ELECTROMAGNETIC THEORY SOLUTIONS

GATE- 2010

Q1. An insulating sphere of radius a carries a charge density

$$\rho(\vec{r}) = \rho_0(a^2 - r^2)\cos\theta; r < a.$$

The leading order term for the electric field at a distance d, far away from the charge distribution, is proportional to

(a) d^{-1} (b) d^{-2} (c) d^{-3} (d) d^{-4}

Ans: (c)

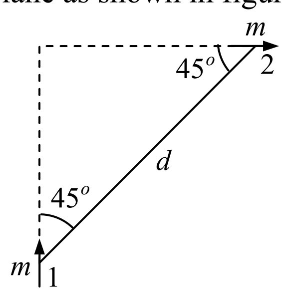
Solution: $V(r) = \left[\frac{1}{r}\int_V \rho d\tau + \frac{1}{r^2}\int \rho\cos\theta d\tau + \cdots\right]$,

Ist term, $\int \rho d\tau = \int_0^a\int_0^\pi\int_0^{2\pi} \rho_0\left(a^2 - r^2\right)\cos\theta \times r^2 \sin\theta dr d\theta d\phi = 0$

IInd term, $\int \rho\cos\theta d\tau = \int_0^a\int_0^\pi\int_0^{2\pi} \rho_0\left(a^2 - r^2\right)\cos^2\theta \times r^2 \sin\theta dr d\theta d\phi \neq 0$.

$\Rightarrow \mathrm{V}\alpha\frac{1}{\mathrm{r}^2} \Rightarrow E\alpha\frac{1}{\mathrm{r}^3}$

Q2. Two magnetic dipoles of magnitude m each are placed in a plane as shown in figure. The energy of interaction is given by



(a) Zero (b) $\frac{\mu_0 m^2}{4\pi d^3}$

(c) $\frac{3\mu_0 m^2}{2\pi d^3}$ (d) $-\frac{3\mu_0 m^2}{8\pi d^3}$

Ans: (d)

Solution: $U = \frac{\mu_0}{4\pi r^3}\left[\vec{m}_1\cdot\vec{m}_2 - 3(\vec{m}_1\cdot\hat{r})(\vec{m}_2\cdot\hat{r})\right]$,

Since $\vec{m}_1 \perp \vec{m}_2 \Rightarrow \vec{m}_1\cdot\vec{m}_2 = 0 \Rightarrow U = \frac{\mu_0}{4\pi d^3}\left[-3\times m\cos 45^0 \times m\cos 45^0\right]$

$\Rightarrow U = -\frac{3\mu_0 m^2}{8\pi d^3}$.

Statement for Linked Answer Questions 3 and 4:

Consider the propagation of electromagnetic waves in a linear, homogeneous and isotropic material medium with electric permittivity ε and magnetic permeability μ.

Q3. For a plane wave of angular frequency ω and propagation vector $\vec{k}$ propagating in the medium Maxwell's equations reduce to

(a) $\vec{k}\cdot\vec{E}=0;\ \vec{k}\cdot\vec{H}=0;\ \vec{k}\times\vec{E}=\omega\varepsilon\vec{H};\ \vec{k}\times\vec{H}=-\omega\mu\vec{E}$

(b) $\vec{k}\cdot\vec{E}=0;\ \vec{k}\cdot\vec{H}=0;\ \vec{k}\times\vec{E}=-\omega\varepsilon\vec{H};\ \vec{k}\times\vec{H}=\omega\mu\vec{E}$

(c) $\vec{k}\cdot\vec{E}=0;\ \vec{k}\cdot\vec{H}=0;\ \vec{k}\times\vec{E}=-\omega\mu\vec{H};\ \vec{k}\times\vec{H}=\omega\varepsilon\vec{E}$

(d) $\vec{k}\cdot\vec{E}=0;\ \vec{k}\cdot\vec{H}=0;\ \vec{k}\times\vec{E}=\omega\mu\vec{H};\ \vec{k}\times\vec{H}=-\omega\varepsilon\vec{E}$

Ans: (d)

Q4. If ε and μ assume negative values in a certain frequency range, then the directions of the propagation vector $\vec{k}$ and the Poynting vector $\vec{S}$ in that frequency range are related as

(a) $\vec{k}$ and $\vec{S}$ are parallel

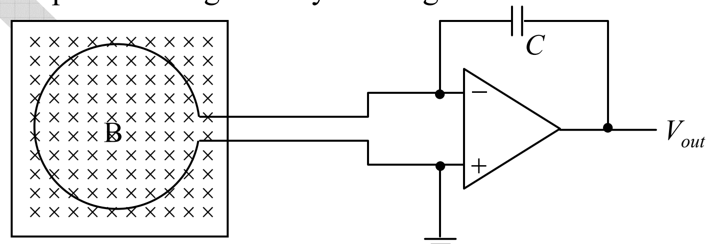
(b) $\vec{k}$ and $\vec{S}$ are anti-parallel

(c) $\vec{k}$ and $\vec{S}$ are perpendicular to each other

(d) $\vec{k}$ and $\vec{S}$ makes an angle that depends on the magnitude of $|\varepsilon|$ and $|\mu|$

Ans: (a)

Q5. Consider a conducting loop of radius a and total loop resistance R placed in a region with a magnetic field B thereby enclosing a flux ϕ_0. The loop is connected to an electronic circuit as shown, the capacitor being initially uncharged



If the loop is pulled out of the region of the magnetic field at a constant speed u, the final output voltage V_{out} is independent of

(a) ϕ_0 (b) u (c) R (d) C

Ans: (a)

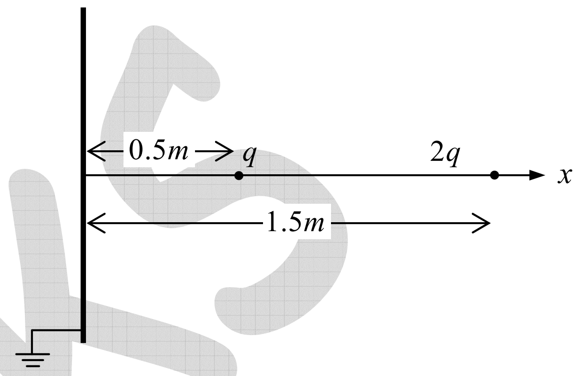
GATE-2011

Q6. If a force $\vec{F}$ is derivable from a potential function $V(r)$, where r is the distance from the origin of the coordinate system, it follows that

(a) $\vec{\nabla}\times\vec{F}=0$ (b) $\vec{\nabla}\cdot\vec{F}=0$ (c) $\vec{\nabla}V=0$ (d) $\nabla^2 V=0$

Ans: (a)

Q7. Two charges q and $2q$ are placed along the x-axis in front of a grounded, infinite conducting plane, as shown in the figure. They are located respectively at a distance of 0.5 m and 1.5 m from the plane. The force acting on the charge q is

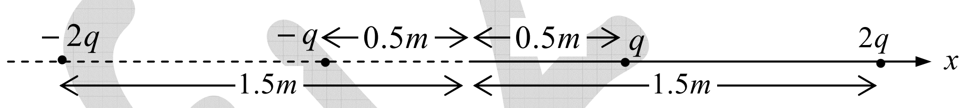


(a) $\frac{1}{4\pi\varepsilon_0}\frac{7q^2}{2}$ (b) $\frac{1}{4\pi\varepsilon_0}2q^2$

(c) $\frac{1}{4\pi\varepsilon_0}q^2$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2}$

Ans: (a)

Solution: Using method of Images we can draw equivalent figure as shown below:



$$F=\frac{q}{4\pi\varepsilon_0}\left[\frac{2q}{(1)^2}+\frac{q}{(1)^2}+\frac{2q}{(2)^2}\right]=\frac{q}{4\pi\varepsilon_0}\times\frac{7q}{2}=\frac{1}{4\pi\varepsilon_0}\frac{7q^2}{2}$$

Q8. A uniform surface current is flowing in the positive y-direction over an infinite sheet lying in x-y plane. The direction of the magnetic field is

