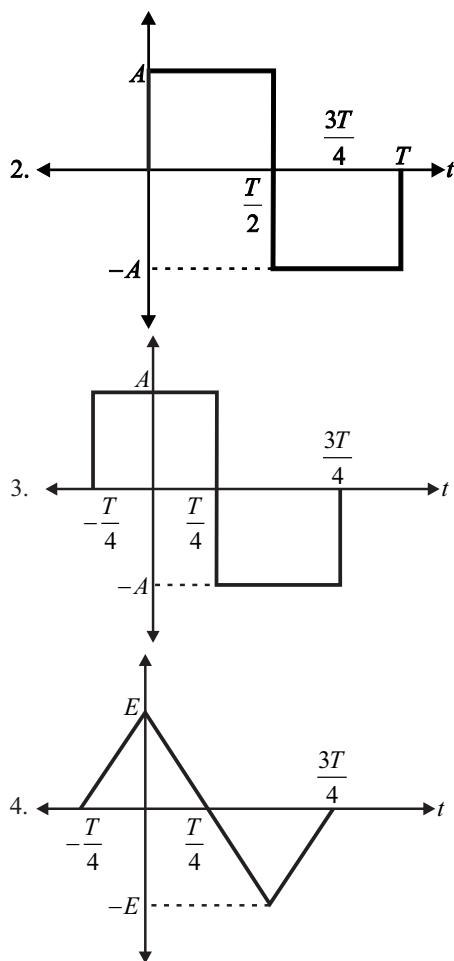
$$x(t) = \begin{cases} A, & 0 < t < T_0 / 2 \\ -A, & T_0 / 2 < t < T_0 \end{cases}$$

It is periodically extended outside this interval. What is the general coefficient  $a_n$  in the Fourier expansion of this wave?

- (A) 0 (B)  $\frac{2A(1 - \cos n\pi)}{n\pi}$   
 (C)  $\frac{2A(1 + \cos n\pi)}{n\pi}$  (D)  $\frac{2A(1 - \cos n\pi)}{(n+1)\pi}$

- Q.22** Which of the following periodic waveform will have only odd harmonics of sinusoidal waveforms? [ESE EC 1993]





Select the correct answer using the codes given below :

- (A) 1 and 2      (B) 1 and 3  
 (C) 1 and 4      (D) 2 and 4

**Q.23** The trigonometric Fourier series of the signal

$$x(t) = \cos 2t + \sin(\pi t)$$

- (A) Contains cosine terms and zero value for the dc component  
 (B) Contains sine terms and zero value for the dc components  
 (C) Contains both sine and cosine terms and positive value for the dc components  
 (D) Does not exist

**Q.24** The trigonometric Fourier series representation of a signal  $x(t)$  with period  $T_0$  is expressed as

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n}{T_0} t\right) + b_n \sin\left(\frac{2\pi n}{T_0} t\right)$$

The signal satisfies the condition  $x\left(t \pm \frac{T_0}{2}\right) = -x(t)$ . Consider the following equations

1.  $a_0 = 0$
2.  $a_n = \frac{4}{T_0} \int_0^{T_0/2} x(t) \cos\left(\frac{2\pi n}{T_0} t\right) dt$ ,  $n$  is even
3.  $b_n = \frac{4}{T_0} \int_0^{T_0/2} x(t) \sin\left(\frac{2\pi n}{T_0} t\right) dt$ ,  $n$  is odd
4.  $a_n = \frac{4}{T_0} \int_0^{T_0/2} x(t) \cos\left(\frac{2\pi n}{T_0} t\right) dt$ ,  $n$  is odd

Which of the above expressions must be true?

- (A) 1 and 4      (B) 1, 3 and 4  
 (C) 3 and 4      (D) 2 and 4

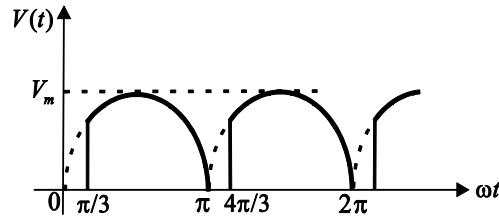
**Q.25** The exponential Fourier series of a periodic function  $x(t)$  is given as following

$$x(t) = \frac{A}{2} + \frac{2A}{\pi^2} \left[ \dots + \frac{e^{-j4t}}{(-4)^2} + \frac{e^{-j2t}}{(-2)^2} + \frac{e^{j2t}}{(2)^2} + \frac{e^{j4t}}{(4)^2} + \dots \right]$$

The trigonometric form of the series will be

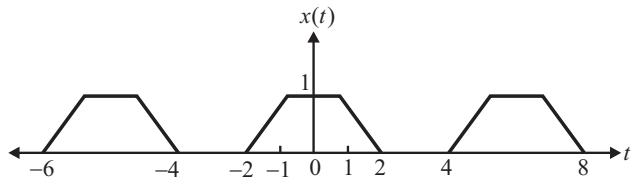
- (A)  $x(t) = \frac{A}{2} + \frac{2A}{\pi} \sum_{m=1}^{\infty} \frac{\cos(mt)}{(m)^2}$ , for all integers  $m$   
 (B)  $x(t) = \frac{A}{2} + \frac{2A}{\pi^2} \sum_{m=1}^{\infty} \frac{\cos(mt)}{(m)^2}$ , for  $m$  even  
 (C)  $x(t) = \frac{A}{2} + \frac{4A}{\pi^2} \sum_{m=1}^{\infty} \frac{\cos(mt)}{(m)^2}$ , for  $m$  even  
 (D)  $x(t) = \frac{A}{2} + \frac{2A}{\pi^2} \sum_{m=1}^{\infty} \frac{\sin(mt)}{(m)^2}$ , for  $m$  odd

**Q.26** The average value of the delayed full-wave rectified sine wave as shown in the given figure is



- (A)  $\frac{V_m}{\pi}$       (B)  $\frac{2V_m}{\pi}$   
 (C)  $\frac{3V_m}{2\pi}$       (D)  $\frac{V_m}{2\pi}$

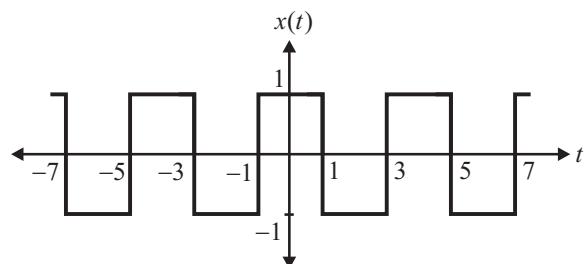
**Q.27** The trigonometric Fourier series of the signal  $x(t)$  shown in figure will contain



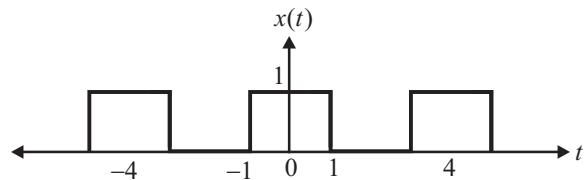
- (A) Non-zero dc component and odd harmonics of cosine function  
 (B) Zero dc component and even harmonics of cosine function  
 (C) Non-zero dc component and odd harmonics of sine function  
 (D) Zero dc components and even harmonics of sine function

**Q.28** Which of the following signal will contain only sine terms and non-zero values of dc components in its trigonometric Fourier series representation?

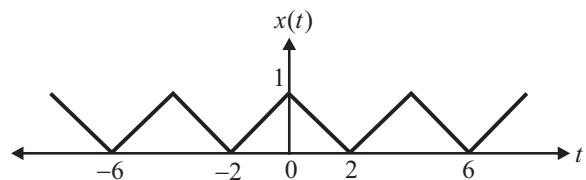
(A)



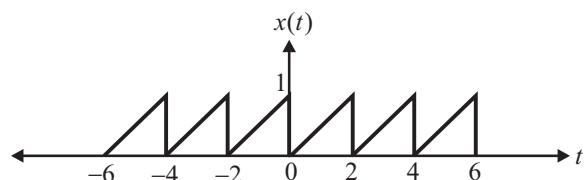
(B)



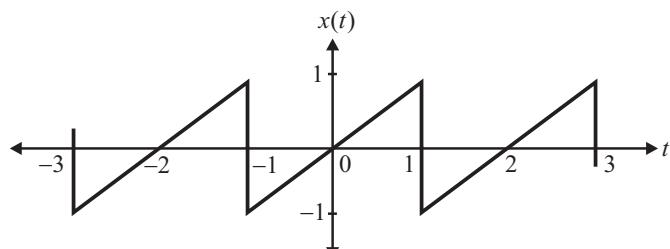
(C)



(D)



**Q.29** Consider a periodic signal  $x(t)$  shown in figure



The trigonometric Fourier series coefficient  $a_0$  and  $a_n$  are respectively

(A)  $0, \frac{2}{\pi} \left[ \frac{-(-1)^n}{n} \right]$  (B)  $0, 0$

(C)  $\frac{1}{2}, 0$  (D)  $\frac{1}{2}, \frac{2}{\pi} \left[ \frac{-(-1)^n}{n} \right]$

**Q.30** Consider a periodic signal  $x(t)$  described for one period as follows

$$x(t) = \begin{cases} 1-t, & 0 < t < 2 \\ t-3, & 2 < t < 4 \end{cases}$$

The trigonometric Fourier series representation of  $x(t)$  equals to

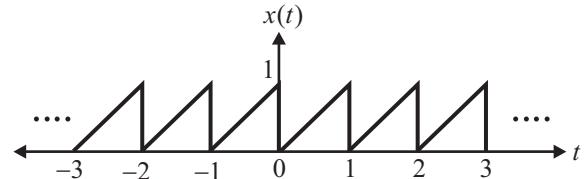
$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

The values of coefficients  $a_0$ ,  $a_1$  and  $b_2$  are respectively

(A)  $0, \frac{8}{\pi^2}, 0$  (B)  $0, 0, 0$

(C)  $\frac{8}{\pi^2}, \frac{4}{\pi^2}, 0$  (D)  $0, \frac{4}{\pi^2}, \frac{8}{\pi^2}$

**Q.31** Trigonometric Fourier series for a periodic function shown in figure below is given as



$$f(t) = \alpha + \sum_{n=1}^{\infty} a_n \cos 2\pi nt + b_n \sin 2\pi nt$$

The constant  $\alpha$ ,  $a_n$  and  $b_n$  are

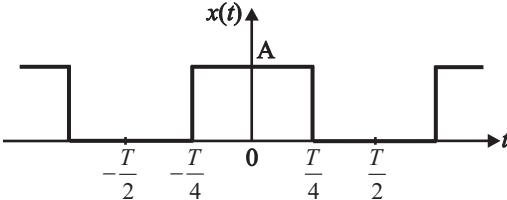
(A)  $\alpha = 0, a_n = 0, b_n = \frac{-1}{\pi n}$

(B)  $\alpha = \frac{1}{2}, a_n = 0, b_n = 0$

(C)  $\alpha = \frac{1}{2}, a_n = \frac{1}{\pi n}, b_n = \frac{1}{\pi n}$

(D)  $\alpha = \frac{1}{2}, a_n = 0, b_n = \frac{-1}{\pi n}$

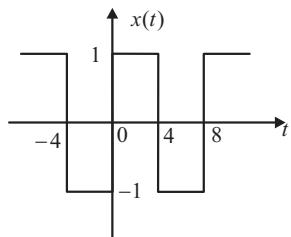
**Q.32** Fourier series for a given below waveform is :



- (A)  $\frac{2A}{\pi} \left[ \sin \omega_0 t - \frac{1}{3} \sin 3\omega_0 t + \frac{1}{5} \sin 5\omega_0 t \dots \right]$
- (B)  $\frac{A}{2} + \frac{2A}{\pi} \left[ \sin \omega_0 t - \frac{1}{3} \sin 3\omega_0 t + \frac{1}{5} \sin 5\omega_0 t \dots \right]$
- (C)  $\frac{2A}{\pi} \left[ \cos \omega_0 t - \frac{1}{3} \cos 3\omega_0 t + \frac{1}{5} \cos 5\omega_0 t \dots \right]$
- (D)  $\frac{A}{2} + \frac{2A}{\pi} \left[ \cos \omega_0 t - \frac{1}{3} \cos 3\omega_0 t + \frac{1}{5} \cos 5\omega_0 t \dots \right]$

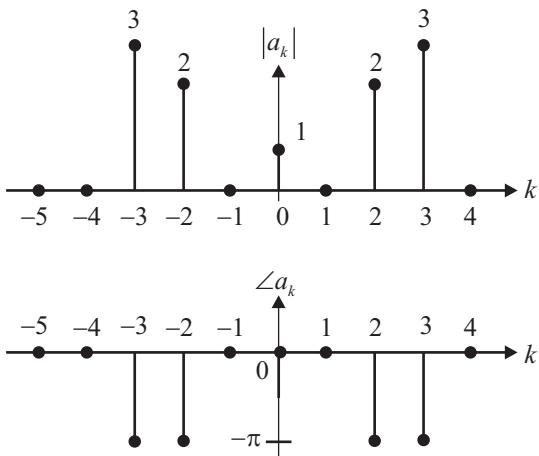
**Q.33** For the periodic signal  $x(t)$  shown below with period  $T = 8\text{s}$ , the power in the 10<sup>th</sup> harmonic is

[GATE IN 2016-Bangalore]



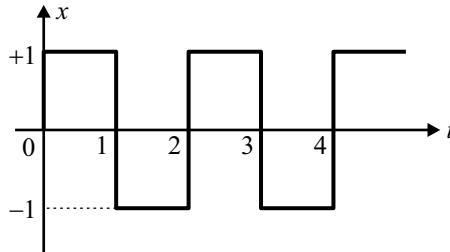
- (A) 0
- (B)  $\frac{1}{2} \left( \frac{2}{10\pi} \right)^2$
- (C)  $\frac{1}{2} \left( \frac{4}{10\pi} \right)^2$
- (D)  $\frac{1}{2} \left( \frac{4}{5\pi} \right)^2$

**Q.34** The magnitude and phase of the complex Fourier series coefficient  $a_k$  of a periodic signal  $x(t)$  are shown in the figure. Choose the correct statement from the four choices given. Notation :  $C$  is the set of complex numbers,  $R$  is the set of purely real numbers, and  $P$  is the set of purely imaginary numbers. [GATE EC 2015-Kanpur]



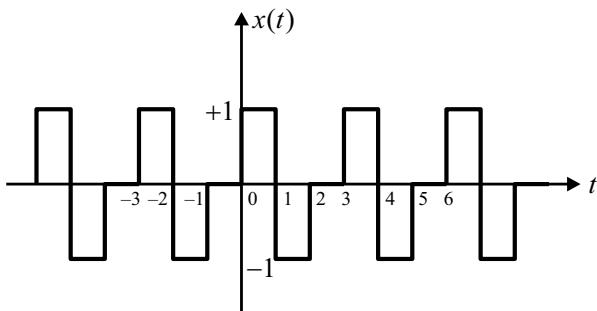
- (A)  $x(t) \in R$
- (B)  $x(t) \in P$
- (C)  $x(t) \in (C - R)$
- (D) the information given is not sufficient to draw any conclusion about  $x(t)$

**Q.35** Consider the periodic square wave in the figure shown. [GATE EC 2014-Kharagpur]



The ratio of the power in the 7<sup>th</sup> harmonic to the power in the 5<sup>th</sup> harmonic for this waveform is closest in value to \_\_\_\_.

**Q.36** Consider a periodic signal  $x(t)$  as shown below



It has a Fourier series representation

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/T)kt} \quad [\text{GATE IN 2011-Madras}]$$

Which one of the following statements is TRUE?

- (A)  $a_k = 0$ , for  $k$  odd integer and  $T = 3$
- (B)  $a_k = 0$ , for  $k$  even integer and  $T = 3$
- (C)  $a_k = 0$ , for  $k$  even integer and  $T = 6$
- (D)  $a_k = 0$ , for  $k$  odd integer and  $T = 6$

**Q.37** The Fourier Series coefficients, of a periodic signal  $x(t)$ , expressed as  $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T}$  are given by  $a_{-2} = 2 - j1$ ;  $a_{-1} = 0.5 + j0.2$ ;  $a_0 = j2$ ;  $a_1 = 0.5 - j0.2$ ;  $a_2 = 2 + j1$ ; and  $a_k = 0$ ; for  $|k| > 2$ . Which of the following is true?

[GATE EE 2009-Roorkee]

- (A)  $x(t)$  has finite energy because only finitely many coefficients are non-zero
- (B)  $x(t)$  has zero average value because it is periodic
- (C) The imaginary part of  $x(t)$  is constant
- (D) The real part of  $x(t)$  is even

**Q.38** Let  $x(t)$  be a periodic signal with time period  $T$ , let  $y(t) = x(t - t_0) + x(t + t_0)$  for some  $t_0$ . The

Fourier series coefficients of  $y(t)$  are denoted by  $b_k$ . If  $b_k = 0$  for all odd  $k$ , then  $t_0$  can be equal to

[GATE EE 2008-Bangalore]

- (A)  $T/8$       (B)  $T/4$   
 (C)  $T/2$       (D)  $2T$

Q.39 A signal  $x(t)$  is given by

[GATE EE 2007-Kanpur]

$$x(t) = \begin{cases} 1 & -T/4 < t \leq 3T/4 \\ -1 & 3T/4 < t \leq 7T/4 \end{cases}$$

Which among the following gives the fundamental Fourier term of  $x(t)$ ?

- (A)  $\frac{4}{\pi} \cos\left(\frac{\pi t}{T} - \frac{\pi}{4}\right)$       (B)  $\frac{4}{\pi} \cos\left(\frac{\pi t}{2T} + \frac{\pi}{4}\right)$   
 (C)  $\frac{4}{\pi} \sin\left(\frac{\pi t}{T} - \frac{\pi}{4}\right)$       (D)  $\frac{4}{\pi} \sin\left(\frac{\pi t}{2T} + \frac{\pi}{4}\right)$

Q.40 The Fourier series expansion of a real periodic signal with fundamental frequency  $f_0$  is given

by  $g_p(t) = \sum_{n=-\infty}^{\infty} C_n \exp(j2\pi n f_0 t)$ . It is given that

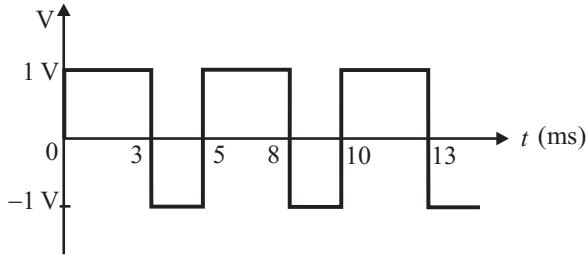
$C_3 = 3 + j5$  then  $C_{-3}$  is

[GATE EC 2003-Madras]

- (A)  $5 + j3$       (B)  $-3 - j5$   
 (C)  $-5 + j3$       (D)  $3 - j5$

Q.41 Consider the voltage waveform  $V$ , shown in figure. Find

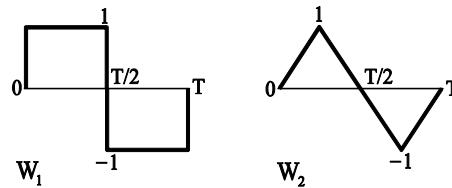
[GATE EE 2001-Kanpur]



- (i) The dc component of  $V$ ,  
 (ii) The amplitude of the fundamental component of  $V$ , and  
 (iii) The rms value of the ac part of  $V$

Q.42 One period ( $0, T$ ) each of two periodic waveforms  $W_1$  and  $W_2$  are shown in figure. The magnitudes of the  $n^{th}$  Fourier series coefficients of  $W_1$  and  $W_2$  for  $n \geq 1$ ,  $n$  odd are respectively proportional to

[GATE EC 2000-Kharagpur]



- (A)  $|n^{-3}|$  and  $|n^{-2}|$       (B)  $|n^{-2}|$  and  $|n^{-3}|$   
 (C)  $|n^{-1}|$  and  $|n^{-2}|$       (D)  $|n^{-4}|$  and  $|n^{-2}|$

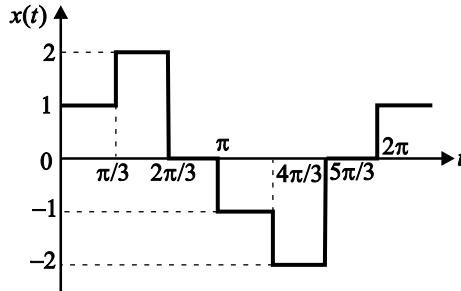
Q.43 The Fourier Series representation of an impulse train denoted by  $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$  is given by :

[GATE EC 1999-Bombay]

- (A)  $\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \exp\left(\frac{-j2\pi nt}{T_0}\right)$       (B)  $\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \exp\left(\frac{-j\pi nt}{T_0}\right)$   
 (C)  $\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \exp\left(\frac{j\pi nt}{T_0}\right)$       (D)  $\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \exp\left(\frac{j2\pi nt}{T_0}\right)$

Q.44 Compute the amplitude of the fundamental component of the waveform given in figure.

[GATE EE 1997-Madras]



Q.45 The complex exponential power form of Fourier series of  $x(t)$  is :

[ESE EC 2016]

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \cdot e^{j \frac{2\pi}{T_0} kt},$$

If  $x(t) = \sum_{b=-\infty}^{\infty} \delta(t - b)$ , then the value of  $a_k$  is

- (A)  $1 - (-1)^k$       (B)  $1 + (-1)^k$   
 (C) 1      (D) -1

Q.46 Consider a continuous time periodic signal  $x(t)$  with fundamental period  $T$  and Fourier series coefficient  $X[K]$ . What is the Fourier series coefficient of the signal

[ESE EC 2015]

$$y(t) = x(t - t_0) + x(t + t_0)?$$

- (A)  $2 \cos\left(\frac{2\pi}{T} K t_0\right) X[K]$       (B)  $2 \sin\left(\frac{2\pi}{T} K t_0\right) X[K]$   
 (C)  $e^{-t_0} X[K] + e^{t_0} X[-K]$       (D)  $e^{-t_0} X[-K] + e^{t_0} X[K]$

- Q.47** The Fourier series of a periodic function  $x_1(t)$  with a period  $T$  is given by

$$\sum_{k=-\infty}^{\infty} X_s(k) e^{jk\omega_0 t}, \text{ where } \omega_0 = \frac{2\pi}{T}$$

and the Fourier coefficient  $X_s(k)$  is defined as,

$$X_s(k) = \frac{1}{T} \int x_T(t) e^{-jk\omega_0 t} dt$$

If  $x_T(t)$  is real and odd, the Fourier coefficients  $X_s(k)$  are

[ESE EC 2015]

- (A) Real and odd      (B) Complex  
 (C) Real                (D) Imaginary
- Q.48** Match list-I with list-II and select the correct answer using the codes given below the lists :

**List-I** [ESE EC 2000]

A.  $f(t) = -f(-t)$

B.  $\sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$

C.  $\int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$

D.  $\int_0^t f_1(\tau) f_2(t-\tau) d\tau$

**List-II**

1. Exponential form of Fourier series
2. Fourier transform
3. Convolution integral
4. Z-transform
5. Odd function wave symmetry

**Codes :** A B C D

(A) 5 1 2 3

(B) 2 1 5 3

(C) 5 4 2 1

(D) 4 5 1 2

- Q.49** If a real valued signal  $x(t)$  can be expressed as

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{j2n\pi}{T_0} t}; -\infty < t < \infty \quad \text{where } T_0 \text{ is the}$$

fundamental period of the signal  $x(t)$ , then

- (A)  $c_n = -c_n^*$       (B)  $c_n = c_{-n}$   
 (C)  $c_{-n} = c_n^*$       (D)  $c_n = -c_{-n}$

- Q.50** Let  $x(t)$  be a periodic signal with period  $T_0$  and  $c_n$  be its Fourier series coefficient. Match List-I (Continuous time signal) with List-II (Fourier series coefficient) and choose the correct answer using the codes given below.

**List I (Continuous time signal)**

- P.  $x(t-t_0)$       Q.  $x(-t)$   
 R.  $x^*(t)$       S.  $e^{jn_0\omega_0 t} x(t)$

**List-II (Fourier series coefficient)**

1.  $c_{-n}$       2.  $c_{-n}^*$   
 3.  $c_{n-n_0}$       4.  $c_n e^{-jn\omega_0 t}$

**Codes** P Q R S

(A) 1 4 3 2

(B) 4 2 1 3

(C) 4 1 2 3

(D) 4 2 3 1

- Q.51** Suppose the periodic signal  $x(t)$  has fundamental period  $T$  and Fourier coefficients  $c_n$ . Let  $d_n$  be the Fourier coefficients of  $y(t)$  where  $y(t) = dx(t)/dt$ . The Fourier coefficient  $c_n$  will be

(A)  $\frac{Td_n}{j2\pi n}, n \neq 0$       (B)  $\frac{-Td_n}{j2\pi n}, n \neq 0$

(C)  $\frac{Td_n}{jn}, n \neq 0$       (D)  $\frac{-Td_n}{jn}, n \neq 0$

- Q.52** The exponential Fourier Series of a real periodic signal  $f(t)$  is given by :

$$f(t) = \dots j \frac{2A}{3\pi} e^{-j3\omega_0 t} + j \frac{2A}{\pi} e^{-j\omega_0 t} - \frac{j2A}{\pi} e^{j\omega_0 t} - j \frac{2A}{3\pi} e^{j3\omega_0 t} \dots$$

The value of  $a_n$  and  $b_n$  are

- (A) 0,  $2A/n\pi$       (B) 0,  $-2A/n\pi$   
 (C) 0,  $4A/n\pi$       (D) 0,  $-4A/n\pi$

- Q.53** The FS coefficients of a signal  $x(t)$  are given as

$$c_1 = \frac{3}{2\sqrt{2}}(1+j) \quad c_{-1} = \frac{3}{2\sqrt{2}}(1-j)$$

$$c_n = 0, \text{ otherwise}$$

The power of signal  $x(t)$  is

- (A) 9 units      (B)  $\sqrt{3}$  units  
 (C)  $\frac{9}{2}$  units      (D)  $\frac{3}{2}$  units

**Statement for Linked Questions 54 & 55**

Consider three continuous-time periodic signals whose Fourier series representations are as follows :

$$x_1(t) = \sum_{n=0}^{100} \left(\frac{1}{3}\right)^n e^{jn\frac{2\pi}{50}t}$$

$$x_2(t) = \sum_{n=-100}^{100} \cos(n\pi) e^{jn\frac{2\pi}{50}t}$$

$$x_3(t) = \sum_{n=-100}^{100} j \sin\left(\frac{n\pi}{2}\right) e^{jn\frac{2\pi}{50}t}$$

**Q.54** The even signals are

- (A)  $x_2(t)$  only      (B)  $x_2(t)$  and  $x_3(t)$   
 (C)  $x_1(t)$  and  $x_3(t)$       (D)  $x_1(t)$  only

**Q.55** The real valued signals are

- (A)  $x_1(t)$  and  $x_2(t)$       (B)  $x_2(t)$  and  $x_3(t)$   
 (C)  $x_3(t)$  and  $x_1(t)$       (D)  $x_1(t)$  and  $x_3(t)$

**Statement for Linked Questions 56 to 63**

Consider a continuous time periodic signal  $x(t)$  with fundamental period  $T$  and Fourier series coefficients  $c_n$ .

**Q.56** The Fourier series coefficient of the signal  $y(t) = x(t - t_0) + x(t + t_0)$  is

- (A)  $2\cos\left(\frac{2\pi}{T}nt_0\right)c_n$       (B)  $2\sin\left(\frac{2\pi}{T}nt_0\right)c_n$   
 (C)  $e^{-t_0}c_n + e^{t_0}c_{-n}$       (D)  $e^{-t_0}c_{-n} + e^{t_0}c_n$

**Q.57** The Fourier series coefficient of the even part of  $x(t)$  i.e.  $y(t) = x_e(t)$  is

- (A)  $\frac{c_n + c_{-n}}{2}$       (B)  $\frac{c_n - c_{-n}}{2}$   
 (C)  $\frac{c_n + c_{-n}^*}{2}$       (D)  $\frac{c_n - c_{-n}^*}{2}$

**Q.58** The Fourier series coefficient of the signal  $y(t) = \text{Re}\{x(t)\}$  is

- (A)  $\frac{c_n + c_{-n}}{2}$       (B)  $\frac{c_n - c_{-n}}{2}$   
 (C)  $\frac{c_n + c_{-n}^*}{2}$       (D)  $\frac{c_n - c_{-n}^*}{2}$

**Q.59** The Fourier series coefficient of the signal  $y(t) = \frac{d^2x(t)}{dt^2}$  is

- (A)  $\left(\frac{2\pi n}{T}\right)^2 c_n$       (B)  $-\left(\frac{2\pi n}{T}\right)^2 c_n$   
 (C)  $j\left(\frac{2\pi n}{T}\right)^2 c_n$       (D)  $-j\left(\frac{2\pi n}{T}\right)^2 c_n$

**Q.60** The Fourier series coefficient of the signal  $y(t) = x(4-t)$  is

- (A)  $c_{-n}e^{-j\omega_0 4n}$       (B)  $c_n e^{j\omega_0 4n}$   
 (C)  $c_{-n}e^{j\omega_0 4n}$       (D)  $c_n e^{-j\omega_0 4n}$

**Q.61** The Fourier series coefficient of the signal  $y(t) = x(t) \otimes x(t)$  is

- (A)  $c_n^2$       (B)  $c_{-n}$   
 (C)  $T \cdot c_n^2$       (D)  $2c_n$

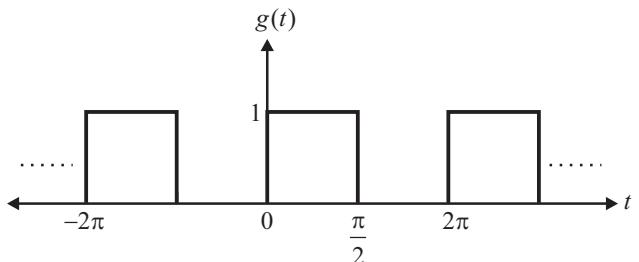
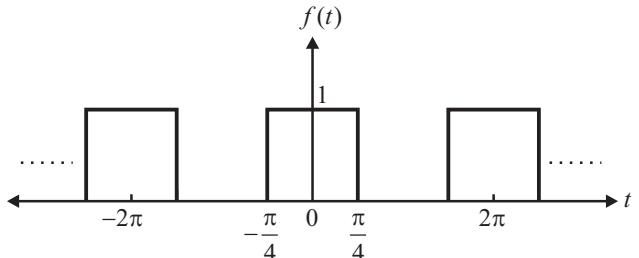
**Q.62** The Fourier series coefficient of the signal  $y(t) = x(t) \cos 4\pi t$  is

- (A)  $c_{n-4} + c_{n+4}$       (B)  $c_{n-4} - c_{n+4}$   
 (C)  $\frac{1}{2}[c_{n-4} - c_{n+4}]$       (D)  $\frac{1}{2}[c_{n-4} + c_{n+4}]$

**Q.63** The Fourier series coefficient of the signal  $y(t) = x(4t-1)$  is

- (A)  $\frac{8\pi}{T}c_n$       (B)  $\frac{4\pi}{T}c_n$   
 (C)  $e^{-jn\left(\frac{8\pi}{T}\right)}c_n$       (D)  $e^{-jn\left(\frac{2\pi}{T}\right)}c_n$

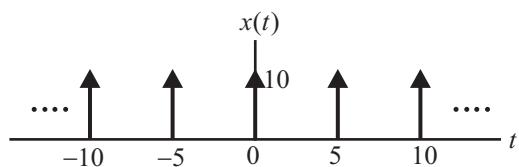
**Q.64** Let  $c_h$  and  $d_n$  denote the CTFS coefficient of waveforms  $f(t)$  and  $g(t)$  shown in figure (A) and figure (B) respectively.



If  $c_2 = \frac{1}{2\pi}$  then the value of coefficient  $d_2$  is

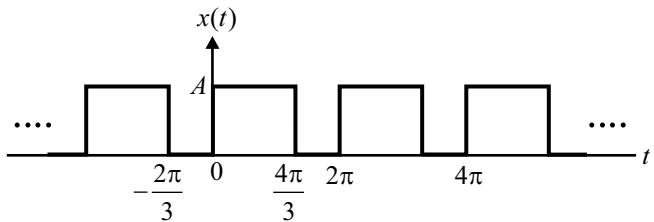
- (A)  $\frac{1}{2\pi}$       (B)  $-\frac{j}{2\pi}$   
 (C)  $-\frac{1}{2\pi}$       (D) Zero

- Q.65** The exponential Fourier series coefficient for the periodic signal shown below is



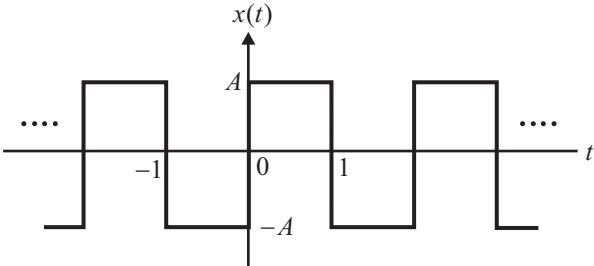
- (A) 1   (B)  $\cos\left(\frac{\pi}{2}n\right)$   
(C)  $\sin\left(\frac{\pi}{2}n\right)$                               (D) 2

- Q.66** The exponential Fourier series coefficient for the periodic signal shown below is



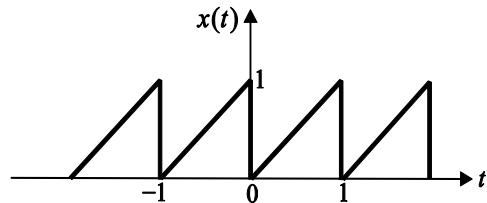
- (A)  $\frac{A}{2\pi n}\left(e^{-j\left(\frac{4\pi n}{3}\right)}-1\right)$       (B)  $j\frac{A}{2\pi n}\left(e^{-j\left(\frac{4\pi n}{3}\right)}-1\right)$   
(C)  $-j\frac{A}{2\pi n}\left(e^{-j\left(\frac{4\pi n}{3}\right)}-1\right)$    (D)  $\frac{-A}{2\pi n}\left(e^{-j\left(\frac{4\pi n}{3}\right)}-1\right)$

- Q.67** The exponential Fourier series coefficient for the periodic signal shown below is



- (A)  $\frac{A}{n\pi}[1-(-1)^n]$       (B)  $\frac{A}{n\pi}[1+(-1)^n]$   
(C)  $\frac{A}{jn\pi}[1-(-1)^n]$       (D)  $\frac{A}{jn\pi}[1+(-1)^n]$

- Q.68** The exponential Fourier series coefficient for  $x(t)$  is

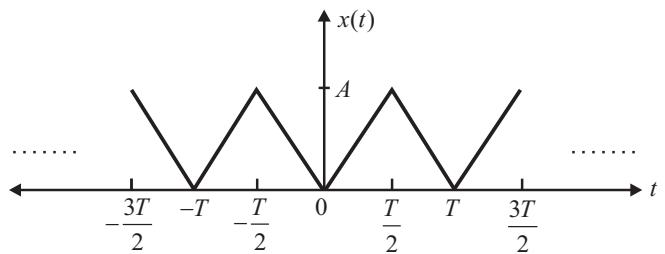


- (A)  $\frac{1}{2\pi n}$     (B)  $\frac{1}{2\pi nj}$   
(C)  $\frac{j}{2\pi n}$     (D) None of these

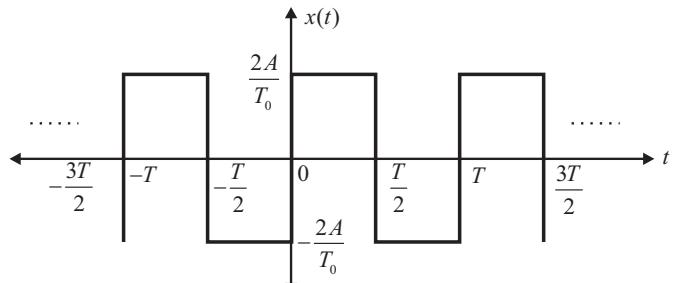
- Q.69** A CT signal is defined as  $x(t) = t^2, -\pi < t < \pi$  and  $x(t + 2\pi) = x(t)$ . The trigonometric Fourier series representation of  $x(t)$  is given by

- (A)  $2\left(\cos t + \frac{1}{2}\cos 2t + \frac{1}{4}\cos 3t + \dots\right)$   
(B)  $\frac{\pi^2}{3} - 4\left(\cos t - \frac{1}{4}\cos 2t + \frac{1}{9}\cos 3t - \dots\right)$   
(C)  $\frac{2}{\pi}\left(\sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t - \dots\right)$   
(D)  $\frac{\pi^2}{2} + 4\left(\cos t + \frac{1}{2}\cos 2t + \frac{1}{3}\sin 3t + \dots\right)$

- Q.70** The exponential CTFS coefficient  $c_n$  of signal  $x(t)$  shown in figure is given as  $c_n = -\frac{2A}{\pi^2 n^2}$

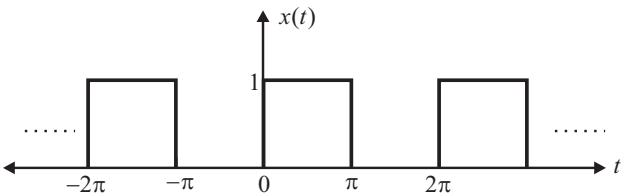


The CTFS coefficient of signal  $y(t)$  shown in figure below will be

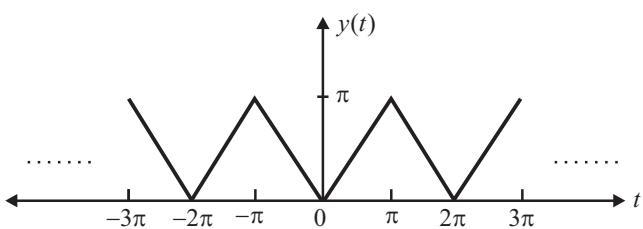


- (A)  $\frac{4A}{j\pi nT}$     (B)  $\frac{-2A}{\pi^2 n^2 T}$   
(C)  $\frac{4A}{j\pi n^2 T}$     (D)  $\frac{2A}{j\pi nT}$

- Q.71** The exponential FS coefficient of function  $x(t)$  shown in figure (A) is denoted as  $c_n$ .



Now, consider the signal  $y(t)$  shown in figure (B), whose CTFS coefficients are denoted as  $d_n$ .



If  $c_3 = -\frac{j}{3\pi}$ , then the value of  $d_3$  is

- (A)  $-\frac{1}{9\pi}$       (B)  $\frac{1}{\pi}$   
 (C)  $\frac{4}{9\pi}$       (D)  $-\frac{2}{9\pi}$

**Common Data for Questions 72 to 75**

Determine Complex exponential Fourier series coefficient  $C_n$  for the following signals :

**Q.72**  $\sin \omega_0 t$

- (A)  $C_1 = \frac{1}{2j}$ ,  $C_{-1} = \frac{1}{2j}$       (B)  $C_1 = \frac{-1}{2j}$ ,  $C_{-1} = \frac{1}{2j}$   
 (C)  $C_1 = \frac{1}{2j}$ ,  $C_{-1} = \frac{-1}{2j}$       (D)  $C_1 = \frac{-1}{2j}$ ,  $C_{-1} = \frac{-1}{2j}$

**Q.73**  $\cos(2t + \pi/4)$

- (A)  $C_1 = \frac{\sqrt{2}}{4}(1+j)$ ,  $C_{-1} = \frac{\sqrt{2}}{4}(1-j)$   
 (B)  $C_1 = \frac{\sqrt{2}}{4}(1-j)$ ,  $C_{-1} = \frac{\sqrt{2}}{4}(1+j)$   
 (C)  $C_1 = \frac{\sqrt{2}}{4}(1+j)$ ,  $C_{-1} = \frac{\sqrt{2}}{4}(1+j)$   
 (D)  $C_1 = \frac{\sqrt{2}}{4}(1-j)$ ,  $C_{-1} = \frac{\sqrt{2}}{4}(1-j)$

**Q.74**  $\sin 6t + \cos 10t$

- (A)  $C_{-5} = C_5 = \frac{1}{2}$ ,  $C_3 = \frac{1}{2j}$ ,  $C_{-3} = \frac{-1}{2j}$   
 (B)  $C_{-1} = C_1 = \frac{1}{2}$ ,  $C_2 = \frac{1}{2j}$ ,  $C_{-2} = \frac{-1}{2j}$   
 (C)  $C_{-5} = C_5 = \frac{1}{2}$ ,  $C_3 = C_{-3} = \frac{1}{2j}$   
 (D)  $C_{-1} = C_1 = \frac{1}{2}$ ,  $C_2 = C_{-2} = \frac{1}{2j}$

**Q.75**  $\cos^4 t$

- (A)  $C_0 = \frac{3}{8}$ ,  $C_2 = C_{-2} = \frac{1}{4}$ ,  $C_4 = C_{-4} = \frac{1}{16}$   
 (B)  $C_0 = \frac{3}{8}$ ,  $C_2 = C_{-2} = \frac{1}{4}$ ,  $C_4 = C_{-4} = \frac{1}{8}$

(C)  $C_0 = \frac{3}{8}$ ,  $C_1 = C_{-1} = \frac{1}{4}$ ,  $C_2 = C_{-2} = \frac{1}{16}$

(D)  $C_0 = \frac{3}{8}$ ,  $C_1 = C_{-1} = \frac{1}{4}$ ,  $C_2 = C_{-2} = \frac{1}{8}$

**Q.76** Exponential FS of a periodic function

$$x(t) = 7 + \sin(4t) + \cos(8t) + \cos(16t)$$

is given as  $x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$ .

The value of coefficient  $c_{-1}$  is

- (A)  $\frac{-1}{2}j$       (B)  $\frac{1}{2}j$   
 (C)  $\frac{1}{2}$       (D)  $\frac{-1}{2}$

**Q.77** For a signal  $x(t) = 5\sin 5t + 7\cos 6t + 3$ , the non-zero coefficients of exponential Fourier series are

- (A)  $c_{-6}, c_{-5}, c_0, c_5, c_6$       (B)  $c_{-3}, c_{-1}, c_0, c_1, c_3$   
 (C)  $c_{-2}, c_{-1}, c_0, c_1, c_2$       (D)  $c_{-4}, c_{-2}, c_0, c_2, c_4$

**Q.78** Consider a continuous time signal  $x(t) = 4\cos(100\pi t)\sin(1000\pi t)$  with fundamental period  $T = \frac{1}{50}$ . The non zero FS coefficient for this function are

- (A)  $c_{-4}, c_4, c_{-7}, c_7$       (B)  $c_{-1}, c_1, c_{-10}, c_{10}$   
 (C)  $c_{-3}, c_3, c_{-4}, c_4$       (D)  $c_{-9}, c_9, c_{-11}, c_{11}$

**Q.79** A real valued continuous time  $x(t)$  has a fundamental period  $T = 8$ . The nonzero Fourier series coefficients for  $x(t)$  are  $c_1 = c_{-1} = 4$ ,  $c_3 = c_{-3} = 4j$ . The signal  $x(t)$  would be

- (A)  $4\cos\left(\frac{\pi}{4}t\right) + 4j\sin\left(\frac{3\pi}{4}t\right)$   
 (B)  $4\cos\left(\frac{\pi}{4}t\right) - 4j\cos\left(\frac{3\pi}{4}t\right)$   
 (C)  $8\cos\left(\frac{\pi}{4}t\right) + 8\cos\left(\frac{3\pi}{4}t + \frac{\pi}{2}\right)$

(D) None of the above

**Q.80** A continuous time signal  $x(t)$  with period 4 has the exponential Fourier series coefficient

$$c_n = \begin{cases} jn, & |n| < 3 \\ 0, & \text{Otherwise} \end{cases}$$

Signal  $x(t)$  is

(A)  $2\cos\left(\frac{\pi}{2}t\right) + 4\sin(\pi t)$

(B)  $-2\sin\left(\frac{\pi}{2}t\right) - 4\sin(\pi t)$

(C)  $2j\sin\left(\frac{\pi}{2}t\right) + 4j\sin(\pi t)$

(D)  $-2\sin\left(\frac{\pi}{4}t\right) - 4\sin\left(\frac{\pi}{2}t\right)$

**Statement for Linked Questions 81 & 82**

The exponential Fourier series coefficient of a signal  $x(t)$  is given by

$$c_n = \frac{1 - \cos(\pi n)}{(\pi n)^2}$$

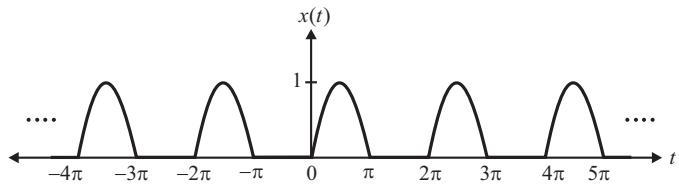
**Q.81** The signal  $x(t)$  is

- (A) Even
- (B) Odd
- (C) Neither even nor odd
- (D) Cannot be determined

**Q.82** The average value of signal  $x(t)$  is

- (A) 0
- (B) 1
- (C)  $\frac{1}{2}$
- (D) None of these

**Q.83** Consider the following statements regarding the Fourier series coefficient  $c_n$  of a signal  $x(t)$  shown in figure

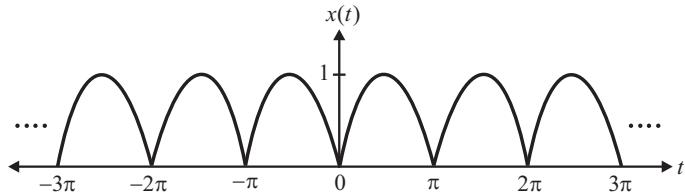


- 1.  $c_{-n} = c_n^*$
- 2.  $c_n$  is purely real
- 3. The amplitude spectrum is even symmetric with respect to vertical axis.

Which of the above statements is/are true?

- (A) 1 and 2
- (B) 2 and 3
- (C) 1, 2 and 3
- (D) 1 and 3

**Q.84** A continuous time signal is shown as



Consider the following statement regarding the Fourier series coefficient  $c_n$  of  $x(t)$

1.  $c_{-n} = c_n^*$
2.  $c_n$  is purely real
3.  $c_n$  is purely imaginary
4. The magnitude spectrum of  $x(t)$  exhibits an even symmetry with respect to vertical axis.

Which of the above statement is/are true?

- (A) 1, 3 and 4
- (B) 1 and 4 only
- (C) 1, 2 and 4
- (D) 1 and 3 only

**Q.85** Consider a continuous time LTI system whose frequency response is

$$H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t} dt = \frac{\sin 4\omega}{\omega}$$

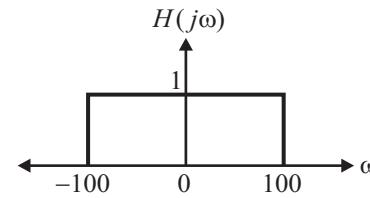
The input of this system is a periodic signal is

$$x(t) = \begin{cases} 2, & 0 \leq t \leq 4 \\ -2, & 4 \leq t \leq 8 \end{cases} \text{ with period } T = 8$$

The output  $y(t)$  will be

- (A)  $1 + \sin^2\left(\frac{\pi t}{4}\right)$
- (B)  $1 + \cos^2\left(\frac{\pi t}{4}\right)$
- (C)  $1 + \sin\left(\frac{\pi t}{4}\right) + \cos\left(\frac{\pi t}{4}\right)$
- (D) 0

**Q.86** A continuous time ideal low-pass filter has the frequency response  $H(j\omega)$  shown in figure below



When an input  $x(t)$  with fundamental period  $T = \frac{\pi}{4}$  and Fourier series coefficient  $c_n$  is applied to the system, the output  $y(t)$  is same as  $x(t)$ . For what values of  $n$  it is guaranteed that  $c_n = 0$ ?

- (A)  $|n| > 8$
- (B)  $|n| > 4$
- (C)  $|n| > 6$
- (D)  $|n| > 12$

**Q.87** Consider a continuous time ideal low pass filter having the frequency response

$$H(j\omega) = \begin{cases} 1, & |\omega| \leq 80 \\ 0, & |\omega| > 80 \end{cases}$$

When the input to this filter is a signal  $x(t)$  with fundamental frequency  $\omega_0 = 12$  and Fourier series coefficients  $c_n$ , it is found that

$$x(t) \xrightarrow{s} y(t) = x(t)$$

The largest value of  $|n|$  for which  $c_n$  is nonzero, is

- |       |        |
|-------|--------|
| (A) 6 | (B) 80 |
| (C) 7 | (D) 12 |

### Practice (objective & Num Ans) Questions :

- Q.1** The trigonometric Fourier series of an even function does not have the

[GATE EC 2011-Madras]

[GATE EC 1996-Bangalore]

- |                |                        |
|----------------|------------------------|
| (A) dc term    | (B) cosine terms       |
| (C) sine terms | (D) odd harmonic terms |

- Q.2** The Fourier series of a real periodic function has only : [GATE EC 2009-Roorkee]

P. cosine terms if it is even

Q. sine terms if it is even

R. cosine terms if it is odd

S. sine terms if it is odd

Which of the above statements are correct?

- |             |             |
|-------------|-------------|
| (A) P and S | (B) P and R |
| (C) Q and S | (D) Q and R |

- Q.3** The RMS value of the voltage  $V(t) = 3 + 4 \cos(3t)$  is [GATE EE 2005-Bombay]

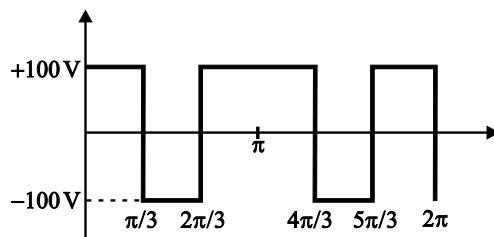
- |                   |                       |
|-------------------|-----------------------|
| (A) $\sqrt{17}$ V | (B) 5 V               |
| (C) 7 V           | (D) $(3+2\sqrt{2})$ V |

- Q.4** Which of the following cannot be the Fourier series expansion of a periodic signal?

[GATE EC 2002-Bangalore]

- |  |
|--|
| (A) $x(t) = 2 \cos t + 3 \cos 3t$            |
| (B) $x(t) = 2 \cos \pi t + 7 \cos t$         |
| (C) $x(t) = \cos t + 0.5$                    |
| (D) $x(t) = 2 \cos 1.5\pi t + \sin 3.5\pi t$ |

- Q.5** What is the rms value of the voltage waveform shown in figure? [GATE EE 2002-Bangalore]



- (A)  $200/\pi$  V      (B)  $100/\pi$  V

- (C) 200 V      (D) 100 V

- Q.6** The RMS value of a half-wave rectified symmetrical square wave current of 2 A is

[GATE EE 1999-Bombay]

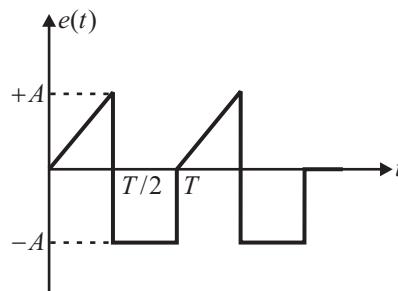
- |                    |                  |
|--------------------|------------------|
| (A) $\sqrt{2}$ A   | (B) 1 A          |
| (C) $1/\sqrt{2}$ A | (D) $\sqrt{3}$ A |

- Q.7** The trigonometric Fourier series of a periodic time function can have only

[GATE EC 1998-Delhi]

- |                           |
|---------------------------|
| (A) Cosine terms          |
| (B) Sine terms            |
| (C) Cosine and sine terms |
| (D) D.C. and cosine terms |

- Q.8** The rms value of the periodic waveform  $e(t)$ , shown in figure is [GATE EE 1995-Kanpur]



- |                           |                           |
|---------------------------|---------------------------|
| (A) $\sqrt{\frac{3}{2}}A$ | (B) $\sqrt{\frac{2}{3}}A$ |
| (C) $\sqrt{\frac{1}{3}}A$ | (D) $\sqrt{2}A$           |

- Q.9** The Fourier series of an odd periodic function contains only [GATE EC 1994-Kharagpur]

- |                   |                    |
|-------------------|--------------------|
| (A) odd harmonics | (B) even harmonics |
| (C) cosine terms  | (D) sine terms     |

- Q.10** A periodic function satisfies Dirichlet's conditions. This implies that the function

[ESE EC 2014]

- |                                  |
|----------------------------------|
| (A) is non-linear                |
| (B) is not absolutely integrable |

- (C) guarantees that Fourier series representation of the function exists  
 (D) has infinite number of maxima and minima within a period

**Q.11** Consider the following statements :

Fourier series of any periodic function  $x(t)$  can be obtained if

[ESE EC 2010]

1.  $\int_0^T |x(t)| dt < \infty$
2. Finite number of discontinuities exist within finite time interval  $t$ .

Which of the above statements is/are correct?

- (A) 1 only                    (B) 2 only  
 (C) Both 1 and 2            (D) Neither 1 nor 2

**Q.12** For half-wave (odd) symmetry, with  $T_0$  = period of  $x(t)$ , which one of the following is correct?

[ESE EC 2004]

- (A)  $x(t \pm T_0 / 2) = -x(t)$     (B)  $x(t \pm T_0 / 2) = x(t)$   
 (C)  $x(t \pm T_0) = -x(t)$       (D)  $x(t \pm T_0) = x(t)$

**Q.13 Assertion (A) :** A periodic function satisfying Dirichlet conditions can be expanded into Fourier series.

[ESE EC 2003]

**Reason (R) :** A periodic function can be reconstructed from

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t \text{ for very large } n,$$

excluding infinity.

- (A) Both A and R are true and R is the correct explanation of A  
 (B) Both A and R are true but R is NOT the correct explanation of A  
 (C) A is true but R is false  
 (D) A is false but R is true

**Q.14** Consider the following statements related to Fourier series of a periodic waveform :

[ESE EC 2002]

1. It expresses the given periodic waveform as a combination of d.c. component, sine and cosine waveforms of different harmonic frequencies.
2. The amplitude spectrum is discrete.
3. The evaluation of Fourier coefficients gets simplified if waveforms symmetries are used.
4. The amplitude spectrum is continuous.

Which of the above statements are correct?

- (A) 1, 2 and 4                (B) 2, 3 and 4  
 (C) 1, 3 and 4               (D) 1, 2 and 3

**Q.15** The Fourier series representation of a periodic current is  $[2 + 6\sqrt{2} \cos \omega t + \sqrt{48} \sin 2\omega t] A$ .

The effective value of the current is

- (A)  $(2 + 6 + \sqrt{24}) A$             (B) 8 A [ESE EC 2000]  
 (C) 6 A                                (D) 2 A

**Q.16** A network consisting of a finite number of linear resistor (R), inductor (L), and capacitor (C) elements, connected all in series or all in parallel, is excited with a source of the form

$$\sum_{k=1}^3 a_k \cos(k\omega_0 t), \text{ where } a_k \neq 0, \omega_0 \neq 0$$

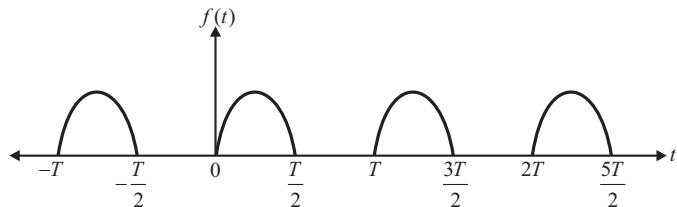
The source has nonzero impedance. Which one of the following is a possible form of the output measured across a resistor in the network?

[GATE EC 2016-Bangalore]

- (A)  $\sum_{k=1}^3 b_k \cos(k\omega_0 t + \phi_k)$ , where  $b_k \neq a_k, \forall k$   
 (B)  $\sum_{k=1}^4 b_k \cos(k\omega_0 t + \phi_k)$ , where  $b_k \neq 0, \forall k$   
 (C)  $\sum_{k=1}^3 a_k \cos(k\omega_0 t + \phi_k)$   
 (D)  $\sum_{k=1}^2 a_k \cos(k\omega_0 t + \phi_k)$

**Q.17 Assertion (A) :** For the half-wave rectified sine wave  $f(t)$  shown in the figure, the only sine term present in its trigonometric form of Fourier series is the fundamental.

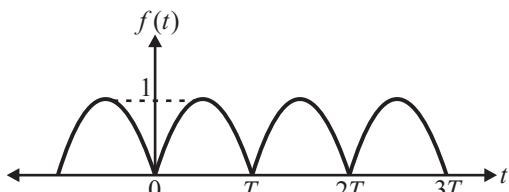
[ESE EC 1998]



**Reason (R) :** The odd part of  $f(t)$  is a pure sinusoid of the fundamental frequency.

- (A) Both A and R are true and R is the correct explanation of A.  
 (B) Both A and R are true but R is not a correct explanation of A.  
 (C) A is true but R is false.  
 (D) A is false but R is true.

- Q.18 Assertion (A) :** The coefficient  $a_n$  and  $b_n$  of the Fourier series for the periodic function shown in the figure is given by [ESE EC 1994]



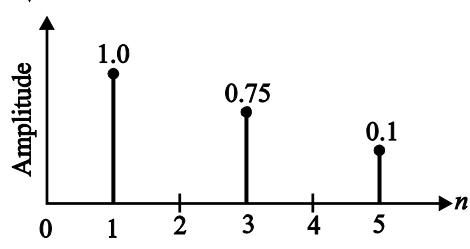
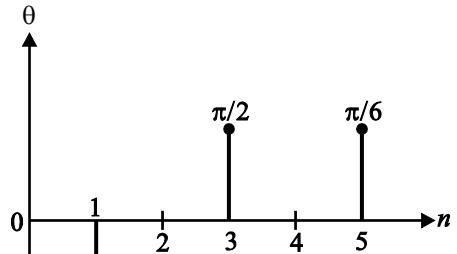
$$f(t) = \sin\left(\frac{\pi}{T}t\right) \text{ for } 0 < t < T$$

$$a_n = \frac{4}{\pi(1-4n^2)} \quad n=1, 2, \dots \quad \text{and} \quad b_n = 0$$

**Reason (R) :** The periodic function is symmetrical about  $t = T$  and  $t = \frac{T}{2}$

- (A) Both A and R are true and R is the correct explanation of A.
- (B) Both A and R are true but R is not a correct explanation of A.
- (C) A is true but R is false.
- (D) A is false but R is true.

