IES MASTER

Institute for Engineers (IES/GATE/PSUs)

GATE 2022

ELECTRONICS & COMMUNICATION ENGINEERING

Detailed Solution

EXAM DATE: 06-02-2022

FORENOON SESSION (09:00 AM-12:00 PM)

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APTITUDE

1. Mr. X speaks ________ Japanese ________ Chinese.

(a) neither / or (b) either / nor

(c) neither / nor (d) also / but

Sol: (c)

Here we will cheake tones.

Mr. X speaks neither Japanese nor Chinese.

2. A sum of money is to be distributed among P, Q, R, and S in the proportion 5 : 2 : 4 : 3, respectively.

If R gets R 1000 more than S, what is the share of Q (in Rs)?

(a) 500 (b) 1000

(c) 1500 (d) 2000

Sol: (d)

P Q R S 1 = 1000 Rs.

5 : 2 : 4 : 3

So, Sharing of Q is = 2 × 1000

= 2000 Rs

3. A trapezium has vertices marked as P, Q, R and S (in that order anticlockwise). The side PQ is parallel to side SR.

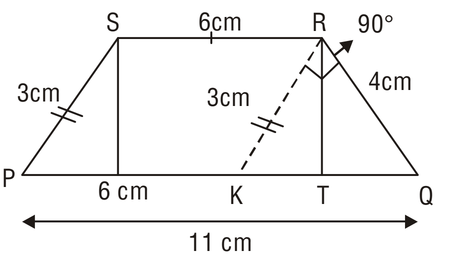
Further, it is given that, PQ = 11 cm, QR = 4 cm, RS = 6 cm and SP = 3 cm.

What is the shortest distance between PQ and SR (in cm)?

(a) 1.80 (b) 2.40

(c) 4.20 (d) 5.76

Sol: (b)



$$\text{Area} = \frac{1}{2} \times B \times H \qquad B \rightarrow \text{Base}$$

$$= \frac{1}{2} \times B \times 4 \qquad H \rightarrow \text{Height}$$

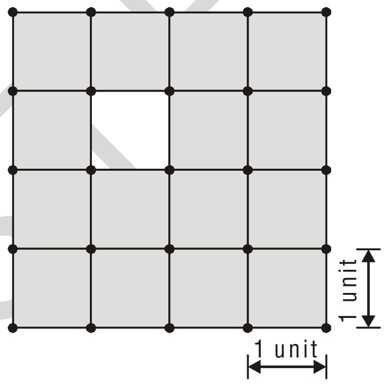
$$= \frac{1}{2} \times 3 \times 4$$

$$= 6 \text{ cm}^2$$

$$\frac{1}{2} \times 5 \times RT = 6 \text{ cm}^2$$

$$RT = 2.4 \text{ cm}$$

4. The figure shows a grid formed by a collection of unit squares. The unshaded unit square in the grid represents a hole.

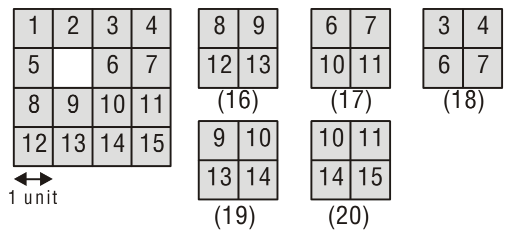


What is the maximum number of squares without a "hole in the interior" that can be formed within the 4 × 4 grid using the unit squares as building blocks?

(a) 15 (b) 20

(c) 21 (d) 26

Sol: (b)

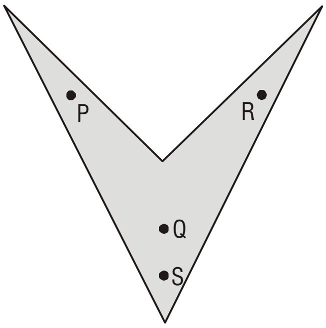


Total number of sqaures without a hole in interior

$$= \underbrace{15}_{\substack{(1\times1)\\ \text{unit}\\ \text{squares}}} + \underbrace{5}_{\substack{(2\times2)\\ \text{unit}\\ \text{squares}}} = 20$$

5. An art gallery engages a security guard to ensure that the items displayed are protected. The diagram below represents the plan of the gallery where the boundary walls are opaque. The location the security guard posted is identified such that all the inner space (shaded region in the plan) of the gallery is within the line of sight of the security guard.

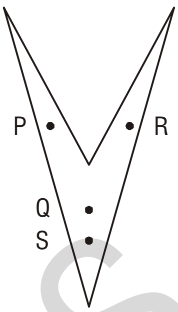
If the security guard does not move around the posted location and has a 360° view, which one of the following correctly represents the set of ALL possible locations among the locations P, Q, R and S, where the security guard can be posted to watch over the entire inner space of the gallery.



(a) P and Q (b) Q

(c) Q and S (d) R and S

Sol: (c)



If person will stand on 'P' than he/she can't see the visibility side of R and vice versa.

But if security guard will be stand at Q and S than he will se whole 360°.

6. Mosquitoes pose a threat to human health. Controlling mosquitoes using chemicals may have undesired consequences. In Florida, authorities have used genetically modified mosquitoes to control the overall mosquito population. It remains to be seen if this novel approach has unforeseen consequences.

Which one of the following is the correct logical inference based on the information in the above passage?

(a) Using chemicals to kill mosquitoes is better than using genetically modified mosquitoes because genetic engineering is dangerous

(b) Using genetically modified mosquitoes is better than using chemicals to kill mosquitoes because they do not have any side effects

(c) Both using genetically modified mosquitoes and chemicals have undesired consequences and can be dangerous

(d) Using chemicals to kill mosquitoes may have undesired consequences but it is not clear if using genetically modified mosquitoes has any negative consequence

Sol: (d)

Using chemicals to kill mosquitoes may have undesired consequence but it is not clear if using genetically modified mosquitoes has any negative consequence.

7. Consider the following inequalities.

(i) $2x - 1 > 7$

(ii) $2x - 9 < 1$

Which one of the following expressions below satisfies the above two inequalities?

(a) $x \le -4$ (b) $-4 < x \le 4$

(c) $4 < x < 5$ (d) $x \ge 5$

Sol: (c)

$$\begin{array}{l|l} \Rightarrow \quad 2x - 1 > 7 & \quad 2x - 9 < 1 \\ \Rightarrow \quad 2x > 8 & \Rightarrow \quad 2x < 10 \\ \Rightarrow \quad x > 4 & \Rightarrow \quad x < 5 \end{array}$$

Combining both inequalities

$4 < x < 5$

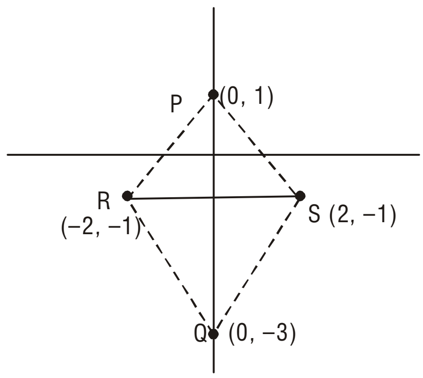
8. Four points P(0, 1), Q(0, –3), R(–2, –1), and S(2, –1) represent the vertices of a quadrilateral.

What is the area enclosed by the quadrilateral?

(a) 4 (b) $4\sqrt{2}$

(c) 8 (d) $8\sqrt{2}$

Sol: (c)



$$PS = \sqrt{\left(\sqrt{2}\right)^4 + (-2)^2}$$
$$= \sqrt{2+4+2}$$
$$= \sqrt{8}$$
$$SQ = \sqrt{4+4} = \sqrt{8}$$
$$SQ = \sqrt{4+4} = \sqrt{8}$$
$$QS = \sqrt{4+4} = \sqrt{8}$$
$$PQ = \sqrt{16} = 4$$
$$RS = \sqrt{16} = 4$$
$$\text{Area} = \sqrt{8} \times \sqrt{8} = 8 \text{ units}$$

9. In a class of five students P, Q, R, S and T, only one student is known to have copied in the exam. The disciplinary committee has investigated the situation and recorded the statements from the students as given below.

Statement of P: R has copied in the exam.

Statement of Q: S has copied in the exam.

Statement of R: P did not copy in the exam.

Statement of S: Only one of us is telling the truth.

Statement of T: R is telling the truth.

The investigating team had authentic information that S never lies.

Based on the information given above, the person who has copied in the exam is

(a) R (b) P

(c) Q (d) T

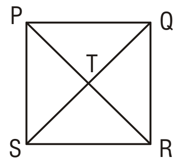
Sol: (b)

Shows cheating done by

P	Q	R	T
↓	↓	↓	↓
R(✓)	P(✓)	P(×)	R(✓)
×	×	×	✓
R(×)	S(×)	P(✓)	

Hence P has copied in the exam.

10. Consider the following square with the four corners and the center marked as P, Q, R, S and T respectively.



Let X, Y and Z represent the following operations:

X: rotation of the square by 180 degree with respect to the S-Q axis.

Y: rotation of the square by 180 degree with respect to the P-R axis.

Z: rotation of the square by 90 degree clockwise with respect to the axis perpendicular, going into the screen and passing through the point T.

Consider the following three distinct sequences of operation (which are applied in the left to right order).

1. XYZZ
2. XY
3. ZZZZ

Which one of the following statements is correct as per the information provided above?

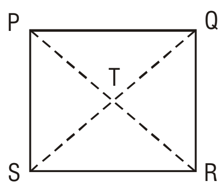
(a) The sequence of operations (1) and (2) are equivalent

(b) The sequence of operations (1) and (3) are equivalent

(c) The sequence of operations (2) and (3) are equivalent

(d) The sequence of operations (1), (2) and (3) are equivalent

Sol: (b)

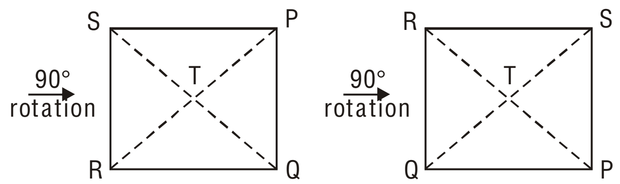


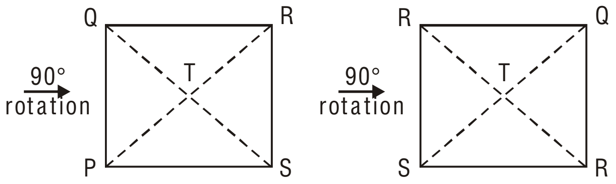
$x \to S - Q \to 180°$

$y \to P - R \to 180°$

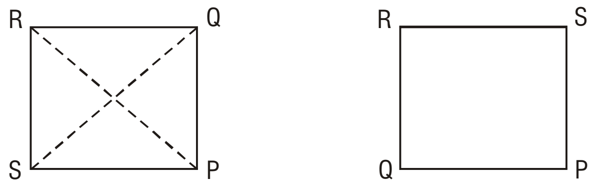
$z \to 90° \, (T)$

(3) Operations - ZZZZ

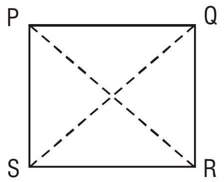




(2) xy



(1) xyzz

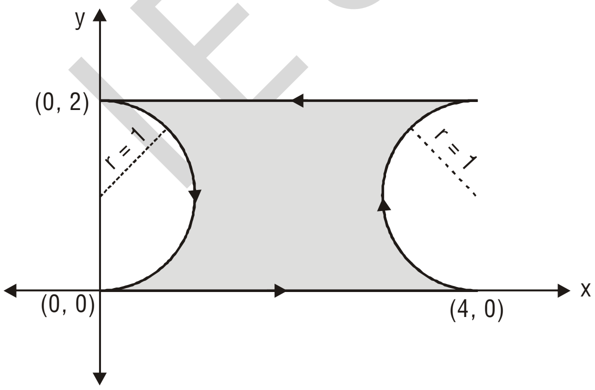


Hence, option (b) is corret

TECHNICAL

11. Consider the two-dimensional vector field $\vec{F}(x,y) = x\vec{i} + y\vec{j}$, where $\vec{i}$ and $\vec{j}$ denote the unit vectors along the x-axis and the y-axis, respectively. A contour C in the x-y plane, as shown in the figure, is composed of two horizontal lines connected at the two ends by two semicircular arcs of unit radius. The contour is traversed in the counter-clockwise sense. The value of the closed path integral

$\oint_c \vec{F}(x,y).(dx\,\vec{i} + dy\,\vec{j})$ is. ________



(a) 0 (b) 1

(c) $8 + 2\pi$ (d) -1

Sol: (a)

$$\oint \vec{F}(x,y)\cdot\left[dx\vec{i} + dy\,\vec{j}\right]$$

Given $\vec{F}(x,y) = x\vec{i} + y\,\vec{j}$

$\therefore \quad \int_c xdx + ydy = 0$

Because here vector is conservative.

If the integral function is the total derivative over the closed contoure then it will be zero.

12. Consider a system of linear equations Ax = b, where

$$A = \begin{bmatrix} 1 & -\sqrt{2} & 3 \\ -1 & \sqrt{2} & -3 \end{bmatrix},\ b = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

This system of equations admits ________.

(a) a unique solution for x

(b) infinitely many solutions for x

(c) no solutions for x

(d) exactly two solutions for x

Sol: (c)

$$A = \begin{bmatrix} 1 & -\sqrt{2} & 3 \\ -1 & \sqrt{2} & -3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

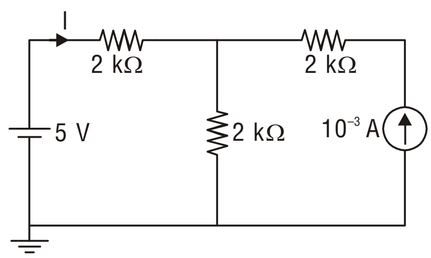
Hence equation will be

$x - \sqrt{2}y + 3z = 1$

$-x + \sqrt{2}y - 3z = 3$

therefore inconsistant solution i.e. there will not be any solution.

