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NETWORK THEORY

Rohini

GATE SYLLABUS

Network graphs: matrices associated with graphs; incidence, fundamental cut set and fundamental circuit matrices. Solution methods: nodal and mesh analysis. Network theorems: superposition, Thevenin and Norton's maximum power transfer, Wye-Delta transformation. Steady state sinusoidal analysis using phasors. Linear constant coefficient differential equations; time domain analysis of simple RLC circuits, Solution of network equations using Laplace transform: frequency domain analysis of RLC circuits. 2-port network parameters: driving point and transfer functions. State equations for networks.

IES SYLLABUS

Network analysis techniques; Network theorems, transient response, steady state sinusoidal response; Network graphs and their applications in network analysis; Tellegen's theorem. Two port networks; Z, Y, h and transmission parameters. Combination of two ports, analysis of common two ports. Network functions : parts of network functions, obtaining a network function from a given part. Transmission criteria : delay and rise time, Elmore's and other definitions effect of cascading. Elements of network synthesis.

NETWORK THEORY CONTENTS

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For

GATE, IES & DRDO

Managing Director
Y.V. Gopala Krishna Murthy

Chapter 1 : Introduction

(Network Elements, $v - i$ characteristics, Energy and Power, Series, Parallel connections of elements, wye – delta transformation, KCL and Node equation, KVL and mesh equation, Linear and constant coefficient differential equation, Source transformation, Magnetic Coupling)

PASSIVE NETWORK ELEMENT :

A two terminal passive network element is shown in Fig. 1 with $v(t)$, across it, and current $i(t)$, through it. The passive element may be Resistance $R (\Omega)$, Inductance $L (H)$ or Capacitance $C (F)$. Basic relationships for these elements are linear as shown in Fig. 2 (a),

(b) and (c) with slopes : R , L and C

$$\begin{aligned} v(t) &= R i(t) && \text{for the Resistance} \\ \phi(t) &= L i(t) && \text{for the Inductance} \end{aligned}$$

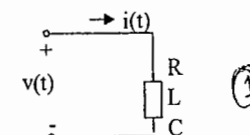


Fig 1.

where $\phi(t)$ is the instantaneous flux (Wb) associated with the inductor current, $i(t)$

$$q(t) = C v(t)$$

where $q(t)$ is the instantaneous charge (Coulomb) associated with the capacitor voltage, $v(t)$.

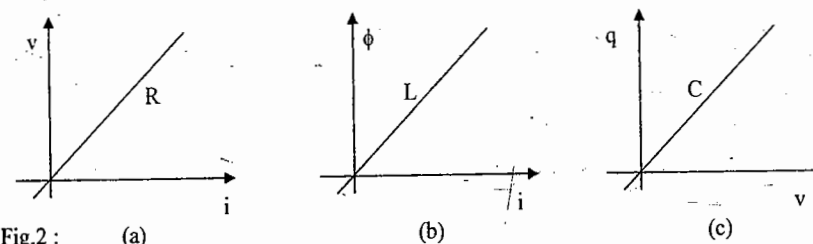


Fig.2 : (a)

(b)

(c)

It is also known that $v(t) = \frac{d\phi(t)}{dt}$ for the Inductor and hence $v(t) = L \frac{di(t)}{dt}$

$$i(t) = \frac{dq(t)}{dt} \text{ for the capacitor and hence } i(t) = C \frac{dv(t)}{dt}$$

The $v - i$ relations for the passive elements are summarized below :

$$v(t) = R i(t) \quad i(t) = G v(t) \text{ where } G = 1/R$$

$$v(t) = L \frac{di(t)}{dt} \quad i(t) = \frac{1}{L} \int_{-\infty}^t v(t) dt$$

$$v(t) = \frac{1}{C} \int_{-\infty}^t i(t) dt \quad i(t) = C \frac{dv(t)}{dt}$$

POWER AND ENERGY :

The instantaneous power $p(t)$ delivered to the passive element shown in Fig. 1 or to any network with $v(t)$ and $i(t)$ as shown in Fig.3 is given by

$$p(t) = v(t) i(t) \text{ Watts}$$

Energy (Joules) delivered up to time t is given by

$$E(t) = \int_{-\infty}^t p(t) dt = \int_{-\infty}^t v(t) i(t) dt$$

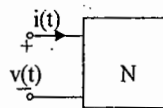


Fig.3

The energy delivered to element or the network from t_1 to t_2 is given by

$$E = \int_{t_1}^{t_2} v(t) i(t) dt$$

Therefore

$$E = R \int_{t_1}^{t_2} i^2(t) dt \quad \text{for resistance } R$$

$$E = \frac{1}{2} L [i^2(t_2) - i^2(t_1)] \quad \text{for Inductance } L$$

$$E = \frac{1}{2} C [v^2(t_2) - v^2(t_1)] \quad \text{for Capacitance } C$$

ACTIVE NETWORK ELEMENT:

The active network element may be an independent voltage source or current source and it may be an ideal, or non ideal source. Symbolic representations of sources and their terminal $v-i$ characteristics are shown in Fig.4 a, b, c and d.

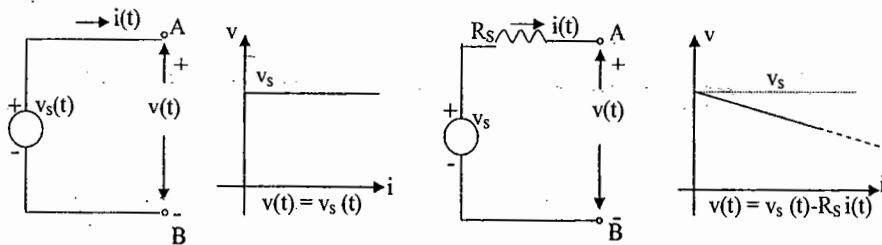
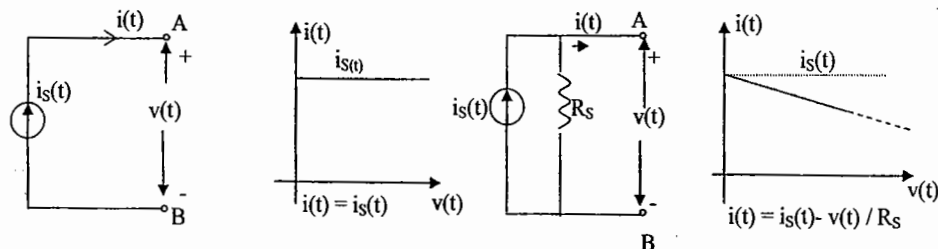


Fig. 4 (a) Ideal voltage source

(b) Non ideal voltage source



(c) Ideal current source

(d) Non ideal current source

The sources may also be dependent or controlled as

VCVS (Voltage controlled voltage source)

VCCS (Voltage controlled current source)

CCVS (Current controlled voltage source)

CCCS (Current controlled current source)

These sources can be identified in the equivalent circuits of BJT and FET as shown in Fig.5 a, b, and c

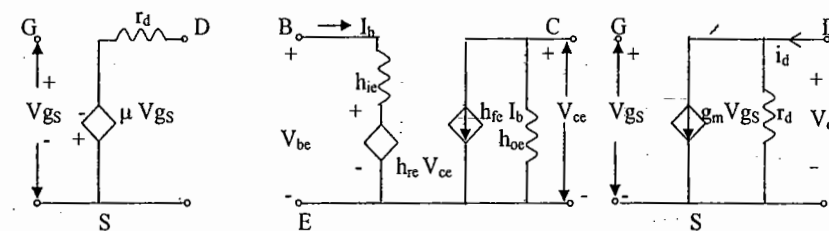


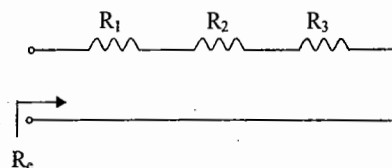
Fig. 5 (a) VCVS

(b) VCVS and CCCS

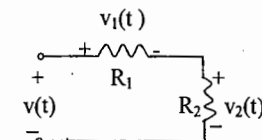
(c) VCCS

EQUIVALENT CIRCUITS:**Resistances in series:**

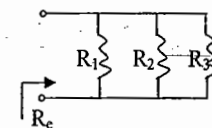
$$R_e = R_1 + R_2 + R_3$$

**Voltage Division:**

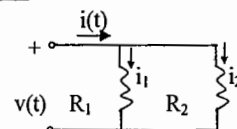
$$v_2(t) = \frac{R_2}{R_1 + R_2} v(t)$$

**Resistances in Parallel:**

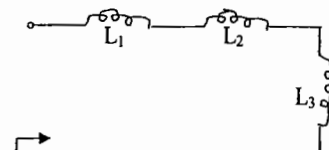
$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

**Current Division:**

$$i_1(t) = \frac{R_2}{R_1 + R_2} i(t) = \frac{G_1}{G_1 + G_2} i(t), \quad G_1 = 1/R_1, G_2 = 1/R_2$$

**Inductors in Series:**

$$L_{eq} = L_1 + L_2 + L_3$$

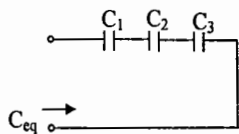
**Inductors in Parallel:**

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$



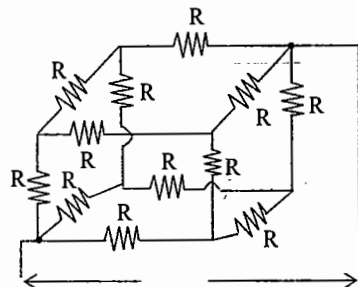
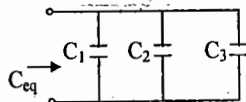
Capacitors in Series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



Capacitors in Parallel:

$$C_{eq} = C_1 + C_2 + C_3$$



i) Equivalent resistance between two opposite diagonal corners of cube shown in figure is given by $R_{eq} = (5R) / 6$

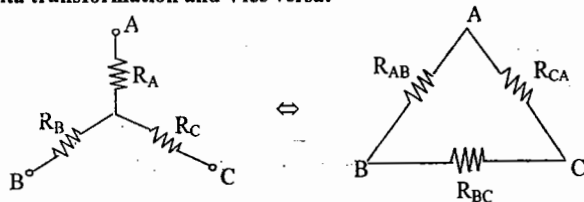
ii) If resistances are replaced with inductors of 'L' Henry, then

$$L_{eq} = (5L) / 6$$

iii) If resistances are replaced with capacitors of 'C' Farad, then

$$C_{eq} = (6C) / 5$$

Star - Delta transformation and Vice versa:



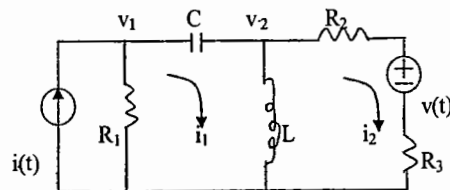
i) Delta to star transformation:

$$R_A = \frac{R_{AB} \times R_{CA}}{D}, \quad R_B = \frac{R_{BC} \times R_{AB}}{D}, \quad R_C = \frac{R_{CA} \times R_{BC}}{D}, \quad D = R_{AB} + R_{BC} + R_{CA}$$

ii) Star to Delta transformation:

$$R_{AB} = N / R_C, \quad R_{BC} = N / R_A, \quad R_{CA} = N / R_B, \quad N = R_A R_B + R_B R_C + R_C R_A$$

KCL AND KVL:



For the circuit shown in fig.

KCL applied at node 2 with voltage v_2 :

$$C \frac{d}{dt} (v_2 - v_1) + \frac{1}{L} \int_{-\infty}^t v_2 dt + v_2 - v(t) = 0$$

Linear and constant coefficient differential equation can be obtained for the variable v_1 (or v_2 or i_1 or i_2) alone as R, L, C elements satisfy Linearity and time invariance properties

KVL applied to mesh 2 with current i_2 :

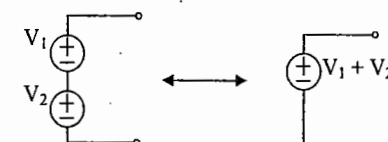
$$L \frac{d}{dt} (i_2 - i_1) + (R_2 + R_3) i_2 + v(t) = 0$$

Features:

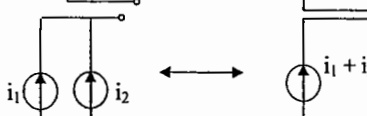
1. Kirchoff's laws are independent of the nature of the circuit elements.
2. KCL expresses the conservation of charge at every node
3. KVL expresses the conservation of energy in every loop.

SOURCE TRANSFORMATION:

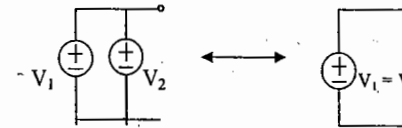
Two voltage sources connected in series:



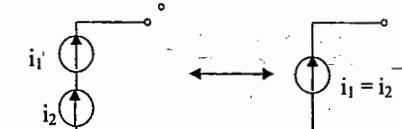
Two current sources in parallel:



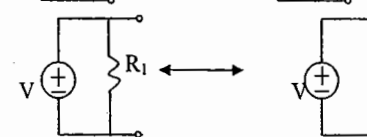
Two voltage sources are in parallel:



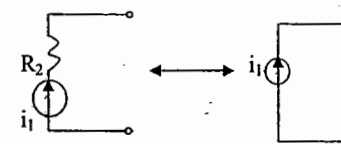
Two current sources in series:



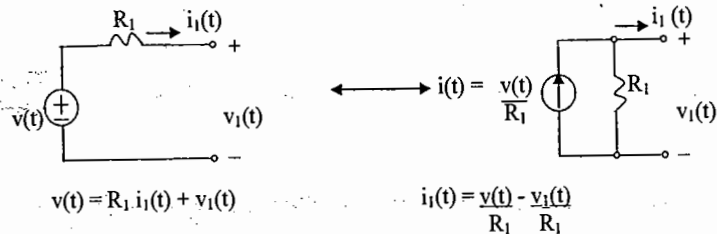
Resistor in parallel with a voltage source:



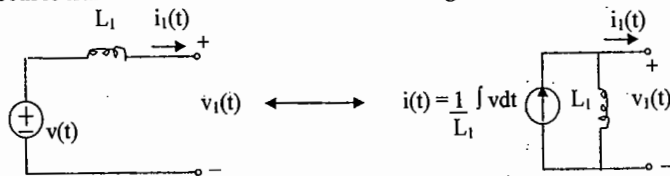
Resistor in series with a current source:



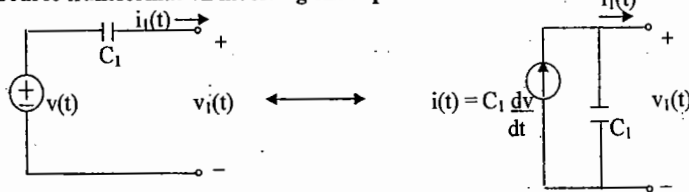
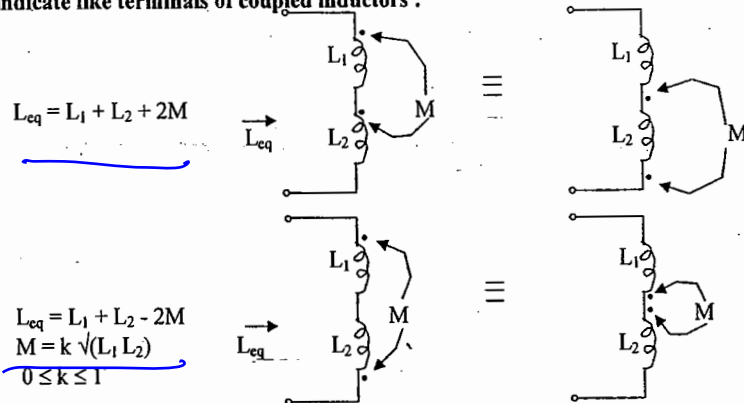
Source transformation involving one resistor:



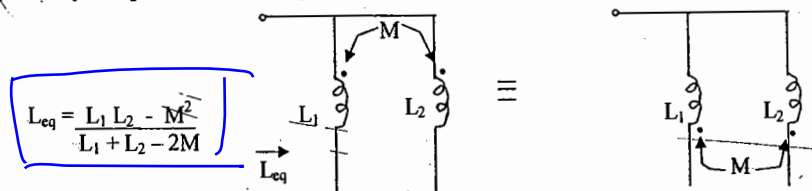
Source transformation for a network with a single inductor:



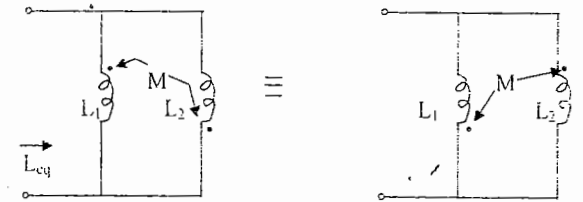
Source transformation involving one capacitor

Magnetically Coupled inductors in series :
Dots indicate like terminals of coupled inductors :

Magnetically Coupled inductors in parallel:

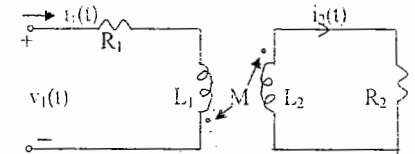


$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$



Transformer Coupling:

Two inductors with self inductances L_1 and L_2 , mutual inductance M , and coefficient of coupling k , are shown in fig. with dot convention.

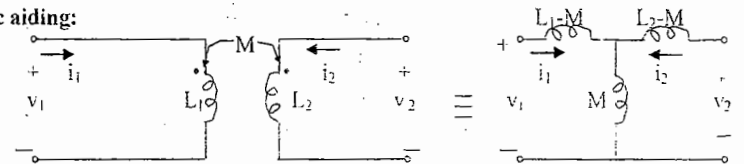


$$\text{KVL: } v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt} + R_1 i_1(t)$$

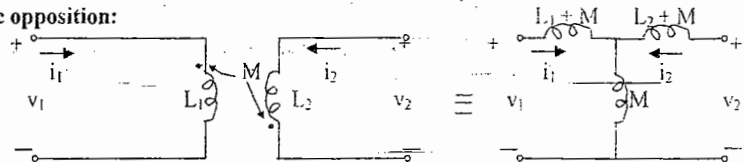
$$0 = \frac{L_2}{dt} \frac{di_2(t)}{dt} + \frac{M}{dt} \frac{di_1(t)}{dt} + R_2 i_2(t)$$

A transformer replaced by its \bar{T} -equivalent:

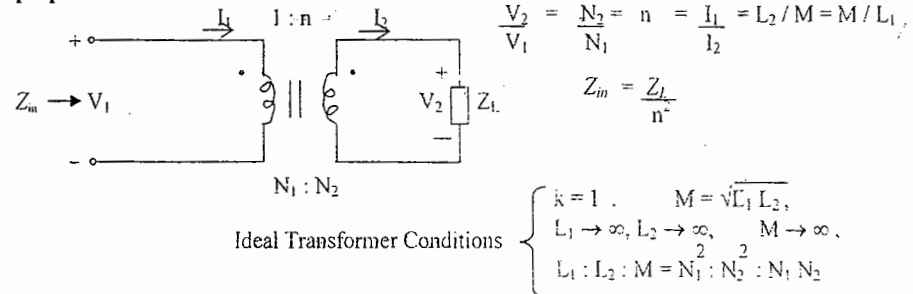
Magnetic aiding:



Magnetic opposition:



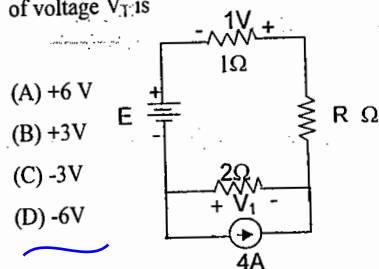
Transformer (Ideal) with voltage and current Transformation and impedance matching properties



SET - A

OBJECTIVE QUESTIONS:

01. For the circuit of Figure., the value of voltage V_1 is



- (A) +6 V
(B) +3V
(C) -3V
(D) -6V

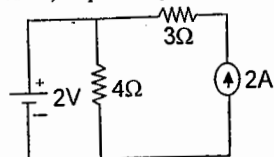
02. A magnetic flux of 400 micro-weber passing through a coil of 1200 turns is reversed in 0.1 sec. The value of emf induced in the coil is

- (A) 9.6 V (B) 4.8 V
(C) 7.2 V (D) None of these

03. Five identical capacitors each of value 2 farads are connected between the pair of nodes (a,c), (c,d), (a,d), (b,d) and (c,b). If all the capacitors are initially uncharged, the equivalent capacitance between the nodes a and b will be

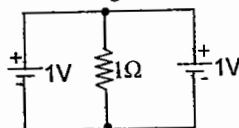
- (A) 3F (B) 5F
(C) 1F (D) 2F

04. The power supplied to the circuit in Figure., by the voltage and current sources, respectively, are



- (A) -3 W, 16 W (B) 3 W, 16 W
(C) 1 W, 28 W (D) 16 W, -3 W

05. The current in the 1 Ohm resistor in the circuit shown in the Fig., is

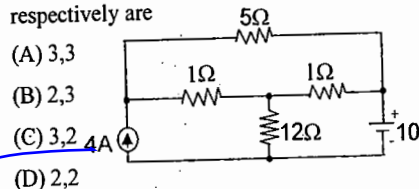


- (A) 0A
(B) 0.5A
(C) 1A
(D) 2A

06. A coil of 1000 turns produces a flux of 2 mWb while carrying a current of 1 A. The energy stored in its magnetic field is

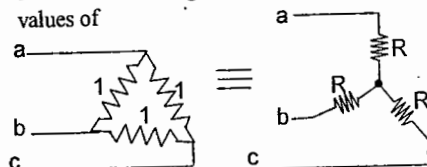
- (A) 0.25J (B) 0.50J
(C) 1J (D) 2J

07. For the network shown in Fig., the minimum number of independent equations to be solved simultaneously for mesh analysis and nodal analysis respectively are



- (A) 3,3
(B) 2,3
(C) 3,2
(D) 2,2

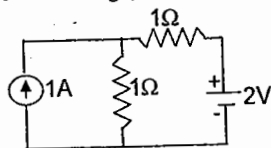
08. The Star equivalent of the Delta circuit shown in Fig., should have R values of



- (A) 2 (B) 2/3
(C) 3/2 (D) 1/3

09. The power expended by the dc current source shown in Fig., is

- (A) 1.5 W
(B) 1.0 W
(C) 0.5 W
(D) 0 W



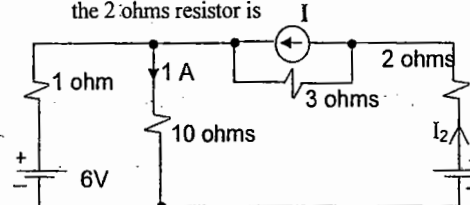
10. The energy stored in a capacitor charged to 10 volts is 10^{-3} Joules. The capacitor value is

- (A) 2 mF (B) 1 mF
(C) 200 μF (D) 100 μF

11. A transformer with a turns ratio of 1:10 is terminated by a 100 ohms resistance on the secondary side. The impedance seen on the primary side will be

- (A) 10 K ohms (B) 100 ohms
(C) 10 ohms (D) 1 ohm

12. In the circuit shown in Fig., the current through the 10 ohm resistor is 1 Ampere. The current I_2 through the 2 ohms resistor is



- (A) 1 A (B) 2A (C) 5A (D) -3A

13. Two identical coils when connected in series present a net inductance of 44 mH. When connections of one of the coils is reversed, the net inductance is 20 mH. The coefficient of coupling (ratio of mutual inductance to the self inductance) is

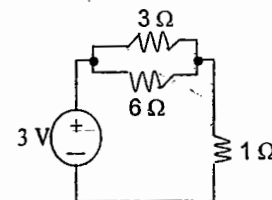
- (A) $\frac{3}{8}$ (B) $\frac{1}{8}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

14. Two coils having self inductances of 4 mH and 9 mH are placed and connected in such a manner to offer minimum possible inductance. The value of the minimum inductance is

- (A) 1mH (B) 7mH
(C) 13 mH (D) 19mH

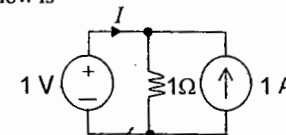
15. The power supplied by the dc voltage source in the circuit shown below is

- (A) 0 W
(B) 1.0 W
(C) 2.5 W
(D) 3.0 W

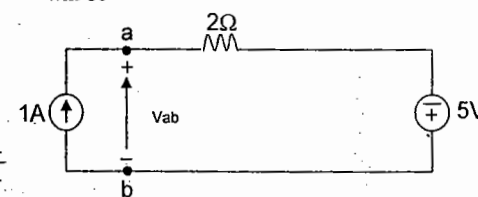


16. The current I supplied by the dc voltage source in the circuit shown below is

- (A) 0 A
(B) 0.5 A
(C) 1 A
(D) 2 A

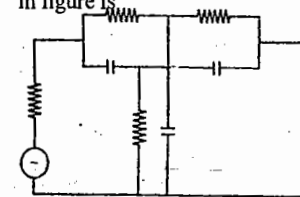


17. Assuming ideal elements in the circuit shown below, the voltage v_{ab} will be



- (A) -3V (B) 0 V
(C) 3 V (D) 5 V

18. The minimum number of equations required to analyze the circuit shown in figure is



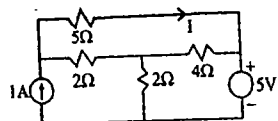
- (A) 5 (B) 4
(C) 6 (D) 7

Key for Set - A:

(Chapter - 1, Network Theory)

01.D	02.A	03.D	04.A	05.C
06.C	07.C	08.D	09.A	10.C
11.D	12.C	13.A	14.A	15.D
16.A	17.A	18.B		

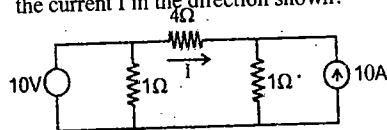
20. Consider the following circuit



What is the value of current I in the 5Ω resistor in the above circuit?

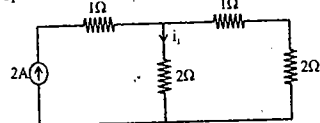
- (A) 0 A (B) 2 A
(C) 3 A (D) 4 A

21. In the network shown below, what is the current I in the direction shown?



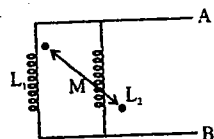
- (A) 0 (B) $1/3$ A
(C) $5/6$ A (D) 4 A

22. i_1 in circuit is



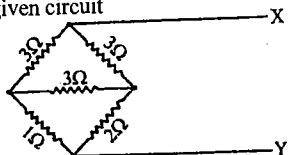
- (A) $4/5$ A (B) $6/5$ A
(C) $2/5$ A (D) $7/5$ A

23. What is equivalent inductance between A & B, where, $L_1 = 2H$, $L_2 = 3H$ and $M = 1$.



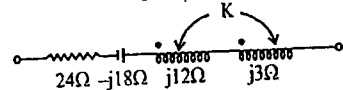
- (A) $5/7$ (B) $5/3$
(C) $-5/7$ (D) zero

24. Equivalent resistance between X and Y in given circuit



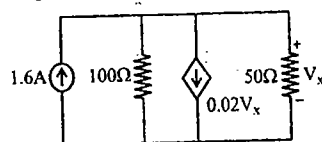
- (A) 9Ω (B) 3Ω
(C) $11/5\Omega$ (D) $5/11\Omega$

25. In series circuit for series resonance, the value of coupling coefficient is



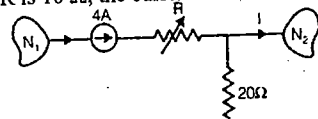
- (A) 1.0 (B) 0.5
(C) 0.25 (D) zero

26. In given network find V_x ?



- (A) 32 V (B) -32 V
(C) 12 V (D) -12 V

27. In the circuit shown in the figure, for $R = 20\Omega$ the current ' I ' is 2 A. When R is 10Ω , the current ' I ' would be



- (A) 1 A (B) 2 A
(C) 2.5 A (D) 3 A

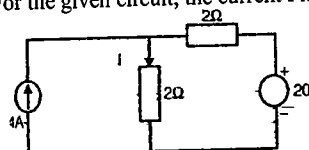
28. Two coils have self-inductances of $0.09H$ and $0.01H$ and a mutual inductance of $0.015H$. the coefficient of coupling between the coils is

- (A) 0.06 (B) 0.5
(C) 1.0 (D) 0.05

29. A network has 10 nodes and 17 branches. The number of different node pair voltages would be

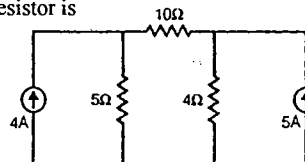
- (A) 7 (B) 9
(C) 10 (D) 45

30. For the given circuit, the current I is



- (A) 2 A (B) 5 A
(C) 7 A (D) 9 A

31. In the circuit shown in the given figure, power dissipated in the 5Ω resistor is



- (A) zero (B) 80 W
(C) 125 W (D) 405 W

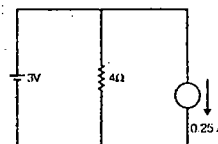
32. Consider the following:

- Energy storage capability of basic passive elements is due to the fact that
1. resistance dissipates energy
2. capacitance stores energy
3. inductance dissipates energy

Which of the above is/are correct?

- (A) 1, 2 and 3 (B) 1 and 3
(C) 3 alone (D) 1 and 2

33. The current flowing through the voltage source in the above circuit



- (A) 1.0 A (B) 0.75 A
(C) 0.5 A (D) 0.25 A

34. The number of edges in a complete graph of n vertices is

- (A) $n(n-1)$ (B) $\frac{n(n-1)}{2}$
(C) n (D) $n-1$

35. Consider the following properties of a particular network theorem:

1. The theorem is not concerned with type of elements.
2. The theorem is only based on the two Kirchhoff's laws.
3. The reference directions of the branch voltages and currents are arbitrary except that they have to satisfy Kirchhoff's laws.

Which of the following theorems has the above characteristics?

- (A) Thevenin's theorem
(B) Norton's theorem
(C) Tellegen's theorem
(D) Superposition theorem

36. A network has 4 nodes and 3 independent loops. What is the number of branches in the network?

- (A) 5 (B) 6
(C) 7 (D) 8

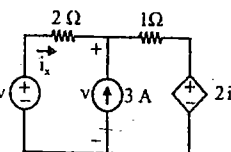
37. In a network containing active components, output voltage

- (A) will always be greater than input voltage
(B) will always be equal to the input voltage
(C) can be less than or greater than input voltage only
(D) will be less than, equal to or greater than input voltage

38. In the circuit

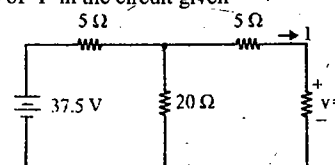
The value of i_x is

- (A) 2 A
(B) -0.6 A
(C) 2.6 A
(D) 1.4 A



39. The value of ' I ' in the circuit given below is

- (A) -5 A
(B) +5 A
(C) -2 A
(D) +2 A

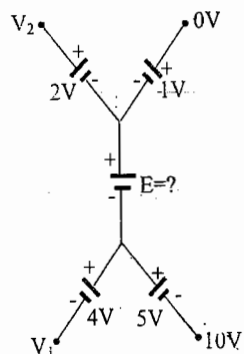


Key for Set - B:

- | | | | | |
|------|------|------|------|------|
| 1.A | 2.C | 3.C | 4.C | 5.B |
| 6.D | 7.C | 8.C | 9.A | 10.C |
| 11.B | 12.D | 13.A | 14.D | 15.A |
| 16.C | 17.C | 18.A | 19.B | 20.A |
| 21.A | 22.B | 23.A | 24.C | 25.C |
| 26.A | 27.B | 28.B | 29.D | 30.C |
| 31.B | 32.D | 33.A | 34.B | 35.C |
| 36.B | 37.C | 38.D | 39.D | |

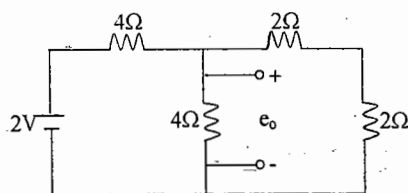
SET - C

01. In the circuit of the figure, the value of the voltage source E is **GATE-2000**



- (a) -16 V (b) 4 V
(c) -6 V (d) 16 V

02. The voltage e_0 in the figure is **GATE - 2001**

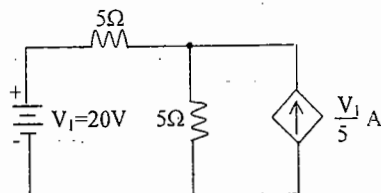


- (a) 2 V (b) 4/3 V
(c) 4 V (d) 8 V

03. If each branch of Delta circuit has impedance $\sqrt{3} Z$, then each branch of the equivalent Wye circuit has impedance. **GATE - 2001**

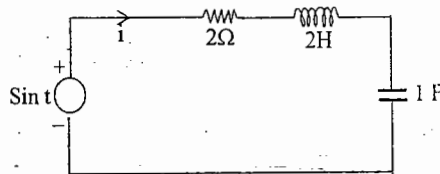
- (a) $\frac{Z}{\sqrt{3}}$ (b) 3 Z
(c) $3\sqrt{3} Z$ (d) $\frac{Z}{3}$

04. The dependent current source shown in the figure **GATE - 2002**



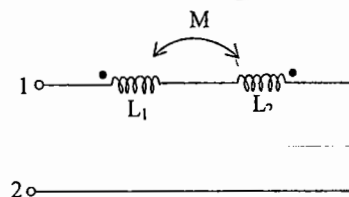
- (a) delivers 80 W (b) absorbs 80 W
(c) delivers 40 W (d) absorbs 40 W

05. The differential equation for the current $i(t)$ in the circuit of the figure is **GATE - 2003**



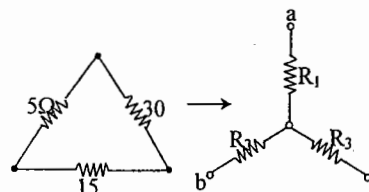
- (a) $2 \frac{d^2 i}{dt^2} + 2 \frac{di}{dt} + i(t) = \sin t$
(b) $2 \frac{d^2 i}{dt^2} + 2 \frac{di}{dt} + 2i(t) = \cos t$
(c) $2 \frac{d^2 i}{dt^2} + 2 \frac{di}{dt} + i(t) = \cos t$
(d) $2 \frac{d^2 i}{dt^2} + 2 \frac{di}{dt} + 2i(t) = \sin t$

06. The equivalent inductance measured between the terminals 1 and 2 for the circuit shown in the figure is **GATE - 2004**



- (a) $L_1 + L_2 + M$ (b) $L_1 + L_2 - M$
(c) $L_1 + L_2 + 2M$ (d) $L_1 + L_2 - 2M$

07. A delta-connected network with its Wye-equivalent is shown in the figure. The resistances R_1 , R_2 and R_3 (in ohms) are respectively **GATE - 1999**

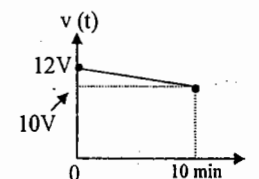


- (a) 1.5, 3 and 9 (b) 3, 9 and 1.5
(c) 9, 3 and 1.5 (d) 3, 1.5 and 9

08. Twelve 1Ω resistances are used as edges to form a cube. The resistance between two diagonally opposite corners of the cube is **GATE - 2003**

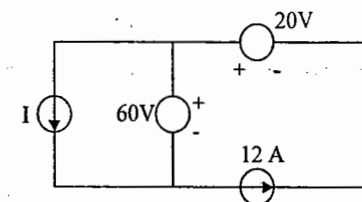
- (a) $5/6 \Omega$ (b) 1Ω
(c) $6/5 \Omega$ (d) $3/2 \Omega$

09. A fully charged mobile phone with a 12 V battery is good for a 10 minute talk time. Assume that, during the talk-time, the battery delivers a constant current of 2A and its voltage drops linearly from 12V to 10V as shown in the figure. How much energy does the battery deliver during this talk time? **GATE - 2009**



- (a) 220 J (b) 12 kJ
(c) 13.2 kJ (d) 14.4 kJ

10. In the interconnection of ideal sources shown in the figure, It is known that the 60 V source is absorbing power. **GATE - 2009**



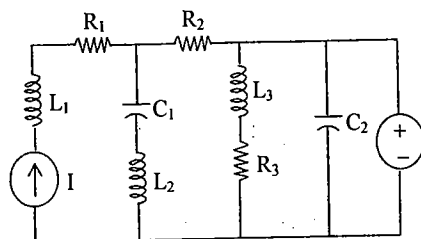
Which of the following can be the value of the current source I ?

- (a) 10 A (b) 13 A
(c) 15 A (d) 18 A

KEY:

01. a 02. c 03. a 04. a 05. c
06. d 07. d 08. a 09. 10.

11. In the circuit shown in the figure, the current source $I = 1$ A, the voltage source $V = 5$ V, $R_1 = R_2 = R_3 = 1\Omega$, $L_1 = L_2 = L_3 = 1$ H, $C_1 = C_2 = 1$ F. The currents (in A) through R_3 and the voltage source V respectively will be

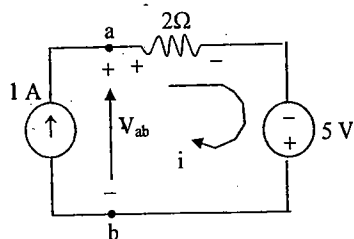


- (a) 1, 4 (b) 5, 1
(c) 5, 2 (d) 5, 4

12. A 3 V dc supply with an internal resistance of 2Ω supplies a passive non-linear resistance characterized by the relation $V_{NL} = I_{NL}^2$. The power dissipated in the non-linear resistance is

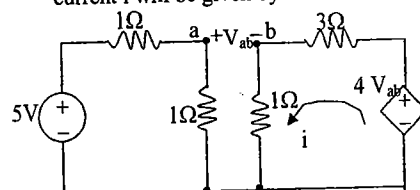
- (a) 1.0 W (b) 1.5 W
(c) 2.5 W (d) 3.0 W

13. Assuming ideal elements in the circuit shown below, the voltage V_{ab} will be



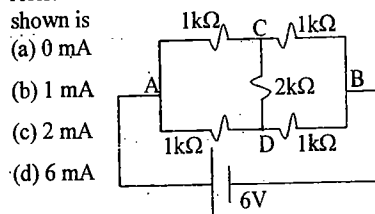
- (a) -3 V (b) 0 V
(c) 3 V (d) 5 V

14. In the circuit shown, the value of the current i will be given by



- (a) 0.31 A (b) 1.25 A
(c) 1.75 A (d) 2.5 A

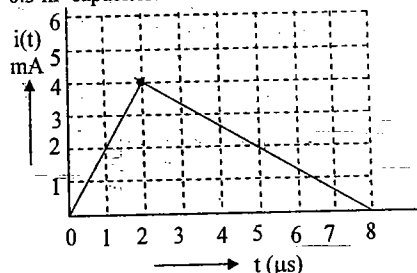
15. The current through the $2k\Omega$ resistance in the circuit shown is



- (a) 0 mA
(b) 1 mA
(c) 2 mA
(d) 6 mA

Statement for Linked Answer Questions 16 & 17.

The current $i(t)$ sketched in the figure flows through an initially uncharged 0.3 nF capacitor.