- Q.19** The amplitude and phase spectra for the few harmonics of a periodic signal of time period 1 sec are shown in the figure below. [ESE EC 1991]

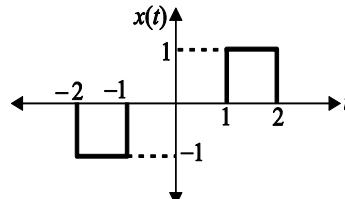


$$(A) \cos\left(2\pi t - \frac{\pi}{4}\right) + 0.75\cos\left(6\pi t + \frac{\pi}{4}\right) + 0.1\cos\left(10\pi t + \frac{\pi}{6}\right) \dots$$

- (B)  $\cos\left(\frac{t}{2\pi} - \frac{\pi}{4}\right) + 0.75\cos\left(\frac{t}{6\pi} - \frac{\pi}{4}\right) + 0.1\cos\left(\frac{t}{10\pi} + \frac{\pi}{6}\right) \dots$
- (C)  $2\cos\left(2\pi t - \frac{\pi}{4}\right) + 1.5\cos\left(6\pi t + \frac{\pi}{2}\right) + 0.2\cos\left(10\pi t + \frac{\pi}{6}\right) \dots$
- (D)  $-\cos\left(2\pi t - \frac{\pi}{4}\right) + 0.75\cos\left(6\pi t + \frac{\pi}{2}\right) + 0.1\cos\left(10\pi t + \frac{\pi}{6}\right) \dots$

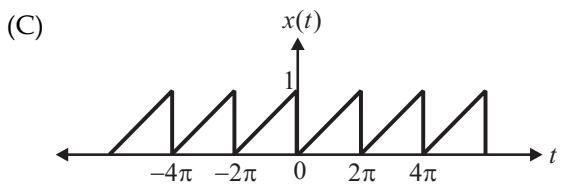
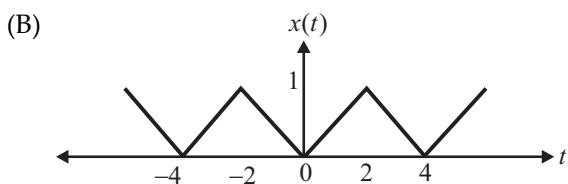
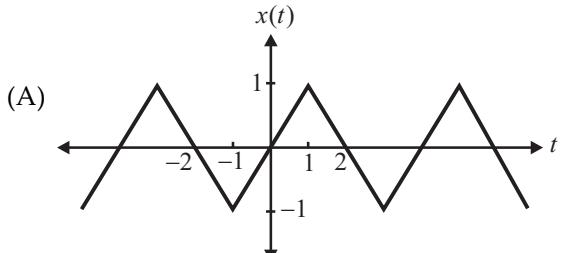
- Q.20** The Fourier series of a periodic function exists, if in one period, the function has
- (A) infinite number of infinite discontinuities
  - (B) infinite number of finite discontinuities
  - (C) finite number of infinite discontinuities
  - (D) finite number of finite discontinuities

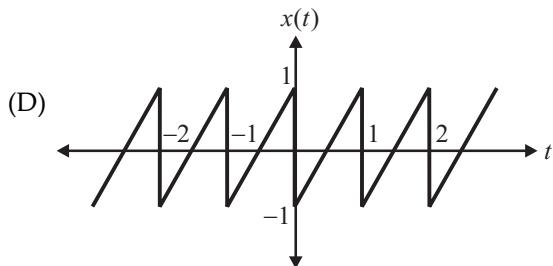
- Q.21** DC value for the given signal is



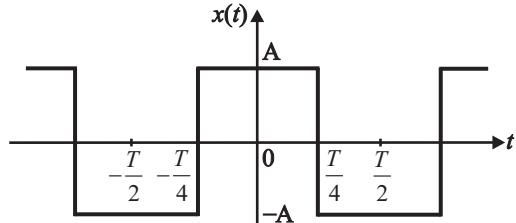
- (A) 0
- (B) 1
- (C) 1/2
- (D) 2

- Q.22** Which of the following signal does not contain sine terms in its trigonometric Fourier series representation?



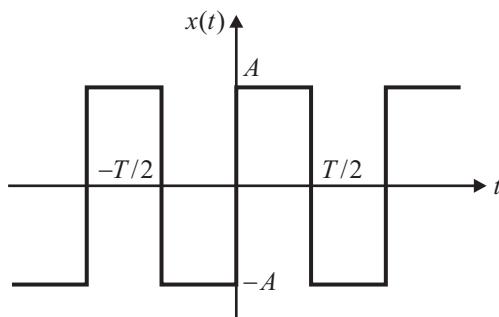


**Q.23** Fourier series for a given below waveform is :



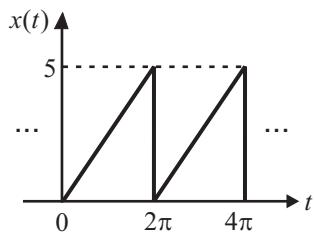
- (A)  $\frac{4A}{\pi} \left[ \cos \omega_0 t - \frac{1}{3} \cos 3\omega_0 t + \frac{1}{5} \cos 5\omega_0 t \dots \right]$
- (B)  $\frac{4A}{\pi} \left[ \cos 2\omega_0 t - \frac{1}{4} \cos 4\omega_0 t + \frac{1}{6} \cos 6\omega_0 t \dots \right]$
- (C)  $\frac{4A}{\pi} \left[ \sin \omega_0 t - \frac{1}{3} \sin 3\omega_0 t + \frac{1}{5} \sin 5\omega_0 t \dots \right]$
- (D)  $\frac{A}{4} + \frac{4A}{\pi} \left[ \cos \omega_0 t - \frac{1}{3} \cos 3\omega_0 t + \frac{1}{5} \cos 5\omega_0 t \dots \right]$

**Q.24** Fourier series for a given below waveform is :



- (A)  $4 \frac{A}{\pi} \left[ \cos \omega_0 t + \frac{1}{3} \cos 3\omega_0 t + \frac{1}{5} \cos 5\omega_0 t \dots \right]$
- (B)  $4 \frac{A}{\pi} \left[ \sin \omega_0 t - \frac{1}{3} \sin 3\omega_0 t + \frac{1}{5} \sin 5\omega_0 t \dots \right]$
- (C)  $4 \frac{A}{\pi} \left[ \sin \omega_0 t + \frac{1}{3} \sin 3\omega_0 t + \frac{1}{5} \sin 5\omega_0 t \dots \right]$
- (D)  $4 \frac{A}{\pi} \left[ \cos \omega_0 t - \frac{1}{3} \cos 3\omega_0 t + \frac{1}{5} \cos 5\omega_0 t \dots \right]$

**Q.25** Fourier series for a given below waveform is :



(A)  $-\frac{5}{\pi} \sin t - \frac{5}{2\pi} \sin 2t - \frac{5}{3\pi} \sin 3t \dots$

(B)  $\frac{5}{2} - \frac{5}{\pi} \sin t - \frac{5}{2\pi} \sin 2t - \frac{5}{3\pi} \sin 3t \dots$

(C)  $\frac{5}{2} + \frac{5}{\pi} \cos t + \frac{5}{2\pi} \cos 2t + \frac{5}{3\pi} \cos 3t \dots$

(D)  $\frac{5}{\pi} \cos t + \frac{5}{2\pi} \cos 2t + \frac{5}{3\pi} \cos 3t \dots$

**Q.26** Consider Fourier representation of continuous and discrete-time systems. The complex exponentials (i.e., signals), which arise in such representation, have [ESE EC 2014]

- (A) same properties always
- (B) different properties always
- (C) non-specific properties
- (D) mostly same properties

**Q.27** Let  $c_n$  is the Fourier series coefficient of a periodic signal  $x(t)$ , then in the Fourier transform  $X(\omega)$  of  $x(t)$  the magnitude of  $n^{\text{th}}$  impulse is

- (A)  $\frac{|c_n|}{2\pi}$
- (B)  $2\pi|c_n|$
- (C)  $|c_n|$
- (D)  $\frac{|c_n|}{2}$

**Q.28**  $x(t)$  and  $y(t)$  are two periodic signals with period  $T_0$  and  $z(t) = x(t) \otimes y(t)$ . If  $a_n$ ,  $b_n$  and  $c_n$  are the  $n^{\text{th}}$  complex exponential Fourier series coefficients of  $x(t)$ ,  $y(t)$  and  $z(t)$  respectively, then

- (A)  $c_n = a_n b_n$
- (B)  $c_n = T_0 a_n b_n$
- (C)  $c_n = \frac{1}{T} a_n b_n$
- (D) None of these

**Q.29** Let  $x(t)$  is a periodic signal and  $x(t) \xrightarrow{\text{CTFS}} c_n$ . If  $|c_n|$  has even symmetry and  $\angle c_n$  has odd symmetry with respect to  $n$ , then  $x(t)$  is

(A) odd symmetric

(B) even-symmetric

(C) complex valued

(D) purely real or purely imaginary

**Q.30** For a real cosine and sine function, the exponential Fourier series coefficient  $c_n$  have the property

- (A)  $c_n = c_{-n}^*$   
 (B)  $c_n = -c_{-n}^*$   
 (C)  $c_n = \frac{1}{2}c_{-n}^*$   
 (D)  $c_n = \frac{-1}{2}c_{-n}^*$

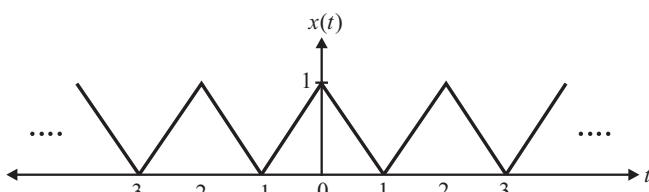
**Q.31** Let  $x_1(t)$  and  $x_2(t)$  be continuous time periodic signals with fundamental frequency  $\omega_1$  and  $\omega_2$ , fourier series coefficients  $c_n$  and  $d_n$  respectively.

Given that  $x_2(t) = x_1(t-1) + x_1(1-t)$

The Fourier coefficient  $d_n$  will be

- (A)  $(c_n - jc_{-n})e^{-j\omega_1 n}$   
 (B)  $(c_{-n} - jc_n)e^{-j\omega_1 n}$   
 (C)  $(c_n + jc_{-n})e^{-j\omega_1 n}$   
 (D)  $(c_n + c_{-n})e^{-j\omega_1 n}$

**Q.32** The exponential Fourier series coefficient of signal  $x(t)$  shown in figure below is



- (A)  $\frac{2}{n^2\pi^2}[\cos n\pi - 1]$   
 (B)  $\frac{j}{2\pi n}$   
 (C)  $\frac{1}{n^2\pi^2}(-1)^n$   
 (D)  $\frac{1}{n^2\pi^2}[1 - (-1)^n]$

**Q.33** A continuous time periodic signal having fundamental period  $T_0$  is define over some interval by

$$x(t) = \begin{cases} -A, & -\frac{T_0}{2} < t < 0 \\ A, & 0 < t < \frac{T_0}{2} \end{cases}$$

The exponential CTFS coefficients of  $\int_{-\infty}^t x(\tau) d\tau$  is

- (A)  $-\frac{jAT_0}{2} \left[ \frac{\sin(n\pi/2)}{(n\pi)^2} \right]$   
 (B)  $\frac{jAT_0}{2} \left[ \frac{\sin(n\pi/2) - 1}{(n\pi)^2} \right]$   
 (C)  $\frac{AT_0}{2} \left( \frac{\cos(n\pi - 1)}{(n\pi)^2} \right)$   
 (D) None of these

**Q.34** Exponential Fourier Series coefficients of  $x(t) = 2 \sin(2\pi t - 3) + \sin 6\pi t$  are

- (A)  $C_1 = \frac{e^{-j3}}{2j}, C_{-1} = \frac{-e^{j3}}{2j}, C_3 = \frac{1}{2j}, C_{-3} = \frac{-1}{2j}$   
 (B)  $C_1 = \frac{e^{-j3}}{j}, C_{-1} = \frac{-e^{j3}}{j}, C_3 = \frac{1}{j}, C_{-3} = \frac{-1}{j}$

- (C)  $C_1 = \frac{e^{-j3}}{2j}, C_{-1} = \frac{-e^{j3}}{2j}, C_3 = \frac{1}{j}, C_{-3} = \frac{-1}{j}$   
 (D)  $C_1 = \frac{e^{-j3}}{j}, C_{-1} = \frac{-e^{j3}}{j}, C_3 = \frac{1}{2j}, C_{-3} = \frac{-1}{2j}$

**Q.35** The exponential Fourier series coefficient for the periodic signal  $x(t) = \sin^2 t$  is

- (A)  $-\frac{1}{4}\delta[n-1] + \frac{1}{2}\delta[n] - \frac{1}{4}\delta[n+1]$   
 (B)  $-\frac{1}{4}\delta[n-1] + \frac{1}{2}\delta[n] - \frac{1}{4}\delta[n+2]$   
 (C)  $-\frac{1}{2}\delta[n-1] + \delta[n] - \frac{1}{2}\delta[n+1]$   
 (D)  $-\frac{1}{2}\delta[n-2] + \delta[n] - \frac{1}{2}\delta[n+2]$

#### Statement for Linked Questions 36 to 38

Consider the following three continuous time signals with a fundamental period of  $T = 1$ .

$$x(t) = \cos 2\pi t$$

$$y(t) = \sin 2\pi t$$

$$z(t) = x(t)y(t)$$

**Q.36** The Fourier series coefficient  $c_n$  of  $x(t)$  are

- (A)  $\frac{1}{2}(\delta[n+1] + \delta[n-1])$  (B)  $\frac{1}{2}(\delta[n+1] - \delta[n-1])$   
 (C)  $\frac{1}{2}(\delta[n-1] - \delta[n+1])$  (D) None of these

**Q.37** The Fourier series coefficient of  $y(t)$ ,  $d_n$  will be

- (A)  $\frac{1}{2}(\delta[n+1] + \delta[n+1])$  (B)  $\frac{j}{2}(\delta[n+1] - \delta[n-1])$   
 (C)  $\frac{j}{2}(\delta[n-1] - \delta[n+1])$  (D)  $\frac{1}{2j}(\delta[n+1] + \delta[n+1])$

**Q.38** The Fourier series coefficient of  $z(t)$ ,  $e_n$  will be

- (A)  $\frac{1}{4j}(\delta[n-2] - \delta[n+2])$   
 (B)  $\frac{1}{2j}(\delta[n-2] - \delta[n+2])$   
 (C)  $\frac{1}{2j}(\delta[n+2] - \delta[n-2])$   
 (D) None of these

**Q.39** A continuous time periodic signal has a fundamental period  $T = 8$ . The nonzero Fourier series coefficients are as,  $c_1 = c_{-1}^* = j, c_5 = c_{-5} = 2$ . The signal will be

(A)  $4\cos\left(\frac{\pi}{4}t\right) - 2\sin\left(\frac{\pi}{4}t\right)$

(B)  $2\cos\left(\frac{\pi}{4}t\right) + 4\sin\left(\frac{\pi}{4}t\right)$

(C)  $2\cos\left(\frac{\pi}{4}t\right) + 2\sin\left(\frac{\pi}{4}t\right)$

(D)  $-2\sin\left(\frac{\pi}{4}t\right) + 4\cos\left(\frac{5\pi}{4}t\right)$

**Q.40** The FS coefficient of time-domain signal  $x(t)$  is

$$c_n = j\delta[n-1] - j\delta[n+1] + \delta[n+3] + \delta[n-3]$$

The fundamental frequency of signal is  $\omega_0 = 2\pi$ .

The signal is

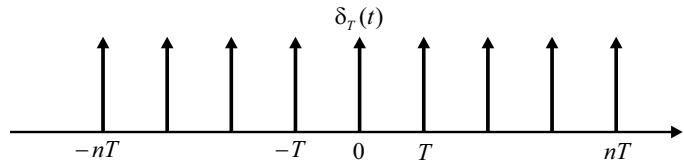
(A)  $2(\cos 3\pi t - \sin \pi t)$     (B)  $-2(\cos 3\pi t - \sin \pi t)$

(C)  $2(\cos 6\pi t - \sin 2\pi t)$     (D)  $-2(\cos 6\pi t - \sin 2\pi t)$

**Q.41** Match list-I (Name of periodic function) with the list-II (Properties of spectrum function) and select the correct answer using the codes given below the lists :

**List-I (Nature of periodic function)**

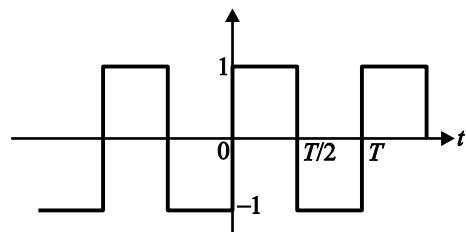
A. Impulse train



B. Full-wave rectified sine function

C.  $2\sin\frac{2\pi t}{6}\cos\frac{4\pi t}{6}$

D.



**List-II (Properties of spectrum function)**

1. Only even harmonics are present
2. Impulse train with strength  $1/T$
3.  $C_3 = \frac{1}{2j}; C_{-3} = -\frac{1}{2j}$
4. Only odd harmonics are present
5. Both even and odd harmonics are present

**Codes :** A B C D

(A) 5 2 3 1

(B) 2 5 3 4

(C) 5 2 4 3

(D) 2 1 3 4

**Q.42** Consider a periodic signal  $x(t)$  whose Fourier series coefficient is defined as below

$$c_n = \begin{cases} 2, & n = 0 \\ j\left(\frac{1}{2}\right)^{|n|}, & \text{Otherwise} \end{cases}$$

Which of the following is true?

(A)  $x(t)$  is a real valued signal

(B)  $x(t)$  is an even signal

(C)  $\frac{dx(t)}{dt}$  is an even signal

(D) Both (B) and (C)

**Q.43** The FS coefficient of time domain signal  $x(t)$  is

$$c_n = \left(\frac{-1}{3}\right)^{|n|}$$

The fundamental frequency of signal is  $\omega_0 = 1$ .

The signal is

(A)  $\frac{4}{5+3\sin t}$     (B)  $\frac{5}{4+3\sin t}$

(C)  $\frac{5}{4+3\cos t}$     (D)  $\frac{4}{5+3\cos t}$

**Q.44** If exponential F.S. coefficient of a periodic signal  $x(t)$  is

$$c_n = \left(\frac{1}{2}\right)^{|n|} e^{\frac{jn\pi}{20}} \quad T_0 = 2$$

Then  $x(t)$  is

(A)  $\frac{4e^{j(\pi t+\frac{\pi}{20})}}{5+\cos(\pi t+\frac{\pi}{20})}$     (B)  $\frac{3}{5-4\cos(\pi t+\frac{\pi}{20})}$

(C)  $\frac{4e^{-j(\pi t+\frac{\pi}{20})}}{5-4\cos(\pi t+\frac{\pi}{20})}$     (D) None of these

**Q.45** Suppose we have given following information about a signal  $x(t)$  :

1.  $x(t)$  is real and odd

2.  $x(t)$  is periodic with  $T = 2$

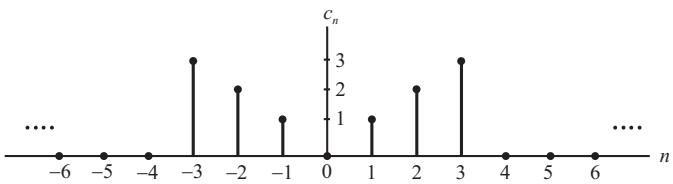
3. Fourier coefficients  $c_n = 0$  for  $|n| > 1$

4.  $\frac{1}{2} \int_0^2 |x(t)|^2 dt = 1$

The signal, that satisfy these condition, is

- (A)  $\sqrt{2} \sin \pi t$  and unique
- (B)  $\sqrt{2} \sin \pi t$  but not unique
- (C)  $2 \sin \pi t$  and unique
- (D)  $2 \sin \pi t$  but not unique

**Q.46** The FS coefficient of time-domain signal  $x(t)$  is shown below

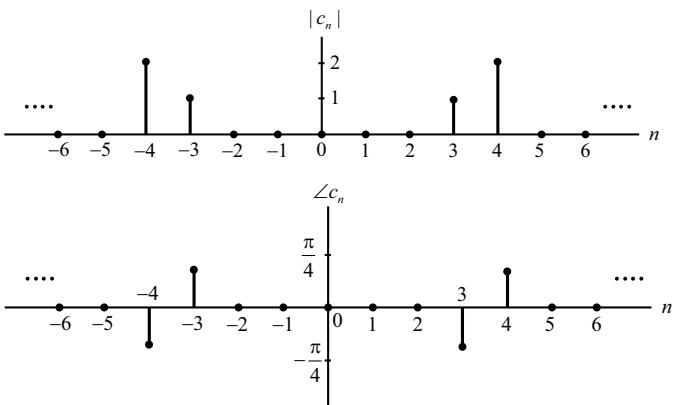


The fundamental frequency of signal is  $\omega_0 = \pi$ .

The signal is

- (A)  $3\cos 3\pi t + 2\cos 2\pi t + \cos \pi t$
- (B)  $3\sin 3\pi t + 2\sin 2\pi t + \sin \pi t$
- (C)  $6\sin 3\pi t + 4\sin 2\pi t + 2\sin \pi t$
- (D)  $6\cos 3\pi t + 4\cos 2\pi t + 2\cos \pi t$

**Q.47** The FS coefficient of time-domain signal  $x(t)$  is shown below

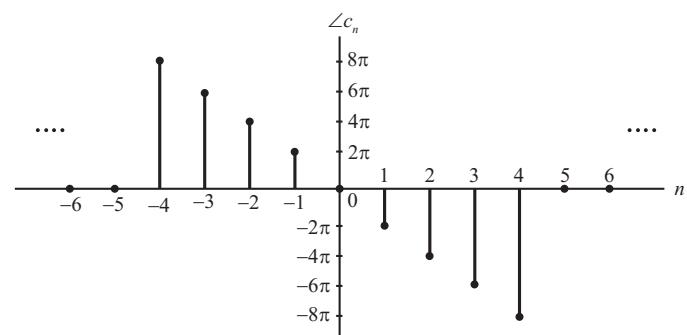
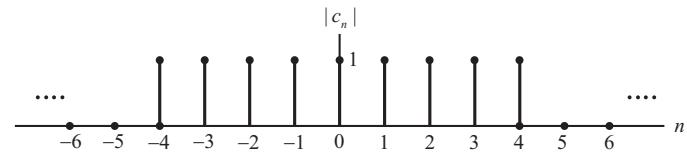


The fundamental frequency of signal is  $\omega_0 = \pi$ .

The signal is

- (A)  $6\cos\left(2\pi t + \frac{\pi}{4}\right) - 3\cos\left(3\pi t - \frac{\pi}{4}\right)$
- (B)  $4\cos\left(4\pi t - \frac{\pi}{4}\right) - 2\cos\left(3\pi t + \frac{\pi}{4}\right)$
- (C)  $2\cos\left(2\pi t + \frac{\pi}{4}\right) - 2\cos\left(3\pi t - \frac{\pi}{4}\right)$
- (D)  $4\cos\left(4\pi t + \frac{\pi}{4}\right) + 2\cos\left(3\pi t - \frac{\pi}{4}\right)$

**Q.48** The FS coefficient of time-domain signal  $x(t)$  is shown below



The fundamental frequency of signal is  $\omega_0 = \pi$ .

The signal  $x(t)$  is

- (A)  $\frac{\sin 9\pi t}{\sin \pi t}$
- (B)  $\frac{\sin 9\pi t}{\pi \sin \pi t}$
- (C)  $\frac{\sin 18\pi t}{2 \sin \pi t}$
- (D)  $\frac{\sin\left(\frac{9\pi t}{2}\right)}{\sin\left(\frac{\pi t}{2}\right)}$

**Q.49** A CT signal  $x(t)$  is given as

$$x(t) = (-1)^m \sum_{m=-\infty}^{\infty} \left[ \delta\left(t - \frac{m}{3}\right) + \delta\left(t - \frac{2m}{3}\right) \right]$$

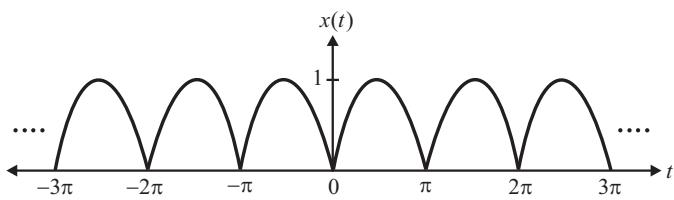
The exponential Fourier series coefficient  $c_n$  of the function is

- (A)  $c_n = \frac{3}{2} \cos\left(\frac{3\pi}{2}n\right)$
- (B)  $c_n = \frac{6}{4} - \frac{3}{2} \cos\left(\frac{4\pi}{3}n\right)$
- (C)  $c_n = \frac{6}{4} + \frac{3}{2} \cos\left(\frac{3\pi}{2}n\right)$
- (D)  $c_n = \frac{6}{4} - \frac{3}{2} \cos\left(\frac{\pi}{2}n\right)$

**Q.50** Let  $y(t)$  is the output of a half wave rectifier circuit when an input  $x(t) = E \sin \omega_0 t$  is applied to it. For even values of  $n$ , the exponential Fourier series coefficient  $c_n$  of  $y(t)$  is

- (A) 0
- (B)  $\frac{E}{\pi(1-n^2)}$
- (C)  $\frac{2E}{\pi(1-n^2)}$
- (D)  $\frac{4E}{\pi(1-n^2)}$

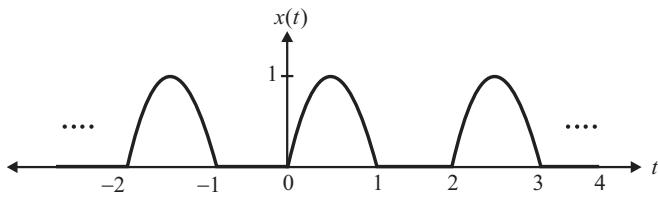
**Q.51** The complex Fourier coefficient  $c_n$  of the full wave rectified sine function shown in figure below is



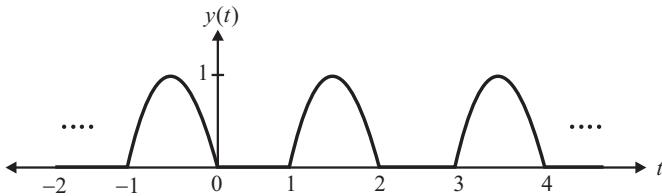
- (A)  $\frac{4}{\pi(1-4n^2)}$       (B)  $\frac{2}{\pi(1-4n^2)}$   
 (C)  $\frac{2}{\pi(1-n^2)}$       (D)  $\frac{4}{n^2\pi^2}(-1)^n$

**Q.52** The Fourier series coefficient of signal  $x(t)$  shown in figure (A) are given as  $c_0 = \frac{1}{\pi}$ ,

$$c_1 = \frac{-j}{4}, c_n = \frac{1}{\pi(1-n^2)}, \text{ for } n \text{ even}$$



Then the Fourier series coefficient  $c'_0, c'_1$  and  $c'_n$  of  $y(t)$  shown in figure (B) are



- (A)  $\frac{1}{\pi}, \frac{-j}{4}, \frac{1}{\pi(1-n^2)}; n \text{ is odd}$   
 (B)  $\frac{-1}{\pi}, \frac{-j}{4}, \frac{1}{\pi(1-n^2)}; n \text{ is odd}$   
 (C)  $\frac{1}{\pi}, \frac{j}{4}, \frac{1}{\pi(1-n^2)}; n \text{ is even}$   
 (D)  $\frac{-1}{\pi}, \frac{j}{4}, \frac{1}{\pi(1-n^2)}; n \text{ is even}$



**Answer Keys**

Objective & Numerical Answer Type Questions									
1.	B	2.	D	3.	C	4.	0.0408	5.	D
6.	C	7.	A	8.	A	9.	C	10.	C
11.	C	12.	D	13.	A	14.	A	15.	C
16.	C	17.	D	18.	A	19.	D	20.	B
21.	A	22.	A	23.	D	24.	B	25.	C
26.	C	27.	A	28.	D	29.	B	30.	A
31.	D	32.	D	33.	A	34.	A	35.	0.5
36.	D	37.	C	38.	B	39.	A	40.	D
41.	0.2, 1.2, 0.98	42.	C	43.	D	44.	1.68	45.	C
46.	A	47.	D	48.	A	49.	C	50.	C
51.	A	52.	C	53.	C	54.	A	55.	B
56.	A	57.	A	58.	C	59.	B	60.	A
61.	C	62.	D	63.	D	64.	B	65.	D
66.	B	67.	C	68.	C	69.	B	70.	A
71.	D	72.	C	73.	A	74.	A	75.	C
76.	B	77.	A	78.	D	79.	C	80.	C
81.	A	82.	C	83.	D	84.	C	85.	D
86.	D	87.	A						

**Practice (Objective & Numerical Answer) Questions**

1.	C	2.	A	3.	A	4.	B	5.	D
6.	A	7.	C	8.	B	9.	D	10.	C
11.	C	12.	A	13.	B	14.	D	15.	B
16.	A	17.	A	18.	A	19.	C	20.	D
21.	A	22.	B	23.	A	24.	C	25.	B
26.	A	27.	B	28.	B	29.	D	30.	A
31.	D	32.	D	33.	C	34.	D	35.	A
36.	A	37.	B	38.	A	39.	D	40.	C
41.	D	42.	B	43.	D	44.	B	45.	B
46.	D	47.	D	48.	D	49.	D	50.	B
51.	B	52.	C						

# 4

# Continuous Time Fourier Transform

## Objective & Numerical Ans Type Questions :

**Q.1** Consider a signal defined by

[GATE EE 2015-Kanpur]

$$x(t) = \begin{cases} e^{j10t} & \text{for } |t| \leq 1 \\ 0 & \text{for } |t| > 1 \end{cases}$$

Its Fourier Transform is

- (A)  $\frac{2\sin(\omega-10)}{\omega-10}$       (B)  $2e^{j10}\frac{\sin(\omega-10)}{\omega-10}$   
 (C)  $\frac{2\sin\omega}{\omega-10}$       (D)  $e^{j10\omega}\frac{2\sin\omega}{\omega}$

**Q.2** Let  $f(t)$  be a continuous time signal and let  $F(\omega)$  be its Fourier transform defined by

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt$$

[GATE EE 2014-Kharagpur]

Define  $g(t)$  by,  $g(t) = \int_{-\infty}^{\infty} F(u)e^{-jut} du$

What is the relationship between  $f(t)$  and  $g(t)$ ?

- (A)  $g(t)$  would always be proportional to  $f(t)$   
 (B)  $g(t)$  would be proportional to  $f(t)$  if  $f(t)$  is an even function  
 (C)  $g(t)$  would be proportional to  $f(t)$  only if  $f(t)$  is a sinusoidal function  
 (D)  $g(t)$  would never be proportional to  $f(t)$

**Q.3** The signal  $x(t)$  is described by

$$x(t) = \begin{cases} 1 & \text{for } -1 \leq t \leq +1 \\ 0 & \text{otherwise} \end{cases}$$

Two of the angular frequencies at which its Fourier transform becomes zero are :

[GATE EC 2008-Bangalore]

- (A)  $\pi, 2\pi$       (B)  $0.5\pi, 1.5\pi$

- (C)  $0, \pi$       (D)  $2\pi, 2.5\pi$

**Q.4** Let  $x(t) = \text{rect}\left(t - \frac{1}{2}\right)$  (where  $\text{rect}(x) = 1$  for  $-\frac{1}{2} \leq x \leq \frac{1}{2}$  and zero otherwise). Then if  $\text{sinc}(x) = \frac{\sin \pi x}{\pi x}$ , the Fourier Transform of  $x(t) + x(-t)$  will be given by :

[GATE EE 2008-Bangalore]

(A)  $\text{sinc}\left(\frac{\omega}{2\pi}\right)$       (B)  $2\text{sinc}\left(\frac{\omega}{2\pi}\right)$

(C)  $2\text{sinc}\left(\frac{\omega}{2\pi}\right)\cos\left(\frac{\omega}{2}\right)$       (D)  $\text{sinc}\left(\frac{\omega}{2\pi}\right)\sin\left(\frac{\omega}{2}\right)$

**Q.5** The Fourier transform of  $e^{-at} u(-t)$ , where  $u(t)$  is the unit step function is :

[GATE IN 2008-Bangalore]

(A) Exists for any real value of  $a$ .

(B) Does not exist for any real value of  $a$ .

(C) Exists if the real value of  $a$  is strictly negative.

(D) Exists if the real value of  $a$  is strictly positive.

**Q.6** Let  $x(t) \xrightarrow{FT} X(j\omega)$  be the Fourier transform pair. The Fourier transform of the signal  $x(5t - 3)$  in terms of  $X(j\omega)$  is given as

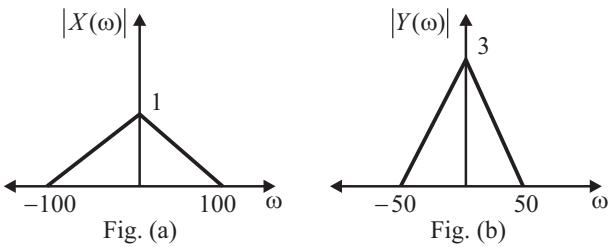
[GATE EC 2006-Kharagpur]

(A)  $\frac{1}{5}e^{-\frac{j3\omega}{5}}X\left(\frac{j\omega}{5}\right)$       (B)  $\frac{1}{5}e^{\frac{j3\omega}{5}}X\left(\frac{j\omega}{5}\right)$

(C)  $\frac{1}{5}e^{-j3\omega}X\left(\frac{j\omega}{5}\right)$       (D)  $\frac{1}{5}e^{j3\omega}X\left(\frac{j\omega}{5}\right)$

- Q.7** The magnitude of Fourier transform  $X(\omega)$  of a function  $x(t)$  is shown below in figure (a). The magnitude of Fourier transform  $Y(\omega)$  of another function  $y(t)$  is shown in figure (b). The phases of  $X(\omega)$  and  $Y(\omega)$  are zero for all  $\omega$ . The magnitude and frequency units are identical in both the figure. The function  $y(t)$  can be expressed in terms of  $x(t)$  as

[GATE IN 2006-Kharagpur]



- (A)  $\frac{2}{3}x\left(\frac{t}{2}\right)$       (B)  $\frac{3}{2}x(2t)$   
 (C)  $\frac{2}{3}x(2t)$       (D)  $\frac{3}{2}x\left(\frac{t}{2}\right)$

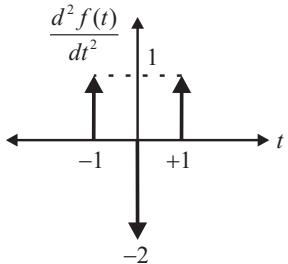
- Q.8** The Fourier transform of a function  $g(t)$  is given as  $G(\omega) = \frac{\omega^2 + 21}{\omega^2 + 9}$ . Then the function  $g(t)$  is given as

[GATE IN 2006-Kharagpur]

- (A)  $\delta(t) + 2 \exp(-3|t|)$       (B)  $\cos 3\omega t + 21 \exp(-3t)$   
 (C)  $\sin 3\omega t + 7 \cos \omega t$       (D)  $\sin 3\omega t + 21 \exp(3t)$

- Q.9** If the waveform, shown in the following figure, corresponds to the second derivative of a given function  $f(t)$ , then the Fourier transform of  $f(t)$  is

[GATE IN 2006-Kharagpur]



- (A)  $1 + \sin \omega$       (B)  $1 + \cos \omega$   
 (C)  $\frac{2(1 - \cos \omega)}{\omega^2}$       (D)  $\frac{2(1 + \cos \omega)}{\omega^2}$

- Q.10** For a signal  $x(t)$  the Fourier transform is  $X(f)$ , then the inverse Fourier transform of  $X(3f + 2)$  is given by

[GATE EC 2005-Bombay]

- (A)  $\frac{1}{3}x\left(\frac{t}{3}\right)e^{-j4\pi t}$       (B)  $\frac{1}{3}x\left(\frac{t}{3}\right)e^{\frac{-j4\pi t}{3}}$   
 (C)  $3x(3t)e^{-j4\pi t}$       (D)  $x(3t + 2)$

- Q.11** The output  $y(t)$  of a linear time invariant system is related to its input  $x(t)$  by the following equation

$$y(t) = 0.5x(t - t_d + T) + x(t - t_d) + 0.5x(t - t_d - T)$$

The filter transfer function  $H(\omega)$  of such a system is given by : [GATE EC 2005-Bombay]

- (A)  $[1 + \cos \omega T]e^{-j\omega t_d}$       (B)  $[1 + 0.5 \cos \omega T]e^{-j\omega t_d}$   
 (C)  $[1 - \cos \omega T]e^{-j\omega t_d}$       (D)  $[1 - 0.5 \cos \omega T]e^{-j\omega t_d}$

- Q.12** The continuous-time signal  $x(t) = \frac{1}{a^2 + t^2}$  has the Fourier transform  $\frac{\pi}{a} \exp(-a|\omega|)$ . The signal  $x(t) \cos bt$  has the Fourier transform

[GATE IN 2005-Bombay]

- (A)  $\frac{\pi}{2a} [\exp(-a|\omega-b|) + \exp(-a|\omega+b|)]$   
 (B)  $\frac{\pi}{2a} [\exp(-a|\omega|) + \exp(-a|\omega|)]$   
 (C)  $\frac{\pi}{a} [\exp(-a|\omega|) \cos b\omega]$   
 (D)  $\frac{\pi}{2a} [\exp(-a|\omega-b|) - \exp(-a|\omega+b|)]$

- Q.13** The Fourier transform  $e^{-t}u(t)$  is equal to  $\frac{1}{1 + j2\pi f}$ . Therefore,  $F\left[\frac{1}{1 + j2\pi t}\right]$  is

[GATE EC 2002-Bangalore]

- (A)  $e^f u(f)$       (B)  $e^{-f} u(f)$   
 (C)  $e^f u(-f)$       (D)  $e^{-f} u(-f)$

- Q.14** The Laplace transform of a continuous time signal  $x(t)$  is  $X(s) = \frac{5-s}{s^2 - s - 2}$ . If the Fourier transform of this signal exists, then  $x(t)$  is

[GATE EC 2002-Bangalore]

- (A)  $e^{2t}u(t) - 2e^{-t}u(t)$       (B)  $e^{2t}u(t) + 2e^{-t}u(t)$   
 (C)  $-e^{2t}u(-t) - 2e^{-t}u(t)$       (D)  $e^{2t}u(-t) - 2e^{-t}u(t)$

- Q.15** The Fourier transform of the signal  $x(t) = e^{-3t^2}$  is of the following form, where  $A$  and  $B$  are constants

[GATE EC 2000-Kharagpur]

- (A)  $Ae^{-B|f|}$       (B)  $Ae^{-Bf^2}$   
 (C)  $A + B|f|^2$       (D)  $A + e^{-Bf}$

**Q.16** The Fourier transform of a voltage signal  $x(t)$  is  $X(f)$ . The unit of  $|X(f)|$  is

[GATE EC 1998-Delhi]

- |              |                       |
|--------------|-----------------------|
| (A) Volt     | (B) Volt-sec          |
| (C) Volt/sec | (D) Volt <sup>2</sup> |

**Q.17** The function  $f(t)$  has the Fourier transform  $g(\omega)$ . The Fourier transform of  $g(t) = \left( \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt \right)$  is

[GATE EC 1992-Delhi]

- |                                |                                 |
|--------------------------------|---------------------------------|
| (A) $\frac{1}{2\pi} f(\omega)$ | (B) $\frac{1}{2\pi} f(-\omega)$ |
| (C) $2\pi f(-\omega)$          | (D) None of these               |

**Q.18**  $G(\omega)$  is the Fourier transform of an odd function  $g(t)$ . The Fourier transform of  $2g\left(\frac{t}{3}\right) + 3g\left(\frac{-t}{2}\right)$  is

[GATE IN 1992-Delhi]

- |   |   |
|---|---|
| (A) $2G\left(\frac{\omega}{3}\right) + 3G\left(\frac{\omega}{2}\right)$ | (B) $2G\left(\frac{\omega}{3}\right) - 3G\left(\frac{\omega}{2}\right)$ |
| (C) $6G(3\omega) - 6G(2\omega)$   | (D) $6G(2\omega) - 6G(3\omega)$   |

**Q.19** The magnitude and phase transfer functions for a distortionless filter should respectively be

[GATE EC 1990-Bangalore]

- | (Magnitude)  | (Phase)  |
|--------------|----------|
| (A) Linear   | Constant |
| (B) Constant | Constant |
| (C) Constant | Linear   |
| (D) Linear   | Linear   |

**Q.20** Which of the following Dirichlet's conditions are correct for convergence of Fourier transform of the function  $x(t)$ ? [ESE EC 2013]

1.  $x(t)$  is square integrable
  2.  $x(t)$  must be periodic
  3.  $x(t)$  should have finite number of maxima and minima within any finite interval
  4.  $x(t)$  should have finite number of discontinuities within any finite interval
- |                     |                     |
|---------------------|---------------------|
| (A) 1, 2, 3 and 4   | (B) 1, 2 and 4 only |
| (C) 1, 3 and 4 only | (D) 2, 3 and 4 only |

**Q.21** If  $X(\omega) = \delta(\omega - \omega_0)$  then  $x(t)$  is [ESE EC 2010]

- |                                      |                 |
|--------------------------------------|-----------------|
| (A) $e^{-j\omega_0 t}$               | (B) $\delta(t)$ |
| (C) $\frac{1}{2\pi} e^{j\omega_0 t}$ | (D) 1           |

**Q.22** The Fourier transform of a function is equal to its two-sided Laplace transform evaluated

- |   |
|---|
| (A) on the real axis of the s-plane [ESE EC 2008]           |
| (B) on a line parallel to the real axis of the s-plane      |
| (C) on the imaginary axis of the s-plane                    |
| (D) on a line parallel to the imaginary axis of the s-plane |

**Q.23** If the Fourier transform of  $x(t)$  is  $\frac{2}{\omega} \sin(\pi\omega)$ , then what is the Fourier transform of  $e^{j\omega t} x(t)$ ?

[ESE EC 2006]

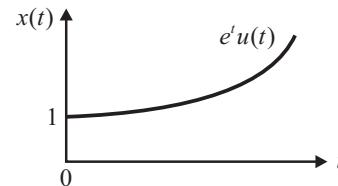
- |  |  |
|--|--|
| (A) $\frac{2}{\omega-5} \sin(\pi\omega)$       | (B) $\frac{2}{\omega} \sin\{\pi(\omega-5)\}$   |
| (C) $\frac{2}{\omega+5} \sin\{\pi(\omega+5)\}$ | (D) $\frac{2}{\omega-5} \sin\{\pi(\omega-5)\}$ |

**Q.24** What is the inverse Fourier transform of  $u(\omega)$ ?

[ESE EC 2006]

- |  |                                  |
|--|----------------------------------|
| (A) $\frac{1}{2} \delta(t) + \frac{j}{2\pi t}$ | (B) $\frac{1}{2} \delta(t)$      |
| (C) $\frac{\delta(t)}{2} + \frac{j}{2\pi t}$   | (D) $2\delta(t) + \text{sgn}(t)$ |

**Q.25** For the signal shown below [ESE EC 2005]



- |  |
|--|
| (A) Only Fourier transform exists                          |
| (B) Only Laplace transform exists                          |
| (C) Both Laplace and Fourier transforms exist              |
| (D) Neither Laplace transform nor Fourier transform exists |

**Q.26** Match List-I (Time function) with List-II (Fourier Spectrum/Fourier Transform) and select the correct answer using the code given below the lists :

**List-I (Time Function)** [ESE EC 2005]

- A. Periodic function
- B. Aperiodic function
- C. Unit Impulse  $\delta(t)$
- D.  $\sin \omega t$

**List-II (Fourier Spectrum/Fourier Transform)**

1. Continuous spectrum at all frequencies
2.  $\delta(\omega)$
3. Line discrete spectrum
4. 1

**Codes :** A B C D

- (A) 4 2 3 1  
 (B) 3 1 4 2  
 (C) 4 1 3 2  
 (D) 3 2 4 1

**Q.27** Consider the following statements [ESE EC 2002]

1. Fourier transform is special case of Laplace transform.
2. Region of convergence need not be specified for Fourier transform.
3. Laplace transform is not unique unless the region of convergence is specified.
4. Laplace transform is a special case of Fourier transform.

Which of these statements are correct?

- (A) 2 and 4 (B) 4 and 1  
 (C) 4, 3 and 2 (D) 1, 2 and 3

**Q.28** The Fourier transform of  $e^{-\pi t^2}$  is  $e^{-\pi f^2}$ ; then the Fourier transform of  $e^{-\alpha t^2}$  is [ESE EC 2002]

- (A)  $\left(\frac{1}{\alpha}\right) \cdot e^{-\alpha f^2}$  (B)  $\sqrt{\frac{\pi}{\alpha}} e^{-\frac{\pi^2 f^2}{\alpha}}$   
 (C)  $\frac{1}{\sqrt{\pi \alpha}} e^{-\alpha \pi^2 f^2}$  (D)  $\sqrt{\pi \alpha} e^{-\frac{f^2}{\pi^2 \alpha}}$

**Q.29** Match List-I (Functions in the time domain) with List-II (Fourier transform of the function) and select the correct answer using the codes given below the lists : [ESE EC 2002]**List-I (Functions in the time domain)**

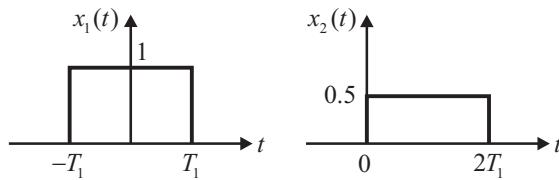
- A. Delta function
- B. Gate function
- C. Normalized Gaussian function
- D. Sinusoidal function

**List-II (Fourier transform of the function)**

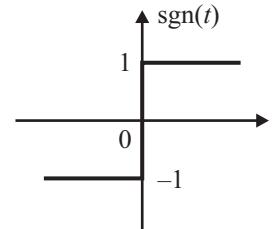
1. Delta function
2. Gaussian function
3. Constant function
4. Sampling function

**Codes :** A B C D

- (A) 1 2 4 3  
 (B) 3 4 2 1  
 (C) 1 4 2 3  
 (D) 3 2 4 1

**Q.30** The Fourier transform of  $x_1(t)$  is  $\frac{2 \sin \omega T_1}{\omega}$ . The Fourier transform of  $x_2(t)$  will be [ESE EC 1994]

- (A)  $\frac{2 \sin \omega T_1}{\omega} e^{j\omega T_1}$  (B)  $\frac{2 \sin \omega T_1}{\omega} e^{-j\omega T_1}$   
 (C)  $\frac{\sin \omega T_1}{\omega} e^{-j\omega T_1}$  (D)  $\frac{\sin \omega T_1}{\omega} e^{j\omega T_1}$

**Q.31** The Fourier transform of the function  $\text{sgn}(t)$  defined in the figure is [ESE EC 1991]

- (A)  $-\frac{2}{j\omega}$  (B)  $\frac{4}{j\omega}$   
 (C)  $\frac{2}{j\omega}$  (D)  $\frac{1}{j\omega} + 1$

**Q.32** If  $x(t)$  is speed then  $|X(f)|$  has dimension of

- (A) Speed (B) Distance  
 (C) Acceleration (D) Time

**Q.33** Shift in time of a certain signal is equivalent to

- (A) Change in magnitude spectrum  
 (B) Change in phase spectrum  
 (C) Change in both  
 (D) Change in phase spectrum keeping magnitude spectrum same

**Q.34** If  $y(t) = x(0.2t)$ , then which of the following is true with reference to the spectrum of  $y(t)$ ?

- (A) Magnitude of the spectrum of  $y(t)$  is 0.2 times the magnitude of the spectrum of  $x(t)$

- (B) Magnitude of the spectrum of  $y(t)$  is 5 times the magnitude of the spectrum of  $x(t)$  but the spectrum is compressed in frequency by a factor of 5.
- (C) Magnitude of the spectrum of  $y(t)$  is 5 times the magnitude of the spectrum of  $x(t)$  and the spectrum is expanded in frequency by a factor of 5.
- (D) Magnitude of the spectrum of  $y(t)$  is 0.2 times the magnitude of the spectrum of  $x(t)$  and the spectrum is expanded in frequency by a factor of 5.

**Q.35** Fourier transform of  $e^{-a|t|} \operatorname{sgn}(t)$  is

- (A)  $\frac{2\omega}{a^2 + \omega^2}$       (B)  $\frac{2j\omega}{a^2 + \omega^2}$   
 (C)  $\frac{-2j\omega}{a^2 + \omega^2}$       (D)  $\frac{2a}{a^2 + \omega^2}$

**Q.36** The spectrum of a signal is  $X(f) = 10j[\delta(f + f_0) - \delta(f - f_0)]$ . The corresponding signal  $x(t)$  is given by

- (A)  $10\sin(2\pi f_0 t)$       (B)  $20\sin(2\pi f_0 t)$   
 (C)  $20\cos(2\pi f_0 t)$       (D)  $10\cos(2\pi f_0 t)$

**Q.37** The Fourier transform of  $x(t) = A \cos(2\pi f_0 t)$  is  $X(j\omega) = 10[\delta(\omega - 4\pi) + \delta(\omega + 4\pi)]$ . The values of  $A$  and  $f_0$  are respectively

- (A) 10, 1      (B)  $\pi, 4$   
 (C)  $\frac{10}{\pi}, 2$       (D)  $20\pi, 2$

**Q.38** The inverse Fourier transform of  $[2\pi\delta(\omega) + \pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)]$  is

- (A)  $2\pi(1 - \cos 4\pi t)$       (B)  $\pi(1 - \cos 4\pi t)$   
 (C)  $1 + \cos 4\pi t$       (D)  $2\pi(1 + \cos 4\pi t)$

**Q.39** A periodic signal has a period of 4 sec. What is the lowest positive frequency at which its CTFT could be non-zero?

- (A)  $\frac{1}{4}$  Hz      (B) 1 Hz  
 (C) 2 Hz      (D) 4 Hz

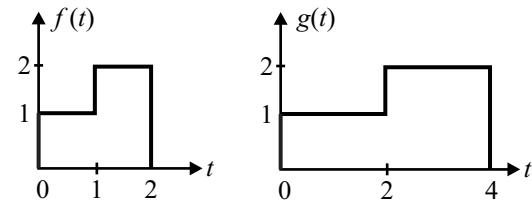
**Q.40** A continuous time signal is defined as

$$x(t) = \begin{cases} e^{-2t}, & t > 0 \\ |t|, & t = 0 \\ -e^{2t}, & t < 0 \end{cases}$$

The Fourier transform of the signal is

- (A)  $\frac{-2j\omega}{4 + \omega^2}$       (B)  $\frac{4}{4 + \omega^2}$   
 (C)  $\frac{2}{j\omega + 4}$       (D) does not exist

**Q.41** Consider two CT signal  $f(t)$  and  $g(t)$  shown in figure.



If CTFT of  $f(t)$  is given by,

$$F(j\omega) = \frac{1}{j\omega} [1 + e^{-j\omega} - 2e^{-j2\omega}]$$

then CTFT of signal  $g(t)$  will be

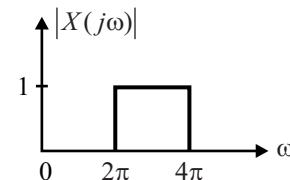
$$(A) \frac{2}{j\omega} [e^{j2\omega} + e^{j\omega} - 2]$$

$$(B) \frac{1}{j\omega} [e^{-j2\omega} + e^{-j3\omega} - 2e^{-j4\omega}]$$

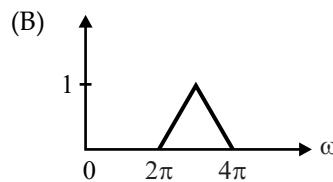
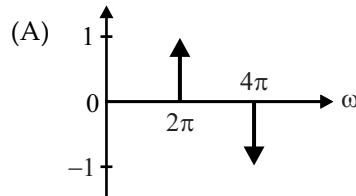
$$(C) \frac{1}{j\omega} [1 + e^{-j2\omega} - 2e^{-j4\omega}]$$

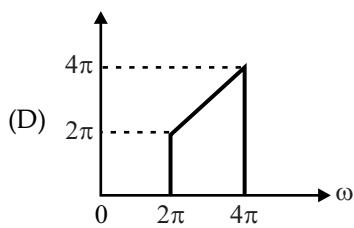
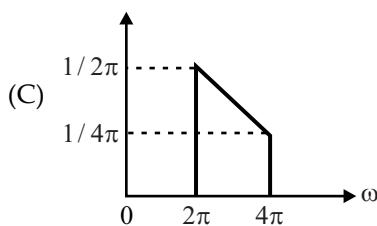
(D) None of the above

**Q.42** Let the Fourier transform of a signal  $x(t)$  be as shown in figure.



Which of the following corresponds to Fourier transform of signal  $\frac{dx(t)}{dt}$ ?





- Q.43** If  $x(t) = u(-2-t) + u(t-2)$ , then what will be the Fourier transform of  $\frac{dx(t)}{dt}$ ?
- (A)  $2 \cos 2\omega$       (B)  $-2j \sin 2\omega$   
 (C)  $2\pi[\delta(\omega+2) + \delta(\omega-2)]$       (D)  $\frac{1}{2+j\omega} + \frac{1}{2-j\omega}$

- Q.44** Fourier transform of the signal  $x(t) = \int_{-\infty}^t e^{-a\tau} u(\tau) d\tau$  is,
- (A)  $\frac{1}{a+j\omega}$       (B)  $\frac{1}{j\omega(a+j\omega)}$   
 (C)  $\frac{1}{j\omega(a+j\omega)} + \frac{\pi}{a} \delta(\omega)$       (D) None of these

- Q.45** The Fourier transform of signal  $te^{-3|t|}$  is
- (A)  $\frac{6e^{-j\omega}}{9+\omega^2} - \frac{12j\omega e^{-j\omega}}{(9+\omega^2)^2}$       (B)  $\frac{12e^{-j\omega}}{(9+\omega^2)^3}$   
 (C)  $\frac{6e^{-j3\omega}}{9+\omega^2} - \frac{12j\omega e^{-3j\omega}}{(9+\omega^2)^2}$       (D)  $\frac{12j\omega e^{-j\omega}}{(9+\omega^2)^3}$

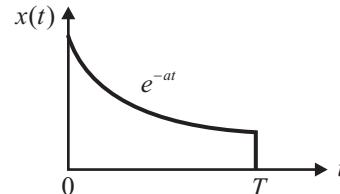
- Q.46** The inverse Fourier transform of  $\frac{6j\omega+16}{(j\omega)^2+5j\omega+6}$  is
- (A)  $-2(e^{-3t} + 2e^{-2t})u(t)$       (B)  $(2e^{-4t} - e^{-t})u(t)$   
 (C)  $(2e^{-3t} + 4e^{-2t})u(t)$       (D)  $(1+5t^2)u(t)$

- Q.47** The inverse Fourier transform of
- $$\frac{-(j\omega)^2 - 4j\omega - 6}{[(j\omega)^2 + 3j\omega + 2](j\omega + 4)}$$
- is
- (A)  $\delta(t) + (e^{-2t} + e^{-4t})u(t)$   
 (B)  $\delta(t) + (e^{-2t} - e^{-4t})u(t)$   
 (C)  $(2e^{-t} - 2e^{-4t})u(t)$   
 (D)  $(e^{-2t} - e^{-t} - e^{-4t})u(t)$

**Q.48** The inverse Fourier transform of  $\frac{j\omega+3}{(j\omega+1)^2}$  is

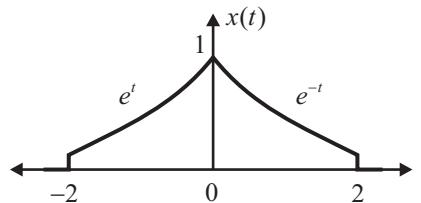
- (A)  $e^{-t}(1+2t)u(t)$       (B)  $e^{-t}(1+2\delta(t))u(t)$   
 (C)  $e^{-t}(1+t)u(t)$       (D) None of these

**Q.49** Determine the Fourier transform at given below figure.



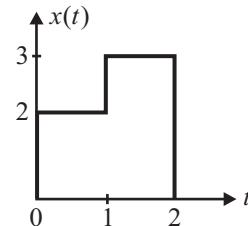
- (A)  $\frac{1}{a+j\omega} [1 - e^{-aT} \cdot e^{j\omega T}]$  (B)  $\frac{1}{a+j\omega} [1 - e^{aT} \cdot e^{j\omega T}]$   
 (C)  $\frac{1}{a+j\omega} [1 - e^{aT} \cdot e^{-j\omega T}]$  (D)  $\frac{1}{a+j\omega} [1 - e^{-aT} \cdot e^{-j\omega T}]$

**Q.50** The Fourier transform of signal shown below is



- (A)  $2 - 2e^{-2} \sin 2\omega + 2\omega e^{-2} \sin 2\omega$   
 (B)  $2 + 2e^{-2} \cos 2\omega - 2\omega e^{-2} \cos 2\omega$   
 (C)  $\frac{2 - 2e^{-2} \cos 2\omega + 2\omega e^{-2} \sin 2\omega}{1 + \omega^2}$   
 (D)  $\frac{2 + 2e^{-2} \cos 2\omega - 2\omega e^{-2} \sin 2\omega}{1 + \omega^2}$

**Q.51** What is the Fourier transform of signal  $x(t)$  shown in figure below?

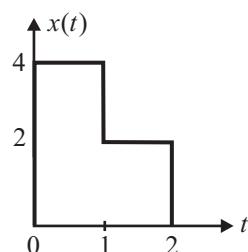


- (A)  $\frac{1}{j\omega} [1 + e^{-j\omega} + 3e^{-j2\omega}]$   
 (B)  $\frac{1}{j\omega} [2e^{-j\omega} - 3e^{-j2\omega}]$

(C)  $\frac{1}{j\omega} \left[ 1 - 2e^{-j\omega} - 3e^{-j\omega/2} \right]$

(D)  $\frac{1}{j\omega} \left[ 2 + e^{-j\omega} - 3e^{-j2\omega} \right]$

**Q.52** Consider a signal  $x(t)$  shown in figure.



The value of its Fourier transform at a frequency  $\omega = \pi$  rad/sec equals to

(A) 1

(B)  $\frac{2}{j\pi}$

(C)  $\frac{4}{j\pi}$

(D) 0

**Q.53** Let  $x(t)$  be a rectangular pulse of unity amplitude and of width 2 centered about zero. Fourier transform of  $x(t+1) + x(t-1)$  will be

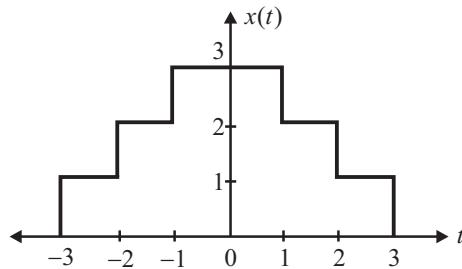
(A)  $4\text{sinc}\left(\frac{2\omega}{\pi}\right)$

(B)  $\text{sinc}\left(\frac{2\omega}{\pi}\right)$

(C)  $2\text{sinc}\left(\frac{2\omega}{\pi}\right)$

(D)  $2\text{sinc}\left(\frac{\omega}{\pi}\right)$

**Q.54** Fourier transform of given below figure is :



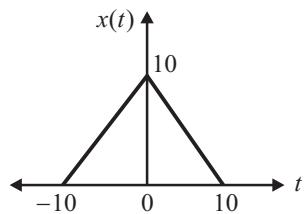
(A)  $\sin c(2f) + 2\sin c(2f) + 3\sin c(3f)$

(B)  $\sin c(2f) + 2\sin c(4f) + 3\sin c(6f)$

(C)  $2\sin c(f) + 4\sin c(2f) + 6\sin c(3f)$

(D)  $2\sin c(2f) + 4\sin c(4f) + 6\sin c(6f)$

**Q.55** Determine the Fourier transform of given below figure.

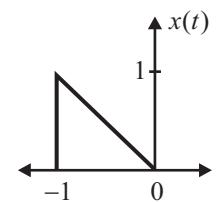


(A)  $50\text{sinc}^2(10f)$  (B)  $100\text{sinc}^2(10f)$

(C)  $50\text{sinc}^2(20f)$  (D)  $100\text{sinc}^2(20f)$

**Statement for Linked Questions 56 & 57**

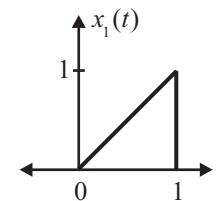
Consider a triangular pulse  $x(t)$  shown in the figure.