(a) along $\hat{i}$ for $z>0$ and along $-\hat{i}$ for $z<0$

(b) along $\hat{k}$ for $z>0$ and along $-\hat{k}$ for $z<0$

(c) along $-\hat{i}$ for $z>0$ and along $\hat{i}$ for $z<0$

(d) along $-\hat{k}$ for $z>0$ and along $\hat{k}$ for $z<0$

Ans: (a)

Q9. A magnetic dipole of dipole moment $\vec{m}$ is placed in a non-uniform magnetic field $\vec{B}$. If the position vector of the dipole is $\vec{r}$, the torque acting on the dipole about the origin is

(a) $\vec{r}\times\left(\vec{m}\times\vec{B}\right)$ (b) $\vec{r}\times\vec{\nabla}\left(\vec{m}\cdot\vec{B}\right)$

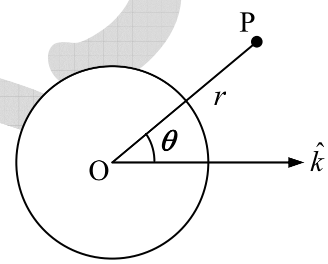
(c) $\vec{m}\times\vec{B}$ (d) $\vec{m}\times\vec{B}+\vec{r}\times\nabla\left(\vec{m}\cdot\vec{B}\right)$

Ans: (c)

Q10. A spherical conductor of radius a is placed in a uniform electric field $\vec{E}=E_0\hat{k}$. The potential at a point $P(r,\theta)$ for $r>a$, is given by

$$\Phi(r,\theta)=\text{constant}-E_0 r\cos\theta+\frac{E_0 a^3}{r^2}\cos\theta$$

where r is the distance of P from the centre O of the sphere and θ is the angle OP makes with the z-axis



The charge density on the sphere at $\theta = 30^o$ is

(a) $3\sqrt{3}\varepsilon_0 E_0/2$ (b) $3\varepsilon_0 E_0/2$

(c) $\sqrt{3}\varepsilon_0 E_0/2$ (d)) $\varepsilon_0 E_0/2$

Ans: (a)

Solution: $\sigma=-\varepsilon_0\left.\frac{\partial V}{\partial r}\right|_{r=a}=-\varepsilon_0\left[-E_0\cos\theta-\frac{2E_0a^3}{r^3}\cos\theta\right]_{r=a}$.

$$\sigma=-\varepsilon_0\left[-E_0\cos\theta-2E_0\cos\theta\right]\Rightarrow\sigma=+3E_0\varepsilon_0\cos\theta=+3E_0\varepsilon_0\cos30^0=\frac{3\sqrt{3}}{2}\varepsilon_0E_0$$

Q11. Which of the following expressions for a vector potential $\vec{A}$ **DOES NOT** represent a uniform magnetic field of magnitude B_0 along the z-direction?

(a) $\vec{A}=\left(0,B_0x,0\right)$ (b) $\vec{A}=\left(-B_0y,0,0\right)$

(c) $\vec{A}=\left(\frac{B_0x}{2},\frac{B_0y}{2},0\right)$ (d) $\vec{A}=\left(-\frac{B_0y}{2},\frac{B_0x}{2},0\right)$

Ans: (c)

Solution: $\vec{B}\neq\vec{\nabla}\times\vec{A}$.

Statement for Linked Questions 12 and 13:

A plane electromagnetic wave has the magnetic field given by

$$\vec{B}(x,y,z,t) = B_0 \sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\hat{k}$$

where k is the wave number and $\hat{i}, \hat{j}$ and $\hat{k}$ are the Cartesian unit vectors in x, y and z directions respectively.

Q12. The electric field $\vec{E}(x,y,z,t)$ corresponding to the above wave is given by

(a) $cB_0 \sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\frac{(\hat{i}-\hat{j})}{\sqrt{2}}$ (b) $cB_0 \sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$

(c) $cB_0 \sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\hat{i}$ (d) $cB_0 \sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\hat{j}$

Ans: (a)

Solution: $\vec{E} = -\frac{c}{k}\left(\vec{k}\times\vec{B}\right) = -\frac{c}{k}\left[-\frac{k\left(\hat{i}+\hat{j}\right)}{\sqrt{2}} \times B_0 \sin\left\{\frac{(x+y)k}{\sqrt{2}} + \omega t\right\}\hat{k}\right]$

$$\vec{E} = cB_0 \sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\frac{\left(\hat{i}-\hat{j}\right)}{\sqrt{2}}$$

Q13. The average Poynting vector is given by

(a) $\frac{cB_0^2}{2\mu_0}\frac{(\hat{i}-\hat{j})}{\sqrt{2}}$ (b) $-\frac{cB_0^2}{2\mu_0}\frac{(\hat{i}-\hat{j})}{\sqrt{2}}$ (c) $\frac{cB_0^2}{2\mu_0}\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$ (d) $-\frac{cB_0^2}{2\mu_0}\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$

Ans: (d)

Solution: $\vec{S} = \frac{cB_0^2}{2\mu_0}\hat{k} = \frac{cB_0^2}{2\mu_0} \times -\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right) = \frac{-cB_0^2}{2\mu_0} \times \left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

GATE-2012

Q14. The space-time dependence of the electric field of a linearly polarized light in free space is given by $\hat{x}E_0\cos(\omega t - kz)$ where E_0, ω and k are the amplitude, the angular frequency and the wavevector, respectively. The time average energy density associated with the electric field is

(a) $\frac{1}{4}\varepsilon_0 E_0^2$ (b) $\frac{1}{2}\varepsilon_0 E_0^2$ (c) $\varepsilon_0 E_0^2$ (d) $2\varepsilon_0 E_0^2$

Ans: (a)

Solution: $u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 E^2\cos^2(wt - kz) \Rightarrow \langle u_E \rangle = \frac{1}{4}\varepsilon_0 E_0^2$

Q15. A plane electromagnetic wave traveling in free space is incident normally on a glass plate of refractive index 3/2. If there is no absorption by the glass, its reflectivity is

(a) 4% (b) 16% (c) 20% (d) 50%

Ans: (a)

Solution: $R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = \left(\frac{1 - 3/2}{1 + 3/2}\right)^2 = \frac{1}{4} \times \frac{4}{25} = .04 \ or \ 4\%$

Q16. The electric and the magnetic field $\vec{E}(z,t)$ and $\vec{B}(z,t)$, respectively corresponding to the scalar potential $\phi(z,t) = 0$ and vector potential $\vec{A}(z,t) = \hat{i}tz$ are

(a) $\vec{E} = \hat{i}z$ and $\vec{B} = -\hat{j}t$ (b) $\vec{E} = \hat{i}z$ and $\vec{B} = \hat{j}t$

(c) $\vec{E} = -\hat{i}z$ and $\vec{B} = -\hat{j}t$ (d) $\vec{E} = -\hat{i}z$ and $\vec{B} = -\hat{j}t$ $+\hat{j}t$

Ans: (d)

Solution: $\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t} = -\frac{\partial \vec{A}}{\partial t} = -\hat{i}z, \quad \vec{B} = \vec{\nabla} \times \vec{A} = +\hat{j}t.$

Q17. A plane polarized electromagnetic wave in free space at time t=0 is given by $\vec{E}(x,z) = 10\hat{j}\exp[i(6x + 8z)]$. The magnetic field $\vec{B}(x,z,t)$ is given by

(a) $\vec{B}(x,z,t) = \frac{1}{c}(6\hat{k} - 8\hat{i})\exp[i(6x + 8z - 10ct)]$

(b) $\vec{B}(x,z,t) = \frac{1}{c}(6\hat{k} + 8\hat{i})\exp[i(6x + 8z - 10ct)]$

(c) $\vec{B}(x,z,t) = \frac{1}{c}(6\hat{k} - 8\hat{i})\exp[i(6x + 8z - ct)]$

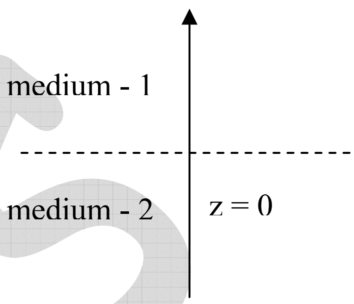
(d) $\vec{B}(x,z,t) = \frac{1}{c}(6\hat{k} + 8\hat{i})\exp[i(6x + 8z + ct)]$