15. The charge stored in the capacitor at $t = 5\mu s$, will be

- (a) 8 nC (b) 10 nC
(c) 13 nC (d) 16 nC

16. The capacitor charged upto $5\mu s$, as per the current profile given in the figure, is connected across an inductor of 0.6 mH. Then the value of voltage across the capacitor after $1\mu s$ will approximately be

- (a) 18.8 V (b) 23.5 V
(c) -23.5 V (d) -30.6 V

Chapter 2 : Network Graphs

(MATRICES ASSOCIATED WITH GRAPHS; INCIDENCE, FUNDAMENTAL CUTSET AND FUNDAMENTAL CIRCUIT MATRICES)

The application of KCL and KVL (the basic laws of Network theory) to a network can be generalized and useful conclusions can be drawn by introducing the concepts of Graph of a Network, Tree and Cotree of a graph, fundamental Cut sets or f-Cut sets and fundamental loops or f-loops (also known as f-circuits or f-tie sets). As KCL and KVL do not depend upon the nature of the elements (the specific $v-i$ relation of the element), the Graph of a given Network is described to show only the interconnection of the various elements with the junctions.

Given a network 'N', the graph 'G' of that is obtained by simply showing each element (R, L, C, coupled inductor, transformer, independent source or dependent source) by a line segment (known as branch or edge in the terminology of Graph theory) and each junction of the elements is shown as thick dot (known as vertex or node in the terminology of Graph theory). A network and its Graph are said to be oriented or directed if reference directions of voltage and current are shown. A network and its oriented Graph are shown in fig. 1(a) and 1(b). The arrows on the Graph indicate the associated reference directions of voltage and current, which are defined for branch k , shown in fig. 2(a) and 2(b).

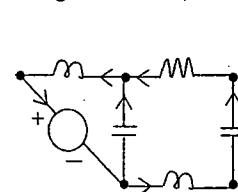


Fig. 1(a)

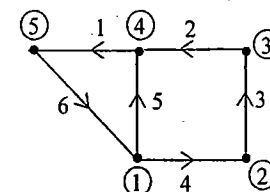


Fig. 1(b)

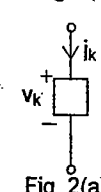


Fig. 2(a)

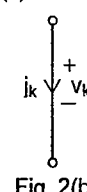


Fig. 2(b)

The meeting of the branches at the nodes can be analytically described in matrix form by node-to-branch incidence matrix A_a of dimension $n_t \times b$, where n_t = total number of nodes and b = total number of branches. For the oriented Graph shown in fig. 1(b), A_a is given by

$$A_a = \begin{matrix} & \text{branches} \\ & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} n \\ o \\ d \\ e \\ s \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 & +1 & +1 & -1 \\ 2 & 0 & 0 & +1 & -1 & 0 & 0 \\ 3 & 0 & +1 & -1 & 0 & 0 & 0 \\ 4 & +1 & -1 & 0 & 0 & -1 & 0 \\ 5 & -1 & 0 & 0 & 0 & 0 & +1 \end{bmatrix} \end{matrix} \longrightarrow (1)$$

5 X 6

Note that each column contains a single +1 and a single -1, with all other elements equal to 0. (convention is branch leaving a node: +1 and entering: -1, no incidence: 0).

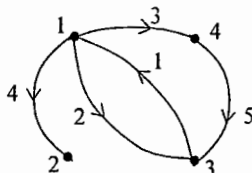
The number of branches incident at a node indicates the degree of that node. For the given example degree of node 1 is 3.

If one of the rows of A_a corresponding to some node is deleted, the resulting $n \times b$ matrix, where $n = n_t - 1$ is called as the reduced incidence matrix, A. Given A , A_a can be obtained by the property of A_a (+1 and -1 in any column). KCL applied at each node of the graph except at the deleted node (known as datum node or reference node) gives n linearly independent node equations:

$$A \vec{j} = 0 \longrightarrow (2)$$

Exa:

Given $A_a = \begin{pmatrix} -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix} \rightarrow (3)$

Fig. (3). Graph corresponding to A_a

Tellegen's theorem applies to any lumped network. Given its oriented graph, if we assign to its branches arbitrary branch voltages v_1, v_2, \dots, v_b subject only to the constraints imposed by KVL, and arbitrary branch currents j_1, j_2, \dots, j_b subject only to the constraints imposed by KCL, then

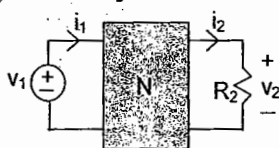
$$\sum_{k=1}^b V_k J_k = 0 \rightarrow (4)$$

Tellegen's theorem guarantees that

$$\sum_{k=1}^b V_k J_k = \sum_{k=1}^b \hat{V}_k \hat{J}_k = 0 \quad \sum_{k=1}^b \hat{V}_k J_k = \sum_{k=1}^b V_k \hat{J}_k = 0 \rightarrow (5)$$

where v_k, j_k and \hat{v}_k, \hat{j}_k correspond to two arbitrary lumped networks with the only constraint that they have the same graph.

Exa: The network, N shown is made of $(n-2)$ LTI resistors. For the measurements shown, determine v_2 .



With $R_2 = 1 \Omega$

$v_1 = 4$ volts

$i_1 = 1$ amp

$v_2 = 1$ volt

With $R_2 = 2 \Omega$

$\hat{v}_1 = 6$ volts

$\hat{i}_1 = 1$ amp

$\hat{v}_2 = ?$

Sub Graph, Connectivity, Separate Part, Cut set, KCL, Loop, KVL

Given a graph 'G', G_1 is a subgraph of 'G' if every node of G_1 is a node of G and if every branch of G_1 is a branch of G. A subgraph consisting of one node only without any branches is called as degenerate subgraph.

A graph is said to be connected if there is at least one path along the branches between any two nodes of the graph. A graph with only one node is considered as connected.

Given an unconnected graph, its connected sub graphs are called separate parts.

Cut set of a connected graph is a set of branches, which cuts the given graph into two separate parts such that (1) the removal of all branches of the set causes the remaining graph to have two separate parts, and (2) the removal of all but any of the branches of the set leaves the remaining graph connected.

KCL: For any lumped network, for any of its cut sets, and at any time, the algebraic sum of all the branch currents traversing the cut-set branches is zero.

Loop (circuit) of a graph is a connected sub graph (closed path) such that precisely two branches are incident at each node or no node is encountered more than once along the closed path.

KVL: For any lumped network, for any of its loops, and at any time, the algebraic sum of the branch voltages around the loop is zero.

Tree and Cotree

Let G be a connected graph and 'T' a subgraph of G. T is a tree of the connected graph G if (1) T is a connected subgraph, (2) it contains all the nodes of G, and (3) it contains no loops. The graph with the branches not in 'T' is called cotree (complementary tree)

Given a connected graph G and a tree T, the branches of T are called tree branches (or twigs), and the branches of G not in T are called links (cotree branches or chords). If a graph has n_t nodes and has a single branch connecting every pair of nodes, then

$$\text{Number of trees} = n_t^{n_t-2} \rightarrow (6)$$

For such graphs, when $n_t = 5$, there are 125 trees; when $n_t = 10$, there are 10^8 trees. In general for any graph

$$\text{Number of trees} = \det [A A^T], \rightarrow (7)$$

where A is the reduced incidence matrix.

Properties

Given a connected graph G of n_t nodes and b branches, and a tree T of G,

1. There is a unique path along the tree between any pair of nodes.

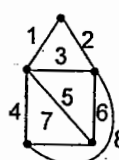
2. Number of tree branches, $n = n_t - 1$ $\rightarrow (8)$
(also known as rank of the tree or tree value of the graph)

$$\text{Number of links, } l = b - n = b - n_t + 1 \rightarrow (9)$$

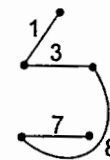
3. Every link of T and the unique tree path between its nodes constitute a unique loop (this is called the fundamental loop associated with the link.)

4. Every tree branch of T together with some links defines a unique cut set of G. This cut set is called the fundamental cut set associated with the tree branch.

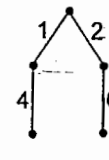
A graph 'G' and four of its Trees are shown in Fig. 4(a) – 4(e).



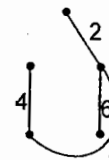
Graph G
Fig. 4(a)



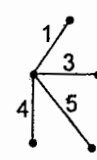
Tree T_1
4(b)



Tree T_2
4(c)

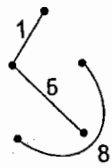


Tree T_3
4(d)



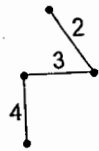
Tree T_4
4(e)

The subgraphs shown in Fig. 5(a) and (b), (c) do not qualify as trees.



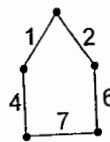
Violates property (1)

Fig. 5(a)



Violates property (2)

5(b)



Violates property (2)

5(c)

Tie set and Cut set matrices

Consider the network and its oriented graph shown in Fig. 6(a) and (b). A Tree: $T(6,7,8,9)$ is selected. Then Links are: 1, 2, 3, 4, 5.

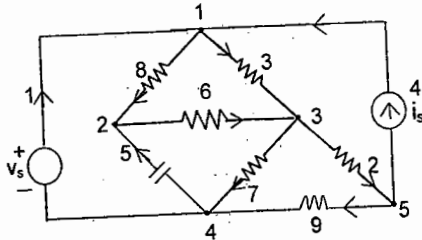


Fig. 6(a)

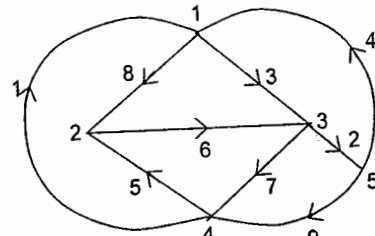


Fig. 6(b)

The twigs are shown in thick lines and links are shown in dotted lines in Fig. 6(c) and 6(d)

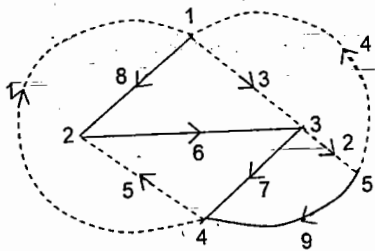


Fig. 6(c)

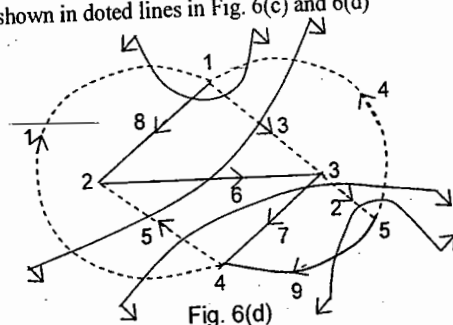


Fig. 6(d)

The unique loops and the KVL equations identified from Fig. 6(c) are given below:

$$\begin{aligned}
 L(1,8,6,7) &: v_1 + v_6 + v_7 + v_8 = 0 \\
 L(2,9,7) &: v_2 - v_7 + v_9 = 0 \\
 L(3,6,8) &: v_3 - v_6 - v_8 = 0 \\
 L(4,8,6,7,9) &: v_4 + v_6 + v_7 + v_8 - v_9 = 0 \\
 L(5,6,7) &: v_5 + v_6 + v_7 = 0
 \end{aligned} \quad \longrightarrow (10)$$

The above equations which are independent can be put in the matrix form as

$$B \vec{v} = 0, \quad \longrightarrow (11)$$

where \vec{v}_{bx1} is a column vector of branch voltages and B_{bx1} is known as f-circuit matrix or tie set matrix. For the given example with the chosen Tree:

$$B_{5 \times 9} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix} \longrightarrow (12)$$

The branch current \vec{j} can be obtained as the superposition of one or more loop (link) currents \vec{i}

$$\vec{j} = B^T \vec{i} \quad \longrightarrow (13)$$

where $i_1 = j_1, i_2 = j_2, i_3 = j_3, i_4 = j_4$ and $i_5 = j_5$.

The fundamental cut sets and the KCL eqns, identified from Fig. 6(d) are given below:

f-cut sets

KCL eqn.

$$\begin{aligned}
 C(6,1,5,3,4) &: -j_1 - j_3 - j_4 - j_5 + j_6 = 0 \\
 C(7,1,5,2,4) &: -j_1 + j_2 - j_4 - j_5 + j_7 = 0 \\
 C(8,1,3,4) &: -j_1 + j_3 - j_4 + j_8 = 0 \\
 C(9,2,4) &: -j_2 + j_4 + j_9 = 0
 \end{aligned} \quad \longrightarrow (14)$$

The above eqns. can be put in the matrix form as

$$Q \vec{j} = 0, \quad \longrightarrow (15)$$

where \vec{j} is column vector of branch currents and Q_{nbx} is known as f-cut set matrix. For the given example and chosen Tree

$$Q_{4 \times 9} = \begin{pmatrix} -1 & 0 & 1 & -1 & -1 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & -1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \longrightarrow (16)$$

Observe that

$$B_{bb} Q_{bn}^T = \vec{0}_{bn} \quad \longrightarrow (17)$$

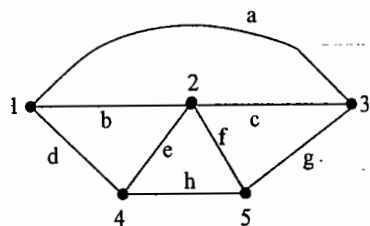
It can identified from Fig. 6(d) that

$$\vec{v} = Q^T \vec{e} \quad \longrightarrow (18)$$

where \vec{e} is the tree branch voltage vector, with $e_1 = v_6, e_2 = v_7, e_3 = v_8, e_4 = v_9$.

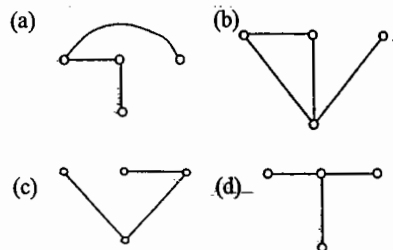
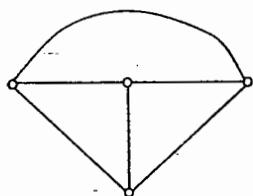
OBJECTIVE QUESTIONS:

01. Identify which of the following is NOT a tree of the graph shown in the figure
GATE-99



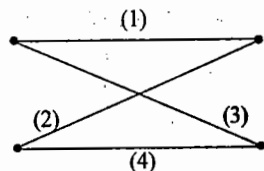
- (a) bcgh
(b) defg
(c) abfg
(d) aegh

2. Consider the network graph shown in the figure. Which one of the following is NOT a 'tree' of this graph?
GATE - 2004



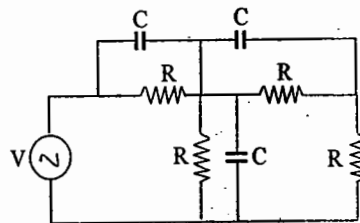
03. In the following graph, the number of trees (P) and the number of cut-sets (Q) are

GATE - 2008



- (a) P = 2, Q = 2
(b) P = 2, Q = 6
(c) P = 4, Q = 6
(a) P = 4, Q = 10

04. The minimum number of equations required to analyze the circuit shown in the figure is
GATE - 2003



- (a) 3
(c) 6
(b) 4
(d) 7

KEY:

01. c 02. b 03. c 04. b

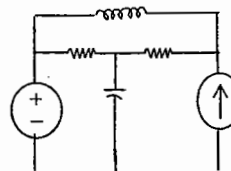
05. The matrix A given below is the node incidence matrix of network. The columns correspond to branches of the network while the rows correspond to nodes. Let $V = [v_1 \ v_2 \ \dots \ v_6]^T$ denote the vector of branch voltages while $I = [i_1 \ i_2 \ \dots \ i_6]^T$ that of branch currents. The vector $E = [e_1 \ e_2 \ e_3 \ e_4]^T$ denotes the vector of node voltages relative to a common ground.

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \end{bmatrix}$$

Which of the following statements is true?

- (a) The equations $v_1 - v_2 + v_3 = 0$, $v_3 + v_4 - v_5 = 0$ are KVL equations for the network for some loops
(b) The equations $v_1 - v_3 - v_6 = 0$, $v_4 + v_5 - v_6 = 0$ are KVL equations for the network for some loops
(c) $E = AV$
(d) $AV = 0$ are KVL equations for the network

06. The number of chords in the graph of the given circuit will be



- (a) 3
(c) 5
(b) 4
(d) 6

07. The number of independent loops for a network with 'n' nodes and 'b' branches is
(Gate 1996)

- (a) n-1
(b) b-n
(c) b-n+1
(d) independent of the number of nodes

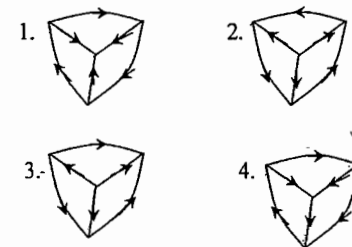
08. A network has 7 nodes and 5 independent loops. The number of branches in the network is
(Gate 1999)

- (a) 13
(b) 12
(c) 11
(d) 10

09. A network has 10 nodes and 17 branches in all. The number of node pair voltages would be (I.E.S -91)

- (a) 7
(b) 9
(c) 10
(d) 45

10. Which of the following oriented graphs have the same fundamental loop matrix? (I.E.S -93)



Select the correct answer using the codes given below:

Codes:

- (a) 1 and 2
(b) 2 and 3
(c) 1, 3 and 4
(d) 2, 3 and 4

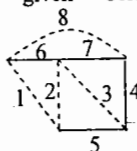
11. Consider the following statements:

(I.E.S -94)

1. One end only one path exists between any pair of vertices of a tree
 2. The number of cutsets are the same as the rank of the graph.
 3. The cut set is a minimal set of edges removal of which from the graph reduces the rank of the graph by one.
 4. The rank of a graph is equal to the number of vertices of the graph.
- Of these statements
- a) 2 and 4 are correct
 - b) 1 and 3 are correct
 - c) 2 and 3 are correct
 - d) 1 and 4 are correct

12. Match List I with List II with reference to the graph shown in the given figure and its particular tree of a circuit and select the correct answer using the codes given below the lists:

(I.E.S -95)



List I (Branches) List II

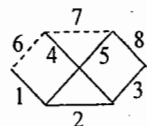
- | | |
|--------------|-----------------------|
| A. 1,2,3,4 | 1. Twigs |
| B. 4,5,6,7 | 2. Links |
| C. 1,2,3,8 | 3. Fundamental cutset |
| D. 1,4,5,6,7 | 4. Fundamental loop |

Codes:

- | | |
|------------|------------|
| a) A B C D | b) A B C D |
| 3 1 2 4 | 2 3 1 4 |
| c) A B C D | d) A B C D |
| 3 2 4 1 | 1 4 3 2 |

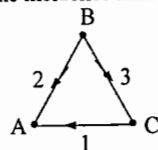
13. In the graph and the tree shown in the given figure, the fundamental cutset for the branch i2 is

(I.E.S -95)



- a) 2,1,5
- b) 2,6,7,8
- c) 2,1,3,4,5
- d) 2,3,4

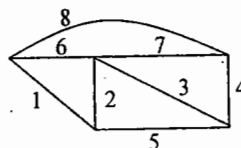
14. For the graph shown in the given figure, the incidence matrix A is given by



- | | |
|---|---|
| (a) $\begin{pmatrix} -1 & -1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & -1 \end{pmatrix}$ | (b) $\begin{pmatrix} 1 & 0 & -1 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$ |
| (c) $\begin{pmatrix} -1 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{pmatrix}$ | (d) $\begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$ |

15. In the graph shown in the figure, one possible tree is formed by the branches 4,5,6,7. Then one possible fundamental cut set is

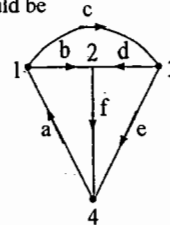
(I.E.S -97)



- a) 1, 2, 3, 8
- b) 1, 2, 5, 6
- c) 1, 5, 6, 8
- d) 1, 2, 3, 7, 8

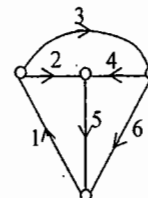
16. For the graph shown in the given figure one set of fundamental cut-set would be

(I.E.S -98)



- a) abc, cde, afe
- b) afdc, cde, abde
- c) cdfe, afe, bdf
- d) cbd, abde, cde

17.



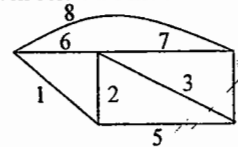
(I.E.S -2000)

Which one of the following is a cut set of the graph shown in the above figure?

- a) 1,2,3 and 4
- b) 2,3,4 and 6
- c) 1,4,5 and 6
- d) 1,3,4 and 5

18. Match List X with List Y for the tree branches 1,2,3 and 8 of the graph shown in the given figure and select the correct answer using the codes given below the lists.

(I.E.S -2001)



List X

- A. Twigs
- B. Links
- C. Fundamental cutset
- D. Fundamental loop

List Y

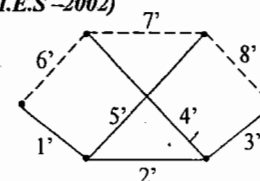
- I. 4,5,6,7
- II. 1,2,3,8
- III. 1,2,3,4
- IV. 6,7,8

Codes:

- | | |
|-------------|-------------|
| a) A B C D | b) A B C D |
| I II III IV | III II I IV |
| c) A B C D | d) A B C D |
| I IV III II | III IV I II |

19. Figure given below shows a graph with 6 vertices and 8 edges.

(I.E.S -2002)



With reference to the above graph, match List-I with List-II and select the correct answer using codes given below the lists:

List-I

- A. Fundamental circuit of chord 6'
- B. Fundamental circuit of chord 7'
- C. Fundamental circuit of chord 8'

List-II

1. The edge set (1', 2', 4', 6')
2. The edge set (2', 4', 5', 7')
3. The edge set (2', 3', 5', 8')
4. The edge set (1', 2', 4', 7')
5. The edge set cannot be determined

Codes:

- | | |
|----------|----------|
| a) A B C | b) A B C |
| 1 2 3 | 4 3 2 |
| c) A B C | d) A B C |
| 2 3 4 | 2 5 3 |

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Chapter 3 : Network Theorems

(Superposition, Thevenin, Norton, Maximum power transfer, Tellegen)

Network theorems simplify the analysis of complicated circuits.

1. Superposition Theorem:

The principle of superposition states that the response (the desired current or voltage) in a linear circuit having more than one independent source can be obtained by adding the responses caused by each independent source acting alone, with other independent sources set to zero. When a voltage source is set equal to zero it becomes a short circuit. When a current source is set equal to zero it becomes an open circuit. Do not set dependent sources to zero.

This theorem is applicable to linear networks (linear time invariant or time varying) consisting of independent sources, linear dependent sources, linear passive elements R, L & C, linear transformer.

Example: Refer to network shown in Fig (1)

Find e_0 by using superposition theorem, when $R_L = 12 \Omega$
(All resistor values are in ohms)

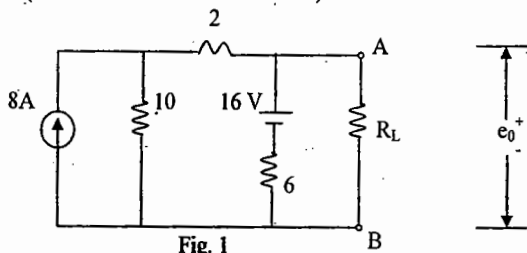


Fig 1

Due to 8 A:

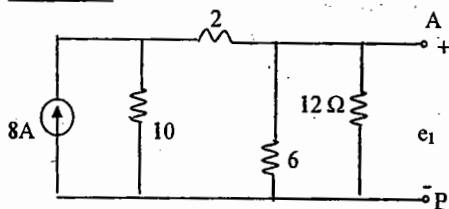


Fig 2

Due to 16 V:

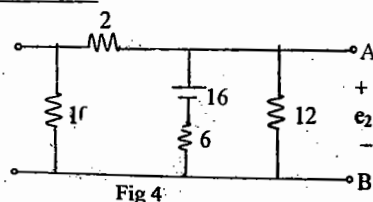


Fig 4

$$\therefore e_0 = e_1 + e_2 = 28 \text{ V}$$

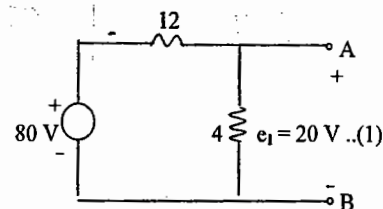


Fig 3

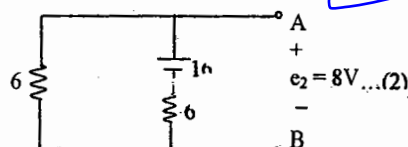


Fig 5

$$\dots\dots(3)$$

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Network Theorems

27

2. Thevenin's and Norton's Theorems:

A network with load resistance R_L connected across two terminals A, B (identified from the NW) is shown in Fig.6

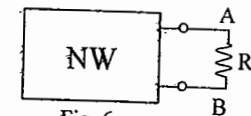


Fig. 6

The load may be linear, time invariant or time varying. The theorems are useful to find out the voltage or current in R_L . They are applicable to linear network with independent as well as dependent sources (not necessarily linear)

Thevenin equivalent network

(At the terminals A & B)

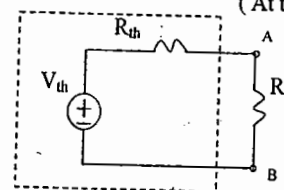


Fig 7

Norton equivalent network

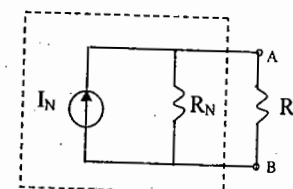


Fig 8

V_{th} = Thevenin's voltage = O.C. voltage across terminals A & B.
 I_N = Norton's current = S.C. current through the terminals A & B
 $R_{th} = R_N = R$; $V_{th} = I_N R$

$$\dots\dots(4) \quad R_T = \frac{V_{OC}}{I_{SC}}$$

Example: Find Thevenin & Norton equivalent networks at the terminals A & B for the network shown in Fig.1.

To find V_T , refer to Fig 9.

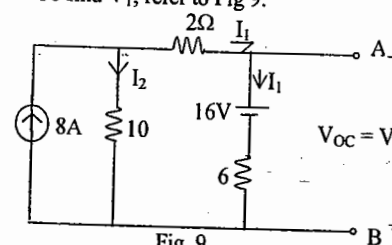


Fig 9

$$I_1 = \frac{V_T - 16}{6} \quad I_2 = 8 - I_1$$

$$10 I_2 = 2 I_1 + V_T, \quad 80 - 10 I_1 = 2 I_1 + V_T$$

$$12 I_1 = 80 - V_T, \quad 2 V_T - 32 = 80 - V_T$$

$$3 V_T = 112, \quad V_T = \frac{112}{3} \text{ V} \dots\dots(5)$$

To find R_T : Set the independent sources to zero as shown in Fig 10

Open Circuit (OC) independent current sources, Short Circuit (SC) independent voltage source. Do not set dependent sources to zero.

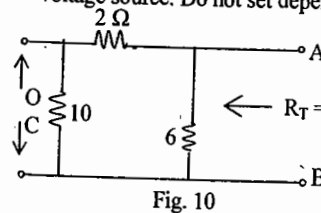


Fig 10

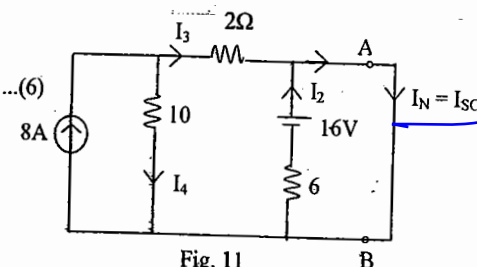


Fig 11

To find $I_N = I_{SC}$, refer to Fig. 11

$$I_2 = \frac{8}{3} \text{ A}, I_3 = -I_2 + I_N, I_4 = 8 - I_3, 10 I_4 = 2 I_3, I_3 = 5 I_4$$

$$6 I_4 = 8, I_4 = \frac{4}{3} \text{ A}, I_3 = \frac{20}{3} \text{ A}, \frac{20}{3} = \frac{-8}{3} + I_N, I_N = \frac{28}{3} \text{ A} \quad \dots\dots(7)$$

$$R_N = R_T = 4 \Omega \quad \text{Note that } R_T = R_N = \frac{V_T}{I_N} = 4 \Omega \quad \dots\dots(8)$$

Thevenin's equivalent circuit

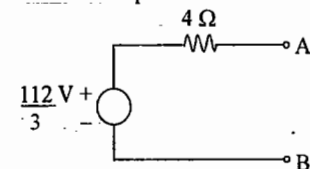


Fig 12

Norton's equivalent circuit

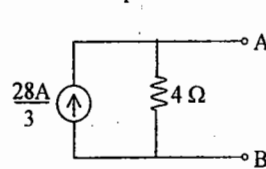


Fig 13

3. Maximum power transfer theorem :- (refer to fig 14)

This is used to find the value of the load resistor R_L (optimum) that absorbs maximum power from a given network

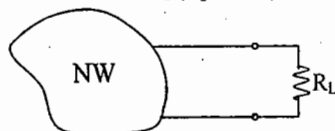


Fig. 14

Example: Find the optimum value of R_L for maximum power transferred to it and also find this maximum power.

Replace the network by Thevenin equivalent circuit as shown in Fig. 15.

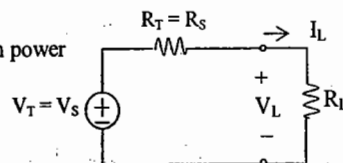


Fig. 15

For maximum power transfer, $R_L = R_s \quad \dots\dots(9)$

Under this condition maximum power transferred to $R_L = \frac{V_s^2}{4R_L} \quad \dots\dots(10)$

and maximum power delivered by $V_s = \frac{V_s^2}{2R_L} \quad \dots\dots(11)$

Under maximum power transfer, efficiency of power transfer = 50% $\dots\dots(12)$

This theorem is applicable to linear networks with independent as well as dependent sources

This is also applicable to linear time invariant networks under sinusoidal steady state. The Thevenin's equivalent circuit is shown in Fig 16

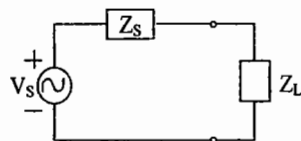


Fig. 16

Where

$Z_L = R_L + jX_L$ is complex load

V_s is Thevenin's voltage phasor

Z_s is Thevenin's equivalent impedance = $R_s + jX_s$

For maximum power transfer to Z_L

Case 1 :

When only X_L is variable in the load, $X_L = -X_s \quad \dots\dots(13)$

then maximum power transferred to Z_L

$$\text{is } \frac{|V_s|^2 R_L}{2(R_s + R_L)^2} \quad \dots\dots(14)$$

where $|V_s|/\sqrt{2}$ is the RMS value and $|V_s|$ is the maximum value of the source

Case 2: When X_L as well as R_L can be varied, then $X_L = -X_s$, $R_L = R_s$ i.e., $Z_L = Z_s^*$ (complex conjugate of Z_s)

$$\text{then maximum power transferred to } Z_L = \frac{|V_s|^2}{8R_s} \quad \dots\dots(15)$$

$$\text{Case 3: When only } R_L \text{ is variable then } R_L (\text{optimum}) = \sqrt{R_s^2 + (X_s + X_L)^2} \quad \dots\dots(16)$$

$$\text{then maximum power transferred to } Z_L = \frac{0.5 |V_s|^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2} \quad \dots\dots(17)$$

For example if $Z_s = 3 \Omega$, $X_L = 4 \Omega$ for maximum power transfer,

$$R_L = \sqrt{R_s^2 + (X_s + X_L)^2} \\ = \sqrt{9 + 16} = 5 \Omega$$

Example: Refer to Fig.1

Find the value of R_L for maximum power transferred to it and find the maximum Power. The Thevenin's equivalent circuit at terminals A & B is shown in Fig.17 (same as Fig. 12).

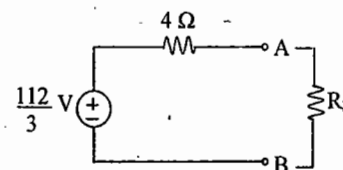


Fig 17

$$\therefore R_L (\text{optimum}) = 4 \Omega$$

$$P_{\max} = \left(\frac{112}{3} \right)^2 \left(\frac{1}{16} \right) \\ = \frac{784}{9} \text{ W}$$

If only R_L optimum is required, note that V_{th} need not be calculated.

Tellegen's Theorem:

This theorem is applicable for any lumped network that contains any elements, linear or non linear, passive or active, time-invariant or time-varying. The theorem depends only on KCL and KVL and hence can be applied to any network for which KCL and KVL are applicable. Note that this theorem is not valid for distributed network like transmission line.

Consider any lumped network with number of branches = b , as shown in Figure 18 with $b = 6$. for branch k , v_k = branch voltage, j_k = branch current with associated reference directions as shown in Fig. 19.

Instantaneous Power in k th branch $p_k = v_k j_k$

If p_k is positive power is delivered to the branch. If p_k is negative power is supplied by the branch. Then that branch contains source, delivering power.

Select any arbitrary values of v_k and j_k for $k = 1$ to b such that v_k 's satisfy KVL, and j_k 's satisfy KCL. Then according to Tellegen's theorem

$$S = \sum_{k=1}^b v_k j_k = 0 \dots \dots \dots (17)$$

Summation of instantaneous powers in all the branches in the network = 0.

Tellegen's theorem implies conservation of energy.

The sum of the powers delivered by the independent sources to the network is equal to the sum of the powers absorbed by all the other branches of the network.

Example : Refer to Fig. 18

Let $v_1 = 3 \text{ V}$; $v_2 = 2 \text{ V}$; $v_3 = 6 \text{ V}$; $v_4 = 4 \text{ V}$; $v_5 = 5 \text{ V}$; $v_6 = 1 \text{ V}$ Satisfying KVL

Similarly, $j_1 = 3$; $j_2 = 4.5$; $j_4 = 1.5$; $j_5 = -1$; $j_3 = -3.5$; $j_6 = 2$ Satisfying KCL

$$S = (3 \times 3) + (2 \times 4.5) + (6 \times -3.5) + (4 \times 1.5) + (5 \times -1) + (1 \times 2) = 9 + 9 - 21 + 6 - 5 + 2 = 0$$

Given two networks, N_1 and N_2 , having the same graph with the same reference direction assigned to

the branches in the two networks, but with different element values and kinds. Let v_{1k} and j_{1k} be the voltages and currents in N_1 , and v_{2k} and j_{2k} similarly be the voltages and currents in N_2 , where all voltages and currents satisfy the appropriate Kirchhoff laws. Then by Tellegen's theorem.

$$\sum_{k=1}^b v_{1k} j_{2k} = 0 \text{ and } \sum_{k=1}^b v_{2k} j_{1k} = 0 \quad \sum v_{1k} j_{2k}, \sum v_{2k} j_{1k} = 0$$

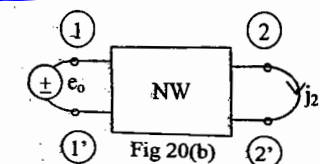
Another Variation of the theorem : If t_1 and t_2 are two different times of observation, it still follows that

$$\sum_{k=1}^b v_k(t_1) j_k(t_2) = 0$$

Reciprocity Theorem:

Consider a linear time invariant network (NW) which consists of resistors, inductors, coupled inductors, capacitors and transformers only. Identify any two pairs of terminals

as shown in Fig. 20 - 22



$$j_2(t) = \hat{j}_1(t) \dots \dots (18) \text{ for all } t$$

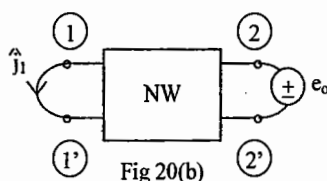


Fig. 18

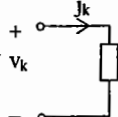


Fig 19

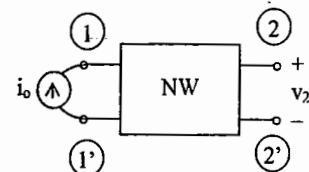


Fig 21(a)

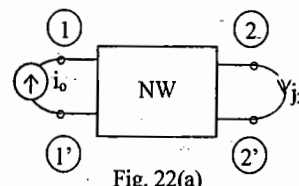


Fig. 22(a)

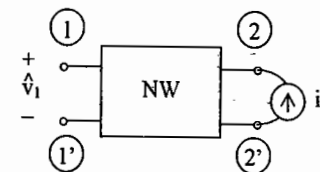


Fig 21(b)

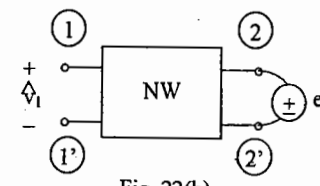


Fig. 22(b)

If $i_o(t)$ and $e_o(t)$ are equal for all 't', then $j_2 = \hat{j}_1$ for all 't':

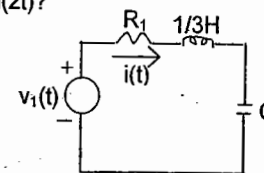
$$\dots \dots \dots (20)$$

In terms of NW functions, Reciprocity theorem for LTI networks asserts that

$$y_{21}(s) = y_{12}(s), \quad z_{21}(s) = z_{12}(s), \quad H_1(s) = H_v(s), \quad h_{12} = h_{21}, \quad AD - BC = 1 \dots \dots \dots (21)$$

A network that obeys the reciprocity theorem is called reciprocal.

Example: In the circuit shown, what is the value of C that will cause maximum power delivery to $R_1 = 1 \Omega$, if $v_1(t) = 2\sqrt{2} \sin(2t)$?



Power delivered to $R_1 = I_{rms}^2 R_1$ is maximum if I_{rms} maximum
 I_{rms} is maximum under Resonance at $\omega_0 = 2$

$$\omega_0^2 = \frac{1}{LC}, \quad C = \frac{1}{\omega_0^2 L} = \frac{3}{4} \text{ F}$$

Example: If $R_2 = 1 \Omega$ is connected across 'C' in the above example, find 'C' that causes maximum power delivery to R_2 .

Voltage across $R_2 = \vec{V}_2 = \vec{V}_1 \frac{Z_2}{Z_1 + Z_2}$

$$\text{where } Z_1 = \left(1 + j\frac{2}{3}\right), \quad Z_2 = 1 \parallel C = \frac{-1}{(1 + j2C)}$$

$$\vec{V}_2 = \frac{2\sqrt{2}}{\left(2 - \frac{4}{3}C\right) + j\left(2C + \frac{2}{3}\right)}$$

$$P = \text{Power delivered to } R_2 = \frac{V_{2rms}^2}{1}$$

$$= \frac{4}{\left(2 - \frac{4}{3}C\right)^2 + \left(2C + \frac{2}{3}\right)^2}$$

P is maximum when $dP/dC = 0$

$$2C + \frac{2}{3} = \frac{2}{3} \left(2 - \frac{4}{3}C\right), C = \frac{3}{13} \text{ F}$$

Transformer adjustment for maximum power transfer

In the circuit shown, if only the transformation Ratio 'a' of the transformer (assumed ideal) is adjustable, to find 'a' such that maximum power is delivered to $R_2 = R_e(Z_2)$

Verify that $Z_2' = Z_2/a^2$

$$a = \sqrt{K}, K = |Z_2/Z_1|$$

The maximum power delivered for this adjustment of 'a' is less than that for a conjugate match $Z_2 = Z_1^*$

Power factor correction

An inductive load with complex power $S = P_{ave} + jQ_L$ (Q_L is +ve) is shown.

