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GATE 2016

Detailed Solutions For Electronics & Communication Engg

Date: 30-01-2016

Forenoon Session

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Q.1 – Q.5 Carry One Mark Each

01. Which of the following is CORRECT with respect to grammar and usage?
Mount Everest is ______
(A) the highest peak in the world
(B) highest peak in the world
(C) one of highest peak in the world
(D) one of the highest peak in the world

01. Ans: (A)

Sol: Before superlative article 'the' has to be used. 'one of ' the expression should take plural noun and so option 'C' and 'D' can't be the answer.

02. The policeman asked the victim of a theft, "What did you ______?"
(A) loose (B) lose (C) loss (D) louse

02. Ans: (B)

Sol: 'lose' is verb.

03. Despite the new medicine's ___________ in treating diabetes, it is not ______ widely
(A) effectiveness --- prescribed (B) availability ---- used
(B) prescription ---- available (D) acceptable--- proscribed

03. Ans: (A)

Sol: 'effectiveness' is noun and 'prescribed' is verb. These words are apt and befitting with the word 'medicine.'

04. In a huge pile of apples and oranges, both ripe and unripe mixed together, 15% are unripe fruits, Of the unripe fruits, 45% are apples, Of the ripe ones, 66% are oranges. If the pile contains a total of 5692000 fruits, how many of them are apples?
(A) 2029198 (B) 2467482 (C) 2789080 (D) 3577422

04. Ans: (A)

Sol: Total no. of fruits = 5692000
Unripe type of apples = 45% of 15% of 5692000

$$= \frac{45}{100}\times\frac{15}{100}\times 5692000$$

$$= 384210$$

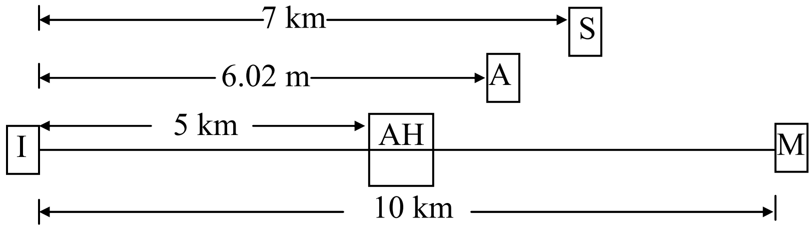
Ripe type of apples $= \frac{34}{100}\times\frac{85}{100}\times 5692000 = 1644988$

$\therefore$ Total no. of apples = 384210 + 1644988 = 2029198

05. Michael lives 10 km way from where I live. Ahmed lives 5km away and Susan lives 7 km away from where I live. Arun is farther away than Ahmed but closer than Susan from where I live. From the information provided here, what is one possible distance (in km) at which I live from Arun's place?
(A) 3.00 (B) 4.99 (C) 6.02 (D) 7.01

05. Ans: (c)

Sol: From given data, the following diagram is possible



I = I live

AH = Ahmed lives; M = Michael lives

S = Susan lives; A= Arun lives

→ Arun lives farthes away than Ahmed means more than 5 km but closer than Susan means less than 7 km, from given alternatives, option 'C' only possible.

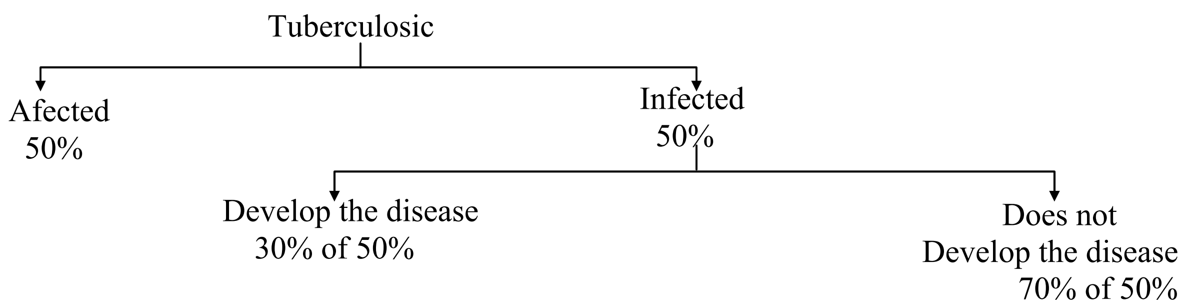
Q.6 – Q.10 Carry two marks each

06. A person moving through a tuberculosis prone zone has a 50% probability of becoming infected. However, only 30% of infected people develop the disease. What percentage of people moving through a tuberculosis prone zone remains infected but does not shows symptoms of disease?

(A) 15 (B) 33 (C) 35 (D) 37

06. Ans: (C)

Sol:



$= \frac{70}{100} \times \frac{50}{100} = \frac{35}{100} = 35\%$

07. In a world filled with uncertainty, he was glad to have many good friends. He had always assisted them in times of need and was confident that they would reciprocate. However, the events of the last week proved him wrong.

Which of the following interference(s) is/are logically valid and can be inferred from the above passage?

(i) His friends were always asking him to help them

(ii) He felt that when in need of help, his friends would let him down.

(iii) He was sure that his friends would help him when in need.

(iv) His friends did not help him last week.

(A) (i) and (ii) (B) (iii) and (iv) (C) (iii) only (D) (iv) only

07. Ans: (B)

Sol: The words 'was confident that they would reciprocate' and 'last week proved him wrong' lead to statements iii and iv as logically valid inferences.

08. Leela is older than her cousin Pavithra, pavithra's brother Shiva is older than Leela. When Pavithra and shiva are visiting Leela, all three like to play chess. Pavithra wins more often than Leela does. Which one of the following statements must be TRUE based on the above?
(A) When Shiva plays chess with Leela and Pavithra. he often loses.
(B) Leela is the oldest of the three.
(C) Shiva is a better chess palyer than Pavithra
(D) Pavithra is the young of the three

08. Ans: (D)

Sol: From given data, the following arrangement is possible
Shiva
Leela
Pavithra
Among four alternatives, option D is TRUE.

09. If $q^{-a} = \frac{1}{r}$ and $r^{-b} = \frac{1}{s}$ and $S^{-C} = \frac{1}{q}$, the value of abc is ____.
(A) $(rqs)^{-1}$ (B) 0 (C) 1 (D) r+q+s

09. Ans: (C)

Sol: $q^{-a} = \frac{1}{r} \Rightarrow \frac{1}{q^a} = \frac{1}{r} \Rightarrow q^a = r$

$r^{-b} = \frac{1}{s} \Rightarrow \frac{1}{r^b} = \frac{1}{s} \Rightarrow s = r^b$

$s^{-c} = \frac{1}{q} \Rightarrow \frac{1}{s^c} = \frac{1}{q} \Rightarrow s^c = q$

$q^a = r \Rightarrow (s^c)^a = r \Rightarrow s^{ac} = r$

$(s^{ac})^b = s$

$s^{abc} = s'$

$\therefore$ abc = 1

10. P,Q,R and S are working on a project. Q can finish the taks in 25 days, working alone for 12 hours a day. R can finish the task in 50 days, working alone for 12 hours per day. Q worked 12 hours a day but took sick leave in the beginning for two days. R worked 18 hours a day on all days. What is the ratio of work done by Q and R after 7 days from the start of the project?
(A) 10:11 (B) 11:10 (C) 20:21 (D) 21:20

10. Ans: (C)

Sol: Q can finish the task = 25 days, 12 hrs/day

= 300 hrs, 1 hr $= \frac{1}{300}$ th

R can finish the task = 50 days, 12 hrs/day

$= 50 \times 12$

$= 600 \text{ hrs}, \; 1\text{hr} = \dfrac{1}{600}\text{th}$

Q working hours $\Rightarrow (7-2) \times 12 = 60$ hrs

R working hours $\Rightarrow 7 \times 18 = 126$ hrs

After 7 days, the ratio of work done by Q and R

$$Q : R$$

$$\frac{60}{300} : \frac{126}{600}$$

$$20 : 21$$

Q.1 – Q.25 Carry one mark each.

01. Let $M^4 = I$ (where I denotes the identity matrix)and $M \neq I$, $M^2 \neq I$ and $M^3 \neq I$. Then, for any natural number k, M^{-1} equals:

(A) M^{4k+1} (B) M^{4k+2} (C) M^{4k+3} (D) M^{4k}

01. Ans: (C)

Sol: $M^4 = I$

$\Rightarrow M^8 = M^4 = I \Rightarrow M^7 = M^{-1}$

$\Rightarrow M^{12} = M^8 = I \Rightarrow M^{11} = M^{-1}$

$\Rightarrow M^{16} = M^{12} = I \Rightarrow M^{15} = M^{-1}$

……………………..

$\therefore M^{-1} = M^{4K+3}$, K is a natural number

02. The second moment of a Poisson-distributed random variables is 2. The mean of the random variable is _____.

02. Ans: $\lambda = 1$

Sol: $E(x^2) = 2$

mean & Var of poisson RV are same

$V(X) = E(X^2) - (E(X))^2$

Let mean of the poission random variable be x

$x = 2 - x^2$

$x^2 + x - 2 = 0$

$x = 1, -2$

$\therefore$ Mean is $\lambda = 1$

03. Given the following statements about a function f: $R \to R$, select the right option:

P : If f(x) is continuous at $x = x_0$, then it is also differentiable at $x = x_0$

Q : If f(x) is continuous at $x = x_0$, then it may not be differentiable at $x = x_0$

R : If f(x) is continuous at $x = x_0$, then it is also different at $x = x_0$

(A) P is true, Q is false, R is false (B) P is false, Q is true, R is true

(C) P is false, Q is true, R is false (D) P is true, Q is false, R is true

03. Ans: (B)

Sol: Since continuous function may not be differentiable. But differentiable function is always continuous.

04. Which one of the following is a property of the solutions to the Laplace equation : $\nabla^2 f = 0$?

(A) The solution have neither maxima nor minima anywhere except at the boundaries

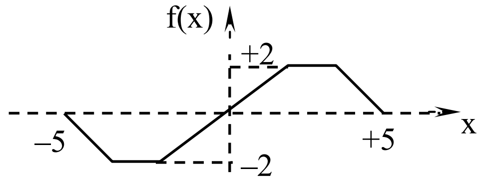
(B) The solution are not separable in the coordinates

(C) The solution are not continuous

(D) The solution are not dependent on the boundary conditions

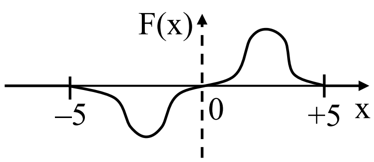
04. Ans: (A)

05. Consider the plot of f(x) versus x as shown below

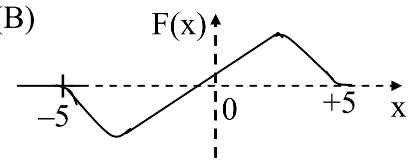


Suppose $F(x) = \int_{-5}^{x} f(y)dy$. which one of the following is a graph of F(x)?