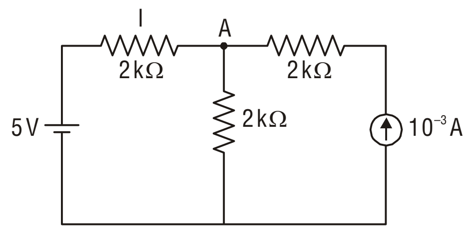
13. The current I in the circuit shown is ________.



(a) 1.25×10^{-3}A (b) 0.75×10^{-3}A

(c) -0.5×10^{-3}A (d) 1.16×10^{-3}A

Sol: (b)



Applying Nodal equation at Node-A

$$\frac{V_A}{2k} + \frac{V_A - 5}{2k} = 10^{-3}$$

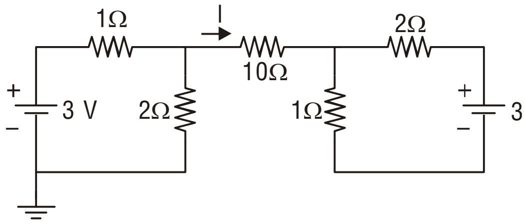
$$\Rightarrow \quad 2V_A - 5 = 2k \times 10^{-3}$$

$$\Rightarrow \quad V_A = \frac{7}{2} V = 3.5\ V$$

Again,

$$I = \frac{5 - V_A}{2k} = \frac{5 - 3.5}{2k} = \frac{1.5}{2k} = \boxed{0.75 \times 10^{-3}\ A}$$

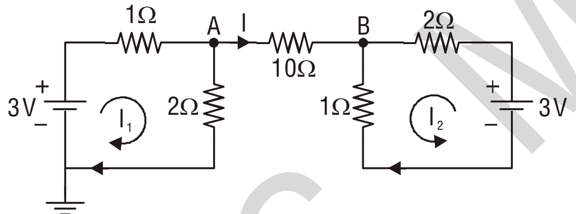
14. Consider the circuit shown in the figure. The current I flowing through the 10 Ω resistor is _________.



(a) 1 A (b) 0 A

(c) 0.1 A (d) –0.1 A

Sol: (b)



- Here, there is no any return closed path for Current (I). Hence I = 0.
- Current always flow in loop.

15. The Fourier transform $X(j\omega)$ of the signal $x(t) = \dfrac{t}{(1+t^2)^2}$ is ______.

(a) $\dfrac{\pi}{2j}\omega e^{-|\omega|}$ (b) $\dfrac{\pi}{2}\omega e^{-|\omega|}$

(c) $\dfrac{\pi}{2j}e^{-|\omega|}$ (d) $\dfrac{\pi}{2}e^{-|\omega|}$

Sol: (a)

$$x(t) = \frac{t}{(1+t^2)^2}$$

As we know that FT of $te^{-|t|} \xleftrightarrow{FT} \dfrac{-j4\omega}{(1+\omega^2)^2}$

Duality $\dfrac{-j4\omega}{(1+t^2)^2} \longleftrightarrow 2\pi(-\omega)e^{-|-\omega|}$

$$\Rightarrow \quad \frac{t}{(1+t^2)^2} \xrightarrow{FT} \frac{-2\pi}{-j4}\omega e^{-|\omega|}$$

$$\Rightarrow \quad \longrightarrow \boxed{\frac{\pi}{j2}\omega e^{-|\omega|}}$$

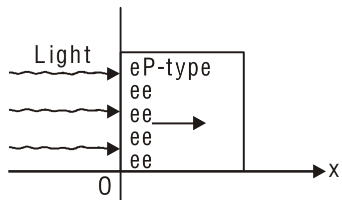
16. Consider a long rectangular bar of direct bandgap p-type semiconductor. The equilibrium hole density is 10^{17} cm^{-3} and the intrinsic carrier concentration is 10^{10} cm^{-3}. Electron and hole diffusion lengths are $2\mu m$ and $1\mu m$, respectively.

The left side of the bar (x = 0) is uniformly illuminated with a laser having photon energy greater than the bandgap of the semiconductor. Excess electron-hole pairs are generated ONLY at x = 0 because of the laser. The steady state electron density at x = 0 is 10^{14} cm^{-3} due to laser illumination. Under these conditions and ignoring electric field, the closest approximation (among the given options) of the steady state electron density at $x = 2\ \mu m$, is __________.

(a) 0.37×10^{14} cm^{-3} (b) 0.63×10^{13} cm^{-3}

(c) 3.7×10^{14} cm^{-3} (d) 10^3 cm^{-3}

Sol: (a)



From continuity equation of electrons

$$\frac{dn}{dt} = n\mu_n\frac{dE}{dx} + \mu_n E\frac{dn}{dx} + G_n - R_n + x_n\frac{d^2x}{dx^2} \quad ...(i)$$

0 + 0

[Because $\vec{E}$ is not mentioned hence

$\dfrac{dE}{dx} = 0\ [\vec{E} = 0]$

For x > 0, G_n is also zero

$$n = \frac{n_i^2}{N_A} = \frac{10^{20}}{10^{17}} = 10^3$$

$$\because \quad n = n_0 + \delta n$$

$$= 10^3 + 10^{14} = 10^{14}$$

at steady state, $\frac{dn}{dt} = 0$

Hence equation (i) becomes:

$$0 = D_n \frac{d^2\delta n}{dx^2} - \frac{\delta n}{\tau_n}$$

$$\Rightarrow \quad \frac{d^2\delta_n}{dx^2} = \frac{\delta_n}{L_n^2} \qquad \text{...(ii)}$$

From solving equation (ii)

$$\delta_n(x) = \delta_n(0)e^{-x/L_n}$$

at $\quad x = 2\mu m$

$$\delta_n(2\mu m) = 10^{14}e^{-\frac{2}{2}} = 10^{14}e^{-1}$$

$$= \boxed{0.37 \times 10^{14}}$$

17. In a non-degenerate bulk semiconductor with electron density $n = 10^{16}\ cm^{-3}$, the value of $E_C - F_{Fn} = 200$ meV, where E_C and E_{Fn} denote the bottom of the conduction band energy and electron Fermi level energy, respectively. Assume thermal voltage as 26 meV and the intrinsic carrier concentration is $10^{10}\ cm^{-3}$. For $n = 0.5 \times 10^{16}\ cm^{-3}$, the closest approximation of the value of $(E_C - E_{Fn})$, among the given options, is __________.

(a) 226 meV (b) 174 meV

(c) 218 meV (d) 182 meV

Sol: (c)

Here we have to find the value of $E_c - E_{fn}$

As we know,

$$E_C - E_F = kT\ell n\left(\frac{N_c}{n}\right) \qquad \text{...(i)}$$

$$E_C - E_{F1} = kT\ell n\left(\frac{N_c}{n_1}\right) \qquad \text{...(ii)}$$

$$E_C - E_{F2} = kT\ell n\left(\frac{N_c}{n_2}\right) \qquad \text{...(iii)}$$

Equation (ii) - Equation (iii)

$$(E_c - E_{F1}) - (E_c - E_{F2}) = kT\,\ell n\left[\frac{\frac{N_c}{n_1}}{\frac{N_c}{n_2}}\right] = kT\ell n\,\frac{n_2}{n_1}$$

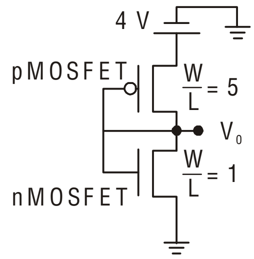
$$\Rightarrow 200\text{meV} - (E_c - E_{F2}) = 26\text{meV} \times \ell n\left(\frac{0.5 \times 10^{16}}{1 \times 10^{16}}\right)$$

$$200\text{mev} - (E_c - E_{F2}) = +26\ \text{mev}\ell n(0.5) = -18$$

$$\Rightarrow \quad (E_c - E_{F2}) = 200 + 8 = 218\ \text{meV}$$

$$= \boxed{218\,\text{meV}}$$

18. Consider the CMOS circuit shown in the figure (substrates are connected to their respective sources). The gate width (W) to gate length (L) ratios $\left(\frac{W}{L}\right)$ of the transistors are as shown. Both the transistors have the same gate oxide capacitance per unit area. For the pMOSFET, the threshold voltage is –1 V and the mobility of holes is 40 cm^2/V.s. For the nMOSFET, the threshold voltage is 1 V and the mobility of electrons is 300 cm^2/V.s. The steady state output voltage V_0 is __________.



(a) equal to 0 V (b) more than 2 V

(c) less than 2 V (d) equal to 2 V

Sol: (c)

Figure.

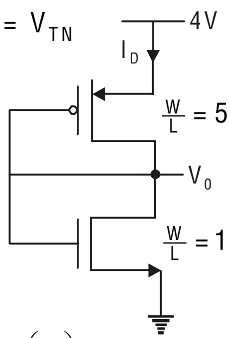
Given data : $|V_{TP}| = 1V = V_{TN}$

CO_x = equal

$\mu_n = 300$

$\mu_P = 40$

$I_{D1} = I_{D2}$



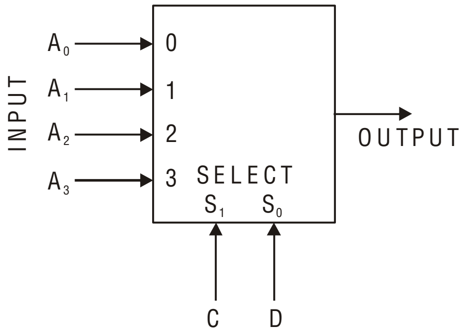
$$\mu_P CO_x\left(\frac{\omega}{L}\right)_1 [4 - V_0 - 1]^2 = \mu_n CO_x\left(\frac{\omega}{L}\right)_2 [V_0 - 0 - 1]^2$$

$$\Rightarrow \frac{300}{40} \times \frac{1}{5}(V_0 - 1)^2 = (3 - V_0)^2$$

$$\Rightarrow \sqrt{1.5}(V_0 - 1) = 3 - V_0$$

$$\Rightarrow \quad V_0 = \frac{3+\sqrt{1.5}}{\sqrt{1.5}+1} = \text{less than } 2V$$

19. Consider the 2-bit multiplexer (MUX) shown in the figure. For OUTPUT to be the XOR of C and D, the values for A_0, A_1, A_2, and A_3 are ____________.



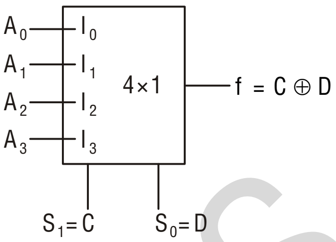
(a) $A_0 = 0,\ A_1 = 0,\ A_2 = 1,\ A_3 = 1$

(b) $A_0 = 1,\ A_1 = 0,\ A_2 = 1,\ A_3 = 0$

(c) $A_0 = 0,\ A_1 = 1,\ A_2 = 1,\ A_3 = 0$

(d) $A_0 = 1,\ A_1 = 1,\ A_2 = 0,\ A_3 = 0$

Sol: (c)



$$f = \bar{C}\bar{D}\underset{0}{\cancel{I_0}} + \bar{C}DI_1 + C\bar{D}I_2 + CD\underset{0}{\cancel{I_3}}$$

$$[C \oplus D]$$

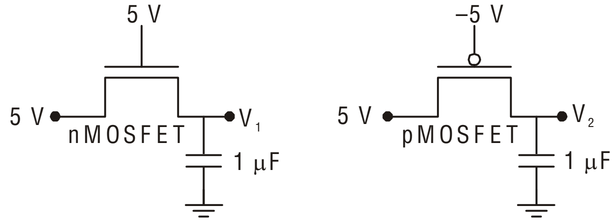
For this

$$A_0 = 0 = A_3$$

and $\quad A_1 = A_2 = 1$

Hence option (c)

20. The ideal long channel nMOSFET and pMOSFET devices shown in the circuits have threshold voltages of 1 V and –1 V, respectively. The MOSFET substrates are connected to their respective sources. Ignore leakage currents and assume that the capacitors are initially discharged. For the applied voltages as shown, the steady state voltages are ________________.