The objective is to introduce negative Q_C by shunting the load with a capacitor so that PF is improved.

$$\text{PF before correction} = \frac{P_{ave}}{\sqrt{P_{ave}^2 + Q_L^2}}$$

$$\text{PF after correction} = \frac{P_{ave}}{\sqrt{P_{ave}^2 + Q_L^2}}, Q_L = Q_L - Q_C$$

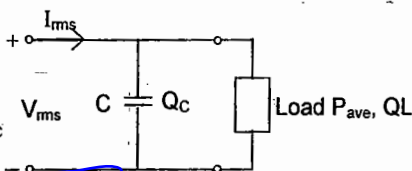
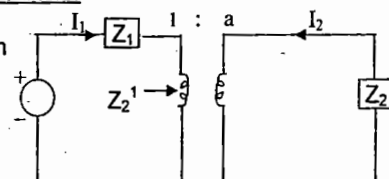
The value of 'C' to be introduced is given by $C = -\frac{Q_C}{V_{rms}^2 \omega}$

Example: Given the load with $P_{ave} = 500\text{W}$ and $Q = +500$ vars, $V_{rms} = 220\text{V}$, $f = 50\text{ Hz}$, find the PF. If the PF is to be improved to 0.9 lagging find the value of 'C' required,

$$\text{PF before correction} = \frac{500}{\sqrt{(500)^2 + (500)^2}} = \frac{1}{\sqrt{2}} = 0.707 \text{ (lagging as } Q \text{ is +ve)}$$

With PF = 0.9, verify that $Q_L = 242$ vars
 $Q_C = 242 - 500 = -258$ vars

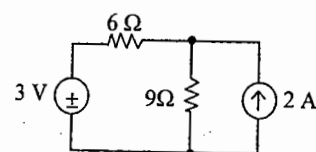
$$C = \frac{2}{2\pi \times 50 \times 220^2} = 17 \mu\text{F}$$



Objective Questions

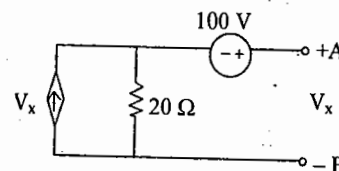
SET - A

1. The current in 9Ω resistor using superposition theorem is



- a) $[(1/5) + (4/5)]\text{A}$
 b) $[(2/5) + (3/5)]\text{A}$
 c) $[(-1/5) + (6/5)]\text{A}$
 d) $[0.5 + 0.5]\text{A}$

2. Thevenin voltage across A and B in the circuit shown below is

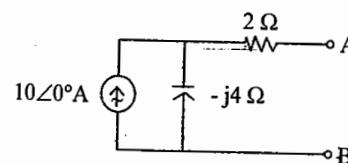


- a) $-(100/19)\text{ V}$
 b) $(50/19)\text{ V}$
 c) $(25/19)\text{ V}$
 d) 10 V

3. Superposition theorem is based on the concept of

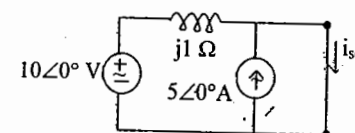
- a) duality
 b) linearity
 c) reciprocity
 d) non-linearity

4. The value of Norton current source for the network shown in figure between terminals A and B is



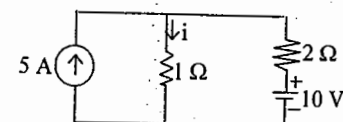
- a) $8(2 - j1)\text{ A}$
 b) $2(2 + j1)\text{ A}$
 c) $4(2 + j1)\text{ A}$
 d) $4(2 - j1)\text{ A}$

05. In the circuit shown below, the short circuit current i_{sc} is



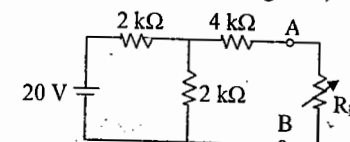
- a) $5(1 - j2)\text{ A}$
 b) $5(1 + j2)\text{ A}$
 c) $10(1 + j2)\text{ A}$
 d) $10(1 + j3)\text{ A}$

06. The value of i in the circuit shown using superposition theorem is



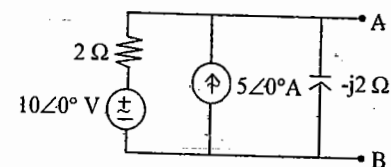
- a) $[(4/3) + (16/3)]\text{ A}$
 b) $[(10/3) + (10/3)]\text{ A}$
 c) $[(10/3) - (10/3)]\text{ A}$
 d) $(1 + 2)\text{ A}$

07. The maximum power that can be transferred to the load resistance in the network shown in figure is,



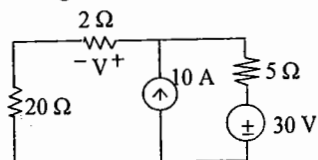
- a) 10 mW
 b) 2.5 mW
 c) 7.5 mW
 d) 5 mW

08. The only two elements in the load connected across terminals A and B of the circuit shown below to derive maximum power from the circuit are



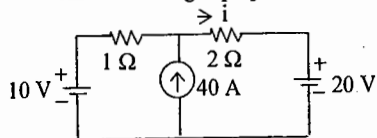
- a) Resistance and capacitance
 b) Resistance and inductance
 c) Inductance and capacitance
 d) Conductance and capacitance

09. In the circuit shown below, if 2Ω resistance is changed to 4Ω , the change in V is



- a) $[320 / (27 \times 29)]$ V
b) $[160 / (27 \times 29)]$ V
c) $[80 / (27 \times 29)]$ V
d) $[640 / (27 \times 29)]$ V

10. In the circuit shown below the value of i using superposition theorem



- is
a) $[(10/3) + (40/3) - (20/3)]$ A
b) $[(10/3) + (40/3) + (20/3)]$ A
c) $[(10/3) - (40/3) - (20/3)]$ A
d) $[-(10/3) + (40/3) - (20/3)]$ A

11. Superposition theorem is applicable to

- a) linear elements only
b) nonlinear elements only
c) linear and nonlinear elements
d) nonlinear and dependent sources

12. The Thevenin impedance of a network between load terminals is $(3 + j5)\Omega$. The load to be connected for maximum power transfer is

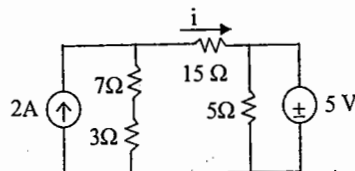
- a) $(-3 - j5)\Omega$ b) $(3 - j5)\Omega$
c) $(-3 + j5)\Omega$ d) $(3 + j5)\Omega$

13. In a circuit $V_{th} = 5V$, $R_{th} = 2\Omega$ and $R_L = 3\Omega$. The value of load current is
a) 2.5 A b) 1.67 A c) 1 A d) $(25/6)$ A

14. In the Thevenin equivalent circuit, V_{th} equals

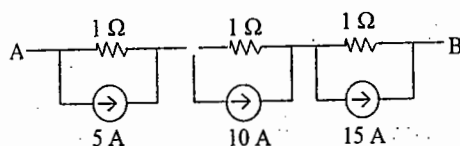
- a) short-circuit terminal voltage
b) open-circuit terminal voltage
c) voltage of the source
d) net voltage available in the circuit

15. Applying superposition theorem the current i in the figure is



- a) $[(4/5) - (1/5)]$ A
b) $[(1/5) - (4/5)]$ A
c) $[(2/5) - (4/5)]$ A
d) $[(4/5) - (3/5)]$ A

16. The Norton's equivalent across terminals A and B in the circuit shown below is

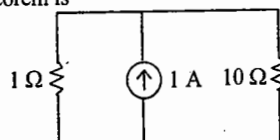


- a) 33 amps in series with 1Ω
b) 66 amps in series with 1Ω
c) 10 amps in shunt with 3Ω
d) 60 amps in shunt with 3Ω

17. Reciprocity theorem is applicable only to

- a) Single source circuits
b) Circuits containing two sources
c) Circuits containing any number of sources
d) Circuits containing only dependent sources

18. In the circuit shown, the change in current if 10Ω resistance is replaced by 11Ω resistance by compensation theorem is



- a) $(1/132)$ A b) $(3/11)$ A
c) $(1/11)$ A d) $(5/132)$ A

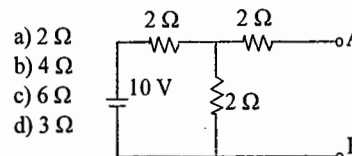
19. Superposition theorem is not applicable to networks containing

- a) nonlinear elements
b) dependent voltage sources
c) dependent current sources
d) transformers

20. In a D.C circuit, Thevenin voltage across load terminals is 15V and Norton current is 3A. The maximum power that can be transferred to the load is

- a) 12.25 W b) 30.5 W
c) 11.25 W d) 22.5 W

21. For the network shown in figure, Thevenin resistance across terminals A and B is

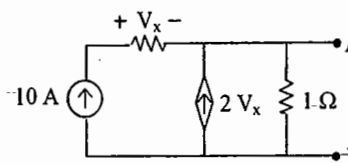


- a) 2Ω
b) 4Ω
c) 6Ω
d) 3Ω

22. Thevenin resistance R_{th} is the resistance

- a) between any two terminals of the network
b) obtained by short circuiting the given two terminals
c) obtained by removing voltage sources only along with their internal resistances
d) between the same open terminals as for finding V_{th}

23. If a resistance R is connected across terminals A and B of the circuit shown below, maximum power that can be dissipated in R is



- a) 250 W b) 125 W
c) 625 W d) 120 W

24. Tellegen's theorem states that, in a circuit if voltage drops and current associated with elements are v 's and i 's then

$$\sum_{k=1}^n v_k i_k = 0 \quad \text{b) } \sum_{k=1}^n (v_k + i_k) = 0$$

$$\sum_{k=1}^n (v_k - i_k) = 0 \quad \text{d) } \sum_{k=1}^n (v_k / i_k) = 0$$

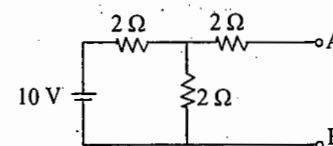
25. In a linear circuit, the superposition principle can be applied to calculate the

- a) Voltage and power
b) Voltage and current
c) Current and power
d) Voltage, current and power

26. A dc voltage source 10V with internal resistance of 2Ω is connected to a resistor R_L . Maximum transfer of power from source to load takes place when:

- a) $R_L = 0\Omega$ b) $R_L = \infty$
c) $R_L = 2\Omega$ d) $R_L = 4\Omega$

27. For the network shown in figure, Thevenin voltage between A and B is

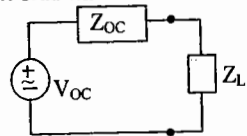


- a) 5 V b) 2.5 V
c) 7.5 V d) 10 V

28. While calculating R_{th} , ideal current sources in the circuit are

- a) replaced by open circuits
b) replaced by shorts circuits
c) not changed in the circuit
d) converted into equivalent voltage sources

29. The maximum power is transferred to load when

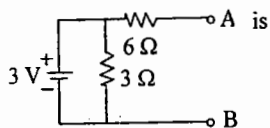


- a) $Z_L = Z_{OC}$ b) $Z_L = Z_{OC}^*$
c) $Z_L = -Z_{OC}$ d) $Z_L = -Z_{OC}^*$

30. Superposition theorem requires as many circuits to be solved as there are:

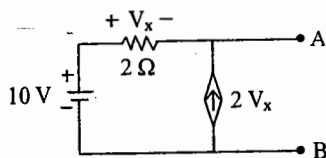
- a) Nodes b) Meshes
c) Sources d) Paths

31. Thevenin resistance of the circuit shown below across the terminals A and B is



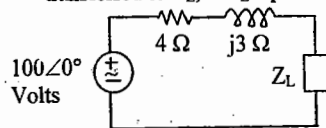
- a) 6 Ω b) 3 Ω
c) 9 Ω d) 2 Ω

32. The current flowing through the short connected across A and B is



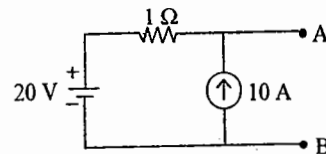
- a) 0 A b) 20 A
c) 15 A d) 25 A

33. If $Z_L = R_L$, maximum power is transferred to Z_L , if Z_L equals



- a) 5 Ω b) 4 Ω
c) 3 Ω d) 1 Ω

34. The Thevenin's equivalent across terminals A and B in the circuit shown below is

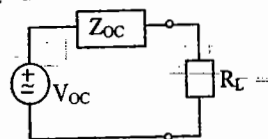


- a) 30 V in series with 1 Ω resistor
b) 30 A in series with 1 Ω resistor
c) 15 V in series with 1 Ω resistor
d) 30 A in series with 2 Ω resistor

35. In a linear system, several sources acting simultaneously produce an effect which is the sum of the separate effects caused by individual source acting at a time. This is:

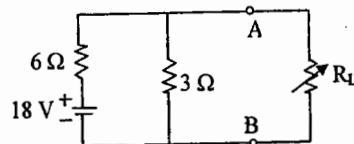
- a) Compensation theorem
b) Superposition theorem
c) Reciprocity theorem
d) Norton's theorem

36. Referring to the equivalent circuit shown, maximum power is transferred to a purely resistive load R_L when



- a) $R_L = |Z_{OC}|$ b) $R_L = Z_{OC}^*$
c) $R_L = -|Z_{OC}|$ d) $R_L = -Z_{OC}^*$

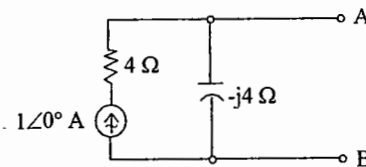
37. The load resistance R_L required to extract maximum power from the



source is

- a) 2 Ω b) 9 Ω
c) 6 Ω d) 18 Ω

38. Equivalent Thevenin voltage source across terminals A and B of the given circuit is:



- a) 4 V b) -j4 V
c) $(4 - j4)$ V d) $(4 + j4)$ V

39. Norton's theorem is the dual of
a) Thevenin's theorem
b) Superposition theorem
c) Maximum power transfer theorem
d) Reciprocity theorem

40. Reciprocity theorem can not be applied to circuits containing

- a) Unilateral elements
b) Independent sources
c) Inductors and capacitors
d) Resistors

41. Tellegen's theorem is based on

- a) Conservation of energy
b) Ohm's law
c) Kirchhoff's law d) Newton's law

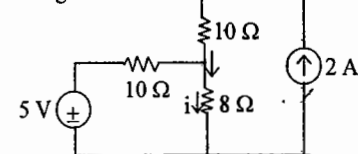
42. The load connected to a source is purely inductive. For maximum transfer of power from source to load, the source impedance should be

- a) Inductive b) Capacitive
c) Resistive d) Zero

43. The reciprocity theorem is applicable to

- a) Linear networks only
b) Bilateral networks only
c) Linear and bilateral networks only
d) Passive networks

44. Which of the following theorems is best suited for finding i in the given circuit?



- a) Thevenin's theorem
b) Reciprocity theorem
c) Compensation theorem
d) Substitution theorem

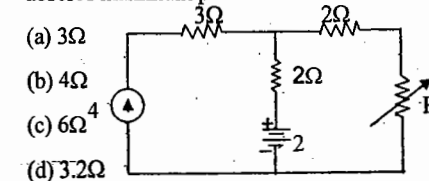
45. An electrical circuit having one or more voltage sources is transformed into an equivalent electrical network with a single voltage source in series with a resistance. This is

- a) Superposition theorem
b) Norton's theorem
c) Thevenin's theorem
d) Reciprocity theorem

46. Application of Norton's theorem results in:

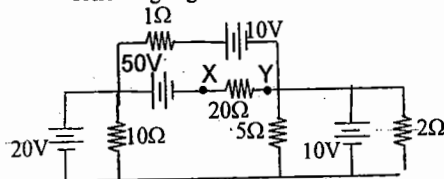
- a) A current source with impedance in parallel
b) A voltage source with impedance in series
c) A voltage source alone
d) A current source alone

47. In Figure., the value of R for which it absorbs maximum power is



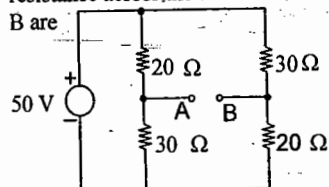
- (a) 3 Ω (b) 4 Ω
(c) 6 Ω (d) 3.2 Ω

48. The Thevenin's voltage (V_{xy}) and resistance across X and Y terminals for the circuit shown in the following figure are

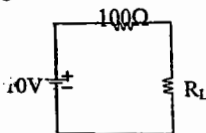


- (a) $-40V$ & 0Ω (b) $+40V$ & 1Ω
 (c) $-40V$ & 1Ω (d) $+40V$ & $\infty\Omega$
49. The condition for maximum power consumption by load impedance $|Z_L| < \theta_L$, where both magnitude and angle are variables, supplied from a source with a fixed internal impedance $|Z_s| < \theta_s$ is:
- (a) $|Z_L| = |Z_s|$ (b) $\theta_L = \theta_s$
 (c) $|Z_L| < \theta_L = Z_s < \theta_s$
 (d) $|Z_L| < \theta_L = |Z_s| < \theta_s$

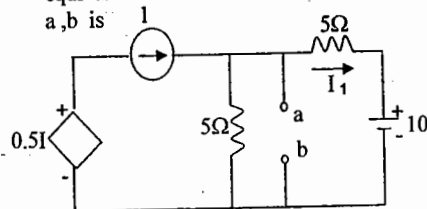
50. For the circuit shown in Fig., the Norton's equivalent current and resistance across the terminals A and B are



- (a) $3/5A$, 2Ω (b) $5A$, 3Ω
 (c) $5/12A$, 24Ω (d) $6A$, 5Ω
51. The maximum power that can be transferred to the load resistor R_L from the voltage source in Fig is
- (a) $1W$.
 (b) $10W$.
 (c) $0.25W$.
 (d) $0.5W$.



52. For the circuit shown in Fig. Thevenin's voltage and Thevenin's equivalent resistance at terminals a, b is

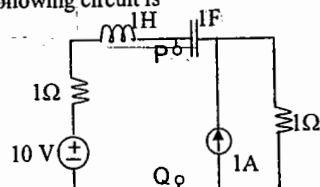


- (a) $5V$ & 2Ω (b) $7.5V$ & 2.5Ω
 (c) $4V$ & 2Ω (d) $3V$ & 2.5Ω

53. An independent voltage source in series with an impedance $Z_s = R_s + jX_s$ delivers a maximum average power to a load impedance Z_L when

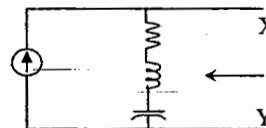
- (a) $Z_L = R_s + jX_s$ (b) $Z_L = R_s$
 (c) $Z_L = jX_s$ (d) $Z_L = R_s - jX_s$

54. The Thevenin's equivalent impedance Z_{th} between the nodes P and Q in the following circuit is

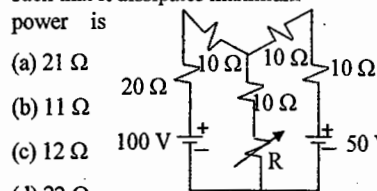


- (a) 1 (b) $1 + s + \frac{1}{s}$
 (c) $2 + s + \frac{1}{s}$ (d) $\frac{s^2 + s + 1}{s^2 + 2s + 1}$

55. In the figure the current source is $1\angle 0A$, $R = 1\Omega$, the impedances are $Z_C = -j\Omega$, and $Z_L = 2j\Omega$. The Thevenin's equivalent looking into the circuit across X - Y is

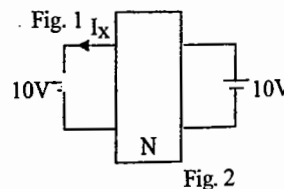
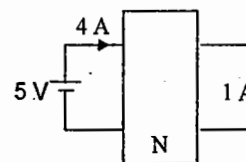


- (a) $\sqrt{2}\angle 0V$, $(1 + 2j)\Omega$
 (b) $2\angle 45^\circ V$, $(1 - 2j)\Omega$
 (c) $2\angle 45^\circ V$, $(1 + j)\Omega$
 (d) $\sqrt{2}\angle 45^\circ V$, $(1 + j)\Omega$
56. In the circuit shown, the value of R such that it dissipates maximum power is



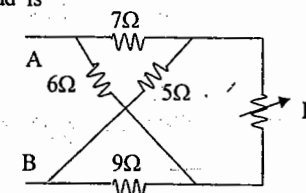
- (a) 21Ω
 (b) 11Ω
 (c) 12Ω
 (d) 22Ω

57. The network N, in fig. 1 and fig. 2, is passive and contains only linear resistors. The port currents in fig. 1 are as marked. The Value of



- (a) $4A$ (b) $6A$
 (c) $5A$ (d) None

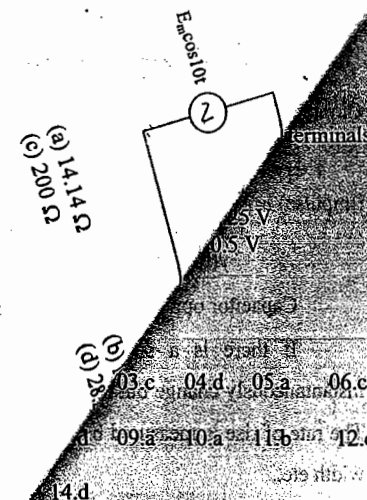
58. In the lattice network, the value of R for maximum power transfer to the load is



- (a) 5Ω (b) 6.5Ω
 (c) 8Ω (d) 9Ω

Key for Set - A

1. a 2. a 3. b 4. d 5. a 6. b
 7. d 8. b 9. a 10. a 11. a 12. b
 13. c 14. b 15. a 16. c 17. a 18. a
 19. a 20. c 21. d 22. d 23. c 24. a
 25. b 26. c 27. a 28. a 29. a
 31. a 32. d



SET - B

01. The maximum power that a 12 V d.c. source with an internal resistance of $2\ \Omega$ can supply to a resistive load is

(A) 12 W (B) 18 W
(C) 36 W (D) 48 W

02. Which one of the following theorems is a manifestation of the law of conservation of energy?

(A) Tellegen's theorem
(B) Reciprocity theorem
(C) Thevenin's theorem
(D) Norton's theorems

03. Which one of the following is not a dual pair?

(A) node, loop
(B) short circuit, open circuit
(C) L, C circuit,
(D) R, C circuit,

04. Superposition theorem is not applicable to networks containing

(A) non linear elements
(B) dependent voltage source
(C) dependent current source
(D) Transformer

05. A certain network consists of two ideal voltage sources and a large number of ideal resistors. The power consumed in one of the resistors is 4W, when either of the two sources is active and the other is replaced by a short circuit. The power consumed by same resistor when both the sources are simultaneously active would be

(A) zero or 16 W (B) 4 W or 8 W
(C) zero or 8 W (D) 8 W or 16 W

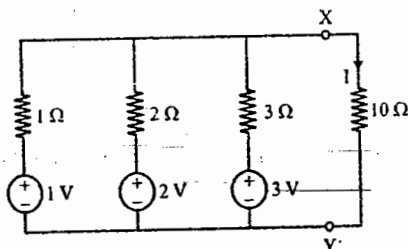
06. The dual of parallel RC circuit is a

(A) Series RC circuit
(B) Series RL circuit
(C) Parallel RC circuit
(D) Parallel RL circuit

07. Which of the following theorems can be applied to any network – linear or non – linear, active or passive, time – variant or time – invariant ?

(A) Thevenin theorem
(B) Norton theorem
(C) Tellegen theorem
(D) Superposition theorem

08. Calculate the load current I in the circuit

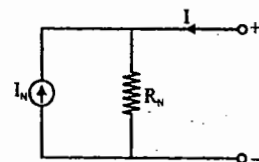
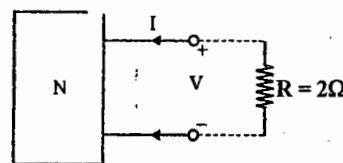


(A) 3 A (B) 6 A
(C) $\frac{9}{58}$ A (D) $\frac{11}{58}$ A

09. If two identical 3 A, $4\ \Omega$ Norton equivalent circuits are connected in parallel with like polarity, the combined Norton equivalent circuit will be

(A) 3 A, $8\ \Omega$ (B) 6 A, $8\ \Omega$
(C) 0 A, $2\ \Omega$ (D) 6 A, $2\ \Omega$

10. The V – I relation for the network shown is given as $V = 4I - 9$



The equivalent circuit to network N

Shown is given by

(A) 2.25 A, $4\ \Omega$ (B) -2.25 A, $4\ \Omega$
(C) 0, $4\ \Omega$ (D) None of these

11. The Norton's equivalent of circuit

shown in Figure – I is drawn in the

circuit shown in Figure – II. The

value of I_{sc} and R_{eq} in Figure – II are respectively

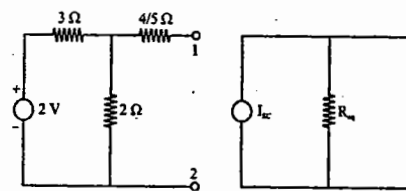


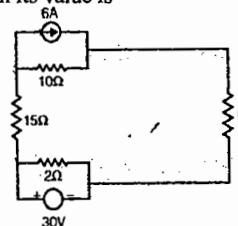
Fig. I

Fig. II

(A) $5/2$ A and $2\ \Omega$
(B) $2/5$ A and $1\ \Omega$
(C) $4/5$ A and $12/5\ \Omega$
(D) $2/5$ A and $2\ \Omega$

12. In the circuit shown in the given figure, R_L will absorb maximum power when its value is

(A) $2.75\ \Omega$
(B) $7.5\ \Omega$
(C) $25\ \Omega$
(D) $27\ \Omega$



13. Match List – I (Property of Network) with List – II (Relevant Theorem) and select the correct answer using the code given below the lists:

List – I

P. Linearity
Q. Structure
R. Equivalent Circuit Theorem
S. Bilateral

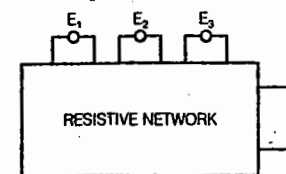
List – II

1. Superposition Theorem
2. Norton's Theorem
3. Tellegen's Theorem
4. Millman's Theorem

Codes:

	P	Q	R	S
(A)	2	5	1	3
(B)	1	3	2	4
(C)	2	3	1	4
(D)	1	5	2	3

14. In the circuit shown in the figure, the power consumed in the resistance R is measured when one source is acting at a time, these values are 18W, 50 W and 98 W. When all the sources are acting simultaneously, the possible maximum and minimum values of power in R will be



(A) 98W & 18W (B) 166W & 18W
(C) 450W & 2W (D) 166W & 2W

15. Consider the following statements:

1. Tellegen's theorem is applicable to any lumped network.
2. The reciprocity theorem is applicable to linear bilateral networks.
3. Thevenin's theorem is applicable to two terminal linear active networks.
4. Norton's theorem is applicable to two terminal linear active networks.

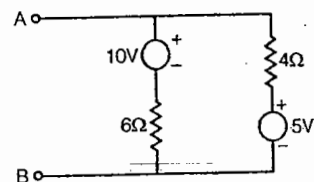
Which of these statements are correct?

- (A) 1, 2 and 3 (B) 1, 2, 3 and 4
(C) 1, 2 and 4 (D) 3 and 4

16. A network N is a dual of network N if

- (A) both of them have same mesh equations
(B) both of them have the same node equations
(C) mesh equations of one are the node equations of the other
(D) KCL and KVL equations are the same

17. In the circuit given below, viewed from A,B, the circuit can be reduced to an equivalent circuit as



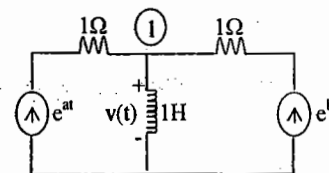
- (A) 5 volt source in series with 10 Ω resistor
(B) 7 volt source in series with 2.4Ω resistor
(C) 15 volt source in series with 2.4Ω resistor
(D) 1 volt source in series with 10 Ω resistor

Key for Set B:

- 1.B 2.A 3.D 4.A 5.C
6.B 7.C 8.C 9.D 10.B
11.D 12.C 13.B 14.C 15.B
16.C 17.B

SET C

01. In the circuit of the figure, the voltage $v(t)$ is GATE-2000

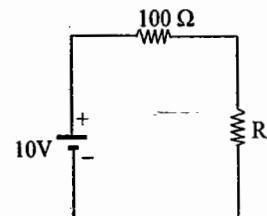


- (a) $e^{at} - e^{bt}$ (b) $e^{at} + e^{bt}$
(c) $ae^{at} - be^{bt}$ (d) $ae^{at} + be^{bt}$

02. A source of angular frequency 1 rad/sec has a source impedance consisting of 1Ω resistance in series with 1H inductance. The load that will obtain the maximum power transfer is GATE - 2003

- (a) 1Ω resistance
(b) 1Ω resistance in parallel with 1H inductance
(c) 1Ω resistance in series with 1F capacitor
(d) 1Ω resistance in parallel with 1 F capacitor.

03. The maximum power that can be transferred to the load resistor R_L from the voltage source in the figure is GATE-2005



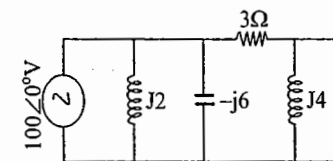
- (a) 1 W (b) 10 W
(c) 0.25 W (d) 0.5 W

04. An independent voltage source in series with an impedance $Z_s = R_s + jX_s$ delivers a maximum average power to a load impedance Z_L when

GATE - 2007

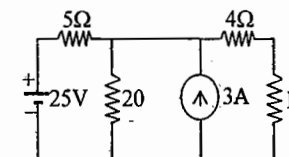
- (a) $Z_L = R_s + jX_s$
(b) $Z_L = R_s$
(c) $Z_L = jX_s$
(d) $Z_L = R_s - jX_s$

05. The Thevenin equivalent voltage V_{TH} appearing between the terminals A and B of the network shown in the figure is given by GATE - 1999



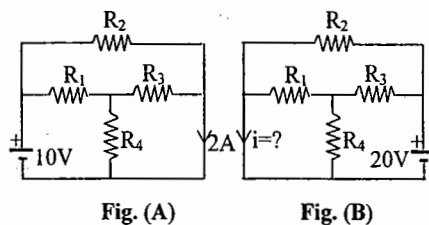
- (a) $j16(3-j4)$ (b) $j16(3+j4)$
(c) $16(3+j4)$ (d) $16(3-j4)$

06. The value of R (in ohms) required for maximum power transfer in the network shown in figure is GATE - 1999



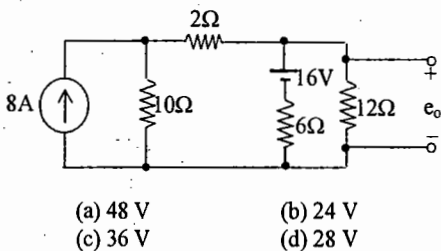
- (a) 2 (b) 4
(c) 8 (d) 16

07. Use the data of the figure (A). The current I in the circuit of the figure (B) is
GATE - 2000



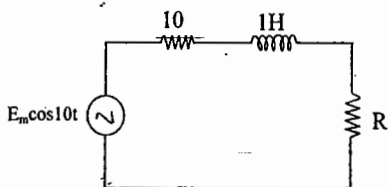
- (a) -2 A
(c) -4 A
(b) 2 A
(d) +4 A

08. The voltage e_o in the figure is
GATE - 2001



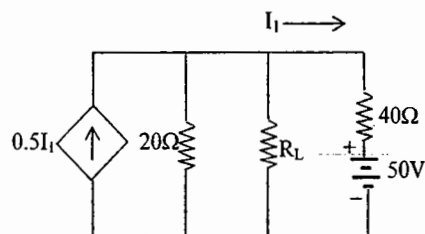
- (a) 48 V
(c) 36 V
(b) 24 V
(d) 28 V

09. In the figure, the value of the load resistor R which maximizes the power delivered to it is
GATE - 2001



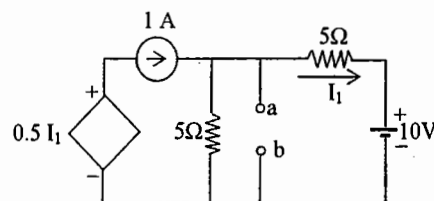
- (a) 14.14 Ω
(c) 200 Ω
(b) 10 Ω
(d) 28.28 Ω

10. In the network of the figure, the maximum power is delivered to R_L if its value is
GATE - 2002



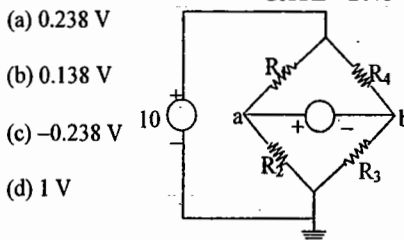
- (a) 16 Ω
(c) 60 Ω
(b) 40/3 Ω
(d) 20 Ω

11. For the circuit shown in the figure, Thevenin's voltage and Thevenin's equivalent resistance at terminals a-b is
GATE - 2005



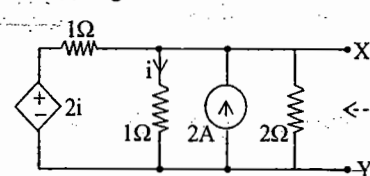
- (a) 5 V and 2 Ω
(c) 4 V and 2 Ω
(b) 7.5 V and 2.5 Ω
(d) 3 V and 2.5 Ω

12. If $R_1 = R_2 = R_4 = R$ and $R_3 = 1.1R$ in the bridge circuit shown in the figure, then the reading in the ideal voltmeter connected between a and b is
GATE - 2005



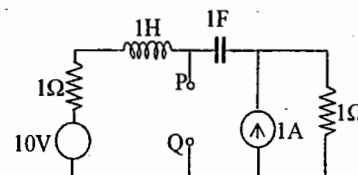
- (a) 0.238 V
(b) 0.138 V
(c) -0.238 V
(d) 1 V

13. For the circuit shown in the figure, the Thevenin voltage and resistance looking into X-Y are
GATE - 2007



- (a) 4/3 V, 2 Ω
(c) 4/3 V, 2/3 Ω
(b) 4 V, 2/3 Ω
(d) 4 V, 2 Ω

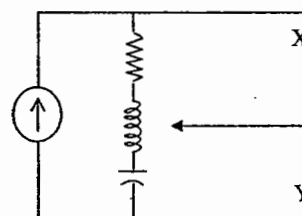
14. The Thevenin equivalent impedance Z_{TH} between the nodes P and Q in the following circuit is
GATE - 2008



- (a) 1
(b) $1 + s + \frac{1}{s}$

- (c) $2 + s + \frac{1}{s}$
(d) $\frac{s^2 + s + 1}{s^2 + 2s + 1}$

15. In the figure the current source is $1\angle 0^\circ$ A, $R = 1\Omega$, the impedances are $Z_c = -j\Omega$, and $Z_1 = 2j\Omega$. The Thevenin equivalent looking into the circuit across X-Y is

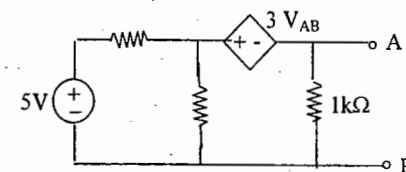


- (a) $\angle 0^\circ$ V, $(1+2j)\Omega$
(b) $2\angle 45^\circ$ V, $(1-2j)\Omega$
(c) $2\angle 45^\circ$ V, $(1+j)\Omega$

16. The Thevenin's equivalent of a circuit operating at $\omega = 5$ rad/s, has $V_{oc} = 3.71\angle -15.9^\circ$ V and $Z_0 = 2.38 - j0.667\Omega$. At this frequency, the minimal realization of the Thevenin's impedance will have a

- a) resistor and a capacitor and an inductor
b) resistor and a capacitor
c) resistor and an inductor
d) capacitor and an inductor

Statement for Linked Answer Questions 17 and 18:



17. For the circuit given above, the Thevenin's resistance across the terminals A and B is

- (A) 0.5 kΩ
(C) 1 kΩ
(B) 0.2 kΩ
(D) 0.11 kΩ

18. For the circuit given above, the Thevenin's voltage across the terminals A and B is

- (A) 1.25 V
(C) 1 V
(B) 0.25 V
(D) 0.5 V

KEY: SET C

- 01.d 02.c 03.c 04.d 05.a 06.c
07.d 08.d 09.a 10.a 11.b 12.c
13.d 14.d

Chapter 4: TRANSIENT RESPONSE

(Time domain analysis, Simple RLC networks,
Solution of network equations using Laplace transform)

Resistor: $v(t) = R i(t)$, (1)

Change in voltage at any instant 't' is instantaneously felt as a change in current at the same instant t.

$i(t) = \frac{1}{L} \int_{-\infty}^t v(t) dt$, **Inductor:** (2)

The current through an inductor at any instant t depends upon the past history of the voltage across it right from $-\infty$ time to the present time t. i.e. the continuous summation of voltage across it upto time t.

The current through an inductor cannot change instantaneously at any time unless infinite voltage (impulse) is applied across it.

$\therefore i(0^-) = i(0^+), i(t^-) = i(t^+) \text{ etc.,}$ (3)

Inductor opposes change in current through it. If there is a sudden jump in voltage across inductor, the current will not instantaneously change but changes fast or slow depending upon the resistance in the circuit. The rate of rise is measured by parameters like time constant, rise time, delay time and band width etc.,

$v(t) = \frac{1}{C} \int_{-\infty}^t i(t) dt$

Capacitor: (4)

The voltage across a capacitor cannot change instantaneously unless infinite current (impulse) is passed through it.

$\therefore v(0^-) = v(0^+), v(t^-) = v(t^+) \text{ etc.,}$ (5)

Capacitor opposes change in voltage across it.

If there is a sudden jump in current through capacitor, the voltage will not instantaneously change but changes fast or slow depending upon the resistance in the circuit. The rate of rise is measured by parameters like time constant, rise time, delay time and band width etc.,

1. D.C. Transients:

i) R-L circuit: In the R-L circuit shown in Fig.1
 $i(t) = V/R [1 - e^{-(R/L)t}]$ (6)

The plot of $i(t)$ with exponential rise is shown in Fig.2

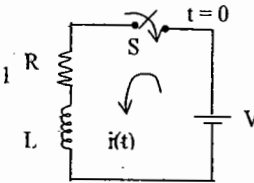


Fig. 1

$i(0^-) = i(0^+) = 0, i(\infty) = V/R = \text{Steady state current}$ (7)

i.e., L behaves as open circuit at $t = 0$ and short circuit as $t \rightarrow \infty$.
 $\tau = L/R$ is known as the time constant.

It is the time at which the exponent of e is unity in (6).

In one time constant $t = \tau$
 $(1 - e^{-1}) = 1 - 0.368 = 0.632$

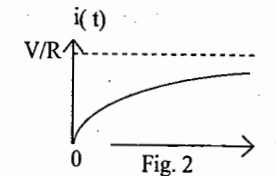


Fig. 2

At this time τ , current will reach 63% of its final value.

The voltage across inductance decays exponentially according to
 $v_L(t) = L \cdot di/dt = V e^{-(R/L)t}$ (8)

Time constant $\tau = L/R$ can also be defined as the time during which this voltage response, the exponential decay reaches 37% of the initial value.