The Fourier transform of  $x(t)$  is given as

$$X(j\omega) = \frac{1}{\omega^2} \left[ e^{j\omega} - j\omega e^{j\omega} - 1 \right]$$

**Q.56** The Fourier transform of  $x_1(t)$  shown in figure is



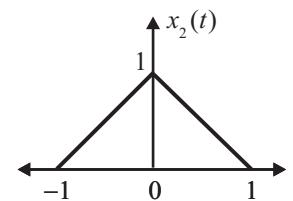
(A)  $\frac{1}{\omega^2} \left[ e^{j\omega} + j\omega e^{j\omega} - 1 \right]$

(B)  $\frac{1}{\omega^2} \left[ e^{-j\omega} + j\omega e^{-j\omega} - 1 \right]$

(C)  $\frac{1}{\omega^2} \left[ -e^{-j\omega} - j\omega e^{-j\omega} - 1 \right]$

(D)  $\frac{1}{\omega^2} \left[ -e^{-j\omega} + j\omega e^{-j\omega} - 1 \right]$

**Q.57** The Fourier transform  $x_2(t)$  shown in figure is



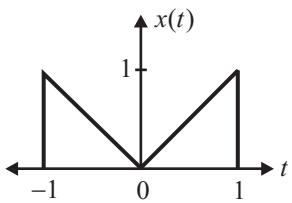
(A)  $Sa^2\left(\frac{\omega}{2}\right)$

(B)  $Sa\left(\frac{\omega}{2}\right)$

(C)  $\frac{2e^{-j\omega}}{\omega^2} [\cos \omega + \omega \sin \omega - 1]$

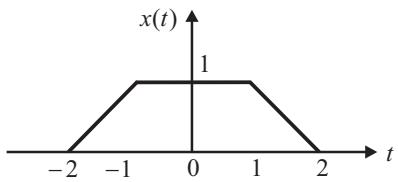
(D)  $\frac{4}{\omega^2} [1 - j2\omega - e^{-j2\omega}]$

**Q.58** What is value of Fourier transform of a signal  $x(t)$  shown in figure below at a frequency of  $\pi$  rad/sec?



- (A)  $-\frac{4}{\pi^2}$       (B) 0  
 (C)  $-\frac{1}{\pi^2}$       (D)  $\frac{4}{\pi}$

**Q.59** The continuous time Fourier transform for the signal  $x(t)$  shown in figure is



- (A)  $2[\cos 2\omega - \cos \omega]$       (B)  $2[\sin 2\omega + \cos \omega]$   
 (C)  $\frac{2}{\omega^2}[\cos \omega - \cos 2\omega]$       (D)  $\frac{2}{\omega^2}[\sin \omega + \sin 2\omega]$

**Q.60** Find the Fourier transform of the function

$$g(t) = \begin{cases} 1 & , \quad |t| < 1 \\ 2 - |t|, & 1 < |t| < 2 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

- (A)  $\text{sinc}^2(f) + 4\text{sinc}^2(2f)$   
 (B)  $3\text{sinc}(3f)\text{sinc}(f)$   
 (C)  $-\text{sinc}^2(f) + \text{sinc}^2(2f)$   
 (D)  $\text{sinc}(3f)\text{sinc}(f)$

**Q.61** The inverse Fourier transform of  $\{e^{-2\omega}u(\omega)\}$  is

- (A)  $\frac{1}{\pi(2+jt)}$       (B)  $\frac{1}{2\pi(2+jt)}$   
 (C)  $\frac{1}{\pi(2-jt)}$       (D)  $\frac{1}{2\pi(2-jt)}$

**Q.62** Fourier transform of signal  $\frac{j}{\pi t}$  is given by

- (A)  $\text{sgn}(f)$       (B)  $-\text{sgn}(f)$   
 (C)  $j\text{sgn}(f)$       (D)  $-j\text{sgn}(f)$

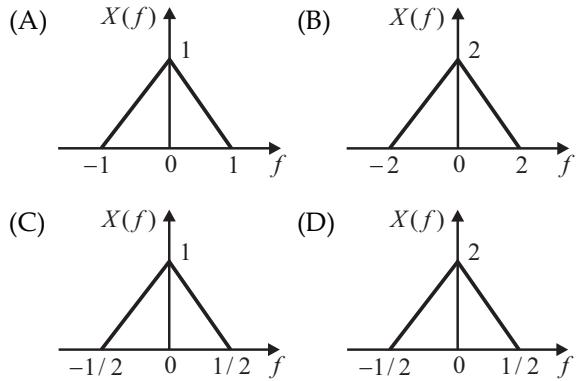
**Q.63** Fourier transform of  $\text{sinc}(2Wt)$  is

- (A)  $2W\text{rect}\left(\frac{f}{2W}\right)$       (B)  $2W\text{rect}(2Wf)$   
 (C)  $\frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$       (D)  $\frac{1}{2W}\text{rect}(2Wf)$

**Q.64** Fourier transform of  $x(t) = \frac{\sin t}{\pi t}$  is

- (A)  $\text{rect}(\omega)$       (B)  $2\text{rect}\left(\frac{\omega}{2}\right)$   
 (C)  $\text{rect}\left(\frac{\omega}{2}\right)$       (D)  $\frac{1}{2}\text{rect}\left(\frac{\omega}{2}\right)$

**Q.65** Fourier transform of  $\sin^2(t)$  is given by



**Q.66** Find the inverse Fourier transform of the function

$$X(f) = 2 \text{tri}\left(\frac{f}{2}\right) e^{-j6\pi f}$$

- (A)  $4\text{sinc}^2[2t+3]$   
 (B)  $4\text{sinc}^2[2(t+3)]$   
 (C)  $4\text{sinc}^2[2t-3]$   
 (D)  $4\text{sinc}^2[2(t-3)]$

**Q.67** The Fourier transform of a signal  $x(t) = e^{-|t|}$  is  $X(j\omega) = 2 / (\omega^2 + 1)$ . Then the Fourier transform of  $y(t) = \frac{4\cos 2t}{t^2 + 1}$  is

- (A)  $\frac{1}{2}e^{-|\omega|}$       (B)  $2\pi[e^{-|\omega|} + e^{|\omega|}]$   
 (C)  $2\pi[e^{-|\omega-2|} + e^{-|\omega+2|}]$       (D)  $\frac{2\pi\delta(\omega)}{\omega^2 + 1}$

**Q.68** The Fourier transform of  $\frac{4t}{(t^2 + 1)^2}$  is

- (A)  $j.2\pi.e^{-|\omega|}$       (B)  $-j.2\pi.e^{-|\omega|}$   
 (C)  $j.2\pi.\omega.e^{-|\omega|}$       (D)  $-j.2\pi.\omega.e^{-|\omega|}$

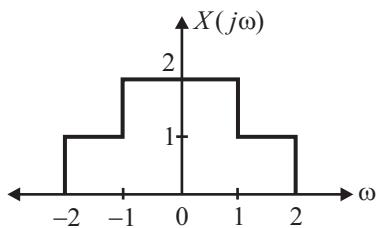
**Q.69** The Fourier transform of  $t\left(\frac{\sin t}{\pi t}\right)^2$  is

- (A)  $\frac{j}{2\pi}[u(\omega+2) - 2u(\omega) + u(\omega-2)]$   
 (B)  $\frac{j}{\pi}[u(\omega+2) - 2u(\omega) + u(\omega-2)]$

(C)  $\frac{j}{2\pi}[u(\omega+2)-u(\omega-2)]$

(D) None of the above

- Q.70** The inverse Fourier transform of  $X(j\omega)$  shown in figure below is



(A)  $\frac{\sin t}{2\pi t}$

(B)  $\frac{\sin 2t}{\pi t}$

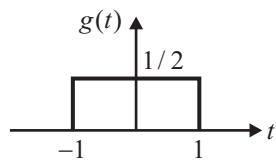
(C)  $\frac{\sin 2t + \sin t}{\pi t}$

(D)  $\frac{\sin 2t + \sin t}{2\pi}$

- Q.71** Inverse Fourier transform of

$$X(\omega) = \left( \frac{\sin \omega}{\omega} \right) \cos \omega$$

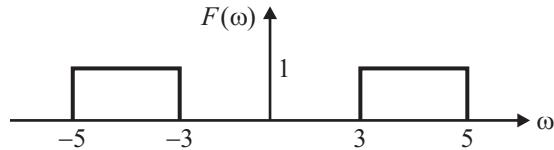
if  $g(t) =$



(A)  $\frac{1}{4}g(t+1) + \frac{1}{4}g(t-1)$  (B)  $\frac{1}{2}g(t+1) + \frac{1}{2}g(t-1)$

(C)  $\frac{1}{4}g(t+1) - \frac{1}{4}g(t-1)$  (D)  $\frac{1}{2}g(t+1) - \frac{1}{2}g(t-1)$

- Q.72** Find the inverse Fourier transform of the spectra depicted in below figure.



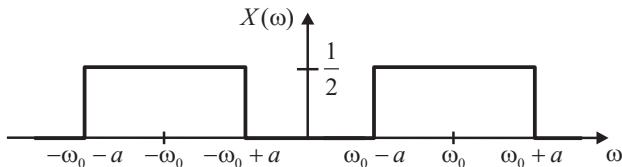
(A)  $\frac{1}{\pi} \text{Sa}(t) \cos 4t$

(B)  $\frac{2}{\pi} \text{Sa}(t) \cos 3t$

(C)  $\frac{2}{\pi} \text{Sa}(t) \cos 4t$

(D)  $\frac{2}{\pi} \text{Sa}(2t) \cos 3t$

- Q.73** The Inverse Fourier transform of the signal given below is



(A)  $\frac{\sin \omega_0 t}{\pi t} \cos \omega_0 t$

(B)  $\frac{\sin \omega_0 t}{\pi t} \cos \omega_0 t$

(C)  $\frac{\sin \omega_0 t}{\pi t} \sin \omega_0 t$

(D)  $\frac{\sin \omega_0 t}{t} \cos \omega_0 t$

- Q.74** The inverse Fourier transform of

$$\frac{4\sin(2\omega-4)}{2\omega-4} - \frac{4\sin(2\omega+4)}{2\omega+4}$$

(A)  $2j \text{rect}\left(\frac{t}{2}\right) \sin(2t)$  (B)  $2j \text{rect}\left(\frac{t}{4}\right) \sin(2t)$

(C)  $2j \text{rect}(2t) \sin\left(\frac{t}{2}\right)$  (D) None of these

- Q.75** The inverse Fourier transform of

$$\frac{d}{d\omega} \left( \frac{4\sin 4\omega \sin 2\omega}{\omega} \right)$$

(A)  $t^2 - 2e^{-t} \text{rect}[2(t-4)]$

(B)  $t^2 + 2e^{-t} \text{rect}[2(t+4)]$

(C)  $-t \left[ \text{rect}\left(\frac{t+4}{4}\right) - \text{rect}\left(\frac{t-4}{4}\right) \right]$

(D)  $t [\text{rect}(2t+8) - \text{rect}(2t-8)]$

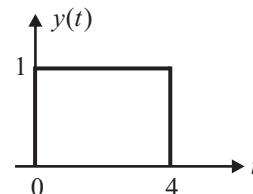
**Statement for Linked Questions 76 to 81**

A Fourier transform pair is as following

$$x(t) \xrightarrow{\text{FT}} X(j\omega) = \frac{2 \sin \omega}{\omega}, \text{ where}$$

$$x(t) = \begin{cases} 1, & |t| < 1 \\ 0, & \text{otherwise} \end{cases}$$

- Q.76** The Fourier transform of signal shown below is



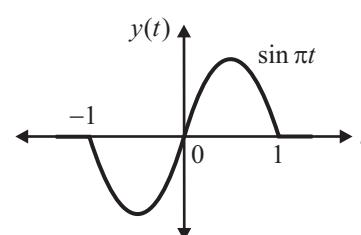
(A)  $\frac{e^{j2\omega} \sin 2\omega}{2\omega}$

(B)  $\frac{2e^{-j2\omega} \sin 2\omega}{\omega}$

(C)  $\frac{4e^{-j2\omega} \sin \frac{\omega}{2}}{\omega}$

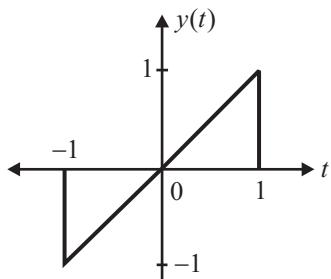
(D)  $\frac{4e^{j2\omega} \sin \frac{\omega}{2}}{\omega}$

- Q.77** The Fourier transform of signal shown below is



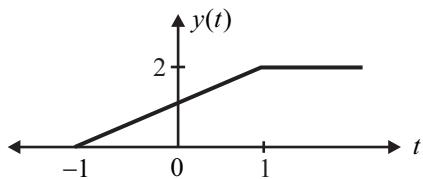
- (A)  $\frac{\sin(\omega-\pi)}{j(\omega-\pi)} - \frac{\sin(\omega+\pi)}{j(\omega+\pi)}$   
 (B)  $\frac{2\sin(\omega-\pi)}{j(\omega-\pi)} - \frac{2\sin(\omega+\pi)}{j(\omega+\pi)}$   
 (C)  $\frac{\sin(\omega-\pi)}{(\omega-\pi)} - \frac{\sin(\omega+\pi)}{(\omega+\pi)}$   
 (D) None of these

Q.78 The Fourier transform of signal shown below is



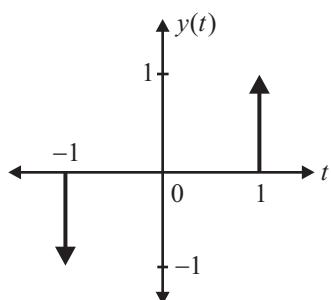
- (A)  $4\pi j \left( \frac{\cos \omega}{\omega} - \frac{\sin \omega}{\omega} \right)$  (B)  $2j \left( \frac{\cos \omega}{\omega} - \frac{\sin \omega}{\omega^2} \right)$   
 (C)  $4\pi j \left( \frac{\cos \omega}{\omega^2} - \frac{\sin \omega}{\omega} \right)$  (D)  $2j \left( \frac{\cos \omega}{\omega^2} - \frac{\sin \omega}{\omega} \right)$

Q.79 The Fourier transform of signal shown below is



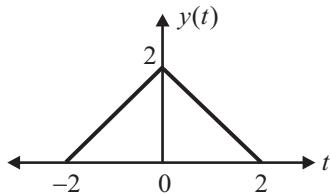
- (A)  $\frac{\sin \omega}{j\pi\omega} + \pi\delta(\omega)$  (B)  $\frac{2\sin \omega}{j\pi\omega} + 2\pi\delta(\omega)$   
 (C)  $\frac{\sin \omega}{\pi\omega^2} + \pi\delta(\omega)$  (D)  $\frac{2\sin \omega}{j\omega^2} + 2\pi\delta(\omega)$

Q.80 The Fourier transform of signal shown below is



- (A)  $4\pi j \sin \omega$  (B)  $2\pi j \sin \omega$   
 (C)  $\frac{j \sin^2 \omega}{\omega^2}$  (D)  $-2j \sin \omega$

Q.81 The Fourier transform of signal shown below is



- (A)  $\frac{4\sin^2 \omega}{\omega^2}$  (B)  $\frac{8\pi \sin^2 \omega}{\omega^2}$

- (C)  $\frac{2\sin^2 \omega}{\pi\omega^2}$  (D) None of these

Q.82 For a function  $g(t)$ , it is given that

$$\int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt = \omega e^{-2\omega^2}$$
 for any real value  $\omega$ . If

$$y(t) = \int_{-\infty}^t g(\tau) d\tau, \text{ then } \int_{-\infty}^{\infty} y(\tau) d\tau \text{ is}$$

[GATE EC 2014-Kharagpur]

- (A) 0 (B)  $-j$   
 (C)  $-\frac{j}{2}$  (D)  $\frac{j}{2}$

Q.83 The Fourier transform of signal  $h(t)$  is

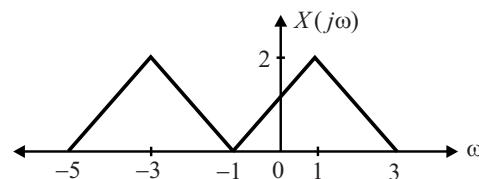
$$H(j\omega) = 2 \cos \omega \cdot \frac{\sin 2\omega}{\omega}$$
. The value of  $h(0)$  is

[GATE EC, EE, IN 2012-Delhi]

- (A)  $1/4$  (B)  $1/2$   
 (C) 1 (D) 2

**Common Data for Questions 84 & 85**

Consider the signal  $X(j\omega)$  shown below whose inverse Fourier transform is  $x(t)$ .



Q.84 The value of  $\int_{-\infty}^{\infty} x(t) dt$  is

- (A) 1 (B)  $2\pi$   
 (C)  $1/2\pi$  (D)  $\infty$

Q.85 The value of  $x(0)$  is

- (A) 0 (B) 8  
 (C) 4 (D)  $4/\pi$

Q.86 If  $F[x(t)] = X(f) = \begin{cases} 1 - \frac{|f|}{B} & |f| \leq B \\ 0 & \text{elsewhere} \end{cases}$

Then value of  $x(0)$  is

- (A) 1 (B) B  
 (C)  $1/2$  (D)  $B/2$

Q.87 A signal  $x(t)$  has Fourier transform  $X(j\omega) =$

$$\frac{j\omega}{(5 + j\omega/10)}. \text{ If } y(t) = \int_{-\infty}^t x(\tau) d\tau, \text{ then the total area under } y(t) \text{ is}$$

- (A) 1/5                                 (B) 10  
 (C) 0                                     (D) 2

**Q.88** If the signal  $x(t) = \frac{\sin(t)}{\pi t} * \frac{\sin(t)}{\pi t}$  with  $*$  denoting the convolution operation, then  $x(t)$  is equal to

[GATE EC 2016-Bangalore]

- (A)  $\frac{\sin(t)}{\pi t}$                              (B)  $\frac{\sin(2t)}{2\pi t}$   
 (C)  $\frac{2\sin(2t)}{2\pi t}$                              (D)  $\left(\frac{\sin(t)}{\pi t}\right)^2$

**Q.89** A real-valued signal  $x(t)$  limited to the frequency band  $|f| \leq \frac{W}{2}$  is passed through a linear time invariant system whose frequency response is

[GATE EC 2014-Kharagpur]

$$H(f) = \begin{cases} e^{-j4\pi f}, & |f| \leq \frac{W}{2} \\ 0, & |f| > \frac{W}{2} \end{cases}$$

The output of the system is

- (A)  $x(t+4)$                              (B)  $x(t-4)$   
 (C)  $x(t+2)$                              (D)  $x(t-2)$

**Q.90** A signal is represented by

$$x(t) = \begin{cases} 1 & |t| < 1 \\ 0 & |t| > 1 \end{cases}$$

The Fourier transform of the convolved signal  $y(t) = x(2t) * x(t/2)$  is

[GATE EE 2014-Kharagpur]

- (A)  $\frac{4}{\omega^2} \sin\left(\frac{\omega}{2}\right) \sin(2\omega)$       (B)  $\frac{4}{\omega^2} \sin\left(\frac{\omega}{2}\right)$   
 (C)  $\frac{4}{\omega^2} \sin(2\omega)$                              (D)  $\frac{4}{\omega^2} \sin^2 \omega$

**Q.91** A linear time invariant causal system has a frequency response given in polar form as  $\frac{1}{\sqrt{1+\omega^2}} \angle -\tan^{-1} \omega$ . For input  $x(t) = \sin t$ , the output is

[GATE IN 2009-Roorkee]

- (A)  $\frac{1}{\sqrt{2}} \cos t$                              (B)  $\frac{1}{\sqrt{2}} \cos\left(t - \frac{\pi}{4}\right)$   
 (C)  $\frac{1}{\sqrt{2}} \sin t$                                      (D)  $\frac{1}{\sqrt{2}} \sin\left(t - \frac{\pi}{4}\right)$

**Q.92** A signal  $x(t) = \text{sinc}(\alpha t)$  where  $\alpha$  is a real constant  $\left[\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}\right]$  is the input to a Linear Time Invariant system whose impulse response  $h(t) = \text{sinc}(\beta t)$  where  $\beta$  is a real constant. If  $\min(\alpha, \beta)$  denotes the minimum of  $\alpha$  and  $\beta$  and similarly  $\max(\alpha, \beta)$  denotes the maximum of  $\alpha$  and  $\beta$ , and  $K$  is a constant, which one of the following statements is true about the output of the system?

[GATE EE 2008-Bangalore]

- (A) It will be of the form  $K \text{sinc}(\gamma t)$  where  $\gamma = \min(\alpha, \beta)$   
 (B) It will be of the form  $K \text{sinc}(\gamma t)$  where  $\gamma = \max(\alpha, \beta)$   
 (C) It will be of the form  $K \sin c(\alpha t)$   
 (D) It cannot be a sinc type of signal

**Q.93** The frequency response of a linear, time-invariant system is given by  $H(f) = \frac{5}{1 + j10\pi f}$ .

The step response of the system is

[GATE EC 2007-Kanpur]

- (A)  $5[1 - e^{-5t}]u(t)$                              (B)  $5[1 - e^{-t/5}]u(t)$   
 (C)  $\frac{1}{5}[1 - e^{-5t}]u(t)$                              (D)  $\frac{1}{5}[1 - e^{-t/5}]u(t)$

**Q.94** Let a signal  $a_1 \sin(\omega_1 t + \phi_1)$  be applied to a stable linear time invariant system. Let the corresponding steady state output be represented as  $a_2 F(\omega_2 t + \phi_2)$ . Then which of the following statements is true?

[GATE EE 2007-Kanpur]

- (A)  $F$  is not necessarily a 'sine' or 'cosine' function but must be periodic with  $\omega_1 = \omega_2$   
 (B)  $F$  must be a 'sine' or 'cosine' function with  $a_1 = a_2$   
 (C)  $F$  must be a 'sine' function with  $\omega_1 = \omega_2$  and  $\phi_1 = \phi_2$   
 (D)  $F$  must be a 'sine' or 'cosine' function with  $\omega_1 = \omega_2$

**Q.95** The transfer function of a system is given by  $\frac{Y(s)}{X(s)} = \frac{e^{-0.1s}}{1+s}$ . If  $x(t)$  is  $0.5 \sin(t)$ , then the phase angle between the output and the input will be

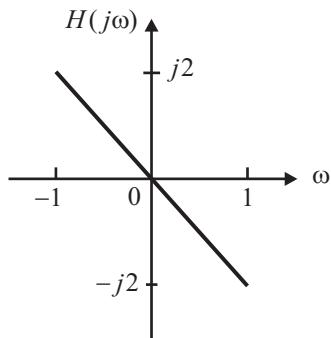
[GATE IN 2003-Madras]

- (A)  $-39.27^\circ$       (B)  $-45^\circ$   
 (C)  $-50.73^\circ$       (D)  $-90^\circ$

**Q.96** Fourier transform of  $g(t) = \text{rect}(4t) \otimes 4\delta(2t)$  is

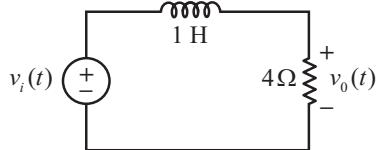
- (A)  $\text{sinc}\left(\frac{f}{4}\right)$       (B)  $\frac{1}{2} \text{sinc}\left(\frac{f}{4}\right)$   
 (C)  $16\text{sinc}(4f)$       (D)  $8\text{sinc}(4f)$

**Q.97** A causal LTI filter has the frequency response  $H(j\omega)$  shown below. For the input signal  $x(t) = e^{-jt}$ , output will be



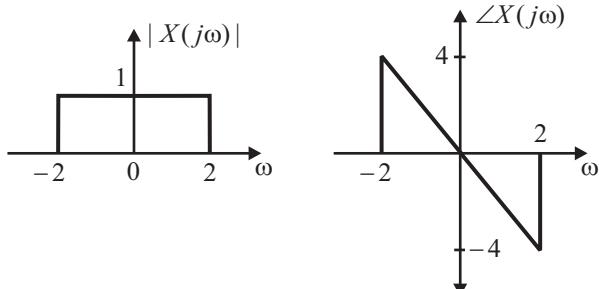
- (A)  $-2je^{-jt}$       (B)  $2je^{-jt}$   
 (C)  $4\pi je^{-jt}$       (D)  $-4\pi je^{-jt}$

**Q.98** Determine  $v_0(t)$  in below figure, if  $v_i(t) = 2 \text{sgn}(t)$ .



- (A)  $4[1-e^{-4t}]u(t)$       (B)  $-2+4[1-e^{-4t}]u(t)$   
 (C)  $2 \text{sgn}(t)-2e^{-4t} \text{sgn}(t)$       (D)  $4[\text{sgn}(t)-e^{-4t} u(t)]$

**Q.99** Inverse Fourier transform of the signals is



- (A)  $\frac{\sin 2(t+2)}{\pi(t-2)}$       (B)  $\frac{\sin 2(t-2)}{\pi(t-2)}$   
 (C)  $\frac{\cos 2(t+2)}{\pi(t-2)}$       (D)  $\frac{\cos 2(t-2)}{\pi(t-2)}$

**Statement for Linked Questions 100 & 101**

Consider a continuous-time signal  $x(t)$  whose magnitude and phase spectra are as follows

$$|X(j\omega)| = 2\{u(\omega+3) - u(\omega-3)\}$$

$$\angle X(j\omega) = \frac{-3\omega}{2} + \pi$$

**Q.100** The time domain signal  $x(t)$  is

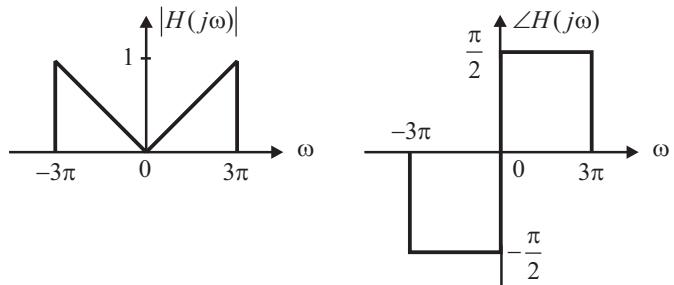
- (A)  $\frac{-6}{\pi} \text{Sa}[3(t-1.5)]$       (B)  $\frac{-6}{\pi} \text{Sa}[(t-1.5)]$   
 (C)  $\frac{-2}{\pi} \text{Sa}[3(t-1.5)]$       (D)  $\frac{-2}{\pi} \text{Sa}[3(t+1.5)]$

**Q.101** The value of  $t$ , for which  $x(t) = 0$ , is

- (A)  $\frac{k\pi}{3} + \frac{3}{2}$ , for  $k = 0, 1, 2, 3, \dots$   
 (B)  $\frac{k\pi}{3} + \frac{3}{2}$ , for  $k = 1, 2, 3, \dots$   
 (C)  $\frac{k\pi}{3} + \frac{2}{3}$ , for  $k = 0, 1, 2, 3, \dots$   
 (D)  $\frac{k\pi}{2} + \frac{2}{3}$ , for  $k = 1, 2, 3, \dots$

**Common Data for Questions 102 & 103**

The frequency response  $H(j\omega)$  of a continuous time filter is shown below.



**Q.102** If the input of this system is  $x(t) = \cos(2\pi t + \theta)$ , then output will be

- (A)  $-\frac{2}{3} \sin(2\pi t + \theta)$       (B)  $\frac{2}{3\pi} \sin(2\pi t + \theta)$   
 (C)  $\frac{2\pi}{3} \sin(2\pi t + \theta)$       (D) None of these

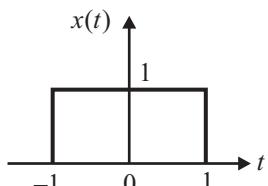
**Q.103** If the input to this system is  $x(t) = \cos(4\pi t + \theta)$ , then output will be

- (A)  $-\frac{4}{3} \cos(4\pi t + \theta)$       (B)  $\frac{4\pi}{3} \cos(4\pi t + \theta)$   
 (C)  $\frac{4}{3\pi} \cos(4\pi t + \theta)$       (D) 0

- Q.104**  $x(t)$  is a positive rectangular pulse from  $t = -1$  to  $t = +1$  with unit height as shown in the figure.

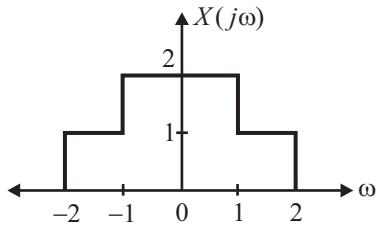
The value of  $\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$  { where  $X(\omega)$  is the Fourier transform of  $x(t)$ } is

[GATE EE 2010-Guwahati]



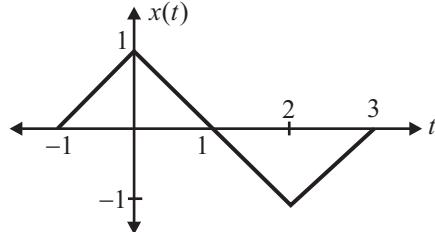
- (A) 2                          (B)  $2\pi$   
 (C) 4                          (D)  $4\pi$

- Q.105** The energy of a signal whose Fourier transform is shown in figure equals to



- (A) 5                          (B)  $10/\pi$   
 (C)  $5/2\pi$                     (D)  $5/\pi$

- Q.106** If  $X(j\omega)$  is the CTFT of a signal  $x(t)$  shown in the figure, then  $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$  equals to



- (A)  $2\pi$                       (B)  $4/3$   
 (C)  $8\pi/3$                     (D)  $3/2\pi$

- Q.107** A signal is given by  $x(t) = \frac{3}{2} \text{rect}\left(\frac{t}{8}\right) \otimes \text{rect}\left(\frac{t}{2}\right)$ .

If  $y(t) = \frac{d}{dt}x(t)$ , then energy of  $y(t)$  is

- (A) 9                          (B) 18  
 (C) 36                        (D) None of these

- Q.108** Find the percentage of the total energy of the signal  $x(t) = 10e^{-at}u(t)$  ( $a$  is positive), contained in frequencies  $|f| < a/2\pi$ .

- (A) 0.5%                    (B) 5%  
 (C) 50%                    (D) 75%

- Q.109** A real and non negative signal  $x(t)$  has Fourier transform  $X(j\omega)$ . The following facts are given :

1.  $F^{-1}\{(1+j\omega)X(j\omega)\} = Ae^{-2t}u(t)$ , where  $A$  is constant

$$2. \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi$$

The signal  $x(t)$  may be

- (A)  $\sqrt{12}[e^{-2t} - e^{-t}]u(t)$     (B)  $\sqrt{12}[e^{-t} - e^{-2t}]u(t)$   
 (C)  $\sqrt{3}[e^{-2t} - e^{-t}]u(t)$     (D)  $\sqrt{3}[e^{-t} - e^{-2t}]u(t)$

- Q.110** The value of  $\int_{-\infty}^{\infty} \frac{8}{(\omega^2 + 4)^2} d\omega$  is

- (A)  $\pi/2$                     (B)  $\pi$   
 (C)  $2\pi$                     (D) None of these

- Q.111** The energy of the signal  $\frac{\sin at}{\pi t}$  is

- (C)  $2a$                     (B)  $a/\pi$   
 (C)  $a\pi$                     (D)  $1/a\pi$

- Q.112** The energy of the signal  $8\sin c(4t)\cos(2\pi t)$  is

- (A) 6                          (B) 8  
 (C) 10                        (D) 12

- Q.113** The energy of the signal  $t\left(\frac{\sin t}{\pi t}\right)^2$  is

- (A)  $\frac{1}{8\pi^3}$                 (B)  $\frac{1}{4\pi^3}$   
 (C)  $\frac{1}{2\pi^3}$                 (D) None of these

- Q.114** Consider signal  $x(t) = \frac{2}{\pi} \text{Sa}(2t)$  whose energy is

$E_x$

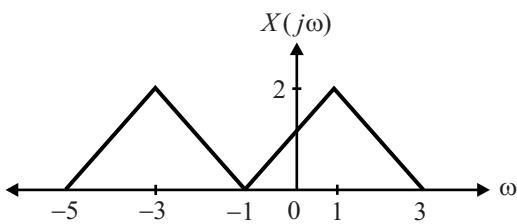
$$\frac{1}{2\pi} \int_{-\omega_p}^{\omega_p} |X(\omega)|^2 d\omega = 0.85E_x$$

Determine the value of  $\omega_p$ .

- (A) 0.9 rad/sec            (B) 1.8 rad/sec  
 (C) 0.85 rad/sec        (D) 1.7 rad/sec

**Common Data for Questions 115 & 116**

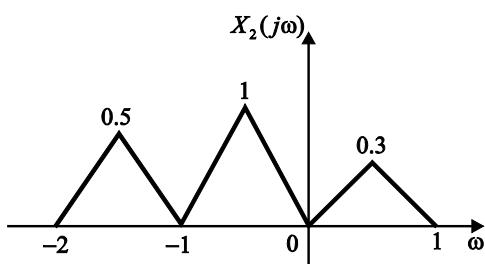
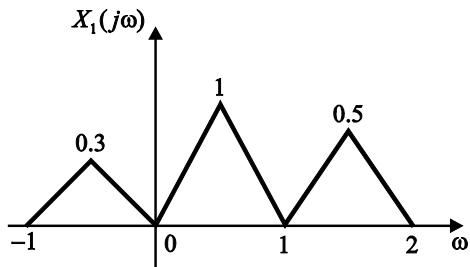
Consider the signal  $X(j\omega)$  shown below whose inverse Fourier transform is  $x(t)$ .



- Q.115** The value of  $\int_{-\infty}^{\infty} |x(t)|^2 dt$  is  
 (A)  $16/3$       (B)  $32/3$   
 (C)  $16/3\pi$       (D)  $32/3\pi$

- Q.116** The value of  $\int_{-\infty}^{\infty} x(t) e^{j3t} dt$  is  
 (A) 2      (B) 1  
 (C) 0      (D) 3

- Q.117** Suppose  $x_1(t)$  and  $x_2(t)$  have the Fourier transforms as shown below.



Which one of the following statements is TRUE?

[GATE EE 2016-Bangalore]

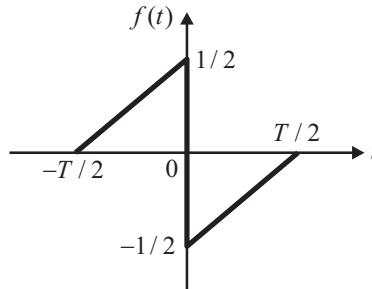
- (A)  $x_1(t)$  and  $x_2(t)$  are complex and  $x_1(t)$   $x_2(t)$  is also complex with nonzero imaginary part  
 (B)  $x_1(t)$  and  $x_2(t)$  are real and  $x_1(t)$   $x_2(t)$  is also real  
 (C)  $x_1(t)$  and  $x_2(t)$  are complex but  $x_1(t)$   $x_2(t)$  is real  
 (D)  $x_1(t)$  and  $x_2(t)$  are imaginary but  $x_1(t)$   $x_2(t)$  is real

- Q.118** A differentiable non constant even function  $x(t)$  has a derivative  $y(t)$ , and their respective Fourier Transforms are  $X(\omega)$  and  $Y(\omega)$ . Which of the following statements is TRUE?

[GATE EE 2014-Kharagpur]

- (A)  $X(\omega)$  and  $Y(\omega)$  are both real  
 (B)  $X(\omega)$  is real and  $Y(\omega)$  is imaginary  
 (C)  $X(\omega)$  and  $Y(\omega)$  are both imaginary  
 (D)  $X(\omega)$  is imaginary and  $Y(\omega)$  is real

- Q.119** A function  $f(t)$  is shown in the figure.



The Fourier transform  $F(\omega)$  of  $f(t)$  is

[GATE EE 2014-Kharagpur]

- (A) real and even function of  $\omega$   
 (B) real and odd function of  $\omega$   
 (C) imaginary and odd function of  $\omega$   
 (D) imaginary and even function of  $\omega$

- Q.120** A real function  $f(t)$  has a Fourier transform  $F(\omega)$ . The Fourier transform of  $[f(t) - f(-t)]$  is

[GATE IN 2003-Madras]

- (A) zero      (B) real  
 (C) real and odd      (D) imaginary

- Q.121** If  $f(t)$  is an even function, then what is its Fourier transform  $F(j\omega)$ ? [ESE EC 2008]

- (A)  $\int_0^{\infty} f(t) \cos(2\omega t) dt$       (B)  $2 \int_0^{\infty} f(t) \cos(\omega t) dt$   
 (C)  $2 \int_0^{\infty} f(t) \sin(\omega t) dt$       (D)  $\int_0^{\infty} f(t) \sin(2\omega t) dt$

- Q.122** A real signal  $x(t)$  has Fourier transform  $X(f)$  which one of the following is correct?

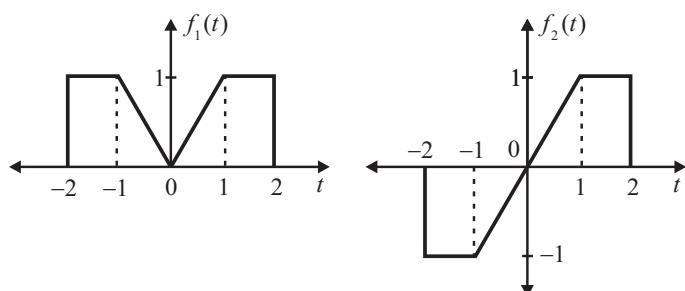
[ESE EC 2007]

- (A) Magnitude of  $X(f)$  has even symmetry while phase of  $X(f)$  has odd symmetry  
 (B) Magnitude of  $X(f)$  has odd symmetry while phase of  $X(f)$  has odd symmetry  
 (C) Both magnitude and phase of  $X(f)$  have even symmetry  
 (D) Both magnitude and phase of  $X(f)$  have odd symmetry

**Q.123** Let  $x_1(t)$  and  $x_2(t)$  be two CT signals whose Fourier transforms are  $X_1(j\omega)$  and  $X_2(j\omega)$  respectively. If  $x_1(t) = e^{-t-1}u(t)$  and  $x_2(t) = e^{-t-j\pi t}u(t)$  then which of the following relations is true?

- (A)  $X_1(-j\omega) = -X_1(j\omega)$  (B)  $X_2(-j\omega) = -X_2(j\omega)$   
 (C)  $X_1(-j\omega) = X_1^*(j\omega)$  (D)  $X_2(-j\omega) = X_2^*(j\omega)$

**Q.124** Consider the following statements regarding CTFT of two functions  $f_1(t)$  and  $f_2(t)$  shown in the figure.

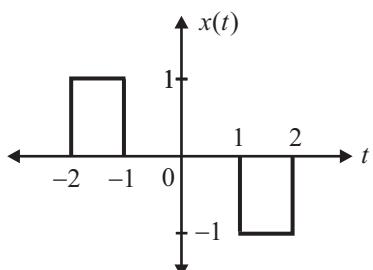


1. CTFT of  $f_1(t)$  is real and even.
2. CTFT of  $f_2(t)$  is real and odd.
3. CTFT of  $f_1(t)$  is imaginary and even.
4. CTFT of  $f_2(t)$  is imaginary and odd.

Which of the above statements are true?

- (A) 1 and 2      (B) 1 and 4  
 (C) 2 and 3      (D) 3 and 4

**Q.125** Let  $X(j\omega)$  be the Fourier transform of a signal  $x(t)$  shown in the figure below.



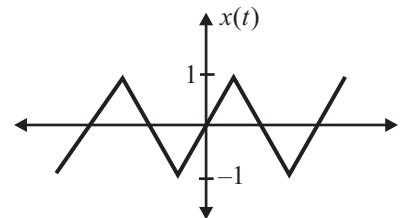
Which of the following conditions is not true for  $X(j\omega)$ ?

- (A)  $\operatorname{Re}\{X(j\omega)\} = 0$   
 (B)  $\int_{-\infty}^{\infty} X(j\omega) d\omega = 0$   
 (C)  $\int_{-\infty}^{\infty} \omega X(j\omega) d\omega = 0$   
 (D) There exists a real  $\alpha$  such that  $e^{j\alpha\omega} X(j\omega)$  is real

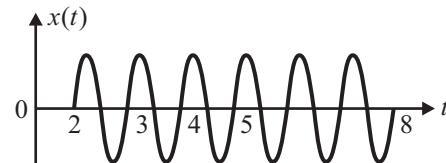
### Statement for Linked Questions 126 to 129

Consider the real signals shown below.

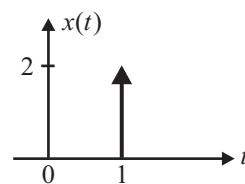
(1)



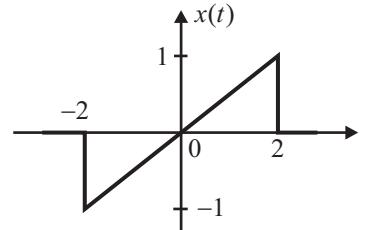
(2)



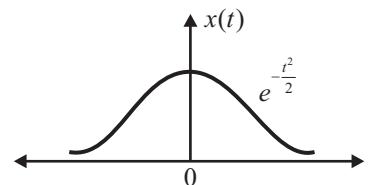
(3)



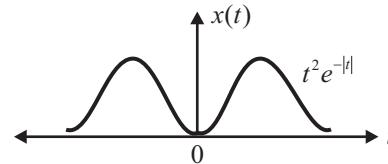
(4)



(5)



(6)



**Q.126** The signals that satisfy  $\operatorname{Re}\{X(j\omega)\} = 0$  are

- (A) 1 and 4      (B) 5 and 6  
 (C) 2 and 3      (D) 2 and 4

**Q.127** The signals that satisfy  $\operatorname{Im}\{X(j\omega)\} = 0$  are

- (A) 1 and 4      (B) 5 and 6  
 (C) 2 and 3      (D) 2 and 4

**Q.128** The signals that satisfy  $\int_{-\infty}^{\infty} X(j\omega) d\omega = 0$  are

- (A) Only 5      (B) 1, 2, 3, 4 and 6  
 (C) 2 and 3      (D) 1, 4, 5 and 6

- Q.129** The signals that satisfy  $\int_{-\infty}^{\infty} \omega X(j\omega) d\omega = 0$  are  
 (A) 1, 2, 3, 4 and 6      (B) Only 5  
 (C) 2, 3, 5 and 6      (D) 2 and 3

**Practice (objective & Num Ans) Questions :**

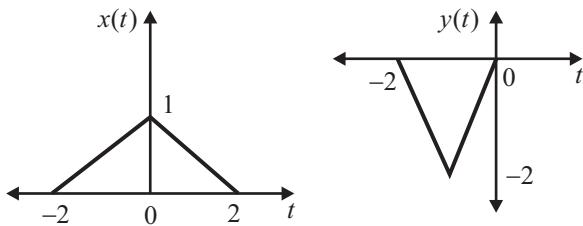
- Q.1** Let the signal  $x(t)$  have the Fourier transform

$X(\omega)$  Consider the signal  $\frac{d}{dt}[x(t-t_d)]$  where  $t_d$

is an arbitrary delay. The magnitude of the Fourier transform of  $y(t)$  is given by the expression : **[GATE IN 2007-Kanpur]**

- (A)  $|X(\omega)|.\omega$       (B)  $|X(\omega)|.\omega$   
 (C)  $\omega^2.|X(\omega)|$       (D)  $|\omega^2|.|X(\omega)|.e^{-j\omega t_d}$

- Q.2** Let  $x(t)$  and  $y(t)$  with Fourier transform  $X(f)$  and  $Y(f)$  respectively be related as shown in figure. Then  $Y(f)$  is **[GATE EC 2004-Delhi]**



- (A)  $-\frac{1}{2} X\left(\frac{f}{2}\right) e^{-j2\pi f}$       (B)  $-\frac{1}{2} X\left(\frac{f}{2}\right) e^{j2\pi f}$   
 (C)  $-X\left(\frac{f}{2}\right) e^{j2\pi f}$       (D)  $-X\left(\frac{f}{2}\right) e^{-j2\pi f}$

- Q.3** The Fourier transform of a function  $x(t)$  is  $X(f)$ . The Fourier transform of  $\frac{dx(t)}{dt}$  will be **[GATE EC 1998-Delhi]**

- (A)  $\frac{dX(f)}{dt}$       (B)  $j2\pi f X(f)$   
 (C)  $jf X(f)$       (D)  $\frac{X(f)}{jf}$

- Q.4** The amplitude spectrum of Gaussian pulse is **[GATE EC 1998-Delhi]**

- (A) Uniform      (B) A sine function  
 (C) Gaussian      (D) An impulse function

- Q.5** If the Fourier Transform of a deterministic signal  $g(t)$  is  $G(f)$ , then **[GATE EC 1997-Madras]**

**Item - 1**

- (1) The Fourier Transform of  $g(t-2)$  is  
 (2) The Fourier Transform  $g(t/2)$  is

**Item - 2**

- (A)  $G(f)e^{-j(4\pi f)}$   
 (B)  $G(2f)$   
 (C)  $2G(2f)$   
 (D)  $G(f-2)$

Match each of the items 1, 2 on the left with the most appropriate item A, B, C or D on the right.

- Q.6** Consider the following transforms :

1. Fourier transform      **[ESE EC 2016]**  
 2. Laplace transform

Which of the above transforms is/are used in signal processing?

- (A) 1 only      (B) 2 only  
 (C) Both 1 and 2      (D) Neither 1 nor 2

- Q.7** The Fourier transform of a rectangular pulse for a period  $t = -\frac{T}{2}$  to  $t = \frac{T}{2}$  is **[ESE EC 2014]**

- (A) a sinc function  
 (B) a sine function  
 (C) a cosine function  
 (D) a sine-squared function

- Q.8** A unit impulse function  $\delta(t)$  is defined by

1.  $\delta(t) = 0$  for all  $t$  except  $t = 0$

2.  $\int_{-\infty}^{\infty} \delta(t) dt = 1$  **[ESE EC 2013]**

The Fourier transform  $F(\omega)$  of  $\delta(t)$  is

- (A) 1      (B)  $\frac{1}{\omega}$   
 (C) 0      (D)  $\frac{1}{j\omega}$

- Q.9** Which one of the following is a Dirichlet condition?

- (A)  $\int_{t_1}^{\infty} |x(t)| dt < \infty$  **[ESE EC 2010]**  
 (B) Signal  $x(t)$  must have a finite number of maxima and minima in the expansion interval

- (C)  $x(t)$  can have an infinite number of finite discontinuities in the expansion interval.  
(D)  $x^2(t)$  must be absolutely summable
- Q.10** Consider the following statements regarding the use of Laplace transforms and Fourier transforms in circuit analysis : [ESE EC 2010]
1. Both make the solution of circuit problems simple and easy.
  2. Both are applicable for the study of circuit behavior for  $t - \alpha$  to  $\alpha$ .
  3. Both convert differential equations to algebraic equations.
  4. Both can be used for transient and steady state analysis.
- Which of the above statements are correct?
- (A) 1, 2, 3 and 4      (B) 2, 3 and 4 only  
(C) 1, 2 and 4 only      (D) 1, 3 and 4 only
- Q.11** For distortionless transmission through LTI system phase of  $H(\omega)$  is [ESE EC 2010]
- (A) constant  
(B) one  
(C) zero  
(D) linearly dependent on  $\omega$
- Q.12** Which one of the following is the correct relation?
- (A)  $F(at) \xleftarrow{FT} aF(\omega/a)$  [ESE EC 2008]  
(B)  $F(at) \xleftarrow{FT} aF(\omega a)$   
(C)  $F(t/a) \xleftarrow{FT} aF(\omega/a)$   
(D)  $F(at) \xleftarrow{FT} (1/a) F(\omega/a)$
- Q.13** Match the List-I (CT function) with List-II (CT Fourier Transform) and select the correct answer using the code given below the lists :
- | List-I<br>(CT Function)   | List-II<br>(CT Fourier Transform) | [ESE EC 2006] |
|---|-----------------------------------|---------------|
| A. $e^{-t}u(t)$   | 1. $\frac{1}{1+\omega^2}$         |               |
| B. $x(t) = \begin{cases} 1, &  t  \leq 1 \\ 0, &  t  > 1 \end{cases}$ | 2. $j\omega X(j\omega)$           |               |
| C. $\frac{dx(t)}{dt}$   | 3. $\frac{1}{1+j\omega}$          |               |
| D. $\frac{e^{- t }}{2}$   | 4. $\frac{2\sin\omega}{\omega}$   |               |

- Codes :** A B C D
- (A) 1 4 2 3  
(B) 3 2 4 1  
(C) 1 2 4 3  
(D) 3 4 2 1
- Q.14** The inverse Fourier transform of  $\delta(f)$  is
- (A)  $u(t)$       (B) 1 [ESE EC 2003]  
(C)  $\delta(t)$       (D)  $e^{j2\pi t}$
- Q.15** Match List-I (Functions) with List-II (Fourier Transforms) and select the correct answer using the codes given below the lists : [ESE EC 2002]
- | List-I<br>(Functions)                  | List-II<br>(Fourier transform)                 |
|--|--|
| A. $\exp(-\alpha t)u(t), \alpha > 0$   | 1. $\frac{1}{(\alpha + j2\pi f)^2}$            |
| B. $\exp(-\alpha  t ), \alpha > 0$     | 2. $\frac{1}{\alpha + j2\pi f}$                |
| C. $t \exp(-\alpha t)u(t), \alpha > 0$ | 3. $\delta\left(f - \frac{\alpha}{t_0}\right)$ |
| D. $\exp(j2\pi\alpha t/t_0)$           | 4. $\frac{2\alpha}{\alpha^2 + (2\pi f)^2}$     |
- Codes :** A B C D
- (A) 3 1 4 2  
(B) 2 4 1 3  
(C) 3 4 1 2  
(D) 2 1 4 3
- Q.16** Match List-I (Fourier transform) with List-II (functions or time) and select the correct answer using the codes given below the lists :
- | List-I                           | [ESE EC 1999]        |
|----------------------------------|----------------------|
| A. $\frac{\sin k\omega}{\omega}$ | B. $e^{-j\omega d}$  |
| C. $\frac{1}{(j\omega + 2)^2}$   | D. $k\delta(\omega)$ |
- List-II**
1. A constant
  2. Exponential function
  3.  $t$ -multiplied exponential function
  4. Rectangular pulse
  5. Impulse function

**Codes :** A B C D

- (A) 4 5 3 1
- (B) 4 5 3 2
- (C) 3 4 2 1
- (D) 3 4 2 5

**Q.17** Given that the Fourier transform of  $f(t)$  is  $F(j\omega)$ , which of the following pairs of functions of time and the corresponding Fourier transforms are correctly matched? [IEC EC 1998]

1.  $f(t+2) \rightarrow e^{j2\omega} F(j\omega)$
2.  $f(-0.5t) \rightarrow 2F(-2j\omega)$
3.  $\int_{-\infty}^t f(t) dt \rightarrow F(j\omega) \left[ \frac{1}{j\omega} + \pi\delta(\omega) \right]$

Select the correct answer using the codes given below :

- (A) 1 and 2
- (B) 1 and 3
- (C) 2 and 3
- (D) 1, 2 and 3

**Q.18** If  $g(t) \leftrightarrow G(f)$  represents a Fourier transform pair, then according to the duality property of Fourier transform [IEC EC 1997]

- (A)  $G(t) \leftrightarrow g(f)$
- (B)  $G(t) \leftrightarrow g^*(f)$
- (C)  $G(t) \leftrightarrow g(-f)$
- (D)  $G(t) \leftrightarrow g^*(-f)$

**Q.19** The Fourier transform of  $v(t) = \cos \omega_0 t$  is given by

- (A)  $V(f) = \frac{1}{2}\delta(f - f_0)$  [ESE EC 1997]
- (B)  $V(f) = \frac{1}{2}\delta(f + f_0)$
- (C)  $V(f) = \frac{1}{2}[\delta(f - f_0) - \delta(f + f_0)]$
- (D)  $V(f) = \frac{1}{2}[\delta(f - f_0) + \delta(f + f_0)]$

**Q.20** Which one of the following is the correct Fourier transform of the unit step signal?

$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

[ESE EC 1997]

- (A)  $\pi\delta(\omega)$
- (B)  $\frac{1}{j\omega}$
- (C)  $\frac{1}{j\omega} + \pi\delta(\omega)$
- (D)  $\frac{1}{j\omega} + 2\pi\delta(\omega)$

**Q.21** The inverse Fourier transform of the function

$$F(\omega) = \frac{1}{j\omega} + \pi\delta(\omega) \text{ is}$$

[ESE EC 1995]

- (A)  $\sin \omega t$
- (B)  $\cos \omega t$
- (C)  $\operatorname{sgn}(t)$
- (D)  $u(t)$

**Q.22** For which of the following signals Fourier transform does not exist?

- (A)  $e^{-at}u(t), a > 0$
- (B)  $e^{-|a|t}, a > 0$
- (C)  $e^{at}u(t), a > 0$
- (D)  $e^{at}u(-t), a > 0$

**Q.23** Fourier Transform of  $e^{-j\omega_0 t}$  is

- (A)  $2\pi\delta(\omega + \omega_0)$
- (B)  $2\pi\delta(\omega - \omega_0)$
- (C)  $\delta(\omega + \omega_0)$
- (D)  $\delta(\omega - \omega_0)$

**Q.24** Fourier transform of  $f(t)$ , if

$$f(t) = \frac{1}{2} \left[ \delta(t+1) + \delta\left(t + \frac{1}{2}\right) + \delta\left(t - \frac{1}{2}\right) + \delta(t-1) \right]$$

- (A)  $\cos \omega + \cos(\omega/2)$
- (B)  $\sin \omega + \sin(\omega/2)$
- (C)  $\cos \omega - \cos(\omega/2)$
- (D)  $\sin \omega - \sin(\omega/2)$

**Q.25** Which of the following corresponds to the CTFT of signal  $x(t) = 2e^{-t}u(t) - 3e^{-3t}u(t)$ ?

- (A)  $\frac{3}{(1+j\omega)(3+j\omega)}$
- (B)  $\frac{6}{(1+j\omega)(3+j\omega)}$
- (C)  $\frac{(3-j\omega)}{(1+j\omega)(3+j\omega)}$
- (D)  $\frac{(5+j\omega)}{(1+j\omega)(3+j\omega)}$

**Q.26** If  $x_1(t) = e^{-t}u(t)$  then the transform of  $x_1(t) - x_1(-t)$  is

- (A)  $\frac{-2j\omega}{1+\omega^2}$
- (B)  $\frac{2}{1+\omega^2}$
- (C)  $\frac{j\omega}{1+\omega^2}$
- (D)  $\frac{1}{1+\omega^2}$

**Q.27** If for a certain  $x(t)$ , the Fourier transform is

$$X(\omega) = \frac{j\omega^2}{\omega^2 + 1}$$

then for  $\frac{1}{2}x(2t)$  the FT will be :

- (A)  $\frac{j\omega^2}{4(\omega^2 + 4)}$
- (B)  $\frac{j\omega^2}{8(\omega^2 + 4)}$
- (C)  $\frac{j\omega^2}{16(\omega^2 + 4)}$
- (D) same as  $X(\omega)$

**Q.28** If  $x(t) \xrightarrow{\text{FT}} X(j\omega)$  be a CTFT pair, then the Fourier transform of signal  $(t+1)x(t+1)$  will be

- (A)  $e^{j\omega}X(j\omega)$
- (B)  $je^{j\omega} \frac{dX(j\omega)}{d\omega}$
- (C)  $j \frac{dX(j\omega)}{d\omega}$
- (D)  $je^{j\omega}X(j\omega)$

- Q.29** Let  $x(t) \xrightarrow{FT} X(j\omega)$  be a Fourier transform pair then, what will be Fourier transform of the signal  $\frac{d^2}{dt^2}[x(t-2)]$ ?

- (A)  $\frac{X(j\omega/2)}{-\omega^2}$       (B)  $-\omega^2 X(j\omega/2)$   
 (C)  $\frac{X(j\omega)e^{j2\omega}}{-\omega^2}$       (D)  $-\omega^2 e^{-j2\omega} X(j\omega)$

- Q.30** The Fourier transform of  $\sin(2\pi t)e^{-t}u(t)$  is

- (A)  $\frac{1}{2} \left[ \frac{1}{1+j(\omega-2\pi)} - \frac{1}{1+j(\omega+2\pi)} \right]$   
 (B)  $\frac{1}{2j} \left[ \frac{1}{1+j(\omega-2\pi)} - \frac{1}{1+j(\omega+2\pi)} \right]$   
 (C)  $\frac{1}{2j} \left[ \frac{1}{1+j(\omega+2\pi)} - \frac{1}{1+j(\omega-2\pi)} \right]$   
 (D)  $\frac{1}{2} \left[ \frac{1}{1+j(\omega+2\pi)} - \frac{1}{1+j(\omega-2\pi)} \right]$

- Q.31** The inverse Fourier transform of

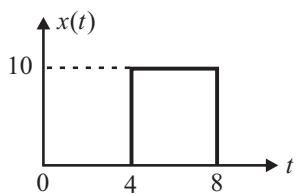
- $\frac{2(j\omega)^2 + 12j\omega + 14}{(j\omega)^2 + 6j\omega + 5}$  is  
 (A)  $2\delta(t) + (e^{-t} - 2e^{-5t})u(t)$   
 (B)  $2t + (e^{-t} - 2e^{-5t})u(t)$   
 (C)  $2\delta(t) + (e^{-t} - e^{-5t})u(t)$   
 (D)  $2t + (e^{-t} - e^{-5t})u(t)$

- Q.32** Fourier Transform of  $t \cos at.u(t)$  is

- (A)  $(\omega^2 + a^2)/(\omega^2 - a^2)^2$   
 (B)  $(\omega^2 + a^2)/(\omega^2 - a^2)$   
 (C)  $-(\omega^2 + a^2)/(\omega^2 - a^2)^2$   
 (D)  $-(\omega^2 + a^2)/(\omega^2 - a^2)$

**Statement for Linked Questions 33 & 34**

Consider the signal  $x(t)$  shown in below figure.