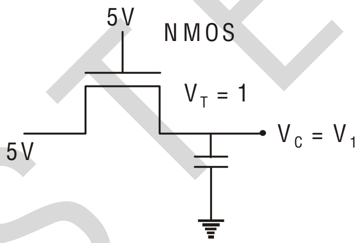
(a) $V_1 = 5\text{ V},\ V_2 = 5\text{ V}$

(b) $V_1 = 5\text{ V},\ V_2 = 4\text{ V}$

(c) $V_1 = 4\text{ V},\ V_2 = 5\text{ V}$

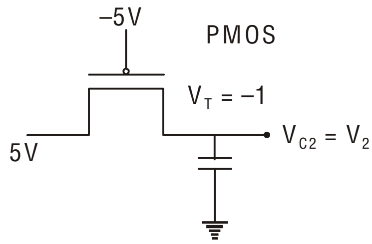
(d) $V_1 = 4\text{ V},\ V_2 = -5\text{ V}$

Sol: (c)



$$V_{C1} = \min(V_G - V_T,\ V_D)$$
$$= \min(4,\ 5)$$

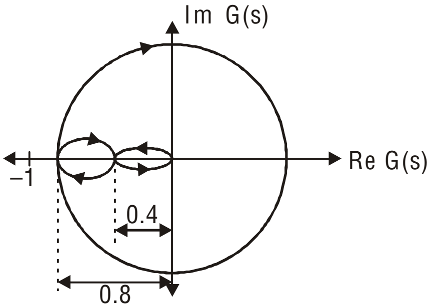
$$\boxed{V_{C1} = 4\,V}$$



$$V_{5G} > |V_{TP}|$$
$$V_{C2} + 4 > 1$$
$$\Rightarrow \quad V_{C2} > -4$$
$$V_{5D} > 0$$
$$V_{C2} \geq 5V$$

$$\boxed{V_{C2} = 5\,V}$$

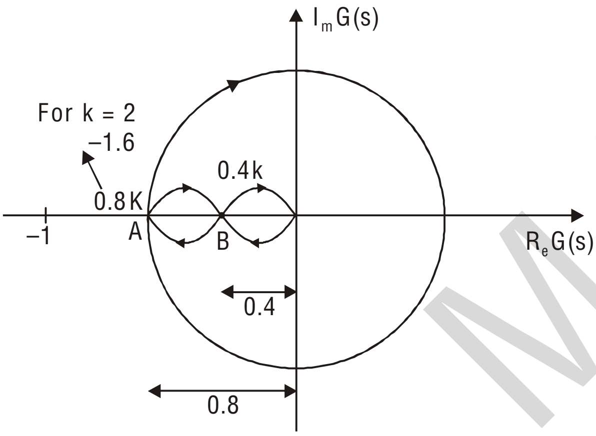
21. Consider a closed-loop control system with unity negative feedback and KG(s) in the forward path, where the gain K = 2. The complete Nyquist plot of the transfer function G(s) is shown in the figure. Note that the Nyquist contour has been chosen to have the clockwise sense. Assume G(s) has no poles on the closed right-half of the complex plane. The number of poles of the closed-loop transfer function in the closed right-half of the complex plane is __________.



(a) 0 (b) 1

(c) 2 (d) 3

Sol: (c)



For K = 2, point A will be –0.8 × 2 = –1.6

Hence N = – 2

P = 0

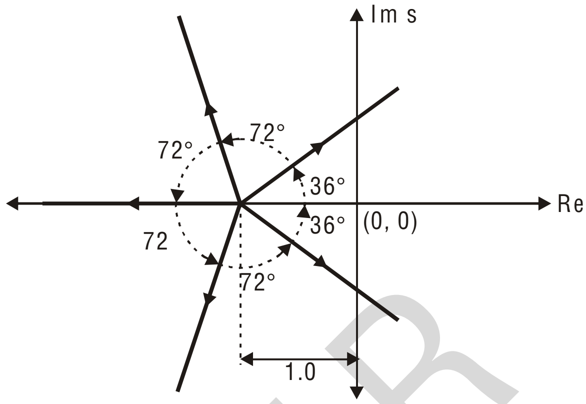
(By default Nyquist contoure is considered in clockwise direction)

$\boxed{P - N = 2}$

Number of closed loop pole in right side of the complex plane.

22. The root-locus plot of a closed-loop system with unity negative feedback and transfer function KG(s) in the forward path is shown in the figure. Note that K is varied from 0 to ∞.

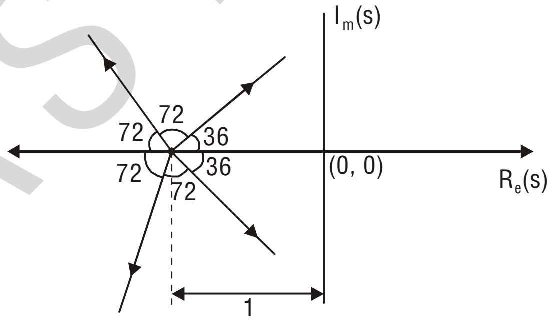
Select the transfer function G(s) that results in the root-locus plot of the closed-loop system as shown in the figure.



(a) $G(s) = \dfrac{1}{(s+1)^5}$ (b) $G(s) = \dfrac{1}{s^5+1}$

(c) $G(s) = \dfrac{s-1}{(s+1)^6}$ (d) $G(s) = \dfrac{s+1}{s^6+1}$

Sol: (a)



Here 5 Root Locus branches are diverging from same point, this can possible only when if we have 5 poles in the system at the same point because Root Locus branch departs from open loop pole and

Number of Root Locus branches = Number of open loop poles or Number of zero (Whichever is greater).

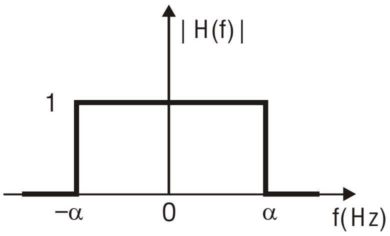
Here, there are 5 multiple Real poles, and matching with option (A).

23. The frequency response H(f) of a linear time-invariant system has magnitude as shown in the figure.

Statement I: The system is necessarily a pure delay system for inputs which are bandlimited to $-\alpha \le f \le \alpha$.

Statement II: For any wide-sense stationary input process with power spectral density $S_X(f)$, the output power spectral density $S_Y(f)$ obeys $S_Y(f) = S_X(f)$ for $-\alpha \le f \le \alpha$.

Which one of the following combinations is true?



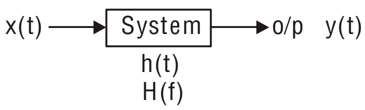
(a) Statement I is correct, Statement II is correct

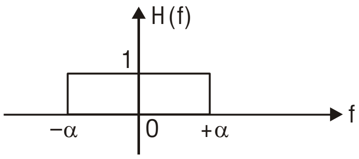
(b) Statement I is correct, Statement II is incorrect

(c) Statement I is incorrect, Statement II is correct

(d) Statement I is incorrect, Statement II is incorrect

Sol: (c)





For the system to be delay system,

$$y(t) = x(t - t_d)$$

$$y(F) = e^{-J\omega t_d} \times (F)$$

$$\Rightarrow \quad H(F) = \frac{Y(F)}{X(F)} = e^{-J\omega t_d}$$

$\Rightarrow$ TF of delay system

- Here given system is constant, hence this is not delay system, therefore statement-I is In correct
- $S_y(f) = S_x(f).|H(f)|^2$

and $|H(f)| = 1$ (given)

Hence, $\boxed{S_y(f) = S_x(f)}$ for $-\alpha \le f \le \alpha$

Statement - II is correct.

24. In a circuit, there is a series connection of an ideal resistor and an ideal capacitor. The conduction current (in Amperes) through the resistor is $2\sin(t+\pi/2)$. The displacement current (in Amperes) through the capacitor is ________.

(a) $2\sin(t)$ (b) $2\sin(t+\pi)$

(c) $2\sin(t+\pi/2)$ (d) 0

Sol: (b)

$$J_d = \in \frac{dE}{dt}$$

$\vec{J}_d$ leads $\vec{J}_c$ by 90°

So, $$i_d = |i_c| \angle i_c + 90°$$

$$i_d = 2\sin\left(t + \frac{\pi}{2} + \frac{\pi}{2}\right)$$

$$i_d = 2\sin(t+\pi)$$

25. Consider the following partial differential equation (PDE)

$$a\frac{\partial^2 f(x,y)}{\partial x^2} + b\frac{\partial^2 f(x,y)}{\partial y^2} = f(x,\ y),$$

where a and b are distinct positive real numbers. Select the combination(s) of values of the real parameters ξ and η such that $f(x, y) = e^{(\xi x+\ \ )}$ is a solution of the given PDE.

(a) $\xi = \frac{1}{\sqrt{2a}},\ \eta - \frac{1}{\sqrt{2b}}$ (b) $\xi = \frac{1}{\sqrt{a}},\ \eta = 0$

(c) $\xi = 0,\ \eta = 0$ (d) $\xi = \frac{1}{\sqrt{a}},\ \eta = \frac{1}{\sqrt{b}}$

Sol: (a, b)

Given: $f(x,\ y) = e^{(\xi x + \eta y)}$

$$\frac{\partial f(x,y)}{\partial x} = \xi \cdot e^{(\xi x+\eta y)}$$

$$\frac{\partial^2 f(x,y)}{\partial x^2} = \xi^2 \cdot e^{(\xi x+\eta y)}$$

Similarly, $$\frac{\partial^2 f(x,y)}{\partial y^2} = \eta^2 \cdot e^{(\xi x+\eta y)}$$

Now, as given

$$a\frac{\partial^2 f(x,y)}{\partial x^2} + b\frac{\partial^2 f(x,y)}{\partial y^2} = f(x,y)$$

or

$$a \times \xi^2 e^{(\xi x+\eta y)} + b \times \eta^2 e^{(\xi x+\eta y)} = e^{(\xi x+\eta y)}$$

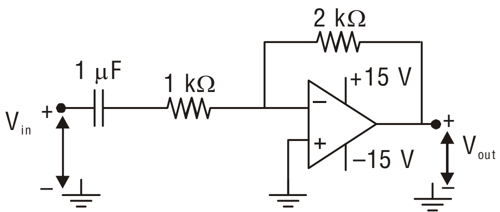
$$\boxed{(a\cdot\xi^2 + b\cdot\eta^2) = 1}$$

Thus, $$\left(\xi = \frac{1}{\sqrt{2a}}, \eta = \frac{1}{\sqrt{2b}}\right)$$

$\left(\xi = \frac{1}{\sqrt{a}},\ \eta = 0\right)$ and $\left(\xi = 0,\ \eta = \frac{1}{\sqrt{b}}\right)$

satisfy the above result.

26. An ideal OPAMP circuit with a sinusoidal input is shown in the figure. The 3 dB frequency is the frequency at which the magnitude of the voltage gain decreases by 3 dB from the maximum value. Which of the options is/are correct?



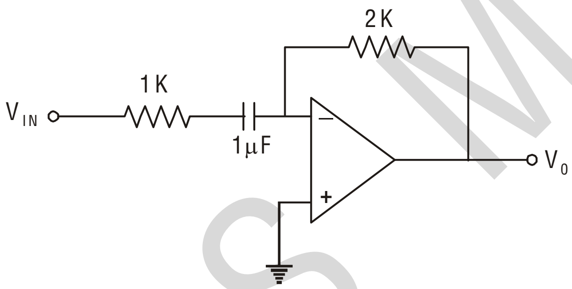
(a) The circuit is a low pass filter.

(b) The circuit is a high pass filter.

(c) The 3 dB frequency is 1000 rad/s.

(d) The 3 dB frequency is $\frac{1000}{3}$ rad/s.

Sol: (b, c)



$$\frac{V_0}{V_{IN}} = \frac{-2000}{1000 + \frac{1}{j\omega \times 10^{-6}}}$$

$$\text{Gain} = \frac{V_0}{V_{IN}} \frac{-2}{1 + \frac{1}{\left(\frac{j\omega}{1000}\right)}}$$

$$\left.\begin{array}{l}\omega \to \infty \Rightarrow \text{gain} = -2 \\ \omega \to 0 \Rightarrow \text{gain} = 0\end{array}\right\} \text{HPF}$$

$\omega_c = 1000$ rad/sec = cutoff frequency

Hence, it is HPF.

27. Select the Boolean function(s) equivalent to x + yz, where x, y, and z are Boolean variables, and + denotes logical OR operation.

(a) x + z + xy (b) (x + y) (x + z)

(c) x + xy + yz (d) x + xz + xy

Sol: (b & c)

A. x + z + xy = x(1 + y) + z = x + z

B. (x + y) (x + z) = x + xz + xy + yz

= x(1 + y + z) + yz

= x + yz

C. x + xy + yz = x(1+y) + yz

= x + yz

D. x + xz + xy = x (1 + z + y)

= x

Only B & C gives x + yz

28. Select the correct statement(s) regarding CMOS implementation of NOT gates.

(a) Noise Margin High (NM_H) is always equal to the Noise Margin Low (NM_L), irrespective of the sizing of transistors.

(b) Dynamic power consumption during switching is zero.

(c) For a logical high input under steady state, the nMOSFET is in the linear regime of operation.

(d) Mobility of electrons never influences the switching speed of the NOT gate.

Sol: (c)

(a) NM_H will not be always equal to NM_L because it depends on transistors parameters like size.

$$NM_H = V_{IL} - V_{OL}$$

$$NM_L = V_{OH} - V_{IH}$$

Condition for $NM_H = NM_L$: When $V_{TN} = |V_{TP}|$

& $V_{IP} = \frac{V_{DD}}{2}$

If $\frac{K_p}{K_n} \lessgtr 1$ then $NM_H \neq NM_L$

(b) Due to capacitive leading of stage, dynamic power consumption during switching will not be zero.

(c) For $V_{DD} - |V_{TP}| \leq V_{in} \leq V_{DD}$ [logic high input]

PMOS $\rightarrow$ cut off

NMOS $\rightarrow$ Linear

(d) Mobility of electrons influences the switching speed because

Propagation delay = $\tau_p = \dfrac{\tau_{PLH} + P_{PHL}}{2}$

$$\tau_{PLH} = \frac{C_L V_{DD}}{\mu_p C_{ox} \frac{W}{L}\left[V_{GS} \quad V_{TP}\right]^2}$$

dependent
on mobality

Therefore (c) is only correct

29. Let H(X) denote the entropy of a discrete random variable X taking K possible distinct real values. Which of the following statements is/are necessarily true?