Voltage across resistor increases exponentially according to
 $v_R(t) = V[1 - e^{-(R/L)t}]$ (9)

The exponential rise of resistor voltage and exponential decay of inductor voltage are shown in Fig. 3

Also, $v_R + v_L = V[1 - e^{-(R/L)t}] + V e^{-(R/L)t}$
 $= V$ (10)

Power in the circuit elements is given by
 $p_R = \frac{V^2}{R} [1 - 2e^{-(R/L)t} + e^{-2(R/L)t}]$ (11)

$p_L = \frac{V^2}{R} [e^{-(R/L)t} - e^{-2(R/L)t}]$ (12)

Total power,

$p = p_R + p_L$
 $= \frac{V^2}{R} [1 - e^{-(R/L)t}] = \frac{V^2}{R} \text{ as } t \rightarrow \infty$ (13)

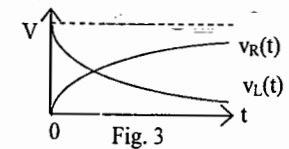


Fig. 3

ii) RC Circuit

In the R-C circuit shown in Fig. 4(a)

$$i(t) = \frac{V}{R} e^{-t/RC} \quad (14)$$

$i(t)$ decays exponentially as shown in Fig. 4(b)

Transient voltages across R and C are given by

$$v_R(t) = V e^{-t/RC} \quad (15)$$

$$v_C(t) = V(1 - e^{-t/RC}) \quad (16)$$

$$v_C(0_+) = v_C(0_-) = 0$$

$$v_C(\infty) = \text{steady state voltage} = V \quad (17)$$

i.e. C behaves as short circuit at $t = 0$ and open circuit as $t \rightarrow \infty$.

Also, the power in circuit elements is given by

$$P_R = V^2/R \cdot e^{-2t/RC} \quad (18)$$

$$P_C = V^2/R \cdot (e^{-t/RC} - e^{-2t/RC}) \quad (19)$$

Total power,

$$P = P_R + P_C = \frac{V^2}{R} e^{-t/RC} = \frac{V^2}{R} \text{ as } t \rightarrow 0 \quad (20)$$

The exponential rise of capacitor voltage and exponential decay of resistor voltage are shown in Fig. 5.

iii) R L C Circuit

In the R-L-C circuit shown in Fig. 6

$$Ri + L \frac{di}{dt} + \frac{1}{C} \int i dt = V \quad (21)$$

$$\text{Let } \alpha = \frac{R}{2L}, \quad \omega_0 = \frac{1}{\sqrt{LC}} \text{ and } Q = \frac{\omega_0}{2\alpha} \quad (22)$$

$$\alpha = \omega_0 \delta, \quad \delta = \text{damping ratio}$$

The response $i(t)$ depends upon the following cases:

CASE -1: Over Damped

$$\alpha > \omega_0 \text{ or } Q < \frac{1}{2}, \delta > 1$$

$$s_1 = -\alpha + \alpha_d, \quad s_2 = -\alpha - \alpha_d$$

$$\text{Where } \alpha_d = \sqrt{\alpha^2 - \omega_0^2} \quad (23)$$

In this case the current is given by

$$C_1 e^{s_1 t} + C_2 e^{s_2 t} \quad i(t) = \dots \quad (24)$$

Response is (not Oscillatory) as shown in Fig. 7.

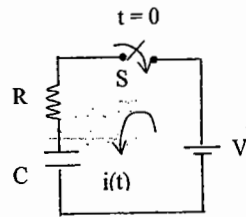


Fig. 4(a)

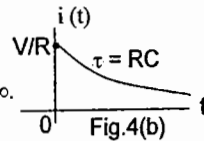


Fig. 4(b)

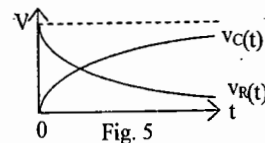


Fig. 5

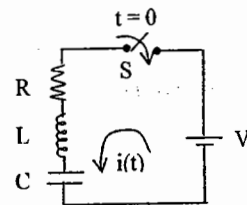


Fig. 6

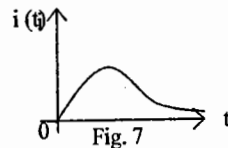


Fig. 7

CASE-2: (Critically damped)

$$\alpha = \omega_0 \text{ or } Q = \frac{1}{2}, \delta = 1$$

$$s_1 = s_2 = -\alpha$$

$$e^{s_1 t} (C_1 + C_2 t) \quad \text{Here, } i(t) = \quad (26)$$

Response is (not Oscillatory) as shown in Fig. 8

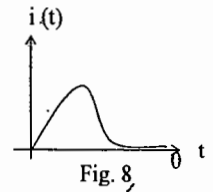


Fig. 8

CASE-3: Under damped

$$\alpha < \omega_0 \text{ or } Q > \frac{1}{2}, \delta < 1$$

$$s_1 = -\alpha + j\omega_d, \quad s_2 = -\alpha - j\omega_d$$

$$\text{Where } \omega_d = \sqrt{\omega_0^2 - \alpha^2} \quad (27)$$

$$C_1 e^{s_1 t} + C_2 e^{s_2 t} \quad i(t) =$$

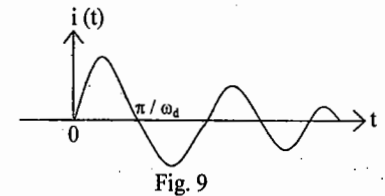


Fig. 9

Current is Oscillatory with damped oscillations with frequency ω_d as shown in Fig. 9

CASE-4: (Undamped (or) loss less)

$$\alpha = 0 \text{ or } Q = \text{infinity } (R=0), \delta = 0$$

$$s_1 = j\omega_0, \quad s_2 = -j\omega_0 \quad (28)$$

$$i(t) = C_1 \sin \omega_0 t + C_2 \cos \omega_0 t$$

$$= A \cos(\omega_0 t + \theta)$$

$$= B \sin(\omega_0 t + \phi)$$

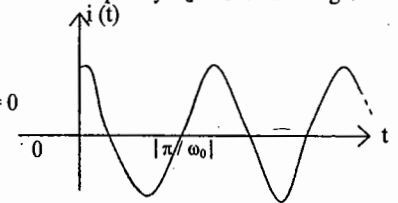


Fig. 10

Response is now steady or Sustained Oscillations with frequency ω_0 and constant amplitude as shown in Fig. 10.

The arbitrary constants C_1 & C_2 depend upon the initial state of the circuit i.e. initial voltage across the capacitor and initial current through the inductor prior to the application of the input to the circuit.

2) A.C Transients:

i) R-L Circuit (Fig. 11), $v(t) = V_m \sin(\omega t + \phi)$

$$Ri + L \frac{di}{dt} = V_m \sin(\omega t + \phi) \quad (29)$$

$$i(t) = k e^{-(R/L)t} + (V_m / D) \sin[\omega t + \phi - \tan^{-1}(\omega L/R)] \quad (30)$$

$$\text{where } D = \sqrt{R^2 + \omega^2 L^2}$$

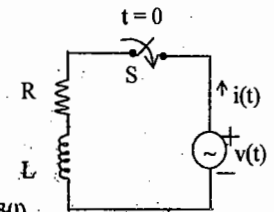


Fig. 11

It may be noted that

⇒ The first part of the above equation contains the factor $e^{-(R/L)t}$ which has a value of nearly zero in a relatively short time, depending upon how small the time constant $\tau = L/R$ is.

⇒ The second part of the above equation is the steady state current which lags the applied voltage by $\tan^{-1}(\omega L/R)$.

The transient disappears when $k = 0$, $\phi = \tan^{-1}(\omega L/R)$ (31)

ii) R-C Circuit (Fig. 12)

$$v(t) = V_m \sin(\omega t + \phi)$$

$$Ri + 1/C \int i dt = V_m \sin(\omega t + \phi) \quad (32)$$

$$i(t) = ke^{-(t/RC)} + (V_m/D) \sin[\omega t + \phi + \tan^{-1}(1/\omega CR)] \quad (33)$$

$$\text{Where } D = \sqrt{R^2 + (1/\omega C)^2}$$

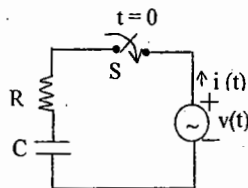


Fig. 12

It may be noted that:

⇒ The first part of the above equation is the transient with decay factor $e^{-t/RC}$

⇒ The second part is the steady current which leads the applied voltage by $\tan^{-1}(1/\omega CR)$.

The transient disappears when $k = 0$, $\phi = -\tan^{-1}(1/\omega CR)$ (34)

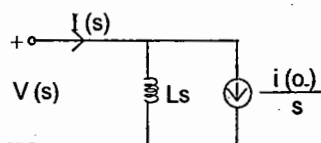
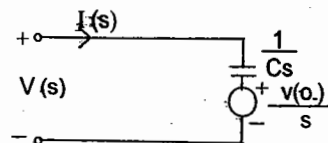
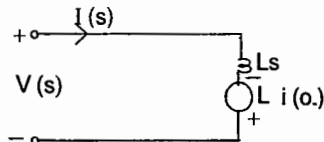
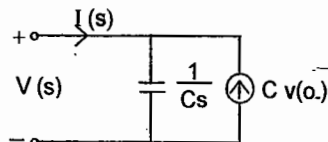
L-Transform Equivalent (s-domain) ckt. for L and C with I.C's

$$i(t) = C \frac{d}{dt} v(t)$$

$$v(t) = L \frac{d}{dt} i(t)$$

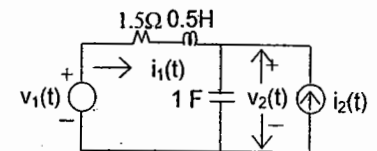
$$I(s) = C[sV(s) - v(o.)]$$

$$V(s) = L[sI(s) - i(o.)]$$



Example:

Using the above equivalents for the circuit shown, verify that



$$V_2(s) = \frac{2}{F(s)} V_1(s) + \frac{s+3}{F(s)} I_2(s) + \frac{s+3}{F(s)} v_2(o.) + \frac{1}{F(s)} i_1(o.), F(s) = s^2 + 3s + 2$$

The contribution due to each source and each I.C is clearly identifiable.

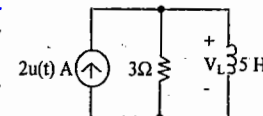
Find ZSR (zero state response) and ZIR (zero input response) in the time domain for

$v_1(t) = u(t)$, $i_2(t) = u(t)$, $v_2(o.) = 1V$, $i_1(o.) = 1A$ by using the above result. Also identify the Natural response and Forced response.

SET - A Objective Questions

01. In the circuit shown, the voltage across the inductor V_L at $t=0^+$ is given by

- (a) 6V
(b) 12V
(c) 15V
(d) -6V

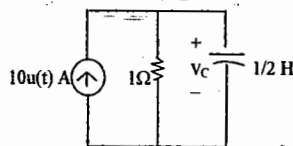


04. In a circuit the transform of current $i(t)$ is $I(s) = 1/(s^2 + 5s + 6)$. The expression of $i(t)$ is

- (a) $(e^{-2t} + e^{-3t})u(t)$
(b) $(e^{-2t} - e^{-3t})u(t)$
(c) $(-e^{-2t} + e^{-3t})u(t)$
(d) $(e^{2t} - e^{3t})u(t)$

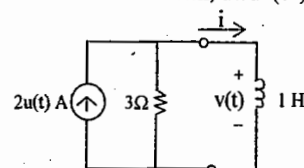
02. The steady state value of v_C in the circuit shown when $v(0^-) = 2$ volts is

- (a) 8V
(b) -8V
(c) $10e^{-2t}$ V
(d) -10V

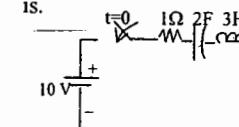


03. In the circuit shown, $dv/dt(0^+)$ is

- (a) 10V
(b) 0V
(c) Infinity
(d) 5V



05. With the switch closed the circuit shown below is in steady state. The switch is opened at $t = 0^+$. The voltage across the capacitor at $t = 0^+$ is.



- (a) $(s+4) / [(s+4)^2 + 4]$
(b) $2 / [(s+4)^2 + 4]$
(c) $s / (s^2 + 4)$
(d) $(s-4) / [(s-4)^2 + 4]$

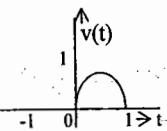
06. The Laplace transform of $v(t) = e^{-4t} \sin 2t u(t)$ is

07. $\int_{-10}^{10} (t+5) \delta(t) dt$ equals

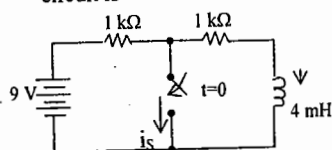
- (a) 1 (b) 0 (c) 5 (d) 20

08. The sinusoidal function shown can be expressed as

- (a) $\sin \pi t [u(t) + u(t-1)]$
 (b) $\sin \pi t [u(t) + u(t+1)]$
 (c) $\sin \pi t [u(t) - u(t-1)]$
 (d) $\sin 2\pi t [u(t) - u(t-1)]$



09. The switch in the circuit shown is closed at $t=0$. The time constant of the circuit is



- (a) 2 μ sec (b) 9 μ sec
 (c) 4 μ sec (d) 1 μ sec

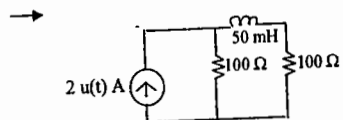
10. Across a series RC circuit

containing $R = 5\Omega$ and

$C = 0.1$ F, a D.C. voltage of

5 V is suddenly applied at $t=0$. Then the current drawn from the source at

$t=0^+$ is



- (a) 0.5 A (b) zero
 (c) 1 A (d) 5 A

11. The Laplace transform of first derivative of a function $f(t)$ is

- (a) $F(s)/s$ (b) $sF(s) - f(0)$
 (c) $F(s) - f(0)$ (d) $f(0)$

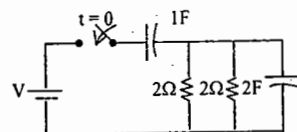
12. The transform of current in a circuit is given by $I(s) = 2s / (s^2 + 16)$. The time domain expression for current is

- (a) $\frac{1}{4} \sin 4t$ amps
 (b) $\frac{1}{4} \times 1 / (s^2 + 16)$ amps
 (c) $2 \cos 4t$ amps
 (d) $\sin t$ amps

13. The Laplace transform of $v(t) = [\sin 2t / t] u(t)$ is

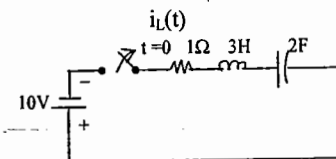
- (a) $\pi/2 - \tan^{-1} s/2$
 (b) $\pi/2 + \tan^{-1} s/2$
 (c) $\tan^{-1} s/2$
 (d) $s/2$

14. In the circuit shown, the switch is closed at $t=0$. The circuit can best be analyzed by



- (a) Finding forced and natural responses
 (b) Finding zero input and zero state responses
 (c) Using conventional methods
 (d) Using Laplace transforms

15. In the circuit below, $i_L(0^+)$ is

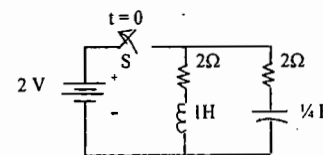


- (a) 0 A (b) 1 A
 (c) -1 A (d) 2 A

16. A unit current step is applied to a parallel RLC circuit. Under steady state, the entire current flows through

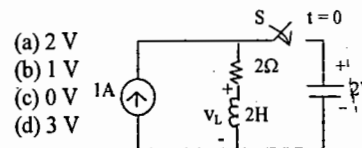
- (a) R only (b) L only
 (c) C only (d) R and L only

17. With the switch 'S' in closed position the circuit is in steady state. The current in the inductor after opening the switch 'S' is



- (a) Under damped
 (b) Critically damped
 (c) Over damped
 (d) Undamped

18. S is open for a long time and steady state is reached. S is closed $t=0$, then $v_L(0^+)$ is

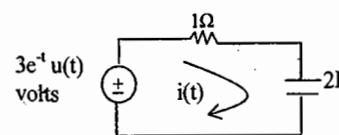


- (a) 2 V
 (b) 1 V
 (c) 0 V
 (d) 3 V

19. The Laplace transform of integral of function $f(t)$ is

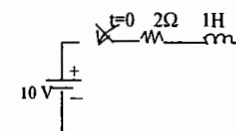
- (a) $(1/s) F(s)$ (b) $sF(s) - f(0)$
 (c) $F(s) - f(0)$ (d) $f(0)$

20. In the circuit shown, the values of $i(0^+)$ and $i(\infty)$ are respectively



- (a) zero and 1.5 A
 (b) 1.5 A and 3 A
 (c) 3 A and zero
 (d) 3 A and 1.5 A

21. With initial conditions zero in the network, the switch is closed at $t=0$. The current drawn from the source when t tends to infinity is



- (a) 0 A (b) 5 A
 (c) 2 A (d) 0 A

22. $d u(t)/dt$ equals

- (a) A ramp function
 (b) An impulse function
 (c) A parabolic function
 (d) An exponential function

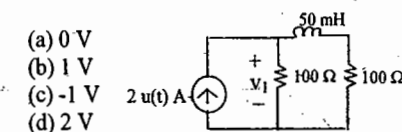
23. When a unit step voltage $u(t)$ is applied to an ideal inductor, the current through the inductor will

- (a) Be zero for all times
 (b) Increase linearly
 (c) Be infinite
 (d) Be constant

24. Zero input response of a circuit is

- (a) The response when time $t=0$
 (b) The response with initial conditions zero
 (c) The response when the input is zero
 (d) The response when transients become zero

25. In the circuit shown below, $v_L(0^+)$ is



- (a) 0 V
 (b) 1 V
 (c) -1 V
 (d) 2 V

26. A unit step voltage is applied across an inductor. The power associated with the inductor will be

- (a) Zero for all time
 (b) A step function
 (c) An exponentially decaying function
 (d) A ramp function

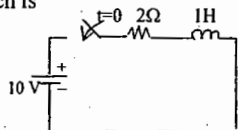
27. When a series RL circuit is connected to a dc voltage V at $t=0$, the current passing through the inductor L at $t=0^+$ is

- (a) V/R amps (b) Infinite
(c) Zero (d) V/L amps

28. When a series RC circuit is connected to a constant voltage V volts at $t=0$, the current passing through the circuit at $t=0^+$ is

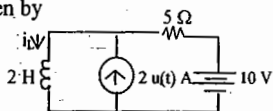
- (a) Infinite (b) Zero
(c) V/R amps (d) $V/\omega C$ amps

29. With initial conditions zero in the network shown, the switch is closed at $t=0$. The voltage across resistance immediately after the closure of the switch is



- (a) 10 V (b) 5 V
(c) 2 V (d) 0 V

30. In the circuit shown below $i_L(0^+)$ is given by



- (a) 1 A (b) 2 A
(c) 3 A (d) 4 A

31. Zero state response of a circuit is

- (a) The response when time $t=0$
(b) The response when transients become zero
(c) The response when the input is zero
(d) The response with initial conditions zero

32. A step current I is applied to a parallel RC circuit. Immediately after the step is applied, the voltage across the capacitor will be

- (a) Zero (b) Infinity
(c) Unity (d) IR

33. The time constant of a series RL circuit is

- (a) LR (b) L/R
(c) R/L (d) $e^{-R/L}$

34. The inverse Laplace transform of $1/s(1-e^{-as})$ is

- (a) $u(t) - u(t-a)$ (b) $u(t)$
(c) $u(t-a)$ (d) Zero

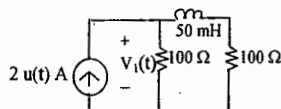
35. The time constant of a series RC circuit is

- (a) $1/RC$ (b) R/C
(c) RC (d) $e^{-R/C}$

36. In over damped circuits,

- (a) The response rises or falls very fast
(b) The response falls very slow
(c) The response is a damped sinusoid
(d) The response is oscillatory

37. In the circuit shown below, $v_1(0^+)$ is



- (a) 100 V (b) 150 V
(c) -100 V (d) 200 V

38. A unit step voltage is applied across a series LC circuit and is removed after some time. Under steady state, circuit current

- (a) Is zero (b) Is constant
(c) Increases Linearly
(d) Is oscillatory

39. The time constant of a circuit having $R=5\Omega$, $L=0.2$ H is

- (a) 1 sec (b) 2.5 sec
(c) 2 sec (d) 0.04 sec

40. The inverse Laplace transform of $6/s^4$ is

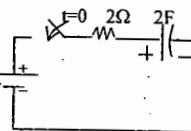
- (a) $3 u(t)$ (b) $t^2 u(t)$
(c) $t^3 u(t)$ (d) $3t u(t)$

41. A series RC circuit has $R=5\Omega$ and $C=2.5 \mu F$. The time constant of the circuit is

- (a) 12.5 μ sec (b) 2 μ sec
(c) 0.5 μ sec (d) $1/12.5 \mu$ sec

42. The initial conditions in the network with switch open are zero. The switch is closed at $t=0$. Voltage across the capacitor when $t \rightarrow \infty$ is

- (a) 0 V (b) 5 V
(c) 10 V (d) 7.5 V



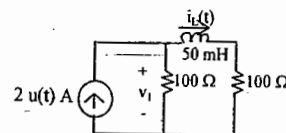
43. $\int_0^t u(t) dt$ equals

- (a) A step function
(b) An impulse function
(c) A doublet function
(d) A ramp function

44. In underdamped circuits,

- (a) The response rises or falls very fast
(b) The response rises or falls very slow
(c) The response is a damped sinusoid
(d) The response is oscillatory

45. In the circuit shown below, $i_L(\infty)$ is



- (a) 0 A (b) 1 A
(c) -1.2 A (d) 2 A

46. A unit step current source is applied to a parallel RLC circuit. Under steady state condition, the input current will

(a) Get shared by all the three components
(b) Get shared between R and C only
(c) Get shared between R and L only
(d) Flow through L only

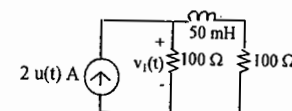
47. $\int_0^1 \delta(t) dt$ equals

- (a) A step function
(b) An impulse function
(c) A doublet function
(d) A ramp function

48. In undamped circuits,

- (a) The response rises or falls very fast
(b) The response falls very slow
(c) The response is a damped sinusoid
(d) The response is oscillatory

49. In the circuit shown below, $v_1(\infty)$ is



- (a) 100 V (b) 150 V
(c) -100 V (d) 200 V

50. The current in a pure inductor with a unit step input voltage is

- (a) Zero
(b) Constant but non-zero
(c) A decaying exponential
(d) A ramp function

51. A series RL circuit is initially relaxed. A step voltage is applied to the circuit. If τ is the time constant of the circuit, the voltages across R and L will be same at time t equal to

- (a) $-\tau \ln 2$ (b) $1/\tau \ln(1/2)$
(c) $-\tau \ln(1/2)$ (d) $-1/\tau \ln 2$

52. The function is said to be periodic when it

- (a) Appears for a particular time interval
(b) Appears for all time
(c) Is of recurring nature
(d) Has only linearly varying components.

53. $\int_0^t r(t) dt$ equals

- (a) A step function
(b) An impulse function
(c) A doublet function
(d) A parabolic function

54. In a series circuit consisting of a single resistor, a single inductor and a single capacitor, if the resistance is more, the response is

- (a) Overdamped (b) Undamped
(c) Oscillatory (d) Underdamped

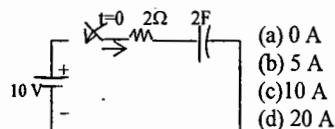
55. The inverse Laplace transform of $1/s$ is

- (a) $\delta(t)$ (b) $u(t)$
(c) $u(t-a)$ (d) t

56. The steady state value of current in a series RLC circuit, when a D.C. voltage of V volts is applied to it at $t = 0$, is

- (a) zero (b) $V/(RL)$
(c) V/R (d) $(VC)/R$

57. The initial conditions in the network, with switch open, are zero. The switch is closed at $t = 0$. The current drawn from the source at $t = 0^+$ is



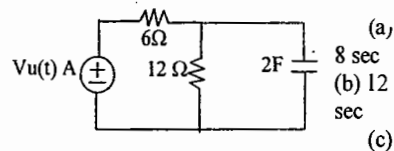
58. $r(t)$ equals

- (a) $t u(t)$ (b) $1/t u(t)$
(c) $t^2 u(t)$ (d) $u(t)$

59. In a parallel circuit consisting of a single resistor, a single inductor and a single capacitor, if the resistance is more, the response is

- (a) Over damped (b) Undamped
(c) Oscillatory (d) Under damped

60. The time constant of the network shown below is

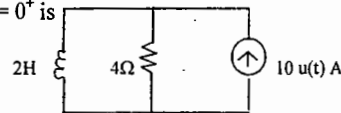


24 sec (d) 36 sec

61. The Laplace transform of ramp function is

- (a) 1 (b) $1/s$ (c) $1/s^2$ (d) $1/s^3$

62. In the circuit, current in 4Ω resistor at $t = 0^+$ is

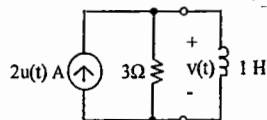


- (a) 10 A (b) 5 A (c) 2.5 A (d) 0 A

63. $\delta(at)$ equals

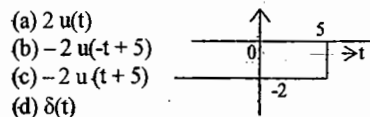
- (a) $a \delta(t)$ (b) $(1/a) \delta(t)$
(c) $a^2 \delta(t)$ (d) $u(t)$

64. In a parallel circuit consisting of a single resistor R , a single inductor L and a single capacitor C , the response is critically damped if



- (a) $R = (1/2) \sqrt{L/C}$
(b) $R < (1/2) \sqrt{L/C}$
(c) $R > (1/2) \sqrt{L/C}$
(d) $R = 0$

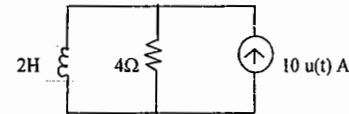
65. The singular function in the diagram can be mathematically expressed as



66. The inverse Laplace transform of ' s ' is

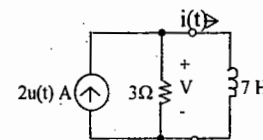
- (a) Impulse (b) Ramp
(c) Step (d) Unit doublet

67. The time constant of the circuit shown is



- (a) 2 sec (b) $1/2$ sec
(c) 4 sec (d) $1/4$ sec

68. In the circuit shown $i(0^+)$ is

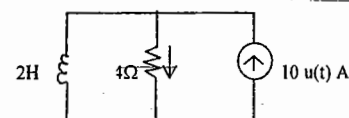


- (a) 0 A (b) 1
(c) 2 A (d) 3 A

69. The derivative of a step function is

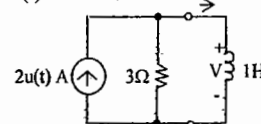
- (a) zero (b) an impulse function
(c) a ramp function (d) a constant

70. In the circuit shown the power supplied by the source in the steady state is



- (a) 40 W (b) 10 W
(c) 5 W (d) 0 W

71. In the circuit shown $v(0^+)$ is



- (a) 1 V (b) 2 V
(c) 4 V (d) 6 V

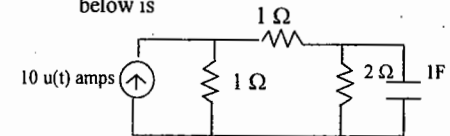
72. The double integration of a unit step function will result in

- (a) a unit step function
(b) ramp function
(c) impulse
(d) parabola

73. The Laplace transform of $f(t) = e^{-\alpha t}$ is

- (a) $1/(s-\alpha)$ (b) $1/(s+\alpha)$
(c) $1/s$ (d) $(1/s^2) + \alpha$

74. The time constant of the circuit shown below is



- (a) $2/3$ sec (b) 1 sec
(c) 2 sec (d) 3 sec

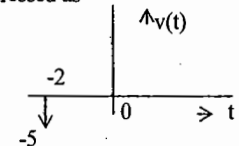
75. $\int_{-\infty}^{\infty} \delta(t) dt$ equals

- (a) 1 (b) 0 (c) ∞ (d) 0.5

76. A ramp current flowing through an initially relaxed capacitor will result in a voltage across it that

- (a) Varies inversely with time
(b) Remains constant
(c) Varies directly with time twice
(d) Varies as the square of time

77. The singular function $v(t)$ in the diagram shown can be mathematically expressed as



- (a) $5 \delta(t+2)$ (b) $5 \delta(t-2)$
(c) $-2 u(t)$ (d) $-5 \delta(t+2)$

78. $\int_{t=-10}^0 \delta(t+5) dt$ equals

- (a) 0 (b) ∞
(c) 0.5 (d) 1

79. If the step response of an initially relaxed circuit is known, then the ramp response can be obtained by

- (a) integrating the step response
(b) Differentiating the step response
(c) Integrating the step response twice
(d) Differentiating the step response twice

80. $\int_{t=-10}^1 2t \delta(t) dt$ equals

- (a) 0 (b) 1 (c) 0.5 (d) ∞

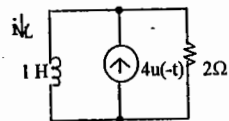
81. The Laplace transform of $\sin 2t u(t)$ is

- (a) $1/(s^2 + 4)$ (b) $s/(s^2 + 4)$
(c) $2/(s^2 + 4)$ (d) $2s/(s^2 + 4)$

82. Laplace transform of a function $f(t) = 5t u(t)$ is

- (a) $1/5s^2$ (b) $s^2/5$
(c) $5/s^2$ (d) $5/s^2$

83. In the circuit, the current flowing through inductor at $t=0^+$ is



- (a) 0 A (b) 2 A
(c) 4 A (d) Infinity

84. $\int_{t=0}^{20} t \delta(t-10) dt$ equals

- (a) 5 (b) 10 (c) 20 (d) 2.5

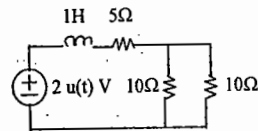
85. In a circuit the response term that tends to zero, when time approaches infinity, is

- (a) Transient response
(b) Steady state response
(c) Forced response
(d) Zero state response

86. The unit step response of a system is given by $(1 - e^{-\alpha t}) u(t)$. Its impulse response is:

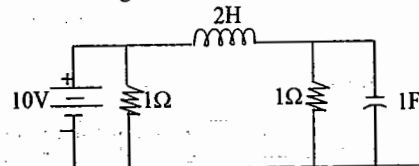
- (a) $e^{-\alpha t} u(t)$ (b) $\alpha e^{-\alpha t} u(t)$
(c) $(1/\alpha) e^{-\alpha t} u(t)$ (d) $-\alpha e^{-\alpha t} u(t)$

87. The time constant of the circuit is



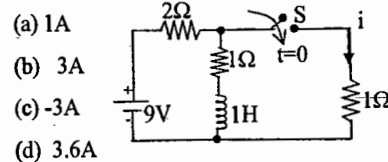
- (a) 0.2 sec (b) 5 sec
(c) 0.1 sec (d) 0.05 sec

88. In Figure., at steady state, the current through the inductor is



- (a) 0 A (b) 10 A
(c) 5 A (d) 20 A

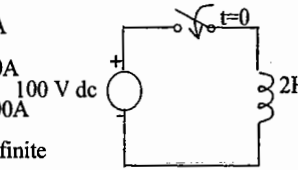
89. The circuit shown in Fig., has reached steady state with the switch S open. The switch is closed at $t=0$. The current i through the switch at $t=0^+$ is



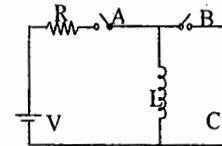
- (a) 1 A (b) 3 A
(c) -3 A (d) 3.6 A

90. In the circuit shown in Fig., the switch is closed at $t=0$ sec. The current flowing in the circuit at $t=2$ sec is

- (a) 0 A
(b) 50 A
(c) 100 A
(d) Infinite

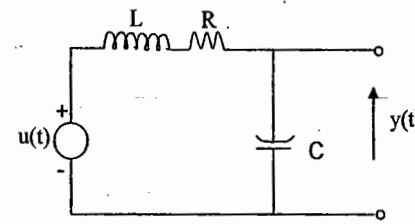


91. For the circuit shown in Fig., the switch A has been in closed position for a long time. If at $t=0$ switch B is closed and switch A is simultaneously opened, the current in the LC circuit in the steady state will be:



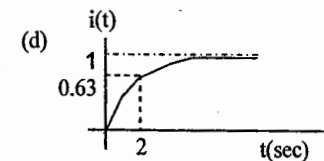
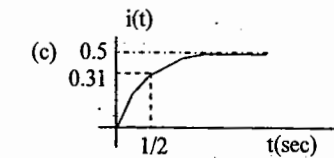
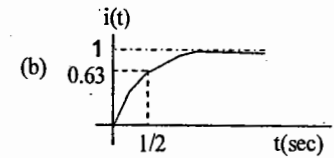
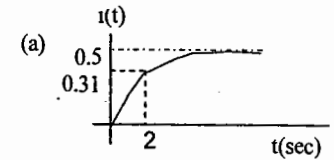
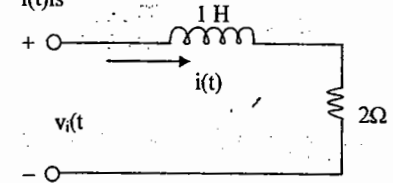
- (a) zero (b) V/R
(c) Sinusoidal (d) infinite

92. The condition on R , L and C such that the step response $y(t)$ in Fig has no oscillations, is



- (a) $R < \frac{1}{2} \sqrt{\frac{L}{C}}$ (b) $R \geq \sqrt{\frac{L}{C}}$
(c) $R \geq 2 \sqrt{\frac{L}{C}}$ (d) $R = \frac{1}{\sqrt{LC}}$

93. For the R-L circuit shown in fig., the input voltage $v_i(t) = u(t)$. The current $i(t)$ is



94. The transfer function $H(s) = \frac{V_0(s)}{V_i(s)}$ of a series R-L-C circuit is given by

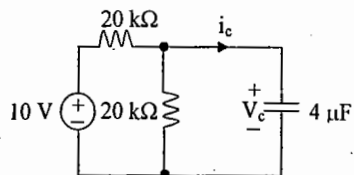
$$H(s) = \frac{10^6}{s^2 + 20s + 10^6} \text{ where } V_0(s) \text{ is}$$

the voltage across the capacitor. The

Quality factor (Q-factor) of this circuit is

- (a) 25 (b) 50
(c) 100 (d) 5000

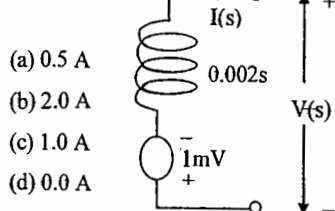
95. In the circuit shown, V_c is 0 volts at $t = 0$ sec. For $t > 0$, the capacitor current $i_c(t)$, where t is in seconds, is given by



- (a) $0.50 \exp(-25t)$ mA
(b) $0.25 \exp(-25t)$ mA
(c) $0.50 \exp(-12.5t)$ mA
(d) $0.25 \exp(-6.25t)$ mA

96. A 2 mH inductor with some initial current can be represented as shown below, where s is the Laplace

Transform variable. The value of initial current is



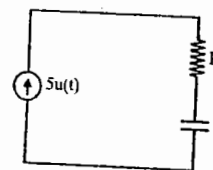
- (a) 0.5 A
(b) 2.0 A
(c) 1.0 A
(d) 0.0 A

Key set - A:

1. a 2. d 3. a 4. b 5. a 6. b
7. c 8. c 9. c 10. c 11. b 12. c
13. a 14. d 15. a 16. b 17. b 18. b
19. a 20. c 21. b 22. b 23. b 24. c
25. a 26. d 27. c 28. c 29. d 30. b
31. b 32. a 33. b 34. a 35. c 36. a
37. d 38. d 39. d 40. c 41. a 42. c
43. d 44. b 45. b 46. d 47. a 48. d
49. a 50. d 51. c 52. a 53. d 54. a
55. b 56. a 57. c 58. a 59. d 60. a
61. c 62. a 63. b 64. a 65. b 66. d
67. b 68. a 69. b 70. d 71. d 72. d
73. b 74. b 75. a 76. d 77. d 78. d
79. a 80. a 81. c 82. d 83. c 84. b
85. a 86. b 87. c 88. b 89. a 90. c
91. c 92. c 93. B 94. B 95. A 96. A

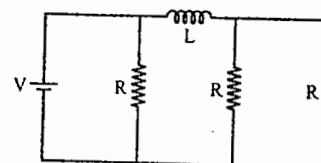
SET - B

01. Voltage across capacitor as a function of time will be



- (A) $(1 - e^{-t/RC}) \times 5R$ (B) $5R e^{-t/RC}$
(C) $5t / C$ (D) $t / 5C$

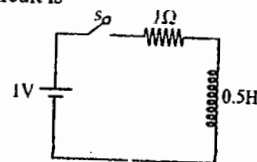
02. Consider the network given below



The time constant of the circuit is

- (A) $\frac{L}{2R}$ (B) $\frac{2L}{R}$
(C) $\frac{L}{3R}$ (D) $\frac{3L}{R}$

03. Steady state value of the current in the circuit is



- (A) 0 (B) $\frac{1}{2}$ (C) 2 (D) 1

04. The capacitor charging current is

- (A) an exponential growth function
(B) an exponential decay function
(C) a linear decay function
(D) a linear rise function

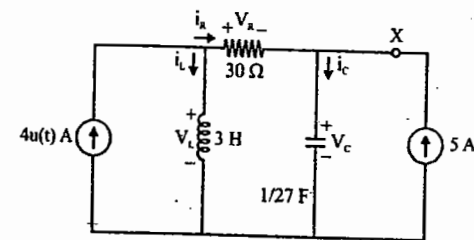
05. The current through a series R-L circuit is $\frac{1}{4}e^{t/2}$, when excited by a unit impulse voltage. The values of R and L are respectively

- (A) 8, 4 (B) 4, 2
(C) 2, 4 (D) 1, 4

06. If a unit step current is passed through a capacitor the voltage across the capacitor will be

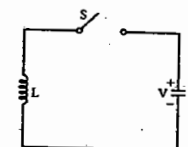
- (A) 0 (B) a step function
(C) a ramp function
(D) An impulse function

07. RLC circuit is as shown below. Assume the circuit is to have been in this state forever, then $\frac{dv_c(0^+)}{dt}$ is



- (A) $\frac{3}{27}$ V/S (B) $\frac{4}{27}$ V/S
(C) 108 V/S (D) 81 V/S

08. In the figure initial voltage on C is V_0 . S is closed at $t = 0$. Determine i_L for $t > 0$



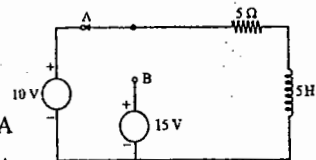
- (A) $-\omega_0 C V_0 \sin \omega_0 t$
(B) $\omega_0 V_0 \sin \omega_0 t$
(C) $-\omega_0 V_0 \sin \omega_0 t$
(D) $\omega_0 C V_0 \sin \omega_0 t$

09. A dc voltage source is connected to a series R - C circuit. When the steady state reaches, the ratio of the energy stored in the capacitor to the total energy supplied by the voltage source is equal to

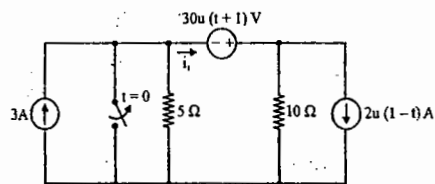
(A) 0.362 (B) 0.500
(C) 0.632 (D) 1.000

10. The circuit is in steady state with switch in position A. At $t = 0$, the switch is moved to position B. The current in the circuit for $t > 0$ is

(A) 2 A
(B) 3 A
(C) $5 + e^{-t}$ A
(D) $3 - e^{-t}$ A



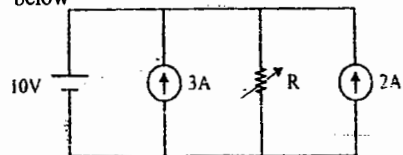
11. For the circuit shown below



i_1 at $t = -2$ s is

(A) $\frac{2}{3}$ A (B) $\frac{4}{3}$ A
(C) 2 A (D) $\frac{8}{3}$ A

12. Consider the electrical network shown below



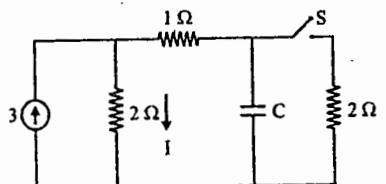
The value of R so that current through it is zero is

(A) 2 Ω (B) 5 Ω
(C) 4 Ω (D) 3 Ω

13. $\sqrt{\frac{L}{C}}$ has the dimension of

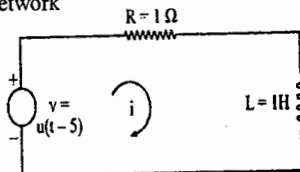
(A) time (B) capacitance
(C) inductance (D) resistance

14. Steady state is reached with S open. S is closed at $t = 0$. The current marked I at $t = 0^+$ is given by



(A) 3.0 (B) 3.8 (C) 4.0 (D) 1.8

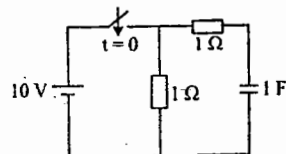
15. A unit step $u(t-5)$ is applied to RL network



The current is given by

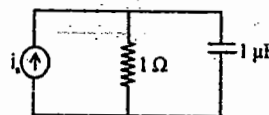
(A) $1 - e^{-t}$
(B) $[1 - e^{-(t-5)}] u(t-5)$
(C) $(1 - e^{-t}) u(t-5)$
(D) $1 - e^{-(t-5)}$

16. In the circuit shown in the given figure, the switch is closed at $t = 0$. The current through the capacitor will decrease exponentially with a time constant of



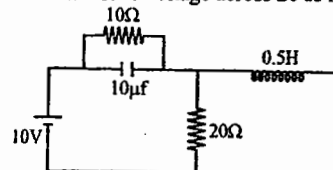
(A) 0.5 s (B) 1 s
(C) 2 s (D) 10 s

17. In the circuit shown in the figure below, if $i_s = u(t)$ A, then what are the initial and steady - state voltages across the capacitor?