Ans: (a)

Solution: $\vec{B} = \frac{1}{c}\left(\hat{k}\times\vec{E}\right) = \frac{1}{c}\left(\frac{\vec{k}}{|\vec{k}|}\times\vec{E}\right) = \frac{1}{c}\left(\frac{6\hat{i}+8\hat{k}}{10}\right)\times 10\hat{j}\exp\left[i\left(\vec{k}.\vec{r}-\omega t\right)\right]$

$\vec{B} = \frac{1}{c}\left(6\hat{k}-8\hat{i}\right)\exp\left[i\left(6x+8z-10ct\right)\right], \quad \omega = 10c.$

Q18. Two infinitely extended homogeneous isotopic dielectric media (medium-1and medium-2 with dielectric constant $\frac{\varepsilon_1}{\varepsilon_0} = 2$ and $\frac{\varepsilon_2}{\varepsilon_0} = 5$, respectively) meet at the z = 0 plane as shown in the figure. A uniform electric field exists everywhere. For $z \geq 0$, the electric field is given by $\vec{E}_1 = 2\hat{i} - 3\hat{j} + 5\hat{k}$. The interface separating the two media is charge free. The electric displacement vector in the medium-2 is given by



(a) $\vec{D}_2 = \varepsilon_0\left[10\hat{i}+15\hat{j}+10\hat{k}\right]$

(b) $\vec{D}_2 = \varepsilon_0\left[10\hat{i}-15\hat{j}+10\hat{k}\right]$

(c) $\vec{D}_2 = \varepsilon_0\left[4\hat{i}-6\hat{j}+10\hat{k}\right]$

(d) $\vec{D}_2 = \varepsilon_0\left[4\hat{i}+6\hat{j}+10\hat{k}\right]$

Ans: (b)

Solution: $\because E_1^{\text{II}} = E_2^{\text{II}} \Rightarrow E_2^{\text{II}} = 2\hat{i}-3\hat{j}$

and $\sigma_f = 0 \Rightarrow D_1^{\perp} = D_2^{\perp} \Rightarrow E_2^{\perp} = \frac{\varepsilon_1}{\varepsilon_2}E_1^{\perp} = \frac{2\times 5}{5}\hat{k} = 2\hat{k} \Rightarrow \vec{E}_2 = 2\hat{i}-3\hat{j}+2\hat{k}$

$\Rightarrow \vec{D}_2 = \varepsilon_2\vec{E}_2 = \varepsilon_0\left[10\hat{i}-15\hat{j}+10\hat{k}\right].$

GATE-2013

Q19. At a surface current, which one of the magnetostatic boundary condition is **NOT** CORRECT?

(a) Normal component of the magnetic field is continuous.

(b) Normal component of the magnetic vector potential is continuous.

(c) Tangential component of the magnetic vector potential is continuous.

(d) Tangential component of the magnetic vector potential is not continuous.

Ans: (d)

Q20. Interference fringes are seen at an observation plane $z = 0$, by the superposition of two plane waves $A_1 \exp\left[i\left(\vec{k}_1 \cdot \vec{r} - \omega t\right)\right]$ and $A_2 \exp\left[i\left(\vec{k}_2 \cdot \vec{r} - \omega t\right)\right]$, where A_1 and A_2 are real amplitudes. The condition for interference maximum is

(a) $\left(\vec{k}_1 - \vec{k}_2\right) \cdot \vec{r} = (2m+1)\pi$ (b) $\left(\vec{k}_1 - \vec{k}_2\right) \cdot \vec{r} = 2m\pi$

(c) $\left(\vec{k}_1 + \vec{k}_2\right) \cdot \vec{r} = (2m+1)\pi$ (d) $\left(\vec{k}_1 + \vec{k}_2\right) \cdot \vec{r} = 2m\pi$

Ans: (b)

Q21. For a scalar function φ satisfying the Laplace equation, $\vec{\nabla}\varphi$ has

(a) zero curl and non-zero divergence

(b) non-zero curl and zero divergence

(c) zero curl and zero divergence

(d) non-zero curl and non-zero divergence

Ans: (c)

Solution: $\nabla^2 \varphi = 0 \Rightarrow \vec{\nabla} . \left(\vec{\nabla}\varphi\right) = 0$ and $\Rightarrow \vec{\nabla} \times \left(\vec{\nabla}\varphi\right) = 0$.

Q22. A circularly polarized monochromatic plane wave is incident on a dielectric interface at Brewaster angle. Which one of the following statements is correct?

(a) The reflected light is plane polarized in the plane of incidence and the transmitted light is circularly polarized.

(b) The reflected light is plane polarized perpendicular to the plane of incidence and the transmitted light is plane polarized in the plane of incidence.

(c) The reflected light is plane polarized perpendicular to the plane of incidence and the transmitted light is elliptically polarized.

(d) There will be no reflected light and the transmitted light is circularly polarized.

Ans: (c)

Q23. A charge distribution has the charge density given by $\rho = Q\{\delta(x - x_0) - \delta(x + x_0)\}$. For this charge distribution the electric field at $(2x_0, 0, 0)$

(a) $\dfrac{2Q\hat{x}}{9\pi\varepsilon_0 x_0^2}$ (b) $\dfrac{Q\hat{x}}{4\pi\varepsilon_0 x_0^3}$ (c) $\dfrac{Q\hat{x}}{4\pi\varepsilon_0 x_0^2}$ (d) $\dfrac{Q\hat{x}}{16\pi\varepsilon_0 x_0^2}$

Ans:

Solution: Potential $V(r)=\dfrac{1}{4\pi\varepsilon_0}\left[\int_{-a}^{a}\dfrac{\rho(x')}{x}dx'+\int_{-a}^{a}\dfrac{\rho(x')}{x^2}x'dx'+\int_{-a}^{a}\dfrac{\rho(x')}{x^3}x'^2dx'+\ldots\right]$

First term, total charge

$$Q_T=\int\rho(x')dx'=Q\int_{-x_0}^{x_0}\delta(x'-x_0)dx'-Q\int_{-x_0}^{x_0}\delta(x'+x_0)dx'=Q-Q=0$$

Second term, dipole moment

$$p=\int x'\rho(x')dx'=Q\int_{-x_0}^{x_0}x'\delta(x'-x_0)dx'-Q\int_{-x_0}^{x_0}x'\delta(x'+x_0)dx'=Qx_0-Q\times -x_0=2Qx_0$$

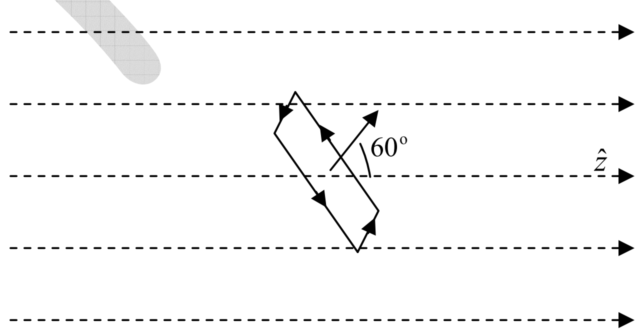
$$V=\frac{2Qx_0}{4\pi\varepsilon_0 x^2}\Rightarrow\vec{E}=-\frac{\partial V}{\partial x}\hat{x}=\frac{4Qx_0}{4\pi\varepsilon_0 x^3}\hat{x}=\frac{4Qx_0}{4\pi\varepsilon_0(2x_0)^3}\hat{x}=\frac{Q}{8\pi\varepsilon_0 x_0^2}\hat{x}$$

Q24. A monochromatic plane wave at oblique incidence undergoes reflection at a dielectric interface. If $\hat{k}_i,\hat{k}_r$ and $\hat{n}$ are the unit vectors in the directions of incident wave, reflected wave and the normal to the surface respectively, which one of the following expressions is correct?