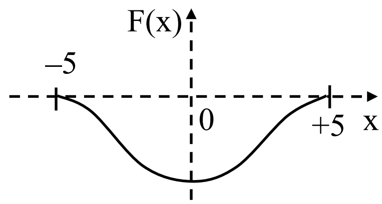
(A)



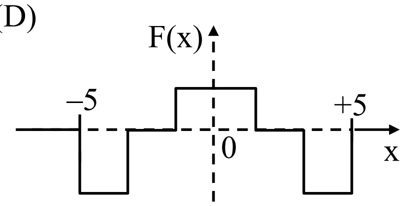
(B)



(C)

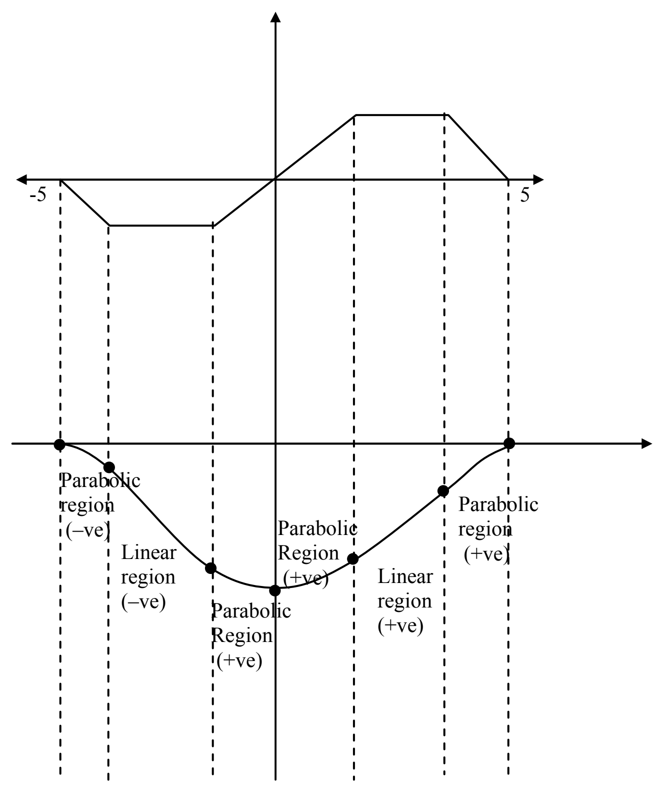


(D)



05. Ans: (C)

Sol: integration of ramp is parabolic, integration of step is ramp.



06. Which one of the following is an eigen function of the class of all continuous-time, linear, time-invariant systems (u(t) denotes the unit-step function) ?

(A) $e^{j\omega_0 t}u(t)$ (B) $\cos(\omega_0 t)$ (C) $e^{j\omega_0 t}$ (D) $\sin(\omega_0 t)$

06. Ans: (C)

Sol: If the input to a system is its eigen signal, the response has the same form as the eigen signal

07. A continuous –time function x(t) is periodic with period T. The function is sampled uniformly with a sampling period T_s. In which one of the following cases is the sampled signal periodic ?

(A) $T = \sqrt{2}\ T_s$ (B) $T = 1.2T_S$ (C) Always (D) Never

07. Ans: (B)

Sol: A discrete time signal x(n) = cos (ω_0n) is said to be periodic if $\frac{\omega_0}{2\pi}$ is a rational number.

08. Consider the sequence $x[n] = a^n u[n] + b^n u[n]$, where u[n] denotes the until-step sequence and $0 < |a| < |b| < 1$. The region of convergence (ROC) of the z-transform of x[n] is

(A) $|z| > |a|$ (B) $|z| > |b|$ (C) $|z| < |a|$ (D) $|a| < |z| < |b|$

08. Ans: (B)

Sol: $x(n) = (a)^n x(n) + (b)^n x(n)$, given $0 < |a| < |b| < 1$

Roc = $(|z|>|a|) \cap (|z| > |b|) = |z| > |b|$

09. Consider a two-port network with the transmission matrix : $T = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$. If the network is reciprocal, then

(A) $T^{-1} = T$ (B) $T^2 = T$ (C) Determinant (T) = 0 (D) Determinant(T) = 1

09. Ans: (D)

Sol: A two port network is reciprocal in transmission parameters if AD – BC = 1

i.e Determinant(T) = 1

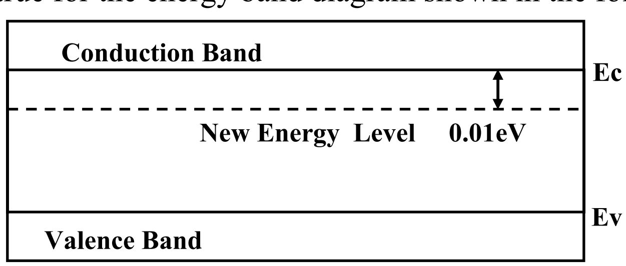
10. A continuous-time sinusoid of frequency 33 Hz is multiplie with a periodic Dirac impulse train of frequency 46 Hz. The resulting signal is passed through an ideal analog low-pass filter with a cutoff frequency of 23Hz. The functional frequency (in Hz) of the output is _____.

10. Ans: 13

Sol: $f_m = 33$Hz, $f_s = 46$Hz

The frequency in sampled signal are = ±33, 13, 79, 59, 125 …… The above frequencies are passed to a LPF of cutoff frequency 23Hz. The output frequency are = 13Hz.

11. A small percentage of impurity is added to intrinsic semiconductor at 300 K. Which one of the following statements is true for the energy band diagram shown in the following figure?



(A) Intrinsic semiconductor doped with pentavalent atoms to form n-types semiconductor
(B) Intrinsic semiconductor doped with trivalent atoms to form n-types semiconductor
(C) Intrinsic semiconductor doped with pentavalent atoms to form p-types semiconductor
(D) Intrinsic semiconductor doped with trivalent atoms to form p-type semiconductor

11. Ans: (A)

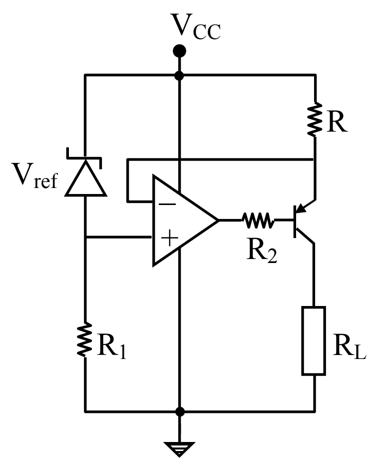
Sol: Donor energy level close to conduction band.

12. Consider the following statements for a metal oxide semiconductor field after effect transistor (MOSFET):
P : As channel length reduces, OFF-state current increases
Q : As channel length reduces, output resistance increases
R : As channel length reduces, threshold voltage remains constant
S: As channel reduces, ON current increases.
Which of the above statements are INCORRECT?
(A) P and Q (B) P and S (C) Q and R (D) R and S

12. Ans: (C)

Sol: P: TRUE
Q: FALSE, As channel length reduces, output resistance reduces
R: FALSE: As channel length reduces, threshold voltage reduces
S: TRUE

13. Consider the constant current source shown in the figure below. Let β represent the current gain of the transistor



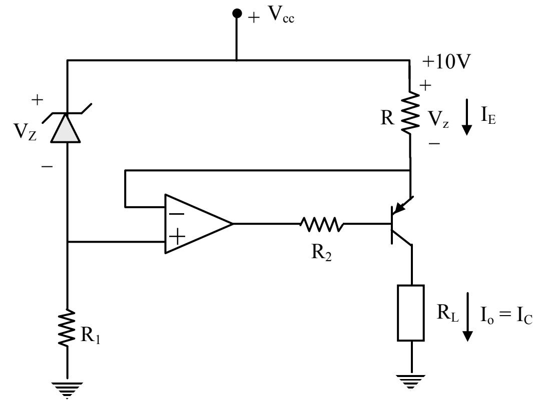
The load current I_0 through R_L is

(A) $I_0 = \left(\frac{\beta+1}{\beta}\right)\frac{V_{ref}}{R}$ (B) $I_0 = \left(\frac{\beta}{\beta+1}\right)\frac{V_{ref}}{R}$

(C) $I_0 = \left(\frac{\beta+1}{\beta}\right)\frac{V_{ref}}{2R}$ (D) $I_0 = \left(\frac{\beta}{\beta+1}\right)\frac{V_{ref}}{2R}$

13. Ans: (B)

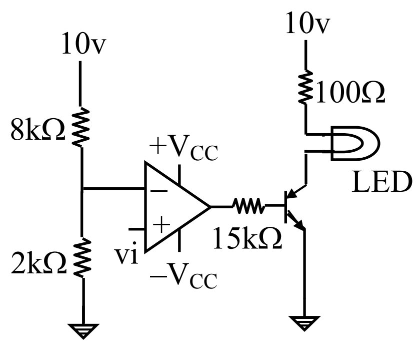
Sol:

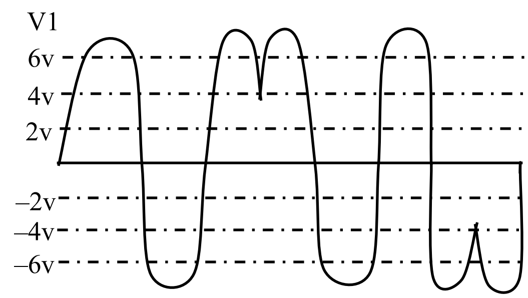


$V_P = V_N$ [Virtual short]

$$I_0 = I_C = \left(\frac{\beta}{\beta+1}\right) I_E = \left(\frac{\beta}{\beta+1}\right)\frac{V_Z}{R}$$

14. The following signal V_i of peak voltage 8V is applied to the non-inverting terminal of an ideal Opamp. The transistor has $V_{BE} = 0.7$ V, $\beta = 100$; $V_{LED} = 1.5$V, $V_{CC} = 10$V and $-V_{CC} = -10$V.