(A) 1 V and 1 V, respectively
(B) 1 V and 0, respectively
(C) 0 and 1 V, respectively
(D) 0 and 0, respectively

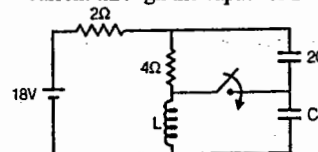
18. At time $t = 0^+$ with zero initial conditions voltage across 20 Ω is



(A) 0 (B) 10 V (C) 2 V
(D) none of these

19. In the circuit shown in the figure below, steady - state was reached when

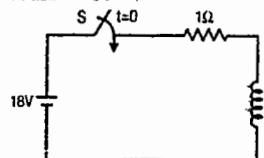
the switch S was open. The switch was closed at $t = 0$. The initial value of the current through the capacitor 2 C is



(A) zero (B) 1 A (C) 2 A (D) 3 A

20. In the circuit below, S was initially open. At time $t = 0$, S is closed. When the current through the inductor is 6 A, the rate of change of current through the resistor is 6 A/s. The value of the inductor would be

(A) 1 H
(B) 2 H
(C) 3 H
(D) 4 H



21. If a unit step current is passed through a capacitor what will be the voltage across the capacitor?

(A) 0
(B) a step function
(C) A ramp function
(D) An impulse function

22. Consider the following statements:

If a network has an impedance of $(1 - j)$ at a specific frequency, the circuit would consist of series

1. R and C 2. R and L

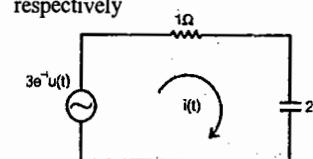
3. R, L and C

Which of these statements are correct?

(A) 1 and 2 (B) 1 and 3
(C) 1, 2 and 3 (D) 2 and 3

23. In the circuit shown in the given figure,

the values of $i(0^+)$ and $I(\infty)$ will be, respectively



(A) zero and 1.5 A
(B) 1.5 A and 3 A
(C) 3 A and zero
(D) 3 A and 1.5 A

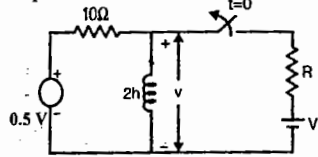
24. Consider the following statements associated with the superposition theorem:

1. It is applicable to d.c. circuits only
2. It can be used to determine the current in a branch or voltage across a branch
3. It is applicable to networks consisting of more than one source
4. It is applicable to networks consisting of linear and bilateral elements

Which of these statements are correct?

(A) 1, 2 and 3 (B) 2, 3 and 4
(C) 3, 4 and 1 (D) 4, 1 and 2

25. The time constant of the circuit after the switch shown in the figure is opened is



- (A) 0.2 s (B) 5 s (C) 0.1 s
(D) dependent of R and hence cannot be determined unless R is known.

26. Match List - I with List - II and select the correct answer:

List - I

P. A series RLC circuit is overdamped when

Q. The unit of the real part of the complex frequency is

R. If $F(s)$ is the Laplace transform of $f(t)$ then $F(s)$ and $f(t)$ are known as

S. If $f(t)$ and its first derivative are Laplace transformable then initial value of $f(t)$ is given by

List - II

1. $f(t) = sF(s) \lim_{t \rightarrow 0} \lim_{s \rightarrow \infty}$

2. $R^2/4L^2 < 1/LC$

3. rad/s

4. inverse functions

5. $R^2/4L^2 > 1/LC$

6. neper sec^{-1}

7. $\lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} sF(s)$

8. transform pairs

Codes:

P	Q	R	S
(A) 5	6	8	1
(B) 5	6	1	8
(C) 6	5	3	4
(D) 6	5	2	7

27. The maximum power that a 12 V d.c. source with an internal resistance of 2Ω can supply to a resistive load is

- (A) 12 W (B) 18 W
(C) 36 W (D) 48 W

28. Match List - I (Quantities) with List - II (units) and select the correct answer

List - I

P. R/L

Q. $1/LC$

R. CR

S. \sqrt{LC}

Codes:

P	Q	R	S
(A) 4	3	1	2
(B) 3	4	2	1
(C) 4	3	2	1
(D) 3	4	1	2

List - II

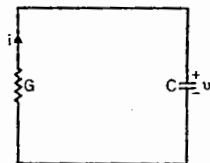
1. Second

2. Ohm

3. (Radian/Second)²

4. (Second)⁻¹

29. Consider the following network



Which one of the following is the

differential equation for 'v' in the above

network?

(A) $C \frac{dv}{dt} + Gv = 0$ (B) $G \frac{dv}{dt} + Cv = 0$

(C) $C \frac{dv}{dt} + Gv = 0$ (D) $C \frac{dv}{dt} - Gv = 0$

Key for Set B:

01.C 02.B 03.D 04.B 05.C

06.C 07.C 08.D 09.B 10.D

11.C 12.A 13.D 14.A 15.C

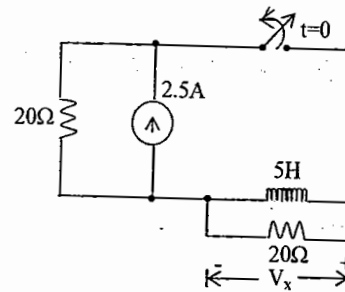
16.B 17.C 18.B 19.C 20.B

21.C 22.B 23.C 24.B 25.A

26.A 27.B 28.A 29.C

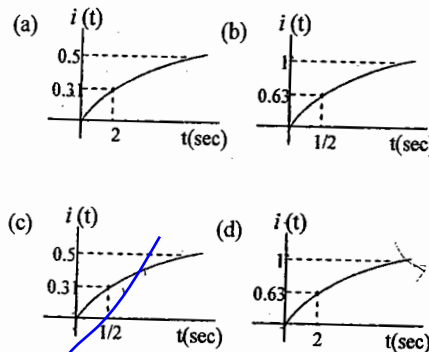
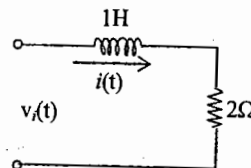
SET - C

01. In the figure, the switch was closed for a long time before opening at $t=0$. the voltage V_x at $t=0^+$ is **GATE - 2002**

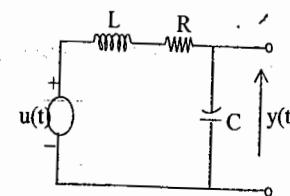


- (a) 25 V (b) 50 V
(c) -50 V (d) 0 V

02. For the R-L circuit shown in the figure, the input voltage $v_i(t) = u(t)$. The current $i(t)$ is **GATE - 2004**

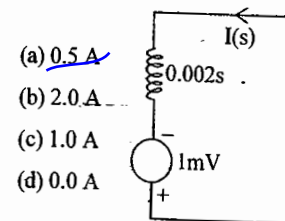


03. The condition on R, L and C such that the step response $y(t)$ in the figure has no oscillations, is **GATE - 2005**



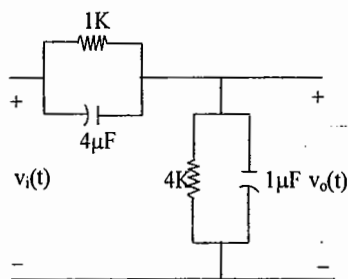
- (a) $R \geq \frac{1}{2} \sqrt{\frac{L}{C}}$
(b) $R \geq \sqrt{\frac{L}{C}}$
(c) $R \geq 2 \sqrt{\frac{L}{C}}$
(d) $R = \frac{1}{\sqrt{LC}}$

04. A 2 mH inductor with some initial current can be represented as shown below, where s is the Laplace Transform variable. The value of initial current is **GATE - 2006**



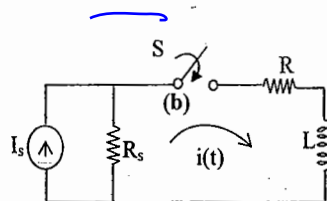
- (a) 0.5 A
(b) 2.0 A
(c) 1.0 A
(d) 0.0 A

05. In the figure shown below, assume that all the capacitors are initial uncharged. If $v_i(t) = 10u(t)$ Volts, $v_o(t)$ is given by **GATE - 2006**



- (a) $8e^{-0.004t}$ Volts (b) $8(1 - e^{-0.004t})$ Volts
(c) $8u(t)$ Volts (d) 8 Volts

06. In the following circuit, the switch S is closed at $t = 0$. The rate of change of current $\frac{di}{dt}(0^+)$ is given by **GATE - 2008**



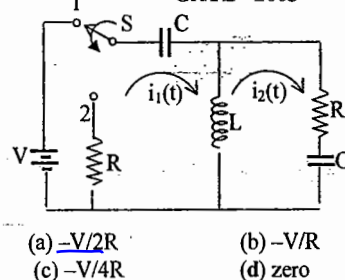
- (a) 0 (b) $\frac{R_s I_s}{L}$
(c) $\frac{(R+R_s)I_s}{L}$ (d) ∞

07. A linear time invariant system has an impulse response e^{2t} , for $t > 0$. If initial conditions are 0 and the input is e^{3t} , the output for $t > 0$ is **GATE - 2000**

- (a) $e^{3t} - e^{2t}$ (b) e^{5t}
(c) $e^{3t} + e^{2t}$ (d) None of the above

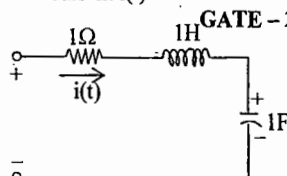
08. At $t=0^+$, the current i_1 is:

GATE - 2003



- (a) $-V/2R$ (b) $-V/R$
(c) $-V/4R$ (d) zero

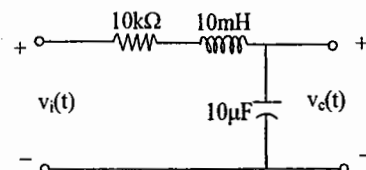
09. The circuit shown in the figure has initial current $i_L(0^-) = 1A$ through the inductor and an initial voltage $v_C(0^-) = -1V$ across the capacitor. For input $v(t) = u(t)$, the Laplace transform of the current $i(t)$ for $t \geq 0$ is **GATE - 2004**



- (a) $\frac{s}{s^2 + s + 1}$ (b) $\frac{s+2}{s^2 + s + 1}$
(c) $\frac{s-1}{s^2 + s + 1}$ (d) $\frac{s-2}{s^2 + s + 1}$

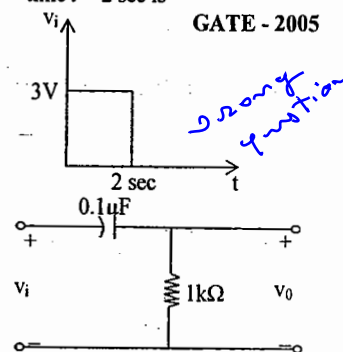
10. For the circuit shown in the figure, the initial conditions are zero. Its transfer

function $H(s) = \frac{V_C(s)}{V_i(s)}$ is **GATE - 2004**



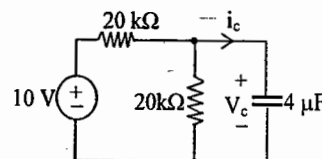
- (a) $\frac{1}{s^2 + 10^6 s + 10^6}$ (b) $\frac{10^6}{s^2 + 10^3 s + 10^6}$
(c) $\frac{10^3}{s^2 + 10^3 s + 10^6}$ (d) $\frac{10^6}{s^2 + 10^6 s + 10^6}$

11. A square pulse of 3 volts amplitude is applied to C-R circuit shown in the figure. The capacitor is initially uncharged. The output voltage v_o at time $t = 2$ sec is **GATE - 2005**



- (a) 3 V (b) -3 V
(c) 4 (d) -4 V

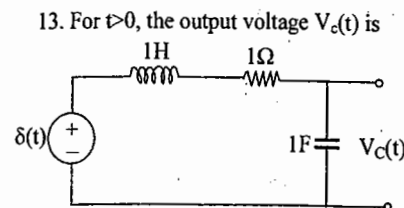
12. In the circuit shown, V_C is 0 volts at $t = 0$ sec. For $t > 0$, the capacitor current $i_c(t)$, where t is in seconds, is given by **GATE - 2007**



- (a) $0.50 \exp(-25t)$ mA
(b) $0.25 \exp(-25t)$ mA
(c) $0.50 \exp(-12.5t)$ mA
(d) $0.25 \exp(-6.25t)$ mA

Common Data for Questions 13 and 14:

The following series RLC circuit with zero initial conditions is excited by a unit impulse function $\delta(t)$ **GATE - 2008**



- (a) $\frac{2}{\sqrt{3}} \left(e^{-\frac{1}{2}t} - e^{-\frac{\sqrt{3}}{2}t} \right)$

- (b) $\frac{2}{\sqrt{3}} t e^{-\frac{1}{2}t}$

- (c) $\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right)$

- (d) $\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right)$

14. For $t > 0$, the voltage across the resistor is

- (a) $\frac{1}{\sqrt{3}} \left(e^{-\frac{\sqrt{3}}{2}t} - e^{-\frac{1}{2}t} \right)$

- (b) $e^{-\frac{1}{2}t} \left[\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{1}{\sqrt{3}} \sin\left(\frac{\sqrt{3}}{2}t\right) \right]$

- (c) $\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right)$

- (d) $\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right)$

15. The circuit shown in the figure is used to charge the capacitor C alternately from two current sources as indicated. The switches S1 and S2 are mechanically coupled and connected as follows:

For $2nT \leq t <$

$(2n+1)T, (n=0,1,2,\dots)$

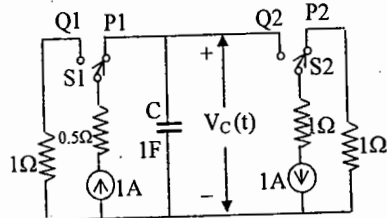
S1 to P1 and S2 to P2.

For $(2n+1)T \leq t < (2n+2)T,$

$(n=0,1,2,\dots)$

S1 to Q1 and S2 to Q2.

GATE - 2008



Assume that the capacitor has zero initial charge. Given that $u(t)$ is a unit step function, the voltage $V_c(t)$ across the capacitor is given by

- (a) $\sum_{n=0}^{\infty} (-1)^n u(t-nT)$
 (b) $u(t) + 2 \sum_{n=1}^{\infty} (-1)^n u(t-nT)$
 (c) $tu(t) + 2 \sum_{n=1}^{\infty} (-1)^n (t-nT)u(t-nT)$
 (d) $\sum_{n=0}^{\infty} [0.5e^{-(t-2nT)} + 0.5e^{-(t-2nT-T)}]$

16. $I_1(s)$ and $I_2(s)$ are the Laplace transforms of $i_1(t)$ and $i_2(t)$ respectively. The equations for the loop currents $I_1(s)$ and $I_2(s)$ for the circuit shown in the figure, after the switch is brought from position 1 to position 2 at $t=0$, are

GATE - 2003

- (a)
$$\begin{bmatrix} R+Ls+1/Cs & -Ls \\ -Ls & R+1/Cs \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} V/s \\ 0 \end{bmatrix}$$

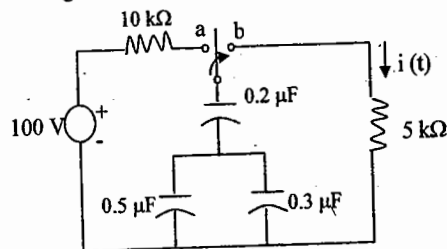
 (b)
$$\begin{bmatrix} R+Ls+1/Cs & -Ls \\ -Ls & R+1/Cs \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} -V/s \\ 0 \end{bmatrix}$$

 (c)
$$\begin{bmatrix} R+Ls+1/Cs & -Ls \\ -Ls & R+Ls+1/Cs \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} V/s \\ 0 \end{bmatrix}$$

 (d)
$$\begin{bmatrix} R+Ls+1/Cs & -Ls \\ -Ls & R+Ls+1/Cs \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} -V/s \\ 0 \end{bmatrix}$$

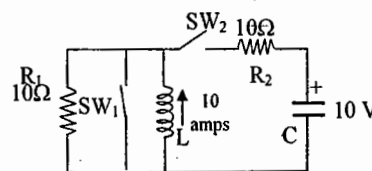
17. The switch in the circuit shown was on position 'a' long time, and is moved to position 'b' at time $t=0$. The current $i(t)$. The current $i(t)$ for $t > 0$ is given by

GATE - 2009



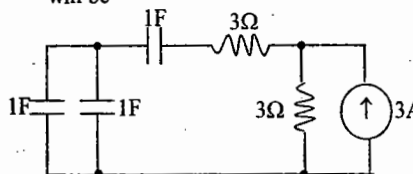
- a) $0.2 e^{-125t} u(t)$ mA
 b) $20 e^{-1250t} u(t)$ mA
 c) $0.2 e^{-1250t} u(t)$ mA
 d) $20 e^{-1000t} u(t)$ mA

18. In the circuit shown in figure switch SW_1 is initially CLOSED and SW_2 is OPEN. The inductor L carries a current of 10 A and the capacitor is charged to 10 V with polarities as indicated. SW_2 is initially CLOSED at $t=0$ and SW_1 OPENED at $t=0$. The current through C and the voltage across L at $t=0+$ is



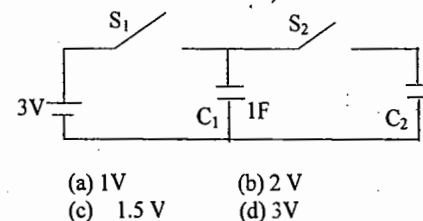
- (a) 55 A, 4.5 V (b) 5.5 A, 45 V
 (c) 45 A, 5.5 V (d) 4.5 A, 55V

19. The time constant for the given circuit will be



- a) 1/9 s b) 1/4 s c) 4 s d) 9 s

20. In the figure shown, all elements used are ideal. For time $t < 0$, S_1 remained closed and S_2 open. At $t=0$, S_1 is opened and S_2 is closed. If the voltage V_{C2} across the capacitor C_2 at $t=0$ is zero, the voltage across the capacitor combination at $t=0+$ will be



- (a) 1V (b) 2V
 (c) 1.5V (d) 3V

KEY SET - C

- | | | | |
|-------|-------|-------|-------|
| 01. c | 02. c | 03. c | 04. a |
| 05. b | 06. b | 07. a | 08. a |
| 09. b | 10. d | 11. b | 12. a |
| 13. d | 14. b | | |

To win the RACE join the ACE

Chapter 5 : Steady State Sinusoidal Response

(Analysis using phasors, Phasors diagrams, Locus plots, Frequency domain analysis of R-L-C circuits, Power calculations, 3 ϕ Circuits)

Sinusoidal excitation to R, L, C elements

Let the excitation be

$$v(t) = V_m \cos(\omega t + \theta) = \text{Re} [V e^{j\omega t}]$$

$$\text{or } v(t) = V_m \sin(\omega t + \theta) = \text{Im} [V e^{j\omega t}]$$

where the complex valued $V = V_m e^{j\theta}$

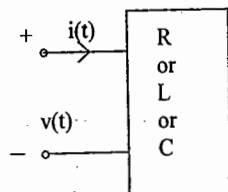
is known as the phasor of $v(t)$ and can be represented in the complex plane by a directed line segment with length, V_m at an angle of θ from the positive real (horizontal) axis.

The current $i(t)$ through the element can be obtained from the $v-i$ relation as

$$i(t) = I_m \cos(\omega t + \theta + \phi) = \text{Re} [I e^{j\omega t}]$$

$$\text{or } i(t) = I_m \sin(\omega t + \theta + \phi) = \text{Im} [I e^{j\omega t}]$$

where the phasor $I = I_m e^{j(\theta + \phi)}$ is displaced from V by ϕ .



Element Value	R(Ω)	L(H)	C(F)
I_m	V_m/R	$V_m/(\omega L)$	$V_m/[1/(\omega C)]$
Phase diff between $i(t)$ and $v(t) = \phi$ (deg)	0	-90	+90
Current $i(t)$	is inphase with $v(t)$	lags $v(t)$ by 90°	leads $v(t)$ by 90°
Phasor impedance $Z = V/I$ (Ω)	R	$j(\omega L)$	$-j[1/(\omega C)]$
Phasor admittance $Y = 1/Z$ (mhos)	$G=1/R$	$-j[1/(\omega L)]$	$j(\omega C)$
PF = $\cos(\phi)$	1	0	0
P_{ave}	$(I_m^2 R)/2$	0	0
Phasor (vector) diagram :			

ωL and $1/(\omega C)$ are called as the reactances of L and C respectively. $1/(\omega L)$ and ωC are called as the susceptances of L and C respectively.

Instantaneous power delivered to the element $p(t) = v(t) i(t)$, $P_{ave} = \frac{1}{T} \int_0^T v(t) i(t) dt$, $T = 2\pi/\omega$

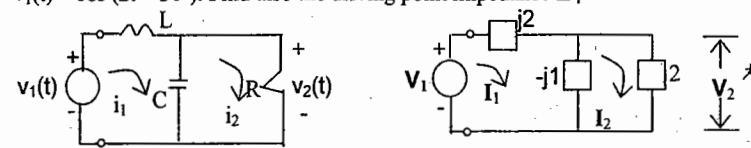
$P_{ave} = (1/2) V_m I_m \cos(\phi) = V_{rms} I_{rms} \cos(\phi)$, $\cos(\phi)$ is called as power factor (PF)

The steady state analysis of RLC networks is simplified by phasor method.

1. Replace the time domain circuit into equivalent phasor circuit. All the sources are represented either in the cosine form or sine form and they are replaced by the corresponding phasors. R, L, C elements are replaced by the respective phasor impedances; R , $j\omega L$, $-j/(\omega C)$.
2. The phasor form of required response, either voltage or current at any point in the circuit is obtained by the usual mesh or nodal analysis.

3. The time domain response is obtained by multiplying the response phasor by $\exp(j\omega t)$ and then taking the Re or Im part depending upon cosine excitations or sine excitations.

Example: Find $i_2(t)$ in the steady state in the network shown for $v_1(t) = \cos(2t + 30^\circ)$. Find also the driving point impedance Z_i



$$L = 1H, C = 0.5F, R = 2\Omega$$

$$V_1 = 1 \exp(j30^\circ)$$

From the mesh equations: $j1I_1 + j1I_2 = 1 \exp(j30^\circ)$, $j1I_1 + (2 - j1)I_2 = 0$

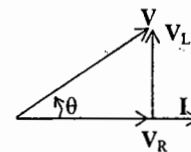
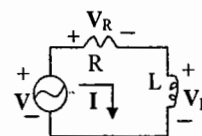
$$I_2 = \frac{1}{-2 + j2} = \frac{1}{2\sqrt{2}} \exp(-j105^\circ), i_2(t) = \frac{1}{2\sqrt{2}} \cos(2t - 105^\circ)$$

$$Z_i = \frac{(-j1) \times 2}{2 - j1} + j2 = \frac{1 + j}{1 - j0.5} = \sqrt{1.6} \exp[j(45^\circ + \tan^{-1}0.5)]$$

Similarly transfer impedances, admittances, voltage and current transfer functions can be defined.

Phasor Diagrams:

RL Series Circuit:



$$V_R = RI, V_L = j\omega LI$$

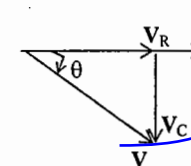
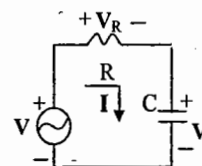
$$V = V_R + V_L = ZI, Z = R + j\omega L$$

$$V = \sqrt{V_R^2 + V_L^2}$$

$$\theta = \tan^{-1}(V_L/V_R) = \tan^{-1}(\omega L/R)$$

$$(PF) = \cos\theta \text{ (lagging), } I \text{ lags } V$$

RC Series Circuit:



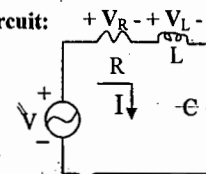
$$V = V_R + V_C = ZI, Z = R - j[1/(\omega C)]$$

$$V = \sqrt{V_R^2 + V_C^2}$$

$$\theta = \tan^{-1}(V_C/V_R) = \tan^{-1}[1/(\omega CR)]$$

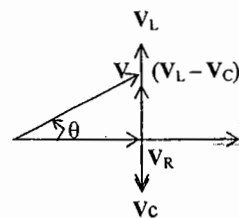
$$(PF) = \cos\theta \text{ (leading), } I \text{ leads } V$$

RLC Series Circuit:



$$V = V_R + V_L + V_C = ZI, Z = R + j\{\omega L - 1/(\omega C)\}$$

For $V_L > V_C$:
 $\omega L > 1/(\omega C)$

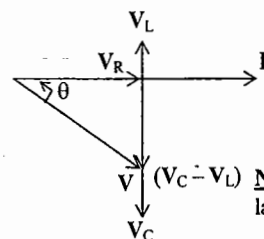


$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\theta = \tan^{-1} [(V_L - V_C) / (V_R)]$$

$$PF = \cos \theta \text{ (lagging), } I \text{ lags } V$$

For $V_C > V_L$:
 $1/(\omega C) > \omega L$



$$V = \sqrt{V_R^2 + (V_C - V_L)^2}$$

$$\theta = \tan^{-1} [(V_C - V_L) / (V_R)]$$

$$PF = \cos \theta \text{ (leading), } I \text{ leads } V$$

Note: By convention PF is said to be leading or lagging depending upon whether I leads or lags V

For $V_L = V_C$ or $\omega L = 1/(\omega C)$, the circuit is

said to be resonant at $\omega = \omega_0 = 1/\sqrt{LC}$

Under this condition, $V_L = jQV$, $V_C = -jQV$

$V = V_R = RI$, where the Quality factor

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

V and I are in phase, PF is unity, Z is purely real = R, Reactance is zero.

Admittance Y is purely real = $G = 1/R$, Susceptance is zero

Frequency response of current I is shown in the figure:

$$f_2, f_1 = f_0 (\sqrt{1 + \delta^2} \pm \delta), \delta = 1/(2Q)$$

$f_1, f_2 = f_0^2$, f_0 is geometric mean of f_1 and f_2 .

$$B = f_2 - f_1 = f_0/Q$$

where f_1 is lower 3-dB frequency

(half power frequency, cutoff frequency)

And f_2 is higher 3-dB frequency

(half power frequency, cutoff frequency)

B is 3-dB Bandwidth.

When δ is small, $\delta^2 < 1$, $f_2, f_1 \approx f_0 (1 \pm \delta)$,

$f_0 = 0.5(f_1 + f_2)$ = Arithmetic mean of f_1 and f_2 .

RL, RC, RLC parallel circuits: Interchange V & I, Z & Y with

$Q = (R/\omega_0 L) = \omega_0 CR = R(\sqrt{C/L})$ in the above relations for series connection.

Power calculations:

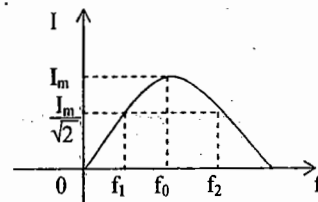
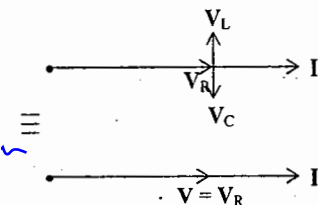
For a periodic signal, $x(t)$ T

$$\text{Average value: } = \frac{1}{T} \int_0^T x(t) dt = \frac{\text{Area of the signal over one period}}{\text{period}}$$

Form factor = RMS value/Average value

$$\text{rms value: } = \sqrt{\frac{1}{T} \int_0^T x^2(t) dt}$$

Peak factor = Maximum value/Average value



The current is maximum equal to $I_m = V_m/R$ at resonant frequency, f_0

Consider the one port RLC network with no independent sources.

Let the driving point impedance, $Z = R + jX = |Z| \exp(j\theta)$

And driving point admittance, $Y = G + jB$

Real part of Z is known as resistance = $R = |Z| \cos \theta$

Imaginary part of Z is known as reactance = $X = |Z| \sin \theta$

Real part of Y is known as conductance = G

Imaginary part of Y is known as susceptance = B

Power delivery to the network:

Complex (or phasor) power, $S = P_{ave} + jQ = V_r I_r^*$

P_{ave} = average (Active, true) power = $V_r I_r \cos \theta = I_r^2 R = V_r^2 G$ Watts, $V_r = I_r |Z|$

Q = reactive power = $\pm V_r I_r \sin \theta = I_r^2 X = -V_r^2 B$ vars

For inductive circuit Q is positive and for capacitive circuit Q is negative.

Apparent power (Volt-Amperes) = $|S| = \sqrt{P_{ave}^2 + Q^2} = V_r I_r$ va

Pure resistor: $P = V_{rms} I_{rms} = I_{rms}^2 R$ W, $Q = 0$ vars, $|S| = P$ va

Pure inductor (or) pure capacitor:

L : $P = 0$ W, $Q = V_{rms} I_{rms}$ vars, $|S| = |Q|$ va

C : $P = 0$ W, $Q = -V_r I_r$ vars, $|S| = |Q|$ va

Example: If $i_1(t) = 5\sqrt{2} \sin(2t)$

for the one port network shown verify:

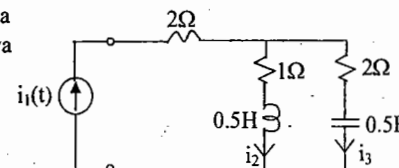
$Z = 3 + j(3)$, $P_{ave} = 75$ W, $Q = 25/3$ vars,

$S = 75 + j(25/3)$, $|S| = 75.4$ va, $V_r = 15.1$ V,

PF = 0.994 leading, $I_1 = 5$ A, $I_2 = 5\sqrt{5}/3$ A,

$I_3 = 5\sqrt{2}/3$ A, $P_{ave} = P_1 + P_2 + P_3 = 50 + (500/36) + (400/36) = 75$ W

$Q = Q_1 + Q_2 = 500/36 - 200/36 = 25/3$ vars

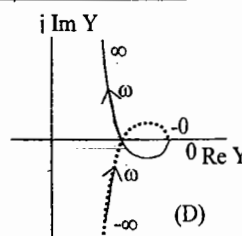
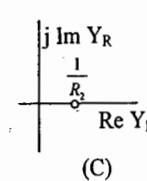
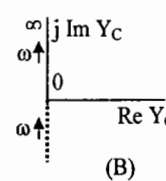
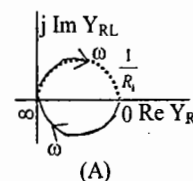
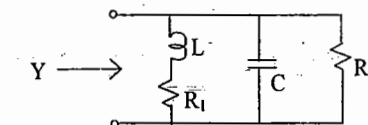


Locus plot

Consider the one port network shown. Sketch the admittance locus.

$$Y = Y_{RL} + Y_C + Y_R, Y_{RL} = \frac{1/L}{(R_1/L) + j\omega}$$

$$Y_C = j\omega C, Y_R = 1/R_2$$



Individual locus plots for (A) R_1, L branch, (B) the C branch, (C) the R_2 branch (the locus being a single point) and (D) the complete locus for Y found by adding the loci for the three parallel branches.

3- ϕ CIRCUITS:

3 coils are 120° electrical apart and the emf's generated in the respective coils also displaced by each other by 120° . All the 3 coils can be either delta connected or star connected.

Star connected load or generator:

For the balanced case, $V_L = \sqrt{3} V_{ph}$, $I_L = I_{ph}$

Observations:

The line voltages are 120° apart.

The line voltages are 30° ahead of the respective phase voltages.

The angle between the line voltage and the respective line current is $(30 + \phi)^\circ$ with current lagging, where ϕ = impedance angle per phase.

Delta connected load or generator:

For the balanced case, $V_L = V_{ph}$, $I_L = \sqrt{3} I_{ph}$

Observations:

The line currents are 120° apart.

The line currents are 30° behind the respective phase currents.

The angle between the line voltage and the respective line current is $(30 - \phi)^\circ$ with current lagging, where ϕ = impedance angle per phase.