(a) $(\hat{k}_i-\hat{k}_r)\times\hat{n}\neq 0$ (b) $(\hat{k}_i-\hat{k}_r)\cdot\hat{n}=0$ (c) $(\hat{k}_i\times\hat{n})\cdot\hat{k}_r=0$ (d) $(\hat{k}_i\times\hat{n})\cdot\hat{k}_r\neq 0$

Ans: (c)

Q25. In a constant magnetic field of 0.6 Tesla along the z direction, find the value of the path integral $\oint\vec{A}\cdot\vec{dl}$ in the units of (Tesla m^2) on a square loop of side length $(1/\sqrt{2})$ meters. The normal to the loop makes an angle of 60^0 to the z-axis, as shown in the figure. The answer should be up to two decimal places. ___________



Ans: 0.15

Solution: $\oint \vec{A}\cdot\vec{dl} = \int_S \left(\vec{\nabla}\times\vec{A}\right)d\vec{a} = \int_S \vec{B}.d\vec{a} = BA\cos 60^0 = 0.6\times\left(\frac{1}{\sqrt{2}}\right)^2\times\frac{1}{2} = 0.15T.m^2$

GATE-2014

Q26. Which one of the following quantities is invariant under Lorentz transformation?

(a) Charge density (b) Charge (c) Current (d) Electric field

Ans: (b)

Q27. An unpolarized light wave is incident from air on a glass surface at the Brewster angle. The angle between the reflected and the refracted wave is

(a) 0^o (b) 45^o (c) 90^o (d) 120^o

Ans: (c)

Q28. The electric field of a uniform plane wave propagating in a dielectric non-conducting medium is given by $\vec{E} = \hat{x}10\cos\left(6\pi\times 10^7 t - 0.4\pi z\right)\ V/m$. The phase velocity of the wave is _________ $10^8\ m/s$

Ans: 1.5

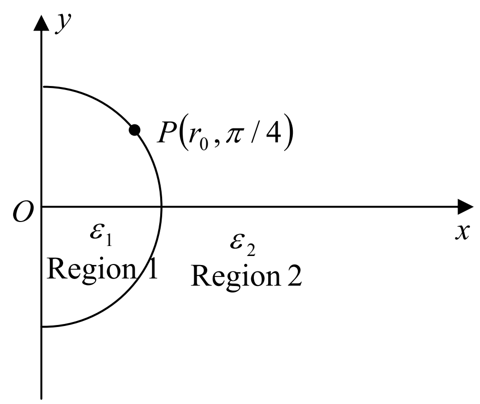
Solution: $v = \frac{\omega}{k} = \frac{6\pi\times 10^7}{0.4\pi} = 1.5\times 10^8\ m/\sec$

Q29. If the vector potential $\vec{A} = \alpha x\hat{x} + 2y\hat{y} - 3z\hat{z}$, satisfies the Coulomb gauge, the value of the constant α is _______

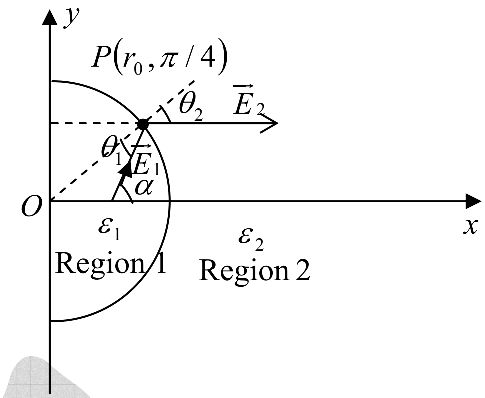
Ans: 1

Solution: Coulomb gauge condition $\vec{\nabla}.\vec{A} = 0 \Rightarrow \alpha + 2 - 3 = 0 \Rightarrow \alpha = 1$

Q30. A ray of light inside Region 1 in the xy-plane is incident at the semicircular boundary that carries no free charges. The electric field at the point $P\left(r_0, \frac{\pi}{4}\right)$ in plane polar coordinates is $\vec{E}_1 = 7\hat{e}_r - 3\hat{e}_\varphi$ where $\hat{e}_r$ and $\hat{e}_\varphi$ are the unit vectors. The emerging ray in Region 2 has the electric field $\vec{E}_2$ parallel to x-axis. If ε_1 and ε_2 are the dielectric constants of Region-1 and Region-2 respectively, then $\frac{\varepsilon_2}{\varepsilon_1}$ is ________



Ans: 2.32



Solution: $\because \vec{E}_1 = 7\hat{e}_r - 3\hat{e}_\varphi$

$$\Rightarrow E_x = \left(7\hat{e}_r - 3\hat{e}_\varphi\right).\hat{x} = 7\cos 45 + 3\sin 45 = \frac{10}{\sqrt{2}}$$

$$\Rightarrow E_y = \left(7\hat{e}_r - 3\hat{e}_\varphi\right).\hat{y} = 7\sin 45 - 3\sin 45 = \frac{4}{\sqrt{2}}$$

Thus $\vec{E}_1$ makes an angle $\alpha = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left(\frac{4}{10}\right) = 21.8^0$

$\because \frac{\tan\theta_2}{\tan\theta_1} = \frac{\varepsilon_2}{\varepsilon_1} \Rightarrow \frac{\varepsilon_2}{\varepsilon_1} = \frac{\tan 45}{\tan 23.2} = 2.32$. where $\theta_1 = \alpha - 45^0$ and $\theta_2 = 45^0$

Q31. The value of the magnetic field required to maintain non-relativistic protons of energy $1MeV$ in a circular orbit of radius 100 mm is ________Tesla

(Given: $m_p = 1.67\times10^{-27}kg,\ \ e = 1.6\times10^{-19}C$)

Ans: 1.44

Solution: $E = \frac{q^2B^2R^2}{2m_p} \Rightarrow 1.6\times10^{-13} = \frac{\left(1.6\times10^{-19}\right)^2 B^2 (0.1)^2}{2\left(1.67\times10^{-27}\right)} \Rightarrow B^2 = \frac{1.6\times10^{-13}\times2\left(1.67\times10^{-27}\right)}{\left(1.6\times10^{-19}\right)^2(0.1)^2}$

$$\Rightarrow B^2 = \frac{10^{-13}\times2\left(1.67\times10^{-27}\right)}{\left(1.6\times10^{-38}\right)(0.01)} = \frac{3.34\times10^{-40}}{1.6\times10^{-40}} = 2.08 \Rightarrow B = \sqrt{2.08}\,Tesla = 1.44Tesla$$

Q32. In an interference pattern formed by two coherent sources, the maximum and minimum intensities are $9I_0$ and I_0 respectively. The intensities of the individual wave are

(a) $3I_0$ and I_0 (b) $4I_0$ and I_0 (c) $5I_0$ and $4I_0$ (d) $9I_0$ and I_0

Ans: (b)

Solution: $I_{\max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ and $I_{\min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

$$9I_0 = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \text{ and } I_0 = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 \Rightarrow I_1 = 4I_0 \quad and \quad I_2 = I_0$$

Q33. The intensity of a laser in free space is $150mW/m^2$. The corresponding amplitude of the electric field of the laser is ________ $\frac{V}{m}$ $\left(\varepsilon_0 = 8.854\times10^{-12}C^2/N.m^2\right)$

Ans: 10.6

Solution: $I=\frac{1}{2}c\varepsilon_0 E_0^2 \Rightarrow E_0=\sqrt{\frac{2I}{c\varepsilon_0}}=\sqrt{\frac{2\times 150\times 10^{-3}}{3\times 10^{8}\times 8.854\times 10^{-12}}}=10.6\,V/m$

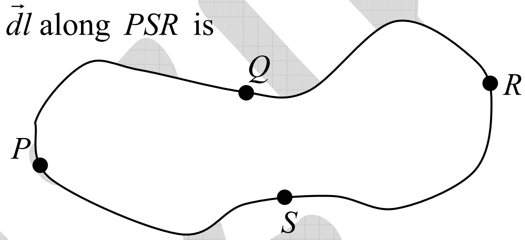
GATE-2015

Q34. A point charge is placed between two semi-infinite conducting plates which are inclined at an angle of 30° with respect to each other. The number of image charges is____________.