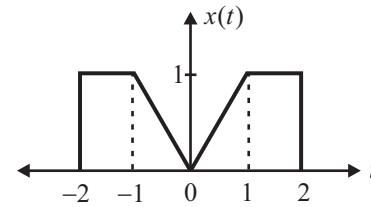
- Q.33** The Fourier transform is

- (A)  $10\text{sinc}(4f)e^{-j8\pi f}$       (B)  $10\text{sinc}(4f)e^{-j4\pi f}$   
 (C)  $40\text{sinc}(4f)e^{-j8\pi f}$       (D)  $40\text{sinc}(4f)e^{-j12\pi f}$

- Q.34** The magnitude spectrum at  $f=0$  is

- (A) 10      (B) 20  
 (C) 40      (D) 0

- Q.35** The Fourier transform of a CT signal shown in figure is



- (A)  $\frac{2}{\omega^2} [\omega \sin 2\omega + \cos \omega + 1]$   
 (B)  $\frac{2}{\omega^2} [\omega \cos 2\omega - \sin \omega + 1]$   
 (C)  $\frac{2}{\omega^2} [\omega \sin \omega + \cos 2\omega - 1]$   
 (D)  $\frac{2}{\omega^2} [\omega \sin 2\omega + \cos \omega - 1]$

- Q.36** Consider a signal  $x(t)$  in the form of a sinc function given by  $A \sin c(2Wt)$ . Mathematical definition of Fourier transform for this sinc pulse is given by

- (A)  $X(f) = A, -W \leq f \leq W$   
 (B)  $X(f) = \frac{A}{W}, -\frac{W}{2} \leq f \leq \frac{W}{2}$   
 (C)  $X(f) = \frac{A}{2W}, -W \leq f \leq W$   
 (D)  $X(f) = \frac{A}{2}, -\frac{W}{2} \leq f \leq \frac{W}{2}$

- Q.37** Inverse Fourier Transform of  $\left(\frac{\sin \omega}{\omega}\right) \cos \omega$  is

- (A)  $\frac{1}{2} \left[ \text{rect}\left(\frac{t}{2} - 1\right) + \text{rect}\left(\frac{t}{2} + 1\right) \right]$   
 (B)  $\frac{1}{4} \left[ \text{rect}\left(\frac{t}{2} - 1\right) + \text{rect}\left(\frac{t}{2} + 1\right) \right]$   
 (C)  $\frac{1}{8} \left[ \text{rect}\left(\frac{t-1}{2}\right) + \text{rect}\left(\frac{t+1}{2}\right) \right]$   
 (D)  $\frac{1}{4} \left[ \text{rect}\left(\frac{t-1}{2}\right) + \text{rect}\left(\frac{t+1}{2}\right) \right]$

- Q.38** Consider the signal [GATE IN 2011-Madras]

$$x(t) = \begin{cases} e^{-t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

- Let  $X(\omega)$  denote the Fourier transform of this signal. The integral  $\frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) d\omega$  is
- (A) 0    (B)  $1/2$   
 (C) 1    (D)  $\infty$
- Q.39** Let  $F(\omega)$  be the Fourier transform of a function  $f(t)$ , then  $F(0)$  is    [ESE EC 1998]
- (A)  $\int_{-\infty}^{\infty} f(t) dt$     (B)  $\int_{-\infty}^{\infty} |f(t)|^2 dt$   
 (C)  $\int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$     (D) None of these
- Q.40** Area of Fourier transform of  $x(t) = 3\text{rect}\left(\frac{t+4}{10}\right)$  is
- (A)  $3/2$     (B) 3  
 (C) 12    (D) 30
- Q.41** The total area under the function  $x(t) = 10\sin c\left[\frac{t+4}{7}\right]$  is
- (A) 10    (B) 40  
 (C) 70    (D) 100
- Statement for Linked Questions 42 & 43**
- The impulse response  $h(t)$  of a linear time-invariant continuous time system is given by  $h(t) = \exp(-2t)u(t)$  where  $u(t)$  denotes the unit step function.    [GATE EC 2008-Bangalore]
- Q.42** The frequency response  $H(\omega)$  of this system in terms of angular frequency  $\omega$ , is given by
- (A)  $\frac{1}{1+j2\omega}$     (B)  $\frac{\sin(\omega)}{\omega}$   
 (C)  $\frac{1}{2+j\omega}$     (D)  $\frac{j\omega}{2+j\omega}$
- Q.43** The output of this system, to the sinusoidal input  $x(t) = 2\cos(2t)$  for all time  $t$ , is
- (A) 0  
 (B)  $2^{-0.25} \cos(2t - 0.125\pi)$   
 (C)  $2^{-0.5} \cos(2t - 0.125\pi)$   
 (D)  $2^{-0.5} \cos(2t - 0.25\pi)$
- Q.44** A signal  $e^{-\omega t} \sin(\omega t)$  is the input to a Linear Time invariant system. Given  $K$  and  $\phi$  are constants, the output of the system will be of the form  $Ke^{-\beta t} \sin(\nu t - \phi)$  where
- [GATE EE 2008-Bangalore]

- (A)  $\beta$  need not be equal to  $\alpha$  but  $\nu$  equal to  $\omega$   
 (B)  $\nu$  need not be equal to  $\omega$  but  $\beta$  equal to  $\alpha$   
 (C)  $\beta$  equal to  $\alpha$  and  $\nu$  equal to  $\omega$   
 (D)  $\beta$  need not be equal to  $\alpha$  and  $\nu$  need not be equal to  $\omega$
- Q.45** Let  $x(t)$  be the input to a linear, time-invariant system. The required output is  $4x(t-2)$ . The transfer function of the system should be
- [GATE EC 2003-Madras]
- (A)  $4e^{j4\pi f}$     (B)  $2e^{-j8\pi f}$   
 (C)  $4e^{-j4\pi f}$     (D)  $2e^{j8\pi f}$
- Q.46** The two inputs to an analogue multiplier are  $x(t)$  and  $y(t)$  with Fourier transforms  $X(f)$  and  $Y(f)$  respectively. The output  $z(t)$  will have a transform  $Z(f)$  given by    [ESE EC 1995]
- (A)  $X(f).Y(f)$     (B)  $X(f)+Y(f)$   
 (C)  $X(f)/Y(f)$     (D)  $\int_{-\infty}^{\infty} X(\lambda)Y(f-\lambda)d\lambda$
- Q.47** The relation between output  $y(t)$  of a causal LTI system and the input  $x(t)$  is given by
- $$\frac{d}{dt}y(t) + 4y(t) = x(t)$$
- The frequency response of this system is
- (A)  $\frac{1}{4-j\omega}$     (B)  $\frac{1}{4+j\omega}$   
 (C)  $\frac{1}{1+j4\omega}$     (D)  $\frac{1}{1-j4\omega}$
- Q.48** Find the CTFT of the convolution of  $10\sin(t)$  with  $2\delta(t+4)$ .
- (A)  $j10[\delta(f-1/2\pi) - \delta(f+1/2\pi)]e^{-j8\pi f}$   
 (B)  $j10[\delta(f+1/2\pi) - \delta(f-1/2\pi)]e^{-j8\pi f}$   
 (C)  $j10[\delta(f-1/2\pi) - \delta(f+1/2\pi)]e^{j8\pi f}$   
 (D)  $j10[\delta(f+1/2\pi) - \delta(f-1/2\pi)]e^{j8\pi f}$
- Q.49** Find the value of  $y_1(t)$  and  $y_2(t)$  if
- $$y_1(t) = \text{rect}(t) \otimes \cos(\pi t)$$
- $$y_2(t) = \text{sinc}(t) \otimes \text{sinc}^2\left(\frac{t}{2}\right)$$
- (A)  $y_1(t) = \frac{2}{\pi} \cos \pi t, \quad y_2(t) = \sin c(t)$   
 (B)  $y_1(t) = 0, \quad y_2(t) = \sin c(t)$

(C)  $y_1(t) = \frac{2}{\pi} \cos \pi t, \quad y_2(t) = \sin c^2 \left( \frac{t}{2} \right)$

(D)  $y_1(t) = 0, \quad y_2(t) = \sin c^2 \left( \frac{t}{2} \right)$

- Q.50** An LTI system is having an impulse response  $h(t) = \frac{\sin 3\pi t}{\pi t}$  for which the input is  $\cos \pi t + \sin 4\pi t$ . Find the steady state output.  
 (A)  $\sin 4\pi t$       (B)  $\cos \pi t$   
 (C)  $\cos \pi t + \sin 4\pi t$       (D) 0

- Q.51** Consider the three LTI systems with impulse response

$$h_1(t) = u(t)$$

$$h_2(t) = -2\delta(t) + 5e^{-2t}u(t)$$

$$h_3(t) = 2te^{-t}u(t)$$

The response to  $x(t) = \cos t$  of above systems are

$$y_1(t) = x(t) \otimes h_1(t)$$

$$y_2(t) = x(t) \otimes h_2(t)$$

$$y_3(t) = x(t) \otimes h_3(t)$$

The same response are

- (A)  $y_1(t), y_2(t)$  and  $y_3(t)$     (B)  $y_1(t)$  and  $y_2(t)$   
 (C)  $y_1(t)$  and  $y_3(t)$     (D)  $y_2(t)$  and  $y_3(t)$

- Q.52** The frequency response of a system is given by

$$H(j\omega) = 1 \text{ for } 2 < |\omega| < 3 \\ = 0 \text{ otherwise}$$

For such a system, which of the following statements are correct?

- (1) Non-invertible      (2) Invertible  
 (3) Causal      (4) Non-causal  
 (A) 1 and 4      (B) 1 and 3  
 (C) 2 and 3      (D) 2 and 4

- Q.53** If  $x(t)$  be a signal with  $X(\omega) = \begin{cases} 1 & |\omega| < 1 \\ 0 & |\omega| > 1 \end{cases}$  and

$$y(t) = \frac{d}{dt}x(t). \text{ The value of } \int_{-\infty}^{\infty} |y(t)|^2 dt \text{ is}$$

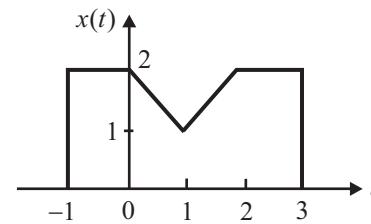
- (A)  $\frac{2}{3\pi}$       (B)  $\frac{1}{3\pi}$   
 (C)  $\frac{1}{\pi}$       (D)  $\pi$

- Q.54** The energy of the signal  $4Sa(2t).\cos(4t)$  is

- (A)  $\pi$       (B)  $2\pi$   
 (C)  $4\pi$       (D)  $8\pi$

**Common Data for Questions 55 to 58**

Consider the signal  $x(t)$  shown below whose Fourier transform is  $X(j\omega)$ .



- Q.55** The value of  $\{X(j0)\}$  is

- (A) 14      (B) 7  
 (C)  $\infty$       (D) 0

- Q.56** The value of  $\int_{-\infty}^{\infty} X(j\omega) d\omega$  is

- (A)  $4\pi$       (B) 2  
 (C)  $2\pi$       (D)  $1/\pi$

- Q.57** The value of  $\int_{-\infty}^{\infty} X(j\omega) \left( \frac{2 \sin \omega}{\omega} \right) e^{j2\omega} d\omega$  is

- (A)  $5\pi$       (B)  $7/2$   
 (C)  $7\pi$       (D) None of these

- Q.58** The value of  $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$  is

- (A)  $19/3\pi$       (B)  $26\pi$   
 (C)  $13/2\pi$       (D) None of these

- Q.59** The Fourier transform of a conjugate symmetric function is always      [GATE EC 2004-Delhi]

- (A) imaginary  
 (B) conjugate anti-symmetric  
 (C) real  
 (D) conjugate symmetric

- Q.60** A signal  $x(t)$  has a Fourier transform  $X(\omega)$ . If  $x(t)$  is a real and odd function of  $t$ , then  $X(\omega)$  is      [GATE EC 1999-Bombay]

- (A) a real and even function of  $\omega$   
 (B) an imaginary and odd function of  $\omega$   
 (C) an imaginary and even function of  $\omega$   
 (D) a real and odd function of  $\omega$

- Q.61** The Fourier transform of a real valued time signal has      [GATE EC 1996-Bangalore]

- (A) Odd symmetry      (B) Even symmetry  
 (C) Conjugate symmetry      (D) No symmetry

**Q.62** If  $G(f)$  represents the Fourier transform of a signal  $g(t)$  which is real and odd symmetric in time, then

- (A)  $G(f)$  is complex [GATE EC 1992-Delhi]
- (B)  $G(f)$  is imaginary
- (C)  $G(f)$  is real
- (D)  $G(f)$  is real and non-negative

**Q.63** If  $f(t)$  is a real and odd function, then its Fourier transform  $F(\omega)$  will be [ESE EC 2013]

- (A) Real and even function of  $\omega$
- (B) Real and odd function of  $\omega$
- (C) Imaginary and odd function of  $\omega$
- (D) Imaginary function of  $\omega$

**Q.64** A signal  $x_1(t)$  and  $x_2(t)$  constitute the real and imaginary parts respectively of a complex valued signal  $x(t)$ . What form of waveform does  $x(t)$  possess? [ESE EC 2009]

- (A) Real symmetric
- (B) Complex symmetric
- (C) Asymmetric
- (D) Conjugate symmetric

**Q.65** Match List-I (Type of signal) with List - II (Property of Fourier transform) and select the correct answer using the codes given below the lists :

**List-I (Type of signal)** [ESE EC 2002]

- A. Real and even symmetric
- B. Real and odd symmetric
- C. Imaginary and even symmetric
- D. Imaginary and odd symmetric

**List-II (Property of Fourier transform)**

1. Imaginary and even symmetric
2. Real and even symmetric
3. Real and odd symmetric
4. Imaginary and odd symmetric

**Codes :** A B C D

- (A) 1 4 2 3
- (B) 2 4 1 3
- (C) 1 3 2 4
- (D) 2 3 1 4

**Q.66** Fourier transform  $F(j\omega)$  of an arbitrary real signal has the property [ESE EC 1995]

- (A)  $F(j\omega) = F(-j\omega)$
- (B)  $F(j\omega) = -F(-j\omega)$
- (C)  $F(j\omega) = F^*(-j\omega)$
- (D)  $F(j\omega) = -F^*(-j\omega)$

**Q.67** Fourier transform of a complex and even signal is

- (A) real and even
- (B) complex and odd
- (C) imaginary and odd
- (D) complex and even

**Q.68** If  $x(t)$  is an odd function of time, then Fourier transform of  $x(t)$  can be evaluated by

- (A)  $X(j\omega) = 2 \int_0^\infty x(t) \cos(\omega t) dt$
- (B)  $X(j\omega) = 2 \int_0^\infty x(t) \sin(\omega t) dt$
- (C)  $X(j\omega) = -2 \int_0^\infty x(t) \sin(\omega t) dt$
- (D)  $X(j\omega) = 2 \int_0^\infty x(t) \cos(\omega t) dt$

**Q.69** Let  $x(t)$  is a real-valued signal whose Fourier Transform is  $X(f)$ , then which of the following is true?

- (A)  $X(f)$  is even symmetric
- (B)  $X(f)$  is odd symmetric
- (C)  $X(f)$  is real-valued
- (D)  $|X(f)| = |X(-f)|$

**Q.70** Inverse Fourier transform of a conjugate anti-symmetric frequency domain function is proportional to

- (A)  $x^*(t)$
- (B) Imaginary  $[x(t)]$
- (C)  $x^*(-t)$
- (D) Real  $[x(t)]$

**Q.71** If CTFT of a signal  $x(t) = e^{-t^2/2}$  is given as  $X(j\omega)$  then which of the following conditions is not satisfied by  $X(j\omega)$  ?

- (A)  $\text{Im}[X(j\omega)] = 0$
- (B)  $\int_{-\infty}^{\infty} X(j\omega) d\omega = 0$
- (C)  $\int_{-\infty}^{\infty} \omega X(j\omega) d\omega = 0$
- (D) There exists a real ' $\alpha$ ' for which  $e^{j\alpha\omega} X(j\omega)$  is real



**Answer Keys****Objective & Numerical Answer Type Questions**

1.	A	2.	B	3.	A	4.	C	5.	C
6.	A	7.	D	8.	A	9.	C	10.	B
11.	A	12.	A	13.	C	14.	C	15.	B
16.	B	17.	C	18.	C	19.	C	20.	C
21.	C	22.	C	23.	D	24.	C	25.	B
26.	B	27.	D	28.	B	29.	B	30.	C
31.	C	32.	B	33.	D	34.	B	35.	C
36.	B	37.	C	38.	C	39.	A	40.	A
41.	C	42.	D	43.	B	44.	C	45.	A
46.	C	47.	D	48.	A	49.	D	50.	C
51.	D	52.	C	53.	A	54.	D	55.	B
56.	B	57.	A	58.	A	59.	C	60.	B
61.	D	62.	A	63.	C	64.	C	65.	A
66.	D	67.	C	68.	D	69.	A	70.	C
71.	B	72.	C	73.	B	74.	B	75.	C
76.	B	77.	A	78.	B	79.	D	80.	D
81.	A	82.	B	83.	C	84.	A	85.	D
86.	B	87.	A	88.	A	89.	D	90.	A
91.	D	92.	A	93.	B	94.	D	95.	C
96.	B	97.	B	98.	B	99.	B	100.	A
101.	B	102.	A	103.	D	104.	D	105.	D
106.	C	107.	A	108.	C	109.	B	110.	A
111.	B	112.	D	113.	C	114.	D	115.	C
116.	A	117.	C	118.	B	119.	C	120.	D
121.	B	122.	A	123.	C	124.	B	125.	D
126.	A	127.	B	128.	B	129.	C		

**Practice (Objective & Numerical Answer) Questions**

1.	A	2.	C	3.	B	4.	C	5.	1-A, 2-C
6.	C	7.	A	8.	A	9.	B	10.	D
11.	D	12.	D	13.	D	14.	B	15.	B
16.	A	17.	D	18.	C	19.	D	20.	C
21.	D	22.	C	23.	A	24.	A	25.	C
26.	A	27.	A	28.	B	29.	D	30.	B
31.	C	32.	C	33.	D	34.	C	35.	D
36.	C	37.	D	38.	C	39.	A	40.	B
41.	C	42.	C	43.	D	44.	C	45.	C
46.	D	47.	B	48.	D	49.	C	50.	B
51.	A	52.	A	53.	B	54.	C	55.	B
56.	A	57.	C	58.	D	59.	C	60.	B
61.	C	62.	B	63.	C	64.	B	65.	B
66.	C	67.	D	68.	C	69.	D	70.	B
71.	B								

# 5

# Laplace Transform

## Objective & Numerical Ans Type Questions :

- Q.1** The Laplace Transform of  $f(t) = e^{2t} \sin(5t)u(t)$  is  
[GATE EE 2016-Bangalore]

(A)  $\frac{5}{s^2 - 4s + 29}$       (B)  $\frac{5}{s^2 + 5}$   
 (C)  $\frac{s-2}{s^2 - 4s + 29}$       (D)  $\frac{5}{s+5}$

- Q.2** The bilateral Laplace transform of a function  $f(t) = \begin{cases} 1 & \text{if } a \leq t \leq b \\ 0 & \text{otherwise} \end{cases}$  is

[GATE EC 2015-Kharagpur]

(A)  $\frac{a-b}{s}$       (B)  $\frac{e^z(a-b)}{s}$   
 (C)  $\frac{e^{-as} - e^{-bs}}{s}$       (D)  $\frac{e^{-(a-b)}}{s}$

- Q.3** Let  $x(t) = \alpha\beta s(t) + \beta s(-t)$  with  $s(t) = e^{-4t}u(t)$  where  $u(t)$  is unit step function. If the bilateral Laplace transform of  $x(t)$  is

[GATE EC 2015-Kharagpur]

$$X(s) = \frac{16}{s^2 - 16} \quad -4 < \operatorname{Re}\{s\} < 4$$

Then the value of  $\beta$  is \_\_\_\_\_.

- Q.4** Consider the function  $g(t) = e^{-t} \sin(2\pi t)u(t)$  where  $u(t)$  is the unit step function. The area under  $g(t)$  is \_\_\_\_\_. [GATE EC 2015-Kanpur]

- Q.5** The Laplace transform of  $f(t) = 2\sqrt{\frac{t}{\pi}}$  is  $s^{-\frac{3}{2}}$ . The

Laplace transform of  $g(t) = \sqrt{\frac{1}{\pi t}}$  is

[GATE EE 2015-Kharagpur]

(A)  $\frac{3s^{-\frac{5}{2}}}{2}$       (B)  $s^{-\frac{1}{2}}$

(C)  $s^{\frac{1}{2}}$       (D)  $s^{\frac{3}{2}}$

- Q.6** The unilateral Laplace transform of  $f(t)$  is  $\frac{1}{s^2 + s + 1}$ . Which one of the following is the unilateral Laplace transform of  $g(t) = t.f(t)$ ?

[GATE EC 2014-Kharagpur]

(A)  $\frac{-s}{(s^2 + s + 1)^2}$       (B)  $\frac{-(2s+1)}{(s^2 + s + 1)^2}$   
 (C)  $\frac{s}{(s^2 + s + 1)^2}$       (D)  $\frac{2s+1}{(s^2 + s + 1)^2}$

- Q.7** Let the Laplace transform of a function  $f(t)$  which exist for  $t > 0$  be  $F_1(s)$  and the Laplace transform of its delayed version  $f(t-\tau)$  be  $F_2(s)$ . Let  $F_1^*(s)$  be the complex conjugate of  $F_1(s)$  with the Laplace variable set as  $s = \sigma + j\omega$ .

If  $G(s) = \frac{F_2(s).F_1^*(s)}{|F_1(s)|^2}$ , then the inverse Laplace

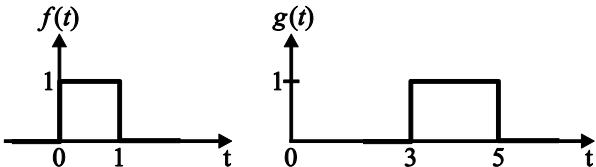
transform of  $G(s)$  is [GATE EE 2011-Madras]

- (A) an ideal impulse  $\delta(t)$   
 (B) an ideal delayed impulse  $\delta(t-\tau)$   
 (C) an ideal step function  $u(t)$   
 (D) an ideal delayed step function  $u(t-\tau)$

## Common Data for Questions 8 & 9

Given  $f(t)$  and  $g(t)$  as shown below :

[GATE EE 2010-Guwahati]



**Q.8**  $g(t)$  can be expressed as

- (A)  $g(t) = f(2t-3)$       (B)  $g(t) = f\left(\frac{t}{2}-3\right)$   
 (C)  $g(t) = f\left(2t-\frac{3}{2}\right)$       (D)  $g(t) = f\left(\frac{t}{2}-\frac{3}{2}\right)$

**Q.9** The Laplace transform of  $g(t)$  is

- (A)  $\frac{1}{s}(e^{3s} - e^{5s})$       (B)  $\frac{1}{s}(e^{-5s} - e^{-3s})$   
 (C)  $\frac{e^{-3s}}{s}(1 - e^{-2s})$       (D)  $\frac{1}{s}(e^{5s} - e^{3s})$

**Q.10** The Laplace transform of  $(t^2 - 2t)u(t-1)$  is

[GATE EE 1998-Delhi]

- (A)  $\frac{2}{s^3}e^{-s} - \frac{2}{s^2}e^{-s}$       (B)  $\frac{2}{s^3}e^{-2s} - \frac{2}{s^2}e^{-s}$   
 (C)  $\frac{2}{s^3}e^{-s} - \frac{1}{s}e^{-s}$       (D) None of these

**Q.11** Consider the following statements [ESE EC 2010]

1. The Laplace transform of the unit impulse function is  $s \times$  Laplace transform of the unit ramp function.
2. The impulse function is a time derivative of the ramp function.
3. The Laplace transform of the unit impulse function is  $s \times$  Laplace transform of the unit step function.
4. The impulse function is a time derivative of the unit step function

Which of the above statements are correct?

- (A) 1 and 2 only      (B) 3 and 4 only  
 (C) 2 and 3 only      (D) 1, 2, 3 and 4

**Q.12** Match List-I with List-II and select the correct answer using the code given below the lists :

**List-I** [ESE EC 2008]

[Function in time domain  $f(t)$ ]

- A.  $\sin \omega_0 t.u(t-t_0)$   
 B.  $\sin \omega_0(t-t_0) u(t-t_0)$   
 C.  $\sin \omega_0(t-t_0) u(t)$   
 D.  $\sin \omega_0 t.u(t)$

**List-II**

[Corresponding Laplace transform  $F(s)$ ]

1.  $\frac{\omega_0}{s^2 + \omega_0^2}$   
 2.  $\left\{ \frac{\omega_0}{s^2 + \omega_0^2} \right\} e^{-t_0 s}$

3.  $\frac{e^{-t_0 s}}{\sqrt{s^2 + \omega_0^2}} \sin \left( \omega_0 t_0 + \tan^{-1} \frac{\omega_0}{s} \right)$

4.  $-\frac{1}{\sqrt{s^2 + \omega_0^2}} \sin \left( \omega_0 t_0 - \tan^{-1} \frac{\omega_0}{s} \right)$

Codes : A B C D

- (A) 3 1 4 2  
 (B) 4 2 3 1  
 (C) 3 2 4 1  
 (D) 4 1 3 2

**Q.13** Match List-I [ $F(s)$ ] with List-II [ $f(t)$ ] and select the correct answer using the codes given below the lists : [ESE EC 2003]

**List-I** [ $F(s)$ ]

- A.  $\frac{10}{s(s+10)}$   
 B.  $\frac{10}{s^2 + 100}$   
 C.  $\frac{(s+10)}{(s+10)^2 + 100}$   
 D. 10

**List-II** [ $f(t)$ ]

1.  $10\delta(t)$   
 2.  $e^{-10t} \cos 10t.u(t)$   
 3.  $(\sin 10t).u(t)$   
 4.  $(1 - e^{-10t}).u(t)$

Codes : A B C D

- (A) 3 4 1 2  
 (B) 4 3 1 2  
 (C) 3 4 2 1  
 (D) 4 3 2 1

**Q.14** In Laplace transform, the variable equals  $(\sigma + j\omega)$ . Which of the following represents the true nature of  $\sigma$  ?

1.  $\sigma$  has a damping effect.
2.  $\sigma$  is responsible for convergence of integral

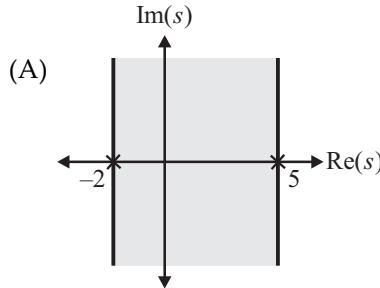
$$\int_0^\infty f(t)e^{-st} dt$$

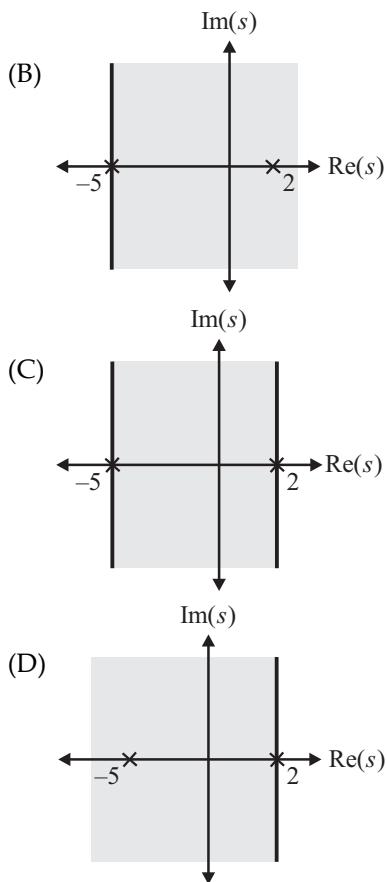
3.  $\sigma$  has a value less than zero

Select the correct answer using the codes given below : [ESE EC 1994]

- (A) 1, 2 and 3      (B) 1 and 2  
 (C) 2 and 3      (D) 1 and 3

**Q.15** Which of the following plot represents correct ROC of a signal  $x(t) = e^{2t}u(-t) - e^{-5t}u(t)$  ?





**Q.16** Laplace transform of  $\text{sgn}(t)$  is

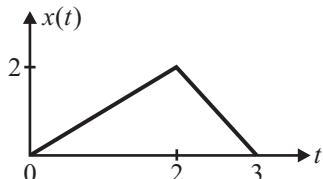
- (A)  $2/s$       (B)  $1/s$   
 (C) 0      (D) None of these

**Q.17** If  $x(t) = u(t) - u(t-a)$  then Laplace transform of  $x(2t)$  is

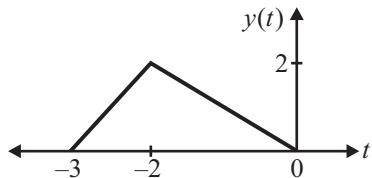
- (A)  $\frac{1-e^{-as/2}}{s}$       (B)  $2\left[\frac{1-e^{-as/2}}{s}\right]$   
 (C)  $\frac{1-e^{-as/2}}{2s}$       (D) None of these

**Q.18** Let  $X(s)$  be the Laplace transform of signal  $x(t)$  shown in figure (A) given as

$$X(s) = \frac{1}{s^2} [1 - 3e^{-2s} + 2e^{-3s}]$$



The Laplace transform of  $y(t)$  shown in figure (B) will be



- (A)  $\frac{1}{s^2} [1 + 3e^{-2s} - 2e^{-3s}]$       (B)  $\frac{-1}{s^2} [1 - 3e^{-2s} + 2e^{-3s}]$   
 (C)  $\frac{1}{s^2} [1 - 3e^{2s} + 2e^{3s}]$       (D)  $\frac{-1}{s^2} [-1 + 3e^{-2s} + 2e^{-3s}]$

**Q.19** The Laplace transform and its ROC of a signal  $x(t) = 3e^{-t}u(t) + 2e^t u(-t)$  is

- (A)  $\frac{s-5}{(s^2-1)}$ , ROC :  $-1 < \text{Re}(s) < 1$   
 (B)  $\frac{5}{(s+1)}$ , ROC :  $-1 < \text{Re}(s) < 1$   
 (C)  $\frac{5s+1}{(s^2-1)}$ , ROC :  $\text{Re}(s) < 1$   
 (D)  $\frac{s-5}{(s^2+1)}$ , ROC :  $\text{Re}(s) > -1$

**Q.20** If  $X_1(s) = \frac{1}{s+5}$  and  $X_2(s) = X_1(s-j4) + X_1(s+j4)$  then  $x_2(t)$  is

- (A)  $2e^{-5t} \sin 4t u(t)$   
 (B)  $2e^{-5t} [\cos 4t + \sin 4t] u(t)$   
 (C)  $2e^{-5t} \cos 4t u(t)$   
 (D)  $e^{-5t} \sin 4t u(t)$

**Q.21** A signal  $x(t)$  has the following Laplace transform  $X(s) = \frac{s}{s^2 + 9}$ ,  $\text{Re}(s) < 0$  the signal  $x(t)$  is

- (A)  $\cos(3t)u(t)$       (B)  $-\cos(3t)u(-t)$   
 (C)  $\sin(3t)u(t)$       (D)  $-\cos(3t)u(t)$

**Q.22** The Laplace transform of the function  $5\sin(2\pi t)u(t-1)$  is

- (A)  $\frac{10\pi e^{-s}}{s^2 + (2\pi)^2}$       (B)  $\frac{10\pi}{s^2 + (2\pi)^2}$   
 (C)  $\frac{10\pi e^s}{s^2 + (2\pi)^2}$       (D)  $\frac{10\pi s e^{-s}}{s^2 + (2\pi)^2}$

**Q.23** The Laplace transform of a signal  $x(t)$  is

$$X(s) = \frac{d^2}{ds^2} \left( \frac{1}{s-3} \right), \quad \text{ROC : } \text{Re}(s) > 3$$

The signal  $x(t)$  is

- (A)  $t^2 e^{3t} u(t)$       (B)  $-t^2 e^{3t} u(t)$   
 (C)  $t^2 u(t-3)$       (D)  $(t-3)^2 u(t-3)$

**Q.24** The Laplace transform of  $e^t \frac{d}{dt} [e^{-2t} u(-t)]$  is

(A)  $\frac{1-s}{s+1}$ ,  $\text{Re}(s) < -1$     (B)  $\frac{1-s}{s+1}$ ,  $\text{Re}(s) > -1$

(C)  $\frac{s-1}{s+1}$ ,  $\text{Re}(s) < -1$     (D)  $\frac{s-1}{s+1}$ ,  $\text{Re}(s) > -1$

**Q.25** Determine the Laplace Transform of the signal

$$t \cdot \frac{d}{dt} [e^{-t} \cos t u(t)]$$

(A)  $-\frac{[s^2 + 4s + 2]}{[s^2 + 2s + 2]^2}$     (B)  $\frac{s^2 + 4s + 2}{[s^2 + 2s + 2]^2}$

(C)  $\frac{s^2 + 2s + 2}{[s^2 + 4s + 2]^2}$     (D)  $-\frac{[s^2 + 2s + 2]}{[s^2 + 4s + 2]^2}$

**Q.26** The time signal corresponding to

$$e^{-2s} \frac{d}{ds} \left[ \frac{1}{(s+1)^2} \right]$$

(A)  $-te^{-t}u(1-t)$     (B)  $-(t+2)^2 e^{-(t+2)}u(t+2)$   
 (C)  $-(t-2)^2 e^{-(t-2)}u(t-2)$     (D)  $te^{-t}u(t-1)$

**Q.27** The time signal corresponding to

$$s \frac{d^2}{ds^2} \left( \frac{1}{s^2 + 9} \right) + \frac{1}{s+3}$$

(A)  $[e^{-3t} + t^2 \sin 3t + 2t \cos 3t]u(t)$   
 (B)  $[e^{-3t} + 2t \sin 3t + t^2 \cos 3t]u(t)$   
 (C)  $\left[ e^{-3t} + \frac{2t}{3} \sin 3t + t^2 \cos 3t \right]u(t)$   
 (D)  $\left[ e^{-3t} + \frac{2t}{3} \sin 3t + \frac{t^2}{9} \cos 3t \right]u(t)$

**Common Data for Questions 28 & 29**

Given a Laplace transform pair

$$x(t)u(t) \xleftarrow{\text{LT}} \frac{3s}{s^2 + 4}$$

Then Laplace transform of given signal  $y(t)$  related to  $x(t)$  as follows :

**Q.28**  $y(t) = x(t-3)$

(A)  $\frac{3se^{3s}}{s^2 + 4}$     (B)  $\frac{3se^{-3s}}{s^2 + 4}$   
 (C)  $\frac{3(s-3)}{s^2 + 4}$     (D)  $\frac{3(s-3)}{(s-3)^2 + 4}$

**Q.29**  $y(t) = x(t) \otimes \frac{dx(t)}{dt}$

(A)  $\frac{9s^3}{(s^2 + 4)^2}$     (B)  $\frac{9s^3}{s^2 + 4}$   
 (C)  $\frac{3s^2}{s^2 + 4}$     (D)  $\frac{3s}{(s^2 + 4)^2}$

**Q.30** If  $X(s)$ , the Laplace transform of signal  $x(t)$  is given by  $X(s) = \frac{(s+2)}{(s+1)(s+3)^2}$ , then the value of  $x(t)$  as  $t \rightarrow \infty$  is \_\_\_\_\_

[GATE IN 2016-Bangalore]

**Q.31** If  $F(s) = L[f(t)] = \frac{2(s+1)}{s^2 + 4s + 7}$  then the initial and final values of  $f(t)$  are respectively

[GATE EC 2011-Madras]

(A) 0, 2    (B) 2, 0  
 (C) 0, 2 / 7    (D) 2 / 7, 0

**Q.32** Given  $f(t) = L^{-1} \left[ \frac{3s+1}{s^3 + 4s^2 + (K-3)s} \right]$ .

If  $\lim_{t \rightarrow \infty} f(t) = 1$ , then the value of  $K$  is

[GATE EC 2010-Guwahati]

(A) 1    (B) 2  
 (C) 3    (D) 4

**Q.33** Consider the function  $f(t)$  having Laplace transform.

[GATE EC 2006-Kharagpur]

$$F(s) = \frac{\omega_0}{s^2 + \omega_0^2} \quad \text{Re}[s] > 0$$

The final value of  $f(t)$  would be

(A) 0    (B) 1  
 (C)  $-1 \leq f(\infty) \leq 1$     (D)  $\infty$

**Q.34** For the equation  $\ddot{x}(t) + 3\dot{x}(t) + 2x(t) = 5$  the solution  $x(t)$  approaches which of the following values as  $t \rightarrow \infty$ ?

[GATE EE 2005-Bombay]

(A) 0    (B) 5 / 2  
 (C) 5    (D) 10

**Q.35** A control system is defined by the following mathematical relationship

$$\frac{d^2x}{dt^2} + 6 \frac{dx}{dt} + 5x = 12[1 - e^{-2t}]$$

The response of the system as  $t \rightarrow \infty$  is

[GATE EE 2003-Madras]

(A)  $x = 6$     (B)  $x = 2$   
 (C)  $x = 2.4$     (D)  $x = -2$

**Q.36** The transfer function of a system is  $\frac{A}{s^2 + \omega^2}$ . The steady-state gain of the system to a unit-step input is

[GATE IN 2003-Madras]

(A)  $\frac{A}{\omega^2}$   
 (B) 0  
 (C)  $\infty$   
 (D) not possible to be determined.

- Q.37** If  $L[f(t)] = \frac{2(s+1)}{s^2 + 2s + 5}$ , then  $f(0^+)$  and  $f(\infty)$  respectively are given by

[GATE EC 1995-Kanpur]

- (A) 0, 2      (B) 2, 0  
 (C) 0, 1      (D)  $\frac{2}{5}, 0$

- Q.38** A voltage having the Laplace transform  $\frac{4s^2 + 3s + 2}{7s^2 + 6s + 5}$  is applied across a 2 H inductor having zero initial current. What is the current in the inductor at  $t = \infty$ ? [ESE EC 2008]

- (A) Zero      (B)  $(1/5)$  A  
 (C)  $(2/7)$  A      (D)  $(2/5)$  A

- Q.39** For the function  $x(t)$ ,  $X(s)$  is given by

$$X(s) = e^{-s} \left[ \frac{-2}{s(s+2)} \right]$$

Then, what are the initial and final values of  $x(t)$ , respectively? [ESE EC 2006]

- (A) 0 and 1      (B) 0 and -1  
 (C) 1 and 1      (D) -1 and 0

- Q.40** The final value of  $F(s) = \frac{1}{(s+j)(s-j)}$  is
- (A)  $\infty$       (B) 1  
 (C) 0      (D) undefined

- Q.41** The initial and final values of the function are

$$F(s) = A \frac{(s+a)\sin\theta + b\cos\theta}{(s+a)^2 + b^2}$$

- (A)  $A\sin\theta$ , undefined      (B) 0, undefined  
 (C) 0, 0      (D)  $A\sin\theta$ , 0

- Q.42** The inverse Laplace transform of the function  $\frac{(s+5)}{(s+1)(s+3)}$  is [GATE EC 1996-Bangalore]

- (A)  $2e^{-t} - e^{-3t}$       (B)  $2e^{-t} + e^{-3t}$   
 (C)  $e^{-t} - 2e^{-3t}$       (D)  $e^{-t} + e^{-3t}$

- Q.43** What is  $F(s) = \frac{8s+10}{(s+1)(s+2)^3}$  equal to?
- (A)  $\frac{2}{s+1} + \frac{4}{(s+2)^3} - \frac{4}{(s+2)^2} - \frac{2}{s+2}$  [ESE EC 2006]

- (B)  $\frac{2}{s+1} + \frac{6}{(s+2)^3} - \frac{2}{(s+2)^2} - \frac{2}{s+2}$   
 (C)  $\frac{2}{s+1} + \frac{6}{(s+2)^3} - \frac{2}{(s+2)^2} + \frac{2}{s+2}$   
 (D)  $\frac{4}{s+1} + \frac{6}{(s+2)^3} - \frac{2}{(s+2)^2} - \frac{4}{s+2}$

- Q.44** If  $F(s) = \frac{2s+5}{s^2 + 5s + 6}$ , then  $f(t)$  is given by

- (A)  $2e^{-2.5t} \cosh(0.5t)$       [ESE EC 2001]  
 (B)  $2e^{2.5t} [\cosh 0.5t + \sinh 0.5t]$   
 (C)  $\sqrt{2}e^{-2.5t} \sin(0.5t + 45^\circ)$   
 (D)  $e^{-2.5t} [\cos 0.5t + \sin 0.5t]$

**Common Data for Questions 45 to 47**

Let  $X(s) = \frac{2s+4}{s^2 + 4s + 3}$ , the inverse LT of  $X(s)$  for

- Q.45** ROC :  $\text{Re}(s) > -1$

- (A)  $-(e^{-t} + e^{-3t})u(-t)$       (B)  $[e^{-t} + e^{-3t}]u(t)$   
 (C)  $-e^{-t}u(-t) + e^{-3t}u(t)$       (D)  $e^{-t}u(t) - e^{-3t}u(-t)$

- Q.46** ROC :  $\text{Re}(s) < -3$

- (A)  $-(e^{-t} + e^{-3t})u(-t)$       (B)  $[e^{-t} + e^{-3t}]u(t)$   
 (C)  $-e^{-t}u(-t) + e^{-3t}u(t)$       (D)  $e^{-t}u(t) - e^{-3t}u(-t)$

- Q.47** ROC :  $-3 < \text{Re}(s) < -1$

- (A)  $-(e^{-t} + e^{-3t})u(-t)$       (B)  $[e^{-t} + e^{-3t}]u(t)$   
 (C)  $-e^{-t}u(-t) + e^{-3t}u(t)$       (D)  $e^{-t}u(t) - e^{-3t}u(-t)$

- Q.48** Let  $x(t)$  be a CT signal and its Laplace transform is

$$X(s) = \frac{s(s+5)}{(s^2 + 16)}$$

If  $x(t) = 5\cos 4t - 4\sin 4t + x_3(t)$ , then  $x_3(t)$  will be

- (A) an impulse function  
 (B) a step function  
 (C) a ramp function  
 (D) a sinusoidal function

- Q.49**  $x(t)$  can be evaluated by taking the inverse Laplace transform of  $X(s)$ . The inverse Laplace transform of  $X(s) = \frac{s^3 + 2s^2 + 6}{s^2 + 3s}$ ,  $\text{Re}(s) > 0$  is :

- (A)  $u(t) - \delta(t) + (2 - e^{-3t})u(t)$   
 (B)  $\delta'(t) - \delta(t) + (2 - e^{-3t})u(t)$   
 (C)  $u(t) - \delta(t) + (2 + e^{-3t})u(t)$   
 (D)  $\delta'(t) - \delta(t) + (2 + e^{-3t})u(t)$

- Q.50** If  $F_1(s) = \frac{1}{s+3}$  and  $F_2(s) = \frac{2}{s^2 + 4}$

What is the inverse Laplace transform of the product  $F_1(s) \cdot F_2(s)$ ?

- (A)  $f(t) = \frac{1}{5} [e^{-t} + 3\cos 2t - 2\sin t]$

(B)  $f(t) = \frac{1}{13} [2e^{-3t} + 3\sin 2t - 2\cos 2t]$

(C)  $f(t) = \frac{1}{7} [e^{-2t} + 2\sin 2t - \cos 2t]$

(D)  $f(t) = \frac{1}{11} [e^{-2t} + \sin t - 2\sin 2t]$

- Q.51** The unilateral Laplace transform of a signal  $x(t)$  is

$$X(s) = \frac{2s^2 + 11s + 16 + e^{-2s}}{(s^2 + 5s + 6)}$$

The time signal  $x(t)$  is

(A)  $2\delta(t) + [3e^{-2t} - 2e^{-3t}]u(t-2)$

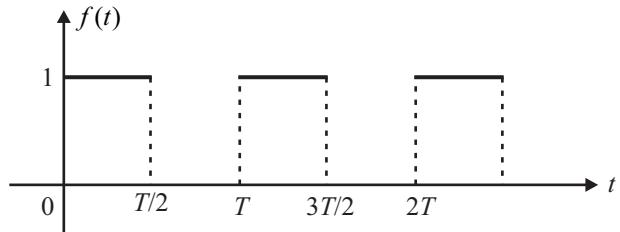
(B)  $2\delta(t) + [2e^{-2t} - 2e^{-3t} + e^{-2(t-2)} + e^{-3(t-2)}]u(t)$

(C)  $2\delta(t) + [2e^{-2t} - e^{-3t}]u(t) + [e^{-2t} - e^{-3t}]u(t-2)$

(D)  $2\delta(t) + [2e^{-2t} - e^{-3t}]u(t) + [e^{-2(t-2)} - e^{-3(t-2)}]u(t-2)$

- Q.52** The Laplace transform of the causal periodic square wave of period  $T$  shown in the figure below is

[GATE EC 2016-Bangalore]



(A)  $F(s) = \frac{1}{1 + e^{-sT/2}}$

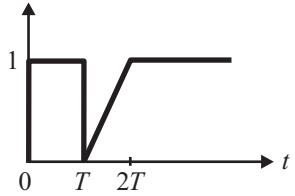
(B)  $F(s) = \frac{1}{s(1 + e^{-sT/2})}$

(C)  $F(s) = \frac{1}{s(1 - e^{-sT})}$

(D)  $F(s) = \frac{1}{1 - e^{-sT}}$

- Q.53** The function shown in the figure can be represented as

[GATE EE 2014-Kharagpur]



(A)

$$u(t) - u(t-T) + \frac{(t-T)}{T} u(t-T) - \frac{(t-2T)}{T} u(t-2T)$$

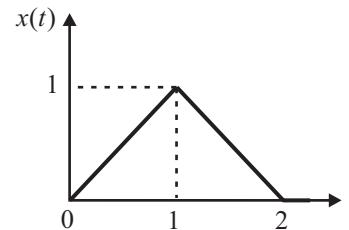
(B)  $u(t) + \frac{t}{T} u(t-T) - \frac{t}{T} u(t-2T)$

(C)  $u(t) - u(t-T) + \frac{(t-T)}{T} u(t) - \frac{(t-2T)}{T} u(t)$

(D)  $u(t) + \frac{(t-T)}{T} u(t-T) - 2 \frac{(t-2T)}{T} u(t-2T)$

- Q.54** The Laplace Transform representation of the triangular pulse shown below is

[GATE EC, EE, IN 2013-Bombay]



(A)  $\frac{1}{s^2} [1 + e^{-2s}]$

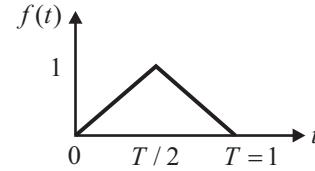
(B)  $\frac{1}{s^2} [1 - e^{-s} + e^{-2s}]$

(C)  $\frac{1}{s^2} [1 - e^{-s} + 2e^{-2s}]$

(D)  $\frac{1}{s^2} [1 - 2e^{-s} + e^{-2s}]$

- Q.55** Laplace transform of the function  $f(t)$  shown in the figure is

[ESE EC 2011]



(A)  $\frac{2}{s^2} [1 - e^{-0.5s}]^2$

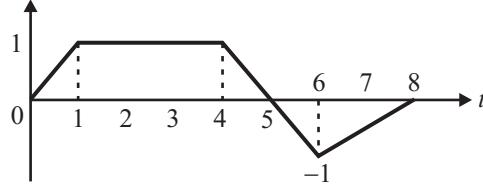
(B)  $\frac{2}{s^2} [1 + e^{-0.5s}]^2$

(C)  $\frac{2}{s^2} [1 - e^{0.5s}]^2$

(D)  $\frac{2}{s^2} [1 + e^{0.5s}]^2$

- Q.56** The Laplace transform of the waveform shown in the figure is

[ESE EC 2006]



$$\frac{1}{s^2} [1 + Ae^{-s} + Be^{-4s} + Ce^{-6s} + De^{-8s}]$$

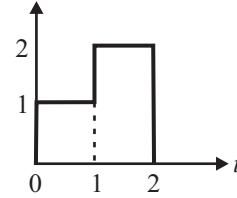
What is the value of D?

(A) -0.5      (B) -1.5

(C) 0.5      (D) 2.0

- Q.57** What is the Laplace transform of the waveform shown below?

[ESE EC 2005]



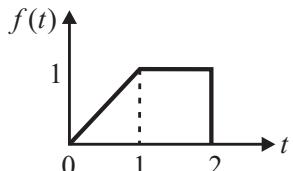
(A)  $F(s) = \frac{1}{s} + \frac{1}{s}e^{-s} - \frac{2}{s}e^{-2s}$

(B)  $F(s) = \frac{1}{s} - \frac{1}{s}e^{-s} + \frac{2}{s}e^{-2s}$

(C)  $F(s) = \frac{1}{s} + \frac{1}{s}e^s + \frac{2}{s}e^{2s}$

(D)  $F(s) = \frac{1}{s} - \frac{1}{s}e^{-2s} - \frac{2}{s}e^{-s}$

- Q.58** The function  $f(t)$  shown in the given figure will have Laplace transform as [ESE EC 1999]

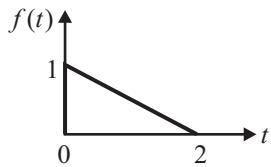


(A)  $\frac{1}{s^2} - \frac{1}{s}e^{-s} - \frac{1}{s^2}e^{-2s}$  (B)  $\frac{1}{s^2}[1 - e^{-s} - e^{-2s}]$

(C)  $\frac{1}{s}[1 - e^{-s} - e^{-2s}]$  (D)  $\frac{1}{s^2}[1 - e^{-s} - se^{-2s}]$

- Q.59** Which one of the following is the correct Laplace transform of the signal in the given figure?

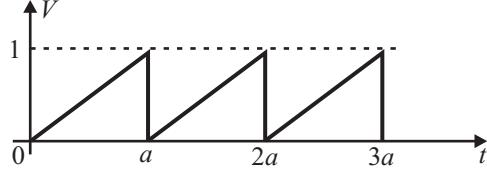
[ESE EC 1994]



(A)  $\frac{1}{2s^2}[1 - e^{-2s}(1 + 2s)]$  (B)  $\frac{1}{2s^2}[e^{-2s} - 1 + 2s]$

(C)  $\frac{1}{2s^2}[e^{-2s} + 1 - 2s]$  (D)  $\frac{1}{2s^2}[1 - e^{-2s} + 2s]$

- Q.60** The Laplace transform of the waveform given below is [ESE EC 1991]



(A)  $V(s) = \frac{1}{(1 - e^{-as})}$

(B)  $V(s) = \left( \frac{1 - e^{-as}}{2s^2} - \frac{e^{-as}}{s} \right)$

(C)  $V(s) = \left( \frac{1 - e^{-as}}{2} \right)$

(D)  $V(s) = \left( \frac{1 - e^{-as}}{as^2} - \frac{e^{-as}}{s} \right) \frac{1}{(1 - e^{-as})}$

- Q.61** Laplace transform of  $x(t) = \sum_{k=0}^{\infty} \delta(t - kT)$  is

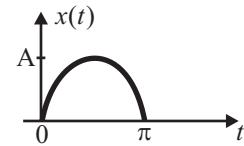
(A)  $\frac{1}{1 - e^{-sT}}$

(B)  $\frac{1}{1 + e^{-sT}}$

(C)  $\frac{1}{1 - e^{sT}}$

(D)  $\frac{1}{1 + e^{sT}}$

- Q.62** The Laplace transform of given function  $x(t)$  will be



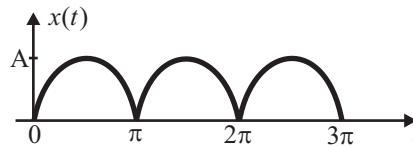
(A)  $\frac{A}{s^2 + 1}[1 + e^{-\pi s}]$

(B)  $\frac{A}{s^2 + 1}[1 - e^{-\pi s}]$

(C)  $\frac{A}{[s^2 + 1][1 + e^{-\pi s}]}$

(D)  $\frac{A}{[s^2 + 1][1 - e^{-\pi s}]}$

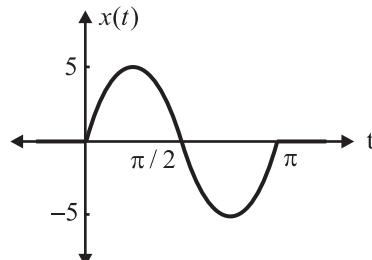
- Q.63** The Laplace transform of given function  $x(t)$  will be



(A)  $\frac{A}{s^2 + 1} \tan h\left(\frac{\pi s}{2}\right)$  (B)  $\frac{A}{s^2 + 1} \tan\left(\frac{\pi s}{2}\right)$

(C)  $\frac{A}{s^2 + 1} \cot h\left(\frac{\pi s}{2}\right)$  (D)  $\frac{A}{s^2 + 1} \cot\left(\frac{\pi s}{2}\right)$

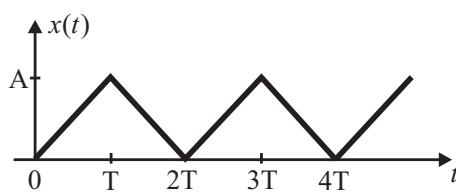
- Q.64** In the figure below, one cycle of a sinusoid for  $0 \leq t \leq \pi$  is shown. The Laplace transform of  $x(t)$  is given by



(A)  $\frac{5e^{-\pi s}}{(s^2 + 4)}$  (B)  $\frac{10(1 - e^{-\pi s})}{(s^2 + 4)}$

(C)  $\frac{5(1 - e^{-\pi s})}{(s^2 + 4)}$  (D)  $\frac{(10e^{-\pi s} - 1)}{(s^2 + 4)}$

- Q.65** The Laplace transform of given function  $x(t)$  for  $T = 1$  will be



(A)  $\frac{A}{s^2} \tanh(s/2)$       (B)  $\frac{A}{s^2} \coth(s/2)$

(C)  $\frac{A}{s^2} \sinh(s/2)$       (D)  $\frac{A}{s^2} \cosh(s/2)$

- Q.66** Consider a causal LTI system characterized by differential equation  $\frac{dy(t)}{dt} + \frac{1}{6}y(t) = 3x(t)$ . The response of the system to the input  $x(t) = 3e^{-\frac{t}{3}}u(t)$ , where  $u(t)$  denotes the unit step function, is      [GATE EE 2016-Bangalore]

(A)  $9e^{-\frac{t}{3}}u(t)$       (B)  $9e^{-\frac{t}{6}}u(t)$

(C)  $9e^{-\frac{t}{3}}u(t) - 6e^{-\frac{t}{6}}u(t)$       (D)  $54e^{-\frac{t}{6}}u(t) - 54e^{-\frac{t}{3}}u(t)$

- Q.67** Input  $x(t)$  and output  $y(t)$  of an LTI system are related by the differential equation  $y''(t) - y'(t) - 6y(t) = x(t)$ . If the system is neither causal nor stable, the impulse response  $h(t)$  of the system is      [GATE EC 2015-Kharagpur]

(A)  $\frac{1}{5}e^{3t}u(-t) + \frac{1}{5}e^{-2t}u(-t)$

(B)  $-\frac{1}{5}e^{3t}u(-t) + \frac{1}{5}e^{-2t}u(-t)$

(C)  $\frac{1}{5}e^{3t}u(-t) - \frac{1}{5}e^{-2t}u(t)$

(D)  $-\frac{1}{5}e^{3t}u(-t) - \frac{1}{5}e^{-2t}u(t)$

- Q.68** A continuous, linear time-invariant filter has an impulse response  $h(t)$  described by

[GATE EC 2014-Kharagpur]

$$h(t) = \begin{cases} 3 & \text{for } 0 \leq t \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

When a constant input of value 5 is applied to this filter, the steady state output is \_\_\_\_.

- Q.69** A stable linear time invariant (LTI) system has a transfer function  $H(s) = \frac{1}{s^2 + s - 6}$ . To make this system causal it needs to be cascaded with another LTI system having a transfer function  $H_1(s)$ . A correct choice for  $H_1(s)$  among the following options is [GATE EC 2014-Kharagpur]

(A)  $s+3$       (B)  $s-2$

(C)  $s-6$       (D)  $s+1$

- Q.70** Consider an LTI system with impulse response  $h(t) = e^{-5t}u(t)$ . If the output of the system is  $y(t) = e^{-3t}u(t) - e^{-5t}u(t)$  then the input,  $x(t)$ , is given by      [GATE EE 2014-Kharagpur]

(A)  $e^{-3t}u(t)$       (B)  $2e^{-3t}u(t)$

(C)  $e^{-5t}u(t)$       (D)  $2e^{-5t}u(t)$

- Q.71** The transfer function of a linear time invariant system is given as  $G(s) = \frac{1}{s^2 + 3s + 2}$ . The steady state value of the output of this system for a unit impulse input applied at instant  $t=1$  will be

[GATE EE 2008-Bangalore]

(A) 0      (B) 0.5

(C) 1      (D) 2

- Q.72** The impulse response of a system is  $h(t) = t.u(t)$ . For an input  $u(t-1)$ , the output is

[GATE EC, EE, IN 2013-Bombay]

(A)  $\frac{t^2}{2}u(t)$       (B)  $\frac{t(t-1)}{2}u(t)$

(C)  $\frac{(t-1)^2}{2}u(t-1)$       (D)  $\frac{t^2-1}{2}u(t-1)$

- Q.73** Which one of the following statements is NOT TRUE for a continuous time causal and stable LTI system? [GATE EC, EE, IN 2013-Bombay]

(A) All the poles of the system must lie on the left side of the  $j\omega$  axis

(B) Zeros of the system can lie anywhere in the  $s$ -plane

(C) All the poles must lie within  $|s| = 1$

(D) All the roots of the characteristic equation must be located on the left side of the  $j\omega$  axis

- Q.74** The impulse response of a continuous time system is given by  $h(t) = \delta(t-1) + \delta(t-3)$ . The value of the step response at  $t=2$  is

[GATE EC, EE, IN 2013-Bombay]

(A) 0      (B) 1

(C) 2      (D) 3

- Q.75** If the unit step response of a network is  $(1 - e^{-\alpha t})$ , then its unit impulse response is

[GATE EC 2011-Madras]

(A)  $\alpha e^{-\alpha t}$       (B)  $\alpha^{-1}e^{-\alpha t}$

(C)  $(1 - \alpha^{-1})e^{-\alpha t}$       (D)  $(1 - \alpha)e^{-\alpha t}$

- Q.76** The response  $h(t)$  of a linear time invariant system to an impulse  $\delta(t)$ , under initially relaxed condition is  $h(t) = e^{-t} + e^{-2t}$ . The response of this system for a unit step input  $u(t)$  is

[GATE EE 2011-Madras]

- (A)  $u(t) + e^{-t} + e^{-2t}$   
 (B)  $(e^{-t} + e^{-2t})u(t)$   
 (C)  $(1.5 - e^{-t} - 0.5e^{-2t})u(t)$   
 (D)  $e^{-t}\delta(t) + e^{-2t}u(t)$

Q.77 For the system  $H(s) = \frac{2}{(s+1)}$ , the approximate

time taken for a step response to reach 98% of its final value is [GATE EE 2010-Guwahati]

- (A) 1 s      (B) 2 s  
 (C) 4 s      (D) 8 s

Q.78 The step response of a linear time invariant system is  $y(t) = 5e^{-10t}u(t)$ , where  $u(t)$  is the unit step function. If the output of the system corresponding to an impulse input  $\delta(t)$  is  $h(t)$  then  $h(t)$  is [GATE IN 2008-Bangalore]

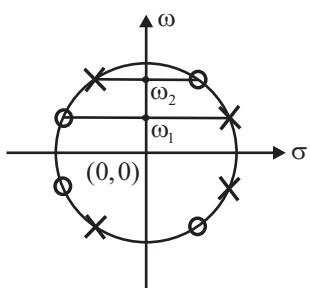
- (A)  $-50e^{-10t}u(t)$       (B)  $5e^{-10t}\delta(t)$   
 (C)  $5u(t) - 50e^{-10t}\delta(t)$       (D)  $5\delta(t) - 50e^{-10t}u(t)$

Q.79 Identify the transfer function corresponding to an all-phase filter from the following.