(a) $H(X) \le \log_2 K$ bits (b) $H(X) \le H(2\,X)$

(c) $H(X) \le H(X^2)$ (d) $H(X) \le H(2^X)$

Sol: (a, b, d)

Let $y = x^2$

X	Y	P(Y)
–1	1	1/4
0	0	1/2
1	1	1/4

Y	0	1
P(Y)	1/2	1/2

$$H(X^2) = H(Y) = \Sigma P_Y(Y_i)\log_2 \frac{1}{P_Y(Y_i)}$$

$$= \frac{1}{2}\log_2 2 + \frac{1}{2}\log_2 2 = 1 \text{ bit/symbol}$$

$$H(X) = \frac{1}{4}\log_2 4 + \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 4 = 1.5 \text{ bit/symbol}$$

$$H(X) > H(X^2)$$

Hence option (c) is not correct

30. Consider the following wave equation,

$$\frac{\partial^2 f(x,t)}{\partial t^2} = 10000\frac{\partial^2 f(x,t)}{\partial x^2}$$

Which of the given options is/are solution(s) to the given wave equation?

(a) $f(x,t) = e^{-(x-100t)^2} + e^{-(x+100t)^2}$

(b) $f(x,t) = e^{-(x-100t)} + 0.5\,e^{-(x+1000t)}$

(c) $f(x,t) = e^{-(x-100t)} + \sin(x+100t)$

(d) $f(x,t) = e^{j100\pi(-100x+t)} + e^{j100\pi(100x+t)}$

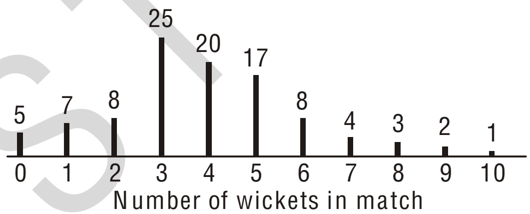
Sol: (a, c)

As we know, wave equation is given by

$$\frac{\partial^2 f(x,t)}{\partial x^2} = \frac{C^2 d^2 f(x,t)}{dt^2}$$

Here, option (a) & (c) are satisfying the above standard wave equation.

31. The bar graph shows the frequency of the number of wickets taken in a match by a bowler in her career. For example, in 17 of her matches, the bowler has taken 5 wickets each. The median number of wickets taken by the bowler in a match is __________ (rounded off to one decimal place).



Sol: (4)

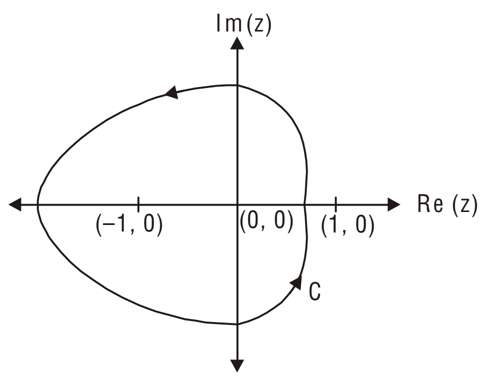
Σ frequency of No. of wickets = 5 +7 + 8 + 25 + 20 +17 + 8 + 4 + 3 + 2 + 1 = 100

Mean = Average of the 50th & 51th matches = 4

32. A simple closed path C in the complex plane is shown in the figure. If

$$\oint_C \frac{2^z}{z^2-1}dz = -i\pi A,$$

where $i = \sqrt{-1}$, then the value of A is ______ (rounded off to two decimal places).



Sol: (1/2 = 0.5)

$$\oint_C \frac{2^z}{z^2-1}dz$$

Roots $(z-1)(z+1) = 0$

$$z = \pm 1$$

$\because$ $z = -1$ is in contour

$$\Rightarrow \oint_c \frac{2^z}{z^2-1}dz = \left[\lim_{z\to -1}\frac{(z+1)\cdot 2^z}{(z+1)(z-1)}\right] * 2\pi i$$

$$= \frac{2^{-1}}{(-1-1)} \times 2\pi i$$

$$= -\frac{1}{2}\pi i$$

$$\Rightarrow \quad A = 1/2 = 0.5$$

33. Let $x_1(t) = e^{-t}u(t)$ and $x_2(t) = u(t) - u(t-2)$, where u(.) denotes the unit step function.

If y(t) denotes the convolution of $x_1(t)$ and $x_2(t)$, then $\lim_{t\to\infty} y(t) =$ _________ (rounded off to one decimal place).

Sol: (0)

$$x_1(t) = e^{-t}U(t) \xleftrightarrow{L.T.} \frac{1}{(s+1)}$$

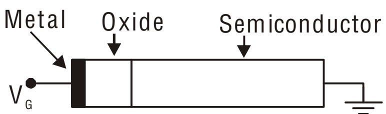
$$x_2(t) = U(t) - U(t-2) \xleftrightarrow{L.T} \frac{1}{s} - \frac{e^{-2s}}{s} = \frac{1-e^{-2s}}{s}$$

$$y(t) = x_1(t) \otimes x_2(t) \longleftrightarrow \frac{(1-e^{-2s})}{s(s+1)}$$

$$y(t) \xleftrightarrow{L.T.} \frac{1-e^{-2s}}{s(s+1)}$$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0}\frac{s\cdot(1-e^{-2s})}{s(s+1)} = 0$$

34. An ideal MOS capacitor (p-type semiconductor) is shown in the figure. the MOS capacitor is under strong inversion with V_G = 2V. the corresponding inversion charge density (Q_{IN}) is 2.2 $\mu C/cm^2$. Assume oxide capacitance per unit area as C_{OX} = 1.7 $\mu F/cm^2$. For V_G = 4V, the value of Q_{IN} is _______ $\mu C/cm^2$ (rounded off to one decimal place).



Sol: (5.6)

$$Q_{IN} = -CO_x(V_G - V_T)$$

$$Q_{IN_1} = -CO_x(V_{G1} - V_T) \quad ...(i)$$

$$Q_{IN_2} = -CO_x(V_{G2} - V_T) \quad ...(ii)$$

(ii) - (i)

$$Q_{IN_2} = Q_{IN_1} = -CO_x(V_{G2} - V_{G1})$$

$$Q_{IN_2} - (-2.2\mu c/cm^2) = -1.7\mu f/cm^2(4-2)$$

$$Q_{IN_2} = 2.2\mu\ c/cm^2 - 3.4\ \mu c/cm^2$$

$$= -5.6\mu c/cm^2$$

35. A symbol stream contains alternate QPSK and 16-QAM symbols. If symbols from this stream are transmitted at the rate of 1 mega-symbols per second, the raw (uncoded) data rate is _______ mega-bits per second (rounded off to one decimal place).

Sol: (3)

BIT RATE = [SYMBOL RATE] * $\log_{20} M$

1. QPSK, M = 4, n = 2

$$R_{b1} = 1 \times 2 = 2\text{mbps}$$

2. $16QAM_1 \Rightarrow$ M = 16, n = 4

$$R_{b2} = 1 \times 4 = 4\text{mbps}$$

$$R_b = \frac{2m + 4m}{2}$$

$$R_b = 3\text{mbps}$$

36. The function $f(x) = 8\log_e x - x^2 + 3$ attains its minimum over the interval [1, e] at x = _________.

(Here $\log_e x$ is the natural logarithm of x.)

(a) 2 (b) 1

(c) e (d) $\frac{1+e}{2}$

Sol: (b)

$$f(x) = 8\log_e x - x^2 + 3 \text{ when } x \in [1, e]$$

Differentiating both side,

$$f(x) = \frac{8}{x} - 2x = 0; \text{ where } x>0$$

$$f'(x) = 0$$

$$\frac{8}{x} - 2x = 0 \Rightarrow 8 - 2x^2 = 0$$

$$2x^2 = 8$$

$$x^2 = 4$$

$$x = \pm 2$$

$x = 2$ which is in $\in[1, e]$

$$f''(x) = \frac{-8}{x^2} - 2$$

$$f''(2) = -6 < 0$$

$\therefore$ f(x) is maximum for x = 2

$\therefore$ Minimum of f(x) will be in [1, e]

= min [f(1), f(e)]

$$f(e) = 8\ell ne - e^2 + 3 = 3.61$$

Hence, minimum value of f(x) occurs at $\boxed{x = 1}$

37. Let α, β be two non-zero real numbers and v_1, v_2 be two non-zero real vectors of size 3 × 1. Suppose that v_1 and v_2 satisfy $v_1^T v_2 = 0$, $v_1^T v_1 = 1$, and $v_2^T v_2 = 1$. Let A be the 3 × 3 matrix given by:

$$A = \alpha v_1 v_1^T + \beta v_2 v_2^T$$

The eigenvalues of A are ________.

(a) $0, \alpha, \beta$ (b) $0, \alpha+\beta, \alpha-\beta$

(c) $0, \frac{\alpha+\beta}{2}, \sqrt{\alpha\beta}$ (d) $0, 0, \sqrt{\alpha^2+\beta^2}$

Sol: (a)

$$A = \alpha\begin{bmatrix} a \\ b \\ c \end{bmatrix}[a\ b\ c] + \beta\begin{bmatrix} d \\ e \\ b \end{bmatrix}[d\ e\ b]$$

$$= \alpha\begin{bmatrix} a^2 & 0 & 0 \\ 0 & b^2 & 0 \\ 0 & 0 & c^2 \end{bmatrix} + \beta\begin{bmatrix} d^2 & 0 & 0 \\ 0 & e^2 & 0 \\ 0 & 0 & p^2 \end{bmatrix}$$

$$\text{Trace} \Rightarrow \alpha(a^2+b^2+c^2) + \beta(d^2+e^2+f^2)$$

$$= \alpha + \beta$$

Alternate:

$$AV_1 = \alpha V V_1^T V_1 + \beta V_2 V_2^T V_1$$

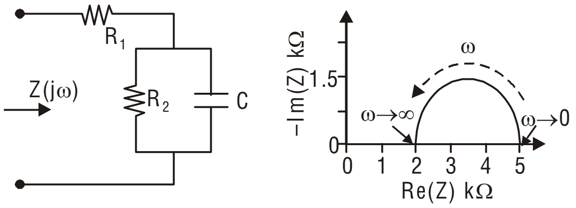
$$AV_1 = \alpha(1)V_1 + \beta(0) = \alpha V_1$$

$$AV_1 = \alpha V_1$$

$$Ax = \lambda X$$

So, eigen value, $\alpha, \beta, 0$

38. For the circuit shown, the locus of the impedance $Z(j\omega)$ is plotted as ω increases from zero to infinity. The values of R_1 and R_2 are:



(a) $R_1 = 2\,k\Omega$, $R_2 = 3\,k\Omega$

(b) $R_1 = 5\,k\Omega$, $R_2 = 2\,k\Omega$

(c) $R_1 = 5\,k\Omega$, $R_2 = 2.5\,k\Omega$

(d) $R_1 = 2\,k\Omega$, $R_2 = 5\,k\Omega$

Sol: (a)

$$Z(j\omega) = R_1 + \frac{R_2 \cdot {}^1\!/_{j\omega c}}{R_2 + {}^1\!/_{j\omega c}} = R_1 + \frac{R_2}{\frac{1}{2} + jR_2\omega c}$$

$$Z(j\omega)_{\omega=0} = R_1 + R_2$$

$$\Rightarrow R_1 + R_2 = 5k\Omega$$

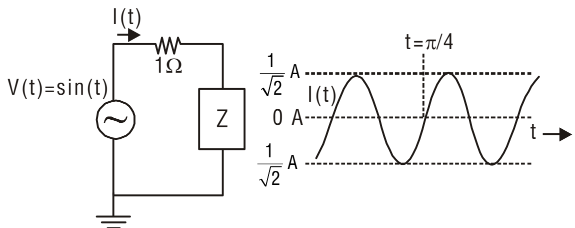
$$Z|j\omega|_{\omega\to\infty} = R_1$$

$$\Rightarrow R_1 = 2k\Omega$$

So, $R_2 = 3k\Omega$

So, $R_1 = 2k\Omega$ & $R_2 = 3k\Omega$

39. Consider the circuit shown in the figure with input V(t) in volts. The sinusoidal steady state current I(t) flowing through the circuit is shown graphically (where t is in seconds). The circuit element Z can be ________.



(a) a capacitor of 1 F

(b) an inductor of 1 H

(c) a capacitor of $\sqrt{3}$ F

(d) an inductor of $\sqrt{3}$ H

Sol: (b)

$$i(t) = \frac{V(t)}{1+Z} \quad \text{where, } V(t) = \sin t$$

$\because$ Given i(t) is lagging (from plot)

$$I_{max} = \frac{V_{max}}{Z} = \frac{1}{Z}$$

$$\Rightarrow \quad Z = 1/\sqrt{2}$$

$$Z = (1 + j\omega L) \Rightarrow L = 1$$

as $\omega = 1\ (\sin \omega t) = \sin t$

40 Consider an ideal long channel nMOSFET (enhancement-mode) with gate length 10 μm and width 100 μm. The product of electron mobility (μ_n) and oxide capacitance per unit area (C_{OX}) is $\mu_n C_{OX}$ = 1 mA/V^2. The threshold voltage of the transistor is 1 V. For a gate-to-source voltage V_{GS} = [2 – sin(2 t)]V and drain-to-source voltage V_{DS} = 1 V (substrate connected to the source), the maximum value of the drain-to-source current is __________.