Power Calculations:

For both the type of connections,

Total active power = 3 x per phase power

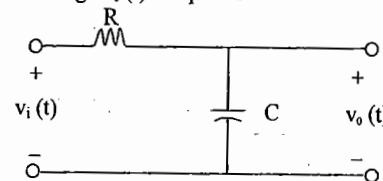
$P = 3 V_{ph} I_{ph} \cos \phi = \sqrt{3} V_L I_L \cos \phi$ W, where ϕ = angle between the phase voltage and current

Total reactive power (Q) = $\sqrt{3} V_L I_L \sin \phi$ Vars

Apparent power (S) = $\sqrt{3} V_L I_L$ VA

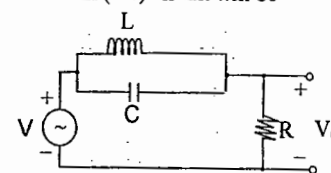
Objective Question SET - A

01. For the circuit shown in fig., the time constant $RC = 1\text{ms}$. The input voltage is $v_i(t) = \sqrt{2} \sin 103t$. The output voltage $v_o(t)$ is equal to



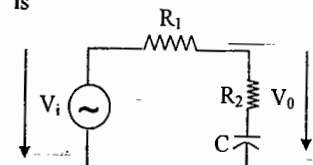
- (A) $\sin(103t - 450^\circ)$
 (B) $\sin(103t + 450^\circ)$
 (C) $\sin(103t - 530^\circ)$
 (D) $\sin(103t + 530^\circ)$

02. In the circuit shown in Fig., voltage V_0 across the resistance R at resonance of the tank (LC) circuit will be



- (A) Zero
 (B) $V/3$
 (C) $V/2$
 (D) V

03. For the circuit shown in figure, the gain ($|V_o/V_i|$) at very high frequencies is

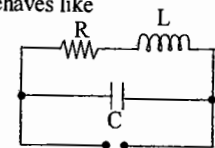


- (A) 1
 (B) 0
 (C) $R_2/(R_1+R_2)$
 (D) R_2/R_1

04. The power factor of an R-L circuit is

- (A) 1
 (B) <1 and leading
 (C) <1 and lagging
 (D) >1

05. The circuit in Fig., at resonance, behaves like

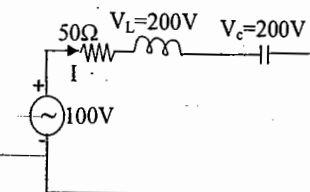


a.c. SOURCE

- (A) an open circuit
 (B) a pure resistor of value R
 (C) a pure resistor of value higher than R
 (D) a pure resistor of value \sqrt{LC}
06. A resistance R , inductance L and capacitance C are connected in parallel across an ac voltage source of output voltage V . At the resonant frequency ω , the source current will be

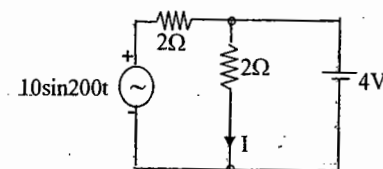
- (A) $\frac{1}{\sqrt{LC}} \frac{V}{R}$
 (B) $\frac{V}{R}$
 (C) $\frac{\omega L}{R} \frac{V}{R}$
 (D) $\frac{R}{\omega L} \frac{V}{R}$

07. In the circuit shown in figure, the rms value of the current 'I' is



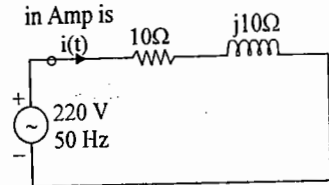
- (A) 6A
 (B) 10A
 (C) 2A
 (D) Cannot be determined

08. In the circuit shown in Fig., the magnitude of the current I flowing through the 2Ω resistor is



- (A) $5\sin 200t$
 (B) $2.5\sin 200t$
 (C) 2A
 (D) $2+2.5\sin 200t$

09. In figure the magnitude of current $i(t)$ in Amp is

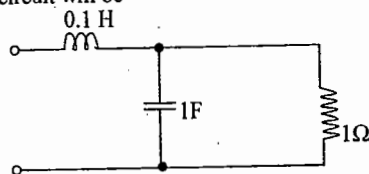


- (A) 11 (B) $11/\sqrt{2}$
(C) $11\sqrt{2}$ (D) 22

10. In a series RLC circuit with quality factor Q at resonance, if the voltages measured across source, resistance, inductance and capacitance are V_s , V_R , V_L and V_C respectively, then at resonance,

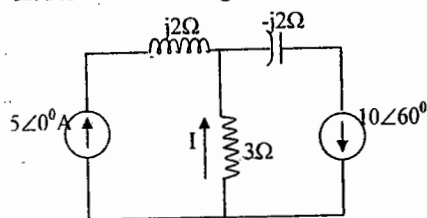
- (A) $V_L = Q V_s$ (B) $V_R = Q V_s$
(C) $V_L = Q V_C$ (D) $V_C = Q V_L$

11. The resonant frequency for the given circuit will be



- (A) 1 rad/s (B) 2 rad/s
(C) 3 rad/s (D) 4 rad/s

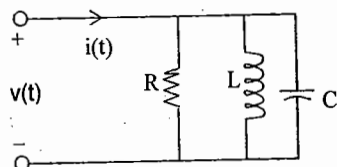
12. For the circuit in Fig. the current I is



- (A) $\frac{10\sqrt{3}}{2} \angle 90^\circ$ Amps
(B) $\frac{10\sqrt{3}}{2} \angle -90^\circ$ Amps

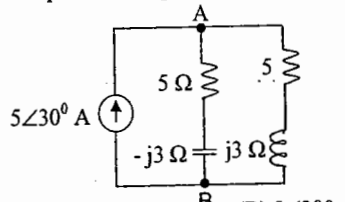
- (C) $5 \angle 600$ Amps
(D) $5 \angle -600$ Amps

13. The circuit shown in fig., with $R = 1/3 \Omega$, $L = 1/4$ H, $C = 3$ F has input voltage $v(t) = \sin 2t$. The resulting current $i(t)$ is



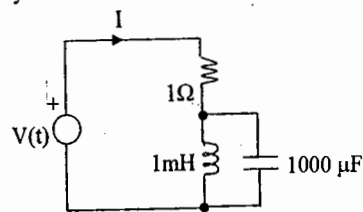
- (A) $5 \sin(2t + 53.10^\circ)$ A
(B) $5 \sin(2t - 53.10^\circ)$ A
(C) $25 \sin(2t + 53.10^\circ)$ A
(D) $25 \sin(2t - 53.10^\circ)$ A

14. In the AC network shown in the figure, the phasor voltage V_{AB} (in volts) is



- (A) 0 (B) $5 \angle 300$
(C) $12.5 \angle 300$ (D) $17 \angle 300$

15. For the circuit shown below the steady-state current I is



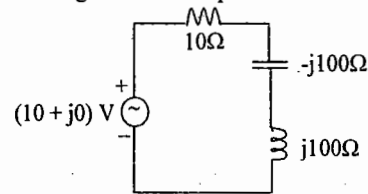
$$v(t) = 5\sqrt{2}e^{-5t} \cos(1000t) \text{ V}$$

- (A) 0 A
(B) $5\sqrt{2} \cos(1000t)$ A
(C) $5\sqrt{2} \cos(1000t - \frac{\pi}{4})$ A
(D) $5\sqrt{2}$ A

16. In a series RLC circuit $R = 2 \text{ k}\Omega$, $L = 1$ H, and $C = 1/400 \mu\text{F}$. The resonant frequency is

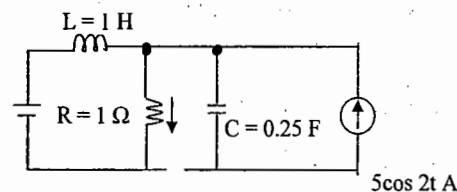
- (A) 2×10^4 Hz (B) $\frac{1}{\pi} \times 10^4$ Hz
(C) 10^4 Hz (D) $2\pi \times 10^4$ Hz

17. For the circuit shown below the voltage across the capacitor is



- (A) $(10 + j0)$ V (B) $(100 + j0)$ V
(C) $(0 + j100)$ V (D) $(0 - j100)$ V

18. In the circuit shown in the following



figure, the current through the 1Ω resistor is

- (A) $(1 + 5 \cos 2t)$ A (B) $(5 + \cos 2t)$ A
(C) $(1 - 5 \cos 2t)$ A (D) 6 A

Key Sheet for SET - A

01. A 02. A 03. C 04. C 05. C
06. B 07. C 08. C 09. C 10. A
11. C 12. A 13. A 14. C 15. A
16. B 17. D 18. C

SET - B

01. A series resonant circuit has an inductive reactance of 1000Ω , a capacitive reactance of 1000Ω and a resistance of 0.1Ω . If the resonant frequency is 10 MHz , then the bandwidth of the circuit will be
(A) 1 kHz (B) 10 kHz
(C) 1 MHz (D) 0.1 kHz

02. For a series RLC circuit, the power factor at the lowest half power frequency is
(A) 0.707 lagging (B) 0.5 leading
(C) unity (D) 0.707 leading

03. The value of current at resonance in a series RLC circuit is governed by

- (A) R (B) L
(C) C (D) all of these

04. A series RLC circuit has resonance at 1 MHz frequency. At $f = 1.1 \text{ MHz}$ the circuit impedance is

- (A) Capacitive (B) inductive
(C) resistive (D) none of these

05. A system function has a pole at $s = 0$ and a zero at $s = -1$. The constant multiplier is unity. For an excitation $\cos t$, the steady state response is given by

- (A) $\sqrt{2} \cos(t + 45^\circ)$
(B) $\sqrt{2} \cos(t - 45^\circ)$ (C) $\cos t$
(D) $\sqrt{2} \cos(t + 45^\circ)$

06. The power in a series RLC circuit will be half of that at resonance when the magnitude of the current is equal to

- (A) $\frac{V}{2R}$ (B) $\frac{V}{\sqrt{3}R}$
(C) $\frac{V}{\sqrt{2}R}$ (D) $\frac{\sqrt{2}V}{R}$

07. In the RLC parallel circuit the impedance at resonance is

- (A) Maximum (B) Minimum
(C) zero (D) infinity

08. If the resistance in a series RC circuit is increased, then the magnitude of the phase

- (A) increases
(B) remains the same
(C) decreases
(D) Changes in an indeterminate manner

09. On increasing the 'Q' of the coil

- (A) its power factor increases
(B) its power factor decreases
(C) its power factor remains unaltered
(D) its power may increase or decreases

10. A circuit with a resistor, inductor and capacitor in series is resonant at f_0 Hz. If all the component values are now doubled, the new resonant frequency is

(A) $2f_0$ (B) still f_0
(C) $f_0/4$ (D) $f_0/2$

11. An RLC series circuit has $R = 1$, $L = 1$ H and $C = 1$ F, then damping ratio of the circuit will be

(A) more than unity (B) unity
(C) 0.5 (D) zero

12. An RLC series circuit has a resistance R of $20\ \Omega$ and a current which lags behind the applied voltage by 45° . If the voltage across the inductor is twice the voltage across the capacitor, what is the value of inductive reactance?

(A) $10\ \Omega$ (B) $20\ \Omega$
(C) $40\ \Omega$ (D) $60\ \Omega$

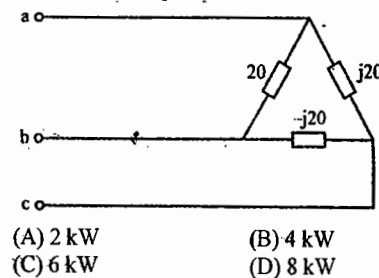
13. A parallel $R-L-C$ circuit resonates at 100 KHz. At frequency of 110 KHz, the circuit impedance will be

(A) Capacitive (B) inductive
(C) Resistive (D) none of these

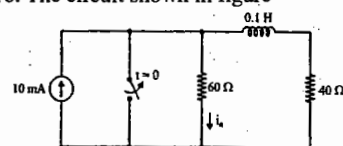
14. For a parallel RLC resonant circuit, the damped frequency is $\sqrt{8}$ r/s and bandwidth is 2 r/s. What is its resonant frequency?

(A) 2 (B) $\sqrt{7}$
(C) $\sqrt{10}$ (D) 3

15. A delta load is connected to a balanced 400 V, 3-phase supply as shown in figure. The total power dissipated in the network is equal to



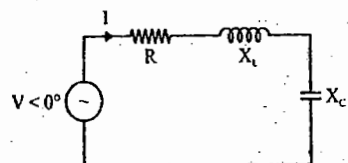
16. The circuit shown in figure



The switch has been closed for a very long time. The switch opens at $t = 0$. i_R at $t = 0^-$ is

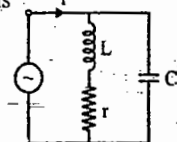
(A) 0 (B) 4 mA
(C) 6 mA (D) 10 mA

17. The resistance of the series RLC circuit is doubled and inductance is halved, then BW becomes



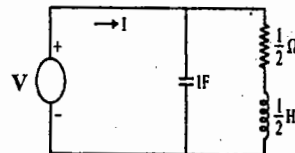
(A) half (B) double
(C) one-fourth (D) four times

18. For the following parallel resonant circuit, the admittance at resonance condition is



(A) ∞ (B) $\frac{1}{r}$
(C) $\frac{1}{Cr}$ (D) $\frac{Cr}{L}$

19. $V_s = \cos t$. The current i is given by

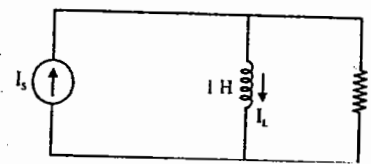


(A) $\sqrt{2} \cos(t - 45^\circ)$
(B) $\sqrt{5} \cos(t - \tan^{-1} 2)$
(C) $\cos t$ (D) None of these

20. An RLC resonant circuit has a resonance frequency of 1.5 MHz and bandwidth of 10 KHz. If $C = 150$ pF, then the effective resistance of the circuit will be

(A) $29.5\ \Omega$ (B) $14.75\ \Omega$
(C) $9.4\ \Omega$ (D) $4.7\ \Omega$

21. $i_s = 5 \cos 2t = 5 \angle 0^\circ$. The phasor current through the inductor i_L is given by



(A) $10\sqrt{2} \angle 45^\circ$ (B) $5 \angle 45^\circ$
(C) $5/\sqrt{2} \angle 45^\circ$ (D) $5/\sqrt{2} \angle -45^\circ$

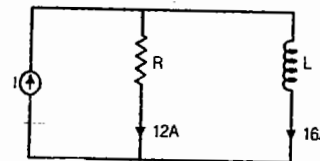
22. Consider the following statements: When a series $R-L-C$ circuit is under resonance

1. current is maximum through R
2. magnitude of the voltage across L is equal to that across C
3. the power factor of the circuit is unity

Which of the statements given above are correct?

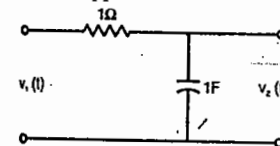
(A) 1, 2 and 3 (B) 1 and 2 only
(C) 2 and 3 only (D) 1 and 3 only

23. In the circuit shown in the figure below, the current supplied by the sinusoidal current source i is



(A) 28 A (B) 4 A
(C) 20 A
(D) not determinable from the data given

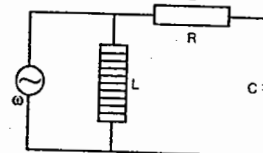
24. For the following circuit a source of $V_1(t) = e^{-2t}$ is applied.



Then the resulting response $V_2(t)$ is given by

(A) $e^{-2t} + e^{-t}$ (B) e^{-t}
(C) $e^{-t} - e^{-2t}$ (D) $e^{-2t}/2$

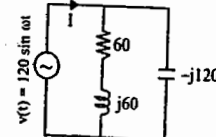
25. Consider the following circuit:



For what value of ω , the circuit shown above exhibits unity power factor?

(A) $\frac{1}{\sqrt{LC}}$ (B) $\frac{1}{\sqrt{LC + R^2C^2}}$
(C) $\frac{1}{\sqrt{LC - R^2C^2}}$ (D) $\frac{1}{RC}$

26. For the a.c. circuit given below, what is the value of I ?



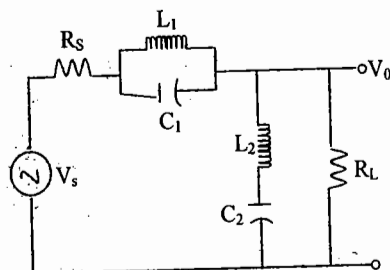
(A) $1 + j1$ (B) $1 + j0$
(C) $0 - j1$ (D) $0 + j0$

Key for Set B:

01.A 02.D 03.A 04.B 05.B 06.C
07.A 08.C 09.B 10.D 11.C 12.C
13.A 14.D 15.D 16.A 17.D 18.D
19.C 20.D 21.D 22.A 23.C 24.C
25.B 26.C

SET - C

01. The circuit of the figure represents a
GATE-2000

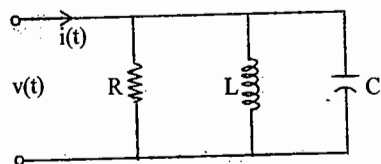


- (a) low pass filter (b) high pass filter
(c) band pass filter (d) band reject filter

02. A series RLC circuit has a resonance frequency of 1kHz and a quality factor $Q = 100$. If each of R , L and C is doubled from its original value, the new Q of the circuit is GATE - 2003

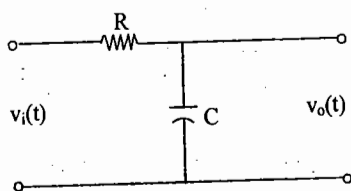
- (a) 25 (b) 50
(c) 100 (d) 200

03. The circuit shown in the figure, with $R=1/3 \Omega$, $L=1/4H$, $C=3F$ has input voltage $v(t)=\sin 2t$. The resulting current $i(t)$ is GATE - 2004



- (a) $5 \sin(2t+53.1^\circ)$ (b) $5 \sin(2t-53.1^\circ)$
(c) $25 \sin(2t+53.1^\circ)$ (d) $25 \sin(2t-53.1^\circ)$

04. For the circuit shown in the figure, the time constant $\tau=1$ ms. The input voltage is $v_i(t)=\sqrt{2} \sin 10^3 t$. The output voltage $v_o(t)$ is equal to GATE - 2004

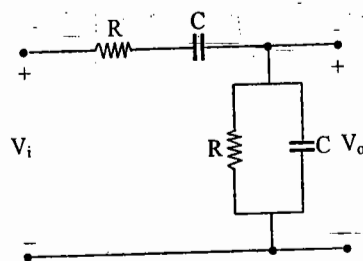


- (a) $\sin(10^3 t - 45^\circ)$ (b) $\sin(10^3 t + 45^\circ)$
(c) $\sin(10^3 t - 53^\circ)$ (d) $\sin(10^3 t + 53^\circ)$

05. In a series RLC circuit, $R=2k\Omega$, $L=1H$, and $C=1/400 \mu F$. The resonant frequency is GATE - 2005

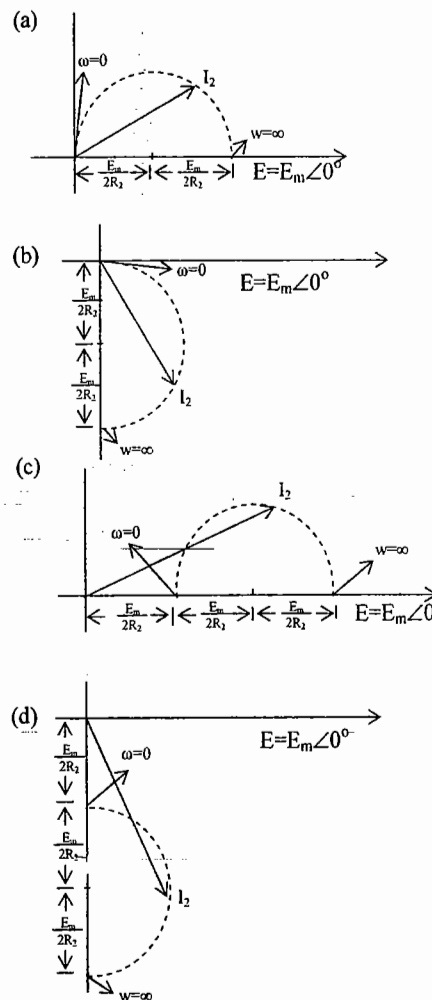
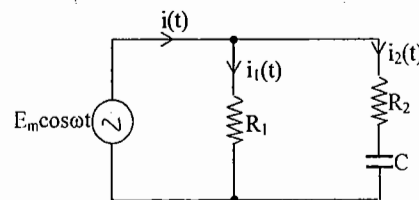
- (a) 2×10^4 Hz (b) $1/\pi \times 10^4$ Hz
(c) 10^4 Hz (d) $2\pi \times 10^4$ Hz

06. The RC circuit shown in the figure is GATE - 2007

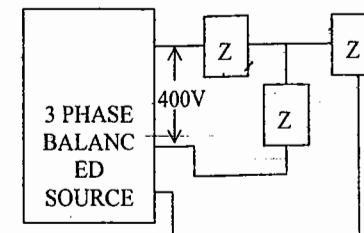


- (a) a low-pass filter
(b) a high-pass filter
(c) a band-pass filter
(d) a band reject filter

07. When the angular frequency ω in the figure is varied from 0 to ∞ , the locus of the current phasor I_2 is given by GATE - 2001

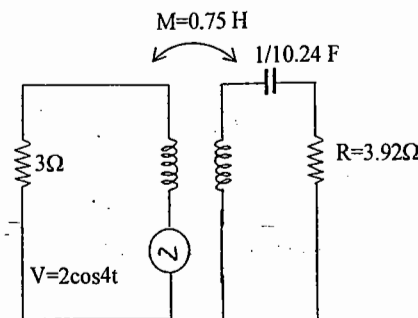


08. If the 3-phase balanced source in the figure delivers 1500 W at a leading power factor 0.844, then the value of Z_L (in ohm) is approximately. GATE - 2002



- (a) $90 \angle 32.44^\circ$ (b) $80 \angle 32.44^\circ$
(c) $80 \angle -32.44^\circ$ (d) $90 \angle -32.44^\circ$

09. The current flowing through the resistance R in the circuit in the figure has the form $P \cos 4t$, where P is GATE - 2003



- (a) $(0.189+j 0.72)$ (b) $(0.46+j 1.90)$
(c) $-(0.18+j 1.90)$ (d) $-(0.192+j 0.144)$

10. An input voltage $v(t)=10\sqrt{2}\cos(t+10^\circ)+10\sqrt{2}\cos(2t+10^\circ)$ V is applied to a series combination of resistance $R=1\Omega$ and an inductance $L=1H$. The resulting steady-state current $i(t)$ in ampere is GATE - 2003

- (a) $10\cos(t+55^\circ)+10\cos(2t+10^\circ+\tan^{-1}2)$
(b) $10\cos(t+55^\circ)+10\sqrt{3}/2\cos(2t+55^\circ)$
(c) $10\cos(t-35^\circ)+10\cos(2t+10^\circ-\tan^{-1}2)$
(d) $10\cos(t-35^\circ)+10\sqrt{3}/2\cos(2t-35^\circ)$

11. The transfer function

of an R-L-C circuit is given by

$$H(s) = \frac{10^6}{s^2 + 20s + 10^6}$$

The Quality factor (Q-factor) of this circuit is

- (a) 25 (b) 50
(c) 100 (d) 5000

12. Consider the following statements

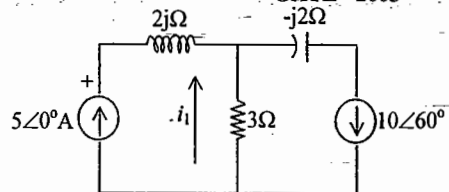
S1 and S2 : At the resonant frequency the impedance of a series R-L-C circuit is zero S2 : In a parallel G-L-C circuit, increasing the conductance G results in increase in its Q factor. Which one of the following is correct?

GATE - 2004

- (a) S1 is FALSE and S2 is TRUE
(b) Both S1 and S2 are TRUE
(c) S1 is TRUE and S2 is FALSE
(d) Both S1 and S2 are FALSE

13. For the circuit shown in the figure, the instantaneous current $i_i(t)$ is

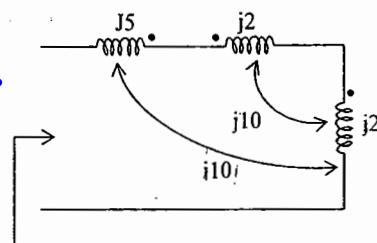
GATE - 2005



- (a) $\frac{10\sqrt{3}}{2} \angle 90^\circ$ (b) $\frac{10\sqrt{3}}{2} \angle -90^\circ$
(c) $5 \angle 60^\circ$ Amps (d) $5 \angle -60^\circ$ Amps

14. Impedance Z as shown in the given figure is

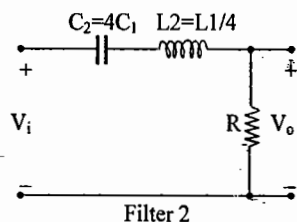
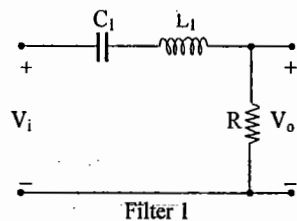
GATE - 2005



- (a) j29Ω (b) j9Ω
(c) j19Ω (d) j39Ω

15. Two series resonant filters are as shown in the figure. Let the 3-dB bandwidth of Filter 1 be B_1 and that of Filter 2 be B_2 . The value of B_1/B_2 is

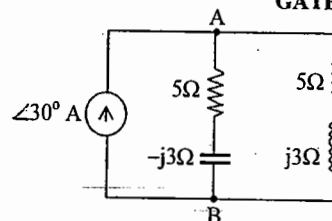
GATE - 2007



- (a) 4 (b) 1
(c) 1/2 (d) 1/4

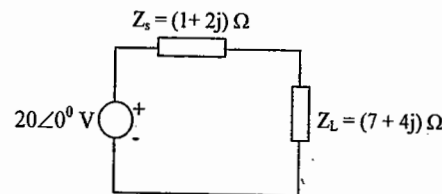
16. In the AC network shown in the figure, the phasor voltage V_{AB} (in Volts) is

GATE - 2007



- (a) 0 (b) $5 \angle 30^\circ$
(c) $12.5 \angle 30^\circ$ (d) $17 \angle 30^\circ$

17. An AC source of RMS voltage 20 V with internal impedance $Z_s = (1 + 2j) \Omega$ feeds a load of impedance $Z_L = (7 + 4j) \Omega$ in the figure below. The reactive power consumed by the load is



- (a) 8 VAR (b) 16 VAR
(c) 28 VAR (d) 32 VAR

18. The time domain behavior of an RL circuit is represented by

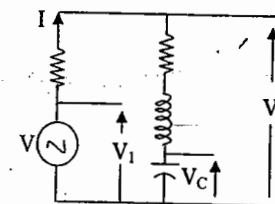
$$L \frac{di}{dt} + Ri = V_0 (1 + B e^{-Rt/L} \sin t) u(t)$$

For an initial current of $i(0) = \frac{V_0}{R}$,

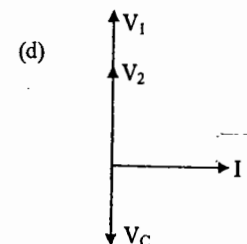
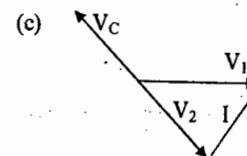
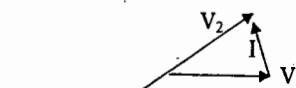
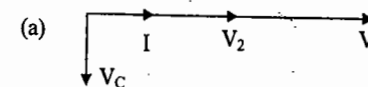
the steady state value of the current is given by

- (a) $i(t) \rightarrow \frac{V_0}{R}$ (b) $i(t) \rightarrow \frac{2V_0}{R}$
(c) $i(t) \rightarrow \frac{V_0}{R} (1 + B)$ (d) $i(t) \rightarrow \frac{2V_0}{R} (1 + B)$

19. The circuit shown in the figure is energized by a sinusoidal voltage source V_1 at a frequency which causes resonance with a current of I.



The phasor diagram which is applicable to the circuit is

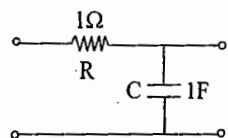


20. An ideal capacitor is charged to a voltage V_0 and connected at $t = 0$ across an ideal inductor L . (The circuit now consists of a capacitor and inductor alone). If we let

the voltage across the capacitor at time $t > 0$ is given by

- (a) V_0 (b) $V_0 \cos(\omega_0 t)$
(c) $V_0 \sin(\omega_0 t)$ (d) $V_0 \cos(\omega_0 t)$

21. The transfer function of the filter shown in the figure and its roll off respectively are

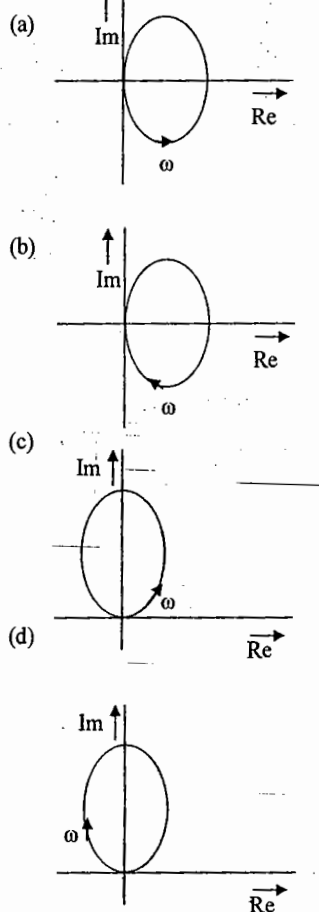
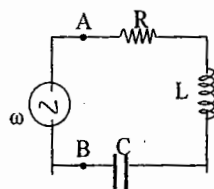


- (a) $1/(1+RCs)$, -20dB/decade
(b) $(1+RCs)$, -40dB/decade
(c) $1/(1+RCs)$, -40dB/decade
(d) $\{RCs/(1+RCs)\}$, -20dB/decade

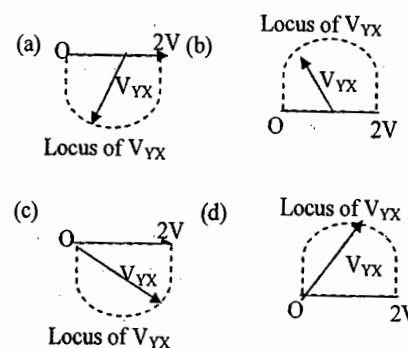
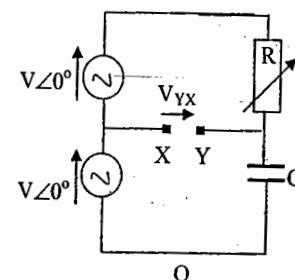
KEY SET -C

01. d 02. b 03. a 04. a 05. b
06. c 07. a 08. d 09. 10. c
11. b 12. d 13. a 14. b 15. d
16. d 17. 18.

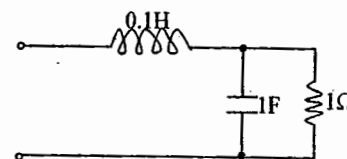
22. The R-L-C series circuit shown is supplied from a variable frequency voltage source. The admittance-locus of the R-L-C network at terminals AB for increasing frequency ω is



23. In the figure given below all phasors are with reference to the potential at point "O". The locus of voltage phasor V_{YX} as R is varied from zero to infinity is shown by

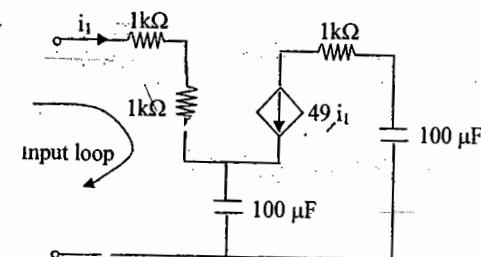


24. The resonant frequency for the given circuit will be



- a) 1 rad/s b) 2 rad/s
c) 3 rad/s d) 4 rad/s

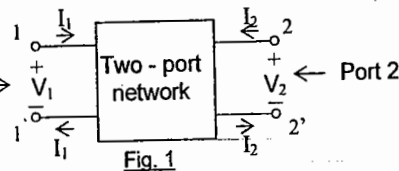
25. The equivalent capacitance of the input loop of the circuit shown is



- (A) 2 μ F (B) 100 μ F
(C) 200 μ F (D) 4 μ F

Chapter 6: Two-port network parameters (Driving point and transfer functions)

A two-port network with standard reference directions for the voltages and currents is shown in Fig. 1. Note that current entering a port is equal to the current leaving that port.



The relation between the four variables V_1 , I_1 , V_2 and I_2 can be described as a set of two equations in six ways by taking two of them as dependent variables, which depend upon the other two as independent variables. The four coefficients of the independent variables in the equations are known as the two-port parameters describing the network. The six sets of two-port parameters are indicated in the following table. The parameters can be calculated from the circuit, which is obtained after implementing either a short circuit ($V_1 = 0$ or $V_2 = 0$) or an open circuit ($I_1 = 0$ or $I_2 = 0$) as shown in Fig. 2

Name of the parameters	Express	In terms of	Equation	Associated Matrix	Condition for passivity, symmetry
Open-circuit Impedance (z)	V_1 V_2	I_1 and I_2 I_1 and I_2	$V_1 = z_{11}I_1 + z_{12}I_2$ $V_2 = z_{21}I_1 + z_{22}I_2$	$[z] = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$z_{12} = z_{21}$ $z_{11} = z_{22}$
Short-circuit Admittance (y)	I_1 I_2	V_1 and V_2 V_1 and V_2	$I_1 = y_{11}V_1 + y_{12}V_2$ $I_2 = y_{21}V_1 + y_{22}V_2$	$[y] = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$y_{12} = y_{21}$ $y_{11} = y_{22}$
Transmission (A, B, C, D)	V_1 I_1	V_2 and I_2 V_2 and I_2	$V_1 = AV_2 - BI_2$ $I_1 = CV_2 - DI_2$	$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$	$AD - BC = 1$ $A = D$
Inverse transmission (A', B', C', D')	V_2 I_2	V_1 and I_1 V_1 and I_1	$V_2 = A'V_1 - B'I_1$ $I_2 = C'V_1 - D'I_1$	$[T'] = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}$	$A'D' - B'C' = 1$ $A' = D'$
Hybrid (h)	V_1 I_2	I_1 and V_2 I_1 and V_2	$V_1 = h_{11}I_1 + h_{12}V_2$ $I_2 = h_{21}I_1 + h_{22}V_2$	$[h] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$h_{12} = -h_{21}$ $\Delta_h = 1$
Inverse hybrid (g)	I_1 V_2	V_1 and I_2 V_1 and I_2	$I_1 = g_{11}V_1 + g_{12}I_2$ $V_2 = g_{21}V_1 + g_{22}I_2$	$[g] = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$	$g_{12} = -g_{21}$ $\Delta_g = 1$

input voltage & output current

Transmission parameters are also called as chain parameters or general circuit parameters.

The symbol Δ indicates the determinant: For example $\Delta_h = \det [h] = h_{11}h_{22} - h_{12}h_{21}$

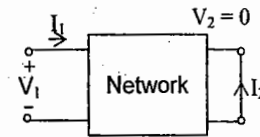


Fig. 2(a)

Port 2 is SC for calculating y_{11} , y_{21} , h_{11} , h_{21} , B & D

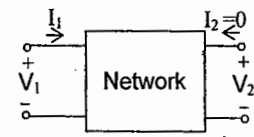


Fig. 2(b)

Port 2 is OC for calculating z_{11} , z_{21} , g_{11} , g_{21} , A & C

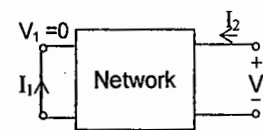


Fig. 2(c)

Port 1 is SC for calculating y_{12} , y_{22} , g_{12} , g_{22} , B' & D'

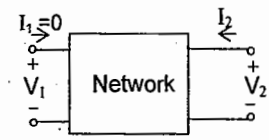


Fig. 2(d)

Port 1 is OC for calculating z_{12} , z_{22} , h_{12} , h_{22} , A' & C'

The parameters of one set can be expressed in terms of parameters of any other set by appropriately using the two sets of equations. For example

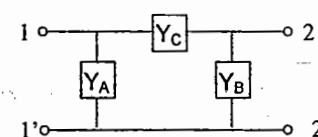
$$[y] = [z]^{-1}, [T'] = [T]^{-1}, [g] = [h]^{-1} \quad \rightarrow (1)$$

$$\Delta_y \Delta_z = 1, z_{11} y_{11} = z_{22} y_{22}, A = z_{11}/z_{21}, h_{11} = B/D = 1/y_{11}, \text{ etc.}$$

Typical networks and parameters are given below:

Fig.(3)

π -network



Reciprocal Nw, $y_{12} = y_{21}$

Symmetric if $Y_A = Y_B$

$$[y] = \begin{bmatrix} Y_A + Y_C & -Y_C \\ -Y_C & Y_B + Y_C \end{bmatrix} \quad \rightarrow (2)$$

If $Y_A = 0$, $Y_B = 0$, $Y_C = Y$, the π -network becomes the series element shown in Fig. (4)

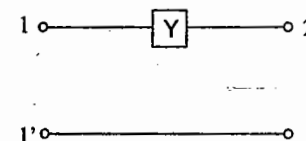


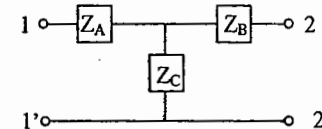
Fig.(4)

$$[y] = \begin{bmatrix} Y & -Y \\ -Y & Y \end{bmatrix} \quad \rightarrow (3)$$

Det $[y] = 0$
Z-parameters are not defined for this network
Reciprocal and Symmetric

Fig.(5)

T-network



Reciprocal Nw, $z_{12} = z_{21}$

$$[z] = \begin{bmatrix} Z_A + Z_C & Z_C \\ Z_C & Z_B + Z_C \end{bmatrix} \quad \rightarrow (4)$$

Symmetric if $Z_A = Z_B$

If $Z_A = 0$, $Z_B = 0$, and $Z_C = Z$, the T-network becomes the shunt element shown in Fig. (6)

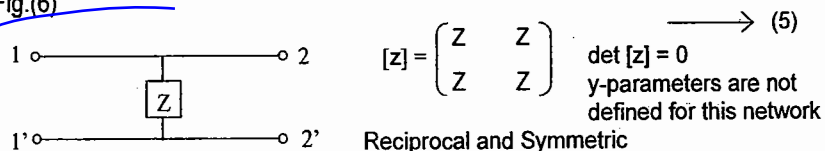


Fig. (6)

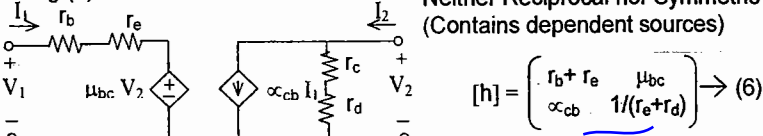


Fig. (7) Model of the CE connected transistor

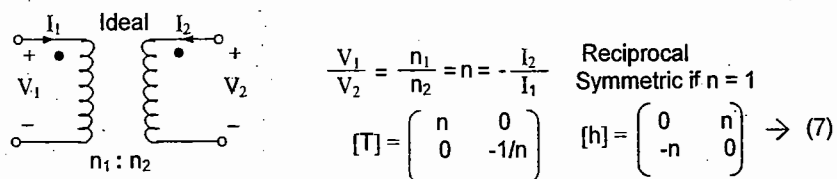


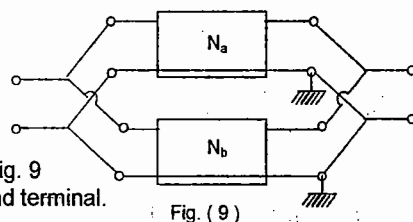
Fig. (8) Ideal Transformer

Neither z -parameters nor y -parameters are defined for this network.

Interconnection of 2-port Networks:

2-port networks with a common terminal between the input and output are quite common. T and π networks (Fig. 3 and 5) are familiar examples.

Parallel connection: This is illustrated in Fig. 9 with 2-port networks having common ground terminal.



For this case the individual y -parameter matrices get added to give the y -parameter matrix of the overall network.

$$[y] = [y_a] + [y_b] \rightarrow (8)$$

The individual y -parameters also get added: $y_{11} = y_{11a} + y_{11b}$, etc.

This relation is true only when the nature of the two networks is not altered by parallel connection. Fig. 10 (a) shows a Twin-T network, which is illustrated in Fig. 10 (b) as the parallel connection of two 2-port (3 terminal) T-networks. Similarly Fig. 11(a) shows a bridged-T network, which is illustrated in Fig. 11 (b) as the parallel connection of a T-network and a π -network with only a series element, Z_a .

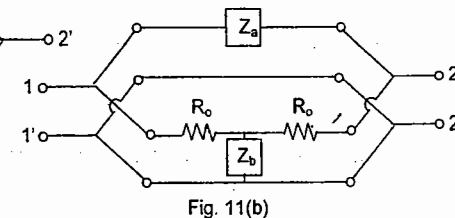
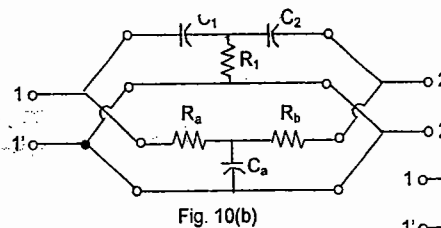
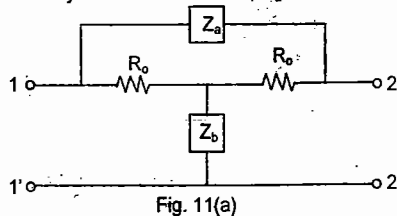
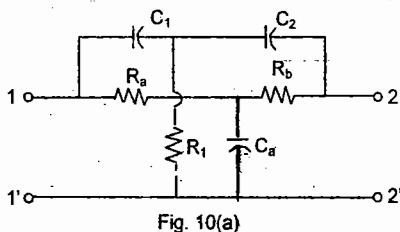
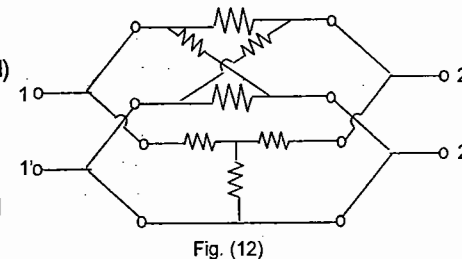


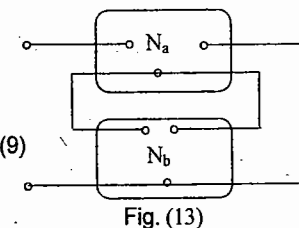
Fig. 12 illustrates a 2-port, 4 terminal network (lattice with no common terminal) in parallel with a 3 terminal T network having a common terminal. Relation (8) is not valid here because the lower resistor of the lattice is shorted and the nature of the original lattice is altered (lattice becoming a π network)



Series connection: This is illustrated in Fig. (13). Here the individual Z -parameter matrices get added.

$$[z] = [z_a] + [z_b] \rightarrow (9)$$

For this relation to be true, the nature of the original networks should not be altered by series connection.