[GATE IN 2005-Bombay]

- (A)  $\frac{1-s\tau}{1+s\tau}$       (B)  $\frac{1+s\tau_1}{1+s\tau_2}$   
 (C)  $\frac{1}{1+s\tau}$       (D)  $\frac{s\tau}{1+s\tau}$

Q.80 A continuous-time LTI system with system function  $H(\omega)$  has the following pole-zero plot. For this system, which of the alternatives is TRUE? [GATE EE 2014-Kharagpur]



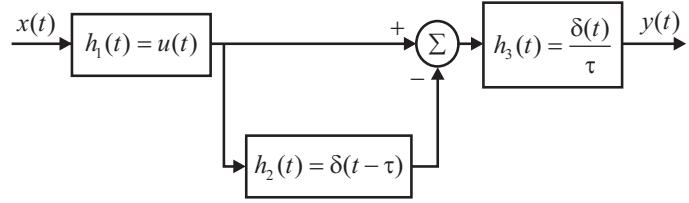
- (A)  $|H(0)| > |H(\omega)|$ ;  $|\omega| > 0$   
 (B)  $|H(\omega)|$  has multiple maxima, at  $\omega_1$  and  $\omega_2$   
 (C)  $|H(0)| > |H(\omega)|$ ;  $|\omega| > 0$   
 (D)  $|H(\omega)| = \text{constant}$ ;  $-\infty < \omega < \infty$

Q.81 Which one of the following transfer function does correspond to a non-minimum phase system? [ESE EC 2001]

- (A)  $\frac{s}{s^2 + 2s + 1}$       (B)  $\frac{s+1}{s^2 + 2s + 1}$   
 (C)  $\frac{s+1}{s^2 + 2s - 1}$       (D)  $\frac{s-1}{s^2 + 2s + 1}$

- Q.82 The impulse response of a single pole system would approach a non-zero constant as  $t \rightarrow \infty$  if and only if the pole is located in the s-plane  
 (A) On the negative real axis [ESE EC 1998]  
 (B) At the origin  
 (C) On the positive real axis  
 (D) On the imaginary axis

Q.83 The impulse response of the system shown in figure is



- (A)  $\frac{1}{\tau}[u(t) - \delta(t - \tau)]$       (B)  $\frac{1}{\tau}[u(t) - u(t - \tau)]$   
 (C)  $\frac{1}{\tau}[u(t - \tau)]$       (D)  $\frac{1}{\tau}[\delta(\tau) - \delta(t - \tau)]$

**Statement for Linked Questions 84 & 85**

A continuous LTI system with an input  $x(t)$  and output  $y(t)$  is described by following differential equation

$$\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t)$$

Q.84 If the system is stable, the impulse response  $h(t)$  of the system will be

- (A)  $\frac{1}{3}e^{2t}u(t) - \frac{1}{3}e^{-t}u(t)$   
 (B)  $-\frac{1}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(-t)$   
 (C)  $-\frac{1}{3}e^{2t}u(-t) - \frac{1}{3}e^{-t}u(t)$   
 (D)  $\frac{1}{3}e^{2t}u(t) + \frac{1}{3}e^{-t}u(t)$

Q.85 If the system is causal, the impulse response  $h(t)$  of the system will be

- (A)  $-\frac{1}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(-t)$   
 (B)  $-\frac{1}{3}e^{-2t}u(-t) - \frac{1}{3}e^{-t}u(t)$   
 (C)  $\frac{1}{3}e^{2t}u(t) + \frac{1}{3}e^{-t}u(t)$   
 (D)  $\frac{1}{3}e^{2t}u(t) - \frac{1}{3}e^{-t}u(t)$

Q.86 A system is excited by a signal  $x(t) = 4 \text{rect}(t/2)$  and its response is

$$y(t) = 10 \left[ (1 - e^{-(t+1)})u(t+1) - (1 - e^{-(t-1)})u(t-1) \right]$$

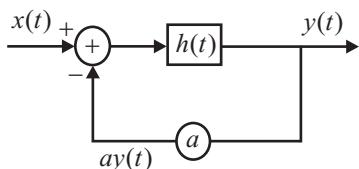
What is its impulse response?

- (A)  $5e^{-t}u(t)$       (B)  $\frac{5}{2}e^{-t}u(t)$   
 (C)  $5[1-e^{-t}]u(t)$       (D)  $\frac{5}{2}[1-e^{-t}]u(t)$

**Q.87** A non-causal signal  $x(t) = e^{-3t}u(t) + e^{-t}u(-t)$  is excitation of a filter whose impulse response is  $h(t) = \delta(t) - e^{-2|t|}$ . Find the response.

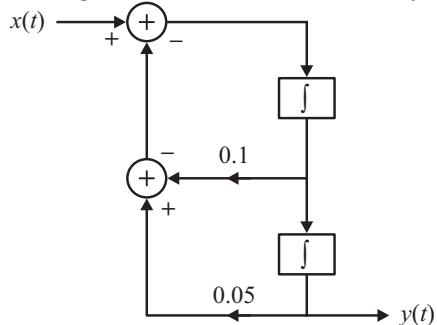
- (A)  $-\frac{1}{3}e^{-t}u(-t) + \frac{9}{5}e^{-3t}u(t) - 2e^{-2t}u(t) + \frac{2}{15}e^{2t}u(-t)$   
 (B)  $\frac{1}{3}e^{-t}u(-t) + \frac{9}{5}e^{-3t}u(t) - 2e^{-2t}u(t) - \frac{2}{15}e^{2t}u(-t)$   
 (C)  $\frac{1}{3}e^{-t}u(t) + \frac{9}{5}e^{-3t}u(t) - 2e^{-2t}u(t) - \frac{2}{15}e^{2t}u(t)$   
 (D) None of the above

**Q.88** A system with impulse response  $h(t) = e^t u(t)$  is connected as a feedback system shown in the figure



For what value of  $a$ , the above system is BIBO stable?

- (A)  $a > 1$       (B)  $a < 1$   
 (C)  $-1 < a < 1$       (D) For all values of  $a$
- Q.89** Find the impulse response of the system in below figure and evaluate its stability.

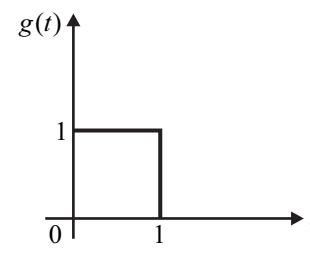


- (A)  $4.589e^{0.05t} \sin(0.2179t)u(t)$ , unstable  
 (B)  $4.589e^{0.05t} \sin(0.2179t)u(t)$ , stable  
 (C)  $4.589e^{-0.05t} \sin(0.2179t)u(t)$ , unstable  
 (D)  $4.589e^{-0.05t} \sin(0.2179t)u(t)$ , stable

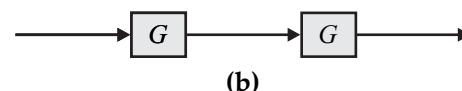
**Q.90** The result of the convolution  $x(-t) * \delta(-t - t_0)$  is  
[GATE EC 2015-Kharagpur]

- (A)  $x(t + t_0)$       (B)  $x(t - t_0)$   
 (C)  $x(-t + t_0)$       (D)  $x(-t - t_0)$

**Q.91** The impulse response  $g(t)$  of a system  $G$ , is as shown in figure (a). What is the maximum value attained by the impulse response of two cascaded blocks of  $G$  as shown in figure (b)?  
[GATE EE 2015-Kharagpur]



(a)



(b)

- (A)  $\frac{2}{3}$       (B)  $\frac{3}{4}$   
 (C)  $\frac{4}{5}$       (D) 1

**Q.92** Given two continuous time signals  $x(t) = e^{-t}$  and  $y(t) = e^{-2t}$  which exist for  $t > 0$ , the convolution  $z(t) = x(t) \otimes y(t)$  is    [GATE EE 2011-Madras]

- (A)  $e^{-t} - e^{-2t}$       (B)  $e^{-3t}$   
 (C)  $e^{+t}$       (D)  $e^{-t} + e^{-2t}$

**Q.93** A linear time invariant system with an impulse response  $h(t)$  produces output  $y(t)$  when input  $x(t)$  is applied. When the input  $x(t-\tau)$  is applied to a system with impulse response  $h(t-\tau)$ , the output will be

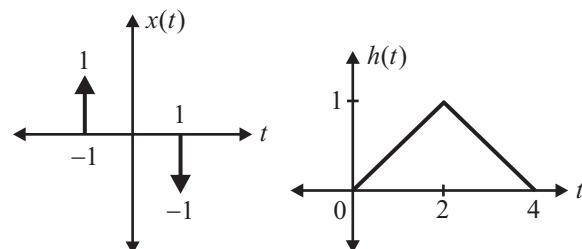
[GATE EE 2009-Roorkee]

- (A)  $y(t)$       (B)  $y(2(t-\tau))$   
 (C)  $y(t-\tau)$       (D)  $y(t-2\tau)$

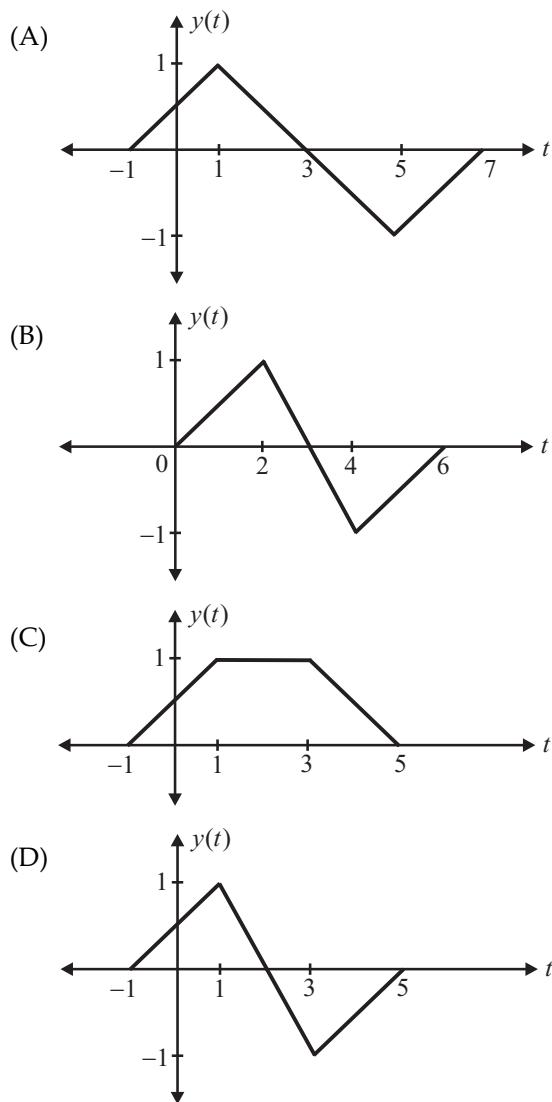
**Q.94** If  $u(t)$ ,  $r(t)$  denote the unit step and unit ramp functions respectively and  $u(t) \otimes r(t)$  their convolution, then the function  $u(t+1) \otimes r(t-2)$  is given by    [GATE EE 2007-Kanpur]

- (A)  $(1/2)(t-1)(t-2)$       (B)  $(1/2)(t-1)^2u(t-1)$   
 (C)  $(1/2)(t-1)^2u(t)$       (D) None of these

**Q.95** The signals  $x(t)$  and  $h(t)$  shown in the figures are convolved to yield  $y(t)$ .



Which one of the figures represents the output  $y(t)$ ?    [GATE IN 2007-Kanpur]



- Q.96** Given  $x(t) \otimes x(t) = t \exp(-2t)u(t)$ . The function  $x(t)$  is [GATE IN 2006-Kharagpur]

(A)  $\exp(-2t)u(t)$       (B)  $\exp(-t)u(t)$   
 (C)  $t \cdot \exp(-t)u(t)$       (D)  $0.5t \cdot \exp(-t)u(t)$

- Q.97** Let  $s(t)$  be the step response of a linear system with zero initial conditions then the response of this system to an input  $u(t)$  is

[GATE EE 2002-Bangalore, EE 1993-Bombay]

- (A)  $\int_0^t s(t-\tau)u(\tau)d\tau$   
 (B)  $\frac{d}{dt} \left[ \int_0^t s(t-\tau)u(\tau)d\tau \right]$   
 (C)  $\int_0^t s(t-\tau) \left[ \int_0^t u(\tau)d\tau \right] d\tau$   
 (D)  $\int_0^t s(t-\tau)^2 u(\tau)d\tau$

- Q.98** The transfer function of the system is given by  $H(s) = \frac{1}{s^2(s-2)}$ . The impulse response of the system [GATE EC 2001-Kanpur]

(A)  $(t^2 \otimes e^{2t})u(t)$       (B)  $(t \otimes e^{2t})u(t)$   
 (C)  $(t^2 e^{-2t})u(t)$       (D)  $(te^{-2t})u(t)$

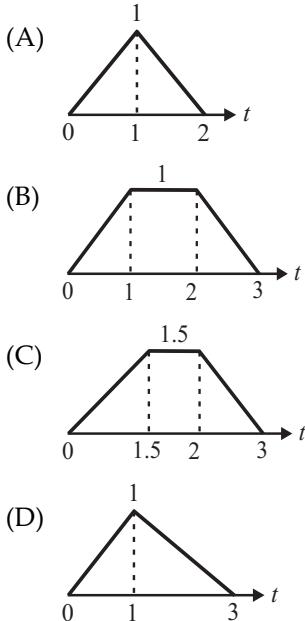
- Q.99** Given a relationship between the input  $u(t)$  and the output  $y(t)$  to be [GATE EE 2001-Kanpur]

$$y(t) = \int_0^t (2+t-\tau)e^{-3(t-\tau)}u(\tau)d\tau$$

The transfer function  $Y(s)/U(s)$  is

- (A)  $\frac{2e^{-2s}}{s+3}$       (B)  $\frac{s+2}{(s+3)^2}$   
 (C)  $\frac{2s+5}{s+3}$       (D)  $\frac{2s+7}{(s+3)^2}$

- Q.100** Let  $u(t)$  be the step function. Which of the following waveforms in below figures corresponds to the convolution of  $u(t) - u(t-1)$  with  $u(t) - u(t-2)$ ? [GATE EC 2000-Kharagpur]



- Q.101** Given that

$$L[f(t)] = \frac{s+2}{s^2+1}; L[g(t)] = \frac{s^2+1}{(s+3)(s+2)}$$

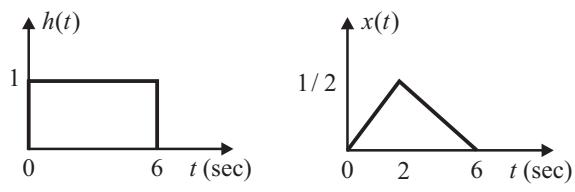
$$h(t) = \int_0^t f(\tau)g(t-\tau)d\tau. \text{ Then } L[h(t)] \text{ is}$$

[GATE EC 2000-Kharagpur]

- (A)  $\frac{s^2+1}{s+3}$       (B)  $\frac{1}{s+3}$   
 (C)  $\frac{s^2+1}{(s+3)(s+2)} + \frac{s+2}{s^2+1}$       (D) None of these

- Q.102** The impulse response and the excitation function of a linear time invariant causal system are shown in figures (a) and (b) respectively. The output of the system at  $t = 2$  sec is equal to

[GATE EC 1990-Bangalore]



- (A) 0                                 (B)  $1/2$   
 (C)  $3/2$                                  (D) 1

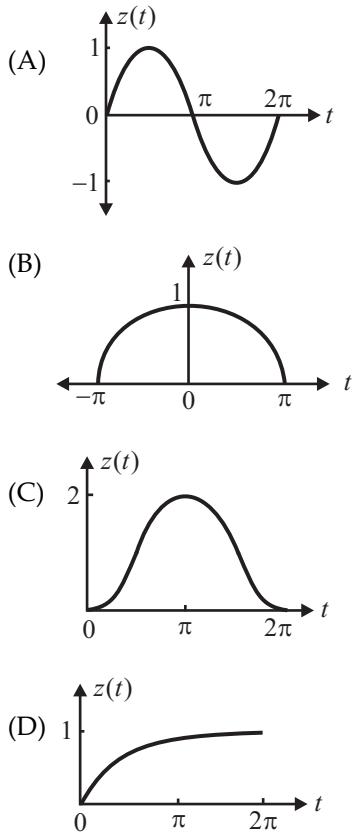
- Q.103** If  $x(t) = \cos(t)u(t)$  and  $h(t) = \sin(t)u(t)$  then  
 $y(t) = x(t) \otimes h(t)$  will be

- (A)  $t \cdot \cos t \cdot u(t)$                      (B)  $-\frac{t}{2} \cdot \cos t \cdot u(t)$   
 (C)  $\frac{t}{2} \cdot \sin t \cdot u(t)$                      (D)  $t \cdot \sin t \cdot u(t)$

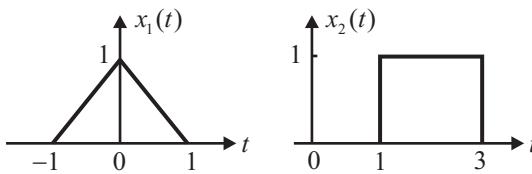
- Q.104** Two continuous time signals are given as follows

$$x(t) = \begin{cases} \sin t, & 0 \leq t \leq 2\pi \\ 0, & \text{elsewhere} \end{cases} \quad \text{and} \quad y(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Convolution  $z(t) = x(t) \otimes y(t)$  is

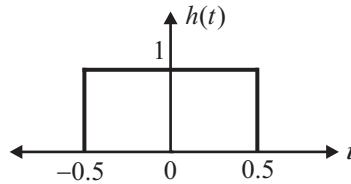


- Q.105** Convolution of  $x_1(t)$  and  $x_2(t)$ , if  $x_1(t)$  and  $x_2(t)$  are the waveforms as shown below, will be



- (A)  $\frac{1}{2} [t^2 - 2(t-1)^2 + 2(t-3)^2 - (t-4)^2]$   
 (B)  $\frac{1}{2} [t^2 - (t-1)^2 + (t-3)^2 - (t-4)^2]$   
 (C)  $\frac{1}{2} [t^2 - 2(t-1)^2 - 2(t-3)^2 - (t-4)^2]$   
 (D) None of the above

- Q.106** A signal  $x(t) = e^{j\omega_0 t}$  is convolved with another signal  $h(t)$  shown in figure.



Let  $y(t) = x(t) \otimes h(t)$ , then for what value of  $\omega_0$ ,  $y(0) = 0$ ?

- (A)  $\pi$                                      (B)  $\pi/2$   
 (C)  $\pi/4$                                      (D)  $2\pi$

- Q.107** The transfer function of a system is  $\frac{Y(s)}{R(s)} = \frac{s}{s+2}$ .

The steady state output  $y(t)$  is  $A \cos(2t + \phi)$  for the input  $\cos(2t)$ . The values of  $A$  and  $\phi$ , respectively are     [GATE EE 2016-Bangalore]

- (A)  $\frac{1}{\sqrt{2}}, -45^\circ$                      (B)  $\frac{1}{\sqrt{2}}, +45^\circ$   
 (C)  $\sqrt{2}, -45^\circ$                              (D)  $\sqrt{2}, +45^\circ$

- Q.108** The solution of the differential equation, for  $t > 0$ ,  $y''(t) + 2y'(t) + y(t) = 0$  with initial condition  $y(0) = 0$  and  $y'(0) = 1$ , is ( $u(t)$  denotes the unit step function),     [GATE EE 2016-Bangalore]

- (A)  $te^{-t}u(t)$                              (B)  $(e^{-t} - te^{-t})u(t)$   
 (C)  $(e^{-t} + te^{-t})u(t)$                      (D)  $e^{-t}u(t)$

- Q.109** Consider a linear time-invariant system with transfer function     [GATE EE 2016-Bangalore]

$$H(s) = \frac{1}{(s+1)}$$

If the input is  $\cos(t)$  and the steady state output is  $A \cos(t + \alpha)$ , then the value of  $A$  is \_\_\_\_\_.

- Q.110** Let  $y(x)$  be the solution of the differential equation  $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$  with initial conditions  $y(0) = 0$  and  $\left.\frac{dy}{dx}\right|_{x=0}$ . Then the value of  $y(1)$  is \_\_\_\_\_.

[GATE EE 2016-Bangalore]

- Q.111** The input  $i(t) = 2 \sin(3t + \pi)$  is applied to a system whose transfer function  $G(s) = \frac{8}{(s+10)^2}$ . The amplitude of the output of the system is \_\_\_\_\_

[GATE IN 2016-Bangalore]

- Q.112** Consider the differential equation  $\frac{dx}{dt} = 10 - 0.2x$  with initial condition  $x(0) = 1$ . The response  $x(t)$  for  $t > 0$  is [GATE EC 2015-Kharagpur]

- (A)  $2 - e^{-0.2t}$       (B)  $2 - e^{0.2t}$   
 (C)  $51 - 49e^{-0.2t}$       (D)  $51 - 49e^{0.2t}$

- Q.113** A moving average function is given by  $y(t) = \frac{1}{T} \int_{t-T}^t u(\tau) d\tau$ . If the input  $u$  is a sinusoidal signal of frequency  $\frac{1}{2T}$  Hz, then in steady state, the output  $y$  will lag  $u$  (in degree) by \_\_\_\_\_.

[GATE EE 2015-Kanpur]

- Q.114** A system is described by the following differential equation, where  $u(t)$  is the input to the system and  $y(t)$  is the output of the system.

$$y'(t) + 5y(t) = u(t)$$

- When  $y(0) = 1$  and  $u(t)$  is a unit step function,  $y(t)$  is [GATE EC 2014-Kharagpur]

- (A)  $0.2 + 0.8e^{-5t}$       (B)  $0.2 - 0.2e^{-5t}$   
 (C)  $0.8 + 0.2e^{-5t}$       (D)  $0.8 - 0.8e^{-5t}$

- Q.115** Consider an LTI system with transfer function

$$H(s) = \frac{1}{s(s+4)} \quad [\text{GATE EE 2014-Kharagpur}]$$

- If the input to the system is  $\cos(3t)$  and the steady state output is  $A \sin(3t + \alpha)$ , then the value of  $A$  is

- (A)  $\frac{1}{30}$       (B)  $\frac{1}{15}$   
 (C)  $\frac{3}{4}$       (D)  $\frac{4}{3}$

- Q.116** The transfer function of a system is given by

$$G(s) = \frac{e^{-s/500}}{s+500} \quad [\text{GATE IN 2014-Kharagpur}]$$

The input to the system is  $x(t) = \sin 100\pi t$ . In periodic steady state the output of the system is found to be  $y(t) = A \sin(100\pi t - \phi)$ . The phase angle ( $\phi$ ) in degree is \_\_\_\_\_.

- Q.117** A system is described by the differential equation

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y(t) = x(t)$$

Let  $x(t)$  be a rectangular pulse given by

$$x(t) = \begin{cases} 1 & 0 < t < 2 \\ 0 & \text{otherwise} \end{cases}$$

Assuming that  $y(0) = 0$ ,  $\frac{dy}{dt} = 0$  at  $t = 0$ , the Laplace transform of  $y(t)$  is

[GATE EC, EE, IN 2013-Bombay]

- (A)  $\frac{e^{-2s}}{s(s+2)(s+3)}$       (B)  $\frac{1-e^{-2s}}{s(s+2)(s+3)}$   
 (C)  $\frac{e^{-2s}}{(s+2)(s+3)}$       (D)  $\frac{1-e^{-2s}}{(s+2)(s+3)}$

- Q.119** A continuous time LTI system is described by

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = 2\frac{dx(t)}{dt} + 4x(t)$$

Assuming zero initial conditions, the response  $y(t)$  of the above system for the input  $x(t) = e^{-2t}u(t)$  is given by

[GATE EC 2010-Guwahati]

- (A)  $[e^t - e^{3t}]u(t)$       (B)  $[e^{-t} - e^{-3t}]u(t)$   
 (C)  $[e^{-t} + e^{-3t}]u(t)$       (D)  $[e^t + e^{3t}]u(t)$

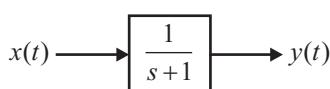
- Q.120** A system with the transfer function  $\frac{Y(s)}{X(s)} = \frac{s}{s+p}$  has an output  $y(t) = \cos\left(2t - \frac{\pi}{3}\right)$

for the input signal  $x(t) = p \cos\left(2t - \frac{\pi}{2}\right)$ . Then, the system parameter  $p$  is

[GATE EC 2010-Guwahati]

- (A)  $\sqrt{3}$       (B)  $\frac{2}{\sqrt{3}}$   
 (C) 1      (D)  $\frac{\sqrt{3}}{2}$

- Q.121** In the system shown below,  $x(t) = \sin t u(t)$ . In the steady state, the response  $y(t)$  will be



[GATE EC 2006-Kharagpur]

- (A)  $\frac{1}{\sqrt{2}} \sin\left(t - \frac{\pi}{4}\right)$       (B)  $\frac{1}{\sqrt{2}} \sin\left(t + \frac{\pi}{4}\right)$   
 (C)  $\frac{1}{\sqrt{2}} e^{-t} \sin(t)$       (D)  $\sin(t) - \cos(t)$

- Q.122** A solution for the differential equation  $\dot{x}(t) + 2x(t) = \delta(t)$  with initial condition  $x(0-) = 0$  is

[GATE EC 2006-Kharagpur]

- (A)  $e^{-2t}u(t)$       (B)  $e^{2t}u(t)$   
 (C)  $e^{-t}u(t)$       (D)  $e^t u(t)$

- Q.123** A system described by the following differential equation  $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = x(t)$  is initially at rest. For input  $x(t) = 2u(t)$ , the output  $y(t)$  is

[GATE EC 2004-Delhi]

- (A)  $[1 - 2e^{-t} + e^{-2t}]u(t)$   
 (B)  $[1 + 2e^{-t} - e^{-2t}]u(t)$   
 (C)  $[0.5 + e^{-t} + 1.5e^{-2t}]u(t)$   
 (D)  $[0.5 + 2e^{-t} + 2e^{-2t}]u(t)$

- Q.124** The unit step response  $y(t)$  of a linear system is  $y(t) = [1 - 3e^{-t} + 3e^{-2t}]u(t)$ . For the system function, the frequency at which the forced response become zero is

[ESE EC 2012]

- (A)  $\frac{1}{\sqrt{2}} \text{ rad/s}$       (B)  $\frac{1}{2} \text{ rad/s}$   
 (C)  $\sqrt{2} \text{ rad/s}$       (D)  $2 \text{ rad/s}$

- Q.125** Which one of the following is an Eigen function of the class of all continuous-time, linear, time-invariant systems ( $u(t)$  denotes the unit-step function)?

[GATE EC 2016-Bangalore]

- (A)  $e^{j\omega_0 t}u(t)$       (B)  $\cos(\omega_0 t)$   
 (C)  $e^{j\omega_0 t}$       (D)  $\sin(\omega_0 t)$

- Q.126** The response of the system  $G(s) = \frac{s-2}{(s+1)(s+3)}$  to the unit step input  $u(t)$  is  $y(t)$ .

The value of  $\frac{dy}{dt}$  at  $t = 0^+$  is \_\_\_\_\_.

[GATE EC 2016-Bangalore]

- Q.127** The output of a continuous-time, linear time-invariant system is denoted by  $T\{x(t)\}$  where  $x(t)$  is the input signal. A signal  $z(t)$  is called

eigen-signal of the system  $T$ , when  $T\{z(t)\} = \gamma z(t)$ , where  $\gamma$  is a complex number, in general, and is called an eigenvalue of  $T$ . Suppose the impulse response of the system  $T$  is real and even. Which of the following statements is TRUE? [GATE EE 2016-Bangalore]

(A)  $\cos(t)$  is an eigen-signal but  $\sin(t)$  is not  
 (B)  $\cos(t)$  and  $\sin(t)$  are both eigen-signals but with different eigenvalues  
 (C)  $\sin(t)$  is an eigen-signal but  $\cos(t)$  is not  
 (D)  $\cos(t)$  and  $\sin(t)$  are both eigen-signals with identical eigenvalues

### Practice (objective & Num Ans) Questions :

- Q.1** Let the signal  $f(t) = 0$  outside the interval  $[T_1, T_2]$ , where  $T_1$  and  $T_2$  are finite. Furthermore,  $|f(t)| < \infty$ . The region of convergence (ROC) of the signal's bilateral Laplace transform  $F(s)$  is

[GATE EC 2015-Kharagpur]

- (A) A parallel strip containing the  $j\Omega$  axis  
 (B) A parallel strip not containing the  $j\Omega$  axis  
 (C) The entire  $s$ -plane  
 (D) A half plane containing the  $j\Omega$  axis

- Q.2**  $u(t)$  represents the unit step function. The Laplace transform of  $u(t-\tau)$  is

[GATE IN 2010-Guwahati]

- (A)  $\frac{1}{s\tau}$       (B)  $\frac{1}{s-\tau}$   
 (C)  $\frac{e^{-s\tau}}{s}$       (D)  $e^{-s\tau}$

- Q.3** Given that  $F(s)$  is the one-sided Laplace transform of  $f(t)$ , the Laplace transform of  $\int_0^t f(\tau) d\tau$  is

[GATE EC 2009-Roorkee]

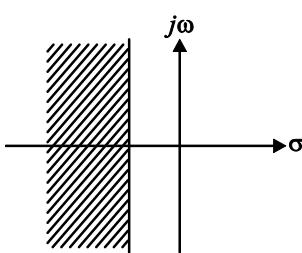
- (A)  $sF(s) - f(0)$       (B)  $\frac{1}{s} F(s)$   
 (C)  $\int_0^s F(\tau) d\tau$       (D)  $\frac{1}{s} [F(s) - f(0)]$

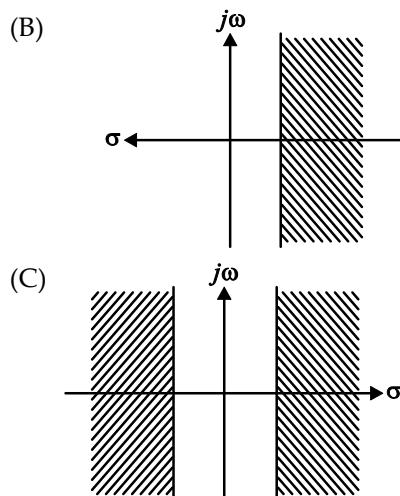
- Q.4** The running integrator given by  $y(t) = \int_{-\infty}^t x(\tau) d\tau$

[GATE EE 2006-Kharagpur]

- (A) has no finite singularities in its double sided Laplace transform  $Y(s)$   
 (B) produces a bounded output for every causal bounded input

- (C) produces a bounded output for every anti causal bounded input  
 (D) has no finite zeros in its double sided Laplace transform  $Y(s)$
- Q.5** In what range should  $\text{Re}(s)$  remain so that the Laplace transform of the function  $e^{(a+2)t+5}$  exist  
**[GATE EC 2005-Bombay]**  
 (A)  $\text{Re}(s) > a + 2$       (B)  $\text{Re}(s) > a + 7$   
 (C)  $\text{Re}(s) < 2$       (D)  $\text{Re}(s) > a + 5$
- Q.6** If  $L[f(t)] = F(s)$ , then  $L[f(t-T)]$  is equal to  
**[GATE EC 1999-Bombay]**  
 (A)  $e^{sT}F(s)$       (B)  $e^{-sT}F(s)$   
 (C)  $\frac{F(s)}{1+e^{sT}}$       (D)  $\frac{F(s)}{1-e^{-sT}}$
- Q.7** The Laplace Transform of  $e^{\alpha t} \cos(\omega t)$  is  
**[GATE EC 1997-Madras]**  
 (A)  $\frac{(s-\alpha)}{(s-\alpha)^2 + \omega^2}$       (B)  $\frac{(s+\alpha)}{(s+\alpha)^2 + \omega^2}$   
 (C)  $\frac{1}{(s-\alpha)^2}$       (D) None of these
- Q.8** If  $x(t)$  is of finite duration and is absolutely integrable, then the 'region of convergence' is  
 (A) Entire  $s$  plane      **[ESE EC 2010]**  
 (B) From  $\sigma = -1$  to  $\sigma = +\infty$   
 (C) From  $\sigma = +1$  to  $\sigma = -\infty$   
 (D) Entire right half plane
- Q.9** What is the inverse Laplace transform of  $e^{-as} / s$ ?  
**[ESE EC 2004]**  
 (A)  $e^{-at}$       (B)  $u(t-a)$   
 (C)  $\delta(t-a)$       (D)  $(t-a)u(t-a)$
- Q.10** Given  $f(t) = F(s) = \int_0^\infty f(t)e^{-st} dt$ ,  
**[ESE EC 2003]**  
 Which of the following expressions are correct?  
 1.  $L[f(t-a)u(t-a)] = F(s)e^{-sa}$   
 2.  $L[t.f(t)] = \frac{-dF(s)}{ds}$   
 3.  $L[(t-a)f(t)] = asF(s)$   
 4.  $L\left[\frac{df(t)}{dt}\right] = sF(s) - f(0)$
- Select the correct answer using the codes given below  
 (A) 1, 2 and 3      (B) 1, 2 and 4  
 (C) 2, 3 and 4      (D) 1, 3 and 4

- Q.11** If  $\delta(t)$  denotes a unit impulse, then the Laplace transform of  $\frac{d^2\delta(t)}{dt^2}$  will be      **[ESE EC 1997]**  
 (A) 1      (B)  $s^2$   
 (C)  $s$       (D)  $s^{-2}$
- Q.12** The following table gives some time functions and their Laplace transforms :      **[ESE EC 1995]**
- | $f(t)$         | $F(s)$  |
|----------------|---------|
| 1. $\delta(t)$ | $s$     |
| 2. $u(t)$      | $1/s$   |
| 3. $tu(t)$     | $2/s^2$ |
| 4. $t^2u(t)$   | $2/s^3$ |
- Of these, the correctly matched pairs are  
 (A) 2 and 4      (B) 1 and 4  
 (C) 3 and 4      (D) 1 and 2
- Q.13** **Assertion (A)** : The Laplace transform of  $e^{-at} \sin \omega t$  is  $\frac{\omega}{(s+a)^2 + \omega^2}$   
**Reason (R)** : If the Laplace transform of  $f(t) = F(s)$ , then Laplace transform of  $e^{-at}f(t)$  is  $F(s+a)$ .  
**[ESE EC 1992]**
- (A) Both A and R are true and R is the correct explanation of A  
 (B) Both A and R are true and R is not a correct explanation of A  
 (C) A is true but R is false  
 (D) A is false but R is true
- Q.14** Which of the following statements are correct?  
 (A) Region of convergence of multiple order pole system is outside the circle of highest pole, for causal system  
 (B) For system to be stable, it's ROC should include  $j\omega$  axis.  
 (C) For all pass filter, poles and zeros exist in reciprocal relationship  
 (D) All are correct
- Q.15** Which of the following represent stable system?  
 (A) 



(D) None of the above

- Q.16** Let  $f(t) \xrightarrow{L} F(s)$  be a Laplace transform pair where  $F(s) = \frac{1}{s^2} [1 - e^{-2s} - 2s.e^{-4s}]$ . The Laplace transform of signal  $f(t-3)$  will be
- (A)  $\frac{3}{s^2} [1 - e^{-2s} - 2s.e^{-4s}]$   
 (B)  $\frac{1}{s^2} [1 - e^{-5s} - 2s.e^{-7s}]$   
 (C)  $\frac{1}{s} [e^{3s} - e^s - 2s.e^{-s}]$   
 (D)  $\frac{1}{s^2} [e^{-3s} - e^{-5s} - 2s.e^{-7s}]$

- Q.17** Let  $X(s)$  be the Laplace transform of a signal

$x(t)$  given as  $X(s) = \frac{4s}{s^2 + 4}$ . The Laplace transform of signal  $e^{-t}x(t)$  is

(A)  $\frac{4(s+1)}{(s^2 + 5)}$   
 (B)  $\frac{4(s+1)}{(s^2 + 2s + 5)}$   
 (C)  $\frac{4(s-1)}{(s^2 - 2s + 5)}$   
 (D)  $\frac{4(s-1)}{(s^2 + 2s + 5)}$

- Q.18** The Laplace transform of a continuous time signal  $x(t)$  is  $X(s) = \frac{s+1}{s^2 + 5s + 7}$ . What will be the Laplace transform of  $y(t) = x(3t-4)u(3t-4)$ ?

(A)  $\left(\frac{s+3}{s^2 + 15s + 63}\right)e^{-(4/3)s}$  (B)  $\left(\frac{s+3}{s^2 + 15s + 63}\right)e^{-4s}$   
 (C)  $\frac{1}{3}\left(\frac{s+1}{s^2 + 5s + 7}\right)e^{-4s}$  (D)  $\left(\frac{s+1}{s^2 + 15s + 21}\right)e^{-(4/3)s}$

- Q.19** The Laplace transform of signal  $x(t) = t^2 \sin t$  is equal to
- (A)  $\frac{2(3s^2 - 1)}{(s^2 + 1)^3}$   
 (B)  $\frac{1}{(s^4 + 1)}$

(C)  $\frac{-2s}{(s^2 + 1)^2}$  (D) None of these

- Q.20** A continuous time signal has the Laplace transform  $X(s) = \frac{s+1}{s^2 + 4s + 5}$ . The Laplace transform of  $\int_0^t x(\tau)d\tau$  is
- (A)  $\frac{s(s+1)}{s^2 + 4s + 5}$   
 (B)  $\frac{(s^2 + 1)}{(s^2 + 4s + 5)}$   
 (C)  $\frac{(s^2 + 1)}{s(s^2 + 4s + 5)}$   
 (D)  $\frac{(s+1)}{s(s^2 + 4s + 5)}$

- Q.21** The Laplace transform of signal  $\int_0^t e^{-3\tau} \cos 2\tau d\tau$  is
- (A)  $\frac{-(s+3)}{s[(s+3)^2 + 4]}$   
 (B)  $\frac{(s+3)}{s[(s+3)^2 + 4]}$   
 (C)  $\frac{s(s+3)}{(s+3)^2 + 4}$   
 (D)  $\frac{-s(s+3)}{(s+3)^2 + 4}$

- Q.22** The Laplace transform of a signal  $x(t)$  is,  $X(s) = s^{-2} \frac{d}{ds} \left( \frac{e^{-3s}}{s} \right)$ , ROC:  $\text{Re}(s) > 0$ . The signal  $x(t)$  is
- (A)  $\frac{1}{6} e^{-3t} [t^3 - 27t + 54] u(t-3)$   
 (B)  $\frac{1}{6} e^{3t} [t^3 - 27t + 54] u(t)$   
 (C)  $-\frac{1}{6} [t^3 - 27t + 54] u(t-3)$   
 (D)  $\frac{1}{6} [t^3 - 27t + 54] u(t-3)$

### Common Data for Questions 23 to 26

Consider the Laplace transform pair given below.

$$\cos(2t)u(t) \xrightarrow{LT} X(s)$$

- Q.23** The time signal corresponding to  $(s+1)X(s)$  is
- (A)  $\delta(t) + [\cos 2t - 2 \sin 2t]u(t)$   
 (B)  $\left(\cos 2t + \frac{\sin 2t}{2}\right)u(t)$   
 (C)  $[\cos 2t - 2 \sin 2t]u(t)$   
 (D)  $\left(\cos 2t - \frac{\sin 2t}{2}\right)u(t)$

**Q.24** The time signal corresponding to  $X(3s)$  is

- (A)  $\cos\left(\frac{2}{3}t\right)u\left(\frac{t}{3}\right)$       (B)  $\frac{1}{3}\cos\left(\frac{2}{3}t\right)u\left(\frac{t}{3}\right)$   
 (C)  $\cos 6tu(t)$       (D)  $\frac{1}{3}\cos 6tu(t)$

**Q.25** The time signal corresponding to  $\frac{X(s)}{s^2}$  is

- (A)  $4\cos 2tu(t)$       (B)  $\frac{1-\cos 2t}{4}u(t)$   
 (C)  $t^2 \cos 2tu(t)$       (D)  $\frac{\cos 2t}{t^2}u(t)$

**Q.26** The time signal corresponding to  $\frac{d}{ds}[e^{-3s}X(s)]$  is

- (A)  $t \cos\{2(t-3)\}u(t-3)$   
 (B)  $t \cos\{2(t-3)\}u(t)$   
 (C)  $-t \cos\{2(t-3)\}u(t-3)$   
 (D)  $-t \cos\{2(t-3)\}u(t)$

**Q.27** Let  $X(s) = \frac{3s+5}{s^2+10s+21}$  be the Laplace

Transform of a signal  $x(t)$ . Then,  $x(0^+)$  is

[GATE EE 2014-Kharagpur]

- (A) 0      (B) 3  
 (C) 5      (D) 21

**Q.28** If the Laplace transform of a signal

$Y(s) = \frac{1}{s(s-1)}$ , then its final value is

[GATE EC 2007-Kanpur]

- (A) -1      (B) 0  
 (C) 1      (D) Unbounded

**Q.29** The Laplace transform of a function  $f(t)$  is

$F(s) = \frac{5s^2 + 23s + 6}{s(s^2 + 2s + 2)}$ . As  $t \rightarrow \infty$ ,  $f(t)$  approaches

[GATE EE 2005-Bombay]

- (A) 3      (B) 5  
 (C) 17/2      (D)  $\infty$

**Q.30** Consider the function,  $F(s) = \frac{5}{s(s^2 + 3s + 2)}$

where  $F(s)$  is the Laplace Transform of the function  $f(t)$ . The initial value of  $f(t)$  is equal to :

[GATE EE 2004-Delhi]

- (A) 5      (B) 5/2  
 (C) 5/3      (D) 0

**Q.31** The Laplace transform of  $i(t)$  is given by

$$I(s) = \frac{2}{s(1+s)} \quad [\text{GATE EC 2003-Madras}]$$

As  $t \rightarrow \infty$ , the value of  $i(t)$  tends to

- (A) 0      (B) 1  
 (C) 2      (D)  $\infty$

**Q.32** The Final value of a function  $y(t)$  whose

Laplace transform  $Y(s) = \frac{4}{s^2 + 2s + 2}$  is

[GATE IN 2001-Kanpur]

- (A) 4      (B) 2  
 (C) 1      (D) 0

**Q.33** The system  $G(s) = \frac{0.8}{s^2 + s - 2}$  is subjected to a step input. The system output  $y(t)$  as  $t \rightarrow \infty$  is

[GATE IN 1999-Bombay]

- (A) 0.8      (B) 0.4  
 (C) -0.4      (D) Unbounded

**Q.34** If  $L[f(t)] = \frac{\omega}{s^2 + \omega^2}$  then the value of  $\lim_{t \rightarrow \infty} f(t)$

[GATE EC 1998-Delhi]

- (A) cannot be determined      (B) is zero  
 (C) is unity      (D) is infinite

**Q.35** The final value theorem is used to find the :

[GATE EC 1995-Kanpur]

- (A) Steady state value of the system output  
 (B) Initial value of the system output  
 (C) Transient behavior of the system output  
 (D) None of the above

**Q.36** The Laplace transformation of  $f(t)$  is  $F(s)$ .

Given  $F(s) = \frac{\omega}{s^2 + \omega^2}$  the final value of  $f(t)$  is

[GATE EE 1995-Kanpur]

- (A) infinity      (B) zero  
 (C) one      (D) None of these

**Q.37** If  $sF(s) = \frac{K}{(s+1)(s^2+4)}$  then,  $\lim_{t \rightarrow \infty} f(t)$  is given by

[GATE EC 1993-Bombay]

- (A)  $\frac{K}{4}$       (B) Zero  
 (C) Infinite      (D) Undefined

**Q.38** If  $\left(\frac{27s+97}{s^2+33s}\right)$  is the Laplace transform of  $f(t)$ ,

then  $f(0^+)$  is

- (A) Zero      (B) 97/33  
 (C) 27      (D) Infinity

**Q.39** If  $x(t)$  and  $\frac{dx(t)}{dt}$  are Laplace transformable and  $\lim_{t \rightarrow \infty} x(t)$  exists, then  $\lim_{t \rightarrow \infty} x(t)$  is equal to  
 $\lim_{t \rightarrow \infty} x(t) = \lim_{t \rightarrow \infty} x(t) + \lim_{t \rightarrow \infty} \frac{dx(t)}{dt}$  [ESE EC 1998]

- (A)  $\lim_{s \rightarrow \infty} sX(s)$       (B)  $\lim_{s \rightarrow 0} sX(s)$   
 (C)  $\lim_{s \rightarrow \infty} \frac{X(s)}{s}$       (D)  $\lim_{s \rightarrow 0} \frac{X(s)}{s}$

**Q.40** The final value of  $L^{-1}\left[\frac{2s+1}{s^4+8s^3+16s^2+s}\right]$  is [ESE EC 1995]

- (A) Infinity      (B) 2  
 (C) 1      (D) zero

**Q.41** Initial value of  $X(s) = \frac{2s}{s^2+2s+2}$  is  
 (A) 2      (B) zero  
 (C) 1      (D)  $\infty$

**Q.42** Final value of  $X(s) = \frac{2}{s^2+2s+2}$  is  
 (A) 2      (B) zero  
 (C) 1      (D)  $\infty$

**Q.43** The value of  $f(0^+)$  and  $f(\infty)$  for  $F(s) = \frac{1}{s-1}$  is  
 (A) 0, 1      (B) 1, 1  
 (C) 1, 0      (D) None of these

**Q.44** The Laplace transform of  $x(t)$  is  $X(s) = \frac{e^{-3s}(2s^2+1)}{s(s+1)(s+4)}$ . The final value of  $x(t)$  is  
 $\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s) = \lim_{s \rightarrow 0} s \cdot \frac{e^{-3s}(2s^2+1)}{s(s+1)(s+4)} = \lim_{s \rightarrow 0} \frac{e^{-3s}(2s^2+1)}{(s+1)(s+4)}$

- (A) 2      (B)  $1/4$   
 (C)  $-3$       (D) Does not exist

**Q.45** Laplace transform of a signal  $x(t)$  is given as

$$X(s) = \frac{3s^2+4s+1}{s^3+2s^2+s+2} . \text{ The final value of } x(t) \text{ equals to}$$

- (A)  $1/2$       (B)  $3/2$   
 (C) 0      (D) Does not exist

**Q.46** A rectangular current pulse of duration T and magnitude 1 has the Laplace transform

[GATE EE 1999-Bombay]

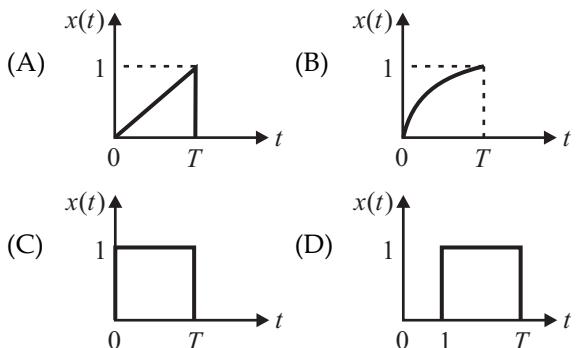
- (A)  $1/s$       (B)  $(1/s)\exp(-Ts)$   
 (C)  $(1/s)\exp(Ts)$       (D)  $(1/s)[1-\exp(-Ts)]$

**Q.47**  $F(s) = (1-e^{-sT})/s$  is the Laplace transform of  
 [GATE IN 1997-Madras]

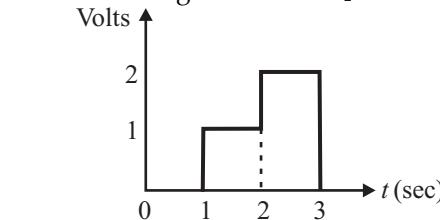
- (A) A pulse of width T  
 (B) A square wave of period T  
 (C) A unit step delayed by T  
 (D) A ramp delayed by T

**Q.48** The Laplace transform  $X(s)$  of a function  $x(t)$  is  $X(s) = \left( \frac{1-e^{-sT}}{s} \right)$  [ESE EC 2006]

What is the wave shape of  $x(t)$ ?

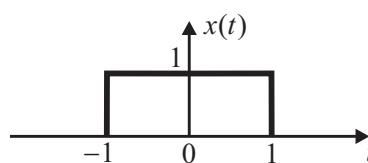


**Q.49** The Laplace transform of the waveform shown in the below figure is [ESE EC 2003]



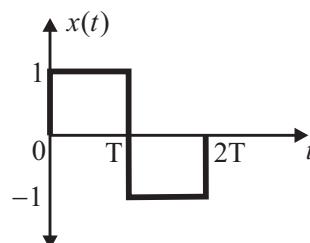
- (A)  $\frac{1}{s}[e^s + e^{2s} + 2e^{3s}]$   
 (B)  $\frac{1}{s}[e^s + e^{2s} + 2e^{-3s}]$   
 (C)  $\frac{1}{s}[e^{-s} + e^{-2s} + 2e^{3s}]$   
 (D)  $\frac{1}{s}[e^{-s} + e^{-2s} - 2e^{-3s}]$

**Q.50** The Laplace transform of given function  $x(t)$  will be



- (A)  $\frac{e^s - e^{-s}}{s}$       (B)  $\frac{e^s + e^{-s}}{s}$   
 (C)  $\frac{e^{-s} - e^s}{s}$       (D)  $\frac{e^{-s} + e^s}{s}$

**Q.51** The Laplace transform of given function  $x(t)$  will be



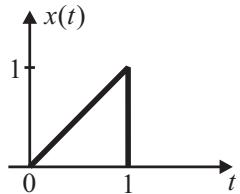
(A)  $\frac{(1+e^{-sT})^2}{s}$

(B)  $\frac{(1-e^{-sT})}{s}$

(C)  $\frac{(1+e^{-sT})}{s}$

(D)  $\frac{(1-e^{-sT})^2}{s}$

**Q.52** The Laplace transform of given function  $x(t)$  will be



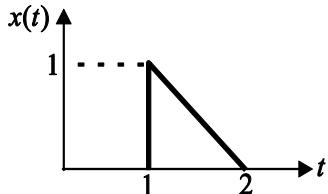
(A)  $\frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s}$

(B)  $\frac{1}{s^2} - \frac{e^{-s}}{s^2} + \frac{e^{-s}}{s}$

(C)  $\frac{1}{s^2} + \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s}$

(D)  $\frac{1}{s^2} + \frac{e^{-s}}{s^2} + \frac{e^{-s}}{s}$

**Q.53** The Laplace transform of the waveform given below is



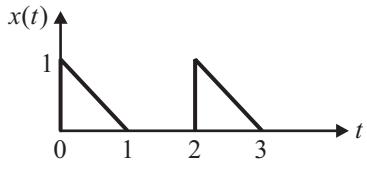
(A)  $\frac{1}{s^2} [e^{-2s} - e^{-s} + se^{-s}]$

(B)  $\frac{1}{s^2} [e^{-2s} - e^s + se^{-s}]$

(C)  $\frac{1}{s^2} [e^{-2s} - e^{-s} - se^{-s}]$

(D)  $\frac{1}{s^2} [e^{-2s} - e^{-s} + se^s]$

**Q.54** Laplace transform of  $x(t)$  will be



(A)  $\frac{s-1-e^{-s}}{s^2(1-e^{-2s})}$

(B)  $\frac{s-1+e^{-s}}{s^2(1-e^{-2s})}$

(C)  $\frac{s-1+e^{-s}}{s^2(1+e^{-2s})}$

(D)  $\frac{s-1-e^{-s}}{s^2(1+e^{-2s})}$

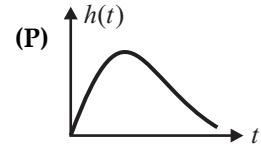
**Q.55** Match the following transfer functions and impulse responses

[GATE EE 1992-Delhi]

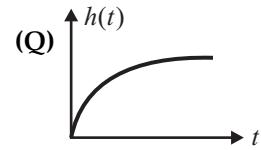
Transfer function

(A)  $\frac{1}{s(s+1)}$

Impulse response

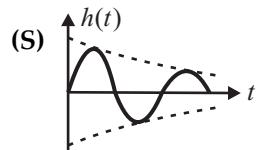
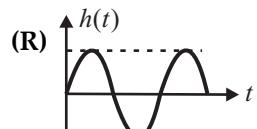


(B)  $\frac{1}{(s+1)^2}$



(C)  $\frac{1}{s(s+1)+1}$

(D)  $\frac{1}{s^2+1}$



Codes : A B C D

(A) Q P S R

(B) Q P R S

(C) P Q S R

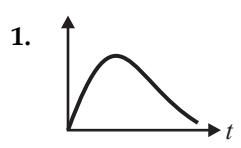
(D) P Q R S

**Q.56** Match List-I (system function) with List-II (Impulse response) and the select the correct answer using the codes given below the lists :

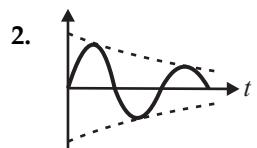
[ESE EC 2000]

List-I

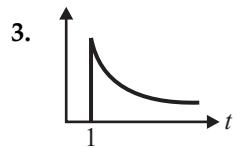
A.  $\frac{e^{-s}}{s+1}$



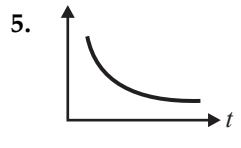
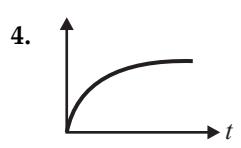
B.  $\frac{1}{s^2+s+1}$



C.  $\frac{1}{(s+1)^2}$



D.  $\frac{1}{s^2+s}$



Codes : A B C D

(A) 3 4 1 2

(B) 5 2 3 4

(C) 3 2 1 4

(D) 5 4 3 2

**Q.57** Inverse Laplace transform of the function

$$\frac{2s+5}{s^2+5s+6}$$

[ESE EC 1999]

(A)  $2 \exp(-2.5t) \cos h(0.5t)$

- (B)  $\exp(-2t) - \exp(-3t)$   
 (C)  $2\exp(-2.5t)\sin h(0.5t)$   
 (D)  $2\exp(-2.5t)\cos(0.5t)$

**Q.58** Given the Laplace transform,  $V(s) = \int_0^\infty e^{-st} v(t) dt$ .

The inverse transform  $v(t)$  is [ESE EC 1993]

- (A)  $\int_{-\infty}^{\sigma+j\infty} V(s) ds$       (B)  $\frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} e^{st} V(s) ds$   
 (C)  $\frac{1}{2\pi j} \int_0^\infty e^{st} V(s) ds$       (D)  $\frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} e^{-st} V(s) ds$

**Common Data for Questions 59 to 61**

Let  $X(s) = \frac{1}{s(s+1)^2}$ , the inverse LT of  $X(s)$

for

**Q.59** ROC :  $\text{Re}(s) > 0$

- (A)  $[1-e^{-t}-te^{-t}]u(t)$       (B)  $[1+e^{-t}+te^{-t}]u(t)$   
 (C)  $[1+e^{-t}-te^{-t}]u(t)$       (D)  $[1-e^{-t}+te^{-t}]u(t)$

**Q.60** ROC :  $-1 < \text{Re}(s) < 0$

- (A)  $u(-t)+[1+t]e^{-t}u(t)$       (B)  $-u(-t)-[1+t]e^{-t}u(t)$   
 (C)  $u(-t)-[1+t]e^{-t}u(t)$       (D)  $-u(-t)+[1+t]e^{-t}u(t)$

**Q.61** ROC :  $\text{Re}(s) < -1$

- (A)  $[1+e^{-t}+te^{-t}]u(-t)$   
 (B)  $[-1-e^{-t}+te^{-t}]u(-t)$   
 (C)  $[-1+e^{-t}+te^{-t}]u(-t)$   
 (D)  $[-1-e^{-t}-te^{-t}]u(-t)$

**Q.62** Let  $x(t)$  be a CT signal and  $X(s)$  be its Laplace transform given as

$$X(s) = \frac{5}{(s^2 + 3s - 4)}$$

Match inverse Laplace transform  $L_1, L_2, L_3$  with Region of convergence  $R_1, R_2, R_3$ .

**Inverse Laplace transform ROC**

- $L_1 : x(t) = -e^{-4t}u(t) - e^t u(-t)$        $R_1 : \text{Re}(s) > 1$   
 $L_2 : x(t) = -e^{-4t}u(t) + e^t u(t)$        $R_2 : \text{Re}(s) < -4$   
 $L_3 : x(t) = e^{-4t}u(-t) - e^t u(-t)$        $R_3 : -4 < \text{Re}(s) < 1$   
 (A)  $(L_1, R_3), (L_2, R_2), (L_3, R_1)$   
 (B)  $(L_1, R_3), (L_2, R_1), (L_3, R_2)$   
 (C)  $(L_1, R_1), (L_2, R_2), (L_3, R_3)$   
 (D)  $(L_1, R_2), (L_2, R_1), (L_3, R_3)$

**Q.63** The unilateral Laplace transform of a signal  $x(t)$  is,  $X(s) = \frac{3s^2 + 10s + 10}{(s+2)(s^2 + 6s + 10)}$ . The time signal  $x(t)$  is

- (A)  $[e^{-2t} + 2e^{-3t} \cos t + 2e^{-3t} \sin t]u(t)$   
 (B)  $[e^{-2t} + 2e^{-3t} \cos t - 6e^{-3t} \sin t]u(t)$   
 (C)  $[e^{-2t} + 2e^{-3t} \cos t - 2e^{-3t} \sin t]u(t)$   
 (D)  $[9e^{-2t} - 6e^{-3t} \cos t + 3e^{-3t} \sin t]u(t)$

**Q.64** Consider a transfer function  $G_p(s) = \frac{ps^2 + 3ps - 2}{s^2 + (3+p)s + (2-p)}$  with  $p$  a positive parameter. The maximum value of  $p$  until which  $G_p$  remain stable is \_\_\_\_.