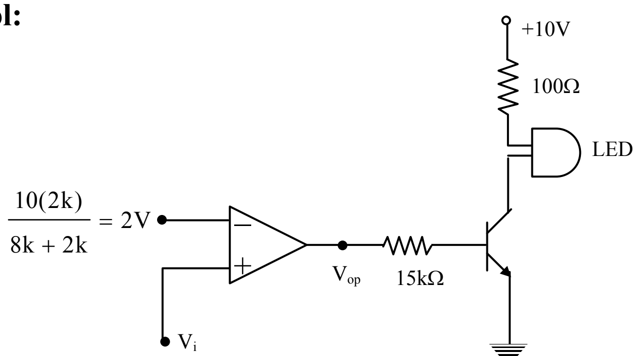


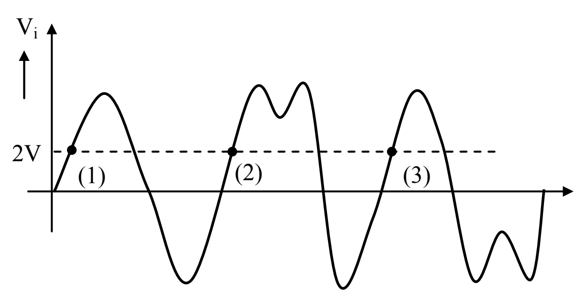


The number of times the LED glows is ———

14. Ans: 3

Sol:

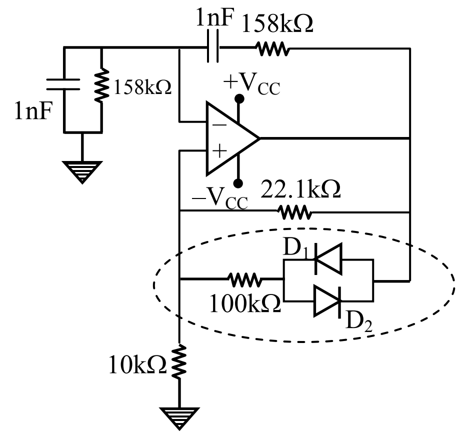




V_i crosses 2V, 3times

Therefore the LED glows 3 times

15. Consider the oscillator circuit shown in the figure. The function of the network (shown in dotted lines) consisting of the 100 kΩ resistor in series with the two diodes connected back-to-back is to:

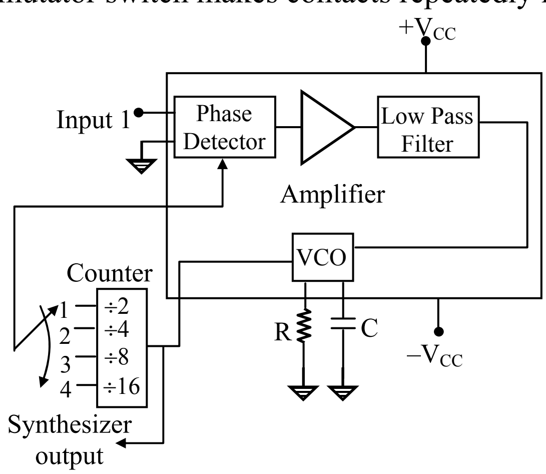


(A) introduce amplitude stabilization by preventing the op-amp from saturating and thus producing sinusoidal oscillations of fixed amplitude
(B) introduce amplitude stabilization by forcing the op-amp to swing between positive and negative saturation and thus producing square wave oscillations of fixed amplitude
(C) introduce frequency stabilization by forcing the circuit to oscillate at a single frequency
(D) enable the loop gain to take on a value that produces square wave oscillations

15. Ans: (A)

Sol: The circuit shown is a wein bridge oscillator. The amplitude of oscillations can be determined and stabilized by using a nonlinear control network. As the oscillations grow, the diodes start to conduct causing the effective resistance in the feedback to decrease. Equilibrium will be reached at the output amplitude that causes the loop gain to be exactly unity.

16. The block diagram of a frequency synthesizer consisting of Phase Locked Loop (PLL) and a divide-by-N counter (comprising ÷2, ÷4, ÷8, ÷16 outputs) is sketched below. The synthesizer is excited with a 5 kHz signal (Input 1). The free-running frequency of the PLL is set to 20kHz. Assume that the commutator switch makes contacts repeatedly in the order 1-2-3-4.

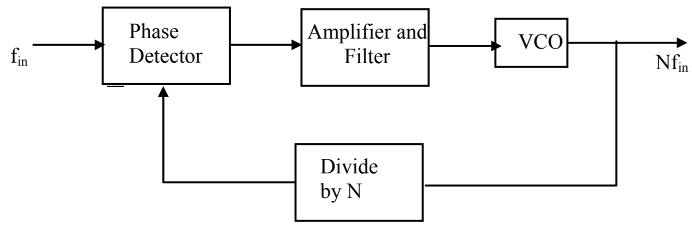


The corresponding frequencies synthesized are:

(A) 10kHz, 20kHz, 40kHz, 80 kHz
(B) 20kHz, 40kHz, 80kHz, 160 kHz
(C) 80kHz, 40kHz, 20kHz, 10kHz
(D) 160kHz, 80kHz, 40kHz, 20kHz

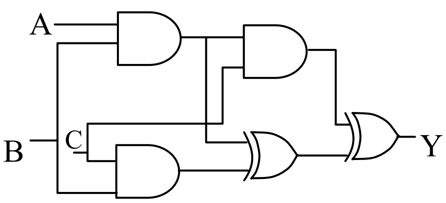
16. Ans: (A)

Sol:



f_{in}	Divide by N	VCO output(Nf_{in})
5kHz	2	10kHz
5kHz	4	20kHz
5kHz	8	40kHz
5kHz	16	80kHz

17. The output of the combinational circuit given below is



(A) A+B+C (B) A(B+C) (C) B(C+A) (D) C(A+B)

17. Ans: (C)

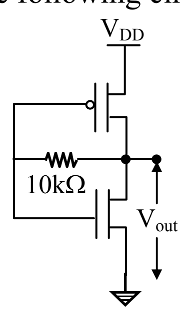
Sol: $Y = ABC \oplus AB \oplus BC = AB\,(C \oplus 1) \oplus BC$

$= AB\overline{C} \oplus BC = B\,(A\overline{C} \oplus C) = B[\,\overline{A\overline{C}}\,C + A\overline{C}\,.\,\overline{C}\,]$

$= B[(\overline{A} + C)\,C + A\overline{C}\,] = B\,[\,\overline{A}\,C + C + A\overline{C}\,]$

$= B\,[C + A\overline{C}\,] = B\,[C + A] \rightarrow Y = B\,(A + C)$

18. What is the voltage V_{out} in the following circuit?



(A) 0V
(B) ($|V_T$ of PMOS$|$ + V_T of NMOS)/2
(C) Switching threshold of inverter
(D) V_{DD}

18. Ans: (D)

19. Match the inferences X, Y, and Z, about a system, to the corresponding properties of the elements of first column in Routh's Table of the system characteristic equation.

X: The system is stable ….. P: … When all elements are positive

Y: The system is unstable … Q: … When any on element is zero

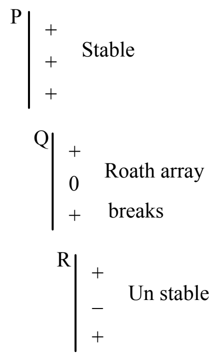
Z: The test breaks down …. R: … When there is a change in sign of coefficients

(A) X →P, Y→Q, Z→R (B) X→Q,Y→P,Z→R

(C) X→R, Y→Q,Z→P (D) X→P, Y→R, Z→Q

19. Ans: (D)

Sol:



X – P, Y – R, Z –Q

20. A closed-loop control system is stable if the Nyquist plot of the corresponding open-loop transfer function

(A) encircles the s-plane point (–1 + j0) in the counterclockwise direction as many times as the number of right-half s-plane poles.

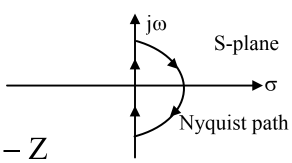
(B) encircles the s-plane point (0 – j1) in the clockwise direction as many times as the number of right-half s-plane poles.

(C) encircles the s-plane point (–1 +j0) in the counterclockwise direction as many times as the number of left-half s-plane poles.

(D) encircles the s-plane point (–1+j0) in the counterclockwise direction as many times as the number of right-half s-plane zeros.

20. Ans: (A)

Sol:



N = P – Z

For closed loop stability Z = 0, N = P

∴ (–1, j0) should be encircled in Counter clock wise direction equaling P poles in RHP.

21. Consider binary data transmission at a rate of 56 kbps using baseband binary pulse amplitude modulation (PAM) that is designed to have a raised-cosine spectrum. The transmission bandwidth (in kHz) required for a roll-off factor of 0.25 is ———.

21. Ans: 35

Sol: R_b = 56 kbps, α = 0.25

$$BW = \frac{R_b}{2}[1+\alpha] = \frac{56}{2}[1+0.25]\text{kHz} = 35 \text{ kHz}$$

22. A superheterodyne receiver operates in the frequency range of 58 MHz-68MHz. The intermediate frequency f_{IF} and local oscillator frequency f_{LO} are chosen such that $f_{IF} \leq f_{LO}$. It is required that the image frequencies fall outside the 59 MHz – 68 MHz band. The minimum required f_{IF} (in MHz) is ________.

22. Ans: 5

Sol: $f_s = 58$ MHz– 68 MHz

When $f_s = 58$ MHz

$f_{si} = f_s + 2IF > 68$ MHz

$2IF > 10$ MHz

$IF \geq 5$ MHz

23. The amplitude of a sinusoidal carrier is modulated by a single sinusoid to obtain the amplitude modulated signal s(t) = 5 cos 1600πt + 20 cos 1800 πt + 5 cos 2000πt. The value of the modulation index is ________.

23. Ans: μ = 0.5

Sol: $S(f) = 5\cos 1600\pi t + 20\cos 1800\pi t + 5\cos 2200\pi t$

$$S(f) = \frac{A_c\mu}{2}\cos 2\pi(f_c - f_m)t + A_c\cos 2\pi f_c t + \frac{A_c\mu}{2}\cos 2\pi(f_c + f_m)\,t$$

$A_c = 20$ $\qquad$ $A_c\mu = 10$

$\frac{A_c\mu}{2} = 5$ $\qquad$ $\mu = \frac{10}{20} = 0.5$

24. Concentric spherical shells of radii 2m, 4m, and 8m carry uniform surface charge densities of 20 nC/m^2, –4 nC/m^2 and ρ_s, respectively. The value of ρ_s (nC/m^2) required to ensure that the electric flux density $\vec{D} = \vec{0}$ at radius 10 m is ———.