Prob : For the parallel connection of two two-port networks shown in

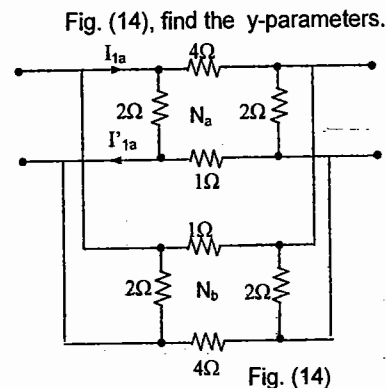


Fig. (14), find the y -parameters.

Prob : For the series connection of two two-port networks shown in

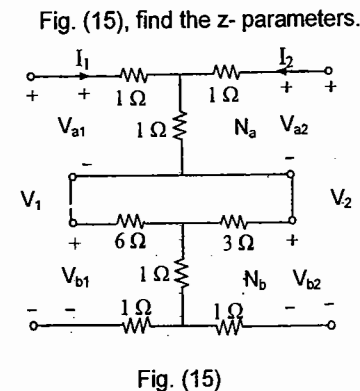


Fig. (15), find the z -parameters.

Cascade (chain or tandem) connection :

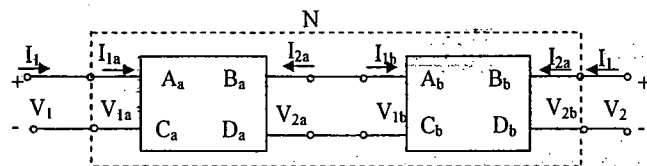


Fig. (16)

Fig. (16) illustrates this connection. Here the individual T- matrices get multiplied.

$$\{T\} = \{T_a\} \times \{T_b\} \rightarrow (10)$$

which is valid irrespective of whether the original networks have a common ground terminal or not.

Note that $A \neq A_a A_b$, etc. $A = A_a A_b + B_a C_b$, etc.

Prob: Find the z-matrix for the circuit in Fig. (17)

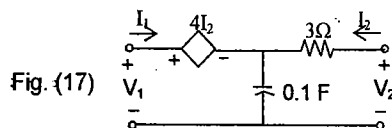
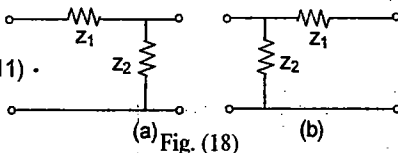


Fig. (17)

$$[Z] = \begin{bmatrix} \frac{10}{s} & \frac{4s+10}{s} \\ \frac{10}{s} & \frac{3s+10}{s} \end{bmatrix} \Omega$$

Prob: Find the T-matrices for the circuits shown in Fig. (18)

$$\{T\} = \begin{bmatrix} 1 + \frac{Z_1}{Z_2} & Z_1 \\ Y_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_1 \\ Y_2 & 1 + \frac{Z_1}{Z_2} \end{bmatrix} \rightarrow (11)$$



(a) Fig. (18)

(b)

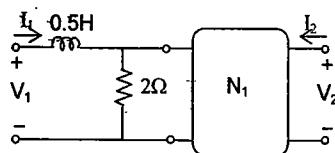


Fig. (19)

Prob: For the circuit shown in Fig. (19) $[T_{N_1}] = \begin{bmatrix} 8 & 4 \\ 2 & 5 \end{bmatrix}$. Find the T-matrix for the

overall 2-port, using the cascade connection and the result of the previous problem.

SET - A

01. The input impedance of a network having transmission parameter matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \text{ is}$$

- (A) A/C (B) A/B
(C) AB/DC (D) D/C

02. The condition for the electrical symmetry in the network is

$$(A) h_{12} = -h_{21} \quad (B) AD - BC = 1$$

$$(C) z_{12} = z_{21} \quad (D) A = D$$

03. Ideal transformer cannot be described by

- (A) h - parameter
(B) ABCD parameter
(C) g - parameter
(D) Z - parameter

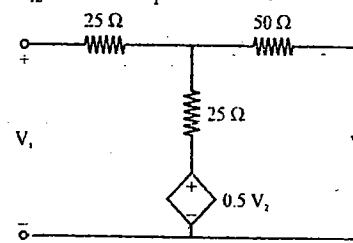
04. For a two port network Y_{22} is given by

- (A) $z_{11}/\Delta y$ (B) $z_{11}/\Delta z$
(C) $z_{22}/\Delta y$ (D) $z_{22}/\Delta z$

05. Two two-port networks are connected in cascade. The combination is to be represented as a single two port network. The parameters of the network are obtained by the multiplying the individuals

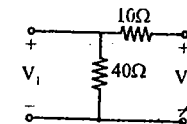
- (A) z - parameter matrix
(B) h - parameter matrix
(C) y - parameter matrix
(D) ABCD parameter matrix

06. z_{12} for the two port shown is



- (A) 25 (B) 50
(C) 75 (D) 100

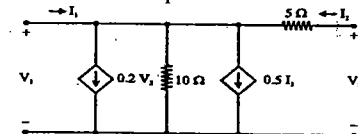
07. For the two - port shown in figure



The value of [h] parameter matrix is

- (A) $\begin{bmatrix} 40 & -40 \\ -40 & 50 \end{bmatrix}$ (B) $\begin{bmatrix} 40 & 40 \\ 40 & 50 \end{bmatrix}$
(C) $\begin{bmatrix} 0.025 & -0.025 \\ -0.025 & 0.02 \end{bmatrix}$ (D) $\begin{bmatrix} 8 & 0.8 \\ -0.8 & 0.02 \end{bmatrix}$

08. For the two - port network shown



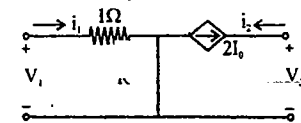
y_{11} and y_{21} are respectively

- (A) 0, 0.6 (B) 0.6, -0.2
(C) 0.6, 0 (D) 0.6, 0.2

09. In a two port network, the condition for reciprocity in terms of 'h' parameters is

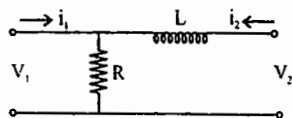
- (A) $h_{12} = h_{21}$ (B) $h_{11} = h_{12}$
(C) $h_{11} = -h_{22}$ (D) $h_{12} = -h_{21}$

10. If the current through 1 ohm is I_0 , the y - parameters of the network shown in matrix form is.



- (A) $\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} -1 & -2 \\ 0 & 0 \end{bmatrix}$
(C) $\begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix}$

11. Match List - I with List - II for the two port network shown and select the correct answer using the codes given below the Lists:



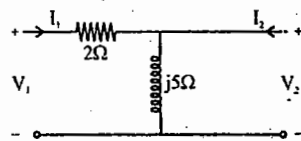
List - I

P. Z_{11}
Q. Z_{12}
R. Z_{21}
S. Z_{22}

Codes:

P	Q	R	S
(A) 1	2	1	4
(B) 2	1	1	3
(C) 1	1	1	4
(D) 2	1	3	4

12. The h-parameters (h_{11} , h_{12}) of the two-port network shown in figure are respectively



- (A) 2, 0
(B) 2, 1
(C) 0, 2
(D) 2, 5

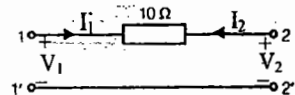
13. The impedance matrices of two, two-port networks are given by

$$\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} \text{ and } \begin{bmatrix} 15 & 5 \\ 5 & 25 \end{bmatrix}$$

If these two networks are connected in series, the impedance matrix of the resulting two-port network will be

- (A) $\begin{bmatrix} 3 & 5 \\ 2 & 25 \end{bmatrix}$ (B) $\begin{bmatrix} 18 & 7 \\ 7 & 28 \end{bmatrix}$
(C) $\begin{bmatrix} 15 & 2 \\ 5 & 3 \end{bmatrix}$ (D) indeterminate

14. The input voltage V_1 and current I_1 for a linear passive network is given by $V_1 = AV_2 - BI_2$ and $I_1 = CV_2 - DI_2$. Now consider the following network:

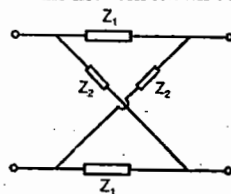


Which one of the following is the

transmission matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ of the network shown above?

- (A) $\begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix}$
(C) $\begin{bmatrix} 0 & 1 \\ 10 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 10 \\ 1 & 0 \end{bmatrix}$

15. What is the expression for h_{12} in respect of the network shown below?



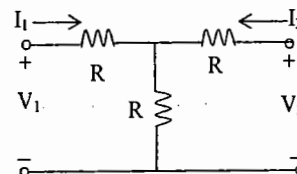
- (A) $\frac{Z_2 - Z_1}{Z_1 + Z_2}$ (B) $\frac{Z_1 + Z_2}{Z_2 - Z_1}$
(C) $\frac{Z_1 + Z_2}{Z_1 - Z_2}$ (D) $\frac{Z_1 - Z_2}{Z_1 + Z_2}$

Key for Set A: (Ch.7 (network theory))

- 01.D 02.D 03.C 04.B 05.D
06.D 07.D 08.B 09.D 10.D
11.C 12.B 13.B 14.B 15.A

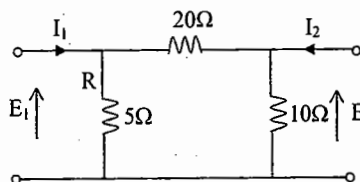
SET - B

01. A 2-port network is shown in the figure. The parameter h_{21} for this network can be given by



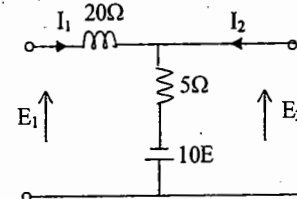
- (a) -1/2 (b) +1/2
(c) -3/2 (d) +3/2

02. The admittance parameter Y_{12} in the 2-port network in figure is



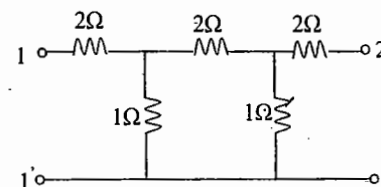
- (a) -0.2 mho (b) 0.1 mho
(c) -0.05 mho (d) 0.05 mho

03. The Z parameters Z_{11} and Z_{21} for the 2-port network in the figure are



- (a) $Z_{11} = \frac{-6}{11} \Omega; Z_{21} = \frac{16}{11} \Omega$
(b) $Z_{11} = \frac{6}{11} \Omega; Z_{21} = \frac{4}{11} \Omega$
(c) $Z_{11} = \frac{6}{11} \Omega; Z_{21} = \frac{-16}{11} \Omega$
(d) $Z_{11} = \frac{4}{11} \Omega; Z_{21} = \frac{4}{11} \Omega$

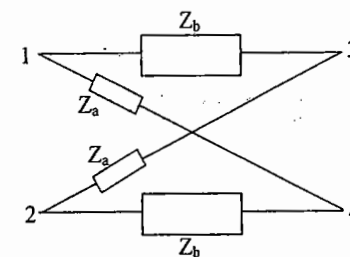
04. The impedance parameters Z_{11} and Z_{12} of the two-port network in the figure are



- (a) $Z_{11} = 2.75 \Omega$ and $Z_{12} = 0.25 \Omega$
(b) $Z_{11} = 3 \Omega$ and $Z_{12} = 0.5 \Omega$
(c) $Z_{11} = 3 \Omega$ and $Z_{12} = 0.25 \Omega$
(d) $Z_{11} = 2.25 \Omega$ and $Z_{12} = 0.5 \Omega$

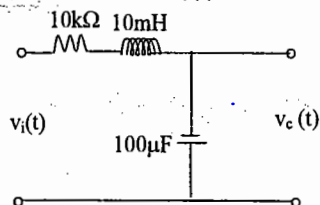
05. For the lattice circuit shown in the figure, $Z_a = j2\Omega$ and $Z_b = 2\Omega$. The values of the open circuit impedance parameters are

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$



- (a) $\begin{bmatrix} 1-j & 1+j \\ 1+j & 1+j \end{bmatrix}$ (b) $\begin{bmatrix} 1-j & 1+j \\ -1+j & 1-j \end{bmatrix}$
(c) $\begin{bmatrix} 1+j & 1+j \\ 1-j & 1-j \end{bmatrix}$ (d) $\begin{bmatrix} 1+j & -1+j \\ -1+j & 1+j \end{bmatrix}$

06. For the circuit shown in the figure, the initial conditions are zero. Its transfer function $H(s) = \frac{V_c(s)}{V_i(s)}$

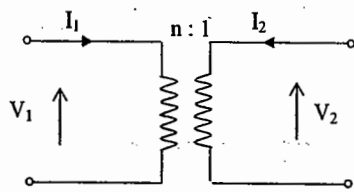


- (a) $\frac{1}{s^2 + 10^6 s + 10^6}$ (b) $\frac{10^6}{s^2 + 10^3 s + 10^6}$
 (c) $\frac{10^3}{s^2 + 10^3 s + 10^6}$ (d) $\frac{10^6}{s^2 + 10^6 s + 10^6}$

07. The ABCD parameters of an ideal n:1 transformer shown in the figure are

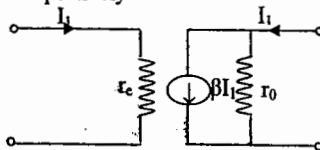
$$\begin{bmatrix} n & 0 \\ 0 & X \end{bmatrix}$$

The value of X will be



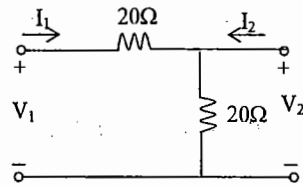
- (a) n (b) 1/n
 (c) n^2 (d) 1/n^2

08. In the two port network shown in the figure below, Z_{12} and Z_{21} are, respectively



- (a) r_c and βr_o (b) 0 and $-\beta r_o$
 (c) 0 and βr_o (d) r_c and $-\beta r_o$

09. The h parameters of the circuit shown in the figure are



- (a) $\begin{bmatrix} 0.1 & 0.1 \\ -0.1 & 0.3 \end{bmatrix}$ (b) $\begin{bmatrix} 10 & -1 \\ 1 & 0.05 \end{bmatrix}$
 (c) $\begin{bmatrix} 30 & 20 \\ 20 & 20 \end{bmatrix}$ (d) $\begin{bmatrix} 10 & 1 \\ -1 & 0.05 \end{bmatrix}$

10. A two-port network is represented by ABCD parameters given by

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

If port - 2 is terminated by R_L , the input impedance seen at port - 1 is given by

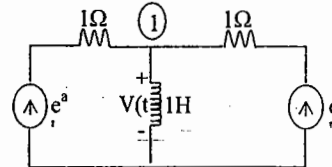
- (a) $\frac{A + BR_L}{C + DR_L}$ (b) $\frac{AR_L + C}{BR_L + D}$
 (c) $\frac{DR_L + A}{BR_L + C}$ (d) $\frac{B + AR_L}{D + CR_L}$

KEYS:

01. a 02. c 03. c 04. a 05. d 06. d
 07. b 08. b 09. d 10. d

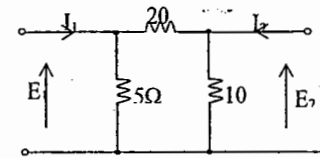
SET - C

01. In the circuit of the figure, the voltage $v(t)$ is GATE-2000



- (a) $e^{at} - e^{bt}$ (b) $e^{at} + e^{bt}$
 (c) $ae^{at} - be^{bt}$ (d) $ae^{at} + be^{bt}$

02. The admittance parameter Y_{12} in the 2-port network in Figure is

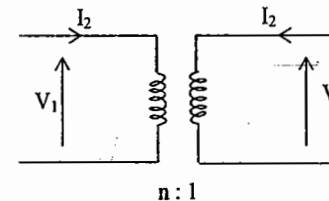


- (a) -0.2 mho (b) 0.1 mho
 (c) -0.05 mho (d) 0.05 mho

03. The ABCD parameters of an ideal n:1 transformer shown in the figure are

$$\begin{bmatrix} n & 0 \\ 0 & X \end{bmatrix}$$

The value of X will be



- (a) n (b) 1/n
 (c) n^2 (d) 1/n^2

04. A two-port network is represented by ABCD parameters given by

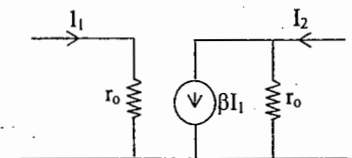
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

If port-2 is terminated by R_L , the input impedance seen at port-1 is given by

GATE - 2006

- (a) $\frac{A + BR_L}{C + DR_L}$ (b) $\frac{AR_L + C}{BR_L + D}$
 (c) $\frac{DR_L + A}{BR_L + C}$ (d) $\frac{B + AR_L}{D + CR_L}$

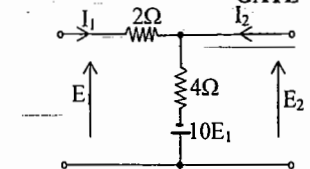
05. In the two port network shown in the figure below, Z_{12} and Z_{21} are, respectively GATE - 2006



- (a) r_c and βr_o (b) 0 and $-\beta r_o$
 (c) 0 and βr_o (d) r_c and $-\beta r_o$

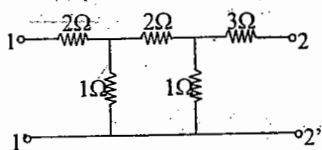
06. The Z parameters Z_{11} and Z_{21} for the 2-port network in the figure are

GATE - 2001



- (a) $Z_{11} = \frac{-6}{11} \Omega$; $Z_{21} = \frac{16}{11} \Omega$;
 (b) $Z_{11} = \frac{6}{11} \Omega$; $Z_{21} = \frac{4}{11} \Omega$;
 (c) $Z_{11} = \frac{6}{11} \Omega$; $Z_{21} = \frac{-16}{11} \Omega$;
 (d) $Z_{11} = \frac{4}{11} \Omega$; $Z_{21} = \frac{4}{11} \Omega$;

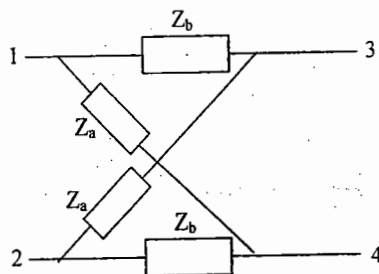
07. The impedance parameters Z_{11} and Z_{12} of the two-port network in the figure are
GATE - 2003



- (a) $Z_{11}=2.75\ \Omega$ and $Z_{12}=0.25\ \Omega$
 (b) $Z_{11}=3\ \Omega$ and $Z_{12}=0.5\ \Omega$
 (c) $Z_{11}=3\ \Omega$ and $Z_{12}=0.25\ \Omega$
 (d) $Z_{11}=2.25\ \Omega$ and $Z_{12}=0.5\ \Omega$

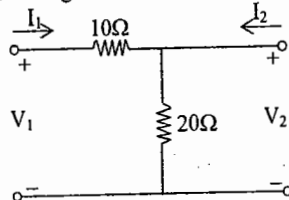
08. For the lattice circuit shown in the figure, $Z_a=j2\ \Omega$ and $Z_b=2\ \Omega$. The values of the open circuit impedance parameters

$$Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \text{ are GATE - 2004}$$



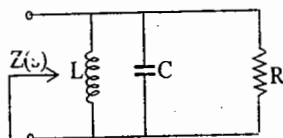
- (a) $\begin{bmatrix} 1-j & 1+j \\ 1+j & 1-j \end{bmatrix}$ (b) $\begin{bmatrix} 1-j & 1+j \\ -1+j & 1-j \end{bmatrix}$
 (c) $\begin{bmatrix} 1+j & 1+j \\ 1-j & 1-j \end{bmatrix}$ (d) $\begin{bmatrix} 1+j & -1+j \\ -1+j & 1+j \end{bmatrix}$

09. The h parameters of the circuit shown in the figure are
GATE - 2005



- (a) $\begin{bmatrix} 0.1 & 0.1 \\ -0.1 & 0.3 \end{bmatrix}$ (b) $\begin{bmatrix} 10 & -1 \\ 1 & 0.05 \end{bmatrix}$
 (c) $\begin{bmatrix} 30 & 20 \\ 20 & 20 \end{bmatrix}$ (d) $\begin{bmatrix} 10 & 1 \\ -1 & 0.05 \end{bmatrix}$

10. The driving point impedance of the following network
GATE - 2008



is given by $Z(s) = \frac{0.2s}{s^2 + 0.1s + 2}$. The component values are

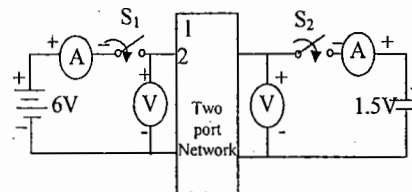
- (a) $L = 5H, R = 0.5\ \Omega, C = 0.1F$
 (b) $L = 0.1H, R = 0.5\ \Omega, C = 5F$
 (c) $L = 5H, R = 2\ \Omega, C = 0.1F$
 (d) $L = 0.1H, R = 2\ \Omega, C = 5F$

- Linked Answer Questions : Q.11 to Q.12 carry two marks each. Statement for linked Answer Questions 11 and 12:

GATE - 2008

A two-port network shown below is excited by external dc sources. The voltages and currents are measured with voltmeters V_1, V_2 and ammeters A_1, A_2 (all assumed to be ideal), as indicated under following switch conditions, the readings obtained are:

- (i) S_1 - Open, S_2 - closed $A_1 = 0A$, $V_1 = 4.5V, V_2 = 1.5V, A_2 = 1A$
 (ii) S_1 - Closed, S_2 - Open, $A_1 = 4A$, $V_1 = 6V, V_2 = 6V, A_2 = 0A$



11. The z-parameter matrix for this network is

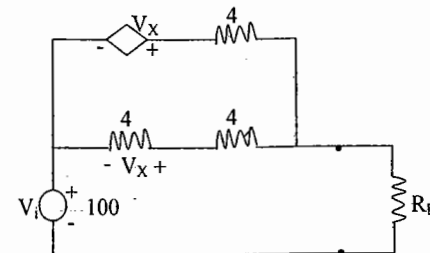
- (a) $\begin{bmatrix} 1.5 & 1.5 \\ 4.5 & 1.5 \end{bmatrix}$ (b) $\begin{bmatrix} 1.5 & 4.5 \\ 1.5 & 4.5 \end{bmatrix}$
 (c) $\begin{bmatrix} 1.5 & 4.5 \\ 1.5 & 1.5 \end{bmatrix}$ (d) $\begin{bmatrix} 4.5 & 1.5 \\ 1.5 & 4.5 \end{bmatrix}$

12. The h-parameter matrix for this network is

- (a) $\begin{bmatrix} -3 & 3 \\ -1 & 0.67 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & -1 \\ 3 & 0.67 \end{bmatrix}$
 (c) $\begin{bmatrix} 3 & 3 \\ 1 & 0.67 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 1 \\ -3 & -0.67 \end{bmatrix}$

13. In the circuit shown, what value of R_L maximizes the power delivered to R_L ?

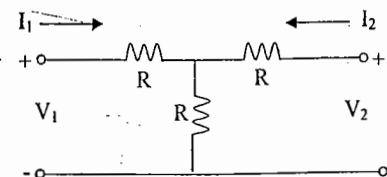
GATE 2009



- a) $2.4\ \Omega$ b) $8/3\ \Omega$ c) $4\ \Omega$ d) $6\ \Omega$

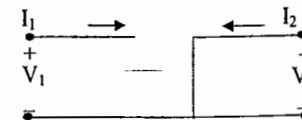
14. A 2-port network is shown in the figure. The parameter h_{21} for this network can be given by

GATE-99



- (a) $-1/2$ (b) $+1/2$
 (c) $-3/2$ (d) $+3/2$

15. The parameter type and the matrix representation of the relevant two port parameters that describe the circuit shown are



- (a) z parameters,
 (b) h parameters,
 (c) h parameters,
 (d) z parameters,

KEY SET - C

- 01.d 02.c 03.b 04.d 05.b 06.c
 07.a 08.d 09.d 10.d 11.c 12.a

Chapter - 7: State equations for network

In the method of Loop (mesh) analysis, network equations are formulated using loop currents as independent variables, which can be determined by solving the equations. In the method of nodal analysis, network equations are formulated using node-to-datum voltages as independent variables, which can be determined by solving the equations. Any voltage and current can be expressed in terms of either loop currents or node-to-datum voltages.

There is a third method of analysis known as state variable analysis, where network equations are formulated using state variables as independent variables. Any voltage or current in a network at any time 't' can be obtained by knowing the initial state of the system (inductor currents and capacitor voltages at $t = 0$). The state variables usually selected are the capacitor voltages and inductor currents. The particular advantage of the state variable formulation of network equations is that it is in a specific form especially suited for computer solution. State variable equations for a second order network with two excitations are given below:

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + b_{11}u_1 + b_{12}u_2, \quad \dot{x}_2 = a_{21}x_1 + a_{22}x_2 + b_{21}u_1 + b_{22}u_2 \quad (1)$$

where $x_1(t)$ and $x_2(t)$ are state variables and $u_1(t)$ and $u_2(t)$ are excitations in the network and the symbol '·' over a variable indicates time differentiation. These equations can be put in a compact way (in vector-matrix form) as

$$\dot{\vec{x}} = \vec{A}\vec{x} + \vec{B}\vec{u} \quad (2)$$

where $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is the state vector, $\vec{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ is the input vector

$$\vec{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad \vec{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \quad (3)$$

\vec{A} is known as Bashkow matrix (\vec{A} is in general known as system matrix where the system in the present case is the network)

Extension to an n th order network with r excitations is straight forward. Then $\vec{x}(t)$ is an $n \times 1$ column vector, $\vec{u}(t)$ is an $r \times 1$ column vector, \vec{A} is an $n \times n$ square matrix and \vec{B} is an $n \times r$ rectangular matrix. If there are m desired outputs (y_1, y_2, \dots, y_m) they can be represented as an $m \times 1$ column vector and the output equation is written as

$$\vec{y} = \vec{C}\vec{x} + \vec{D}\vec{u} \quad (4)$$

where \vec{C} is an $m \times n$ matrix and \vec{D} is an $m \times r$ matrix.

The following steps are followed in formulating state equations:

1. Select a tree containing all capacitors but no inductors.
2. The state variables are the branch (capacitor) voltages in this tree and the inductor currents in the chords (links).
3. Write a node equation for each capacitor.
4. Write a loop equation using each inductor as a chord in the tree of (1).

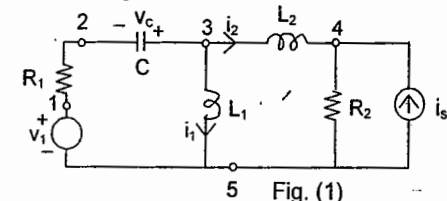
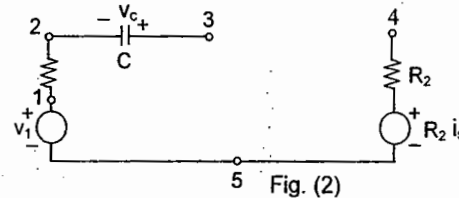
5. Manipulate the equations in (3) and (4) as may be necessary until they appear in the standard form of equation (2).

Exa: 1. Obtain the state equations for the network shown in Fig. 1.

The state and input vectors are taken as

$$\vec{x} = \begin{bmatrix} v_c \\ i_1 \\ i_2 \end{bmatrix}, \quad \vec{u} = \begin{bmatrix} v_1 \\ i_s \end{bmatrix}$$

A tree is selected as shown in Fig. 2



At node 3 connected to C (Fig. 3), apply KCL to get node equation:

$$C \dot{v}_c + i_1 + i_2 = 0 \quad (5)$$

Apply KVL to the loop (Fig. 4), formed by the chord L_1 to get the loop equation:

$$L_1 \dot{i}_1 - v_1 + R_1 i_1 - v_c = 0 \quad (6)$$

Apply KVL to the loop (Fig. 5), formed by the chord L_2 to get the loop equation:

$$L_2 \dot{i}_2 + R_2(i_2 + i_s) - v_1 + R_1 i_2 - v_c = 0 \quad (7)$$

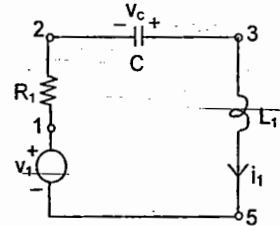


Fig. (4)

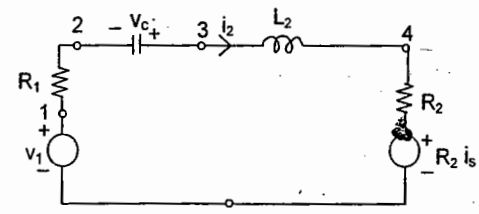


Fig. (5)

Equations (5), (6) and (7) are put in the standard form of state equations (1).

$$\dot{v}_c = 0 v_c - \frac{1}{C} i_1 - \frac{1}{C} i_2 + 0 v_1 + 0 i_s \quad (8)$$

$$\dot{i}_1 = \frac{1}{L_1} v_c - \frac{R_1}{L_1} i_1 + 0 i_2 + \frac{1}{L_1} v_1 + 0 i_s \quad (9)$$

$$\dot{i}_2 = \frac{1}{L_2} v_c + 0 i_1 - \frac{R_1 + R_2}{L_2} i_2 + \frac{1}{L_2} v_1 - \frac{R_2}{L_2} i_s \quad (10)$$

Identifying these equations in vector-matrix form of state equation (2),

$$\vec{\dot{x}} = \vec{A}\vec{x} + \vec{B}\vec{u}$$

$$A = \begin{bmatrix} 0 & \frac{1}{C} & \frac{1}{C} \\ \frac{1}{L_1} & -\frac{R_1}{L_1} & 0 \\ \frac{1}{L_2} & 0 & -\frac{R_1+R_2}{L_2} \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ \frac{1}{L_1} & 0 \\ \frac{1}{L_2} & -\frac{R_2}{L_2} \end{bmatrix}$$

Exa: 2. Obtain the state model for the network shown:

The state variables are $x_1 = v_c$, $x_2 = i_L$
Input is $u_1 = v_s$

KCL applied at node 3 gives

$$C \dot{v}_c = i_L, \quad \dot{v}_c = 0 v_c + \frac{1}{C} i_L + 0 v_s \longrightarrow (11)$$

KVL applied to the loop gives

$$L \dot{i}_L - v_s + R i_L + v_c = 0, \quad \dot{i}_L = -\frac{1}{L} v_c - \frac{R}{L} i_L + \frac{1}{L} v_s \quad (12)$$

Identifying equations (11) and (12) in the form of state equation (2)

$$\dot{\vec{x}} = \begin{bmatrix} \dot{v}_c \\ \dot{i}_L \end{bmatrix} \quad A = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} \quad (13)$$

Exa: 3 Obtain the state model of the circuit shown in Fig. (7)

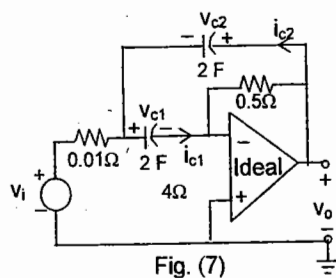


Fig. (7)

Current through the capacitor, C_1

$$2 \dot{v}_{c1} = i_{c1}$$

Current through the capacitor, C_2

$$2 \dot{v}_{c2} = i_{c2}$$

voltage across $0.5 \Omega = v_{c1} + v_{c2}$

Current through 0.5Ω is i_{c1} as the opamp is ideal

$$i_{c1} = -(v_{c1} + v_{c2}) / 0.5$$

Applying KCL at the junction of 0.01Ω and $2 F$

$$i_{c2} = i_{c1} + (v_{c1} - v_i) / 0.01$$

Putting the above equations in the standard form

$$\dot{v}_{c1} = -v_{c1} - v_{c2} + 0 v_i \quad (14)$$

$$\dot{v}_{c2} = 49 v_{c1} - v_{c2} - 50 v_i \quad (15)$$

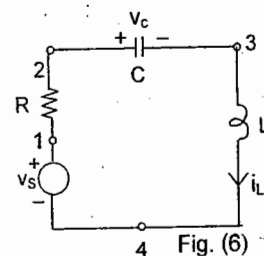


Fig. (6)

$$A = \begin{bmatrix} -1 & -1 \\ 49 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ -50 \end{bmatrix} \quad (16)$$

If $y = v_o$ is the output desired, the output equation in the standard form of equation (4) is given by

$$y = C \vec{x}, \quad C = [1 \ 1] \quad (17)$$

Exa: 4. Obtain the state model of the circuit shown in Fig. (8)

Take $x_1 = v_c$ and $x_2 = i_L$ as the state variables and $u_1 = v_i$ as the input

Nodal equation with capacitor current is

$$C \dot{v}_c = i_L + i_1 - i_2 \quad (18)$$

Loop equation with inductor current is

$$2 i_L + L \dot{i}_L + 4 C \dot{v}_c + v_c - v_i = 0 \quad (19)$$

$$\text{voltage of node 1} = 4 C \dot{v}_c + v_c \quad (20)$$

$$i_2 = (1/4) (4 C \dot{v}_c + v_c) \quad (21)$$

$$i_1 = (1/8) (v_i - 4 C \dot{v}_c - v_c) \quad (22)$$

The above equations can be manipulated to give the state equations

$$\dot{v}_c = -1.5 v_c + 4 i_L + 0.5 v_i \quad (23)$$

$$\dot{i}_L = -2 v_c - 18 i_L + 4 v_i \quad (24)$$

Prob: 1. Taking v , i_1 and i_2 as state variables in the circuit shown in Fig. (9), show that the state equation is (25)

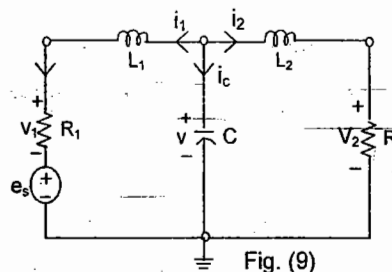


Fig. (9)

$$\begin{bmatrix} \frac{dv}{dt} \\ \frac{di_1}{dt} \\ \frac{di_2}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{C} & -\frac{1}{C} \\ \frac{1}{L_1} & -\frac{R_1}{L_1} & 0 \\ \frac{1}{L_2} & 0 & -\frac{R_2}{L_2} \end{bmatrix} \begin{bmatrix} v \\ i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -1/4 \\ 0 \end{bmatrix} e_s$$

Equation (25)

Taking the desired outputs as $y_1 = i_c$, $y_2 = v_1$ find the matrices C and D in the standard output equation (4). Taking the state variables as charge q in C , ϕ_1 and ϕ_2 as fluxes in the inductors L_1 and L_2 show that the state equation is (26).

$$\begin{bmatrix} \dot{q} \\ \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L_1} & -\frac{1}{L_2} \\ \frac{1}{C} & -\frac{R_1}{L_1} & 0 \\ \frac{1}{C} & 0 & -\frac{R_2}{L_2} \end{bmatrix} \begin{bmatrix} q \\ \phi_1 \\ \phi_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} e_s$$

Equation (26)

Prob: 2. Obtain the state models for the circuits in Fig. (10) and Fig. (11)

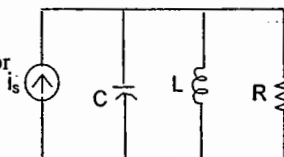


Fig. (10)

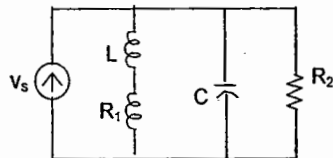


Fig. (11)

Solution of state equation:

$$\dot{\vec{x}} = \vec{A}\vec{x} + \vec{B}\vec{u} \quad (2)$$

with n state variables and r inputs. Let $\vec{x}(t) \xrightarrow{LT} \vec{X}(s)$, $\vec{u}(t) \xrightarrow{LT} \vec{U}(s)$

Then $\vec{x}(t) \rightarrow s \vec{X}(s) - \vec{x}(0)$ (27) where $\vec{x}(0)$ is the initial state vector at $t=0$.

Eqn. (2) is converted to $(sI - \vec{A})\vec{X}(s) = \vec{B}\vec{U}(s) + \vec{x}(0)$ (28)

where I is identity matrix. $\vec{X}(s) = \phi(s)\vec{B}\vec{U}(s) + \phi(s)\vec{x}(0)$ (29)

$$\text{where } \phi(s) = (sI - \vec{A})^{-1} \quad (30)$$

Taking inverse L.T.

$$\vec{x}(t) = \phi(t) * \vec{B}\vec{u}(t) + \phi(t)\vec{x}(0) \quad (31)$$

$$= \text{Z.S.R} + \text{Z.I.R} \quad (32)$$

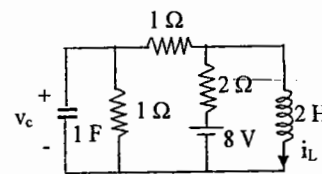
where $\phi(t)$ = Inverse LT of $\phi(s)$ is known as the $n \times n$ State Transition Matrix (STM)

Z.S.R is Zero State Response, Z.I.R is Zero Input Response

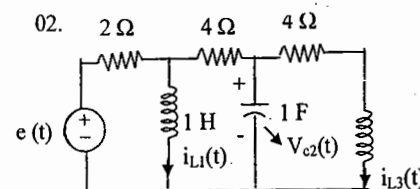
The outputs are calculated using $\vec{y}(t) = \vec{C}\vec{x}(t) + \vec{D}\vec{u}(t)$ (4)

OBJECTIVE QUESTIONS

1. For the circuit in the figure write the state equations using v_c and i_L as state variables. (Gate 2000) (EE)



02.



For the circuit shown in the figure above choose state variables X_1, X_2, X_3 to be $i_{L1}(t), V_{c2}(t), i_{L3}(t)$ (Gate 1997) (EC)

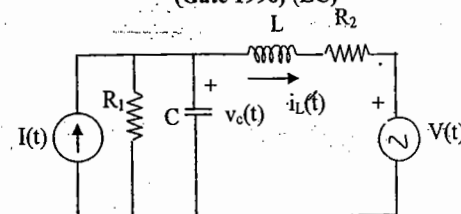
(a) Write the state equations,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \vec{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \vec{B} e(t)$$

(b) If $e(t) = 0$, $t \geq 0$, $i_{L1}(0) = 0$, $V_{c2}(0) = 0$, $i_{L3}(0) = 1A$. Then what would be the total energy dissipated in the resistors in the interval $(0, \infty)$ be?

3. Refer to the circuit shown,

(Gate 1996) (EC)



Choosing the voltages $v_c(t)$ across the capacitor, and the current $i_L(t)$ through the inductor as state variables, ie

$$\begin{bmatrix} \dot{x}(t) \end{bmatrix} = \begin{bmatrix} v_c(t) \\ i_L(t) \end{bmatrix}$$

Write the state equation in the form

$$\frac{d}{dt}[\vec{x}(t)] = [\vec{A}][\vec{x}(t)] + [\vec{B}][\vec{u}(t)]$$

and find $[\vec{A}]$, $[\vec{B}]$, and $[\vec{u}(t)]$

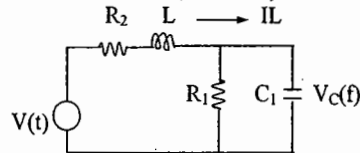
04. Which one of the following is NOT a correct statement about the state-space model of a physical system?

(I.E.S -2003)

- State-space model can be obtained only for a linear system
- Eigenvalues of the system represent the roots of the characteristic equation.
- $\dot{X} = AX + BU$ represents linear state-space model of a physical system.
- $X(t)$ represents the state vector of the system.