(a) 40 mA (b) 20 mA

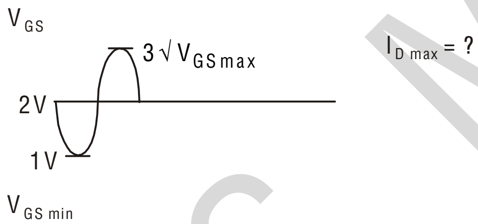
(c) 15 mA (d) 5 mA

Sol: (c)

Given

$\mu_n Co_x = 1m\ A/V^2;\ W = 100\mu m;\ L = 10\mu m$

$V_T = \perp V;\ V_{GS} = [2 - \sin 2t] V;\ V_{DS} = 1V$



Let, V_{GS} = 3V (max)

$\Rightarrow \quad V_{DS} < V_{GS} - V_t$

$\because \quad 1 < (3 - 1)$

MOSFET in triode region

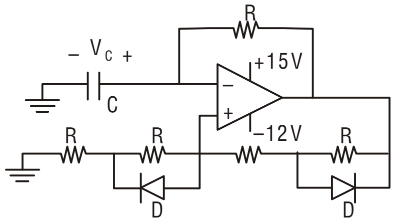
$$I_{D\,max} = \mu_n Co_x \left(\frac{\omega}{L}\right)\left\{\left(V_{as\,max} - V_t\right)V_{DS} - \frac{1}{2}V_{DS}^2\right\}$$

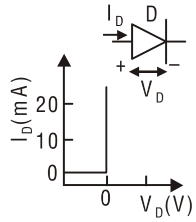
$$= 1 \times \left(\frac{100}{10}\right)\left\{(3-1)\times 1 - \frac{1}{2}\times 1^2\right\} mA$$

$$= 10\ (2 - 1/2)$$

$$I_{Dmax} = 15\ mA$$

41. For the following circuit with an ideal OPAMP, the difference between the maximum and the minimum values of the capacitor voltage (V_c) is __________.

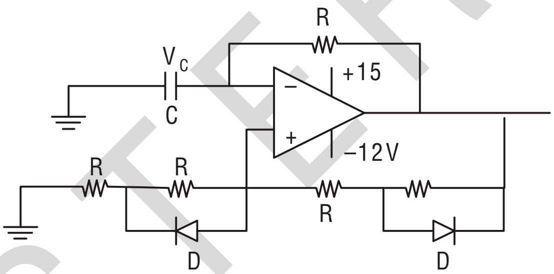


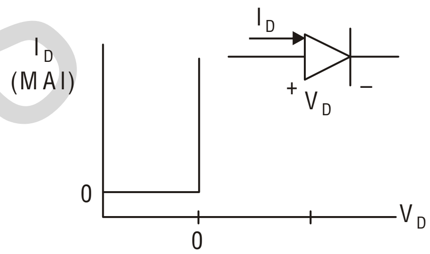


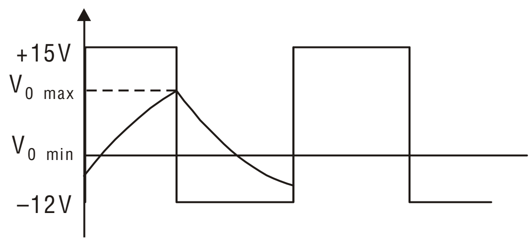
(a) 15 V (b) 27 V

(c) 13 V (d) 14 V

Sol: (c)





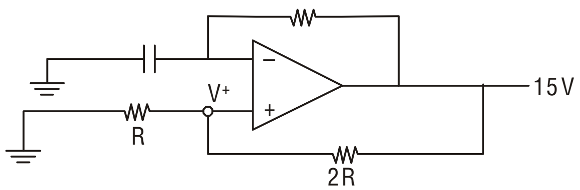


When

$$V_0 = +15V$$

$$V^+ = \frac{R}{3R} \times 15 = 5\ V$$

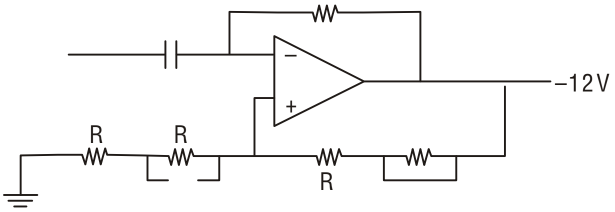
$$V_{c\ max} = 5\ V$$



When $V_0 = -12\ V$

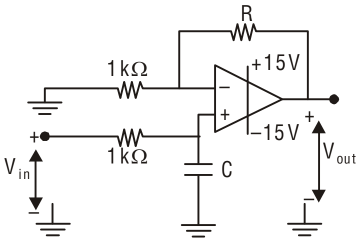
$$V^+ = \frac{2R}{3R} \times (-12)$$

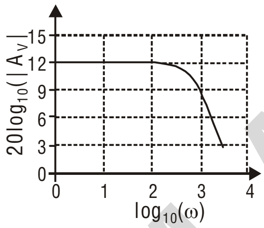
$$V_{c\,min} = -8V$$



$$V_{c\,max} = V_{c\,min}$$
$$= 5 - (-8) = 13\ V$$

42. A circuit with an ideal OPAMP is shown. The Bode plot for the magnitude (in dB) of the gain transfer function $\left(A_V(j\omega) = V_{out}(j\omega)/V_{in}(j\omega)\right)$ of the circuit is also provided (here, ω is the angular frequency in rad/s). The values of R and C are ________.





(a) $R = 3\,k\Omega,\ C = 1\,\mu F$

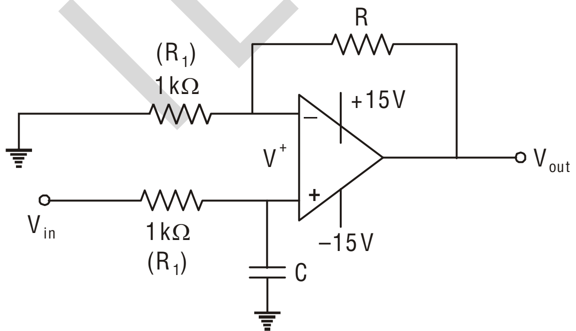
(b) $R = 1\,k\Omega,\ C = 3\,\mu F$

(c) $R = 4\,k\Omega,\ C = 1\,\mu F$

(d) $R = 3\,k\Omega,\ C = 2\,\mu F$

Sol: (a)

$$Av(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$$



$$V^+ = \frac{1}{1 + j\omega(1K)\times C} V_{in}$$

$$\&\qquad V^+ = \frac{1k}{1k + R}\times V_{out}$$

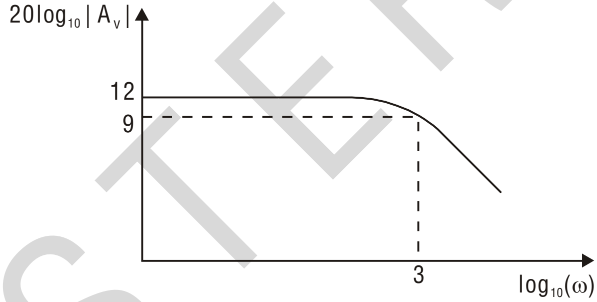
$$\frac{1(k)}{1k + R} V_{out} = \frac{1}{1 + j\omega(1k)\cdot C} V_{in}$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{1k + R}{1k + j\omega(1k)\cdot(1k)C} = \frac{(R+1)}{1 + j\omega c}$$

$$|A_v| = \frac{(R+1)^2\times 10^6}{\sqrt{10^6 + \omega^2\left(10^6\times 10^{-6}\right)^2 C^2}}$$

Let, R in kΩ

& C in μF

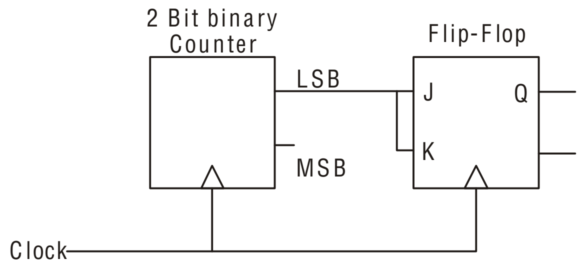


$\because$ Gain decreases by 3dB from 12 to 9

So, $\log_{10}\omega_c = 3$ is 3-dB freq.

$$20\log_{10}A_{max} = 12$$
$$\Rightarrow \quad A_{max} \simeq 4$$
$$1 + \frac{R_2}{R_1} = 4$$
$$R_2 = 3R_1 = 3k\Omega$$
$$\because \quad \log_{10}\omega_c = 3$$
$$\Rightarrow \quad \omega_c = 10^3\ \text{rad/sec}$$
$$\omega_c = \frac{1}{R_1 C} \Rightarrow C = \frac{1}{10^3\times 10^3} = 1\mu F$$

43. For the circuit shown, the clock frequency is f_o and the duty cycle is 25%. For the signal at the Q output of the Flip-Flop, _______.



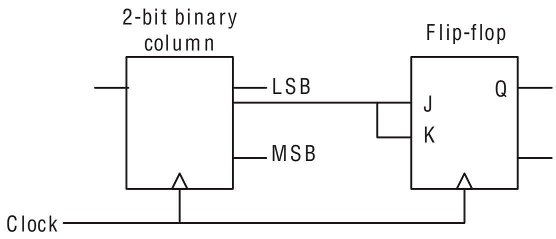
(a) Frequency is $f_0/4$ and duty cycle is 50%

(b) Frequency of $f_0/4$ and duty cycle is 25%

(c) Frequency is $f_0/2$ and duty cycle is 50%

(d) Frequency is f_0 and duty cycle is 25%

Sol: (a)



$\because$ 2 bit Counter

MSB LSB
0 0
0 1
1 0
1 1

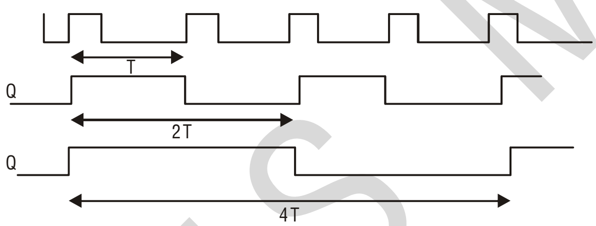
LSB 0, 1, 0, 1

For JK flip-flop (FF), 00 will not change the state

So, output frequency $f_0/2$

$\because$ Two time change of state and duty cycle = 50%.

e.g.



$$f = f_0/4$$

$$\text{Duty cycle} = 50\%$$

44. Consider an even polynomial p(s) given by

$p(s) = s^4 + 5s^2 + 4 + K,$

where K is an unknown real parameter. The complete range of K for which p(s) has all its roots on the imaginary axis is ________.

(a) $-4 \le K \le \frac{9}{4}$ (b) $-3 \le K \le \frac{9}{2}$

(c) $-6 \le K \le \frac{5}{4}$ (d) $-5 \le K \le 0$

Sol: (a)

$$p(s) = s^4 + 5s^2 + 4 + k$$

$$\begin{array}{llll} s^4 & 1 & 5 & (4+k) \\ s^3 & 0 & 0 & \\ s^2 & & & \\ s^1 & & & \\ s_0 & & & \end{array}$$

$$s^4 + 5s^2 + (4+k) = 0$$

$$4s^3 + 10s = 0$$

$$\& \quad (4s^2 + 10) = 0$$

$$s = \pm j\sqrt{5/2}$$

$$\begin{array}{llll} s^4 & 1 & 5 & (4+k) \\ s^3 & 4 & 10 & \\ s^2 & 5/2 & (4+k) & \\ s^1 & 10 - \dfrac{4(4+k)}{5/2} & & \\ s^0 & (4+k) & & \end{array}$$

$$(4+K) > 0$$

$$K > -4$$

$$10 \times \frac{5}{2} > 4(4+K)$$

$$(4+K) < \frac{25}{4}$$

$$K < 9/4$$

$\Rightarrow$ All roots be on imaginary axis $-4 \le k \le 9/4$

45. Consider the following series:

$$\sum_{n=1}^{\infty} \frac{n^d}{c^n}$$

For which of the following combinations of c, d values does this series converge ?

(a) c = 1, d = – 1 (b) c = 2, d = 1

(c) c = 0.5, d = – 10 (d) c = 1, d = – 2

Sol: (b, d)

$\sum_{n=1}^{\infty} \frac{n^d}{c^n}$ series converage for

Ratio test $\lim_{n\to\infty} \frac{U_{n+1}}{U_n}$

Option (b) c = 2, d = 1 $\Rightarrow$ $\Sigma U_n \sum U_n = \sum \frac{n}{2^n}$

$$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \lim_{n\to\infty} \frac{(n+1)}{2^{n+1}} \times \frac{2^n}{n} = \frac{1}{2} < 1$$

convergence

Option (a) c = 1, d = – 1

$$\Rightarrow \sum U_n = \sum \frac{n^{-1}}{1^n} = \sum \frac{1}{n}$$

$\sum \frac{1}{n}$ is divergent by p-test

Option (c) c = 0.5, d = – 10

$$\Rightarrow \sum U_n = \sum \frac{n^{-10}}{\left(\frac{1}{2}\right)^n}$$

$$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \lim_{n\to\infty} \frac{(n+1)^{-10}}{\left(\frac{1}{2}\right)^{n+1}} \times \frac{(1/2)^n}{n^{-10}} = 2 > 1$$

Divergent by ratio test.

Option (d) c = 1, d = – 2

$$\Rightarrow \sum U_n = \sum \frac{n^{-2}}{1^n} = \sum \frac{1}{n^2}$$

$\therefore \sum \frac{1}{n^2}$ is convergent by p-test

$\because \sum \frac{1}{n^p}$ p > 1.

46. The outputs of four systems (S_1, S_2, S_3 and S_4) corresponding to the input signal sin(t), for all time t, are shown in the figure.