24. Ans: – 0.25

Sol: Given:

$\rho s_1 = 20$ nc/m^2

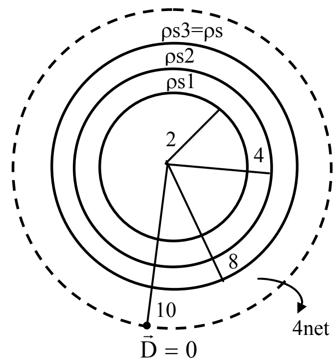
$\rho s_1 = -4$nc/m^2

ρs_3 = ? (unknown)

Electric flux density at r = 10 m

in given by

$$\vec{D} = \left(\frac{\psi \text{ net leaving the sphere of radian } r = 10\text{m}}{\text{Area of sphere of radius } r = 10\text{m}}\right)\hat{a}_r$$



but $\vec{D} = 0$

$\psi_{net}|_{at\ r = 10} = 0 \quad 20 \times 10^{-9} \times 4\pi(2)^2 + (-4\times10^{-9}) \times 4\pi(4)^2 + \rho_{s3}4\pi(8)^2 = 0$

$\therefore\ \rho_{s3} = \rho_s = -0.25$ nc/m^2

25. The propagation constant of a lossy transmission line is (2 +j5) m^{-1} and its characteristic impedance is (50 + j0) Ω at ω = 10^6 rad S^{-1}. The values of the line constants L,C,R,G are, respectively,

(A) L = 200 μH/m, C = 0.1 μF/m, R = 50 Ω/m, G = 0.02 S/m

(B) L = 250 μH/m, C = 0.1 μF/m, R = 100 Ω/m, G = 0.04 S/m

(C) L = 200 μH/m, C = 0.2 μF/m, R = 100 Ω/m, G = 0.02 S/m

(D) L = 250 μH/m, C = 0.2 μF/m, R = 50 Ω/m, G = 0.04 S/m

25. Ans: (B)

Sol: Given

Propagation contact, P = (2 + j5) m^{-1},

characteristic impedance z_0 = 50 Ω,

angular frequency ω = 10^6 rad/sec,

$$P = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$z_0 = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}}$$

$Pz_0 = R + j\omega L$

$\Rightarrow R + j\omega L = (100 + j250)$

$\therefore\ R = 100\ \Omega/m$

$$L = \frac{250}{10^6} = 250\,\mu H/m$$

$$\frac{P}{z_0} = G + j\omega C$$

$$G + j\omega C = \left(\frac{2}{50} + j\frac{5}{50}\right)$$

$\therefore G = 0.04$ s/m

$$C = \frac{5}{50\times10^{6}} = 0.1\mu F/m$$

Therefore line constants L, C, R & G are respectively L = 250 μH/m, C = 0.1 μF/m, R = 100 Ω/m, G = 0.04 s/m

Q.26 – Q.55 carry two marks each.

26. The integral $\frac{1}{2\pi}\iint_D (x+y+10)dxdy$, where D denotes the disc: $x^2+y^2 \le 4$, evaluates to ———.

26. Ans: 20

Sol: Converting to polar coordinates, we get

$$\frac{1}{2\pi}\iint_D (x+y+10)dxdy = \frac{1}{2\pi}\int_{r=0}^{2}\int_{\theta=0}^{2\pi} (r\cos\theta + r\sin\theta + 10)\ \rho dr d\theta$$

$$= \frac{1}{2\pi}\int_{r=0}^{2}\int_{\theta=0}^{2\pi} (r^2\cos\theta + r^2\sin\theta + 10r)\ dr\ d\theta$$

$$= \frac{1}{2\pi}\int_{\theta=0}^{2\pi}\left\{\frac{r^3}{3}\cos\theta + \frac{r^3}{3}\sin\theta + 5r^2\right\}_0^2 d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\left\{\frac{8}{3}\cos\theta + \frac{8}{3}\sin\theta + 20\right\}d\theta$$

$$= \frac{1}{2\pi}\left\{\frac{8}{3}\sin\theta - \frac{8}{3}\cos\theta + 20\theta\right\}_0^{2\pi}$$

$$= \frac{1}{2\pi}\left\{\left(-\frac{8}{3}+40\pi\right)-\left(-\frac{8}{3}\right)\right\}$$

$$= 20$$

27. A sequence x[n] is specified as

$$\begin{bmatrix} x[n] \\ x[n-1] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \text{for} \quad n \ge 2.$$

The initial conditions are x[0] = 1, x[1] = 1 and x[n] = 0 for n < 0. The value of x[12] is ——

27. Ans: 233

Sol: $$\begin{bmatrix} x(n) \\ x(n-1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}, n \ge 2$$

n = 2

$$\begin{bmatrix} x(2) \\ x(1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$x(2) = 2,\ x(1) = 1$

$n = 3$

$$\begin{bmatrix} x(3) \\ x(2) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$x(3) = 3,\ x(2) = 2$

From the above values we can write the recursive relation as

$x(n) = x(n-1) + x(n-2)$

$x(2) = x(1) + x(0) = 1 + 1 = 2$

$x(3) = x(2) + x(1) = 2 + 1 = 3$

$x(4) = x(3) + x(2) = 3 + 2 = 5$

$x(5) = x(4) + x(3) = 5 + 3 = 8$

$x(6) = x(5) + x(4) = 8 + 5 = 13$

$x(7) = x(6) + x(5) = 13 + 8 = 21$

$x(8) = x(7) + x(6) = 21 + 13 = 34$

$x(9) = x(8) + x(7) = 34 + 21 = 55$

$x(10) = x(9) + x(8) = 55 + 34 = 89$

$x(11) = 89 + 55 = 144$

$x(12) = 144 + 89 = 233$

28. In the following integral, the contour C encloses the points $2\pi j$ and $-2\pi j$. The value of the integral $-\frac{1}{2\pi}\oint_C \frac{\sin z}{(z-2\pi j)^3}dz$ is ———.

28. Ans: – 133.87

Sol: $$-\frac{1}{2\pi}\oint_c \frac{\sin z}{(z-2\pi j)^3}dz = -\frac{1}{2\pi}\times 2\pi i\frac{f''(2\pi j)}{2!}$$

$f(z) = \sin z$

$f''(z) = -\sin z$

$\therefore\ f''(z_0) = -\sin 2\pi j$

$$\frac{1}{2\pi}\oint_c \frac{\sin z}{(z-2\pi j)^3}dz = -\frac{1}{2\pi}\times 2\pi j\left(\frac{-\sin(2\pi j)}{2}\right)$$

$$= j\times j\frac{\sinh 2\pi}{2}$$

$$= -\frac{1}{2}(\sinh 2\pi)$$

$$= -133.87$$

29. The region specified by $\{(\rho,\varphi,Z): 3 \le \rho \le 5, \frac{\pi}{8} \le \phi \le \frac{\pi}{4},\ 3 \le z \le 4.5\}$ in cylindrical coordinates has volume of ———.

29. Ans: 4.714

Sol: Given region of cylinder

$3 \le \rho \le 5,$

$\frac{\pi}{8} \le \phi \le \frac{\pi}{4},$

$3 \le z \le 4.5$

The differential volume of cylinder in given by

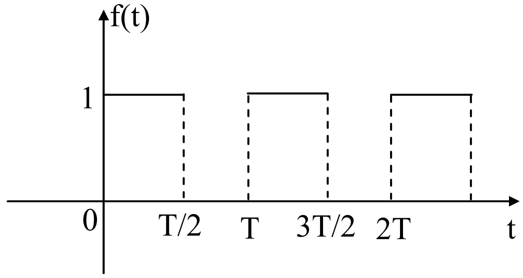
$dv = \rho d\rho d\phi dz$

$$\text{Volume, } v = \int_{\rho=3}^{5} \int_{\phi=\frac{\pi}{8}}^{\frac{\pi}{4}} \int_{z=3}^{4.5} \rho\, d\rho\, d\phi\, dz$$

$$= \frac{\rho^2}{2}\Bigg|_3^5 \times \phi\Bigg|_{\frac{\pi}{8}}^{\frac{\pi}{8}} \times z\Big|_3^{4.5} \quad = \frac{1}{2}(25-9)\times\left(\frac{\pi}{4}-\frac{\pi}{8}\right)\times(4.5-3)$$

$\therefore v = 4.71\ m^3$

30. The Laplace transform of the causal periodic square wave of period T shown in the figure below is



(A) $F(s)=\dfrac{1}{1+e^{-sT/2}}$ (B) $F(s)=\dfrac{1}{s\left(1+e^{-\frac{sT}{2}}\right)}$

(C) $F(s)=\dfrac{1}{s(1-e^{-sT})}$ (D) $F(s)=\dfrac{1}{1-e^{-sT}}$

30. Ans: (B)

Sol: One period of signal $x_1(t) = u(t) - u(t-T/2)$

$$X_1(s)=\frac{1}{s}-\frac{e^{-sT/2}}{s}=\frac{1-e^{-sT/2}}{s}$$

$$X(s)=\frac{1}{1-e^{-sT}}X_1(s)=\frac{1-e^{-sT/2}}{s(1-e^{-sT})}=\frac{1}{s(1+e^{-sT/2})}$$

31. A network consisting of a finite number of linear resistor (R), inductor (L), and capacitor (C) elements, connected all in series or all in parallel, is excited with a source of the form $\sum_{k=1}^{3} a_k \cos(k\omega_0 t)$, where $a_k \neq 0$, $\omega_0 \neq 0$. The source has nonzero impedance. Which one of the following is a possible form of the output measured across a resistor in the network?