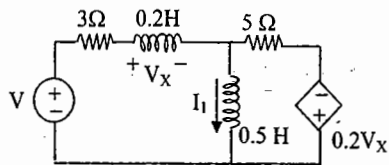
05. Which one of the following is the correct state model for the network shown in the given figure with $x_2(t) = I_L(t)$ and $x_1(t) = v_C(t)$?

(I.E.S -94)



- a) $\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1/R_1 C_1 & 1/C_1 \\ -1/L_2 & -R_2/L_2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L_2 \end{bmatrix} v(t)$
- b) $\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -R_2/L_2 & 1/C_1 \\ -1/L_2 & -1/R_1 C_1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L_2 \end{bmatrix} v(t)$
- c) $\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -R_2/L_2 & -1/L_2 \\ -1/C_1 & -1/R_1 C_1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1/L_2 \\ 0 \end{bmatrix} v(t)$
- d) $\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1/R_1 C_1 & -1/L_1 \\ -1/C_1 & -R_2/L_2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1/L_2 \\ 0 \end{bmatrix} v(t)$

06. The state equation for the current I_1 shown in the network shown below in terms of the voltage V_x and the independent source V , is given by



- (a)
- (b)
- (c)
- (d)

07. Which one of the following state-space models is the correct representation of the physical system described by the differential equation

$$\frac{d^2}{dt^2} y(t) + 4 \frac{dy(t)}{dt} + 6y(t) \quad (\text{I.E.S -97})$$

a) $\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 8 \end{bmatrix} u(t)$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

b) $\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 8 \end{bmatrix} u(t)$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

c) $\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 8 \end{bmatrix} u(t)$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

d) $\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 8 \end{bmatrix} u(t)$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

08. With the conventional notation $\dot{X} = AX + BU$ for the state description of a linear time-invariant network, examine the validity of the following statements relating to the matrix A: (I.E.S -2003)

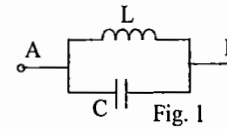
1. A is symmetrical if the network is reciprocal
 2. The sum of the natural frequencies of the network is equal to the determinant of A.
- Which of these statements is/are true?
- a) Both 1 and 2 b) 1 only
c) 2 only d) neither 1 nor 2

"To win the RACE join the ACE"

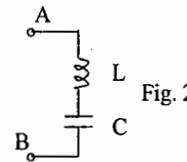
Chapter 8: Elements of Network Synthesis (LC, RC, RL, Foster and Cauer Forms)

Parallel & Series Structures

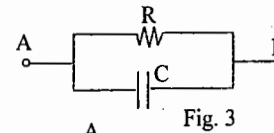
DP Z(s) and Y(s) across the terminals A, B



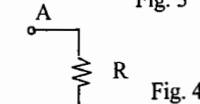
$$Y(s) = Cs + \frac{1}{Ls}, \quad Z(s) = \frac{1}{\frac{1}{Ls} + Cs} \quad (1)$$



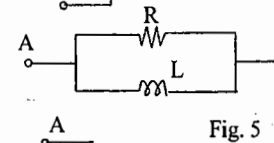
$$Z(s) = Ls + \frac{1}{Cs}, \quad Y(s) = \frac{1}{Ls + \frac{1}{Cs}} \quad (2)$$



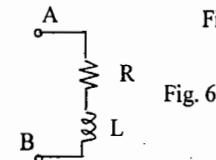
$$Y(s) = \frac{1}{R} + Cs, \quad Z(s) = \frac{1}{\frac{1}{R} + Cs} \quad (3)$$



$$Z(s) = R + \frac{1}{Cs}, \quad Y(s) = \frac{1}{R + \frac{1}{Cs}} \quad (4)$$



$$Y(s) = \frac{1}{R} + \frac{1}{Ls}, \quad Z(s) = \frac{Rs}{s + \frac{R}{L}} \quad (5)$$



$$Z(s) = R + Ls, \quad Y(s) = \frac{1}{R + Ls} \quad (6)$$

Observe the similarity between the relations 3 & 6 ; 4 & 5

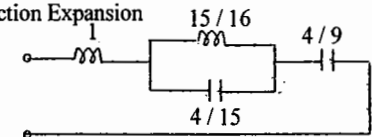
LC NETWORK SYNTHESIS

Foster Forms

$$Z(s) = \frac{(s^2 + 1)}{s} \frac{(s^2 + 9)}{(s^2 + 4)}, \quad \text{Partial Fraction Expansion}$$

$$= K_1 s + \frac{K_2}{s} + \frac{K_3 s}{s^2 + 4}$$

Fig. 7



FOSTER - I FORM

SET - A

01. The poles and zeros of a driving-point function of a network are simple and interlace on the negative real axis with a pole closest to the origin. It can be realized
- by an LC network
 - as an RC driving-point impedance
 - only by an RLC network.
 - None

02. Consider the following from the point of view of possible realization as driving-point impedances using two passive elements : L & C or R & L or R & C.

$$(1) \frac{1}{s(s+5)} \quad (2) \frac{(s+3)}{s^2(s+5)}$$

$$(3) \frac{(s^2+3)}{s^2(s^2+5)} \quad (4) \frac{(s+5)}{s(s+3)}$$

Among these

- 1, 2 and 4 are realizable
- 1, 2 and 3 are realizable
- 3 and 4 are realizable
- None is realizable

03. The driving point admittance of a network is given by

$$Y(s) = \frac{s^2 + 4s + 3}{s(s+2)}$$

The minimum number of elements required to realise this network is

- 5
- 2
- 3
- 4

04. Consider the following statements regarding the driving-point admittance function

$$Y(s) = (s^2 + 2.5s + 1) / (s^2 + 4s + 3)$$

- It is an admittance of RL network
- Poles and zeros alternate on the negative real axis of s-plane
- lowest critical frequency is a pole
- $Y(0) = 1/3$

Which of these statements are correct?

- 1, 2 & 3
- 2 & 4
- 1 and 3
- 1, 2, 3 and 4

05. Match List-I with List-II for the driving-point impedance synthesis and select the correct answer using the codes given below the lists:

List - I (form)

P. Cauer I

Q. Cauer II

R. Foster I

S. Foster II

List - II (Network)

1. L in series arms and C in shunt arms of a ladder

2. C in series arms and L in shunt arms of a ladder

3. Series combination of L and C in parallel

4. Parallel combination of L and C in series

Codes:

	P	Q	R	S
(A) 1	2	3	4	
(B) 1	2	4	3	
(C) 2	1	4	3	
(D) 2	1	3	4	

06. The network function

$$F(s) = \frac{(s+2)}{(s+1)(s+3)}$$
 represents an

- RC impedance
- RL impedance
- RC impedance and an RL admittance
- RC admittance and an RL impedance

07. The driving - point impedance of a one - port reactive network is given by

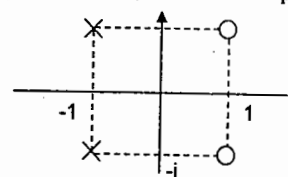
$$(A) \frac{(s^2 + 1)(s^2 + 2)}{s(s^2 + 3)(s^2 + 4)}$$

$$(B) \frac{(s^2 + 1)(s^2 + 3)}{s(s^2 + 2)(s^2 + 4)}$$

$$(C) \frac{s(s^2 + 1)}{(s^2 + 2)(s^2 + 3)}$$

$$(D) \frac{1}{s+1}$$

08. If an impedance has the pole-zero pattern shown., it must be composed of



- RC elements only
- RL elements only
- LC elements only
- RLC elements only

09. For an RC driving-point impedance function the poles and zeros

- should alternate on real axis
- should alternate only on the negative real axis
- should alternate on the imaginary axis
- can lie anywhere in the left half plane

10. Match List-I with List-II using the codes given below :

List - I [F(s)]

P. $(s^2 - s + 4) / (s^2 + s + 4)$

Q. $(s + 4) / (s^2 + 3s - 4)$

R. $(s + 4) / (s^2 + 6s + 5)$

S. $(s^3 + 3s) / (s^4 + 2s^2 + 1)$

List - II [Type of F(s)]

- Non - positive real
- Non-minimum phase
- RC - impedance
- Unstable
- RL - impedance

	P	Q	R	S
(A) 1	2	3	4	
(B) 1	2	4	5	
(C) 2	4	3	1	
(D) 2	4	1	5	

11. Which one of the following is a positive real function ?

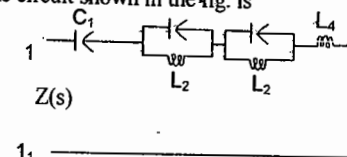
(A) $s(s^2 + 4) / (s^2 + 1)(s^2 + 6)$

(B) $s(s^2 - 4) / (s^2 + 1)(s^2 + 6)$

(C) $(s^3 + 3s^2 + 2s + 1) / 4s$

(D) $s(s^4 + 3s^2 + 1) / (s+1)(s+2)(s+3)(s+4)$

12. The circuit shown in the fig. is

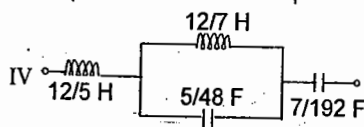
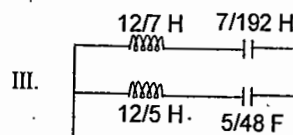
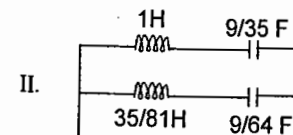
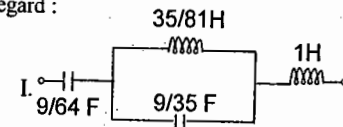


- Cauer I form
- Foster I form
- Cauer II form
- Foster II form

13. The driving point impedance function of a reactive network is :

$$Z(s) = (s^2 + 4)(s^2 + 16) / s(s^2 + 9)$$

Consider the following circuit in this regard :



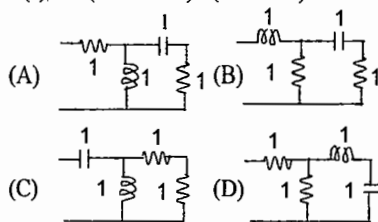
The first and second Foster forms will be as in figures

- I and III respectively
- II and IV respectively
- I and II respectively
- III and IV respectively

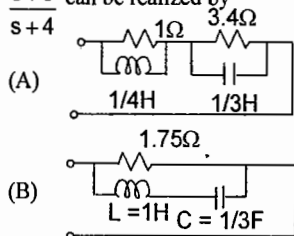
14. The first critical frequency nearest the origin of the complex frequency plane for an R - L driving point impedance function will be

- a zero in the left-half plane
- a zero in the right-half plane
- a pole in the left-half plane
- either a pole or zero in the left-half plane depending on the connection

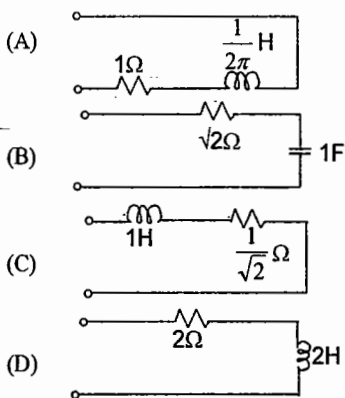
15. Which one of the following circuits has a driving-point impedance of $Z(s) = 2(s^2 + s + \frac{1}{2}) / (s^2 + s + 1)$?



16. The driving point impedance function $\frac{s+3}{s+4}$ can be realized by



- (C) neither (A) nor (B) above
(D) both (A) and (B) above
17. The driving point impedance of a network at a frequency of 1 Hz is $\sqrt{2}j$. The impedance can be realized as :

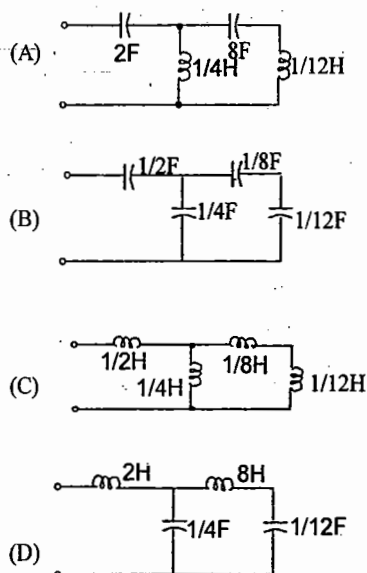


18. An RC driving-point impedance function has zeros at $s = -2$ and $s = -5$. The admissible poles for the function would be
(A) $s = 0$; $s = -6$ (B) $s = -1$; $s = -3$
(C) $s = 0$; $s = -1$ (D) $s = -3$; $s = -4$

19. The driving-point impedance function of a reactive network is:

$$Z_D(s) = \frac{2(s^2 + 1)(s^2 + 3)}{s(s^2 + 2)}$$

Which one of the following diagrams realizes the Cauer network for the above $Z_D(s)$?



20. Match List - I and List - II and select the correct answer using the codes given below the lists:

List - I

- P. $(s^2 - s + 1) / (s^2 + s + 1)$
Q. $(s^2 + s + 1) / (s^2 - s + 1)$
R. $(s^2 + 4s + 3) / (s^2 + 6s + 8)$

List - II

1. RL admittance
2. RL impedance
3. Unstable
4. Non-minimum phase

	P	Q	R
(A)	1	2	3
(B)	1	4	2
(C)	4	3	2
(D)	4	3	1

21. The driving point impedance of a network is given by

$$Z(s) = \frac{s^2 + 4s + 3}{s(s + 2)}$$

The number of energy storing elements present in the network is

- (A) 1 (B) 2 (C) 3 (D) 4

Key for Test 6:

1. B 2. D 3. D 4. B 5. A 6. C
7. B 8. D 9. B 10. C 11. A 12. B
13. A 14. A 15. A 16. C 17. A 18. B
19. D 20. C 21. B

SET - B

01. A pole of driving point admittance function implies
(A) zero current for a finite value of driving voltage
(B) zero voltage for a finite value of driving current
(C) an open circuit condition
(D) none of (A), (B) and (C) mentioned in the question
02. Cauer and foster forms of realizations are used only for
(A) driving point reactance functions
(B) transfer reactance functions
(C) driving point impedance functions
(D) transfer impedance functions

03. Driving point impedance

$$Z(s) = \frac{s(s^2 + 1)}{s^2 + 4}$$

is not realizable because the

- (A) number of zeros is more than the number of poles.
(B) poles and zeros lie on the imaginary axis.
(C) poles and zeros do not alternate on imaginary axis.
(D) poles and zeros are not located on the real axis.

04. Poles and zeros of a driving point

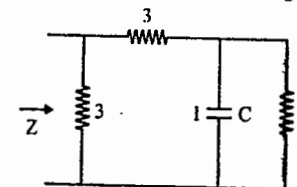
function of a network are simple and interlace on the $j\omega$ axis. The network consists of elements

- (A) R and C (B) L and C
(C) R and L (D) R, L and C

05. A Hurwitz polynomial has

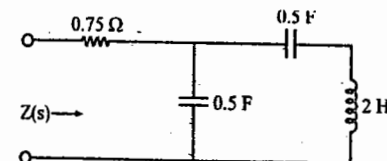
- (A) zeros only in the left half of the s -Plane.
(B) poles only in the left half of the s -Plane.
(C) zeros anywhere in the s -plane.
(D) poles on the $j\omega$ axis only.

06. The driving point impedance Z is given by



- (A) $K_1 \frac{s+2}{s+3}$ (B) $K_2 \frac{s+3}{s+2}$
(C) $K_3 \frac{s + \frac{1}{2}}{s + \frac{1}{3}}$ (D) $K_4 \frac{s + \frac{1}{3}}{s + \frac{1}{2}}$

07. The driving point function of the circuit shown in the given figure when $s \rightarrow 0$ and $s \rightarrow \infty$ (the elements are normalized) will respectively be



- (A) $1/s$ and $2/s$ (B) $1/s$ and 0.75
(C) 0.75 and $2/s$ (D) $2/s$ and 0.75

08. For a driving point impedance function

$Z(s) = \frac{s + \alpha}{s + \beta}$; the voltage will lead the current sinusoidal input, if

- (A) α & β real positive and $\alpha > \beta$
 (B) α is real positive and β is real negative and $\alpha > \beta$
 (C) α and β are real positive and $\beta > \alpha$
 (D) α and β are real negative and $\beta > \alpha$

09. The driving point impedance function

$Z(s) = \frac{s^2 + 2s + 2}{s^2 + s + 1}$ can be realized as a

- (A) R - C network
 (B) R - L network
 (C) L - C network
 (D) R - L - C network

10. Voltage transfer function of a simple RC integrator has

- (A) a finite zero and a pole at infinity
 (B) a finite zero and a pole at the origin
 (C) a zero at the origin and a finite pole
 (D) a zero at infinity and a finite pole

11. Match the List - I (Network) with List - II (Poles of driving - point impedance) and select the correct answer using the codes given below the

Lists:

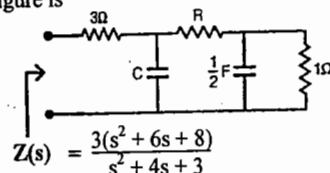
List - I
 P. LC
 Q. RC
 R. RLC
 S. RL

List - II
 1. Negative real
 2. imaginary
 3. Either real or complex

Codes:

P	Q	R	S
(A) 1	2	3	1
(B) 1	2	1	3
(C) 2	1	1	3
(D) 2	1	3	1

12. $Z(s)$ for the network shown in the figure is



The value of C and R are, respectively

- (A) 1/6 F and 4 Ω
 (B) 2/9 F and 9/2 Ω
 (C) 2/3 F and 1/2 Ω
 (D) 1/2 F and 1 Ω

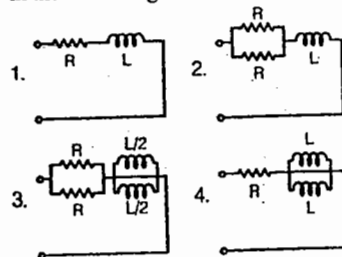
13. The function $s + 2 + \frac{3}{s}$ can be realized

- (A) both as a driving point impedance and as a driving point admittance
 (B) as an impedance, but not as an admittance
 (C) as an admittance, but not as an impedance
 (D) neither as an impedance nor as an admittance

14. Which one of the following statements is not a property of R - L driving point impedance?

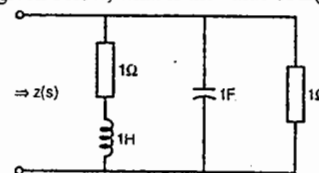
- (A) The first critical frequency at the origin is a zero
 (B) The last critical frequency is a pole
 (C) the impedance at $S = \infty$ is always less than the impedance at $S = 0$
 (D) The slope of the impedance curve is positive at all points

15. The correct sequence of the time constants of the circuits shown below in the increasing order is



- (A) 1 - 2 - 3 - 4
 (B) 4 - 1 - 2 - 3
 (C) 4 - 3 - 1 - 2
 (D) 4 - 3 - 2 - 1

16. For the network shown in the figure given below, what is the value of $Z(s)$?



- (A) $\frac{s^2 + 2s + 2}{s + 2}$
 (B) $\frac{s + 2}{(s + 1)^2}$
 (C) $\frac{s + 1}{s^2 + 2s + 2}$
 (D) $\frac{(s + 1)^2}{(s + 2)}$

17. Which one of the following functions is an RC driving point impedance?

- (A) $\frac{s(s + 3)(s + 4)}{(s + 1)(s + 2)}$
 (B) $\frac{(s + 3)(s + 4)}{(s + 1)(s + 2)}$
 (C) $\frac{(s + 3)(s + 4)}{s(s + 1)(s + 2)}$
 (D) $\frac{(s + 2)(s + 4)}{(s + 1)(s + 3)}$

18. $P(s) = s^4 + s^3 + 2s^2 + 4s + 3$

$Q(s) = s^5 + 3s^3 + s$

Which one of the following statements is correct for above $P(s)$ and $Q(s)$ polynomials?

- (A) Both $P(s)$ and $Q(s)$ are Hurwitz
 (B) both $P(s)$ and $Q(s)$ are non-Hurwitz
 (C) $P(s)$ is Hurwitz but $Q(s)$ is non-Hurwitz
 (D) $P(s)$ is non-Hurwitz but $Q(s)$ is Hurwitz

19. Consider the following expression for the driving point impedance:

$$Z = \frac{(s + 3)(s + 4)}{s(s + 1)(s + 2)}$$

- (1) It represents an LC circuit.
 (2) it represents an RLC circuit.
 (3) it has poles lying on the $j\omega$ axis.
 (4) it has a pole at infinite frequency and a zero at zero frequency.

the correct statement are

- (A) 2 and 4
 (B) 1 and 3
 (C) 1 and 4
 (D) 2 and 3

20. Match List - I (form) with List - II (Method) with respect to the synthesis of R - C driving point function $Z(s) = 1/y(s)$ and select the correct answer using the code given below the lists:

List - I

P. Foster I form
 Q. Foster II form
 R. Cauer I form
 S. Cauer II form

List - II

1. Continued fraction expansion of $Z(s)$ around $s = \infty$
 2. Partial fraction expansion of $Y(s)/s$
 3. Continued fraction expansion of $Z(s)$ around $s = 0$
 4. partial fraction expansion of $Z(s)$

Codes:

	P	Q	R	S
(A) 1	2	4	3	
(B) 4	3	1	2	
(C) 1	3	4	2	
(D) 4	2	1	3	

21. The lowest and the highest critical frequency of an R - L driving - point impedance are, respectively

- (A) a zero, a pole
 (B) a pole, a pole
 (C) a zero, a zero
 (D) a pole, a zero

Key Set B:

01.B	02.C	03.C	04.B	05.B	06.C
07.B	08.C	09.D	10.D	11.D	12.A
13.A	14.C	15.C	16.C	17.D	18.D
19.B	20.D	21.A			

SET - C

01. The first and the last critical frequency of an RC-driving point impedance function must respectively be

GATE - 2005

- (a) a zero and a pole
(b) a zero and a zero
(c) a pole and a pole
(d) a pole and a zero

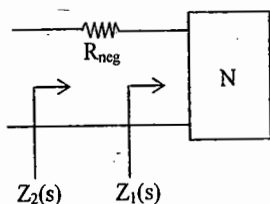
02. The first and the last critical frequencies (singularities) of a driving point impedance function of a passive network having two kinds of elements, are a pole and a zero respectively. The above property will be satisfied by

GATE - 2006

- (a) RL network only
(b) RC network only
(c) LC network only
(d) RC as well as RL networks

03. A negative resistance R_{neg} is connected to a passive network N having driving point impedance $Z_1(s)$ as shown below. For $Z_2(s)$ to be positive real,

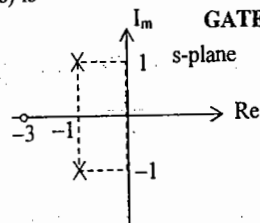
GATE - 2006



- (a) $|R_{neg}| \leq \operatorname{Re} Z_1(j\omega), \forall \omega$
(b) $|R_{neg}| \leq |Z_1(j\omega)|, \forall \omega$
(c) $|R_{neg}| \leq \operatorname{Im} Z_1(j\omega), \forall \omega$
(d) $|R_{neg}| \leq \angle Z_1(j\omega), \forall \omega$

04. The driving-point impedance $Z(s)$ of a network has the pole-zero locations are shown in the figure. If $Z(0)=3$, then $Z(s)$ is

GATE - 2003



- (a) $\frac{3(s+3)}{s^2+2s+3}$
(b) $\frac{2(s+3)}{s^2+2s+2}$
(c) $\frac{3(s-3)}{s^2-2s-2}$
(d) $\frac{2(s-3)}{s^2-2s-3}$

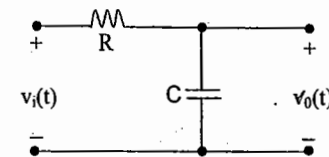
KEY- SET C

01. d 02. b 03. a 04. b

Chapter 9: Transmission Criteria

Consider the first order system:

A simple R-C low pass filter as shown in Fig. 1



When the input is a step voltage as shown in fig(2), with an instantaneous jump of voltage by one volt at $t = 0$, the capacitor output voltage $v_o(t)$ will not rise instantaneously at $t = 0$ from zero initial capacitor voltage to one volt. As the capacitor voltage cannot change instantaneously the output rises from 0 to 1V, according to the exponential rise as shown in Fig (3). $v_o(t) = 1 - e^{-t/\tau}$, for $t \geq 0$, $\tau = RC$ (1)

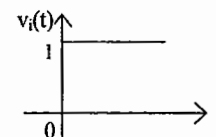


Fig. 2

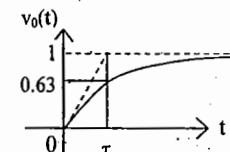


Fig. 3

Rate at which the capacitor voltage rises is specified by parameters like time constant (τ), rise time (t_r) and delay time (t_d).

Time constant (τ) is defined as time during which the response rises to 63% of its final value. It is also defined as the time during which the response reaches the final value, if it is assumed to rise with its initial slope, $1/\tau$. Rise time (t_r) is defined as the time taken for the response to rise from 10% to 90% of its final value.

$$t_r = t_2 - t_1, \quad 1 - e^{-t_2/\tau} = 0.9, \quad \text{and} \quad 1 - e^{-t_1/\tau} = 0.1$$

$$t_2 = 2.3\tau, \quad t_1 = 0.1\tau, \quad t_r = 2.2\tau = 2.2 RC \quad (2)$$

Delay time (t_d) is defined as the time during which the response reaches 50% of its final value.

$$t_d = \ln(2) \tau = 0.693 \tau \quad (3)$$

τ , t_r & t_d are known as transient response or time domain response specifications.

Consider the frequency response $H(\omega)$ or $H(f)$ of the same system.

$$\frac{V_o(\omega)}{V_i} = H(\omega) = H(f) = \frac{1}{1 + jRC\omega}, \quad \omega = 2\pi f \quad (4)$$

$$\text{Mag. Response } A(\omega) = |H(\omega)| = \frac{1}{\sqrt{1 + R^2 C^2 \omega^2}} \quad (5)$$

is shown in Fig. 4

$$\text{Phase Response } \theta(\omega) = \angle H(\omega) = -\tan^{-1}(\omega RC) \quad (6)$$

is shown in Fig. 5

For distortion less transmission of a signal through a system, the system should respond to input such that

$$v_o(t) = K v_i(t - t_0) \quad (7)$$

in the time domain and

$$H(f) = H(\omega) = K \exp(-j\omega t_0) \quad (8)$$

where K is constant and t_0 is time delay

$$\text{i.e. } A(\omega) = K \text{ (constant with frequency)} \quad (9)$$

$$\text{and } \theta(\omega) = -t_0\omega, \text{ varying linearly with frequency} \quad (10)$$

For the RC low pass system considered above $A(\omega)$ is not constant and $\theta(\omega)$ is not a linear function of ω and hence the output is a distorted version of the input.

The rate at which of the magnitude response is decreasing with frequency is specified by 3dB cut-off frequency ω_c . ω_c is defined as the frequency at which the magnitude response reaches $1/\sqrt{2}$ times the initial value (or at which the response is down by 3dB from the initial value at $\omega = 0$).

$$\omega_c = 1/(RC) = 1/\tau, \quad f_c = 1/(2\pi RC) = 1/(2\pi\tau) \quad (11)$$

$$\text{Note that rise time } t_r = 2.2\tau = 2.2/\omega_c = 0.35/f_c, \quad t_r \propto 1/f_c \quad (12)$$

A pulse input and the response is shown in Fig. 6. Observe the distortion caused by the system on the input.

Let t_r be the rise time for exponential rise during the pulse width from $t = 0$ to t_p . To minimize distortion $t_r \ll t_p$.

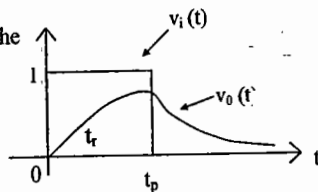


Fig. 6

Distortion is said to be tolerable or the output is reasonable reproduction of the input, if $t_r = 0.35/f_c = 0.35 t_p$, or $f_c = 1/t_p$

$$\text{As a rule of thumb pulse shape is preserved if } f_c \approx 1/\text{pulse width} \quad (13)$$

If a $0.5 \mu\text{s}$ pulse is to be reasonably reproduced, f_c of the filter should be approximately 2 MHz

For a cascade of two systems of rise times, t_{r1} and t_{r2} , the overall rise time is given by the empirical relationship.

$$t_r = 1.05 \sqrt{t_{r1}^2 + t_{r2}^2} \quad (14)$$

$$= 1.05 t_{r1} \sqrt{1 + k^2}, \quad (15)$$

where $k = t_{r2}/t_{r1}$

$$= 1.49 t_{r1}, \quad (16)$$

for $t_{r2} = t_{r1}$

If the rise time t_{r1} of an input wave form is measured by C.R.O with rise time t_{r2} , the observed rise is more than the actual rise time by approximately 50% for $t_{r2} = t_{r1}$ according to the above formula. The actual measurement shows that the observed rise time is 53% longer than the rise time of the input waveform. If $t_{r2} < \frac{1}{3} t_{r1}$, the observed rise time differs from the rise time of input wave form by less than 10%. Hence CRO used for rise time measurement should have a bandwidth at least three times the bandwidth of the circuit under test.

Referring to an LTI system with transfer function (or system function) $H(\omega)$ or $H(f)$, both equations (9) & (10) should be satisfied. For a bandpass system with center frequency, f_c and bandwidth, $2W$, Let $A(\omega)$ be constant but $\theta(\omega)$ is nonlinear as shown in Fig. 7.

For such a system causing distortion on the input,

$$\text{Phase delay or Carrier delay, } \tau_p = \frac{-\theta(\omega_c)}{\omega_c} = \frac{-\theta(f_c)}{2\pi f_c} \quad (17)$$

$$\text{A single tone (frequency) } \cos(\omega_c t), \text{ undergoes delay as } \cos[\omega_c(t - \tau_p)] \quad (18)$$

Whereas a low pass message signal $m(t)$ with a group of frequencies, undergoes Group delay $= \tau_g = -\frac{d\theta(\omega)}{d\omega} = -\frac{d\theta(f)}{2\pi df}$ (19)

$$\text{A bandpass signal } m(t) \cos(\omega_c t) \text{ undergoes delays as } m(t - \tau_g) \cos[\omega_c(t - \tau_p)] \quad (20)$$

$\tau_p \neq \tau_g$ in general.

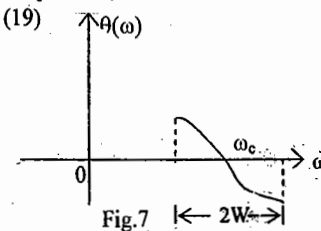
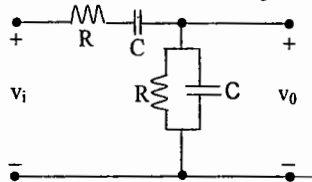


Fig. 7

SET - A

01. The RC circuit shown in the figure is



- (A) a low-pass filter
(B) a high-pass filter
(C) a band-pass filter
(D) a band-reject filter

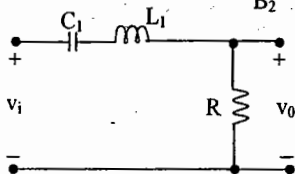
02. A two-port network is said to be reciprocal if

- (A) $y_{12} = -y_{21}$ (B) $h_{12} = h_{21}$
(C) $BC - AD = -1$ (D) $A = D$

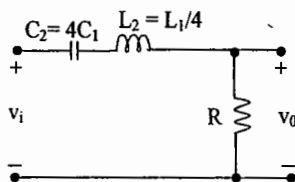
03. For a 2-port reciprocal network, the output open circuit voltage divided by the input current is equal to

- (A) B (B) z_{12}
(C) $1/y_{21}$ (D) h_{12}

04. Two series resonant filters are as shown in the figure. Let the 3-dB band width of filter 1 be
- B_1
- and that of filter 2 be
- B_2
- . The value of
- $\frac{B_1}{B_2}$
- is



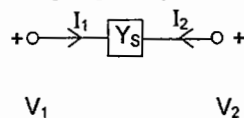
Filter 1



Filter 2

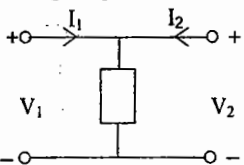
- (A) 4 (B) 1
(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

05. The y-parameters of the network shown in fig. is given by the matrix



- (A) $\begin{bmatrix} Y_s & Y_s \\ Y_s & Y_s \end{bmatrix}$ (B) $\begin{bmatrix} Y_s & -Y_s \\ -Y_s & Y_s \end{bmatrix}$
(C) $\begin{bmatrix} -Y_s & Y_s \\ Y_s & Y_s \end{bmatrix}$ (D) None

06. The z-parameters of the network shown in fig., is given by the matrix



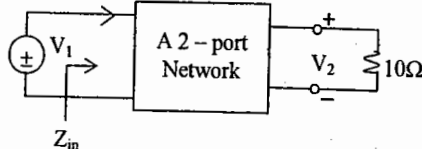
- (A) $\begin{bmatrix} Z_p & Z_p \\ Z_p & Z_p \end{bmatrix}$ (B) $\begin{bmatrix} Z_p & -Z_p \\ -Z_p & Z_p \end{bmatrix}$
(C) $\begin{bmatrix} Z_p & 0 \\ 0 & Z_p \end{bmatrix}$ (D) None

07. The transfer function

$$H(s) = \frac{(s^2 - 5s + 100)}{(s^2 + 5s + 100)}$$

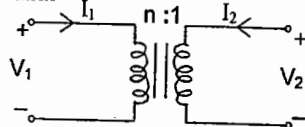
- (A) a high-pass filter
(B) a band elimination filter
(C) a low-pass filter
(D) an all pass filter

08. If the transmission parameters of the network shown in figure are
- $A = C = 1$
- ,
- $B = 2$
- and
- $D = 3$
- , then the value of
- Z_{in}
- in ohms is



- (A) $\frac{12}{13}$ (B) $\frac{13}{12}$
(C) 3 (D) 4

09. The ABCD parameters of the ideal transformer in figure is given by the matrix



- (A) $\begin{bmatrix} 0 & 4 \\ (1/n) & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & (1/n) \\ n & 0 \end{bmatrix}$
(C) $\begin{bmatrix} (1/n) & 0 \\ 0 & n \end{bmatrix}$ (D) $\begin{bmatrix} n & 0 \\ 0 & (1/n) \end{bmatrix}$

10. The transmission parameters of a two port network is given by the matrix,
- $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$
- . If a
- 1Ω
- resistor is connected in series with one of the input leads, then the transmission parameters of the overall 2 port network will be

- (A) $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 3 \\ 1 & 2 \end{bmatrix}$
(C) $\begin{bmatrix} 1 & 2 \\ 3 & 3 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 3 \\ 1 & 3 \end{bmatrix}$

11. The transfer function,
- $T(s) = s/(s+a)$
- is that of a

- (A) low-pass filter
(B) notch filter
(C) high-pass filter
(D) band-pass filter

12. The transfer function of a low pass RC network is

- (A) $RCs/(1+RCs)$ (B) $1/(1+RCs)$
(C) $RC/(1+RCs)$ (D) $S/(1+RCs)$

13. An electric circuit contains R, L and C in series with a voltage source. The current through the circuit at the resonant frequency is
- I_0
- . The frequencies at which the current would reduce to
- $0.707I_0$
- are
- f_{01}
- and
- f_{02}
- . The resonant frequency of the circuit is the

- (A) geometric mean of f_{01} and f_{02}
(B) arithmetic mean of f_{01} and f_{02}
(C) difference of f_{01} and f_{02}
(D) harmonic mean of f_{01} and f_{02}

14. A transfer function having open right-half plane zero is a/an

- (A) minimum phase function
(B) non-minimum phase function
(C) unstable function
(D) constant phase function

15. Frequency response of the function
- $T(s) = (s+1)/(s+2)$
- exhibits maximum phase at a frequency (in radian/sec.) of

- (A) 0 (B) $1/\sqrt{2}$
(C) $\sqrt{2}$ (D) ∞

16. Two networks are cascaded through an ideal buffer. If
- t_{d1}
- and
- t_{d2}
- are the delay times of the networks, then the overall delay of the networks together will be

- (A) $\sqrt{t_{d1}^2 + t_{d2}^2}$ (B) $\sqrt{t_{d1}^2 + t_{d2}^2}$
(C) $t_{d1} + t_{d2}$ (D) $(t_{d1} + t_{d2})/2$

17. Two networks are cascaded through an ideal buffer. If
- t_{r1}
- and
- t_{r2}
- are the rise times of the two networks, then the overall rise time of the two networks together will be

- (A) $\sqrt{t_{r1}^2 + t_{r2}^2}$ (B) $\sqrt{t_{r1}^2 + t_{r2}^2}$
(C) $t_{r1} + t_{r2}$ (D) $(t_{r1} + t_{r2})/2$

18. For a reciprocal network, the two port h - parameters are related as follows:

- (A) $h_{12} = h_{21}$
 (B) $h_{12} = -h_{21}$
 (C) $h_{11} h_{22} - h_{21} h_{12} = 1$
 (D) None

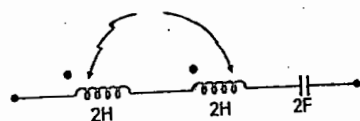
Key Set A

01. C 02. C 03. B 04. D 05. B
 06. A 07. D 08. A 09. D 10. B
 11. C 12. B 13. A 14. B 15. C
 16. C 17. B 18. B

SET - B

01. The resonant frequency of the given series circuit is

$$M = 1H$$



- (A) $1/2\pi \sqrt{3}$ Hz (B) $1/4\pi \sqrt{3}$ Hz
 (C) $1/4\pi \sqrt{2}$ Hz (D) $1/4\pi \sqrt{2}$ Hz

02. If the numerator of a second - order transfer function $F(s)$ is a constant, then the filter is a
 (A) band - pass filter
 (B) band - stop filter
 (C) high - pass filter
 (D) low - pass filter

03. Voltage transfer function of a simple RC integrator has

- (A) a finite zero and a pole at infinity.
 (B) a finite zero and a pole at the origin.
 (C) a zero at the origin and a finite pole.
 (D) a zero at infinity and a finite pole.

04. For an all - pass function

- (A) Zeros are in the right half plane (RHP) and poles in the left half plane (LHP)
 (B) Zeros are in the LHP and poles in the LHP
 (C) Zeros are in the LHP and poles in the RHP
 (D) Zeros are in the RHP and poles in the RHP

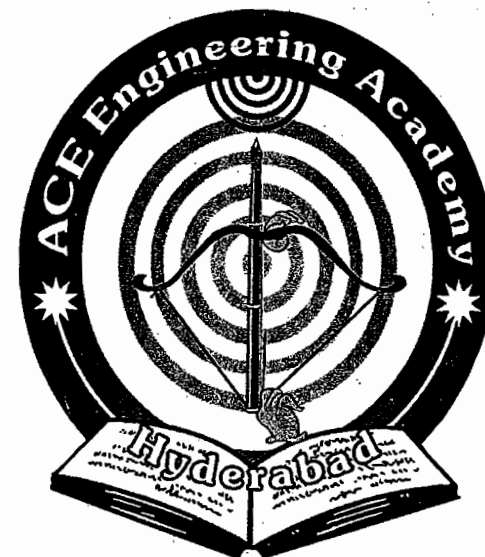
Key Set B

01. B 02. D 03. D 04. A

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