[GATE EC 2014-Kharagpur]

**Q.65** The input  $-3e^{2t}u(t)$ , where  $u(t)$  is the unit step function, is applied to a system with transfer function  $\frac{s-2}{s+3}$ . If the initial value of the output is  $-2$ , then the value of the output at steady state is \_\_\_\_. [GATE EC 2014-Kharagpur]

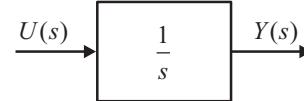
**Q.66** A causal LTI system has zero initial conditions and impulse response  $h(t)$ . Its input  $x(t)$  and output  $y(t)$  are related through the linear constant-coefficient differential equation

$$\frac{d^2y(t)}{dt^2} + \alpha \frac{dy(t)}{dt} + \alpha^2 y(t) = x(t)$$

Let another signal  $g(t)$  be defined as,  $g(t) = \alpha^2 \int_0^t h(\tau) d\tau + \frac{dh(t)}{dt} + \alpha h(t)$ . If  $G(s)$  is the Laplace transform of  $g(t)$ , then the number of poles of  $G(s)$  is \_\_\_\_.

[GATE EC 2014-Kharagpur]

**Q.67** Assuming zero initial condition, the response  $y(t)$  of the system given below to a unit step input  $u(t)$  is [GATE EC, EE, IN 2013-Bombay]



- (A)  $u(t)$       (B)  $tu(t)$   
 (C)  $\frac{t^2}{2}u(t)$       (D)  $e^{-t}u(t)$

**Q.68** An input  $x(t) = \exp(-2t)u(t) + \delta(t-6)$  is applied to an LTI system with impulse response  $h(t) = u(t)$ . The output is

[GATE EC 2011-Madras]

- (A)  $[1 - \exp(-2t)]u(t) + u(t+6)$   
 (B)  $[1 - \exp(-2t)]u(t) + u(t-6)$   
 (C)  $0.5[1 - \exp(-2t)]u(t) + u(t+6)$   
 (D)  $0.5[1 - \exp(-2t)]u(t) + u(t-6)$

**Q.69** A linear, time-invariant, causal continuous time system has a rational transfer function with simple poles at  $s = -2$  and  $s = -4$ , and one simple zero at  $s = -1$ . A unit step  $u(t)$  is applied at the input of the system. At steady state, the output has constant value of 1. The impulse response of this system is

[GATE EC 2008-Bangalore]

- (A)  $[\exp(-2t) + \exp(-4t)]u(t)$   
 (B)  $[-4\exp(-2t) + 12\exp(-4t) - \exp(-t)]u(t)$   
 (C)  $[-4\exp(-2t) + 12\exp(-4t)]u(t)$   
 (D)  $[-0.5\exp(-2t) + 1.5\exp(-4t)]u(t)$

**Q.70** A function  $y(t)$  satisfies the following differential equation  $\frac{dy(t)}{dt} + y(t) = \delta(t)$  where  $\delta(t)$  is the delta function. Assuming zero initial condition, and denoting the unit step function by  $u(t)$ ,  $y(t)$  can be of the form

[GATE EE 2008-Bangalore]

- (A)  $e^t$  (B)  $e^{-t}$   
 (C)  $e^t u(t)$  (D)  $e^{-t} u(t)$

**Q.71** The unit step response of a system starting from rest is given by  $c(t) = 1 - e^{-2t}$  for  $t \geq 0$ . The transfer function of the system is

[GATE EC 2006-Kharagpur]

- (A)  $\frac{1}{1+2s}$  (B)  $\frac{2}{2+s}$   
 (C)  $\frac{1}{2+s}$  (D)  $\frac{2s}{1+2s}$

**Q.72** A causal system having the transfer function  $H(s) = \frac{1}{s+2}$  is excited with  $10u(t)$ . The time at which the output reaches 99% of its steady state value is

[GATE EC 2004-Delhi]

- (A) 2.7 sec (B) 2.5 sec  
 (C) 2.3 sec (D) 2.1 sec

**Q.73** The transfer function of the system described by  $\frac{d^2y}{dt^2} + \frac{dy}{dt} = \frac{du}{dt} + 2u$  with  $u$  as input and  $y$  as output is

[GATE EE 2002-Bangalore]

- (A)  $\frac{s+2}{s^2+s}$  (B)  $\frac{s+1}{s^2+s}$   
 (C)  $\frac{2}{s^2+s}$  (D)  $\frac{2s}{s^2+s}$

**Q.74** A linear time invariant system has an impulse response  $e^{2t}$ ,  $t > 0$ . If the initial conditions are zero and the input is  $e^{3t}$ , the output for  $t > 0$  is

[GATE EC 2000-Kharagpur, ESE EC 2012]

- (A)  $e^{3t} - e^{2t}$  (B)  $e^{5t}$   
 (C)  $e^{3t} + e^{2t}$  (D) None of these

**Q.75** A linear time invariant system initially at rest, when subjected to a unit step input, gives a response  $y(t) = t e^{-t}$ ,  $t > 0$ . The transfer function of the system is

[GATE EE 2000-Kharagpur]

- (A)  $\frac{1}{(s+1)^2}$  (B)  $\frac{1}{s(s+1)^2}$   
 (C)  $\frac{s}{(s+1)^2}$  (D)  $\frac{1}{s(s+1)}$

**Q.76** The unit impulse response of a linear time invariant system is the unit step function  $u(t)$ . For  $t > 0$ , the response of the system to an excitation  $e^{-at}u(t)$ ,  $a > 0$  will be

[GATE EC 1998-Delhi]

- (A)  $a e^{-at}$  (B)  $\left(\frac{1}{a}\right)[1 - e^{-at}]$   
 (C)  $a[1 - e^{-at}]$  (D)  $1 - e^{-at}$

**Q.77** The output of a linear time-invariant control system is  $c(t)$  for a certain input  $r(t)$ . If  $r(t)$  is modified by passing it through a block whose transfer function is  $e^{-s}$  and then applied to the system, the modified output of the system would be

[GATE EE 1998-Delhi]

- (A)  $\frac{r(t)}{1+e^t}$  (B)  $\frac{r(t)}{1+e^{-t}}$   
 (C)  $r(t-1)u(t-1)$  (D)  $r(t)u(t-1)$

**Q.78** The transfer function of a system is the Laplace transform of its

[GATE IN 1998-Delhi]

- (A) Square wave response  
 (B) Step response  
 (C) Ramp response  
 (D) Impulse response

**Q.79** A differentiator has transfer function whose

[GATE EE 1997-Madras]

- (A) phase increases linearly with frequency  
 (B) amplitude remains constant  
 (C) amplitude increases linearly with frequency  
 (D) amplitude decreases linearly with frequency

**Q.80** The unit impulse response of a system is given as  $-4e^{-t} + 6e^{-2t}$ . The step response of the system for  $t \geq 0$  is equal to

[GATE EE 1996-Bangalore]

- (A)  $-3e^{-2t} + 4e^{-t} + 1$       (B)  $-3e^{-2t} + 4e^{-t} - 1$   
 (C)  $3e^{-2t} + 4e^{-t} + 1$       (D)  $-3e^{-2t} - 4e^{-t} - 1$
- Q.81** The transfer function of a linear system is the  
[GATE EC 1995-Kanpur]  
 (A) Ratio of the output  $v_o(t)$ , and input  $v_i(t)$   
 (B) Ratio of the derivatives of the output and the input  
 (C) Ratio of the Laplace transform of the output and that of the input with all initial conditions zeros  
 (D) None of the above
- Q.82** Non minimum phase transfer function is defined as the transfer function  
[GATE EC 1995-Kanpur]  
 (A) which has zeros in the right half s-plane  
 (B) which has zeros only in the left half s-plane  
 (C) which has poles in the right half s-plane  
 (D) which has poles in the left half s-plane
- Q.83** The impulse response of an initially relaxed linear system is  $e^{-2t}u(t)$ . To produce a response of  $te^{-2t}u(t)$ , the input must be equal to  
[GATE EE 1995-Kanpur]  
 (A)  $2e^{-t}u(t)$       (B)  $\frac{1}{2}e^{-2t}u(t)$   
 (C)  $e^{-2t}u(t)$       (D)  $e^{-t}u(t)$
- Q.84** Indicate whether the following statement is TRUE/FALSE. Give reason for your answer. If  $G(s)$  is a stable transfer function, then  
 $F(s) = \frac{1}{G(s)}$  is always a stable transfer function  
[GATE EC 1994-Kharagpur]
- Q.85** The response of an initially relaxed linear constant parameter network to a unit impulse applied at  $t=0$  is  $4e^{-2t}u(t)$ . The response of this network to a unit step function will be  
[GATE EC 1990-Bangalore]  
 (A)  $2[1-e^{-2t}]u(t)$       (B)  $4[e^{-t}-e^{-2t}]u(t)$   
 (C)  $\sin 2t$       (D)  $[1-4e^{-4t}]u(t)$
- Q.86** The unit impulse response of a system is  $-4e^{-t} + 6e^{-2t}$ . The step response of the same system for  $t \geq 0$  is  $Ae^{-t} + Be^{-2t} + C$ , where  $A$ ,  $B$  and  $C$  are respectively [ESE EC 2014]  
 (A)  $-4, -3$  and  $+1$       (B)  $+4, -3$  and  $-1$   
 (C)  $-4, -3$  and  $-1$       (D)  $+4, -3$  and  $+1$
- Q.87** A continuous time system will be BIBO stable if all the Eigen values are [ESE EC 2013]  
 (A) One  
 (B) Distinct and their real parts negative

- (C) Negative  
 (D) Zero
- Q.88** What is the unit impulse response of the system shown in figure for  $t \geq 0$ ? [ESE EC 2011]  

- (A)  $1+e^{-t}$       (B)  $1-e^{-t}$   
 (C)  $e^{-t}$       (D)  $-e^{-t}$
- Q.89** The response of a linear, time-invariant system to a unit step is  $s(t) = [1 - e^{-t/RC}]u(t)$ , where  $u(t)$  is the unit step. What is the impulse response of this system? [ESE EC 2007]  
 (A)  $e^{-t/RC}$       (B)  $e^{-t/RC}u(t)$   
 (C)  $1/RC[e^{-t/RC}u(t)]$       (D)  $\delta(t)$
- Q.90** The relationship between the input  $x(t)$  and the output  $y(t)$  of a system is [ESE EC 2003]  

$$\frac{d^2y}{dt^2} = x(t-2)u(t-2) + \frac{d^2x}{dt^2}$$
 The transfer function of the system is  
 (A)  $1 + \frac{s^2}{e^{2s}}$       (B)  $1 + \frac{e^{-2s}}{s^2}$   
 (C)  $1 + \frac{e^{2s}}{s^2}$       (D)  $1 + \frac{s^2}{e^{-2s}}$
- Q.91** The transfer function of an active network with gain 'K' is given by : [ESE EC 1998]  

$$\frac{V_2(s)}{V_1(s)} = \frac{k}{s^2c^2R^2 + sCR + (3-K)+1}$$
**Assertion (A) :** The networks is unstable for all values of K.  
**Reason (R) :** The poles of the network function depend on the parameter K.  
 (A) Both A and R are true and R is the correct explanation of A  
 (B) Both A and R are true are R is not a correct explanation of A  
 (C) A is true but R is false  
 (D) A is false but R is true
- Q.92** Let  $x(t)$  be the input and  $y(t)$  be the output of a continuous time LTI system described by following differential equation,  

$$\frac{d^2y(t)}{dt^2} + \frac{9y(t)}{dt} + 2y(t) = 5\frac{dx(t)}{dt} + 2x(t)$$
 The transfer function of the system is :  
 (A)  $\frac{2}{(s+9)}$       (B)  $\frac{1}{(s^2+9s+2)}$

(C)  $\frac{5s^2}{(9s+2)}$

(D)  $\frac{(5s+2)}{(s^2+9s+2)}$

- Q.93** The output  $y(t)$  of an LTI system is related to its input  $x(t)$  by the following equation

$$\frac{d^2y(t)}{dt^2} + 11\frac{dy(t)}{dt} + 24y(t) = 5\frac{dx(t)}{dt} + 3x(t)$$

The transfer function  $H(s)$  of the system is given by

(A)  $\frac{1}{(s^2+11s+24)}$

(B)  $\frac{(5s+3)}{(s^2+11s+24)}$

(C)  $\frac{(5s+3)}{(s+11)}$

(D)  $\frac{3}{(s+11)}$

- Q.94** Find out the response of the system to unit step input signal. Impulse response is given as :

$$h(t) = \frac{R}{L} e^{-tR/L} u(t)$$

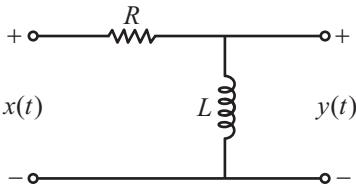
(A)  $[1 - e^{-tR/L}]u(t)$

(B)  $\left[1 - \frac{R}{L} e^{-tR/L}\right]u(t)$

(C)  $\frac{R}{L} [1 - e^{-tR/L}]u(t)$

(D)  $\frac{1}{L} [1 - e^{-tR/L}]u(t)$

- Q.95** An electric network is shown in the figure below



Which of the following statement regarding the stability of the system is true?

- (A) The system is BIBO stable.
- (B) The system is BIBO unstable
- (C) The system stability depends on the values of R and L
- (D) None of the above

- Q.96** Consider the statements  $S_1$  and  $S_2$  for a causal and stable system with an impulse response  $h(t)$ .

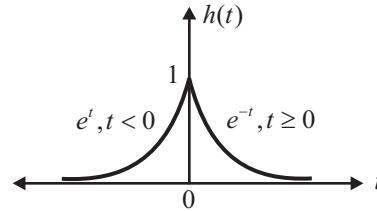
$S_1$  : The system with impulse response  $dh(t)/dt$  must be causal and stable.

$S_2$  : The system with impulse response  $\int_{-\infty}^t h(\tau)d\tau$  must be causal and stable.

Which of above statements is/are true?

- (A) Only  $S_1$
- (B) Only  $S_2$
- (C) Both  $S_1$  and  $S_2$
- (D) None of these

- Q.97** The impulse response of an LTI system is shown in figure. The step response is



- (A)  $2 + e^t - e^{-t}$
- (B)  $e^t u(-t+1) + 2 - e^{-t}$
- (C)  $e^t u(-t+1) + [2 - e^{-t}]u(t)$
- (D)  $e^t + [2 - e^{-t} - e^t]u(t)$

- Q.98** For a causal LTI system having a transfer function  $H(s) = \frac{1}{s+1}$ . Find the output  $y(t)$  if the input  $f(t)$  is given by  $e^{-t/4}u(t) + e^{-t/2}u(-t)$ .

- (A)  $\left(\frac{2}{3}e^{-t} + \frac{4}{3}e^{-t/4}\right)u(t) + 2e^{-t/2}u(-t)$
- (B)  $\left(\frac{2}{3}e^{-t} + \frac{4}{3}e^{-t/4}\right)u(-t) + 2e^{-t/2}u(t)$
- (C)  $\left(\frac{2}{3}e^{-t}\right)u(t) + 2e^{-t/2}u(-t)$
- (D)  $\left(\frac{2}{3}e^{-t} + \frac{4}{3}e^{-t/4}\right)u(t) - 2e^{-t/2}u(-t)$

- Q.99** The transfer function  $H(s)$  of a stable system is

$$H(s) = \frac{s^2 + 5s - 9}{(s+1)(s^2 - 2s + 10)}$$

The impulse response is

- (A)  $-e^{-t}u(t) + [e^t \sin 3t + 2e^t \cos 3t]u(t)$
- (B)  $-e^{-t}u(t) - [e^t \sin 3t + 2e^t \cos 3t]u(-t)$
- (C)  $-e^{-t}u(t) - [e^t \sin 3t + 2e^t \cos 3t]u(t)$
- (D)  $-e^{-t}u(t) + [e^t \sin 3t + 2e^t \cos 3t]u(-t)$

- Q.100** A causal LTI system with impulse response  $h(t)$  satisfies the following properties :

- (1) When the input of the system is  $x(t) = e^{2t}$

then output is  $y(t) = \frac{1}{6}e^{2t}$ .

- (2)  $h(t)$  satisfies the differential equation

$$\frac{d}{dt}h(t) + 2h(t) = e^{-4t}u(t) + bu(t)$$

Determine the impulse response of the system.

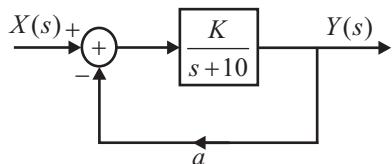
- (A)  $\frac{1}{2}[1 + e^{-4t}]u(t)$
- (B)  $\frac{1}{2}[1 - e^{-4t}]u(t)$

(C)  $\frac{1}{2}[1-e^{-4t}-2e^{-2t}]u(t)$

(D)  $\frac{1}{2}[1-e^{-4t}+2e^{-2t}]u(t)$

**Common Data for Questions 101 & 102**

The figure shows block diagram representation of a continuous time system



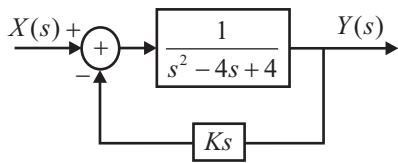
- Q.101** If  $a=1$ , then for what values of  $K$  the system will be stable?

(A)  $K > -10$       (B)  $K < 10$   
 (C)  $K < 0$       (D)  $0 < K < 10$

- Q.102** If  $a=-1$ , then for what values of  $K$  the system will be stable?

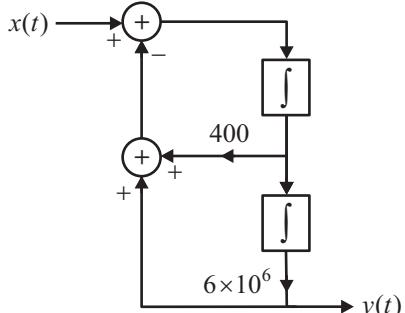
(A)  $K > -10$       (B)  $K > 10$   
 (C)  $K < 10$       (D)  $K > 0$

- Q.103** The system whose block diagram representation is shown in the figure will be stable if



(A)  $K > 4$       (B)  $0 < K < 8$   
 (C)  $K > 0$       (D)  $K < 2$

- Q.104** The impulse response for the system shown



(A)  $2441.3e^{-200t} \sin(2457.7t)u(t)$   
 (B)  $2457.7e^{-200t} \sin(2441.3t)u(t)$   
 (C)  $2441.3e^{-200t} \cos(2457.7t)u(t)$   
 (D)  $2457.7e^{-200t} \cos(2441.3t)u(t)$

- Q.105**  $x(t)$  is non-zero only for  $T_x < t < T_x'$ , and similarly,  $y(t)$  is nonzero only for  $T_y < t < T_y'$ .

Let  $z(t)$  be convolution of  $x(t)$  and  $y(t)$ . Which one of the following statements is TRUE?

[GATE EE 2014-Kharagpur]

(A)  $z(t)$  can be nonzero over an unbounded interval

(B)  $z(t)$  is nonzero for  $t < T_x + T_y$

(C)  $z(t)$  is zero outside of  $T_x + T_y < t < T_x' + T_y'$

(D)  $z(t)$  is nonzero for  $t > T_x' + T_y'$

- Q.106** Two systems with impulse responses  $h_1(t)$  and  $h_2(t)$  are connected in cascade. Then the overall impulse response of the cascaded system is given by [GATE EC, EE, IN 2013-Bombay]

(A) product of  $h_1(t)$  and  $h_2(t)$

(B) sum of  $h_1(t)$  and  $h_2(t)$

(C) convolution of  $h_1(t)$  and  $h_2(t)$

(D) subtraction of  $h_1(t)$  and  $h_2(t)$

- Q.107** Convolution of  $x(t+5)$  with impulse function  $\delta(t-7)$  is equal to [GATE EC 2002-Bangalore]

(A)  $x(t-12)$       (B)  $x(t+12)$

(C)  $x(t-2)$       (D)  $x(t+2)$

- Q.108** Let  $h(t)$  be the impulse response of a linear time invariant system. Then the response of the system for any input  $u(t)$  is

[GATE EC 1995-Kanpur]

(A)  $\int_0^t h(\tau)u(t-\tau)d\tau$       (B)  $\frac{d}{dt} \int_0^t h(\tau)u(t-\tau)d\tau$

(C)  $\int_0^t h(\tau)u(\tau-t)d\tau$       (D)  $\int_0^t h^2(\tau)u(t-\tau)d\tau$

- Q.109** The convolution of the functions  $f_1(t) = e^{-2t}u(t)$  and  $f_2(t) = e^t u(t)$  is equal to

[GATE EE 1995-Kanpur]

(A)  $\frac{1}{3}[e^t + e^{-2t}]u(t)$       (B)  $\frac{1}{3}[e^t - e^{-2t}]u(t)$

(C)  $\frac{1}{3}[-e^t - e^{-2t}]u(t)$       (D)  $\frac{1}{3}[-e^t + e^{-2t}]u(t)$

- Q.110** If  $f(t)$  is the step response of a linear time invariant system, then its impulse response is given by [GATE EE 1994-Kharagpur]

(A)  $h(t) = \int_{-\infty}^t f(\tau)d\tau$       (B)  $h(t) = f(t) - f(t-1)$

(C)  $h(t) = \frac{d}{dt}f(t)$       (D)  $h(t) = \frac{d^2}{dt^2}f(t)$

- Q.111** The voltage across an impedance in a network is  $V(s) = Z(s)I(s)$ , where  $V(s)$ ,  $Z(s)$  and  $I(s)$  are the Laplace transforms of the corresponding time functions  $v(t)$ ,  $z(t)$  and  $i(t)$ . The voltage  $v(t)$  is

[GATE EC 1991-Madras]

(A)  $v(t) = z(t) \cdot i(t)$

(B)  $v(t) = \int_0^t i(\tau) z(t-\tau) d\tau$

(C)  $v(t) = \int_0^t i(\tau) z(t+\tau) d\tau$

(D)  $v(t) = z(t) + i(t)$

**Q.112** What is the Laplace transform of  $x(t) = -e^{2t} u(t) \otimes [tu(t)]$ ? [ESE EC 2006]

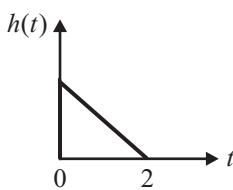
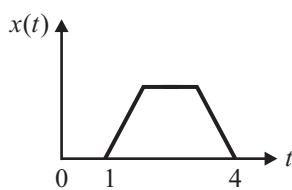
(A)  $\frac{-1}{s^2(s+2)}$

(B)  $\frac{-1}{s^2(s-2)}$

(C)  $\frac{1}{s^2(s-2)}$

(D)  $\frac{-1}{s(s-2)}$

**Q.113** Figure-1 and Figure-2 show respectively the input  $x(t)$  to a linear time-invariant system and the impulse response  $h(t)$  of the system.



The output of the system is zero everywhere except for the time interval [ESE EC 1999]

(A)  $0 < t < 4$

(B)  $0 < t < 5$

(C)  $1 < t < 5$

(D)  $1 < t < 6$

**Q.114** The impulse response of a causal linear, time-invariant, continuous time system is  $h(t)$ . The output  $y(t)$  of the same system to an input  $x(t)$ , where  $x(t) = 0$  for  $t < -2$ , is [ESE EC 1998]

(A)  $\int_0^t h(\tau) x(t-\tau) d\tau$

(B)  $\int_{-2}^t h(\tau) x(t-\tau) d\tau$

(C)  $\int_{-2}^{t-2} h(\tau) x(t-\tau) d\tau$

(D)  $\int_0^{t+2} h(\tau) x(t-\tau) d\tau$

**Q.115** If  $f_1(t)$  and  $f_2(t)$  are duration limited signal such that [ESE EC 1998]

$$f_1(t) \neq 0 \text{ for } 1 \leq t \leq 3 \quad f_2(t) \neq 0 \text{ for } 5 \leq t \leq 7 \\ = 0 \text{ elsewhere} \quad = 0 \text{ elsewhere}$$

Then the convolution of  $f_1(t)$  and  $f_2(t)$  is zero everywhere except for :

$$(A) 1 \leq t \leq 7 \quad (B) 3 \leq t \leq 5 \\ (C) 5 \leq t \leq 21 \quad (D) 6 \leq t \leq 10$$

**Q.116** Given that  $h(t) = 10e^{-10t} u(t)$ , and

$e(t) = \sin(10t) u(t)$ , the Laplace transform of the

$$\text{signal } f(t) = \int_0^t h(t-\tau) e(\tau) d\tau \text{ is given by}$$

[ESE EC 1997]

(A)  $\frac{10}{(s+10)(s^2+100)}$

(B)  $\frac{10(s+10)}{(s^2+100)}$

(C)  $\frac{100}{(s+10)(s^2+100)}$

(D)  $\frac{1}{(s+10)(s^2+100)}$

**Q.117** If  $f(t) \otimes g(t) = c(t)$  then which of the following is correct?

(A)  $f(at) \otimes g(at) = \left| \frac{1}{a^2} \right| c(at)$

(B)  $f(at) \otimes g(at) = \left| \frac{1}{a} \right| c\left( \frac{t}{a} \right)$

(C)  $f(at) \otimes g(at) = \left| \frac{1}{a^2} \right| c\left( \frac{t}{a} \right)$

(D)  $f(at) \otimes g(at) = \left| \frac{1}{a} \right| c(at)$

**Q.118** Let  $y(t) = x(t) \otimes g(t)$ , then, with regard to the following relations :

(P)  $y(t-2) = x(t-2) \otimes g(t)$

(Q)  $y(t-1) = g(t-1) \otimes x(t-1)$

(R)  $y(-t) = g(-t) \otimes x(t)$

(S)  $y(-t) = g(-t) \otimes x(-t)$

(A) All are correct

(B) All are incorrect

(C) Only P & S are correct

(D) Only Q & R are correct

**Q.119** The continuous time convolution integrals  $y(t) = e^{-3t} u(t) \otimes u(t+3)$  is

(A)  $\frac{1}{3} [1 - e^{-3(t+3)}] u(t+3)$  (B)  $\frac{1}{3} [1 - e^{-3(t+3)}] u(t)$

(C)  $\frac{1}{3} [1 - e^{-3t}] u(t)$  (D)  $\frac{1}{3} [1 - e^{-3t}] u(t+3)$

**Q.120** The continuous time convolution integrals  $y(t) = x(t) \otimes h(t)$  where  $x(t) = u(t)$  and

$$h(t) = \begin{cases} e^{2t} & t < 0 \\ e^{-3t} & t \geq 0 \end{cases}$$

(A)  $\frac{1}{2} e^{-2t} u(-t-1) + \frac{5}{6} - \frac{1}{3} e^{-3t} u(-t)$

(B)  $\frac{1}{2} e^{2t} u(-t-1) + \frac{5}{6} - \frac{1}{3} e^{-3t} u(-t)$

(C)  $\frac{1}{2} e^{2t} + \frac{1}{6} [5 - 3e^{2t} - 2e^{-3t}] u(t)$

(D)  $\frac{1}{2} e^{2t} + \frac{1}{6} [5 - 3e^{2t} - 2e^{-3t}] u(-t)$

**Q.121** The continuous time convolution integrals  $y(t) = \cos(\pi t) [u(t+1) - u(t-1)] \otimes u(t)$  is

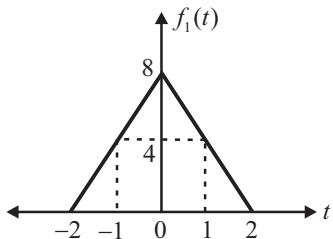
(A)  $\frac{\sin(\pi t)}{\pi} [u(t+1) - u(t-1)]$

(B)  $\frac{\sin(\pi t)}{\pi} u(t-1)$

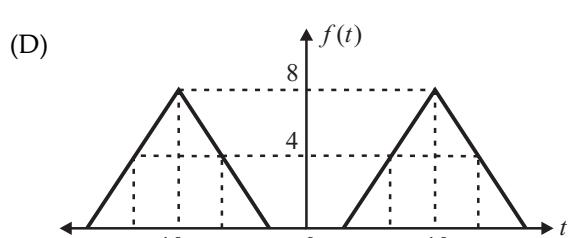
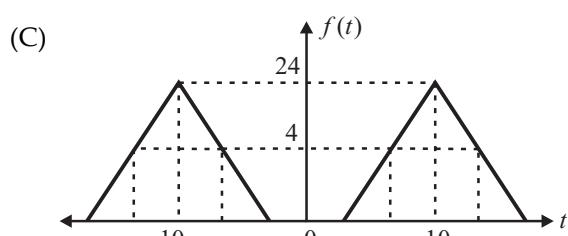
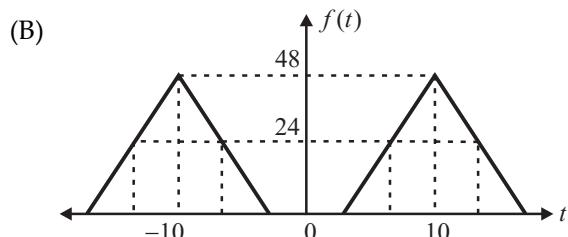
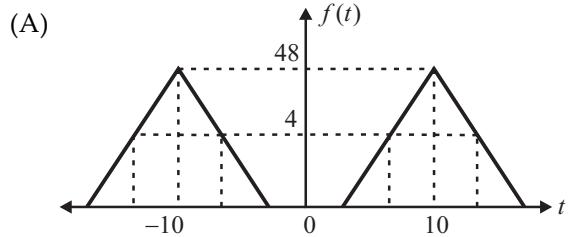
(C)  $\frac{\sin(\pi t)}{\pi} u(t+1)$

(D)  $\frac{\sin(\pi t)}{\pi} u(t)$

**Q.122** For a given convolution function  $f(t) = f_1(t) \otimes f_2(t)$  where  $f_1(t)$  is :

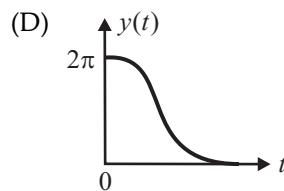
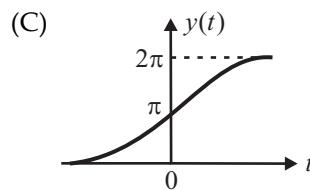
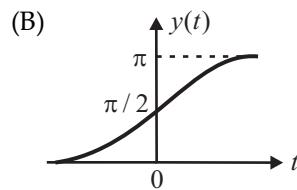
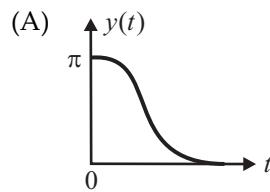


and  $f_2(t) = 6\delta(t-10) + 6\delta(t+10)$ . The function  $f(t)$  is

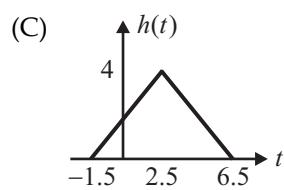
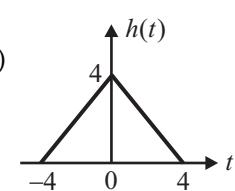
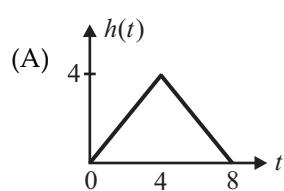


**Q.123** If impulse response is given by  $h(t) = \frac{1}{t^2 + 1}$ .

Which of the following sketch is correct for output response  $y(t) = h(t) \otimes u(t)$ .

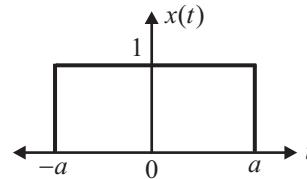


**Q.124** Two systems have impulse responses  $h_1(t) = u(t) - u(t-4)$  and  $h_2(t) = \text{rect}\left[\frac{t-2}{4}\right]$ . If these two systems are cascaded, then its impulse response is

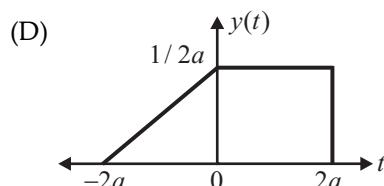
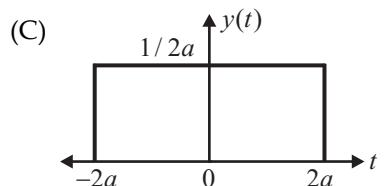
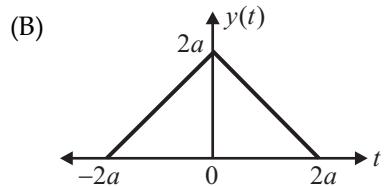
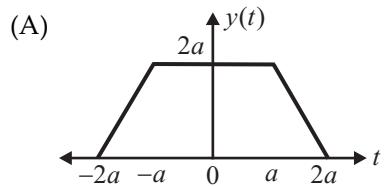


(D) None of these

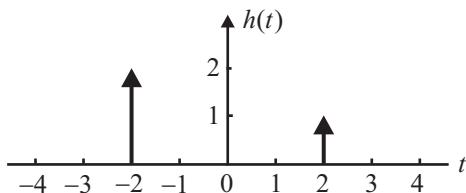
**Q.125** Consider a rectangular pulse shown in the figure below.



The graph for  $y(t) = x(t) \otimes x(t)$  will be



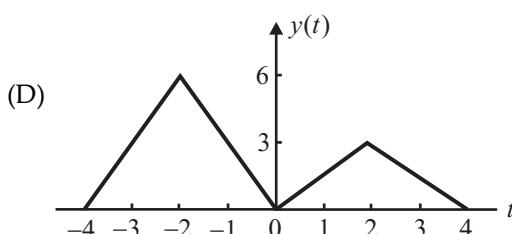
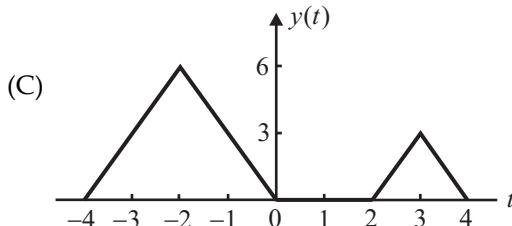
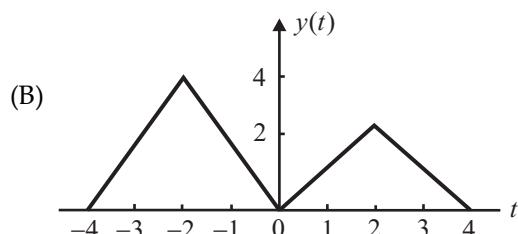
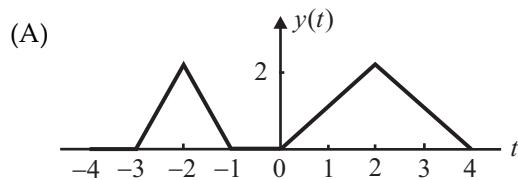
- Q.126** Impulse response of a CT system is shown in figure below.



The system is excited by an input given as

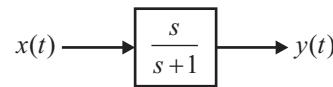
$$x(t) = \begin{cases} 3 - 1.5|t|, & |t| < 2 \\ 0, & |t| \geq 2 \end{cases}$$

The correct graph for output  $y(t)$  is



- Q.127** In the system shown in figure, the input  $x(t) = \sin t$ . In the steady state, the response  $y(t)$  will be

[GATE EE 2004-Delhi]



- (A)  $\frac{1}{\sqrt{2}} \sin(t - 45^\circ)$       (B)  $\frac{1}{\sqrt{2}} \sin(t + 45^\circ)$   
 (C)  $\sin(t - 45^\circ)$       (D)  $\sin(t + 45^\circ)$

- Q.128** Statement (I) : Sinusoidal signals are used as basic function in electrical systems.

Statement (II) : The response of a linear system to a sinusoidal input function remains sinusoidal.

[ESE EC 2015]

- (A) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).  
 (B) Both Statement (I) and Statement (II) are individually true and Statement (II) is the NOT the correct explanation of Statement (I).  
 (C) Statement (I) is true but Statement (II) is false.  
 (D) Statement (I) is false but Statement (II) is true.

- Q.129** Statement (I) : Zero input response is the natural response with zero initial conditions.

Statement (II) : Zero state response is the response with given input with zero initial conditions.

Codes :

[ESE EC 2013]

- (A) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I)  
 (B) Both Statement (I) and Statement (II) are individually true but Statement (II) is NOT the correct explanation of Statement (I)

- (C) Statement (I) is true but Statement (II) is false  
 (D) Statement (I) is false but Statement (II) is true

**Q.130** A system described by the following differential equation is initially at rest and then excited by the input  $x(t) = 3u(t)$   $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = x(t)$ .

The output  $y(t)$  is [ESE EC 2012]

- (A)  $1 - 1.5e^{-t} + 0.5e^{-3t}$     (B)  $1 - 0.5e^{-t} + 1.5e^{-3t}$   
 (C)  $1 + 1.5e^{-t} - 0.5e^{-3t}$     (D)  $1 + 0.5e^{-t} - 1.5e^{-3t}$

**Q.131** An electrical system transfer function has a pole as  $s = -2$  and a zero at  $s = -1$  with system gain 10. For sinusoidal current excitation, voltage response of the system [ESE EC 2011]

- (A) Is zero  
 (B) Is in phase with the current  
 (C) Leads the current  
 (D) Lags behind the current

**Q.132** The relation between input  $x(t)$  and output  $y(t)$  of a continuous time system is given by

$$\frac{dy(t)}{dt} + 3y(t) = x(t)$$

What is the forced response of the system when  $x(t) = k$  (a constant)? [ESE EC 2007]

- (A)  $k$                          (B)  $k/3$   
 (C)  $3k$                          (D) 0

**Q.133** Consider an LTI system which is described by following differential equation

$$\frac{dy(t)}{dt} + 5y(t) = x(t)$$

With initial condition  $y(0) = -2$ . For an input  $x(t) = 3e^{-2t}u(t)$ , response will be

- (A)  $[e^{-2t} - e^{-5t}]u(t)$     (B)  $[e^{-2t} - 3e^{-5t}]u(t)$   
 (C)  $[e^{-t} - 3e^{-2t}]u(t)$     (D)  $[e^{-2t} + e^{-5t}]u(t)$

**Q.134** A differential equation for a LTI system with specified input and initial conditions is

$$\frac{d^3y(t)}{dt^3} + 4\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} = x(t)$$

All initial condition are zero,  $x(t) = 10e^{-2t}$ . The system output is

- (A)  $\left[\frac{5}{3} + 5e^{-t} - 5e^{-2t} + \frac{5}{3}e^{-3t}\right]u(t)$   
 (B)  $\left[\frac{5}{3} - 5e^{-t} + 5e^{-2t} - \frac{5}{3}e^{-3t}\right]u(t)$

(C)  $\frac{5}{3}u(t) - 5u(t-1) + 5u(t-2) + \frac{5}{3}u(t-3)$

(D)  $\frac{5}{3}u(t) - 5u(t-1) - 5u(t-2) + \frac{5}{3}u(t-3)$

**Q.135** A differential equation for a LTI system with specified input and initial conditions is

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 5y(t) = \frac{dx(t)}{dt};$$

$$y(0^-) = 2; \quad \left.\frac{dy(t)}{dt}\right|_{t=0} = 0; \quad x(t) = u(t)$$

The system is

- (A)  $\frac{1}{2}e^{-t} \sin t.u(t)$   
 (B)  $2e^{-t} \cos t.u(t)$   
 (C)  $2e^{-t} \cos 2t.u(t) + \frac{3}{2}e^{-t} \sin 2t.u(t)$   
 (D)  $\frac{1}{2}e^{-t} \cos t.u(t-1) + 2e^{-t} \sin t.u(t-1)$



**Answer Keys**

Objective & Numerical Answer Type Questions									
1.	A	2.	C	3.	-2	4.	0.155	5.	B
6.	D	7.	B	8.	D	9.	C	10.	C
11.	B	12.	C	13.	D	14.	B	15.	C
16.	D	17.	A	18.	C	19.	A	20.	C
21.	B	22.	A	23.	A	24.	A	25.	A
26.	C	27.	C	28.	B	29.	A	30.	0
31.	B	32.	D	33.	C	34.	B	35.	C
36.	D	37.	B	38.	B	39.	B	40.	D
41.	D	42.	A	43.	B	44.	A	45.	B
46.	A	47.	C	48.	A	49.	D	50.	B
51.	D	52.	B	53.	A	54.	D	55.	A
56.	A	57.	A	58.	D	59.	B	60.	D
61.	A	62.	A	63.	C	64.	B	65.	A
66.	D	67.	B	68.	45	69.	B	70.	B
71.	A	72.	C	73.	C	74.	B	75.	A
76.	C	77.	C	78.	D	79.	A	80.	D
81.	D	82.	B	83.	B	84.	C	85.	D
86.	B	87.	A	88.	A	89.	A	90.	D
91.	D	92.	A	93.	D	94.	B	95.	D
96.	A	97.	B	98.	B	99.	D	100.	B
101.	B	102.	B	103.	C	104.	C	105.	A
106.	D	107.	B	108.	A	109.	0.707	110.	7.389
111.	0.146	112.	C	113.	90	114.	A	115.	B
116.	67 to 69	117.	B	118.	B	119.	B	120.	B
121.	A	122.	A	123.	A	124.	C	125.	C
126.	1	127.	D						
Practice (Objective & Numerical Answer) Questions									
1.	C	2.	C	3.	B	4.	D	5.	A
6.	B	7.	A	8.	A	9.	B	10.	B
11.	B	12.	A	13.	A	14.	D	15.	D
16.	D	17.	B	18.	A	19.	A	20.	D
21.	B	22.	C	23.	A	24.	B	25.	B
26.	C	27.	B	28.	D	29.	A	30.	D
31.	C	32.	D	33.	D	34.	A	35.	A
36.	D	37.	D	38.	C	39.	B	40.	C
41.	A	42.	B	43.	D	44.	B	45.	D

46.	D	47.	A	48.	C	49.	D	50.	A
51.	D	52.	A	53.	A	54.	B	55.	A
56.	C	57.	A	58.	B	59.	A	60.	B
61.	C	62.	B	63.	B	64.	2	65.	0
66.	1	67.	B	68.	D	69.	C	70.	D
71.	B	72.	C	73.	A	74.	A	75.	C
76.	B	77.	C	78.	D	79.	C	80.	B
81.	C	82.	A	83.	C	84.	False	85.	A
86.	B	87.	B	88.	B	89.	C	90.	B
91.	D	92.	D	93.	B	94.	A	95.	A
96.	A	97.	D	98.	A	99.	B	100.	B
101.	A	102.	C	103.	A	104.	B	105.	C
106.	C	107.	C	108.	A	109.	B	110.	C
111.	B	112.	B	113.	D	114.	D	115.	D
116.	C	117.	D	118.	C	119.	A	120.	C
121.	A	122.	B	123.	B	124.	A	125.	B
126.	D	127.	B	128.	A	129.	D	130.	A
131.	C	132.	B	133.	B	134.	B	135.	C

# 6

# Discrete Time Fourier Transform

## Objective & Numerical Ans Type Questions :

- Q.1** A 5-point sequence  $x[n]$  is given as  $x[-3]=1$ ,  $x[-2]=1/2$ ,  $x[-1]=0$ ,  $x[0]=5$ ,  $x[1]=1$ . Let  $X(e^{j\omega})$  denote the discrete-time Fourier transform of  $x[n]$ . The value of  $\int_{-\pi}^{\pi} X(e^{j\omega}) d\omega$  is

[GATE EC 2007, IIT Kanpur]

- (A) 5    (B)  $10\pi$   
 (C)  $16\pi$                                       (D)  $5 + j10\pi$

- Q.2** Let  $x[n] = \left(\frac{1}{2}\right)^n u[n]$ ,  $y[n] = x^2[n]$  and  $Y(e^{j\omega})$  be the Fourier transform of  $y[n]$  then  $Y(e^{j\omega})$  is

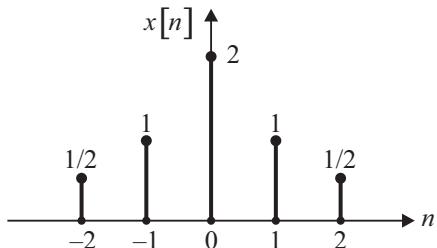
[GATE EC 2005, IIT Bombay]

- (A)  $1/4$                                         (B) 2  
 (C) 4    (D)  $4/3$

### Statement for Linked Answer Questions 3 & 4

A sequence  $x[n]$  has non-zero values as shown in the figure.

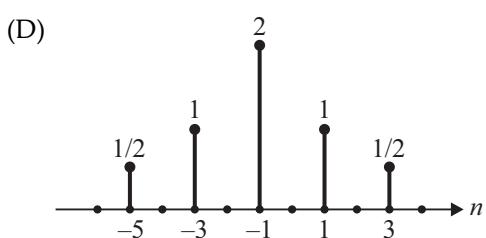
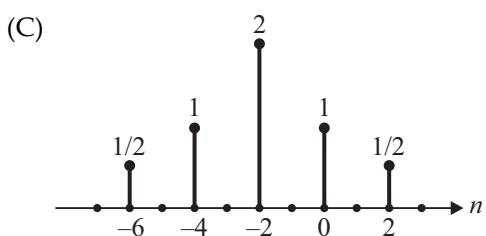
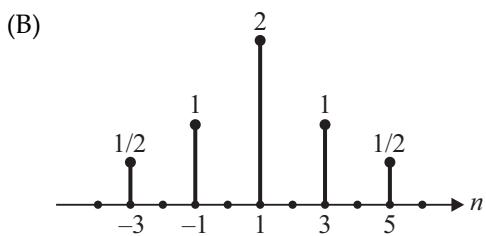
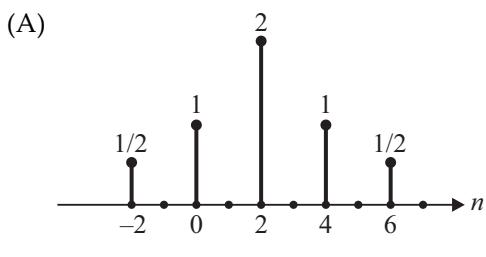
[GATE EC 2005, IIT Bombay]



- Q.3** The sequence

$$y[n] = \begin{cases} x\left[\frac{n}{2} - 1\right]; & \text{for } n \text{ even} \\ 0; & \text{for } n \text{ odd} \end{cases}$$

will be



- Q.4** The Fourier transform of  $y[2n]$  will be

- (A)  $e^{-2j\omega} [\cos 4\omega + 2 \cos 2\omega + 2]$   
 (B)  $[\cos 2\omega + 2 \cos 2\omega + 2]$   
 (C)  $e^{-j\omega} [\cos 2\omega + 2 \cos \omega + 2]$   
 (D)  $e^{-\frac{j\omega}{2}} [\cos 2\omega + 2 \cos \omega + 2]$

- Q.5** Given  $x(n) = \frac{\sin \omega_c n}{\pi n}$ , the energy of the signal given by  $\sum_{n=-\infty}^{\infty} |x(n)|^2$  is \_\_\_\_\_.

[GATE IN 2003, IIT Madras]

(A)  $\frac{\omega_c}{\pi}$       (B)  $\pi \omega_c$

(C) Infinite      (D)  $2\pi \omega_c$

- Q.6** If the Fourier transform of  $x(n)$  is  $X(e^{j\omega})$ , then the Fourier transform of  $(-1)^n x(n)$  is \_\_\_\_\_.

[GATE IN 2004, IIT Delhi]

(A)  $(-j)^{\omega} X(e^{j\omega})$       (B)  $(-1)^{\omega} X(e^{j\omega})$

(C)  $X(e^{j(\omega-\pi)})$       (D)  $\frac{d}{d\omega} X(e^{j\omega})$

- Q.7** Given the discrete-time sequence,

$$x(n) = [2, 0, -1, \underset{\uparrow}{-3}, 4, 1, -1], X(e^{j\pi}) \text{ is } \underline{\hspace{2cm}}$$

[GATE IN 2005, IIT Bombay]

(A) 8      (B)  $6\pi$

(C)  $8\pi$       (D) 6

- Q.8** The signal  $x[n] = \frac{\sin(\pi n/6)}{\pi n}$  is processed through a linear filter with the impulse response  $h[n] = \frac{\sin(\omega_c n)}{\pi n}$  where  $\omega_c > \frac{\pi}{6}$ . The output of the filter is \_\_\_\_\_.

[GATE IN 2015, IIT Kanpur]

(A)  $\sin \frac{(2\omega_c n)}{\pi n}$       (B)  $\sin \frac{(\pi n/3)}{\pi n}$

(C)  $\left[ \sin \frac{(\pi n/6)}{\pi n} \right]^2$       (D)  $\sin \frac{(\pi n/6)}{\pi n}$

- Q.9** Let  $h[n]$  be the impulse response of a discrete-time linear time invariant (LTI) filter. The impulse response is given by

$$h[0] = \frac{1}{3}, h[1] = \frac{1}{3}, h[2] = \frac{1}{3} \text{ and}$$

$$h[n] = 0 \text{ for } n < 0 \text{ and } n > 2.$$

Let  $H(\omega)$  be the discrete-time Fourier transform (DTFT) of  $h[n]$ , where  $\omega$  is the normalized angular frequency in radians. Given that  $H(\omega_0) = 0$  for  $0 < \omega_0 < \pi$ , the value of  $\omega_0$  (in radians) is equal to \_\_\_\_\_.

[GATE EC 2017 Set – 01, IIT Roorkee]

- Q.10** Consider the signal  $x[n] = 6\delta[n+2] + 3\delta[n+1] + 8\delta[n] + 7\delta[n-1] + 4\delta[n-2]$ . If  $X(e^{j\omega})$  is the discrete-time Fourier transform of  $x[n]$ , then  $\frac{1}{\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) \sin^2(2\omega) d\omega$  is equal to \_\_\_\_\_.

[GATE EC 2016 Set – 01, IISc Bangalore]

- Q.11** A Fourier transform pair is given by,

$$\left( \frac{2}{3} \right)^n u[n+3] \xleftrightarrow{F.T.} \frac{A e^{j6\pi f}}{1 - \left( \frac{2}{3} \right) e^{-j2\pi f}}$$

where  $u[n]$  denotes the unit step sequence. The value of  $A$  is \_\_\_\_\_.

[GATE EC 2014 Set – 04, IIT Kharagpur]

- Q.12** Let  $h[n]$  be a length-7 discrete-time finite impulse response filter, given by  $h[0] = 4, h[1] = 3, h[2] = 2, h[3] = 1, h[-1] = -3, h[-2] = -2, h[-3] = -1$  and  $h[n]$  is zero for  $|n| \geq 4$ . A length-3 finite impulse response approximation  $g[n]$  of  $h[n]$  has to be obtained such that

$$E(h, g) = \int_{-\pi}^{\pi} |H(e^{j\omega}) - G(e^{j\omega})|^2 d\omega$$

is minimized, where  $H(e^{j\omega})$  and  $G(e^{j\omega})$  are the discrete-time Fourier transforms of  $h[n]$  and  $g[n]$ , respectively. For the filter that minimizes  $E(h, g)$ , the value of  $10g[-1] + g[1]$ , rounded off to 2 decimal places, is \_\_\_\_\_.

[GATE EC 2019, IIT Madras]

### Practice (objective & Num Ans) Questions :

- Q.1** The impulse response  $h[n]$  of a linear time invariant system is given as

$$h[n] = \begin{cases} -2\sqrt{2} & n = 1, -1 \\ 4\sqrt{2} & n = 2, -2 \\ 0 & \text{otherwise} \end{cases}$$

If the input to the above system is the sequence  $e^{j\pi n/4}$ , then the output is \_\_\_\_\_.

[GATE EC 2004, IIT Delhi]

- (A)  $4\sqrt{2}e^{j\pi n/4}$       (B)  $4\sqrt{2}e^{-j\pi n/4}$   
 (C)  $4e^{j\pi n/4}$       (D)  $-4e^{j\pi n/4}$

- Q.2** The discrete time Fourier transform (DTFT) of  $x[n] = 2(3)^n u[-n]$  is equal to [ESE EC 2005]

(A)  $\frac{2}{1-e^{j\Omega}/3}$       (B)  $\frac{2}{1+e^{j\Omega}/3}$   
 (C)  $2\left(\frac{1+e^{j\Omega}/3}{1-e^{j\Omega}/3}\right)$       (D)  $2\left(\frac{1-e^{j\Omega}/3}{1+e^{j\Omega}/3}\right)$

- Q.3** The impulse response of discrete LTI system is  $h(n) = \frac{1}{\pi n} \sin\left(\frac{\pi n}{4}\right)$ . The output response of the

system when  $x(n) = \frac{1}{n} \sin\left(\frac{\pi n}{8}\right)$  is  
 (A)  $\frac{1}{n} \sin\left(\frac{\pi n}{4}\right)$       (B)  $\frac{1}{\pi n} \sin\left(\frac{\pi n}{2}\right)$   
 (C)  $\frac{1}{\pi n} \sin\left(\frac{\pi n}{4}\right)$       (D)  $\frac{1}{n} \sin\left(\frac{\pi n}{8}\right)$

- Q.4** If z-transform of  $x(n)$  includes unit circle in its ROC, then the Fourier transform of  $x(n)$  can be expressed as

(A)  $\sum_{n=-\infty}^{\infty} x(n) z^{-n} \Big|_{z=e^{-j\omega}}$       (B)  $\sum_{n=0}^{\infty} x(n) z^{-jn} \Big|_{z=e^{-j\omega}}$   
 (C)  $\sum_{n=-\infty}^{\infty} x(n) z^n \Big|_{z=\omega}$       (D)  $\sum_{n=-\infty}^{\infty} x(n) z^{-n} \Big|_{z=e^{j\omega}}$

- Q.5** The Fourier transform of correlation sequence of two discrete time signals  $x_1(n)$  and  $x_2(n)$  is given by

(A)  $X_1(e^{j\omega})X_2(e^{j\omega})$       (B)  $X_1(e^{j\omega})X_2(e^{-j\omega})$   
 (C)  $X_1(e^{-j\omega})X_2(e^{-j\omega})$       (D) none of the above

- Q.6** Let  $x[n] = a^n u[n]$ ,  $h[n] = b^n u[n]$   
 What is the expression for  $y[n]$ , for a discrete-time system ?

(A)  $\sum_{k=-\infty}^{\infty} a^k u[k] b^{n-k} u[n-k]$   
 (B)  $\sum_{k=-\infty}^{\infty} a^n u[k] b^{n-k} u[n-k]$   
 (C)  $\sum_{k=-\infty}^{\infty} a^k u[n-k] b^n u[k]$   
 (D)  $\sum_{k=-\infty}^{\infty} a^{n-k} u[k] b^{n-k} u[n-k]$

- Q.7** Discrete time fourier transform of  $2^n u[n]$  is :

(A)  $\frac{1}{1-2e^{-j\omega}}$       (B)  $\frac{2}{1-2e^{-j\omega}}$   
 (C)  $\frac{2}{1-e^{-j\omega}}$       (D) None of these

- Q.8** The DTFT of the signal  $x[n] = u[n] - u[n-6]$  is :

(A)  $\frac{1-e^{-j5\omega}}{1-e^{-j\omega}}$       (B)  $\frac{1-e^{-6j\omega}}{1-e^{-j\omega}}$   
 (C)  $\frac{1-e^{-j5\omega}}{1+e^{-j\omega}}$       (D)  $\frac{1-e^{-6j\omega}}{1+e^{-j\omega}}$

- Q.9** The discrete time Fourier transform of the signal,  $x(n) = 0.5^{(n-1)} u(n-1)$  is :

(A)  $\frac{e^{-j\omega}}{1-0.5e^{-j\omega}}$       (B)  $e^{-j\omega}(1-0.5e^{-j\omega})$   
 (C)  $\frac{0.5e^{-j\omega}}{1-0.5e^{-j\omega}}$       (D)  $\frac{0.5e^{j\omega}}{1-0.5e^{-j\omega}}$

**Common Data Questions 10 to 13**

Given  $x(n) = [1, -2, 3, -4]$   
 ↑

$X(e^{j\omega})$  denotes fourier transform of  $x(n)$

- Q.10**  $X(e^{j0})$  is

(A) 10      (B) -2  
 (C) 2      (D) -10

- Q.11**  $X(e^{j\pi})$  is

(A) 10      (B) -2  
 (C) 2      (D) -10

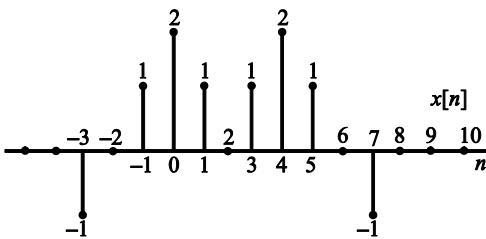
- Q.12**  $\frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) d\omega$  is

(A)  $6\pi$       (B)  $2\pi$   
 (C) 3      (D)  $3\pi$

- Q.13**  $\int_{-\pi}^{\pi} \left| \frac{d}{d\omega} X(e^{j\omega}) \right|^2 d\omega$  is

(A) 12      (B) 24  
 (C)  $48\pi$       (D)  $12\pi$

- Q.14** For the signal  $x[n]$  shown in figure,  $x[n] = 0$  for  $n < -3$  and  $n > 7$ .



If  $X(\omega)$  is the Fourier transform of  $x[n]$ , which one of the following is TRUE ?