Based on the given information, which of the four systems is/are definitely NOT LTI (linear and time-invariant)?

$$\sin(t) \rightarrow \boxed{S_1} \rightarrow \sin(-t)$$
$$\sin(t) \rightarrow \boxed{S_2} \rightarrow \sin(t+1)$$
$$\sin(t) \rightarrow \boxed{S_3} \rightarrow \sin(2t)$$
$$\sin(t) \rightarrow \boxed{S_4} \rightarrow \sin^2(t)$$

(a) S_1 (b) S_2

(c) S_3 (d) S_4

Sol: (c & d)

Linearity:

$$ax_1(t) + bx_2(t) \xrightarrow{\text{sys}} a\,y_1(t) + by_2(t)$$

Time - Invariant:

$$x(t+t_0) \xrightarrow{\text{sys}} y(t+t_0)$$

$$S_1 : x(t+t_0) = \sin(t+t_0) \xrightarrow{S_1} \sin(-(t+t_0))$$

$\because$ sin (–t) = – sin(t)

May be LTI sys

$$S_2: \; a\sin t + b\sin t \rightarrow \boxed{S_2} \rightarrow a\sin(t+1) + b\sin(t+1)$$

$$\sin(t+t_0) \rightarrow \boxed{S_2} \rightarrow \sin(t+t_0+1)$$

LTI sys

$$S_3: \; \sin(t+t_0) \rightarrow \boxed{S_3} \rightarrow \sin(2(t+t_0))$$

$\because$ sin $(2t + t_0) \neq$ sin $(2(t + t_0))$

Time-variant (Not LTI sys)

S_4 :

$$\sin(t+t_0) \rightarrow \boxed{S_4} \rightarrow \sin^2(t) = \frac{1 \quad \cos 2t}{2}$$

Linear and Time - variant (not LTI)

47. Select the CORRECT statement(s) regarding semiconductor devices.

(a) Electrons and holes are of equal density in an intrinsic semiconductor at equilibrium.

(b) Collector region is generally more heavily doped than Base region in a BJT.

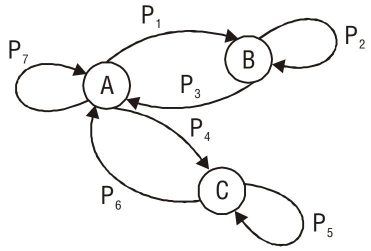
(c) Total current is spatially constant in a two terminal electronic device in dark under steady state condition.

(d) Mobility of electrons always increases with temperature in Silicon beyond 300 K.

Sol: (a, c)

- At equilibrium n = p = n_i for intrinsic semiconductor
- Collector region is generally lightly doped then base region in BJT. Hence option B is wrong.
- By increasing temperature above 300K, mobality of electrons decreases hence option (d) is also wrong

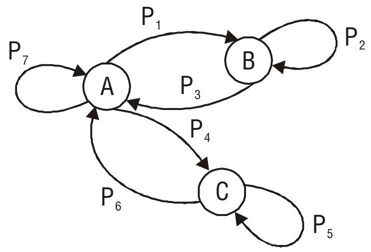
48. A state transition diagram with states A, B and C, and transition probabilities p_1, p_2, ...,p_7 is shown in the figure (e. g., p_1 denotes the probability of transition from state A to B). For this state diagram, select the statement(s) which is/are universally true.



(a) $p_2 + p_3 = p_5 + p_6$ (b) $p_1 + p_3 = p_4 + p_6$

(c) $p_1 + p_4 + p_7 = 1$ (d) $p_2 + p_5 + p_7 = 1$

Sol: (a, c)



$$\left.\begin{array}{lll} A \to & A & P_7 \\ \to & B & P_1 \\ \to & C & P_4 \end{array}\right\} \Rightarrow P_1 + P_4 + P_7 = 1$$

$$\left.\begin{array}{llll} B \to & A & \to & P_2 \\ \to & B & \to & P_3 \end{array}\right\} \Rightarrow P_2 + P_3 = 1$$

$$\left.\begin{array}{llll} C \to & A & \to & P_6 \\ \to & C & \to & P_5 \end{array}\right\} \Rightarrow P_5 + P_6 = 1$$

$\boxed{P_2 + P_3 = P_5 + P_6} \Rightarrow$ Option (a) correct

$\boxed{P_1 + P_4 + P_7 = 1} \Rightarrow$ Option (c) correct

49. Consider a Boolean gate (D) where the output Y is related to the inputs A and B as, $Y = A + \overline{B}$, where + denotes logical OR operation. The Boolean inputs '0' and '1' are also available separately. Using instances of only D gates and inputs '0' and '1', __________ (select the correct option(s))

(a) NAND logic can be implemented

(b) OR logic cannot be implemented

(c) NOR logic can be implemented

(d) AND logic cannot be implemented

Sol: (a & c)

$$y = A + \overline{B} = \overline{\overline{A} \cdot B}$$

$$f(A, B) = A + \overline{B}$$

$$f(0, B) = 0 + \overline{B} = \overline{B} \Rightarrow \text{NOT Gate}$$

$$f(A, \overline{B}) = A + \overline{\overline{B}} = A + B \Rightarrow \text{OR Gate}$$

$$f(0, f(A, \overline{B})) = 0 + (\overline{A + B}) \Rightarrow \text{NOR Gate}$$

$$f(\overline{A}, B) = \overline{A} + \overline{B} = \overline{A \cdot B} \Rightarrow \text{NAND Gate}$$

$$f(0, f(\overline{A}, B)) = 0 + (\overline{\overline{A \cdot B}}) = A \cdot B \Rightarrow \text{AND Gate}$$

Since, we can implement NOT Gate we can also implement

OR, AND, NOR and NAND Gate.

50. Two linear time-invariant systems with transfer functions

$$G_1(s) = \frac{10}{s^2 + s + 1} \text{ and } G_2(s) = \frac{10}{s^2 + s\sqrt{10} + 10}$$

have unit step responses $y_1(t)$ and $y_2(t)$, respectively. Which of the following statements is/are true?

(a) $y_1(t)$ and $y_2(t)$ have the same percentage peak overshoot.

(b) $y_1(t)$ and $y_2(t)$ have the same steady-state value

(c) $y_1(t)$ and $y_2(t)$ have the same damped frequency of oscillation.

(d) $y_1(t)$ and $y_2(t)$ have the same 2% settling time.

Sol: (a)

$$G_1(s) = \frac{10}{s^2 + s + 1}$$

$$\omega_{n_1}^2 = 1$$

$$2\xi_1 \times 1 = 1$$

$$\xi_1 = 1/2$$

$$G_2(s) = \frac{10}{s^2 + s\sqrt{10} + 10}$$

$$\omega_{n_2}^2 = 10$$

$$2 \times \xi_2 \times \sqrt{10} = \sqrt{10}$$

$$\xi_2 = 1/2$$

$\because$ M_P depends on ξ only

$y_1(t)$ & $y_2(t)$ have same percentage peak overshoot.

$$\omega_{d_1} = \omega_{n_1}\sqrt{1-\xi_1^2}$$

$$\Rightarrow \quad \omega_{d_1} \neq \omega_{d_2} \quad \because \quad \omega_{n_1} \neq \omega_{n_2}$$

damped frequency of oscillation is not same.

$$T_s = \frac{4}{\xi\omega_n} \quad \text{(2\% setting time)}$$

$$T_{s_1} \neq T_{s_2} \quad \because \quad \omega_{n_1} \neq \omega_{n_2}$$

$$C_1(s) = \frac{10 \times 1/s}{s^2+s+1}$$

$$C_{ss} = \lim_{s\to 0} s.C_1(s)$$

$$= 10$$

$$C_2(s) = \frac{10 \times 1/s}{s^2+\sqrt{10}s+10}$$

$$C_{ss} = \lim_{s\to 0} s.C_2(s)$$

$$\frac{10}{10}1 \Rightarrow \frac{10}{10} = 1$$

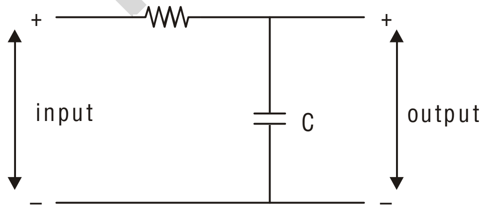
Steady - state value is not same.

51. Consider an FM broadcast that employs the pre-emphasis filter with frequency response

$$H_{pe}(\omega) = 1 + \frac{j\omega}{\omega_0},$$

where $\omega_0 = 10^4$ rad/sec.

For the network shown in the figure to act as a corresponding de-emphasis filter, the appropriate pair(s) of (R, C) values is/are __________.



(a) $R = 1\,k\Omega$, $C = 0.1\,\mu F$

(b) $R = 2\,k\Omega$, $C = 1\,\mu F$

(c) $R = 1\,k\Omega$, $C = 2\,\mu F$

(d) $R = 2\,k\Omega$, $C = 0.5\,\mu F$

Sol: (a)

$$H_{pe}(f) = \frac{1}{H_{de}(f)}$$

$$\Rightarrow \quad |H_{Pe}(f)|^2 = \frac{1}{|H_{de}(f)|^2} \qquad ...(i)$$

$$H_{Pe}(\omega) = 1 + j\frac{\omega}{\omega_0} \quad \text{where} \quad \omega_0 = 10^4$$

$$|H_{Pe}(\omega)| = \sqrt{1+(\omega/\omega_0)^2}$$

$$\Rightarrow |H_{Pe}(\omega)|^2 = 1+(\omega/\omega_0)^2 \qquad ...(ii)$$

$$H_{de}(\omega) = \frac{1}{1+j\omega RC}$$

$$\Rightarrow \quad |H_{de}(\omega)|^2 = \frac{1}{1+(\omega RC)^2} \qquad ...(iii)$$

from (i), (ii) & (iii) $\qquad \omega_0 = \frac{1}{RC} = 10^4$

$\Rightarrow$ $R = 1\,k\Omega$ & $C = 0.1\,\mu F$ satisfies only

52. A waveguide consists of two infinite parallel plates (perfect conductors) at a separation of 10^{-4} cm, with air as the dielectric. Assume the speed of light in air to be 3×10^8 m/s. The frequency/frequencies of TM waves which can propagate in this waveguide is/are ________.

(a) 6×10^{15} Hz (b) 0.5×10^{12} Hz

(c) 8×10^{14} Hz (d) 1×10^{13} Hz

Sol: (a, c)

Cut-off frequency.

$$f_c = \frac{c}{2a} \quad (m = 1)$$

$$= \frac{3\times 10^8}{2\times 10^{-4}\times 10^{-2}} \quad \because a = 10^{-4}\text{ cm (given)}$$

$$= 1.5 \times 10^{14}\text{ Hz}$$

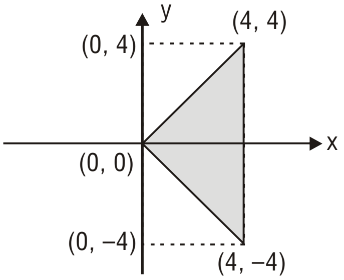
$f > f_c$ will only propagate

$\Rightarrow$ A & C will propage

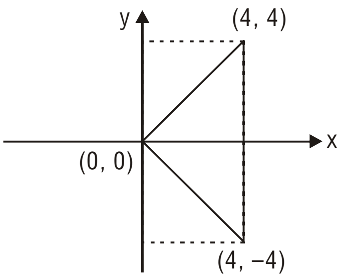
53. The value of the integral

$$\iint_D 3\left(x^2+y^2\right)dxdy,$$

where D is the shaded triangular region shown in the diagram, is __________ (rounded off to the nearest integer).



Sol: (512)



$$I = \int_0^4\int_{-x}^{x}\left(3x^2+3y^2\right)dy\,dx$$

$$= \int_0^4\left[3x^2y+\frac{3y^3}{3}\right]_{-x}^{x}dx$$

$$= \int_0^4\left[3x^2(2x)+2x^3\right]dx$$

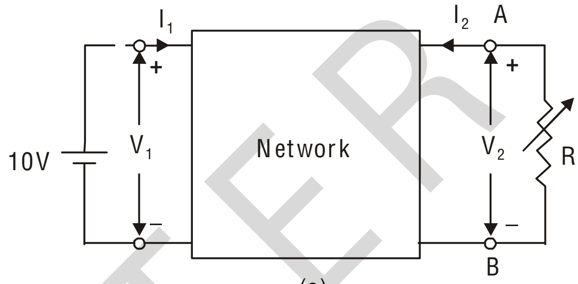
$$= \int_0^4 8x^3$$

$$= 2\times 4^4$$

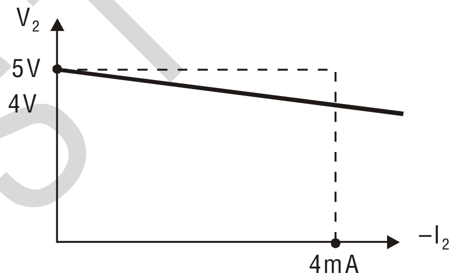
$$= 512$$

54. A linear 2-port network is shown in figure (a). An ideal DC voltage source of 10 V is connected across Port 1. A variable resistance R is connected across Port 2. As R is varied, the measured voltage and current at Port 2 is shown in figure (b) as a V_2 versus $-I_2$ plot. Note that for V_2 = 5 V, I_2 = 0 mA, and for V_2 = 4 V, I_2 = – 4 mA.

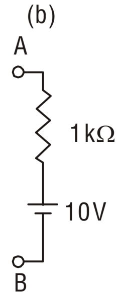
When the variable resistance R at Port 2 is replaced by the load shown in figure (c), the current I_2 is __________ mA (rounded off to one decimal place).