(A) $\sum_{k=1}^{3} b_k \cos(k\omega_0 t+\phi_k)$, where $b_k \neq a_k, \forall k$ (B) $\sum_{k=1}^{4} b_k \cos(k\omega_0 t+\phi_k)$, where $b_k \neq 0, \forall k$

(C) $\sum_{k=1}^{3} a_k \cos(k\omega_0 t+\phi_k)$ (D) $\sum_{k=1}^{3} a_k \cos(k\omega_0 t+\phi_k)$

31. Ans: (A)

Sol: Consider a series RLC-Circuit with voltage source

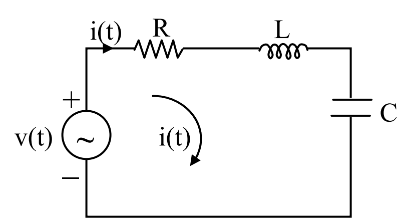
Here

$V(t) = a_1\cos\omega_0 t+a_2\cos 2\omega_0 t+a_3\cos 3\omega_0 t$

$i(t) = b_1\cos(\omega_0 t+\phi_2)+b_3\cos(2\omega_0 t+\phi_2) +b_3\cos(3\omega_0 t+\phi_3)$

$$i(t)=\sum_{k=1}^{3} b_k \cos(k\omega_0 t+\phi_k)$$

Where $b_k \neq a_k$ for all k



32. A first-order low-pass filter of time constant T is excited with different input signals (with zero initial conditions up to t = 0). Match the excitation signals X,Y, Z with the corresponding time responses for $t \leq 0$:

X: Impulse	P: $1 - e^{-t/T}$
Y: Unit step	Q: $t - T(1 - e^{-t/T})$
Z: Ramp	R: $e^{-t/T}$

(A) $X \to R, Y \to Q, Z \to P$
(B) $X \to Q, Y \to P, Z \to R$
(C) $X \to R, Y \to P, Z \to Q$
(D) $X \to P, Y \to R, Z \to Q$

32. Ans: (C)

Sol: $H(s) = \dfrac{1}{1+s\tau}$

$V_0(s) = H(s).\ V_I(s)$

(I) if $v_i(t) = \delta(t)$

$V_I(s) = 1$

$V_0(s) = H(s).V_I(s)$

$= \dfrac{1}{1+s\tau}$

$\upsilon_0(t) = \dfrac{1}{\tau} e^{-\frac{t}{\tau}}$

(II) if $v_i(t) = u(t)$

$V_I(s) = 1/s$

$V_0(s) = \dfrac{1}{s(1+s\tau)} = \dfrac{1}{s} - \dfrac{1}{s+\frac{1}{\tau}}$

$\upsilon_0(t) = (1 - e^{-t/\tau})$

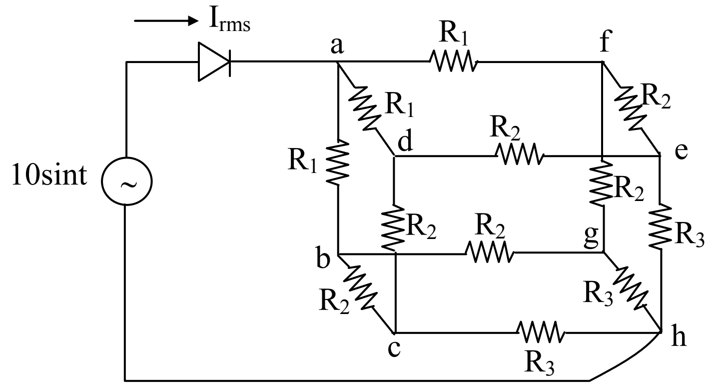
(III) $v_i(t) = r(t) \Rightarrow V_I(s) = \dfrac{1}{s^2}$

$V_0(s) = H(s).\ V_I(s) = \dfrac{1}{s^2(1+s\tau)}$

$= \dfrac{1}{s^2} - \dfrac{\tau}{s} + \dfrac{\tau}{s+\frac{1}{\tau}}$

$V_0(t) = t - \tau(1 - e^{-t/\tau})$

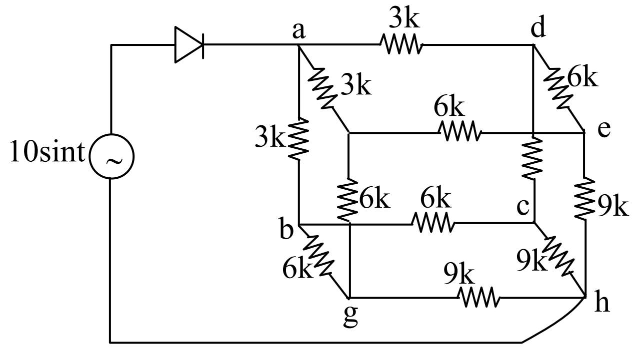
33. An AC voltage source V = 10 sin (t) volts is applied to the following network. Assume that $R_1 = 3\ k\Omega$, $R_2 = 6k\Omega$ and $R_3 = 9k\Omega$, and that the diode is ideal.



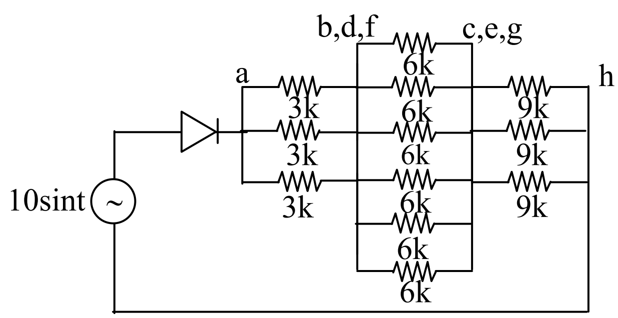
Rms current I_{rms} (in mA) through the diode is _____

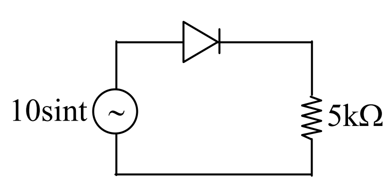
33. Ans: 1

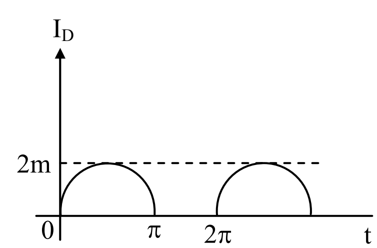
Sol:



We can join nodes that are at same potential so network becomes

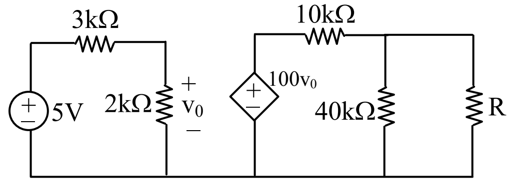






$$I_{D(RMS)} = \frac{2\,m}{2} = 1mA$$

34. In the circuit shown in the figure, the maximum power (in watt) delivered to the resistor R is ——.



34. Ans: 0.8

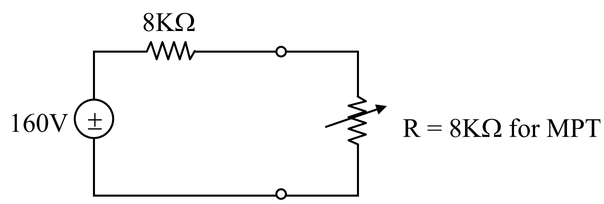
Sol: $V_0 = \left(\dfrac{5}{5k}\right).2k = 2v$

$$v_{0c} = v_{th} = v_{40k} = i_{40k}.40k = \left(\frac{200}{50k}\right).40k = 160V$$

$$\Rightarrow I_{sc} = \frac{200}{10k} = 20mA$$

So, $R_{th} = \dfrac{v_{th}}{I_{sc}} = \dfrac{160v}{20mA} = 8k\Omega$

$$P_{max} = \frac{v_s^2}{4R_L} w = \frac{(160)^2}{4\times 8k} = 0.8W$$



35. Consider the signal x[n] =6δ[n +2] +3δ[n +1]+8δ[n]+7δ[n – 1] +4δ[n – 2].
If $X(e^{j\omega})$ is the discrete-time Fourier transform of x[n],
then $\dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} X(e^{j\omega})\sin^2(2\omega)\, d\omega$ is equal to ——.

35. Ans: 8

Sol: Plancheral's relation is

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right).Y\left(e^{j\omega}\right)d\omega = \sum_{n=-\infty}^{\infty} x(n)y(n)$$

$$Y\left(e^{j\omega}\right) = \sin^2(2\omega) = \frac{1-\cos(4\omega)}{2}$$

$$= \frac{1}{2} - \frac{1}{4}e^{j4\omega} - \frac{1}{4}e^{-j4\omega}$$

$$y(n) = \frac{1}{2}\delta(n) - \frac{1}{4}\delta(n+4) - \frac{1}{4}\delta(n-4)$$

$$y(n) = \left\{-\frac{1}{4}, 0, 0, 0, \underset{\uparrow}{\frac{1}{2}}, 0, 0, 0, -\frac{1}{4}\right\}$$

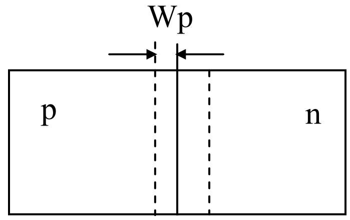
$$x(n) = \left\{6, 3, \underset{\uparrow}{8}, 7, 4\right\}$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right).Y\left(e^{j\omega}\right)d\omega = 2\sum_{n=-\infty}^{\infty} x(n)y(n)$$

$$2\sum_{n=-\infty}^{\infty} x(n)y(n) = 2\times 8\times\frac{1}{2} = 8$$

36. Consider a silicon p-n junction with a uniform acceptor doping concentration of 10^{17} cm^{-3} on the p-side and a uniform donor doping concentration of 10^{16} cm^{-3} on the n-side. No external voltage is applied to the diode. Given: kT/q = 26 mV, $n_i = 1.5 \times 10^{10} cm^{-3}$, $\varepsilon_{si} = 12\ \varepsilon_0$, $\varepsilon_0 = 8.85 \times 10^{-14}$ F/m, and $q = 1.6 \times 10^{-19}$ C.
The charge per unit junction area (nC cm^{-2}) in the depletion region on the p-side is ———.

36. Ans: 4.836



Sol: $\varepsilon = 12\varepsilon_0$

$= 12\times8.85\times10^{-14}$F/m

$N_D = 10^{16} cm^{-3}$

$= 10^{22} m^{-3}$

$N_A = 10^{17}\, cm^{-3}$

$= 10^{23}\, m^{-3}$

$$V_0 = \frac{kT}{q}\ell n\left[\frac{N_A N_D}{ni^2}\right] = 0.026\ell n\left[\frac{10^{23}\times 10^{22}}{(1.5\times 10^{16})^2}\right] = 0.757V$$

$$W = \sqrt{\frac{2\varepsilon}{q}V_0\left(\frac{1}{N_A}+\frac{1}{N_D}\right)} = \sqrt{\frac{2\times12\times8.85\times10^{-14}}{1.6\times10^{-19}}\times 0.757\left(\frac{1}{10^{23}}+\frac{1}{10^{22}}\right)}$$

$= 3.325\times10^{-8}$m

$= 3.325\times10^{-6}$cm

$$W_p = \frac{N_D}{N_A + N_D}\omega = \frac{10^{22}}{10^{22}+10^{23}}\times 3.325\times10^{-8} = 3.023\times10^{-9}\, m$$

$= 3.023\times10^{-7}$cm

$Q = W_P N_A eA$

$$\Rightarrow \frac{Q}{A} = W_P N_A e = 3.023\times10^{-7}\times10^{17}\times1.6\times10^{-19}$$

$= 4.836\times10^{-9}cm^{-2}$

$= 4.836nc\text{-}cm^{-2}$

37. Consider an n-channel metal oxide semiconductor field effect transistor (MOSFET) with a gate-to-source voltage of 1.8V. Assume that $\frac{W}{L} = 4$, $\mu_N C_{ox} = 70 \times 10^{-6} AV^{-2}$, the threshold voltage is 0.3V, and the channel length modulation parameter is 0.09 V^{-1}, In the saturation region, the drain conductance (in micro siemens)is ———.