Ans.: 11

Solution: $n=\frac{360}{\theta}-1=\frac{360}{30}-1=11$

Q35. Given that the magnetic flux through the closed loop $PQRSP$ is ϕ. If $\int_P^R \vec{A}\cdot d\vec{l}=\phi_1$ along PQR, the value of $\int_P^R \vec{A}\cdot d\vec{l}$ along PSR is



(a) $\phi-\phi_1$ (b) $\phi_1-\phi$ (c) $-\phi_1$ (d) ϕ_1

Ans.: (b)

Solution: $\phi=\int_s \vec{B}.d\vec{a}=\oint \vec{A}.d\vec{l}=\int_P^R \vec{A}\cdot d\vec{l}+\int_R^P \vec{A}\cdot d\vec{l} \Rightarrow \phi=\phi_1-\int_P^R \vec{A}\cdot d\vec{l} \Rightarrow \int_P^R \vec{A}\cdot d\vec{l}=\phi_1-\phi$

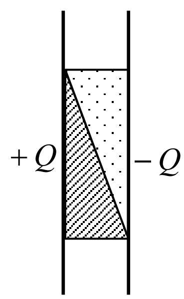
Q36. The space between two plates of a capacitor carrying charges $+Q$ and $-Q$ is filled with two different dielectric materials, as shown in the figure. Across the interface of the two dielectric materials, which one of the following statements is correct?

(a) $\vec{E}$ and $\vec{D}$ are continuous

(b) $\vec{E}$ is continuous and $\vec{D}$ is discontinuous

(c) $\vec{D}$ is continuous and $\vec{E}$ is discontinuous

(d) $\vec{E}$ and $\vec{D}$ are discontinuous



Ans.: (d)

Q37. Four forces are given below in Cartesian and spherical polar coordinates

(i) $\vec{F}_1 = K\exp\left(\frac{-r^2}{R^2}\right)\hat{r}$ (ii) $\vec{F}_2 = K\left(x^3\hat{y} - y^3\hat{z}\right)$

(iii) $\vec{F}_3 = K\left(x^3\hat{x} + y^3\hat{y}\right)$ (iv) $\vec{F}_4 = K\left(\frac{\hat{\phi}}{r}\right)$

where K is a constant Identify the correct option

(a) (iii) and (iv) are conservative but (i) and (ii)are not

(b) (i) and (ii) are conservative but (iii) and (iv) are not

(c) (ii) and (iii) are conservative but (i) and (iv) are not

(d) (i) and (iii) are conservative but (ii) and (iv) are not

Ans.: (d)

Solution: $\vec{\nabla}\times\vec{F}_1 = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ \text{k}\exp\left(-\frac{r^2}{R^2}\right) & 0 & 0 \end{vmatrix} = 0$

$$\vec{\nabla}\times\vec{F}_2 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & kx^3 & -ky^3 \end{vmatrix} = \hat{x}\left(-3ky^2 - 0\right) + \hat{z}\left(3kx^2 - 0\right) = -3ky^2\hat{x} + 3kx^2\hat{z}$$

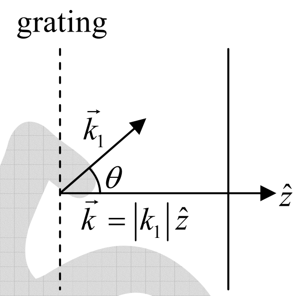
$$\vec{\nabla}\times\vec{F}_3 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ kx^3 & ky^3 & 0 \end{vmatrix} = 0$$

$$\vec{\nabla}\times\vec{F}_4 = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 0 & 0 & r\sin\theta\times\frac{k}{r} \end{vmatrix} = \hat{r}\left[k\cos\theta\right]\times\frac{1}{r^2\sin\theta}$$

Q38. A monochromatic plane wave (wavelength $=600\,nm$) $E_0\exp[i(kz-\omega t)]$ is incident normally on a diffraction grating giving rise to a plane wave $E_1\exp\left[i\left(\vec{k}_1\cdot\vec{r}-\omega t\right)\right]$ in the first order of diffraction. Here $E_1<E_0$ and $\vec{k}_1=\left|\vec{k}_1\right|\left[\frac{1}{2}\hat{x}+\frac{\sqrt{3}}{2}\hat{z}\right]$. The period (in μm) of the diffraction grating is ______________ (upto one decimal place)

Ans.: 1.2

Solution: $d\sin\theta=n\lambda\Rightarrow d=\dfrac{\lambda}{\sin\theta}\qquad\because n=1$



and $\vec{k}_1=\left|\vec{k}_1\right|\left[\frac{1}{2}\hat{x}+\frac{\sqrt{3}}{2}\hat{z}\right]$

$$\Rightarrow\sin\theta=\frac{\vec{k}\times\vec{k}_1}{\left|\vec{k}_1\right|\left|\vec{k}\right|}=\frac{\hat{z}\times\left(\frac{1}{2}\hat{x}+\frac{\sqrt{3}}{2}\hat{z}\right)}{\sqrt{\frac{1}{4}+\frac{3}{4}}\times\sqrt{\frac{1}{4}+\frac{3}{4}}}=\frac{1}{2}\Rightarrow\theta=30^0$$

$$\Rightarrow d=\frac{600}{\sin 30}nm=1200\ nm=1.2\ \mu m$$

Q39. A long solenoid is embedded in a conducting medium and is insulated from the medium. If the current through the solenoid is increased at a constant rate, the induced current in the medium as a function of the radial distance r from the axis of the solenoid is proportional to

(a) r^2 inside the solenoid and $\frac{1}{r}$ outside
(b) r inside the solenoid and $\frac{1}{r^2}$ outside
(c) r^2 inside the solenoid and $\frac{1}{r^2}$ outside
(d) r inside the solenoid and $\frac{1}{r}$ outside

Ans.: (d)

Solution: $\oint\vec{E}\cdot d\vec{l}=-\int\frac{\partial\vec{B}}{\partial t}\cdot d\vec{a}$;

$$For\quad r<R,\ \left|\vec{E}\right|2\pi r=-\mu_0 n\frac{dI}{dt}\int_{r'=0}^{r}2\pi r'dr'=-\mu_0 n\frac{dI}{dt}\frac{2\pi r^2}{2}\Rightarrow\left|\vec{E}\right|=-\frac{1}{2}\mu_0 n\frac{dI}{dt}r$$

$$For\quad r>R,\ \left|\vec{E}\right|2\pi r=-\mu_0 n\frac{dI}{dt}\int_{r'=0}^{R}2\pi r'dr'=-\mu_0 n\frac{dI}{dt}\frac{2\pi R^2}{2}\Rightarrow\left|\vec{E}\right|=-\frac{1}{2r}\mu_0 n\frac{dI}{dt}R^2$$

Q40. A plane wave $(\hat{x}+i\hat{y})E_0 \exp[i(kz-\omega t)]$ after passing through an optical element emerges as $(\hat{x}-i\hat{y})E_0 \exp[i(kz-\omega t)]$, where k and ω are the wavevector and the angular frequency, respectively. The optical element is a

(a) quarter wave plate
(b) half wave plate
(c) polarizer
(d) Faraday rotator

Ans.: (b)

Solution: Incident wave: $\left(\hat{x}+i\hat{y}\right)E_0 e^{i\theta} = \left[E_0\cos\theta\hat{x} - E_0\sin\theta\hat{y}\right]$

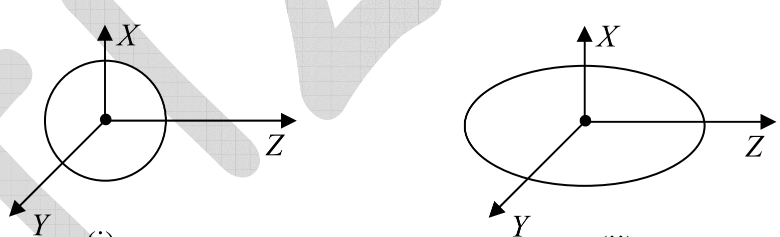
Left circular polarization with phase angle $\phi_1 = -\theta = \theta e^{i\pi}$

Emergent wave: $\left(\hat{x}-i\hat{y}\right)E_0 e^{i\theta} = \left[E_0\cos\theta\hat{x} + E_0\sin\theta\hat{y}\right]$

Right circular polarization with phase angle $\phi_1 = +\theta = \theta e^{i0}$

Thus there is phase change of π and hence path difference is $\frac{\lambda}{2}$.