(A)  $X(0) = 5$

(B)  $\int_{-\pi}^{\pi} X(\omega) d\omega = 2\pi$

- (C) The phase  $\angle X(\omega) = -2\omega$   
(D)  $X(\omega) = X(-\omega)$

**Statement for Linked Answer Questions 15 & 16**

A sequences of length six is given by

$$x[n] = [2, \underset{\uparrow}{1}, -1, 0, 3, -2]$$

having the Fourier transform  $X(e^{j\omega})$

- Q.15** If  $Y(e^{j\omega}) = \frac{d}{d\omega} X(e^{j\omega})$ ,  $|y[n]|$  is given by

(A)  $[4, \underset{\uparrow}{1}, 1, 0, -6, 6]$

(B)  $[4, \underset{\uparrow}{1}, 1, 0, 6, 6]$

(C)  $[4, \underset{\uparrow}{1}, 0, 0, -6, 6]$

(D)  $[4, \underset{\uparrow}{1}, 0, 0, 6, 6]$

- Q.16** If  $I = \int_0^{2\pi} |Y(e^{j\omega})|^2 d\omega$ , the value of I is

(A)  $180\pi$

(B)  $178\pi$

(C)  $196\pi$

(D)  $200\pi$

**Statement for Linked Answer Questions 17 & 18**

4-sample sequence is given by

$$x[n] = [1, \underset{\uparrow}{2}, -2, -1]$$

- Q.17** The Fourier transform  $X(e^{j\omega})$  of this sequence is given by :

(A)  $j2e^{-j1.5\omega}[\sin(1.5\omega) + 2\sin(0.5\omega)]$

(B)  $j2e^{-j1.5\omega}[\sin(1.5\omega) - 2\sin(0.5\omega)]$

(C)  $j2e^{j1.5\omega}[\sin(1.5\omega) + 2\sin(0.5\omega)]$

(D)  $j2e^{j1.5\omega}[\sin(1.5\omega) - 2\sin(0.5\omega)]$

- Q.18** An eight length sequence is defined by

$$y[n] = [-1, \underset{\uparrow}{-2}, 2, 1, 1, 2, -2, -1]$$

$Y(e^{j\omega})$  is given by :

(A)  $e^{-4j\omega}X(e^{j\omega}) + e^{+3j\omega}X(e^{-j\omega})$

(B)  $e^{-4j\omega}X(e^{j\omega}) + e^{-3j\omega}X(e^{-j\omega})$

(C)  $e^{4j\omega}X(e^{j\omega}) + e^{-3j\omega}X(e^{j\omega})$

(D) None of the above

**Q.19** Inverse DTFT of  $X(e^{j\omega}) = \frac{1 - \frac{1}{3}e^{-j\omega}}{1 - \frac{1}{4}e^{-j\omega} - \frac{1}{8}e^{-2j\omega}}$

(A)  $\left[ \frac{2}{9} \left( \frac{1}{2} \right)^n + \frac{7}{9} \left( \frac{-1}{4} \right)^n \right] u(n)$

(B)  $\left[ \frac{2}{9} \left( \frac{-1}{2} \right)^n - \frac{7}{9} \left( \frac{-1}{4} \right)^n \right] u(n)$

(C)  $\left[ \frac{2}{9} \left( \frac{-1}{2} \right)^n + \frac{7}{9} \left( \frac{1}{4} \right)^n \right] u(n)$

(D)  $\left[ \frac{2}{9} \left( \frac{1}{2} \right)^n - \frac{7}{9} \left( \frac{-1}{4} \right)^n \right] u(n)$

**Q.20** If  $X(e^{j\omega}) = \begin{cases} 1 & |\omega| \leq \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$  then  $x[n]$  is :

(A)  $\frac{1}{\pi n} \sin \omega_c n$       (B)  $\frac{1}{n} \sin \omega_c n$

(C)  $\frac{n}{\pi} \sin \omega_c n$       (D)  $\frac{\pi}{n} \sin \omega_c n$

- Q.21** If  $X(e^{j\omega}) = e^{-j\omega}$  for  $-\pi \leq \omega \leq \pi$ , then the discrete time signal  $x(n)$  is :

(A)  $\frac{\sin 2\pi(n-1)}{2\pi(n-1)}$       (B)  $\sin \pi(n-1)$

(C)  $\frac{\sin \pi(n-1)}{\pi(n-1)}$       (D)  $\frac{\sin \pi(2n-1)}{\pi(2n-1)}$

- Q.22** The frequency response of a causal and stable LTI system is

$$H(e^{j\omega}) = \frac{2e^{j2\omega}}{e^{j2\omega} - 2e^{j\omega} + 1}$$

The time delay of the system is

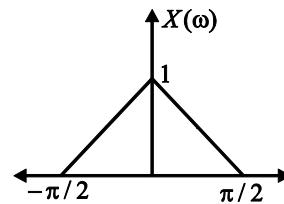
(A) 0      (B) -1

(C) 1      (D) None of these

- Q.23** The center of gravity of a signal  $x(n)$  is defined as :

$$C = \frac{\sum_{n=-\infty}^{\infty} nx(n)}{\sum_{n=-\infty}^{\infty} x(n)}$$

and provides, a measure of the 'time delay' of the signal. Compute C for the signal  $x(n)$  whose FT is shown :



- (A) 0                          (B) 1  
 (C)  $\omega$                       (D)  $\pi\omega$

**Q.24** The signal energy of  $x(n) = \frac{1}{5} \sin c\left(\frac{n}{100}\right)$  is :

- (A) 4                           (B) 100  
 (C) 400                        (D) none of these

**Q.25** A signal  $x[n] = \sin(\omega_0 n + \phi)$  is the input to a linear time invariant system having a frequency response  $H(e^{j\omega})$ . If the output of the system is  $Ax[n - n_0]$ , then the most general form of  $\angle H(e^{j\omega})$  will be

- (A)  $-n_0\omega_0 + \beta$  for any arbitrary real  $\beta$ .  
 (B)  $-n_0\omega_0 + 2\pi k$  for any arbitrary integer.  
 (C)  $n_0\omega_0 + 2\pi k$  for any arbitrary  $k$ .  
 (D)  $-n_0\omega_0\phi$ .

**Q.26** Match the following and choose the correct combination.

**Group 1**

- E. Continuous and aperiodic signal  
 F. Continuous and periodic signal  
 G. Discrete and aperiodic signal  
 H. Discrete and periodic signal

**Group 2**

- Fourier representation is continuous and aperiodic.
  - Fourier representation is discrete and aperiodic.
  - Fourier representation is continuous and periodic.
  - Fourier representation is discrete and periodic.
- (A) E-3, F-2, G-4, H-1  
 (B) E-1, F-3, G-2, H-4  
 (C) E-1, F-2, G-3, H-4  
 (D) E-2, F-1, G-4, H-3



**Answer Keys**

Objective & Numerical Answer Type Questions									
1.	B	2.	D	3.	A	4.	C	5.	A
6.	C	7.	D	8.	D	9.	2.093	10.	8
11.	3.375	12.	- 27						
Practice (Objective & Numerical Answer) Questions									
1.	D	2.	A	3.	D	4.	D	5.	B
6.	A	7.	D	8.	B	9.	A	10.	B
11.	A	12.	C	13.	C	14.	C	15.	D
16.	B	17.	A	18.	A	19.	A	20.	A
21.	C	22.	B	23.	A	24.	A	25.	B
26.	C								

# 7

# Z - Transform

## Objective & Numerical Ans Type Questions :

- Q.1** Consider the sequence  $x[n] = a^n u[n] + b^n u[n]$ , where  $u[n]$  denotes the unit-step sequence and  $0 < |a| < |b| < 1$ . The region of convergence (ROC) of the z-transform of  $x[n]$  is

[GATE EC 2016-Bangalore]

- (A)  $|z| > |a|$       (B)  $|z| > |b|$   
 (C)  $|z| < |a|$       (D)  $|a| < |z| < |b|$

- Q.2** A discrete-time signal  $x[n] = \delta[n-3] + 2\delta[n-5]$  has Z-transform  $X(z)$ . If  $Y(z) = X(-z)$  is the Z-transform of another signal  $y(n)$ , then

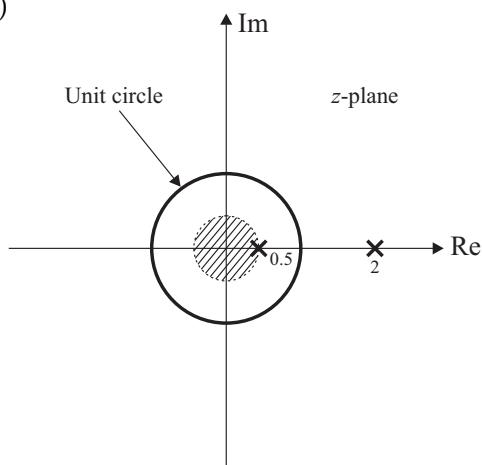
[GATE EC 2016-Bangalore]

- (A)  $y(n) = x(n)$       (B)  $y(n) = x(-n)$   
 (C)  $y(n) = -x(n)$       (D)  $y(n) = -x(-n)$

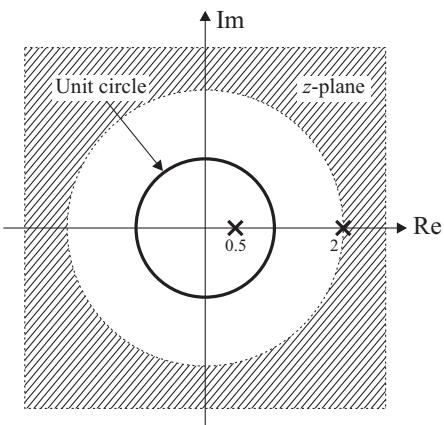
- Q.3** The ROC (region of convergence) of the z-transform of a discrete-time signal is represented by the shaded region in the z-plane. If the signal  $x[n] = (2.0)^{|n|}$ ,  $-\infty < n < +\infty$ , then the ROC of its z-transform is represented by

[GATE EC 2016-Bangalore]

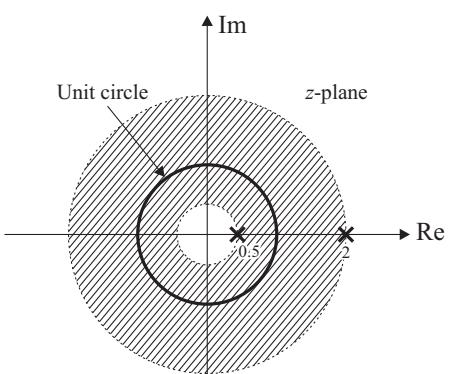
- (A)



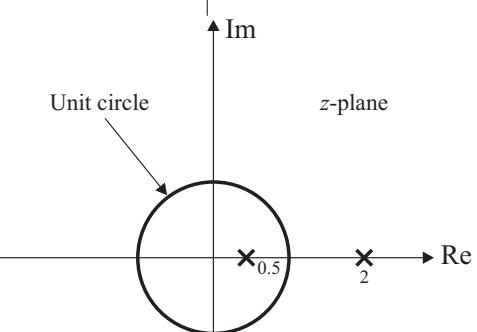
(B)



(C)

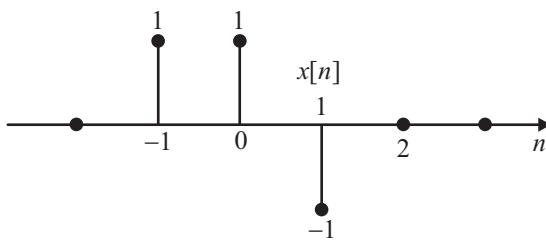


(D)



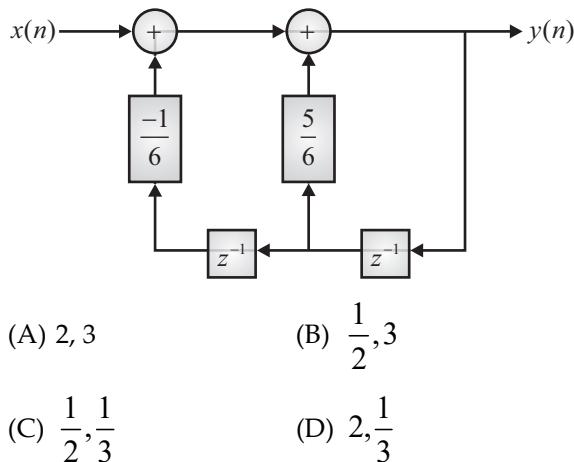
(ROC does not exist)

- Q.4** The signal  $x[n]$  shown in the figure below is convolved with itself to get  $y[n]$ . The value of  $y[-1]$  is \_\_\_\_\_. [GATE IN 2016-Bangalore]



- Q.5** The value of  $\sum_{n=0}^{\infty} n \left(\frac{1}{2}\right)^n$  is \_\_\_\_\_. [GATE EC 2015-Kanpur]

- Q.6** For the discrete-time system shown in the figure, the poles of the system transfer function are located at [GATE EC 2015-Kanpur]



- Q.7** Two causal discrete-time signals  $x[n]$  and  $y[n]$  are related as  $y[n] = \sum_{m=0}^n x[m]$ . If the z-transform of  $y[n]$  is  $\frac{2}{z(z-1)^2}$ , the value of  $x[2]$  is \_\_\_\_\_. [GATE EC 2015-Kanpur]

- Q.8** Suppose  $x[n]$  is an absolutely summable discrete-time signal. Its z-transform is a rational function with two poles and two zeroes. The poles are at  $z = \pm 2j$ . Which one of the following statements is TRUE for the signal  $x[n]$ ?

[GATE EC 2015-Kanpur]

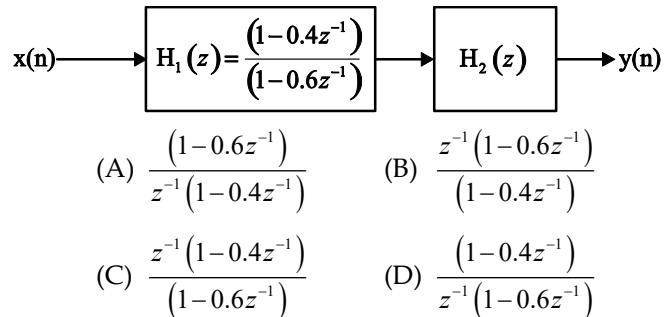
- (A) It is a finite duration signal
- (B) It is a causal signal
- (C) It is a non-causal signal
- (D) It is a periodic signal

- Q.9** The Z-transform of a sequence  $x[n]$  is given as  $X(z) = 2z + 4 - \frac{4}{z} + \frac{3}{z^2}$ . If  $y[n]$  is the first difference of  $x[n]$ , then  $Y[z]$  is given by

[GATE EE 2015-Kanpur]

- (A)  $2z + 2 - \frac{8}{z} + \frac{7}{z^2} - \frac{3}{z^3}$
- (B)  $-2z + 2 - \frac{6}{z} + \frac{1}{z^2} - \frac{3}{z^3}$
- (C)  $-2z - 2 + \frac{8}{z} - \frac{7}{z^2} + \frac{3}{z^3}$
- (D)  $4z - 2 - \frac{8}{z} - \frac{1}{z^2} + \frac{3}{z^3}$

- Q.10** Two systems  $H_1(z)$  and  $H_2(z)$  are connected in cascade as shown below. The overall output  $y(n)$  is the same as the input  $x(n)$  with a one unit delay. The transfer function of the second system  $H_2(z)$  is [GATE EC 2011-Madras]



- Q.11** Consider the difference equations  $y[n] - \frac{1}{3}y[n-1] = x[n]$  and suppose that  $x[n] = \left(\frac{1}{2}\right)^n u[n]$ . Assuming the conditions of initial rest, the solution of  $y[n]$ ,  $n \geq 0$  is

[GATE IN 2011-Madras]

- (A)  $3\left(\frac{1}{3}\right)^n - 2\left(\frac{1}{2}\right)^n$
- (B)  $-2\left(\frac{1}{3}\right)^n + 3\left(\frac{1}{2}\right)^n$
- (C)  $2\left(\frac{1}{3}\right)^n + \frac{1}{3}\left(\frac{1}{2}\right)^n$
- (D)  $\frac{1}{3}\left(\frac{1}{3}\right)^n + \frac{2}{3}\left(\frac{1}{2}\right)^n$

- Q.12** Two discrete time systems with impulse responses  $h_1[n] = \delta[n-1]$  and  $h_2[n] = \delta[n-2]$  are connected in cascade. The overall impulse response of the cascaded system is

[GATE EC 2010-Guwahati]

- (A)  $\delta[n-1] + \delta[n-2]$
- (B)  $\delta[n-4]$
- (C)  $\delta[n-3]$
- (D)  $\delta[n-1].\delta[n-2]$

- Q.13** The transfer function of a discrete time LTI system is given by [GATE EC 2010-Guwahati]

$$H(z) = \frac{2 - \frac{3}{4}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

Consider the following statements :

S1 : The system is stable and causal for

$$\text{ROC} : |z| > \frac{1}{2}$$

S2 : The system is stable but not causal for

$$\text{ROC} : |z| < \frac{1}{4}$$

S3 : The system is neither stable nor causal for

$$\text{ROC} : \frac{1}{4} < |z| < \frac{1}{2}$$

Which one of the following statements is valid?

(A) Both S1 and S2 are true

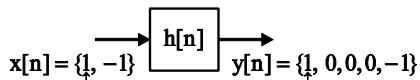
(B) Both S2 and S3 are true

(C) Both S1 and S3 are true

(D) S1, S2 and S3 are all true

- Q.14** Given the finite length input  $x[n]$  and the corresponding finite length output  $y[n]$  of an LTI system as shown below, the impulse response  $h[n]$  of the system is

[GATE EE 2010-Guwahati]



(A)  $h[n] = \{1, 0, 0, 1\}$       (B)  $h[n] = \{1, 0, 1\}$

(C)  $h[n] = \{1, 1, 1, 1\}$       (D)  $h[n] = \{1, 1, 1\}$

- Q.15** A discrete time linear shift-invariant system has an impulse response  $h[n]$  with  $h[0]=1$ ,  $h[1]=-1$ ,  $h[2]=2$ , and zero otherwise. The system is given an input sequence  $x[n]$  with  $x[0]=x[2]=1$ , and zero otherwise. The number of nonzero samples in the output sequence  $y[n]$ , and the value of  $y[2]$  are, respectively

[GATE EC 2008-Bangalore]

- (A) 5, 2      (B) 6, 2  
(C) 6, 1      (D) 5, 3

- Q.16** Given  $X(z) = \frac{z}{(z-a)^2}$  with  $|z| > a$ , the residue of  $X(z)z^{n-1}$  at  $z = a$  for  $n \geq 0$  will be

[GATE EE 2008-Bangalore]

- (A)  $a^{n-1}$       (B)  $a^n$   
(C)  $na^n$       (D)  $na^{n-1}$

- Q.17** Consider a discrete-time system for which the input  $x[n]$  and the output  $y[n]$  are related as

$$y[n] = x[n] - \frac{1}{3}y[n-1]. \text{ If } y[n] = 0 \text{ for } n < 0 \text{ and}$$

$x[n] = \delta[n]$ , then  $y[n]$  can be expressed in terms of the unit step  $u[n]$  as

[GATE IN 2008-Bangalore]

- (A)  $\left(\frac{-1}{3}\right)^n u[n]$       (B)  $\left(\frac{1}{3}\right)^n u[n]$   
(C)  $(3)^n u[n]$       (D)  $(-3)^n u[n]$

- Q.18** A linear discrete time system has the characteristic equation  $z^3 - 0.81z = 0$ . The system is

[GATE EC 2007-Kanpur, EC 1992-Delhi]

- (A) Stable  
(B) Marginal stable  
(C) Unstable  
(D) Can't be determined from the given information

- Q.19** The Z-Transform  $X(z)$  of a sequence  $x[n]$  is given by  $X(z) = \frac{0.5}{1-2z^{-1}}$ . It is given that the region of convergence of  $X(z)$  includes the unit circle. The value of  $x[0]$  is

[GATE EC 2007-Kanpur]

- (A) -0.5      (B) 0  
(C) 0.25      (D) 0.5

- Q.20**  $X(z) = 1 - 3z^{-1}$ ,  $Y(z) = 1 + 2z^{-2}$  are Z-Transform of two signals  $x[n]$ ,  $y[n]$  respectively. A linear time invariant system has the impulse response  $h[n]$  defined by these two signals as

$$h[n] = x[n-1] \otimes y[n]$$

[GATE EE 2006-Kharagpur]

Where  $\otimes$  denotes discrete time convolution then the output of the system for the input  $\delta[n-1]$ .

(A) has Z-Transform :  $z^{-1} X(z) Y(z)$

(B) equals  $\delta[n-2] - 3\delta[n-3] + 2\delta[n-4] - 6\delta[n-5]$

- (C) has Z-Transform :  $1 - 3z^{-1} + 2z^{-2} - 6z^{-3}$   
(D) does not satisfy any of the above three.
- Q.21**  $y[n]$  denotes the output and  $x[n]$  denotes the input of a discrete-time system given by the difference equation  $y[n] - 0.8y[n-1] = x[n] + 1.25x[n+1]$ . Its right-sided impulse response is  
**[GATE EE 2006-Kharagpur]**
- (A) Causal (B) Unbounded  
(C) Periodic (D) Non-negative
- Q.22** The region of convergence of Z-Transform of the sequence  $\left(\frac{5}{6}\right)^n u[n] - \left(\frac{6}{5}\right)^n u[-n-1]$  must be  
**[GATE EC 2005-Bombay]**
- (A)  $|z| < \frac{5}{6}$  (B)  $|z| > \frac{5}{6}$   
(C)  $\frac{5}{6} < |z| < \frac{6}{5}$  (D)  $\frac{6}{5} < |z| < \infty$
- Q.23** If  $u(t)$  is the unit step and  $\delta(t)$  is the unit impulse function, the inverse z-transform of  $F(z) = \frac{1}{z+1}$  for  $k > 0$  is  
**[GATE EE 2005-Bombay]**
- (A)  $(-1)^k \delta(k)$  (B)  $\delta(k) - (-1)^k u(k)$   
(C)  $(-1)^k u(k)$  (D)  $u(k) - (-1)^k \delta(k)$
- Q.24** A discrete-time signal  $x[n]$ , suffered a distortion modeled by an LTI system with  $H(z) = (1 - az^{-1})$ ,  $a$  is real and  $|a| > 1$ . The impulse response of a stable system that exactly compensates the magnitude of the distortion is  
**[GATE IN 2004-Delhi]**
- (A)  $\left(\frac{1}{a}\right)^n u[n]$  (B)  $-\left(\frac{1}{a}\right)^n u[-n-1]$   
(C)  $a^n u[n]$  (D)  $a^n u[-n-1]$
- Q.25** Given  $X(z) = \frac{1/2}{1 - az^{-1}} + \frac{1/3}{1 - bz^{-1}}$ ,  $|a| < 1$  and  $|b| < 1$  with the ROC specified as  $|a| < |z| < |b|$ ,  $x[0]$  of the corresponding sequence is given by  
**[GATE IN 2004-Delhi]**
- (A)  $\frac{1}{3}$  (B)  $\frac{5}{6}$   
(C)  $\frac{1}{2}$  (D)  $\frac{1}{6}$

- Q.26** A causal LTI system is described by the difference equation  
 $2y[n] = \alpha y[n-2] - 2x[n] + \beta x[n-1]$   
The system is stable only if  
**[GATE EC 2004-Delhi]**
- (A)  $|\alpha| = 2, |\beta| < 2$   
(B)  $|\alpha| > 2, |\beta| > 2$   
(C)  $|\alpha| < 2$ , any value of  $\beta$   
(D)  $|\beta| < 2$ , any value of  $\alpha$
- Q.27** A sequence  $x[n]$  with the Z-Transform  $X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$  is applied as an input to a linear, time-invariant system with the impulse response  $h[n] = 2\delta[n-3]$  where  
 $\delta[n] = \begin{cases} 1, & n=0 \\ 0, & \text{otherwise} \end{cases}$
- The output at  $n = 4$  is  
**[GATE EC 2003-Madras]**
- (A) -6 (B) 0  
(C) 2 (D) -4
- Q.28** The sequence  $x[n]$  whose Z-Transform is  $X(z) = e^{1/z}$  is  
**[GATE IN 2003-Madras]**
- (A)  $\frac{1}{n!} u[n]$  (B)  $\frac{1}{n!} u[-n]$   
(C)  $(-1)^n \frac{1}{n!} u[n]$  (D)  $\frac{1}{-(n+1)!} u[-n-1]$
- Q.29** Given  $h[n] = [1, 2, 2]$ ,  $f[n]$  is obtained by convolving  $h[n]$  with itself and  $g[n]$  by correlating  $h[n]$  with itself. Which one of the following statements is true?  
**[GATE IN 2003-Madras]**
- (A)  $f[n]$  is causal and its maximum value is 9  
(B)  $f[n]$  is non-causal and its maximum value is 8  
(C)  $g[n]$  is causal and its maximum value is 9  
(D)  $g[n]$  is non-causal and its maximum value is 9
- Q.30** Given  $x = [a, b, c, d]$  as the input, a linear time invariant system produces an output  $y = [x, x, x, \dots, < \text{repeated } N \text{ times} >]$ . The impulse response of the system is  
**[GATE IN 2003-Madras]**

(A)  $\sum_{i=0}^{N-1} \delta[n-4i]$       (B)  $u[n] - u[n-N]$

(C)  $u[n] - u[n-N-1]$       (D)  $\sum_{i=0}^{N-1} \delta[n-i]$

- Q.31** The Z-Transform of a signal is given by  $X(z) = \frac{z^{-1}(1-z^{-4})}{4(1-z^{-1})^2}$ . Its final value is

[GATE EC 1999-Bombay]

- (A) 1 / 4      (B) 0  
 (C) 1      (D)  $\infty$

- Q.32** Z-transform deals with discrete time systems for their :

[ESE EC 2016]

1. Transient behavior
2. Steady-state behavior

Which of the above behaviours is/are correct?

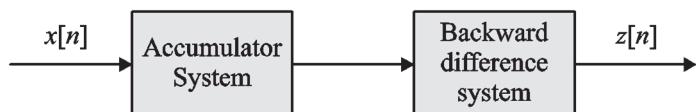
- (A) 1 only      (B) 2 only  
 (C) Both 1 and 2      (D) Neither 1 nor 2

- Q.33** When is a function  $f(n)$  said to be left sided ?

[ESE EC 2016]

- (A)  $f(n) = 0$  for  $n < 0$   
 (B)  $f(n) < 0$  for  $n > 0$   
 (C)  $f(n) = 0$  for  $n > n_0$   
 (D)  $f(n) = \infty$  for  $n < n_0$   
 $(n_0 \rightarrow \text{Positive or negative integer})$

- Q.34** Consider a discrete time accumulator system  $y[n] = \sum_{k=-\infty}^n x[k]$  and the backward difference system  $y[n] = x[n] - x[n-1]$  where  $x[\cdot]$  represents the input and  $y[\cdot]$  represents the output of the individual systems.



When these two systems are cascaded as in figure, the impulse response of combined system with output  $z[n]$  is

[ESE EC 2015]

- (A) Unit impulse sequence  
 (B) Unit step sequence  
 (C) Unit ramp sequence  
 (D) None of the above

- Q.35** Consider two infinite duration input sequences  $\{x_1[n], x_2[n]\}$ . When will the Region of Convergence [ROC] of Z transform of their superposition i.e.  $\{x_1[n] + x_2[n]\}$  be entire Z plane except possibly at  $Z = 0$  or  $Z = \infty$ ? [ESE EC 2015]

- (A) When their linear combination is of finite duration  
 (B) When they are left sided sequences  
 (C) When they are right sided sequences  
 (D) When their linear combination is causal

- Q.36** Z-transform and Laplace transform are related by

[ESE EC 2010]

- (A)  $s = \ln z$       (B)  $s = \frac{\ln z}{T}$   
 (C)  $s = z$       (D)  $s = \frac{T}{\ln z}$

- Q.37** Algebraic expression for Z transform of  $x[n]$  is  $X[z]$ . What is the algebraic expression for Z-transform of  $e^{j\omega_0 n} x[n]$ ? [ESE EC 2007]

- (A)  $X(z - z_0)$       (B)  $X(e^{-j\omega_0} z)$   
 (C)  $X(e^{j\omega_0} z)$       (D)  $X(z)e^{j\omega_0} z$

- Q.38** Let  $x[n] = a^n u[n]$ ,  $h[n] = b^n u[n]$ . What is the expression for  $y[n]$ , for a discrete-time system ?

- (A)  $\sum_{k=-\infty}^{\infty} a^k u[k] b^{n-k} u[n-k]$       [ESE EC 2005]  
 (B)  $\sum_{k=-\infty}^{\infty} a^n u[k] b^{n-k} u[n-k]$   
 (C)  $\sum_{k=-\infty}^{\infty} a^k u[n-k] b^n u[k]$   
 (D)  $\sum_{k=-\infty}^{\infty} a^{n-k} u[k] b^{n-k} u[n-k]$

- Q.39** If  $X(z) = \frac{z+z^{-3}}{z+z^{-1}}$  then  $x(n)$  series has

[ESE EC 2002]

- (A) Alternate 0's      (B) alternate 1's  
 (C) alternate 2's      (D) alternate - 1's

- Q.40** Which is true for Z-Transform?

- (A) It converts differential equation into algebraic equation.  
 (B) It converts difference equation into algebraic equation.

- (C) Solve integro differential equation.  
 (D) Solve algebraic equation.

- Q.41** ROC of unilateral z-transform is always  
 (A) inside of the circle in the z-plane  
 (B) outside of the circle in the z-plane  
 (C) both (A) and (B)  
 (D) on the circle

- Q.42** The ROC for the signal  $x[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{3}\right)^n u[-n-1]$  is  
 (A)  $|z| > \frac{1}{2}$       (B)  $|z| < \frac{1}{3}$   
 (C)  $\frac{1}{3} < |z| < \frac{1}{2}$       (D) None of these

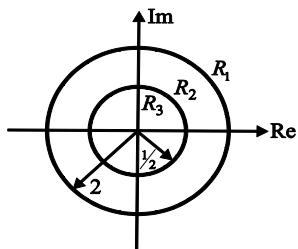
- Q.43** Consider three different signals

$$x_1[n] = \left[ 2^n - \left(\frac{1}{2}\right)^n \right] u[n]$$

$$x_2[n] = -2^n u[-n-1] + \frac{1}{2^n} u[-n-1]$$

$$x_3[n] = -2^n u[-n-1] - \frac{1}{2^n} u[n]$$

Below figure shows the three different regions. Choose the correct option for the ROC of signals.



- |  |                |                |
|--|----------------|----------------|
| R <sub>1</sub>   | R <sub>2</sub> | R <sub>3</sub> |
| (A) x <sub>1</sub> [n] x <sub>2</sub> [n] x <sub>3</sub> [n] |                |                |
| (B) x <sub>2</sub> [n] x <sub>3</sub> [n] x <sub>1</sub> [n] |                |                |
| (C) x <sub>1</sub> [n] x <sub>3</sub> [n] x <sub>2</sub> [n] |                |                |
| (D) x <sub>3</sub> [n] x <sub>2</sub> [n] x <sub>1</sub> [n] |                |                |

- Q.44** If  $X(z) = e^{az/z}$  for  $|z| > 0$  then  $x[n]$  is

- |                           |                                 |
|---------------------------|---------------------------------|
| (A) $\frac{a^n}{n!} u[n]$ | (B) $n! a^n u[n]$               |
| (C) $(n+1)! a^n u[n]$     | (D) $\frac{1}{(n+1)!} a^n u[n]$ |

- Q.45** The Z-transform of sequence  $x(n) = \frac{(\ln \alpha)^n}{n!} u(n)$  is  
 (A)  $\alpha^{1/z}$       (B)  $\alpha^z$   
 (C)  $\alpha^{-z}$       (D)  $\alpha^{-1/z}$

**Common Data for Questions 46 to 49**

If  $x[n] \xrightarrow{ZT} X(z)$  and  $X(z) = \frac{z}{z^2 + 4}; |z| < 2$

then find  $Y(z)$  for following  $y[n]$

- Q.46**  $y[n] = 2^n x[n]$

- |                               |                          |
|-------------------------------|--------------------------|
| (A) $\frac{z/2}{(z/2)^2 + 2}$ | (B) $\frac{2z}{z^2 + 4}$ |
| (C) $\frac{z/2}{(z/2)^2 + 4}$ | (D) $\frac{2z}{z^2 + 8}$ |

- Q.47**  $y[n] = nx[n]$

- |                                |                                    |
|--------------------------------|------------------------------------|
| (A) $\frac{z^3 - 4z}{z^2 + 4}$ | (B) $\frac{z^3 - 4z}{(z^2 + 4)^2}$ |
| (C) $\frac{z^3 + 4z}{z^2 + 4}$ | (D) $\frac{z^3 + 4z}{(z^2 + 4)^2}$ |

- Q.48**  $y[n] = x[n+1] + x[n-1]$

- |                                      |                                      |
|--------------------------------------|--------------------------------------|
| (A) $(z - z^{-1}) \frac{z}{z^2 + 4}$ | (B) $(z + z^{-1}) \frac{z}{z^2 + 4}$ |
| (C) $(z^2 + 1) \frac{z}{z^2 + 4}$    | (D) $(1 + z^{-2}) \frac{z}{z^2 + 4}$ |

- Q.49**  $y[n] = [n-3]x[n-2]$

- |                                   |                                    |
|-----------------------------------|------------------------------------|
| (A) $\frac{8z^{-1}}{(z^2 + 4)^2}$ | (B) $\frac{-4z^{-1}}{(z^2 + 4)^2}$ |
| (C) $\frac{4z^{-1}}{(z^2 + 4)^2}$ | (D) $\frac{-8z^{-1}}{(z^2 + 4)^2}$ |

**Common Data for Questions 50 to 53**

Given the Z-Transform pair

$$3^n n^2 u[n] \xrightarrow{ZT} X(z)$$

Determine the discrete time signal  $y[n]$  for corresponding Z-Transform  $Y(z)$

- Q.50**  $Y(z) = X(2z)$

- |                        |                         |
|------------------------|-------------------------|
| (A) $n^2 3^n u[2n]$    | (B) $(-3/2)^n n^2 u[n]$ |
| (C) $(3/2)^n n^2 u[n]$ | (D) $6^n n^2 u[n]$      |

- Q.51**  $Y(z) = X(z^{-1})$

- |                        |                                 |
|------------------------|---------------------------------|
| (A) $n^2 3^{-n} u[-n]$ | (B) $n^2 3^{-n} u[-n+1]$        |
| (C) $-n^2 3^n u[-n]$   | (D) $\frac{1}{n^2} 3^{-n} u[n]$ |

**Q.52**  $Y(z) = \frac{d}{dz} X(z)$

- (A)  $(n-1)^3 3^{n-1} u[n-1]$  (B)  $n^3 3^n u[n-1]$   
 (C)  $(n-1)^3 3^{n-1} u[n-1]$  (D)  $(1-n)^3 3^{n-1} u[n-1]$

**Q.53** If  $a^n u[n] \xleftarrow{ZT} \frac{z}{z-a}; |z| > |a|$  then Z-

Transform of  $x[n] = n(n-1)a^{n-2} u[n]$  is

- (A)  $\frac{2z}{(z-a)^3}$  (B)  $\frac{2az}{(z-a)^3}$   
 (C)  $\frac{az}{(z-a)^3}$  (D)  $\frac{2a^2 z}{(z-a)^3}$

**Q.54** Determine the initial and final value of  $x[n]$  for a given below z transform.

$$X(z) = \frac{z}{2z^2 - 3z + 1} \quad \text{ROC: } |z| > 1$$

- (A)  $x[0] = 0, x[\infty] = 0$  (B)  $x[0] = 0, x[\infty] = 1$   
 (C)  $x[0] = 1, x[\infty] = 1$  (D)  $x[0] = 1, x[\infty] = 0$

**Q.55** If  $X(z) = 2 + 3z^{-1} + 4z^{-2}$  then initial & final values of  $x[n]$  are

- (A) 1, 0 (B) 2, 0  
 (C) 2, 9 (D) 0, 2

**Q.56** Inverse Z-Transform of  $X(z) = \frac{0.5z}{z^2 - z + 0.5}$  is

- (A)  $\left(\frac{1}{\sqrt{2}}\right)^n \sin(n\pi/4) u[n]$   
 (B)  $\left(\frac{1}{\sqrt{2}}\right)^n \sin(n\pi/3) u[n]$   
 (C)  $\left(\sqrt{2}\right)^n \sin(n\pi/4) u[n]$   
 (D)  $\left(\sqrt{2}\right)^n \sin(n\pi/3) u[n]$

**Q.57** The z-transform  $X(z)$  of a sequence  $x[n]$  is given by

$$X(z) = \frac{z^{20}}{\left(z - \frac{1}{2}\right)(z-2)(z+3)}$$

If  $X(z)$  converges for  $|z|=1$  then  $x[-18]$  is

- (A)  $-\frac{1}{9}$  (B)  $-\frac{2}{21}$   
 (C)  $-\frac{1}{10}$  (D)  $-\frac{2}{27}$

**Q.58** The Z-transform of a discrete sequence  $f(n)$  is given by  $F(z) = \frac{z^2(7z-2)}{(z-0.2)(z-0.5)(z-1)}$ . The value of  $f(2)$  is

- (A) 9.9 (B) 11.23  
 (C) 11.87 (D) 7

**Q.59** The discrete-time convolution sum of

$$y[n] = \beta^n u[n] \otimes u[n-3]; |\beta| < 1$$

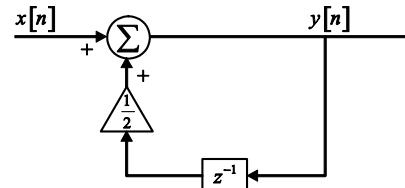
will be

- (A)  $\frac{1-\beta^{n-2}}{1-\beta}$  (B)  $\frac{1-\beta^{n-2}}{\beta-1}$   
 (C)  $\frac{1-\beta^{n-1}}{1-\beta}$  (D)  $\frac{1-\beta^{n-1}}{\beta-1}$

**Q.60** The unit impulse response of the system described by  $y[n] - y[n-1] = x[n] - x[n-1]$  is

- (A)  $\delta[n]$  (B)  $u[n]$   
 (C) 1 (D) 0

**Q.61** Consider the system shown in below figure. Find the impulse response  $h[n]$ .



- (A)  $\left(\frac{1}{2}\right)^n u[n]$  (B)  $(2)^n u[n]$   
 (C)  $\left(\frac{1}{3}\right)^n u[n]$  (D)  $\left(\frac{1}{2}\right)^n u[-n]$

**Common Data for Questions 62 & 63**

Consider a discrete-time LTI system whose function  $H(z)$  is given by

$$H(z) = \frac{z}{z - \frac{1}{2}} \quad \text{ROC: } |z| > \frac{1}{2}$$

**Q.62** Find the step response  $s[n]$ .

- (A)  $\left[2 - \left(\frac{1}{2}\right)^n\right] u[n]$  (B)  $\left[2 + \left(\frac{1}{2}\right)^n\right] u[n]$   
 (C)  $\left[1 - \left(\frac{1}{2}\right)^n\right] u[n]$  (D)  $\left[2 - \left(\frac{1}{2}\right)^{\frac{1}{n}}\right] u[n]$

**Q.63** Find the output  $y[n]$  to the input  $x[n] = n u[n]$ .

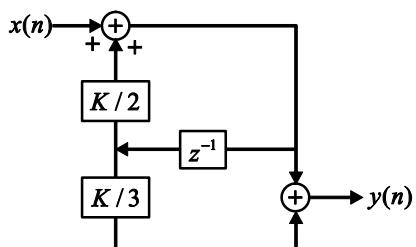
(A)  $2 \left[ \left( \frac{1}{2} \right)^n + n - 1 \right] u[-n]$

(B)  $2 \left[ \left( \frac{1}{2} \right)^n + n - 1 \right] u[n]$

(C)  $2 \left[ \left( \frac{1}{2} \right)^n + n + 1 \right] u[n]$

(D)  $2 \left[ \left( \frac{1}{2} \right)^n - n - 1 \right] u[n]$

**Q.64**



Value of 'K' for which the system will be stable is

(A)  $K < 1$       (B)  $K > 1$

(C)  $K < 2$       (D)  $K > 2$

**Q.65** If length of  $x_1[n]$  and  $x_2[n]$  are 3 and 2 respectively, then length of  $x_1[n] \otimes x_2[n]$  may be

(A) 3      (B) 5

(C) 6      (D) 7

**Q.66** If

$$x_1[n] = [1, 1, 2, 2] \text{ and } x_2[n] = [1, 2, 3, 4]$$

↑                                  ↑

then  $x_1[n] \otimes x_2[n]$  is

(A)  $[1, 3, 7, 13, 14, 14, 8]$  (B)  $\{1, 3, 7, 13, 14, 14, 8\}$

↑                                  ↑

(C)  $[1, 3, 7, 13, 14, 14, 8]$  (D)  $[1, 3, 7, 13, 14, 14, 8]$

↑                                  ↑

**Q.67** The value of  $y[2]$  if  $y[n] = x[n] \otimes h[n]$

$$x[n] = [1, 1, 0, 1, 1] \quad h[n] = [1, -2, -3, 4]$$

↑                                  ↑

(A) 2      (B) 4

(C) -5      (D) 3

**Q.68** For an LTI system with input  $x[n] = [2, 5, 0, 4]$  and output  $y[n] = [8, 22, 11, 31, 4, 12]$  the impulse response is

(A)  $[16, 84, 132, 117, 251, 88, 182, 16, 48]$

(B)  $[16, 84, 132, 117, 253, 88, 184, 16, 48]$

(C)  $[16, 84, 132, 117, 251, 88, 184, 16, 48]$

(D) None of the above

**Q.69**  $\cos\left(\frac{n\pi}{2}\right) \otimes \left(\frac{1}{2}\right)^n$  for  $n = 0, 1, 2, 3$  is

(A)  $1, \frac{1}{2}, \frac{-3}{4}, \frac{-3}{8}, \frac{-1}{4}, \frac{-1}{8}, 0$

(B)  $1, \frac{1}{2}, \frac{-3}{4}, \frac{3}{8}, \frac{-1}{4}, \frac{-1}{8}, 0$

(C)  $1, \frac{1}{2}, \frac{3}{4}, \frac{-3}{8}, \frac{1}{4}, \frac{-1}{8}, 0$

(D)  $1, \frac{1}{2}, \frac{3}{4}, \frac{3}{8}, \frac{1}{4}, \frac{-1}{8}, 0$

**Q.70** If  $x(n) = [1, -1, 2, 1]$

Then, auto correlation sequence  $r_{xx}(0)$  is

(A) 7      (B) 3

(C) 4      (D) 0

**Q.71** If  $x_1(n) = [1, 2, 3]$

$$x_2(n) = [1, -2, 1]$$

then, pick off the correct relation.

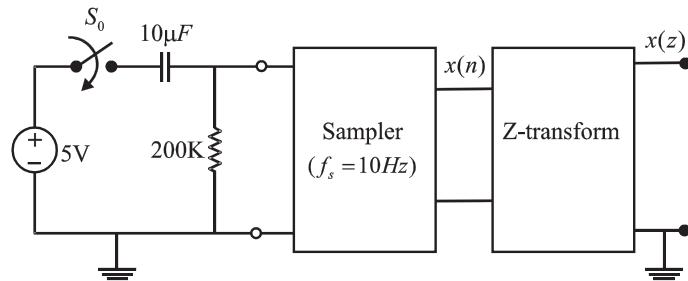
(A)  $r_{xy}(0) \geq r_{xy}(l)$       (B)  $r_{xy}(0) > r_{xy}(l)$

(C)  $r_{xy}(0) = 0$       (D)  $r_{xy}(0) \leq r_{xy}(l)$

### Statement for Linked Questions 72 & 73

In the following network, the switch is closed at  $t = 0^-$  and the sampling starts from  $t = 0$ . The sampling frequency is 10 Hz.

**[GATE EC 2008-Bangalore]**



**Q.72** The samples  $x[n]$  ( $n = 0, 1, 2, \dots$ ) are given by

(A)  $5(1 - e^{-0.05n})$       (B)  $5e^{-0.05n}$

(C)  $5(1 - e^{-5n})$       (D)  $5e^{-5n}$

**Q.73** The expression and the region of convergence of the Z-Transform of the sampled signal are

(A)  $\frac{5z}{z-e^{-5}}, |z|<e^{-5}$

(B)  $\frac{-5z}{z-e^{-0.05}}, |z|<e^{-0.05}$

(C)  $\frac{5z}{z-e^{-0.05}}, |z|>e^{-0.05}$

(D)  $\frac{5z}{z-e^{-5}}, |z|>e^{-5}$

### Practice (objective & Num Ans) Questions :

**Q.1** Consider a discrete time signal given by

$$x[n] = (-0.25)^n u[n] + (0.5)^n u[-n-1]$$

The region of convergence of its Z-transform would be **[GATE EE 2015-Kanpur]**

- (A) The region inside the circle of radius 0.5 and centered at origin
- (B) The region outside the circle of radius 0.25 and centered at origin
- (C) The annular region between the two circles, both centered at origin and having radii 0.25 and 0.5.
- (D) The entire Z plane

**Q.2** The z-transform of  $x[n] = \alpha^{|n|}, 0 < |\alpha| < 1$ , is  $X(z)$ .

The region of convergence of  $X(z)$  is

**[GATE IN 2015-Kanpur]**

(A)  $|\alpha| < |z| < \frac{1}{|\alpha|}$

(B)  $|z| > \alpha$

(C)  $|z| > \frac{1}{|\alpha|}$

(D)  $|z| < \min\left[|\alpha|, \frac{1}{|\alpha|}\right]$

**Q.3** Consider the Z-transform  $X(z) = 5z^2 + 4z^{-1} + 3;$

$0 < |z| < \infty$ . The inverse Z-transform  $x[n]$  is

**[GATE EC 2010-Guwahati]**

(A)  $5\delta[n+2] + 3\delta[n] + 4\delta[n-1]$

(B)  $5\delta[n-2] + 3\delta[n] + 4\delta[n+1]$

(C)  $5u[n+2] + 3u[n] + 4u[n-1]$

(D)  $5u[n-2] + 3u[n] + 4u[n+1]$

**Q.4** The ROC of Z-transform of the discrete time

sequence  $x(n) = \left(\frac{1}{3}\right)^n u(n) - \left(\frac{1}{2}\right)^n u(-n-1)$  is

**[GATE EC 2009-Roorkee]**

(A)  $\frac{1}{3} < |z| < \frac{1}{2}$       (B)  $|z| > \frac{1}{2}$

(C)  $|z| < \frac{1}{3}$       (D)  $2 < |z| < 3$

**Q.5** Consider a discrete-time LTI system with input  $x[n] = \delta[n] + \delta[n-1]$  and impulse response  $h[n] = \delta[n] - \delta[n-1]$ . The output of the system will be given by **[GATE IN 2008-Bangalore]**

- (A)  $\delta[n] - \delta[n-2]$
- (B)  $\delta[n] - \delta[n-1]$
- (C)  $\delta[n-1] + \delta[n-2]$
- (D)  $\delta[n] + \delta[n-1] + \delta[n-2]$

**Q.6** If the region of convergence of  $x_1[n] + x_2[n]$  is  $\frac{1}{3} < |z| < \frac{2}{3}$  then the region of convergence of  $x_1[n] - x_2[n]$  includes

**[GATE EC 2006-Kharagpur]**

(A)  $\frac{1}{3} < |z| < 3$       (B)  $\frac{2}{3} < |z| < 3$

(C)  $\frac{3}{2} < |z| < 3$       (D)  $\frac{1}{3} < |z| < \frac{2}{3}$

**Q.7** The Z-Transform of a system is  $H(z) = \frac{z}{z-0.2}$ . If the ROC is  $|z| < 0.2$ , then the impulse response of the system is **[GATE EC 2004-Delhi]**

- (A)  $(0.2)^n u[n]$
- (B)  $(0.2)^n u[-n-1]$
- (C)  $-(0.2)^n u[n]$
- (D)  $-(0.2)^n u[-n-1]$

**Q.8** A discrete-time signal,  $x[n]$ , suffered a distortion modeled by an LTI system with  $H(z) = (1-az^{-1})$ ,  $a$  is real and  $|a| > 1$ . The impulse response of a stable system that exactly compensates the magnitude of the distortion is

**[GATE IN 2004-Delhi]**

(A)  $\left(\frac{1}{a}\right)^n u[n]$       (B)  $-\left(\frac{1}{a}\right)^n u[-n-1]$

(C)  $a^n u[n]$       (D)  $-a^n u[-n-1]$

**Q.9** If the impulse response of a discrete-time system is  $h[n] = -5^n u[-n-1]$ , then the system function  $H(z)$  is equal to **[GATE EC 2002-Bangalore]**

(A)  $\frac{-z}{z-5}$  and the system is stable

- (B)  $\frac{z}{z-5}$  and the system is stable  
 (C)  $\frac{-z}{z-5}$  and the system is unstable  
 (D)  $\frac{z}{z-5}$  and the system is unstable
- Q.10** The impulse response of a discrete LTI system is  $a^n u[n]$ . Its response is given by
- [GATE IN 2001-Kanpur]**
- (A)  $\sum_{j=0}^n a^j$       (B)  $\sum_{j=0}^{\infty} a^j$   
 (C)  $\frac{1}{1-a}$       (D)  $\sum_{j=-\infty}^{\infty} a^j$
- Q.11** The region of convergence of the Z-Transform of a unit step function is [GATE EC 2001-Kanpur]
- (A)  $|z| > 1$   
 (B)  $|z| < 1$   
 (C) (Real part of z)  $> 0$   
 (D) (Real part of z)  $< 0$
- Q.12** The Z-Transform F(z) of the function  $f[nT] = a^{nT}$  is [GATE EC 1999-Bombay]
- (A)  $\frac{z}{z-a^T}$       (B)  $\frac{z}{z+a^T}$   
 (C)  $\frac{z}{z-a^{-T}}$       (D)  $\frac{z}{z+a^{-T}}$
- Q.13** The lengths of two discrete time sequences  $x_1[n]$  and  $x_2[n]$  are 5 and 7, respectively. The maximum length of a sequence  $x_1[n] \otimes x_2[n]$  is
- [GATE IN 1999-Bombay]**
- (A) 5      (B) 6  
 (C) 7      (D) 11
- Q.14** The Z-Transform of the time function  $\sum_{k=0}^{\infty} \delta[n-k]$  is [GATE EC 1998-Delhi]
- (A)  $\frac{z-1}{z}$       (B)  $\frac{z}{z-1}$   
 (C)  $\frac{z}{(z-1)^2}$       (D)  $\frac{(z-1)^2}{z}$
- Q.15** The transfer function which corresponds to an unstable system is [GATE EC 1997-Madras]

- (A)  $\frac{z-0.5}{z+0.5}$       (B)  $\frac{z+0.5}{z-0.5}$   
 (C)  $\frac{z}{z+2}$       (D)  $\frac{z}{z-1}$
- Q.16** Consider a two-sided discrete-time signal (neither left sided, nor right sided). The region of convergence (ROC) of the Z transform of the sequence is : [ESE EC 2016]
1. All region of z-plane outside a unit circle (in z-plane)
  2. All region of z-plane inside a unit circle (in z-plane)
  3. Ring in z-plane
- Which of the above is/are correct ?
- (A) 1 only      (B) 2 only  
 (C) 3 only      (D) 1 and 3
- Q.17** The impulse response for the discrete-time system : [ESE EC 2016]
- $y[n] = 0.24(x[n] + x[n-1] + x[n-2] + x[n-3])$  is given by
- (A) 0 for  $0 \leq n \leq 3$  and 0.24 otherwise  
 (B) 0.24 for  $0 \leq n \leq 3$  and 0 otherwise  
 (C) 0.24 for  $n = 0$  to  $n = \infty$   
 (D) 0 for  $n = 0$  to  $n = \infty$
- Q.18** The response of a linear, time-invariant discrete-time system to a unit step input  $u[n]$  is  $\delta[n]$ . The system response to a ramp input  $n u[n]$  would be [ESE EC 2016]
- (A)  $\delta[n-1]$       (B)  $u[n-1]$   
 (C)  $n \delta[n-1]$       (D)  $n u[n-1]$
- Q.19** If the lower limit of Region of Convergence (ROC) is greater than the upper limit of ROC, the series  $X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$  [ESE EC 2015]
- (A) Converges      (B) Zero  
 (C) Does not converge      (D) None of the above
- Q.20** Unit step response of the system described by the equation  $y(n) + y(n-1) = x(n)$  is [ESE EC 2010]
- (A)  $\frac{z^2}{(z+1)(z-1)}$       (B)  $\frac{z}{(z+1)(z-1)}$   
 (C)  $\frac{z+1}{z-1}$       (D)  $\frac{z(z-1)}{(z+1)}$

**Q.21** What is the inverse Z-transform of  $X(z)$ ?  
[ESE EC 2006]

- (A)  $\frac{1}{2\pi j} \oint X(z)z^{n-1} dz$    (B)  $\frac{1}{2\pi j} \oint X(z)z^{n+1} dz$   
 (C)  $\frac{1}{2\pi j} \oint X(z)z^{1-z} dz$    (D)  $2\pi j \oint X(z)z^{-(n+1)} dz$

**Q.22**



For the system shown,  $x[n] = k\delta[n]$ , and  $y[n]$  is related to  $x[n]$  as  $y[n] - \frac{1}{2}y[n-1] = x[n]$ . What is  $y[n]$  equal to? [ESE EC 2006]

- (A)  $k$                                   (B)  $(1/2)^n k$   
 (C)  $nk$                                   (D)  $2^n$

**Q.23** Match List-I (Discrete Time Signal) with List-II (Transform) and select the correct answer using the code given below the lists : [ESE EC 2005]

**List-I**

(Discrete time Signal)

- A. Unit Step Function  
 B. Unit impulse function  
 C.  $\sin \omega t$ ,  $t = 0, T, 2T, \dots$   
 D.  $\cos \omega t$ ,  $t = 0, T, 2T \dots$

**List-II**

(Transform)

1.  $1$   
 2.  $\frac{z - \cos \omega T}{z^2 - 2z \cos \omega T + 1}$   
 3.  $\frac{z}{z-1}$   
 4.  $\frac{z \sin \omega T}{z^2 - 2z \cos \omega T + 1}$

**Code :**    **A**    **B**    **C**    **D**

- (A) 2    4    1    3  
 (B) 3    1    4    2  
 (C) 2    1    4    3  
 (D) 3    4    1    2

**Q.24** Which one of the following is the inverse z-transform of

$$X(z) = \frac{z}{(z-2)(z-3)}, \text{ ROC: } |z| < 2 ?$$

- (A)  $[2^n - 3^n] u(-n-1)$                                   [ESE EC 2005]

(B)  $[3^n - 2^n] u(-n-1)$

(C)  $[2^n - 3^n] u(n+1)$

(D)  $[2^n - 3^n] u(n)$

**Q.25** What is the output of the system with  $h[n] = \left(\frac{1}{2}\right)^n u(n)$  in response to the input

$$x[n] = 3 + \cos\left(\pi n + \frac{\pi}{3}\right) ?$$
                                  [ESE EC 2004]

(A)  $y[n] = 3 + \frac{1}{3} \left[ \cos\left(\frac{\pi n}{pn} + \frac{\pi}{3}\right) \right]$

(B)  $y[n] = 3 + \frac{2}{3} \left[ \cos\left(\pi n + \frac{\pi}{3}\right) \right]$

(C)  $y[n] = 1 + \frac{2}{3} \left[ \sin\left(\pi n + \frac{\pi}{3}\right) \right]$

(D)  $y[n] = 6 + \frac{2}{3} \left[ \cos\left(\pi n + \frac{\pi}{3}\right) \right]$

**Q.26** What is the number of roots of the polynomial  $F(z) = 4z^3 - 8z^2 - z + 2$ , lying outside the unit circle? [ESE EC 2004]

- (A) 0    (B) 1  
 (C) 2    (D) 3

**Q.27** Which one of the following gives the cross-correlation ( $Y_{xy}(k)$ ) of two finite length sequences  $x(n) = [1, 3, 1, 2]$  and  $y(n) = [1, 2, 1, 3]$ ?

- (A) [3, 10, 8, 14, 7, 5, 2]                                  [ESE EC 2004]  
 (B) [2, 10, 7, 14, 6, 6, 3]  
 (C) [3, 9, 8, 14, 7, 5, 2]  
 (D) None of the above

**Q.28** Match I List-I with List-II and select the correct answer using the codes given below the lists :

**List – I**

**List – II** [ESE EC 2000]

A.  $\alpha^n u(n)$     1.  $\frac{az^{-1}}{(1-az^{-1})^2}$   
 $ROC: |z| > |\alpha|$

B.  $-\alpha^n u(-n-1)$     2.  $\frac{1}{(1-az^{-1})}$   
 $ROC: |z| > |\alpha|$

C.  $-n\alpha^n u(-n-1)$     3.  $\frac{1}{(1-az^{-1})}$   
 $ROC: |z| < |\alpha|$

D.  $n\alpha^n u(n)$

4.  $\frac{az^{-1}}{(1-az^{-1})^2}$   
ROC:  $|z| < |\alpha|$

Codes : A B C D

(A) 2 4 3 1

(B) 1 3 4 2

(C) 1 4 3 2

(D) 2 3 4 1

Q.29 If  $x[n] = \{3, 5, 6, 1, 2, 0, 4\}$  then  $X(z)$  is  
 $\uparrow$ 

(A)  $3z^3 + 5z^2 + 6z + 1 + 2z^{-1} + 4z^{-3}$ ; ROC:  $|z| \geq 0$

(B)  $3z^3 + 5z^2 + 6z + 1 + 2z^{-1} + 4z^{-3}$ ; ROC:  $|z| \neq 0$

(C)  $3z^3 + 5z^2 + 6z + 1 + 2z^{-1} + 4z^{-3}$ ; ROC:  $0 < |z| < \infty$

(D)  $3z^3 + 5z^2 + 6z + 1 + 2z^{-1} + 4z^{-3}$ ; ROC:  $|z| \neq \infty$

Q.30 The output  $y[n]$  of a discrete time LTI system is related to the input  $x[n]$  as given below

$$y[n] = \sum_{k=0}^{\infty} x[k]$$

Which one of the following correctly relates the Z-Transforms of the input and output, denoted by  $X(z)$  and  $Y(z)$  respectively?

(A)  $Y(z) = (1 - z^{-1})X(z)$  (B)  $Y(z) = z^{-1}X(z)$

(C)  $Y(z) = \frac{X(z)}{1 - z^{-1}}$  (D)  $Y(z) = \frac{dX(z)}{dz}$

Q.31 Z-Transform and associated ROC for the sequence  $x[n] = a^{-n}u[-n]$  is

(A)  $\frac{1}{1-az}; |z| > \frac{1}{|a|}$  (B)  $\frac{a}{a-z}; |z| < |a|$

(C)  $\frac{1}{1-az}; |z| < \frac{1}{|a|}$  (D)  $\frac{a}{a-z}; |z| > |a|$

Q.32 What is the number of roots of the polynomial  $F(z) = 4z^3 - 8z^2 - z + 2$ , lying outside the unit circle?