(a)

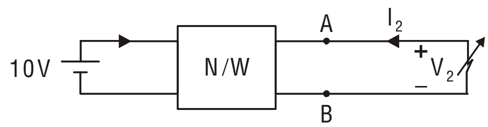


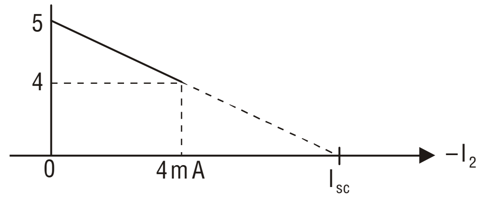
(b)

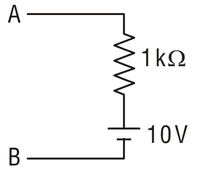


(c)

Sol: (4)







Case -1: (I_2 = 0)

$$V_{oc} = 5V$$

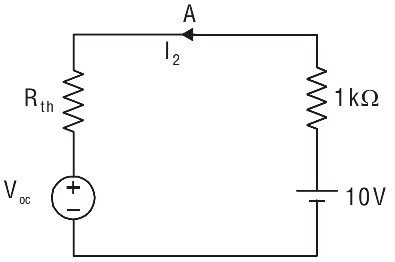
Case - 2: With help of linearity

$$\frac{4-5}{4-0} = \frac{I_{sc}-5}{I_{sc}-0}$$

$$I_{sc} = 20 \text{ mA}$$

$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{5}{20}k = 250\Omega = 0.25\ k\Omega$$

Case -3:



$$I_2 = \frac{10-5}{1.25} mA$$

$$I_2 = 4mA$$

55. For a vector $\overline{x}$ = [x[0], x [1],...,x[7]], the 8-point discrete Fourier transform (DFT) is denoted by $\overline{X} = DFT(\overline{x})$ = [X[0],X[1],...,X[7]], where

$$X[k] = \sum_{n=0}^{7} x[n]\exp\left(-j\frac{2\pi}{8}nk\right).$$

Here, j = $\sqrt{-1}$. If $\overline{x}$ = [1, 0, 0, 0, 2, 0, 0, 0] and $\overline{y}$ = DFT (DFT($\overline{x}$)), then the value of y[0] is ______ (rounded off to one decimal place).

Sol: **(8)**

$$X[K] = \sum_{n=0}^{7} x[n]\exp\left(-j\frac{2\pi}{8}mk\right)$$

$\overline{y} = DFT(DFT(\overline{x}))$ where

$\overline{x} = \left[x[0], x[1],...x[7]\right]$

y(0) = ?   x[n] = [1, 0, 0, 0, 2, 0, 0, 0]

$$x[n] \xrightarrow{DFT} \xrightarrow{DFT} N \cdot x(-k) = \overline{y}(n)$$

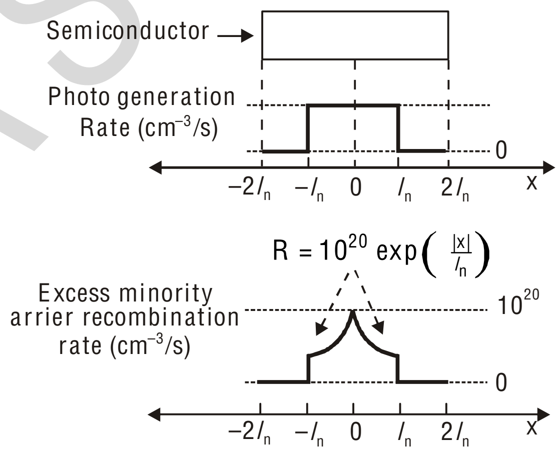
$$\overline{y}(n) = N \cdot x(-k)|_{mod.N} = N.x(N-K),\ N = 8$$

$$\overline{y}(n) = 8[1, 0, 0, 0, 2, 0, 0, 0]$$

$$\overline{y}(n) = [8, 0, 0, 0, 16, 0, 0, 0]$$

$$y(0) = 8$$

56. A p-type semiconductor with zero electric field is under illumination (low level injection) in steady state condition. Excess minority carrier density is zero at $x = \pm 2\ell_n$, where $\ell_n = 10^{-4}$cm is the diffusion length of electrons. Assume electronic charge, q = – 1.6 × 10^{-19}C. the profiles of photo-generation rate of carriers and the recombination rate of excess minority carriers (R) are shown. Under these conditions, the magnitude of the current density due to the photo-generated electrons at x = + $2\ell_n$ is ______ mA/cm² (rounded off to two decimal places).



Sol: **(0.59)**

Given

$$\delta_n(x) = R\tau_n = 10^{20}e^{-|x|/\ell_n} \cdot \tau_n$$

$$\delta_n(\ell_n) = 10^{20}e^{-1}\tau_n \qquad ...(i)$$

for $\ell_n \le x \le 2\ell_n$

Continuity equation is given by

$$D_n\frac{\partial^2\delta_n}{\partial x^2} + G - R = 0 \qquad ...(ii)$$

G & R both are zero - for $\left[\ell_n \le x \le 2\ell_n\right]$

Hence Equation (i) reduced to $D_n\frac{\partial^n\delta_n}{\partial x^2} = 0$

$$\Rightarrow \quad \delta_n(x) = Ax + B$$

For calculating A & B we use Boundary condition

$$\delta_n(2\ell_n) = 0 \quad \Rightarrow \boxed{A = \frac{-B}{2\ell_n}}$$

$$\therefore \quad \delta_n(x) = \frac{-B}{2\ell_n}x + B = B\left[1-\frac{x}{2\ell_n}\right] \quad ...(iii)$$

At $x = \ell_n$

$$\Rightarrow 10^{20}e^{-1}\tau_n = B\left[1-\frac{\ell_n}{2\ell_n}\right]$$

$$\Rightarrow \quad B = 2\times 10^{20}e^{-1}\tau_n$$

$$\therefore \quad \delta_n(x) = 2\times 10^{20}e^{-1}\tau_n\left[1-\frac{x}{2\ell_n}\right]$$

for $\ell_n \le x \le 2\ell_n$

Electron diffusion current density is given by

$$|J_n|_{diff} = qD_n\frac{d\eta}{dx} = qD_n\times 2\times 10^{20}\times e^{-1}\times\tau_n\left[0-\frac{1}{2\ell_n}\right]$$

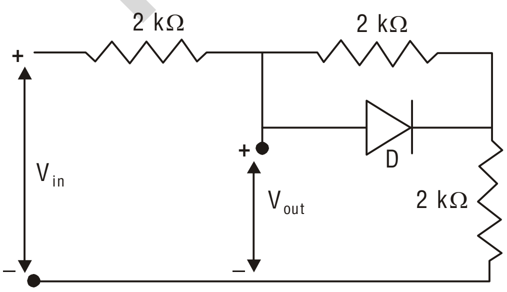
$$= \frac{1.6\times 10^{-19}\times \ell_n{}^2\times 2\times 10^{20}\times e^{-1}}{2\ell_n}$$

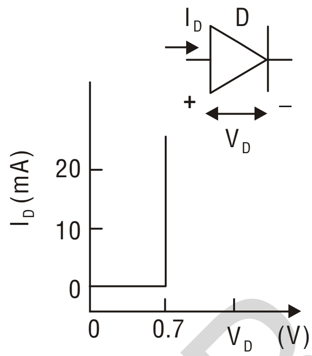
$$= 1.6\times 10^{-19}\times \ell_n\times 10^{20}\times e^{-1}$$

$$= 1.6\times 10\times 1\times 10^{-4}\times e^{-1}$$

$$= \boxed{0.59\ \text{mA/cm}^2}$$

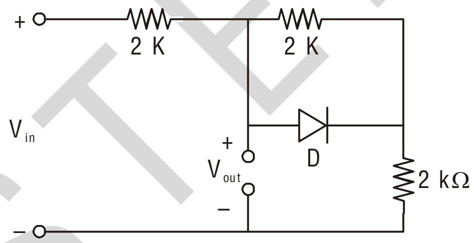
57. A circuit and the characteristics of the diode (D) in it are shown. The ratio of the minimum to the maximum small signal voltage gain $\frac{\partial V_{out}}{\partial V_{in}}$ is ________ (rounded off to two decimal places).



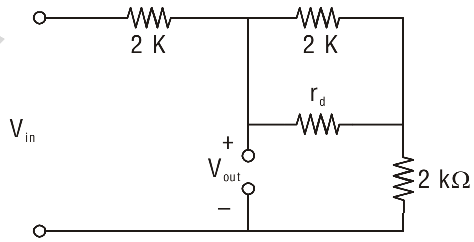


Sol: (0.75)

Given circuit shown below



Replacing the circuit with small signal equivalent.



Case-I when diode is ON

As $r_d(ON) = 0$, the $2\,k\Omega$ resistor in parallel to the diode becomes open circuit.

$$\therefore \quad V_{out} = V_{IN}\times\frac{2}{4} = \frac{V_{in}}{2}$$

$$\therefore \quad \left.\frac{\partial V_{out}}{\partial V_{in}}\right|_{max} = \frac{1}{2} \quad ...(i)$$

Case-I: When diode is off:

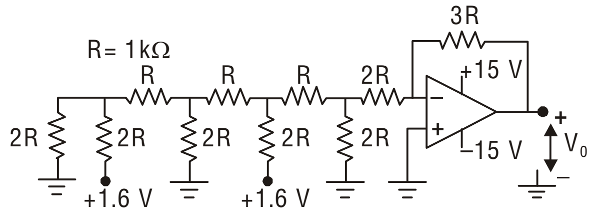
r_d (off) $= \infty \Rightarrow$ total $R_{eq} = 2 + 2 + 2 = 6\ k\Omega$

$$\therefore \quad V_{out} = \frac{V_{in}\times 4}{6} = \frac{2}{3}V_{in} \Rightarrow \left.\frac{\partial V_{out}}{\partial V_{in}}\right|_{min} = \frac{2}{3} \quad ..(ii)$$

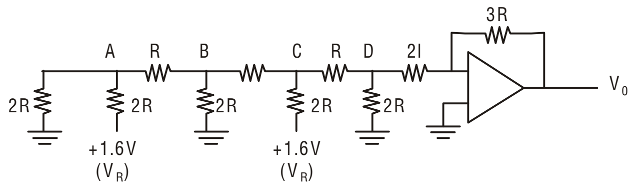
From (i) and (ii)

$$\frac{\left(\frac{\partial V_{out}}{\partial V_{in}}\right)_{min}}{\left(\frac{\partial V_{out}}{\partial V_{in}}\right)_{max}} = \frac{1/2}{2/3} = \frac{1}{2}\times\frac{3}{2} = 0.75$$

58. Consider the circuit shown with an ideal OPAMP. The output voltage V_0 is ________V (rounded off to two decimal places).

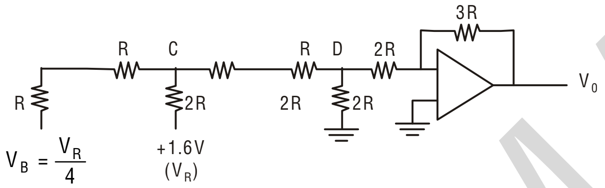


Sol: **(–0.5)**



$$V_A = \frac{V_R}{2} \;\&\; R_{th} = R$$

$$V_B = \frac{V_A}{2}$$

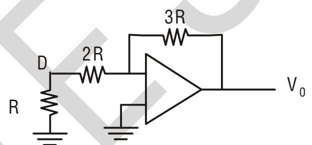


$$V_C = \frac{\left(V_R + V_R/4\right)}{2} = \frac{5}{8}V_R$$

$$R_{th} = 2R\,||\,2R = R$$

Similarly, $VD = \frac{V_c}{2} = \frac{5}{16}V_R \;\&\; R_{th} = R$

So,

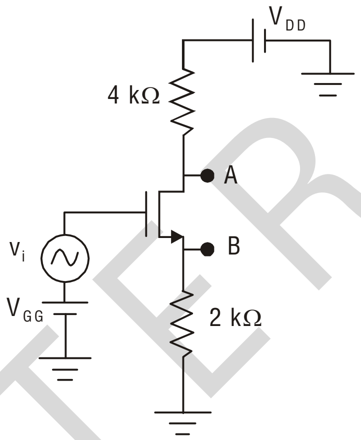


$$\frac{S}{16}V_R = \frac{5}{16}\times 1.6\text{ V}$$

$$V_0 = \frac{-3R}{(2R+R)}\times\frac{5}{16}\times 1.6 = -\frac{3}{2}\times\frac{5}{16}\times 1.6$$

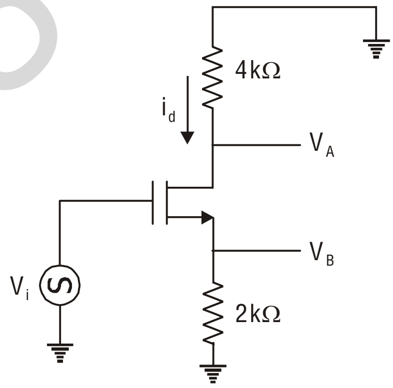
$$V_0 = -\,0.5\text{ V}$$

59. Consider the circuit shown with an ideal long channel nMOSFET (enhancement-mode, substrate is connected to the source). The transistor is appropriately biased in the saturation region with V_{GG} and V_{DD} such that it acts as a linear amplifier. v_i is the small-signal ac input voltage. v_A and v_B represent the small-signal voltages at the nodes A and B, respectively. The value of $\frac{v_A}{v_B}$ is ________ (rounded off to one decimal place).