Q41. A charge $-q$ is distributed uniformly over a sphere, with a positive charge q at its center in (i). Also in (ii), a charge $-q$ is distributed uniformly over an ellipsoid with a positive charge q at its center. With respect to the origin of the coordinate system, which one of the following statements is correct?



(i) (ii)

(a) The dipole moment is zero in both (i) and (ii)
(b) The dipole moment is non-zero in (i) but zero in (ii)
(c) The dipole moment is zero in (i) but non-zero in (ii)
(d) The dipole moment is non-zero in both (i) and (ii)

Ans.: (a)

Solution: $\vec{p} = \sum q_i \vec{r}_i = 0$ in both cases.

GATE-2016

Q42. Which of the following magnetic vector potentials gives rise to a uniform magnetic field $B_0\hat{k}$?

(a) $B_0 z\,\hat{k}$ (b) $-B_0 x\,\hat{j}$ (c) $\frac{B_0}{2}\left(-y\hat{i}+x\hat{j}\right)$ (d) $\frac{B_0}{2}\left(y\hat{i}+x\hat{j}\right)$

Ans.: (c)

Solution: (a) $\vec{\nabla}\times\vec{A}=0$

(b) $\vec{\nabla}\times\vec{A}=-B_0\hat{k}$

(c) $\vec{\nabla}\times\vec{A}=B_0\hat{k}$

(d) $\vec{\nabla}\times\vec{A}=0$

Q43. The magnitude of the magnetic dipole moment associated with a square shaped loop carrying a steady current I is m. If this loop is changed to a circular shape with the same current I passing through it, the magnetic dipole moment becomes $\frac{pm}{\pi}$. The value of p is ______.

Ans.: 4

Solution: Magnetic dipole moment associated with a square shaped loop (let side is *a*) carrying a steady current I is $m = Ia^2$.

Magnetic dipole moment associated with a circular shaped loop (let radius is *r*) carrying a steady current I is $m' = I\pi r^2$.

Here $4a = 2\pi r \Rightarrow r = \frac{2a}{\pi} \Rightarrow m' = I\pi r^2 = I\pi\left(\frac{2a}{\pi}\right)^2 = \frac{4Ia^2}{\pi} = \frac{4m}{\pi}$

Q44. In a Young's double slit experiment using light, the apparatus has two slits of unequal widths. When only slit-1 is open, the maximum observed intensity on the screen is $4I_0$. When only slit-2 is open, the maximum observed intensity is I_0. When both the slits are open, an interference pattern appears on the screen. The ratio of the intensity of the principal maximum to that of the nearest minimum is ________.

Ans.: 9

Solution: $\dfrac{I_{\max}}{I_{\min}} = \dfrac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2} = \dfrac{\left(\sqrt{4I_0}+\sqrt{I_0}\right)^2}{\left(\sqrt{4I_0}-\sqrt{I_0}\right)^2} = \dfrac{\left(2\sqrt{I_0}+\sqrt{I_0}\right)^2}{\left(2\sqrt{I_0}-\sqrt{I_0}\right)^2} = \dfrac{9I_0}{I_0} = 9$

Q45. An infinite, conducting slab kept in a horizontal plane carries a uniform charge density σ. Another infinite slab of thickness *t*, made of a linear dielectric material of dielectric constant k, *is* kept above the conducting slab. The bound charge density on the upper surface of the dielectric slab is

(a) $\dfrac{\sigma}{2k}$ (b) $\dfrac{\sigma}{k}$ (c) $\dfrac{\sigma(k-2)}{2k}$ (d) $\dfrac{\sigma(k-1)}{k}$

Ans.: (d)

Solution:

Electric field due to infinite, conducting slab inside the dielectric is $\vec{E} = \dfrac{\sigma}{\varepsilon}\hat{z} = \dfrac{\sigma}{\varepsilon_0 k}\hat{z}$

Polarisation $\vec{P} = \varepsilon_0\chi_e\vec{E} = \varepsilon_0(k-1)\dfrac{\sigma}{\varepsilon_0 k}\hat{z} = \dfrac{\sigma(k-1)}{k}\hat{z} \Rightarrow \sigma_1 = \vec{P}.\hat{z} = \dfrac{\sigma(k-1)}{k}$

Q46. The electric field component of a plane electromagnetic wave travelling in vacuum is given by $\vec{E}(z,t) = E_0\cos(kz-\omega t)\hat{i}$. The Poynting vector for the wave is

(a) $\left(\dfrac{c\varepsilon_0}{2}\right)E_0^2\cos^2(kz-\omega t)\hat{j}$ (b) $\left(\dfrac{c\varepsilon_0}{2}\right)E_0^2\cos^2(kz-\omega t)\hat{k}$

(c) $c\varepsilon_0 E_0^2\cos^2(kz-\omega t)\hat{j}$ (d) $c\varepsilon_0 E_0^2\cos^2(kz-\omega t)\hat{k}$

Ans.: (d)

Solution: $\vec{E}(z,t) = E_0\cos(kz-\omega t)\hat{i} \Rightarrow \vec{B} = \dfrac{1}{c}\hat{z}\times\vec{E}(z,t) = \dfrac{E_0}{c}\cos(kz-\omega t)\hat{j}$

The Poynting vector for the wave is

$$\vec{S} = \frac{1}{\mu_0}\left(\vec{E}\times\vec{B}\right) = \frac{E_0^2}{\mu_0 c}\cos^2(kz-\omega t)\hat{k} = c\varepsilon_0 E_0^2\cos^2(kz-\omega t)\hat{k}$$

Q47. The $x-y$ plane is the boundary between free space and a magnetic material with relative permeability μ_r. The magnetic field in the free space is $B_x\hat{i} + B_z\hat{k}$. The magnetic field in the magnetic material is

(a) $B_x\hat{i} + B_z\hat{k}$ (b) $B_x\hat{i} + \mu_r B_z\hat{k}$ (c) $\dfrac{1}{\mu_r}B_x\hat{i} + B_z\hat{k}$ (d) $\mu_r B_x\hat{i} + B_z\hat{k}$

Ans.: (d)

Solution: $B_1^{\perp} = B_z \hat{k} = B_2^{\perp}$ and $H_1^{\parallel} = H_2^{\parallel} \Rightarrow \frac{B_1^{\parallel}}{\mu_0} = \frac{B_2^{\parallel}}{\mu_0 \mu_r} \Rightarrow B_2^{\parallel} = \mu_r B_1^{\parallel} = \mu_r B_x \hat{i}$

The magnetic field in the magnetic material is $\mu_r B_x \hat{i} + B_z \hat{k}$

GATE- 2017

Q48. Identical charges q are placed at five vertices of a regular hexagon of side a. The magnitude of the electric field and the electrostatic potential at the centre of the hexagon are respectively

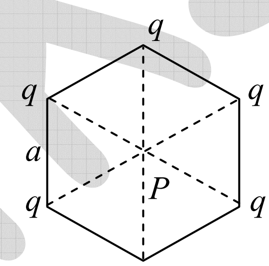
(a) $0, 0$ (b) $\frac{q}{4\pi\varepsilon_0 a^2}, \frac{q}{4\pi\varepsilon_0 a}$

(c) $\frac{q}{4\pi\varepsilon_0 a^2}, \frac{5q}{4\pi\varepsilon_0 a}$ (d) $\frac{\sqrt{5}q}{4\pi\varepsilon_0 a^2}, \frac{\sqrt{5}q}{4\pi\varepsilon_0 a}$

Ans. : (c)