37. Ans: 28.35

Sol: Drain conductance in saturation region is, $g_d = \frac{1}{r_d} = \lambda.I_D$

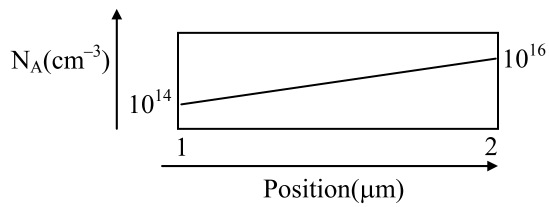
$$I_D = \frac{1}{2}\mu_m C_{ox}\frac{W}{L}\left[V_{gs} - V_T\right]^2 = \frac{1}{2}\times 70\times10^{-6}\times 4[1.8V - 0.3V]^2$$

$= 0.315$ mA

$g_d = 0.09\times 0.315\, mA/V$

$g_d = 28.35 \times 10^{-6}$ A/v = 28.35 μ Seimens

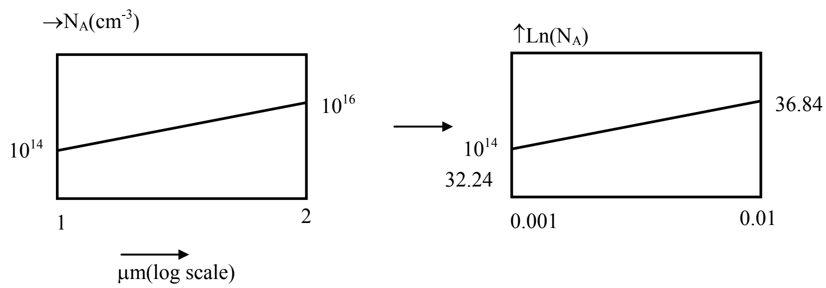
38. The figure below shows the doping distribution in a P-type semiconductor in log scale.



The magnitude of the electric field (in kV/cm) in the semiconductor due to non uniform doping is ______.

38. Ans: 0.0133

Sol:



$$qD_p \frac{dp}{dx} = q\mu_p p\varepsilon$$

$$\mu_p V_T \frac{dp}{dx} = \mu_p p\varepsilon$$

$$\varepsilon = \frac{V_T}{p}\frac{dp}{dx} \qquad p \cong N_A$$

$$\varepsilon = \frac{V_T}{N_A}\frac{dN_A}{dx} \qquad \Rightarrow \varepsilon = V_T \frac{d}{dx}\ln[N_A(x)]$$

$\log_{10} x_1 = 1\,\mu m \;\Rightarrow x_1 = 10^1\,\mu m = 0.001$ cm

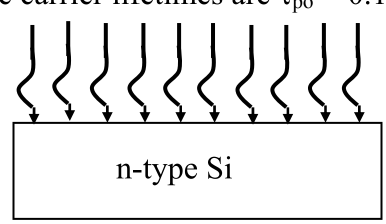
$\log_{10} x_2 = 2\,\mu m \;\Rightarrow x_2 = 10^2\,\mu m = 0.01$ cm

$\ln(10^{14}) = 32.23$

$\ln(10^{16}) = 36.84$

$$\varepsilon = 0.026\left[\frac{36.84 - 32.23}{0.01 - 0.001}\right] = 0.0133\ /\text{cm}$$

39. Consider a silicon sample at T = 300 K, with a uniform donor density $N_d = 5\times10^{16}$ cm^{-3} illuminated uniformly such that the optical generation rate is $G_{opt} = 1.5\times10^{20}$ $cm^{-3}s^{-1}$ through out the sample. The incident radiation is turned off at t = 0. Assume low-level injection to be valid and ignore surface effects. The carrier lifetimes are $\tau_{po} = 0.1$ and $\tau_{no} = 0.5$ μs.



The hole concentration at t = 0 and the hole concentration at t = 0.3 μs, respectively, are
(A) 1.5×10^{13} cm^{-3} and 7.47×10^{11} cm^{-3}
(B) 1.5×10^{13} cm^{-3} and 8.23×10^{11} cm^{-3}
(C) 7.5×10^{13} cm^{-3} and 3.73×10^{11} cm^{-3}
(D) 7.5×10^{13} cm^{-3} and 4.12×10^{11} cm^{-3}

39. Ans: (C)

Sol: $P_n(t) = P_{n_0} + P_n(0)e^{-t/\tau_p}$

at low level injuction $\Rightarrow$ P_{n_0} neglective

$$GR = \frac{P_n(0)}{\tau_{n_0}}$$

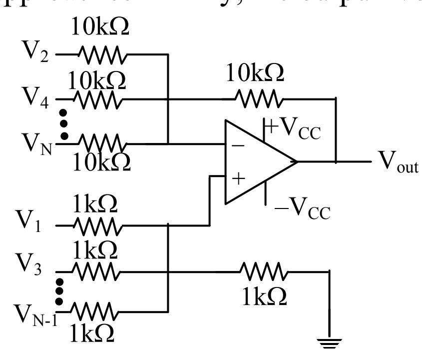
$$\Rightarrow P_{n_0}(0) = GR\times\tau_{n_0} = 1.5\times10^{20}\times0.5\times10^{-6} = 7.5\times10^{13}/\text{cm}^3$$

incorrect

At t = 0 $\Rightarrow$ $P(t) = P_n(0).\ e^0 = 7.5\times10^{13}/\text{cm}^3$

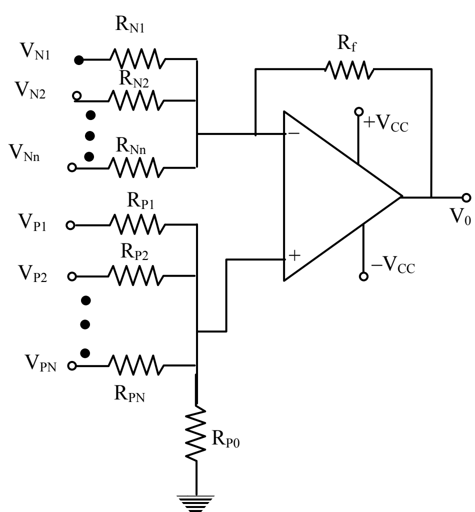
At t = 0.3 μs $\Rightarrow$ $P(t) = P_n(0)e^{-\frac{0.3}{0.1}} = 3.73\times10^{11}/\text{cm}^3$

40. An ideal opamp has voltage source $V_1, V_2, V_3, V_5, \ldots, V_{N-1}$ connected to the non-inverting input and $V_2, V_4, V_6 \ldots V_N$ connected to the inverting input as shown in the figure below ($+V_{CC}$ = 15 volt, $-V_{CC}$ = –15 volt). The voltage $V_1, V_2, V_3, V_4, V_5, V_6 \ldots$ are 1, –1/2, 1/3, –1/4, 1/5, –1/6, ….. volt, respectively. As N approaches infinity, the output voltage (in volt) is ________



40. Ans: V_0 = 15

Sol:



Using superposition it can shown that the output

$$V_0 = \left[1+\frac{R_f}{R_N}\right]\left[\frac{R_p}{R_{p1}}V_{p1} + \frac{R_p}{R_{p2}}V_{p2} + \ldots\ldots\ldots \frac{R_P}{R_{PN}}V_{pn}\right] - \left[\frac{R_f}{R_{N1}}V_{N1} + \frac{R_f}{R_{N2}}V_{N2} + \ldots\ldots\ldots \frac{R_f}{R_{Nn}}V_{Nn}\right]$$

Where $R_N = R_{N1}||R_{N2}||\ldots\ldots||R_{Nn}$ and $R_p = R_{p1}||R_{p2} \ldots\ldots R_{PN}||R_{PO}$

In the problem given

$R_f = R_{N1} = R_{N2} = \ldots\ldots\ldots = R_{Nn} = 10k\Omega$

$R_{p1} = R_{P2} = R_{P3} = \ldots\ldots\ldots = R_{PN} = R_{PO} = 1k\Omega$

$$\therefore V_0 = \left[1+\frac{10k}{\left(\frac{10k}{n}\right)}\right]\left[\frac{\frac{1k}{(1+n)}}{1k}V_{P1} + \frac{\left(\frac{1k}{1+n}\right)}{1k}V_{P2} + \ldots\ldots\right] - \left[\frac{10k}{10k}V_{N1} + \frac{10k}{10k}V_{N2} + \ldots\ldots\right]$$

$$\therefore V_0 = (V_{P1} + V_{P2} + \ldots\ldots V_{Pn}) - (V_{N1} + V_{N2} + \ldots\ldots\ldots\ldots V_{Nn})$$

If the series approaches ∞ then

$$V_0 = \left(1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}\ldots\ldots\right) - \left(\frac{-1}{2}\frac{-1}{4}\frac{-1}{6}.-\ldots\ldots\ldots\right)$$

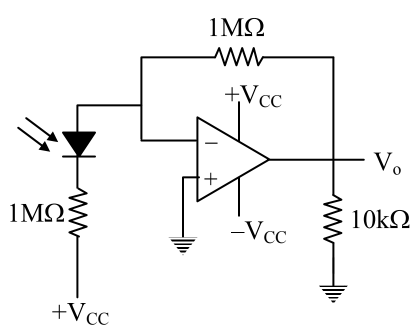
$$= 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\ldots\ldots$$

$$= \infty$$

This series is called harmonic series which is a divergent infinite series

$\therefore V_0 = +\infty = +V_{sat} = +V_{CC} = +15V$

41. A p-i-n photo diode of responsivity 0.8A/W is connected to the inverting input of an ideal opamp as shown in the figure, +Vcc = 15V, –Vcc = –15V, Load resistor R_L = 10 kΩ. If 10μW of power is incident on the photodiode, then the value of the photocurrent (in μA) through the load is _____.