(A) 0 (B) 1

(C) 2 (D) 3

Q.33 If  $x[n] \xrightarrow{ZT} X(z)$  and  $y[n] = \begin{cases} x\left(\frac{n}{2}\right) & ; n = \text{even} \\ 0 & ; n = \text{odd} \end{cases}$   
then  $Y(z)$  is

(A)  $X(2z)$  (B)  $X(z^{-2})$

(C)  $X(z^2)$  (D)  $X(z^{1/2})$

Q.34  $Y(z) = |X(z)|^2$

(A)  $x^2[n]$  (B)  $x[n] \otimes x[n]$

(C)  $x[n] \otimes x[-n]$  (D)  $x[-n] x[n]$

Q.35 Given the Z-Transform  $X(z) = \frac{z(8z-7)}{4z^2 - 7z + 3}$

The value of  $x[\infty]$  is

(A) 1 (B) 2

(C) 0 (D) None of these

Q.36 Determine the initial and final value of  $x[n]$  for a given below Z transform.

$$X(z) = \frac{2z\left(z - \frac{5}{12}\right)}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{3}\right)}$$
 ROC:  $|z| > \frac{1}{2}$

(A)  $x[0] = 2, x[\infty] = 0$  (B)  $x[0] = 0, x[\infty] = 0$

(C)  $x[0] = 2, x[\infty] = 2$  (D)  $x[0] = 0, x[\infty] = 2$

Q.37 The inverse Z-Transform of  $X(z) = \frac{3}{z-2}$ ;  
ROC:  $|z| > 2$  is

(A)  $3[2^{n-1}u[n]]$  (B)  $3[2^n u[n-1]]$

(C)  $1 \cdot 5[2^n u[n]]$  (D)  $3[2^{n-1}u[n-1]]$

Q.38 Which one of the following is the inverse Z-Transform of  $X(z) = \frac{z}{(z-2)(z-3)}$ ;  
ROC:  $|z| < 2$ ?

(A)  $[2^n - 3^n] u[-n-1]$  (B)  $[3^n - 2^n] u[-n-1]$

(C)  $[2^n - 3^n] u[n+1]$  (D)  $[2^n - 3^n] u[n]$

Q.39 Determine  $x[n]$  for  $X(z) = \frac{z^{-1}}{1-3z^{-1}}$  with  
ROC:  $|z| < 3$ 

(A)  $-3^{n-1}u[n]$  (B)  $-3^{n-1}u[-n-1]$

(C)  $-3^{n-1}u[-n]$  (D)  $3^{n-1}u[n-1]$

Q.40 Consider the difference equation of LTI causal discrete time system  $y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n)$ . The impulse response of the system is

- (A)  $\left[2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right]u(n)$  (B)  $\left[2\left(\frac{1}{2}\right)^n + \left(\frac{1}{4}\right)^n\right]u(n)$   
 (C)  $\left[2\left(\frac{1}{4}\right)^n - \left(\frac{1}{2}\right)^n\right]u(n)$  (D)  $\left[2\left(\frac{1}{4}\right)^n + \left(\frac{1}{2}\right)^n\right]u(n)$

**Common Data for Questions 41 & 42**

Consider a causal discrete-time system whose output  $y[n]$  and input  $x[n]$  are related by

$$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = x[n]$$

- Q.41** Find the system function  $H(z)$ .

- (A)  $H(z) = \frac{z^2}{(z - \frac{1}{2})(z - \frac{1}{3})}, ROC : |z| < \frac{1}{2}$   
 (B)  $H(z) = \frac{z^2}{(z + \frac{1}{2})(z + \frac{1}{3})}, ROC : |z| > \frac{1}{2}$   
 (C)  $H(z) = \frac{z^2}{(z + \frac{1}{2})(z + \frac{1}{3})}, ROC : |z| < \frac{1}{2}$   
 (D)  $H(z) = \frac{z^2}{(z - \frac{1}{2})(z - \frac{1}{3})}, ROC : |z| > \frac{1}{2}$

- Q.42** Find its impulse response  $h[n]$ .

- (A)  $h[n] = \left[3\left(\frac{1}{2}\right)^n + 2\left(\frac{1}{3}\right)^n\right]u[n]$   
 (B)  $h[n] = \left[2\left(\frac{1}{2}\right)^n - 3\left(\frac{1}{3}\right)^n\right]u[n]$   
 (C)  $h[n] = \left[3\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{3}\right)^n\right]u[n]$   
 (D)  $h[n] = \left[2\left(\frac{1}{2}\right)^n + 3\left(\frac{1}{3}\right)^n\right]u[n]$

- Q.43** The difference equation is given by  $y(n+2) - 5y(n+1) + 6y(n) = 3f(n+1) + 5f(n)$ . If

the initial conditions are  $y(-1) = \frac{11}{6}, y(-2) = \frac{37}{36}$ .

Determine the discrete response for the input  $f(n) = (2)^{-n}u(n)$ .

- (A)  $y(n) = \left(\frac{26}{15}(0.5)^n + \frac{7}{3}(2)^n + \frac{18}{5}(3)^n\right)u(n)$   
 (B)  $y(n) = \left(\frac{26}{15}(0.5)^n - \frac{7}{3}(2)^n - \frac{18}{5}(3)^n\right)u(n)$

(C)  $y(n) = \left(-\frac{26}{15}(0.5)^n - \frac{7}{3}(2)^n - \frac{18}{5}(3)^n\right)u(n)$

(D)  $y(n) = \left(\frac{26}{15}(0.5)^n - \frac{7}{3}(2)^n + \frac{18}{5}(3)^n\right)u(n)$

**Q.44** Consider the system :  $H(z) = \frac{\frac{2}{z-1} + \frac{1}{z-2}}{1 - \frac{3}{5}z^{-1} + \frac{2}{25}z^{-2}}$

The step response of the systems if  $y(-1) = 1$  and  $y(-2) = 2$  is

- (A)  $\left[\frac{25}{8} + \frac{7}{8}\left(\frac{1}{5}\right)^n - 3\left(\frac{2}{5}\right)^n\right]u(n)$   
 (B)  $\left[\frac{25}{8} + \frac{33}{200}\left(\frac{1}{5}\right)^n - \frac{87}{25}\left(\frac{2}{5}\right)^n\right]u(n)$   
 (C)  $\left[\frac{15}{8} + \frac{29}{200}\left(\frac{1}{5}\right)^n - \frac{83}{25}\left(\frac{2}{5}\right)^n\right]u(n)$   
 (D) None of the above

**Common Data for Questions 45 & 46**

The difference equation relating  $y(n)$  and  $x(n)$

$$\text{is } y(n+2) - \frac{5}{6}y(n+1) + \frac{1}{6}y(n) = 5x(n+1) - x(n)$$

- Q.45** For  $x(n) = u(n)$  and initial conditions  $y(-1) = 2$  and  $y(-2) = 0$ , the value of  $y(n)$  is

- (A)  $\left[12 - 15\left(\frac{1}{2}\right)^n + \frac{14}{3}\left(\frac{1}{3}\right)^n\right]u(n)$   
 (B)  $\left[12 + 15\left(\frac{1}{2}\right)^n - \frac{14}{3}\left(\frac{1}{3}\right)^n\right]u(n)$   
 (C)  $\left[12 - 15\left(\frac{1}{3}\right)^n + \frac{14}{3}\left(\frac{1}{2}\right)^n\right]u(n)$   
 (D)  $\left[12 + 15\left(\frac{1}{3}\right)^n - \frac{14}{3}\left(\frac{1}{2}\right)^n\right]u(n)$

- Q.46** The zero state component of  $y(n)$  is

- (A)  $\left[12 - 18\left(\frac{1}{2}\right)^n + 6\left(\frac{1}{3}\right)^n\right]u(n)$   
 (B)  $\left[12 + 18\left(\frac{1}{2}\right)^n - 6\left(\frac{1}{3}\right)^n\right]u(n)$   
 (C)  $\left[12 - 18\left(\frac{1}{3}\right)^n + 6\left(\frac{1}{2}\right)^n\right]u(n)$   
 (D)  $\left[12 + 18\left(\frac{1}{3}\right)^n - 6\left(\frac{1}{2}\right)^n\right]u(n)$

**Answer Keys**

Objective & Numerical Answer Type Questions									
1.	B	2.	C	3.	D	4.	2	5.	2
6.	C	7.	0	8.	C	9.	A	10.	B
11.	B	12.	C	13.	C	14.	C	15.	D
16.	D	17.	A	18.	A	19.	B	20.	B
21.	D	22.	C	23.	B	24.	D	25.	C
26.	C	27.	B	28.	A	29.	D	30.	A
31.	C	32.	C	33.	C	34.	A	35.	A
36.	B	37.	B	38.	A	39.	A	40.	B
41.	B	42.	D	43.	C	44.	A	45.	A
46.	C	47.	B	48.	B	49.	D	50.	C
51.	A	52.	D	53.	A	54.	B	55.	B
56.	A	57.	B	58.	B	59.	A	60.	A
61.	A	62.	A	63.	B	64.	C	65.	A
66.	B	67.	B	68.	*	69.	A	70.	A
71.	C	72.	B	73.	C				

Practice (Objective & Numerical Answer) Questions									
1.	C	2.	A	3.	A	4.	A	5.	A
6.	D	7.	D	8.	D	9.	B	10.	B
11.	A	12.	A	13.	D	14.	B	15.	C
16.	C	17.	B	18.	B	19.	C	20.	A
21.	A	22.	B	23.	B	24.	A	25.	D
26.	B	27.	D	28.	D	29.	C	30.	C
31.	C	32.	B	33.	C	34.	B	35.	A
36.	A	37.	D	38.	A	39.	C	40.	A
41.	D	42.	C	43.	D	44.	B	45.	A
46.	A								

# 8

## DFT & FFT

### Objective & Numerical Ans Type Questions :

- Q.1** The 4-point Discrete Fourier Transform (DFT) of a discrete time sequence  $[1, 0, 2, 3]$  is

[GATE EC 2009, IIT Roorkee]

- (A)  $[0, -2 + 2j, 2, -2, -2 - 2j]$
- (B)  $[2, 2 + 2j, 6, 2 - 2j]$
- (C)  $[6, 1 - 3j, 2, 1 + 3j]$
- (D)  $[6, -1 + 3j, 0, -1 - 3j]$

- Q.2** Let  $X[k] = k+1$ ,  $0 \leq k \leq 7$  be 8-point DFT of a sequence  $x[n]$ , where

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi nk/N}$$

The value (correct to two decimal places) of  $\sum_{n=0}^3 x[2n]$  is \_\_\_\_\_.  
**[GATE EC 2018, IIT Guwahati]**

- Q.3** The  $N$ -point DFT  $X$  of a sequence  $x[n]$ ,  $0 \leq n \leq N-1$  is given by

$$X[k] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}nk}, \quad 0 \leq k \leq N-1$$

Denote this relation as  $X = \text{DFT}(x)$ . For  $N = 4$ , which one of the following sequences satisfies  $\text{DFT}[\text{DFT}(x)] = x$ ?

[GATE EC 2014 Set-04, IIT Kharagpur]

- (A)  $x = [1 \ 2 \ 3 \ 4]$
- (B)  $x = [1 \ 2 \ 3 \ 2]$
- (C)  $x = [1 \ 3 \ 2 \ 2]$
- (D)  $x = [1 \ 2 \ 2 \ 3]$

- Q.4** The DFT of a vector  $[a \ b \ c \ d]$  is the vector  $[a \ \beta \ \gamma \ \delta]$ . Consider the product

$$[p \ q \ r \ s] = [a \ b \ c \ d] \begin{bmatrix} a & b & c & d \\ d & a & b & c \\ c & d & a & b \\ b & c & d & a \end{bmatrix}$$

The DFT of vector  $[p \ q \ r \ s]$  is a scaled version of [GATE EC 2013, IIT Bombay]

- (A)  $[\alpha^2 \ \beta^2 \ \gamma^2 \ \delta^2]$
- (B)  $[\sqrt{\alpha} \ \sqrt{\beta} \ \sqrt{\gamma} \ \sqrt{\delta}]$
- (C)  $[\alpha + \beta \ \beta + \delta \ \delta + \gamma \ \gamma + \alpha]$
- (D)  $[\alpha \ \beta \ \gamma \ \delta]$

- Q.5** Two sequences  $[a, b, c]$  and  $[A, B, C]$  are related as,

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^{-1} & W_3^{-2} \\ 1 & W_3^{-2} & W_3^{-4} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

where,  $W_3 = e^{j\frac{2\pi}{3}}$

If another sequence  $[p, q, r]$  is derived as,

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^1 & W_3^2 \\ 1 & W_3^2 & W_3^4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & W_3^2 & 0 \\ 0 & 0 & W_3^4 \end{bmatrix} \begin{bmatrix} A/3 \\ B/3 \\ C/3 \end{bmatrix}$$

then the relationship between the sequence  $[p, q, r]$  and  $[a, b, c]$  is

[GATE EC 2014 Set - 01, IIT Kharagpur]

- (A)  $[p, q, r] = [b, a, c]$
- (B)  $[p, q, r] = [b, c, a]$
- (C)  $[p, q, r] = [c, a, b]$
- (D)  $[p, q, r] = [c, b, a]$

- Q.6** Consider two real sequences with time-origin marked by the bold value,

$$x_1[n] = \{1, 2, 3, 0\}, x_2[n] = \{1, 3, 2, 1\}$$

Let  $X_1(k)$  and  $X_2(k)$  be 4-point DFTs of  $x_1[n]$  and  $x_2[n]$ , respectively.

Another sequence  $x_3[n]$  is derived by taking 4-point inverse DFT of  $X_3(k) = X_1(k)X_2(k)$ .

The value of  $x_3[2]$  is \_\_\_\_\_.

[GATE EC 2014 Set – 02, IIT Kharagpur]

- Q.7** The Discrete Fourier Transform (DFT) of the 4-point sequence

$$x[n] = \{x[0], x[1], x[2], x[3]\} = \{3, 2, 3, 4\}$$

$$X[k] = \{X[0], X[1], X[2], X[3]\}$$

$$X[k] = \{12, 2j, 0, -2j\}$$

If  $X_1[k]$  is the DFT of the 12-point sequence  $x_1[n] = \{3, 0, 0, 2, 0, 0, 3, 0, 0, 4, 0, 0\}$ , the value of  $\left| \frac{X_1[8]}{X_1[11]} \right|$  is \_\_\_\_\_.

[GATE EC 2016 Set – 02, IISc Bangalore]

- Q.8**  $X(k)$  is the Discrete Fourier Transform of a 6-point real sequence  $x(n)$ .

If  $X(0) = 9 + j0$ ,  $X(2) = 2 + j2$ ,

$X(3) = 3 - j0$ ,  $X(5) = 1 - j1$ ,  $x(0)$  is

[GATE IN 2014, IIT Kharagpur]

- (A) 3                          (B) 9  
 (C) 15                        (D) 18

- Q.9** For an  $N$ -point FFT algorithm with  $N = 2^m$ , which one of the following statements is TRUE?

[GATE EC 2010, IIT Guwahati]

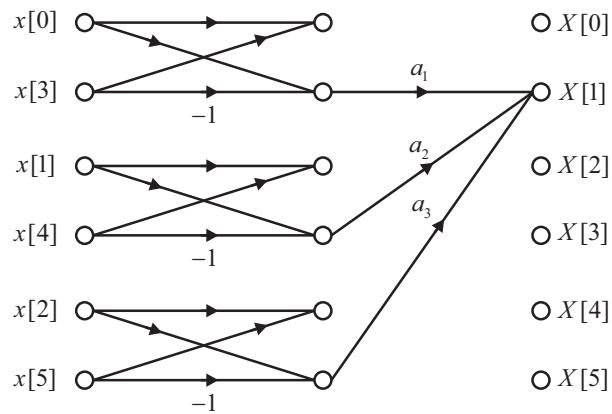
- (A) It is not possible to construct a signal flow graph with both input and output in normal order.  
 (B) The number of butterflies in the  $m^{\text{th}}$  stage is  $N/m$ .  
 (C) In-place computation requires storage of only  $2N$  node data.  
 (D) Computation of a butterfly requires only one complex multiplication.

- Q.10** A continuous-time speech signal  $x_a(t)$  is sampled at a rate of 8 kHz and the samples are subsequently grouped in blocks, each of size  $N$ . The DFT of each block is to be computed in real time using the radix-2 decimation-in-frequency FFT algorithm. If the processor performs all operations sequentially, and takes 20  $\mu\text{s}$  for computing each complex multiplication (including multiplications by 1 and -1) and the time required for addition / subtraction is negligible, then the maximum value of  $N$  is \_\_\_\_\_.

[GATE EC 2016 Set – 03, IISc Bangalore]

- Q.11** Consider a six-point decimation-in-time Fast Fourier Transform (FFT) algorithm, for which the signal-flow graph corresponding to  $X[1]$  is shown in the figure. Let  $W_6 = \exp\left(-\frac{j2\pi}{6}\right)$ . In the figure, what should be the values of the coefficients  $a_1, a_2, a_3$  in terms of  $W_6$  so that  $X[1]$  is obtained correctly?

[GATE EC 2019, IIT Madras]



- (A)  $a_1 = -1, a_2 = W_6^2, a_3 = W_6$   
 (B)  $a_1 = 1, a_2 = W_6^2, a_3 = W_6$   
 (C)  $a_1 = 1, a_2 = W_6, a_3 = W_6^2$   
 (D)  $a_1 = -1, a_2 = W_6, a_3 = W_6^2$

#### Practice (objective & Num Ans) Questions :

- Q.1** 4-point DFT of a real discrete-time signal  $x(n)$  of length 4 is given by  $X(k)$ ,  $n = 0, 1, 2, 3$  and  $k = 0, 1, 2, 3$ . It is given that  $X(0) = 5$ ,  $X(1) = 1 + j1$ ,  $X(2) = 0.5$ .  $X(3)$  and  $x(0)$  respectively are

- (A)  $1-j, 1.875$       (B)  $1-j, 1.500$   
 (C)  $1+j, 1.875$       (D)  $0.1-j0.1, 1.500$
- Q.2** A discrete-time signal  $x[n]$  is obtained by sampling an analog signal at 10 kHz. The signal  $x[n]$  is filtered by a system with impulse response  

$$h[n] = 0.5 \{ \delta[n] + \delta[n-1] \}.$$
- The 3dB cut-off frequency of the filter is  
 (A) 1.25 kHz      (B) 2.50 kHz  
 (C) 4.00 kHz      (D) 5.00 kHz
- Q.3**  $x(n)$  is a real valued periodic sequence with a period  $N$ .  $x(n)$  and  $X(k)$  form  $N$ -point Discrete Fourier Transform (DFT) pairs. The DFT  $Y(k)$  of the sequence  $y(n) = \frac{1}{N} \sum_{r=0}^{N-1} x(r)x(n+r)$  is  
 (A)  $|X(k)|^2$       (B)  $\frac{1}{N} \sum_{r=0}^{N-1} X(r)X^*(k+r)$   
 (C)  $\frac{1}{N} \sum_{r=0}^{N-1} X(r)X(k+r)$       (D) 0
- Q.4** Three DFT coefficients, out of the DFT coefficients of a five-point real sequence are given as :  $X(0) = 4$ ,  $X(1) = 1-j1$  and  $X(3) = 2+j2$ . The zeroth value of the sequence  $x(n)$ ,  $x(0)$  is  
 (A) 1      (B) 2  
 (C) 3      (D) 4
- Q.5** For the sequence  $x[n] = \{1, -1, 1, -1\}$ , with  $n = 0, 1, 2, 3$  the DFT is computed as  $X(k) = \sum_{n=0}^3 x(n) e^{-j\frac{2\pi nk}{4}}$ , for  $k = 0, 1, 2, 3$ . The value of  $k$  for which  $X(k)$  is not zero is  
 (A) 0      (B) 1  
 (C) 2      (D) 3
- Q.6** Match List-I (Fourier series and Fourier transforms) with List-II (Their properties) and select the correct answer using the codes given below the lists :

**List-I** [ESE EC 2002]**(Fourier series and Fourier transforms)**

- A. Fourier series
- B. Fourier transform
- C. Discrete time fourier transform
- D. Discrete fourier transform

**List-II****(Their properties)**

1. Discrete, periodic
2. Continuous, periodic
3. Discrete, aperiodic
4. Continuous, aperiodic

**Codes :** **A** **B** **C** **D**

- (A) 3 4 2 1
- (B) 1 2 4 3
- (C) 3 2 4 1
- (D) 1 4 2 3

**Q.7** Which of the following is a non-linear transform ?

- (A) Fourier transform
- (B) Z-transform
- (C) DFT
- (D) None of these

**Q.8** The inverse DFT of  $x[n]$  can be expressed as :

- (A)  $x[n] = \frac{1}{N} \sum_{k=0}^N X[k] e^{-\frac{j2\pi kn}{N}}$
- (B)  $x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{\frac{j2\pi kn}{N}}$
- (C)  $x[n] = \frac{1}{N} \sum_{n=0}^{N-1} X[n] e^{-\frac{j2\pi kn}{N}}$
- (D)  $x[n] = N \sum_{n=0}^{N-1} X[k] e^{-\frac{j2\pi kn}{N}}$

**Q.9** The  $N$ -point DFT of a finite duration sequence can be obtained as

- (A)  $X(k) = X(z) \Big|_{z=e^{\frac{j2\pi k}{N}}}$
- (B)  $X(k) = X(z) \Big|_{z=e^{-\frac{j2\pi k}{N}}}$
- (C)  $X(k) = X(z) \Big|_{z=e^{-\frac{j2\pi k}{N}}}$
- (D)  $X(k) = X(z) \Big|_{z=e^{\frac{j2\pi k}{N}}}$

**Q.10** If DFT of  $x[n] = X[k]$ , then DFT of  $x(n+m)$  is

- (A)  $X(k) e^{\frac{-j2\pi km}{N}}$
- (B)  $X(k) e^{\frac{-j2\pi mk}{N}}$
- (C)  $X(k) e^{\frac{j2\pi km}{N}}$
- (D)  $X(k) e^{\frac{j2\pi mk}{N}}$

**Q.11** The DFT of product of two discrete time sequences  $x_1(n)$  and  $x_2(n)$  is equivalent to (where  $\otimes$  represents circular convolution)

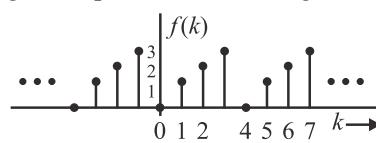
- (A)  $\frac{1}{N} [X_1(k) \otimes X_2(k)]$
- (B)  $\frac{1}{N} [X_1(k) X_2(k)]$
- (C)  $\frac{1}{N} [X_1(k) \otimes X_2^*(k)]$
- (D)  $X_1(k) \otimes X_2(k)$

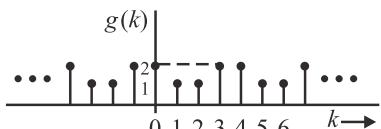
**Q.12** By correlation property, the DFT of circular correlation of two sequences  $x[n]$  and  $y[n]$  is :

- (A)  $X(k)Y^*(k)$       (B)  $X(k) \otimes Y(k)$   
 (C)  $X(k) \otimes Y^*(k)$       (D)  $X(k)Y(k)$
- Q.13** The complex valued phase factor/twiddle factor,  $W_N$  can be represented as  
 (A)  $e^{-j2\pi N}$       (B)  $e^{-j\frac{2\pi}{N}}$   
 (C)  $e^{-j2\pi}$       (D)  $e^{-j2\pi kN}$
- Q.14** The 4-point Discrete Fourier Transform (DFT) of a discrete time sequence  $x[n] = [1, 2, 3, 4]$  is  
 (A)  $[10, -2 + 2j, -2, -2 - 2j]$   
 (B)  $[10, -2 - 2j, -2, -2 + 2j]$   
 (C)  $[10, -2 + 2j, -2, 2 + 2j]$   
 (D) None of the above
- Q.15** The 4-point discrete time sequence of a Discrete Fourier Transform  $X[k] = [1, 2, 3, 4]$  is :  
 (A)  $[2.5, 0.5(1+j), -0.5, 0.5(1-j)]$   
 (B)  $[2.5, -0.5(1+j), -0.5, -0.5(-1+j)]$   
 (C)  $[2.5, -0.5(1+j), -0.5, -0.5(1-j)]$   
 (D) None of the above
- Q.16** Determine the IDFT of  $X[k]=[3, 2+j, 1, 2-j]$   
 (A)  $[1, 2, 3, 4]$       (B)  $[1, 1, 2, 2]$   
 (C)  $[2, 0, 0, 1]$       (D)  $[2, 2, 0, 0]$
- Common Data Questions 17 to 20**  
 $X[k]$  is the DFT of  $x[n]$ . Given  $x[n] = [1, 2, 3, 4]$
- Q.17**  $X[0]$  is  
 (A) 0      (B) 10  
 (C)  $16\pi$       (D)  $20\pi$
- Q.18**  $\sum_{k=0}^3 X[k]$  is  
 (A) 1      (B) 2  
 (C) 3      (D) 4
- Q.19**  $\sum_{k=0}^3 |X[k]|^2$  is  
 (A) 30      (B) 60  
 (C) 90      (D) 120
- Q.20**  $\sum_{k=0}^3 (-1)^k X[k]$  is  
 (A) 4      (B) 8  
 (C) 12      (D) 16

**Common Data Questions 21 & 22**

- $X[k]$  is the DFT of  $x[n]$ . Given  $x[n] = [4, 3, 2, 1]$
- Q.21**  $\sum_{k=0}^3 |X[k]|^2$  is  
 (A) 30      (B) 60  
 (C) 90      (D) 120
- Q.22**  $\sum_{k=0}^3 (-1)^k X[k]$  is  
 (A) 4      (B) 8  
 (C) 12      (D) 16
- Q.23** The first 5-point of an 8-point DFT are  $[2.5, -1+2j, 0, -2-2j, 0]$ . The other 3 samples are :  
 (A)  $[-2+2j, 0, -1-2j]$       (B)  $[2j, 2.5, -2j]$   
 (C)  $[-2-2j, 2.5, -1+2j]$       (D) None of these
- Q.24** For the given 4 point DFT, the value of  $W_N W_N^*$  is  
 (A) Identity matrix  
 (B) Unit matrix  
 (C) N times identity matrix  
 (D)  $N^2$
- Q.25** If  $W$  is the twiddle factor then value of  $W_8^{25} + W_8^{19}$  is  
 (A) 0      (B)  $\sqrt{2} e^{j\pi/2}$   
 (C)  $\sqrt{2} e^{j\pi}$       (D) None of these
- Q.26** If  $x_1[n] = x_2[n] = [1, 0, 1, 2]$ , the value of  $y[n] = x_1[n] \textcircled{\times} x_2[n]$  where  $\textcircled{\times}$  represents circular convolution is :  
 (A)  $[2, 4, 5, 4]$       (B)  $[2, 5, 4, 5]$   
 (C)  $[2, 4, 4, 5]$       (D) None of these
- Q.27** Compute the circular convolution of the two discrete-time sequence  $x_1[n] = [1, 2, 1, 2]$  and  $x_2[n] = [3, 2, 1, 4]$   
 (A)  $[16, 14, 16, 14]$       (B)  $[16, 14, 17, 14]$   
 (C)  $[16, 14, 14, 16]$       (D)  $[16, 17, 16, 14]$
- Q.28** Find the circular convolution of the sequence  $f(k)$  and  $g(k)$ , depicted in below figure.





- (A) [7, 9, 11, 9]      (B) [7, 11, 9, 11]  
 (C) [7, 9, 9, 11]      (D) [7, 11, 11, 9]

**Common Data Questions 29 & 30**

A sequence of length 4 is given by  
 $x[n] = [1 \quad 0 \quad -2 \quad 3]$   
 ↑

- Q.29** If  $y_1[n] = x[n]$  convolved with itself, the maximum value of  $y_1[n]$  is :  
 (A) 4      (B) 9  
 (C) 2      (D) 6

- Q.30** If  $g[n]$  is defined as a periodic sequence  $g[n] = \sum_{i=-\infty}^{\infty} x[n-4i]$ , the maximum positive value of the periodic convolution of  $g[n]$  with itself is  
 (A) 5      (B) 6  
 (C) 12      (D) 10

- Q.31** Determine circular convolution of the following two sequences :

$$x[n] = [1, 0.5] \text{ and } h[n] = [0.5, 1]$$

(A) [1, 1.25]      (B) [0.5, 1.25, 0.5]  
 (C) [1.25, 0.5, 0.5]      (D) [1.25, 1]


**Answer Keys**

Objective & Numerical Answer Type Questions									
1.	D	2.	3	3.	B	4.	A	5.	C
6.	11	7.	6	8.	A	9.	D	10.	4096
11.	C								
Practice (Objective & Numerical Answer) Questions									
1.	A	2.	B	3.	A	4.	B	5.	C
6.	A	7.	D	8.	B	9.	B	10.	C
11.	A	12.	A	13.	B	14.	A	15.	C
16.	C	17.	B	18.	D	19.	D	20.	C
21.	D	22.	B	23.	A	24.	C	25.	B
26.	D	27.	A	28.	A	29.	B	30.	B
31.	A								

# 9

# Digital Filters

## Objective & Numerical Ans Type Questions :

**Q.1** A system with transfer function  $H(z)$  has impulse response  $h[n]$  defined as  $h[2]=1$ ,  $h[3]=-1$  and  $h[k]=0$  otherwise. Consider the following statements.

**S1 :**  $H(z)$  is a low-pass filter.

**S2 :**  $H(z)$  is an FIR filter.

Which of the following is correct?

[GATE EC 2009, IIT Roorkee]

- (A) Only S2 is true.
- (B) Both S1 and S2 are false.
- (C) Both S1 and S2 are true and S2 is a reason for S1.
- (D) Both S1 and S2 are true but S2 is not a reason for S1.

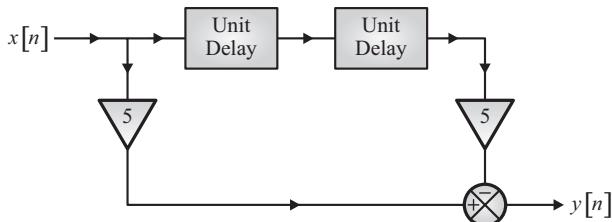
**Q.2** For an all-pass system  $H(z) = \frac{(z^{-1} - b)}{(1 - az^{-1})}$ , where  $|H(e^{-j\omega})| = 1$ , for all  $\omega$ . If  $\text{Re}(a) \neq 0$ ,  $\text{Im}(a) \neq 0$ , then  $b$  equals

[GATE EC 2014 Set – 03, IIT Kharagpur]

- (A)  $a$
- (B)  $a^*$
- (C)  $1/a^*$
- (D)  $1/a$

**Q.3** The direct form structure of an FIR (finite impulse response) filter is shown in the figure.

[GATE EC 2016 Set – 03, IISc Bangalore]



The filter can be used to approximate a

- (A) low-pass filter.
- (B) high-pass filter.
- (C) band-pass filter.
- (D) band-stop filter.

**Q.4** Consider a four point moving average filter defined by the equation  $y[n] = \sum_{i=0}^3 \alpha_i x[n-i]$ .

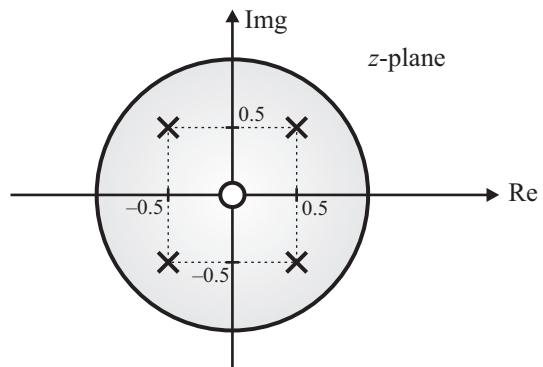
The condition on the filter coefficient that results in a null at zero frequency is

[GATE EC 2015 Set – 03, IIT Kanpur]

- (A)  $\alpha_1 = \alpha_2 = 0; \alpha_0 = -\alpha_3$
- (B)  $\alpha_1 = \alpha_2 = 1; \alpha_0 = -\alpha_3$
- (C)  $\alpha_0 = \alpha_3 = 0; \alpha_1 = \alpha_2$
- (D)  $\alpha_1 = \alpha_2 = 0; \alpha_0 = \alpha_3$

**Q.5** The pole-zero diagram of a causal and stable discrete-time system is shown in the figure. The zero at the origin has multiplicity 4. The impulse response of the system is  $h[n]$ . If  $h[0]=1$ , we can conclude

[GATE EC 2015 Set - 01, IIT Kanpur]



- (A)  $h[n]$  is real for all  $n$ .  
 (B)  $h[n]$  is purely imaginary for all  $n$ .  
 (C)  $h[n]$  is real for only even  $n$ .  
 (D)  $h[n]$  is purely imaginary for only odd  $n$ .

**Q.6** An LTI system with unit sample response

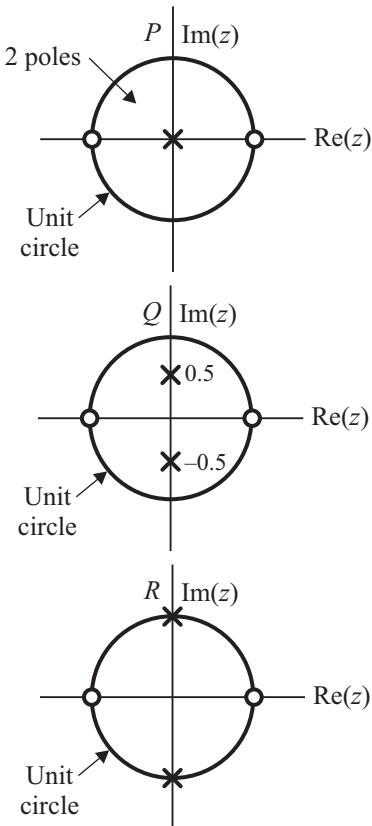
$$h[n] = 5\delta[n] - 7\delta[n-1] + 7\delta[n-3] - 5\delta[n-4]$$

is a

[GATE EC 2017 Set – 02, IIT Roorkee]

- (A) low-pass filter.  
 (B) high-pass filter.  
 (C) band-pass filter.  
 (D) band-stop filter.

**Q.7** The pole-zero plots of three discrete-time systems  $P$ ,  $Q$  and  $R$  on the  $z$ -plane are shown below.



Which one of the following is TRUE about the frequency selectivity of these systems?

[GATE EE 2006, IIT Kharagpur]

- (A) All three are high-pass filters  
 (B) All three are band-pass filters  
 (C) All three are low-pass filters  
 (D)  $P$  is a low-pass filter,  $Q$  is a band-pass filter and  $R$  is a high-pass filter

**Q.8** A discrete time all-pass system has two of its poles at  $2\angle30^\circ$  and  $0.25\angle0^\circ$ . Which one of the following statements about the system is TRUE?

[GATE EC 2018, IIT Guwahati]

- (A) It has two more poles at  $0.5\angle30^\circ$  and  $4\angle0^\circ$ .  
 (B) It is stable only when the impulse response is two sided.  
 (C) It has constant phase response over all frequencies.  
 (D) It has constant phase response over the entire  $z$ -plane.

**Q.9** If  $H_a(s) = \frac{1}{(s+1)(s+2)}$ , then the corresponding  $H(z)$  using impulse invariance method for sampling frequency of 5 samples/sec is

- (A)  $\frac{0.148z}{z^2 + 1.48z + 0.548}$     (B)  $\frac{0.148z}{z^2 - 1.48z + 0.548}$   
 (C)  $\frac{0.148z}{z^2 - 1.48z - 0.548}$     (D)  $\frac{0.148z}{z^2 + 1.48z - 0.548}$

**Q.10** The system function of the analog filter is given as

$$H_a(s) = \frac{s + 0.1}{(s + 0.1)^2 + 9}$$

The system function of IIR digital filter using impulse invariance method is

- (A)  $\frac{1 + (e^{-0.1T} \cos 3T)z^{-1}}{1 - (2e^{-0.1T} \cos 3T)z^{-1} + e^{-0.2T}z^{-2}}$   
 (B)  $\frac{1 - (e^{-0.1T} \cos 3T)z^{-1}}{1 + (2e^{-0.1T} \cos 3T)z^{-1} + e^{-0.2T}z^{-2}}$   
 (C)  $\frac{1 - (e^{-0.1T} \cos 3T)z^{-1}}{1 - (2e^{-0.1T} \cos 3T)z^{-1} + e^{-0.2T}z^{-2}}$   
 (D)  $\frac{1 + (e^{-0.1T} \cos 3T)z^{-1}}{1 + (2e^{-0.1T} \cos 3T)z^{-1} + e^{-0.2T}z^{-2}}$

**Q.11** The total number of delay elements required to realize a linear phase FIR filter with the impulse response

$$h(n) = \delta(n) + \frac{1}{2}\delta(n-1) - \frac{1}{4}\delta(n-2) + \delta(n-4) + \frac{1}{2}\delta(n-3)$$

is \_\_\_\_\_.

**Q.12** The total number of delay elements required to realize the cascade form structure of a linear phase FIR filter with the system function

$$H(z) = 1 + \frac{3}{4}z^{-1} + \frac{17}{8}z^{-2} + \frac{3}{4}z^{-3} + z^{-4}$$

is \_\_\_\_\_.

- Q.13** A linear phase FIR system with impulse response real has a zero at  $z = \frac{1}{2}e^{j\frac{\pi}{4}}$ . The largest of remaining zeros that can be obtained from the above information is

[GATE IN 2003, IIT Madras]

(A)  $\frac{1}{2}e^{j\frac{\pi}{2}}, \frac{1}{2}e^{j\frac{3\pi}{4}}, \frac{1}{2}e^{j\frac{\pi}{2}}$

(B)  $\frac{1}{2}e^{-j\frac{\pi}{2}}, \frac{1}{2}e^{-j\frac{3\pi}{4}}, \frac{1}{2}e^{-j\frac{\pi}{2}}$

(C)  $2e^{-j\frac{\pi}{4}}, 2e^{j\frac{\pi}{4}}, \frac{1}{2}e^{-j\frac{\pi}{4}}$

(D)  $2e^{j\frac{\pi}{4}}, \frac{1}{2}e^{-j\frac{\pi}{4}}$

 Scan for Video Solution



- Q.14** An analog signal is sampled at 9 kHz. The sequence so obtained is filtered by an FIR filter with transfer function  $H(z) = 1 - z^{-6}$ . One of the analog frequencies for which the magnitude response of the filter is zero is

[GATE IN 2009, IIT Roorkee]

- (A) 0.751 kHz      (B) 1 kHz  
 (C) 1.5 kHz      (D) 2 kHz

 Scan for Video Solution



### Practice (objective & Num Ans) Questions :

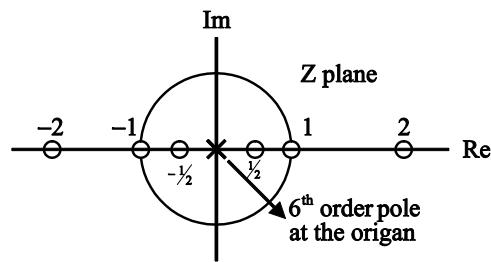
- Q.1** An FIR system is described by the system function

$$H(z) = 1 + \frac{7}{2}z^{-1} + \frac{3}{2}z^{-2}$$

The system is [GATE EC 2014 (Set-02)]

- (A) maximum phase      (B) minimum phase  
 (C) mixed phase      (D) zero phase

- Q.2** Shown below is the pole-zero plot of a digital filter



Which one of the following statement is TRUE?

- (A) This is a low pass filter [GATE IN 2011]  
 (B) This is a high pass filter  
 (C) This is an IIR filter  
 (D) This is an FIR filter

- Q.3** The  $z$ -transform of a signal  $x(n)$  is given by  $4z^{-3} + 3z^{-1} + 2 - 6z^2 + 2z^3$ . It is applied to a system, with a transfer function  $H(z) = 3z^{-1} - 2$ . Let the output be  $y(n)$ . Which of the following is true ?

- (A)  $y(n)$  is non causal with finite support  
 (B)  $y(n)$  is causal with infinite support  
 (C)  $y(n) = 0; |n| > 3$  [GATE EE 2009]  
 (D)  $\text{Re}[Y(z)]_{z=e^{j\theta}} = -\text{Re}[Y(z)]_{z=e^{-j\theta}}$   
 $\text{Im}[Y(z)]_{z=e^{j\theta}} = \text{Im}[Y(z)]_{z=e^{-j\theta}}; -\pi \leq \theta < \pi$

- Q.4** The transfer function  $H(z)$  of a fourth-order linear phase FIR system is given by

[GATE IN 2009]

$$H(z) = (1 + 2z^{-1} + 3z^{-2})G(z)$$

The  $G(z)$  is :

- (A)  $3 + 2z^{-1} + z^{-2}$       (B)  $1 + \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2}$   
 (C)  $\frac{1}{3} + 2z^{-1} + z^{-2}$       (D)  $1 + 2z + 3z^2$

### Statement for Linked Answer Questions 5 & 6

- Q.5** A signal is processed by a causal filter with transfer function  $G(s)$ . For a distortion free output signal waveform,  $G(s)$  must

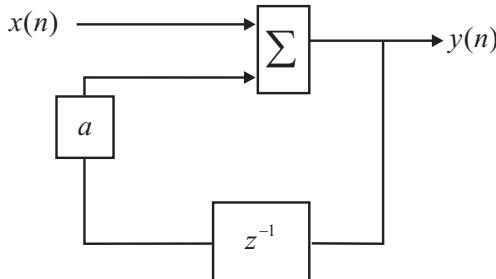
[GATE EE 2007]

- (A) provide zero phase shift for all frequency  
 (B) provide constant phase shift for all frequency  
 (C) provide linear phase shift that is proportional to frequency  
 (D) provide a phase shift that is inversely proportional to frequency

- Q.6**  $G(z) = \alpha z^{-1} + \beta z^{-3}$  is all pass digital filter with a phase characteristics same as that of the above question if [GATE EE 2007]  
 (A)  $\alpha = \beta$       (B)  $\alpha = -\beta$   
 (C)  $\alpha = \beta^{(1/3)}$       (D)  $\alpha = \beta^{-(1/3)}$

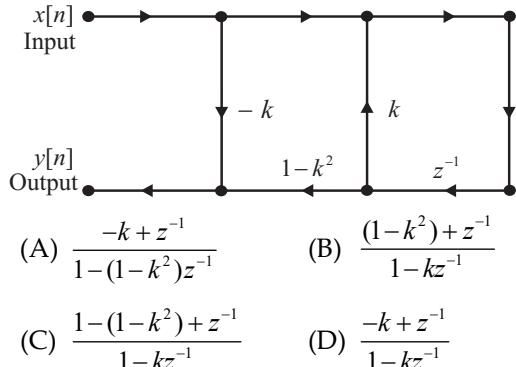
- Q.7** A discrete real all pass system has a pole at  $2\angle 30^\circ$ , it therefore [GATE EC 2006]  
 (A) also has a pole at  $0.5\angle 30^\circ$   
 (B) has a constant phase response over the z-plane,  $\arg |H(z)|$  is constant  
 (C) is stable only if it is anticausal  
 (D) has a constant phase response over the unit circle,  $\arg |H(e^{j\Omega})|$  is constant

- Q.8** In the IIR filter shown below,  $a$  is a variable gain. For which of the following cases, the system will transit from stable to unstable condition? [GATE IN 2006]



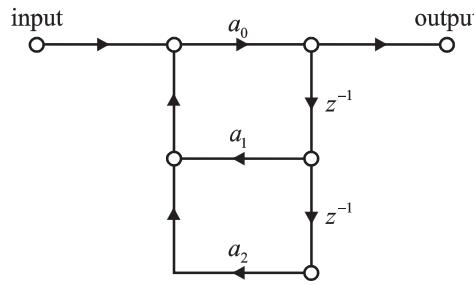
- (A)  $0.1 < a < 0.5$       (B)  $0.5 < a < 1.5$   
 (C)  $1.5 < a < 2.5$       (D)  $2 < a < \infty$

- Q.9** A discrete-time system is shown in the figure. The system function,  $H(z)$ , of the network is given by [GATE IN 2005]



- (A)  $\frac{-k + z^{-1}}{1 - (1 - k^2)z^{-1}}$       (B)  $\frac{(1 - k^2) + z^{-1}}{1 - kz^{-1}}$   
 (C)  $\frac{1 - (1 - k^2) + z^{-1}}{1 - kz^{-1}}$       (D)  $\frac{-k + z^{-1}}{1 - kz^{-1}}$

- Q.10** A direct form implementation of an LTI system with  $H(z) = \frac{1}{1 - 0.7z^{-1} + 0.13z^{-2}}$  is shown in figure. The value of  $a_0$ ,  $a_1$  and  $a_2$  are respectively [GATE IN 2004]

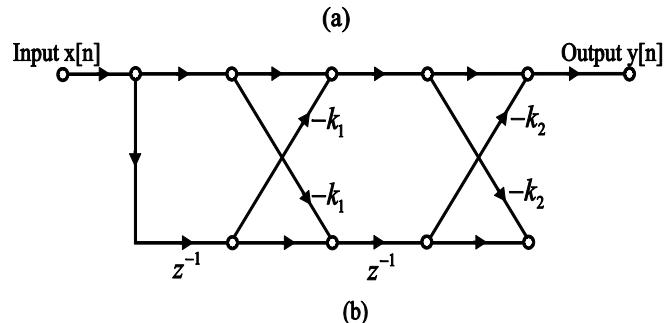
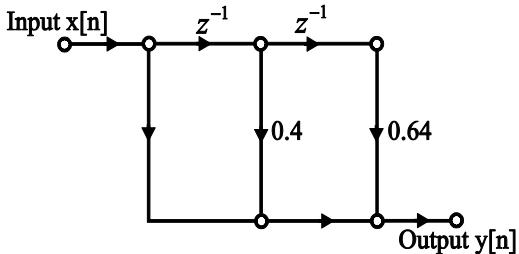


- (A) 1.0, 0.7 and -0.13      (B) -0.13, 0.7 and 1.0  
 (C) 1.0, -0.7 and 0.13      (D) 0.13, -0.7 and 1.0

- Q.11** A discrete-time system with input  $x[n]$  & output  $y[n]$  where  $y[n] = x[n] - 2x[n-1] + x[n-2]$  is a good approximation to a [GATE IN 2003]  
 (A) high-pass filter

- (B) band-stop filter blocking  $\frac{\pi}{8} \leq |\omega| \leq \frac{\pi}{4}$   
 (C) low-pass filter  
 (D) band-pass passing  $\frac{\pi}{8} \leq |\omega| < \frac{\pi}{4}$

- Q.12** Direct form implementation of the FIR structure shown in figure (a) is also implemented in figure (b) in lattice form. The coefficients  $k_1$  and  $k_2$  are respectively : [GATE IN 2003]



- (A) 0.640 and -0.244      (B) 0.640 and 0.244  
 (C) 0.244 and 0.640      (D) -0.244 and -0.640

- Q.13** A linear phase channel with phase delay  $T_p$  and group delay  $T_g$  must have [GATE EC 2002]  
 (A)  $T_p = T_g = \text{constant}$   
 (B)  $T_p \propto f$  and  $T_g \propto f$

- (C)  $T_p = \text{constant}$  and  $T_g \propto f$   
 (D)  $T_p \propto f$  and  $T_g = \text{constant}$  ( $f$  denotes frequency)

**Q.14** Which one of the following digital filters does have a linear phase response? [ESE EC 2003]

- (A)  $y(n) + y(n-1) = x(n) - x(n-1)$   
 (B)  $y(n) = \frac{1}{6}[3x(n) + 2x(n-1) + x(n-2)]$   
 (C)  $y(n) = \frac{1}{6}[x(n) + 2x(n-1) + 3x(n-2)]$   
 (D)  $y(n) = \frac{1}{4}[x(n) + 2x(n-1) + x(n-2)]$

**Q.15** The poles of a digital filter with linear phase response can lie [ESE EC 2001]

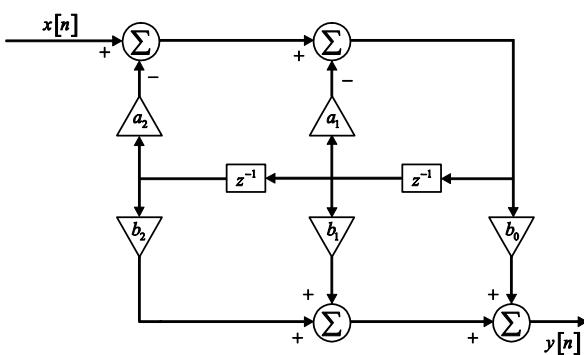
- (A) only at  $z = 0$   
 (B) only on the unit circle  
 (C) only inside the unit circle but not at  $z = 0$   
 (D) on the left side of Real( $z$ ) = 0 line

**Q.16** The minimum number of delay elements required in realizing a digital filter with the transfer function

$$H(z) = \frac{a + az^{-1} + bz^{-2}}{1 + cz^{-1} + dz^{-2} + ez^{-3}}$$

(A) 2                          (B) 3 [ESE EC 2001]  
 (C) 4                          (D) 5

**Q.17** Consider the system shown in the below figure.



Find the system function  $H(z)$ .

- (A)  $\frac{b_0 + b_1z^{-1} + b_2z^{-2}}{1 - a_1z^{-1} - a_2z^{-2}}$   
 (B)  $\frac{b_0 + b_1z^{-1} + b_2z^{-2}}{1 + a_1z^{-1} + a_2z^{-2}}$   
 (C)  $\frac{b_0 - b_1z^{-1} - b_2z^{-2}}{1 + a_1z^{-1} + a_2z^{-2}}$   
 (D)  $\frac{1 + b_1z^{-1} + b_2z^{-2}}{b_0 + a_1z^{-1} + a_2z^{-2}}$

**Q.18** A discrete-time signal  $x[n]$  is obtained by sampling an analog signal at 10 kHz. The signal  $x[n]$  is filtered by a system with impulse response  $h[n] = 0.5\{\delta[n] + \delta[n-1]\}$ . The 3dB cut-off frequency of the filter is [GATE IN 2014]

- (A) 1.25 kHz                          (B) 2.50 kHz  
 (C) 4.00 kHz                          (D) 5.00 kHz



**Answer Keys**

Objective & Numerical Answer Type Questions									
1.	A	2.	B	3.	C	4.	A	5.	A
6.	C	7.	B	8.	B	9.	B	10.	C
11.	4	12.	4	13.	C	14.	C		
Practice (Objective & Numerical Answer) Questions									
1.	C	2.	B	3.	A	4.	A	5.	C
6.	A	7.	C	8.	B	9.	D	10.	A
11.	A	12.	D	13.	A	14.	D	15.	A
16.	B	17.	A	18.	B				

# 10

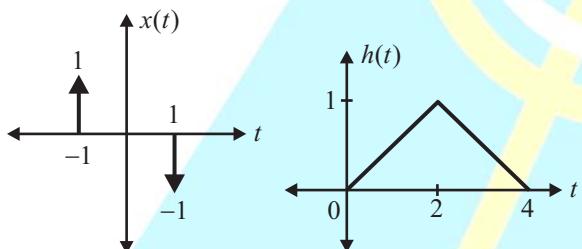
# Continuous & Discrete Time Convolution

TM

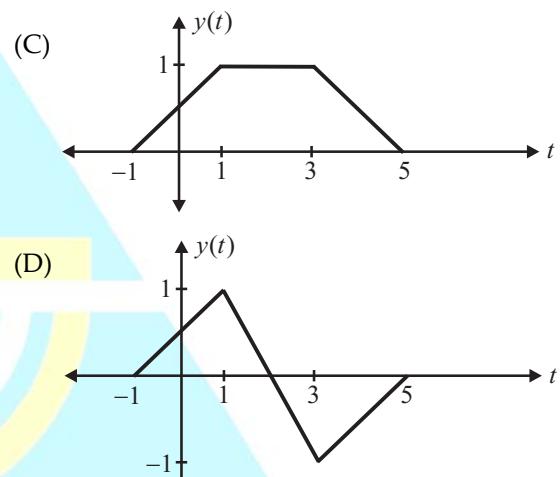
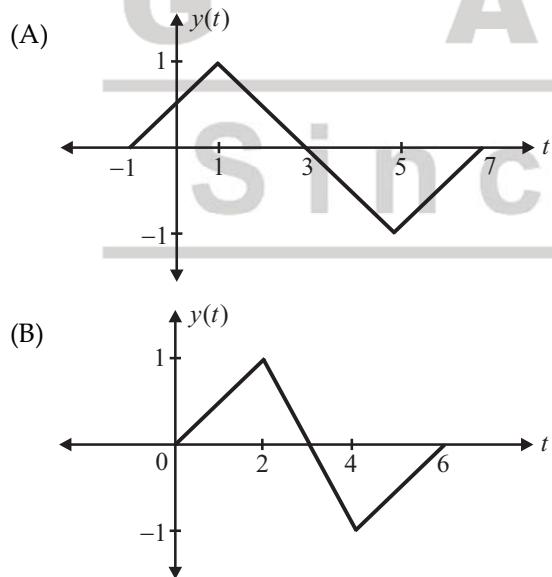
## Objective & Numerical Ans Type Questions :

- Q.1** Given two continuous time signals  $x(t) = e^{-t}$  and  $y(t) = e^{-2t}$  which exist for  $t > 0$ , the convolution  $z(t) = x(t) \otimes y(t)$  is [GATE EE 2011-Madras]
- (A)  $e^{-t} - e^{-2t}$       (B)  $e^{-3t}$   
 (C)  $e^{+t}$       (D)  $e^{-t} + e^{-2t}$

- Q.2** The signals  $x(t)$  and  $h(t)$  shown in the figures are convolved to yield  $y(t)$ .

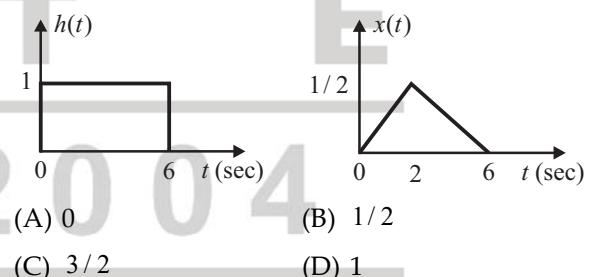


Which one of the figures represents the output  $y(t)$ ? [GATE IN 2007-Kanpur]



- Q.3** The impulse response and the excitation function of a linear time invariant causal system are shown in figures (a) and (b) respectively. The output of the system at  $t = 2$  sec is equal to

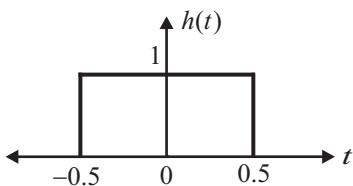
[GATE EC 1990-Bangalore]



- Q.4** If  $x(t) = \cos(t)u(t)$  and  $h(t) = \sin(t)u(t)$  then  $y(t) = x(t) \otimes h(t)$  will be

- (A)  $t \cdot \cos t \cdot u(t)$       (B)  $-\frac{t}{2} \cdot \cos t \cdot u(t)$   
 (C)  $\frac{t}{2} \cdot \sin t \cdot u(t)$       (D)  $t \cdot \sin t \cdot u(t)$

- Q.5** A signal  $x(t) = e^{j\omega_0 t}$  is convolved with another signal  $h(t)$  shown in figure.



Let  $y(t) = x(t) \otimes h(t)$ , then for what value of  $\omega_0$ ,  $y(0) = 0$ ?

- (A)  $\pi$     (B)  $\pi/2$   
 (C)  $\pi/4$     (D)  $2\pi$

Q.6 Let  $s(t)$  be the step response of a linear system with zero initial conditions then the response of this system to an input  $u(t)$  is

[GATE EE 2002-Bangalore, EE 1993-Bombay]

- (A)  $\int_0^t s(t-\tau)u(\tau)d\tau$   
 (B)  $\frac{d}{dt} \left[ \int_0^t s(t-\tau)u(\tau)d\tau \right]$   
 (C)  $\int_0^t s(t-\tau) \left[ \int_0^\tau u(\tau)d\tau \right] d\tau$   
 (D)  $\int_0^t s(t-\tau)^2 u(\tau)d\tau$

Q.7 The continuous time convolution integrals  $y(t) = \cos(\pi t)[u(t+1)-u(t-1)] \otimes u(t)$  is

- (A)  $\frac{\sin(\pi t)}{\pi}[u(t+1)-u(t-1)]$   
 (B)  $\frac{\sin(\pi t)}{\pi}u(t-1)$   
 (C)  $\frac{\sin(\pi t)}{\pi}u(t+1)$   
 (D)  $\frac{\sin(\pi t)}{\pi}u(t)$

Q.8 The discrete-time convolution sum of

$$y[n] = \beta^n u[n] \otimes u[n-3]; \quad |\beta| < 1 \text{ will be}$$

- (A)  $\frac{1-\beta^{n-2}}{1-\beta}$     (B)  $\frac{1-\beta^{n-2}}{\beta-1}$

- (C)  $\frac{1-\beta^{n-1}}{1-\beta}$     (D)  $\frac{1-\beta^{n-1}}{\beta-1}$

Q.9  $\cos\left(\frac{n\pi}{2}\right) \otimes \left(\frac{1}{2}\right)^n$  for  $n = 0, 1, 2, 3$  is

- (A)  $1, \frac{1}{2}, \frac{-3}{4}, \frac{-3}{8}, \frac{-1}{4}, \frac{-1}{8}, 0$   
 (B)  $1, \frac{1}{2}, \frac{-3}{4}, \frac{3}{8}, \frac{-1}{4}, \frac{-1}{8}, 0$   
 (C)  $1, \frac{1}{2}, \frac{3}{4}, \frac{-3}{8}, \frac{1}{4}, \frac{-1}{8}, 0$   
 (D)  $1, \frac{1}{2}, \frac{3}{4}, \frac{3}{8}, \frac{1}{4}, \frac{-1}{8}, 0$

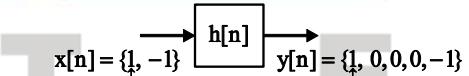
Q.10 A discrete time linear shift-invariant system has an impulse response  $h[n]$  with  $h[0]=1$ ,  $h[1]=-1$ ,  $h[2]=2$ , and zero otherwise. The system is given an input sequence  $x[n]$  with  $x[0]=x[2]=1$ , and zero otherwise. The number of nonzero samples in the output sequence  $y[n]$ , and the value of  $y[2]$  are, respectively

[GATE EC 2008-Bangalore]

- (A) 5, 2    (B) 6, 2  
 (C) 6, 1    (D) 5, 3

Q.11 Given the finite length input  $x[n]$  and the corresponding finite length output  $y[n]$  of an LTI system as shown below, the impulse response  $h[n]$  of the system is

[GATE EE 2010-Guwahati]



- (A)  $h[n] = \{1, 0, 0, 1\}$                                   (B)  $h[n] = \{1, 0, 1\}$   
 (C)  $h[n] = \{1, 1, 1, 1\}$     (D)  $h[n] = \{1, 1, 1\}$

### Answer Keys

#### Objective & Numerical Answer Type Questions

1.	A	2.	D	3.	B	4.	C	5.	D
6.	B	7.	A	8.	A	9.	A	10.	D
11.	C								