Solution: The resultant field at P is $E = \frac{q}{4\pi\varepsilon_0 a^2}$

The electrostatic potential at P is $V = \frac{5q}{4\pi\varepsilon_0 a}$



Q49. A parallel plate capacitor with square plates of side $1m$ separated by 1 micro meter is filled with a medium of dielectric constant of 10. If the charges on the two plates are $1C$ and $-1C$, the voltage across the capacitor is………….. kV. (up to two decimal places). ($\varepsilon_0 = 8.854 \times 10^{-12}\ F/m$)

Ans. : 11.29

Solution: $q = CV = \frac{\varepsilon_0 \varepsilon_r A}{d} V \Rightarrow V = \frac{qd}{\varepsilon_0 \varepsilon_r A} = \frac{1 \times 1 \times 10^{-6}}{8.854 \times 10^{-12} \times 10 \times 1} \approx 11.29 kV$

Q50. Light is incident from a medium of refractive index $n = 1.5$ onto vacuum. The smallest angle of incidence for which the light is not transmitted into vacuum is………….. degrees. (up to two decimal places)

Ans. : 41.8

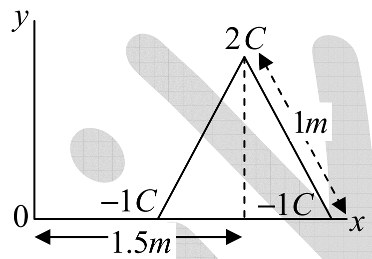
Solution: $\sin\theta_C = \frac{n_2}{n_1} = \frac{1}{1.5} \Rightarrow \theta_C = \sin^{-1}\left(\frac{1}{1.5}\right) \Rightarrow \theta_C = 41.8$

Q51. A monochromatic plane wave in free space with electric field amplitude of $1\,V/m$ is normally incident on a fully reflecting mirror. The pressure exerted on the mirror is.................$\times 10^{-12}\,Pa$. (up to two decimal places) ($\varepsilon_0 = 8.854\times 10^{-12}\,F/m$)

Ans. : 8.85

Solution: $P = \frac{2I}{c} = \frac{2}{c}\times\frac{1}{2}c\varepsilon_0 E_0^2 = \varepsilon_0 E_0^2 = 8.854\times 10^{-12}\times(1)^2 = 8.85\times 10^{-12}\,Pa$

Q52. Three charges $(2C, -1C, -1C)$ are placed at the vertices of an equilateral triangle of side $1m$ as shown in the figure. The component of the electric dipole moment about the marked origin along the $\hat{y}$ direction is........$C\,m$.



Ans. : 1.73

Solution: $\vec{p} = -1(1\hat{x}) - 1(2\hat{x}) + 2\left(1.5\hat{x} + \sqrt{1-0.25}\,\hat{y}\right)$

Along the $\hat{y}$ direction $= 2\times\sqrt{1-0.25} = 1.73$

Q53. An infinite solenoid carries a time varying current $I(t) = At^2$, with $A \neq 0$. The axis of the solenoid is along the $\hat{z}$ direction. $\hat{r}$ and $\hat{\theta}$ are the usual radial and polar directions in cylindrical polar coordinates. $\vec{B} = B_r\hat{r} + B_\theta\hat{\theta} + B_z\hat{z}$ is the magnetic field at a point outside the solenoid. Which one of the following statements is true?

(a) $B_r = 0, B_\theta = 0, B_z = 0$

(b) $B_r \neq 0, B_\theta \neq 0, B_z = 0$

(c) $B_r \neq 0, B_\theta \neq 0, B_z \neq 0$

(d) $B_r = 0, B_\theta = 0, B_z \neq 0$

Ans. : (d)

Q54. A uniform volume charge density is placed inside a conductor (with resistivity $10^{-2}\,\Omega m$). The charge density becomes $\frac{1}{(2.718)}$ of its original value after time.......Fermi seconds (up to two decimal places) ($\varepsilon_0 = 8.854\times 10^{-12}\,F/m$)

Ans. : 88.54

Solution: $\rho(t)=\rho(0)e^{-\sigma t/\varepsilon_0} \Rightarrow -\sigma t/\varepsilon_0 = \ln\frac{\rho(t)}{\rho(0)} = \ln\frac{1}{2.718} = 1$

$$\Rightarrow t = \frac{\varepsilon_0}{\sigma} = 8.854\times10^{-12}\times10^{-2} = 88.54\times10^{-15}\sec = 88.54\,fs$$

Q55. Consider a metal with free electron density of $6\times10^{22}\,cm^{-3}$. The lowest frequency of electromagnetic radiation to which this metal is transparent, is $1.38\times10^{16}\,Hz$. If this metal had a free electron density of $1.8\times10^{23}\,cm^{-3}$ instead, the lowest frequency electromagnetic radiation to which it would be transparent is…………….$\times10^{16}\,Hz$ (up to two decimal places).

Ans. : 2.39

Solution: Cut-off frequency is $f \propto \sqrt{n}$.

$$\text{Thus } \frac{f_2}{f_1} = \sqrt{\frac{n_2}{n_1}} \Rightarrow f_2 = f_1\sqrt{\frac{n_2}{n_1}} \Rightarrow f_2 = 1.38\times10^{16}\sqrt{\frac{1.8\times10^{23}}{6\times10^{22}}} = 2.39\times10^{16}\,Hz$$

GATE- 2018

Q56. Among electric field ($\vec{E}$), magnetic field ($\vec{B}$), angular momentum ($\vec{L}$) and vector potential ($\vec{A}$), which is/are **odd** under parity (space inversion) operation?

(a) $\vec{E}$ only

(b) $\vec{E}$ and $\vec{A}$ only

(c) $\vec{E}$ and $\vec{B}$ only

(d) $\vec{B}$ and $\vec{L}$ only

Ans. : (b)

Solution: Under parity operation $r \to -r$

$$E = -\frac{\partial V}{\partial r} \quad ; \quad E : P \to -E$$

$$B = \vec{I}\times\vec{r} \quad ; \quad B : P \to +B$$

$$L = \vec{r}\times\vec{p} \quad ; \quad L : P \to +L$$

$$E = -\frac{\partial A}{\partial t} \quad ; \quad A : P \to -A$$

Q57. An infinitely long straight wire is carrying a steady current I. The ratio of magnetic energy density at distance r_1 to that at $r_2\left(=2r_1\right)$ from the wire is ___________.

Ans. : 4

Solution: $u_B=\dfrac{B^2}{2\mu_0}\propto\dfrac{1}{r^2}\Rightarrow\dfrac{u_{B1}}{u_{B2}}=\dfrac{r_2^2}{r_1^2}=\dfrac{\left(2r_1\right)}{r_1^2}=4$

Q58. A light beam of intensity I_0 is falling normally on a surface. The surface absorbs 20% of the intensity and the rest is reflected. The radiation pressure on the surface is given by $X\,I_0/c$, where X is __________ (up to one decimal place). Here c is the speed of light.

Ans. : 1.8

Solution: Radiation pressure $=\dfrac{I_0}{c}-\left(-0.8\dfrac{I_0}{c}\right)=1.8\dfrac{I_0}{c}$

Q59. The number of independent components of a general electromagnetic field tensor is__________

Ans. : 6

Solution: In Cartesian co-ordinate, three Independent coordinate for electric field, $\left(E_x,E_y,E_z\right)$ and three Independent co-ordinate for magnetic field $\left(B_x,B_y,B_z\right)$.

Q60. Consider an infinitely long solenoid with N turns per unit length, radius R and carrying a current $I\left(t\right)=\alpha\cos\omega t$, where α is a constant and ω is the angular frequency. The magnitude of electric field at the surface of the solenoid is

(a) $\dfrac{1}{2}\mu_0 NR\omega\alpha\sin\omega t$ (b) $\dfrac{1}{2}\mu_0\omega NR\cos\omega t$

(c) $\mu_0 NR\omega\alpha\sin\omega t$ (d) $\mu_0\omega NR\cos\omega t$

Ans. : (a)

Solution: $\vec{B}=\begin{cases}\mu_0 NI\left(t\right)\hat{z}, & \text{inside}\\ 0 & \text{, outside}\end{cases}$

Since, $\oint\limits_{line}\vec{E}\cdot d\vec{l}=-\int\dfrac{\partial\vec{B}}{\partial t}\cdot d\vec{a}$

$\Rightarrow\left|\vec{E}\right|\times 2\pi R=-\mu_0 N\left(-\alpha\omega\sin\omega t\right)\times\pi R^2$

$$\Rightarrow \left|\vec{E}\right| = \frac{1}{2}\mu_0 NR\omega\alpha \sin\omega t$$

Q61. A constant and uniform magnetic field $\vec{B} = B_0\hat{k}$ pervades all space. Which one of the following is the correct choice for the vector potential in Coulomb gauge?