41. Ans: – 800μA

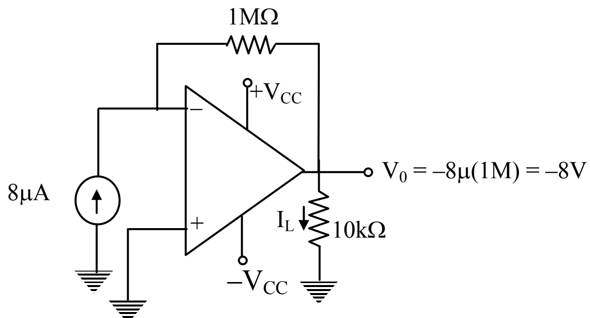
Sol: The photo diode with Responsivity 0.8A/W

$\therefore$ Diode current = $0.8A/W[10\mu W]$

$= 8 \times 10^{-6}A$

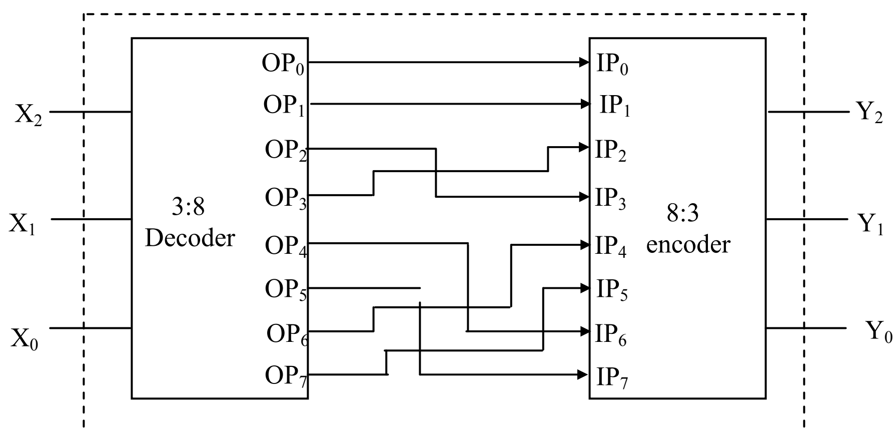
$V_0 = -8\mu\,(1M) = -8V$

$$I_L = \frac{-8}{10k} = -8\times10^{-4}\,A = -800\times10^{-6}\,A$$
$$= -800\mu A$$



Therefore the value of photo current throughput the load is –800 μA

42. Identify the circuit below.



(A) Binary to Gray code converter
(B) Binary to XS3 converter
(C) Gray to Binary converter
(D) XS3 Binary converter

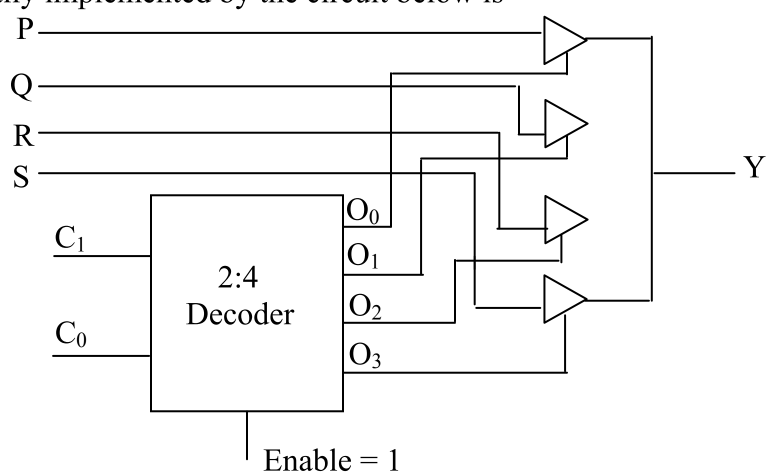
42. Ans: (A) (No Answer) if considering $OP_6 \rightarrow IP_5$, $OP_7 \rightarrow IP_4$

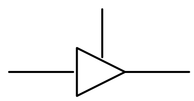
Sol:

X_2	X_1	X_0	OP_0	OP_1	OP_2	OP_3	OP_4	OP_5	OP_6	OP_7	IP_0	IP_1	IP_2	IP_3	IP_4	IP_5	IP_6	IP_7	Y_2	Y_1	Y_0
0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
0	1	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1
0	1	1	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0
				.	.								.	.						.	
				.	.								.	.						.	
1	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	1	0	1
1	1	1	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	1	0	0

Thus it is a Binary to Gray code converter

43. The functionally implemented by the circuit below is





(A) 2-to-1 multiplexer (B) 4-to-1 multiplexer
(C) 7-to-1 multiplexer (D) 6-to-1 multiplexer

43. Ans: (B)

Sol:

C_1	C_0	Y
0	0	P
0	1	Q
1	0	R
1	1	S

Hence it is a "4 to 1 multiplexer"

44. In a 8085 system, a PUSH operation requires more clock cycles than a POP operation, which one of the following options is the correct reason for this?

(A) For POP, the data transceivers remain in the same direction as for instruction fetch (memory to processor), whereas for PUSH their direction has to be reversed

(B) Memory write operations are slower than memory read operations in an 8085 bases system.

(C) The stack pointer needs to be pre-determined before writing registers in a PUSH, whereas a POP operation uses the address already in the stack pointer.

(D) Order of register has to be interchanged for a PUSH operation, whereas POP uses their natural order.

44. Ans: (C)

Sol: Push takes 12T states due to pre decrement and pop takes 10T states.

45. The open-loop transfer function of a unity-feedback control system is

$$G(S) = \frac{K}{s^2 + 5s + 5}$$

The value of K at the breakaway point of the feedback control system's root-locus plot is ____

45. Ans: 1.25

Sol: Break away point $\frac{dk}{ds} = 0$

$$\frac{d}{ds}\left(\frac{1}{s^2+5s+5}\right)=0$$

$0 - (2s+5) = 0$

$s = -2.5$ is a break away point

K Value is Obtain From Magnitude Condition

$$\left|\frac{K}{s^2+5s+5}\right|_{s=-2.5}=1$$

$$\Rightarrow \left|\frac{K}{6.25-12.5+5}\right|=1$$

$\Rightarrow K = 1.25$

46. The open-loop transfer function of a unity-feedback control system is given by

$$G(S) = \frac{K}{s(s+2)}$$

For the peak overshoot of the closed-loop system to a until step input to be 10% , the value of K is ________.

46. Ans: 2.87

Sol: Given % $M_p = 10\%$

$M_p = 0.1$

$\Rightarrow M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$

$0.1 = e^{-\pi\xi/\sqrt{1-\xi^2}}$

$$\Rightarrow \ln(0.1) = \frac{-\pi\xi}{\sqrt{1-\xi^2}}$$

$$\Rightarrow 2.3 = \frac{\pi\xi}{\sqrt{1-\xi^2}}$$

$\xi = 0.59$

Given $G(s) = \frac{K}{s(s+2)}$

CE:-

$1+G(s) = 0 \Rightarrow s^2 + 2s + K = 0$

$2\varepsilon\omega_n = 2$

$2\times 0.59 \times \omega_n = 2$

$\omega_n = 1.69$ r/sec

$K = \omega_n^2 = 2.87$

47. The transfer function of a linear time invariant systems is given by $H(s) = 2s^4 – 5s^3 + 5s – 2$
The number of zeros in the right half of the s-plane is _____.

47. Ans: 3

Sol: TF $H(s) \Rightarrow 2s^4 - 5s^3 + 5s - 2$
RH – Criteria

$$\begin{array}{cc|ccc} & +S^4 & 2 & 0 & -2 \\ 1 & & & & \\ & -S^3 & -5 & +5 & \\ 2 & & & & \\ & +S^2 & 2 & -2 & \\ & & & & \\ 3 & +S^1 & 0(2) & & \\ & -S^0 & -2 & & \end{array}$$

3 Sign Changes
3 Roots (Zeros) in the RH -S-Plane.

48. Consider a discreet memoryless source with alphabet $S = \{s_0, s_1, s_2, s_3, s_4 \ldots..\}$ and respective probabilities of occurrence $P = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots.\right\}$. The entropy of the source (in bits) is _____.

48. Ans: 2

Sol: $H = \frac{1}{2}\log_2^2 + \frac{1}{4}\log_2^4 + \frac{1}{8}\log_2^8 + \frac{1}{16}\log_2^{16} + \ldots\ldots..$

$H = \frac{1}{2}\times 1 + \frac{1}{4}\times 2 + \frac{1}{8}\times 3 + \frac{1}{16}\times 4$

$= \sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n = \dfrac{\frac{1}{2}}{\left(1-\frac{1}{2}\right)^2} = 2$

49. A digital communication system uses a repetition code for channel encoding/decoding. During transmission, each bit is repeated three times (0 is transmitted as 000, and 1 is transmitted as 111). It is assumed that the source puts out symbols independently and with equal probability . The decoder operates as follows: In a block of three received bits, if the number of zeros exceeds the number of ones, the decoder decides in favor of a 0, and if the number of ones exceeds the number of zeros, the decoder decides in favor of a 1,Assuming a binary symmetric channel with crossover probability p = 0.1, the average probability of error is ________.

49. Ans: 0.028

Sol: $P_e = P^3 + 3P^2 (1 – P)$
$P = 0.1$
$P_e = (0.1)^3 + 3 \times (0.1)^2 (1–0.1) = 0.001 + 3 \times 0.01 \times 0.9 = 0.001 + 0.027 = 0.028$

50. An analog pulse s(t) is transmitted over an additive white Gaussian (AWGN) channel. The received signal is r(t) = s(t) + n(t), where n(t) is additive white Gaussian noise with power spectral density $\frac{N_0}{2}$. The received signal is passed through a filter with impulse response h(t). Let E_s and E_n denote the energies of the pulse s(t) and the filter h(t), respectively. When the signal-to-noise ratio (SNR) is maximized at the output of the filter (SNR_{max}), which of the following holds?