Sol: **(–2)**

For ac analysis



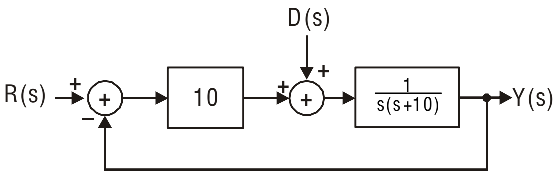
$$V_A = -i_d.4k$$

$$V_B = i_d.2k$$

$$\frac{V_A}{V_B} = \frac{-4}{2}$$

$$\frac{V_A}{V_B} = -2$$

60. The block diagram of a closed-loop control system is shown in the figure. R(s), Y(s), and D(s) are the Laplace transforms of the time-domain signals r(t), y(t), and d(t), respectively. Let the error signal be defined as e(t) = r(t) – y(t). Assuming the reference input r(t) = 0 for all t, the steady-state error $e(\infty)$, due to a unit step disturbance d(t), is __________ (rounded off to two decimal places).



Sol: (–0.1)

$$Y(s) = \frac{R(s).10/s(s+10)}{1+10/s(s+10)} + \frac{D(s).10/s(s+10)}{1+10/s(s+10)}$$

when, $r(t) = 0$ & $d(t) = U(t) \xleftrightarrow{\text{L.T.}} \frac{1}{s}$

$$e(\infty) = -\lim_{s\to 0} \frac{s\cdot\frac{1}{s}\times 1}{s(s+10)+10}$$

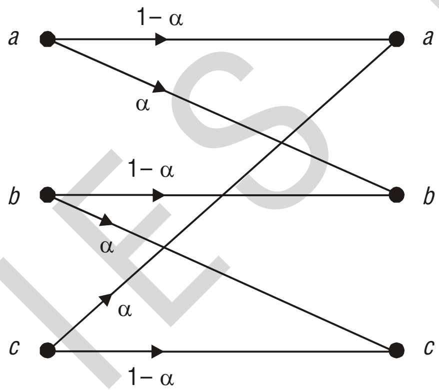
$[\because\ e(t) = r(t) - y(t)]$

$= -1/10$

$= -0.1$

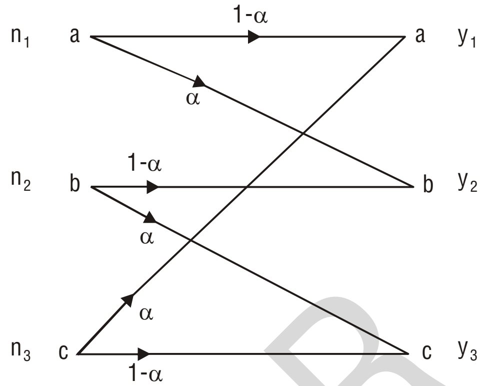
61. The transistion diagram of a discrete memoryless channel with three input symbols and three output symbols is shown in the figure. The transition probabilities are as marked.

The parameter α lies in the interval [0.25, 1]. The value of α for which the capacity of this channel is maximized, is _______ (rounded off to two decimal places).



Sol: (1)

$\alpha \in [0.25, 1]$



$I(x : y) = H(y) - H(y/x)$

where,

$$H(y/x) = \sum_{i=1}^{3}\sum_{j=1}^{3} P(x_i, y_i)\log_2 P(y_i / x_i)$$

$$[P(y/x)] = \begin{bmatrix} 1-\alpha & \alpha & 0 \\ 0 & 1-\alpha & \alpha \\ \alpha & 0 & 1-\alpha \end{bmatrix}$$

$$[P(x/y)] = [P(x)]_d \cdot [P(y/x)]$$

$$[P(x)]_d = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$

$$\Rightarrow [P(x,y)] = \begin{bmatrix} (1-\alpha)/3 & \alpha/3 & 0 \\ 0 & (1-\alpha)/3 & \alpha/3 \\ \alpha/3 & 0 & (1-\alpha)/3 \end{bmatrix}$$

$$H(y/x) = -\left\{\frac{(1-\alpha)}{3}\log_2(1-\alpha) + \frac{\alpha}{3}\log_2(\alpha)\right\}\times 3$$

$$= -\{(1-\alpha)\log_2(1-\alpha) + \alpha\log_2(\alpha)\}$$

$I(x ; y) = H(y) - H(y/x)$

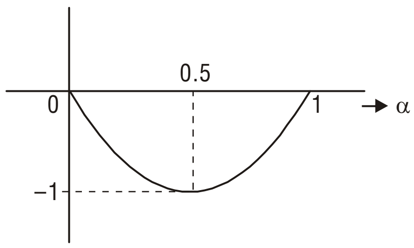
$= H(y) + (1-\alpha)\log_2(1-\alpha) + \alpha\log_2(\alpha)$

$C_s = \max\{I\ (x; y)\}$

$= \max\{H(y)\} + \alpha\log_2(\alpha) + (1-\alpha)\log_2(1-\alpha)$

$= \log_2^3 + \alpha\log_2(\alpha) + (1-\alpha)\log_2(1-\alpha)$

Plot of, $\alpha\log_2(\alpha) + (1-\alpha)\log_2(1-\alpha)$



Channel capacity will be maximum for $\alpha = 0$ or $\alpha = 1$. Otherwise it will be lesser.

$C_s\,(\max) = \log_2 3$

$C_s(\max)$ at $\alpha = 1$ in $\alpha \in [0.25, 1]$

62. Consider communication over a memoryless binary symmetric channel using a (7, 4) Hamming code. Each transmitted bit is received correctly with probability $(1-\epsilon)$, and flipped with probability ϵ. For each codeword transmission, the receiver performs minimum Hamming distance decoding, and correctly decodes the message bits if and only if the channel introduces at most one bit error.

For $\epsilon = 0.1$, the probability that a transmitted codeword is decoded correctly is ________ (rounded off to two decimal places).

Sol: (0.85)

Here (7, 4) Hamming code is given

P(0/1) = P(1/0) (due to bindary symmetry channel)

= 0.1

When n bits are transmitted then probability of getting error in γ bits = ${}^{n}C_r P^r (1-p)^{n-r}$

P:Bit error probability

$$P_c = C_0 (0.1)^0 [1-0.1]^{7-0} + {}^7C_1 (0.1)(1-0.1)^{7-1}$$

$= (0.9)^7 + 7 \times 0.1 \times (0.9)^6$

$= 0.85$

P_C: Prob of all most one bit error

63. Consider a channel over which either symbol x_A or symbol x_B is transmitted. Let the output of the channel Y be the input to a maximum likelihood (ML) detector at the receiver. The conditional probability density functions for Y given x_A and x_B are:

$$f_{Y|x_A}(y) = e^{-(y+1)} u(y+1),$$

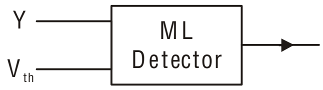
$$f_{Y|x_B} = e^{(y-1)} (1 - u(y-1)),$$

where $u(\cdot)$ is the standard unit step function.

The probability of symbol error for this system is __________ (rounded off to two decimal places).

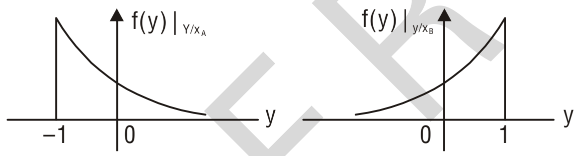
Sol: (0.367)

Here source is transmitting two symbols X_A & X_B



$$f_{y/x_A}(y) = e^{-(y+1)} u(y+1)$$

$$f_{y/x_B}(y) = e^{(y-1)} [1 - u(x-1)]$$



Calculation V_{th} optimum

$$f_{Y/X_A}(Y)\Big|_{Y=V_{th}} = f_{Y/X_B}(y)\Big|_{Y=V_{th}}$$

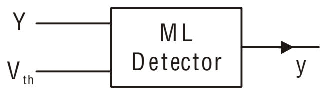
$$e^{-(V_{th}+1)} = e^{V_{th}-1} \Rightarrow \boxed{V_{th} = 0}$$

Problem of error = $P_e = P(X_A)P_{eXA} + P(X_B)\,P_{eXB}$

P_{eXA} = Problem of error when X_A is transmitted

P_{eXB} = Problem of error when X_B is transmitted

For decision making



$$Y = \begin{cases} X_A \ ; \ \text{If } Y < V_{th} \\ X_B \ ; \ \text{If } Y > V_{th} \end{cases}$$

$$P_e = P(X_A)P(Y > V_{th})$$

$$= P(X_B)P(Y < V_{th})$$

$$P_e = P(X_A)\int_0^{\infty} e^{-(Y+1)}dy + P(X_B)\int_{-\infty}^{0} e^{Y-1}\,dy$$

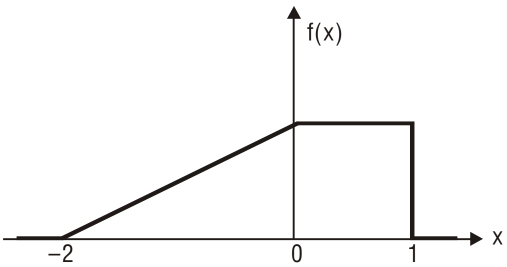
$$= P(X_A).e^{-1} + P(X_B)e^{-1}$$

$$= e^{-1}[P(X_A) + P(X_B)]$$

$$= e^{-1}$$

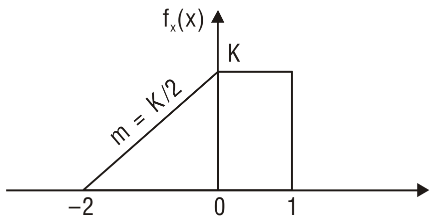
$$= 0.367$$

64. Consider a real valued source whose samples are independent and identically distributed random variables with the probability density function, f(x), as shown in the figure.



Consider a 1 bit quantizer that maps positive samples to value α and other to value β. If α^* and β^* are the respective choices for α and β that minimize the mean square quantization error, then $(\alpha^* - \beta^*)$ = _________ (rounded off to two decimal places).

Sol: (1.167)



$$\frac{1}{2}\times K\times 2+1\times K = 1 \Rightarrow K = 0.5$$

$$f_X(x) = mx + C$$

$$= 0.25 + C \quad \text{for} \quad (-2\le x\le 0)$$

When $x = -2 \Rightarrow f_X(x) = 0$

$$0 = 0.25\ x - 2 + C$$

$$\Rightarrow \quad C = 0.5$$

$$f_X(x) = \frac{1}{4}x+\frac{1}{2} = -2\le x\le 0$$

$$f_X(x) = 0.5\ ;\ 0\le x\le 1$$

$$x_q = \alpha;\ \text{for}\ 0\le x\le 1$$

$$x_q = \beta;\ \text{for}\ -2\le x\le 0$$

Again, $MSQ[Q_e] = E\left[\left(X-X_q\right)^2\right]$

Quantization noise power = N_o

$$= MSQ\left[Q_e\right] = \int (X-X_a)^2 f_X(x)dx$$

$$\text{for } -2\le x\le 0 \Rightarrow N_Q = \int_{-2}^{0}(x-\beta)^2\times\left(\frac{1}{4}x+\frac{1}{2}\right)dx$$

$$= \int_{-2}^{0}\left[x^2+\beta^2-2x\beta\right]\left[\frac{x}{4}+\frac{1}{2}\right]dx$$

$$\Rightarrow \quad N_Q = \frac{\beta^2}{2}+\frac{2}{3}\beta-\frac{1}{3}$$

N_Q to be minimum:

$$\frac{dN_Q}{d\beta} = 0$$

$$\Rightarrow \quad \frac{1}{2}\times 2\beta+\frac{2}{3} = 0$$

$$\beta = -\frac{2}{3}$$

for $0\le x\le 1$

$$\Rightarrow N_Q = \int_0^1 (x-\alpha)^2\times\frac{1}{2}dx = \frac{1}{6}\left[(1-\alpha)^3+\alpha^3\right]$$

Similarly for 'α'

$$\frac{dN_Q}{d\alpha} = 0$$

$$\Rightarrow \quad \frac{1}{6}\left[3(1-\alpha)^2(-1)+3\alpha^2\right] = 0$$

$$\alpha = 1/2$$

For N_Q to be minimum

$$\alpha-\beta = \frac{1}{2}-\left(-\frac{2}{3}\right)$$

$$= \frac{7}{6} = 1.167$$

65. In an electrostatic field, the electric displacement density vector, $\vec{D}$, is given by

$$\vec{D}(x,y,z) = \left(x^3\,\vec{i}+y^3\,\vec{j}+xy^2\,\vec{k}\right)C/m^2,$$

where $\vec{i},\vec{j},\vec{k}$ are the unit vectors along x-axis, y-axis, and z-axis, respectively. Consider a cubical region R centered at the origin with each side of length 1m, and vertices at $(\pm 0.5\,m, \pm 0.5\,m, \pm 0.5\,m)$. The electric charge enclosed within R is _________ C (rounded off to two decimal places).

Sol: (0.5)

$$\vec{D}(x,y,z) = \left(x^3\vec{i} + y^3\vec{j} + xy^2\vec{k}\right) c/m^2$$

$$Q_{enc.} = \int_v \rho_v \cdot dV = \int\left(\nabla\cdot\vec{D}\right)dV$$

$$\nabla\cdot\vec{D} = 3x^2 + 3y^2$$

$$dV = dxdydz$$

$$\therefore\ Q_{enc.} = \int_v 3\left(x^2+y^2\right)dx\,dy\,dz$$

$$= 3\left[\int_{-0.5}^{0.5} x^2dx\int_{-0.5}^{0.5}dy\int_{-0.5}^{0.5}dz + \int_{-0.5}^{0.5}dx\int_{-0.5}^{0.5}y^2dy\int_{-0.5}^{0.5}dz\right]$$

$$= 3\left[\left.\frac{x^3}{3}\right|_{-0.5}^{0.5}\times 1\times 1 + \left.\frac{y^3}{3}\right|_{-0.5}^{0.5}\times 1\times 1\right]$$

$$= 0.25 + 0.25$$

$$Q_{enc} = 0.5\ C$$