(A) $E_s = E_h$; $SNR_{max} = \frac{2E_s}{N_0}$
(B) $E_s = E_h$; $SNR_{max} = \frac{E_s}{2N_0}$
(C) $E_s > E_h$; $SNR_{max} > \frac{2E_s}{N_0}$
(D) $E_s < E_h$; $SNR_{max} = \frac{2E_h}{N_0}$

50. Ans: (A)

Sol: The impulse response of the filter is same on the signal so $E_s = E_h$

$$SNR = \frac{2E_s}{No}$$

$$E_s = E_s$$

$$SNR = \frac{2E_s}{No}$$

51. The current density in a medium is given by

$$\vec{J} = \frac{400\sin\theta}{2\pi(r^2+4)}\,\hat{a}_r \quad Am^{-2}$$

The total current and the average current density flowing through the portion of a spherical surface r = 0.8m, $\frac{\pi}{12} \le \theta \le \frac{\pi}{4},\ 0 \le \phi \le 2\pi$ are given, respectively, by

(A) 15.09 A, 12.86 Am^{-2}
(B) 18.73 A, 13.65 Am^{-2}
(C) 12.86A, 9.23 Am^{-2}
(D) 10.28 A, 7.56 Am^{-2}

51. Ans: correct option is not given

Sol: Current density,

$$\vec{J} = \frac{400\sin\theta}{2\pi\left(r^2+4\right)}\bar{a}_r\ A/m^2$$

current passing through the portion of sphere of radius r = 0.8 m is given by

$$I = \int_s \vec{J}.d\vec{s}\ (r = \text{constant})$$

$$d\vec{s} = r^2\sin\theta\, d\theta\, d\phi\, \hat{a}r\ d\ (\because\ r = 0.8\text{ m})$$

$$I = \int_{\theta=\frac{\pi}{2}}^{\frac{\pi}{4}}\int_{\phi=0}^{2\pi}\frac{400\sin\theta}{2\pi\left(r^2+4\right)}r^2\sin\theta\, d\theta\, d\phi$$

$$= \frac{400(0.8)^2}{2\pi\left(0.8^2+4\right)}\left[\left(\frac{\pi}{4}-\frac{\pi}{12}\right)-\left(\sin\left(\frac{\pi}{2}\right)-\sin\left(\frac{\pi}{6}\right)\right)\right]\times\left(2\pi\right)$$

$\therefore$ I = 7.45 Amp

The average current density through the given sphere surface is

$$J = \frac{I}{\text{Area of } r = 0.8\text{m sphere}}$$

$$= \frac{7.45}{(0.8)^2 \int_{\theta=\pi/2}^{\pi/4} \int_{\phi=0}^{2\pi} \sin\theta \, d\theta \, d\phi}$$

$$= \frac{7.45}{1.04}$$

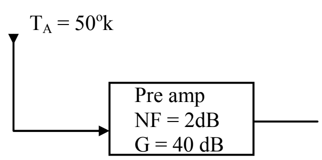
$\therefore$ J = 7.15 A/m^2

52. An antenna pointing in a certain direction has a noise temperature of 50K. The ambient temperature is 290K. The antenna is connected to pre-amplifier that has a noise figure of 2dB and an available gain of 40 dB over an effective bandwidth of 12 MHz. The effective input noise temperature T_e for the amplifier and the noise power P_{ao} at the output of the preamplifier, respectively, are

(A) T_e = 169.36K and $P_{ao} = 3.73\times10^{-10}$ W
(B) T_e = 170.8K and $P_{ao} = 4.56\times10^{-10}$ W
(C) T_e = 182.5K and $P_{ao} = 3.85\times10^{-10}$ W
(D) T_e = 160.62K and $P_{ao} = 4.6\times10^{-10}$ W

52. Ans: (A)

Sol:



10 $\log_{10}$ NF = 2dB

$\log_{10}$ NF = 0.2

NF = $10^{0.2}$

Noise temperature = (F – 1) T_o

= $(10^{0.2} - 1)$ 290o = 169.36 K

Noise power i/p = k T_eB

= $1.38 \times 10^{-23} \times (169.36 + 50) \times 12 \times 10^6$

Noise power at o/p = $(3.632 \times 10^{-14}) \times 10^4$

= 3.73×10^{-10} watts

53. Two lossless X-band horn antennas are separated by a distance of 200λ. The amplitude reflection coefficients at the terminals of the transmitting and receiving antennas are 0.15 and 0.18, respectively. The maximum directivities of the transmitting and receiving antennas (over the isotropic antenna) are 18dB and 22dB, respectively. Assuming that the input power in the lossless transmission line connected to the antenna is 2 W, and that the antennas are perfectly aligned and polarization matched, the power (in mW) delivered to the load at the receiver is ____

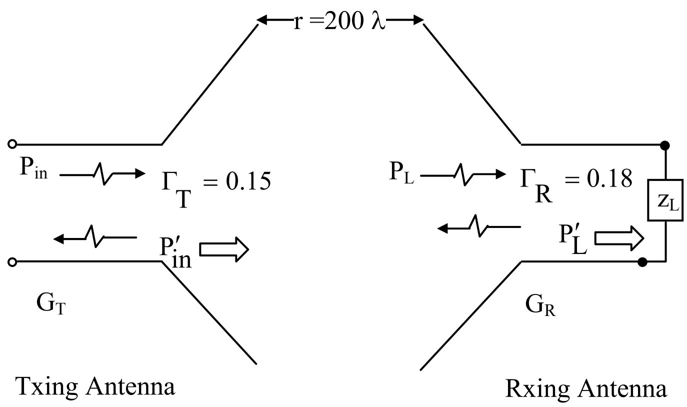
53. Ans: 2.99

Sol: Given

Lossless horn antennas

$\eta_T = \eta_R = 1$

Power Gain = Directivity
Directivity of Txing antenna, D_T = 18 dB
10 logD_T = 18
G_T (or) D_T = 63.09
Directivity of Rxing antenna, D_R = 22 dB
10logD_R = 22
G_R(or) D_R = 158.48
input power P_{in} = 2 W
Spacing, r = 200 λ



Friis transmission formula in given by

$$P_L = G_T G_R \left[\frac{\lambda}{4\pi r}\right]^2 P'_{in}$$

where:
P'_{in} : Input power (prime indicates power due to reflection)

$$P'_{in} = \left|1-\Gamma_T{}^2\right| P_{in}$$

$$= \left|1-(0.15)^2\right| \times 2$$

$$P'_{in} = 1.955\,\text{W}$$

$$P_L = 63.09 \times 158.48 \left[\frac{\lambda}{4\pi \times 200\,\lambda}\right]^2 \times 1.955$$

$$= 3.1 \times 10^{-3}$$

As there is a reflection at the terminals of Rxing antenna power delivered to the load in given by

$$P'_L = \left\{1-\left|\Gamma_R{}^2\right|\right\} \times P_L$$

$$= \{1-(0.18)^2\} \times 3.1 \times 10^{-3}$$

$$\therefore P'_L = 2.99\,\text{mW}$$

54. The electric filed of a uniform plane wave travelling along the negative z direction is given by the following equation:

$\vec{E}_w^i = (\hat{a}_x + j\hat{a}_y)E_0 e^{jkz}$

This wave is incident upon a receiving antenna placed at the origin and whose radiated electric field towards the incident wave is given by the following equation:

$\vec{E}_a = (\hat{a}_x + 2\hat{a}_y)E_I \frac{1}{r} e^{-jkr}$

The polarization of the incident wave, the polarization of the antenna and losses due to the polarization mismatch are, respectively,

(A) Linear, Circular (clockwise), –5dB
(B) Circular (clockwise), Linear, –5dB
(C) Circular (clockwise), Linear, –3dB
(D) Circular (anti clockwise), Linear, –3dB

54. Ans: (C)

Sol: Given

Electric field of incident wave is

$E_W^i = (\hat{a}_x + j\hat{a}_y)E_0 e^{jkz}$

at z = 0;

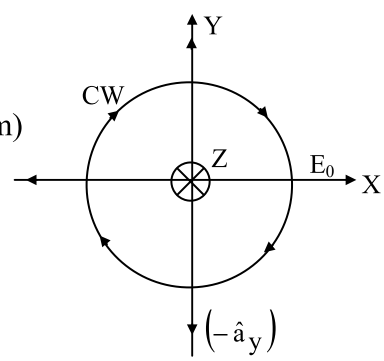
$\vec{E}_w^i = E_0 \cos\omega t\, \hat{a}_x - E_0 \sin\omega t\, \hat{a}_y$ (in time varying form)

at $\omega t = 0$

$\vec{E}_w^i = E_0 \hat{a}_x$

at $\omega t = \frac{\pi}{2}$

$\vec{E}_w^i = E_0(-\hat{a}_y)$



As a tip of electric field intensity is tracing a circle when time varies, hence the wave is said to be circularly polarized in clockwise direction (or) RHCP. Polarizing vector of incident wave is given by,

$\hat{P}_i = \frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}}$

radiated electric field from the antenna is

$\vec{E}_a = (\hat{a}_x + 2\hat{a}_y)E_I \frac{1}{\gamma} e^{-jk\gamma}$

at r = 0

$\vec{E}_a = E_I \cos\omega t\, \hat{a}_x + 2E_I \cos\omega t\, \hat{a}_y$ (in time varying form)

As both x & y components are in-phase, hence the wave is said to be linear polarized. Polarizing vector of radiated field is $\hat{P}_a = \frac{(\hat{a}_x + 2\hat{a}_y)}{\sqrt{5}}$ polarizing mismatch; The polarizing mismatch is said to have, if the polarization of receiving antenna is not same on the polarization of the incident wave. The polarization loss factor (PLF) characterizes the loss of EM power due to polarization mismatch.

$PLF = \left|\hat{P}_{i}.\hat{P}_a\right|^2$

in dB; PLF (dB) = 10 log (PLF)

$$\text{PLF} = \left| \left(\frac{\hat{a}_x + j\hat{a}_y}{2} \right) . \left(\frac{\hat{a}_x + 2\hat{a}_y}{\sqrt{5}} \right) \right|^2 = \left| \frac{1 + j2}{\sqrt{2}\sqrt{5}} \right|^2 = \frac{1}{2} (\text{or}) 0.5$$

PLF(dB) = 10 log 0.5 = – 3.0102

55. The far-zone power density radiated by a helical antenna is approximated as:

$$\vec{W}_{rad} = \vec{W}_{average} \approx \hat{a}_r C_0 \frac{1}{r^2} \cos^4 \theta$$

The radiated power density is symmetrical with respect to ϕ and exists only in the upper hemisphere: $0 \le \theta \le \frac{\pi}{2}$; $0 \le \phi \le 2\pi$; C_0 is a constant. The power radiated by the antenna (in watts) and the maximum directivity of the antenna, respectively, are

(A) $1.5C_0$, 10dB
(B) $1.256C_0$, 10dB
(C) $1.256C_0$, 12dB
(D) $1.5C_0$, 12dB

55. Ans: (B)

Sol: Given

Power density radiated by the antenna

$$\vec{W}_{rad} = \frac{C_0'}{r^2} \cos^4 \theta \, \hat{a}_r \ \ W/m^2$$

Power radiated (or) average power radiated by the antenna in given by

$$P_{rad} = \oint_s \vec{W}_{rad} . d\vec{s}$$

$$= \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} \frac{C_0'}{r^2} \cos^4 \theta \, r^2 \sin \theta \, d\theta \, d\phi \ \ (\because \text{ radiated only in the upper hemisphere})$$

$$= C_0' (2\pi) \frac{1}{5}$$

$\therefore P_{rad} = 1.256 C_0'$ Watt

Maximum directivity of the antenna in given by

$$D = 4\pi \frac{U_{max}}{P_{rad}}$$

$U = r^2 W_{rad}$

$$U = r^2 \times \frac{C_0'}{r^2} \cos^4 \theta$$

$U = C_0' \cos^4 \theta$

$U_{max} = C_0'$

$$\therefore D = \frac{4\pi C_0'}{1.256 C_0}$$

$= 10$

$D_{(dB)} = 10 \log 10$

$\therefore D = 10$ dB