(a) $-B_0\left(x+y\right)\hat{i}$ (b) $B_0\left(x+y\right)\hat{j}$ (c) $B_0 x\hat{j}$ (d) $-\frac{1}{2}B_0\left(x\hat{i}-y\hat{j}\right)$

Ans. : (c)

Solution: Check option (c),

$$\vec{\nabla}\cdot\vec{A} = 0,\ \vec{B} = \vec{\nabla}\times\vec{A} = B_0\hat{k}$$

Q62. A long straight wire, having radius a and resistance per unit length r, carries a current I. The magnitude and direction of the Poynting vector on the surface of the wire is

(a) $I^2 r/2\pi a$, perpendicular to axis of the wire and pointing inwards

(b) $I^2 r/2\pi a$, perpendicular to axis of the wire and pointing outwards

(c) $I^2 r/\pi a$, perpendicular to axis of the wire and pointing inwards

(d) $I^2 r/\pi a$, perpendicular to axis of the wire and pointing outwards

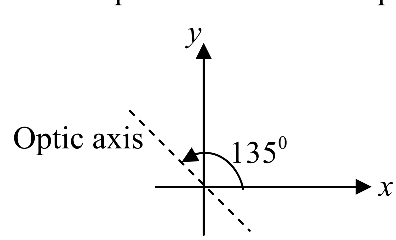
Ans. : (a)

Solution: $\left|\vec{S}\right| = \frac{1}{\mu_0}\left|\left(\vec{E}\times\vec{B}\right)\right| = \frac{1}{\mu_0}\frac{V}{l}\times\frac{\mu_0 I}{2\pi a} = \frac{IR}{l}\times\frac{I}{2\pi a}$

$$\because V = IR,\ r = \frac{R}{l} \Rightarrow \left|\vec{S}\right| = \frac{I^2 r}{2\pi a}$$

Q63. A quarter wave plate introduces a path difference of $\lambda/4$ between the two components of polarization parallel and perpendicular to the optic axis. An electromagnetic wave with $\vec{E} = \left(\hat{x}+\hat{y}\right)E_0 e^{i(kz-\omega t)}$ is incident normally on a quarter wave plate which has its optic axis making an angle 135^0 with the x - axis as shown.

The emergent electromagnetic wave would be



(a) elliptically polarized

(b) circularly polarized

(c) linearly polarized with polarization as that of incident wave

(d) linearly polarized but with polarization at 90^0 to that of the incident wave

Ans. : (c)

Q64. An electromagnetic plane wave is propagating with an intensity $I = 1.0\times10^5\, Wm^{-2}$ in a medium with $\in = 3\in_0$ and $\mu = \mu_0$. The amplitude of the electric field inside the medium is ________$\times10^3\, Vm^{-1}$ (up to one decimal place).

$(\in_0 = 8.85\times10^{-12} C^2N^{-1}m^{-2},\ \mu_0 = 4\pi\times10^{-7} NA^{-2}, c = 3\times10^8 ms^{-1})$

Ans. : 6.6

Solution: $I = \frac{1}{2}v\in E^2 \Rightarrow E^2 = \frac{2I}{v\in} = \frac{2I}{\frac{1}{\sqrt{\mu\in}}\in} = 2I\sqrt{\frac{\mu}{\in}}$

$$\Rightarrow E^2 = 2\times10^5\sqrt{\frac{\mu_0}{3\in_0}} = 2\times10^5\sqrt{\frac{4\pi\times10^{-7}}{3\times8.8\times10^{-12}}} \approx 4363.4\times10^4$$

$$\Rightarrow E \approx 66\times10^2 \approx 6.6\times10^3\, V/m$$

GATE-2019

Q65. The electric field of an electromagnetic wave is given by $\vec{E} = 3\sin(kz-\omega t)\hat{x} + 4\cos(kz-\omega t)\hat{y}$. The wave is

(a) linearly polarized at an angle $\tan^{-1}\left(\frac{4}{3}\right)$ from the x - axis

(b) linearly polarized at an angle $\tan^{-1}\left(\frac{3}{4}\right)$ from the x - axis

(c) elliptically polarized in clockwise direction when seen travelling towards the observer

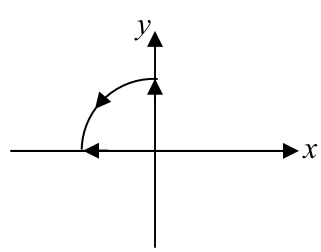
(d) elliptically polarized in counter-clockwise direction when seen travelling towards the observer

Ans. : (d)

Solution: At $z = 0,\ E_x = -3\sin\omega t,\ E_y = 4\cos\omega t$

At $\omega t = 0,\ E_x = 0,\ E_y = 4$

At $\omega t = \frac{\pi}{2},\ E_x = -3,\ E_y = 0$



Q66. An infinitely long thin cylindrical shell has its axis coinciding with the z-axis. It carries a surface charge density $\sigma_0 \cos\phi$, where ϕ is the polar and σ_0 is a constant. The magnitude of the electric field inside the cylinder is

(a) 0 (b) $\frac{\sigma_0}{2\in_0}$ (c) $\frac{\sigma_0}{3\in_0}$ (d) $\frac{\sigma_0}{4\in_0}$

Ans. : (b)

Solution: $dE = \frac{d\lambda}{2\pi\in_0 R} = \frac{(\sigma_0\cos\phi)(Rd\phi)}{2\pi\in_0 R} = \frac{\sigma_0\cos\phi}{2\pi\in_0}$

Along axis of cylinder $dE_x = dE\cos\phi \Rightarrow E_x = \frac{\sigma_0}{2\pi\in_0}\int_0^{2\pi}\cos^2\phi\, d\phi = \frac{\sigma_0}{2\in_0}$

Q67. A circular loop made of a thin wire has radius $2\,cm$ and resistance 2Ω. It is placed perpendicular to a uniform magnetic field of magnitude $|\vec{B}_0| = 0.01$ Tesla. At time $t=0$ the field starts decaying as $\vec{B} = \vec{B}_0 e^{-t/t_0}$, where $t_0 = 1s$. The total charge that passes through a cross section of the wire during the decay is Q. The value of Q in μC (rounded off to two decimal places) is____________

Ans. : 6.28

Solution: $\varepsilon = -\frac{d\phi}{dt} = -\frac{AdB}{dt}, I = \frac{\varepsilon}{R} = -\frac{d\phi}{dt}\frac{1}{R}$

$\Rightarrow -\frac{d\phi}{dt} = -\pi r^2 \frac{d}{dt}\left(B_0 e^{-t/t_0}\right) = \pi r^2 B_0 e^{-t}\,(t_0 = 1)$

$Q = \int_0^\infty I(t)\,dt = \int_0^\infty \frac{\pi r^2}{R} B_0 e^{-t}\,dt = \frac{\pi r^2 B_0}{R}\left|\frac{e^{-t}}{-1}\right|_0^\infty$

$= 3.14\times\left(2\times10^{-2}\right)^2\times 0.01 = 6.28\mu C$

Q68. The electric field of an electromagnetic wave in vacuum is given by

$$\vec{E} = E_0\cos\left(3y + 4z - 1.5\times10^9 t\right)\hat{x}$$

The wave is reflected from the $z=0$ surface. If the pressure exerted on the surface is $\alpha \in E_0^2$, the value of α (rounded off to one decimal place) is___________

Ans. : 0.8

Solution: $\vec{K} = 3\hat{y} + 4\hat{z}$

$$\tan\theta_R = \frac{K_y}{K_z} = \frac{3}{4}$$

$$P = 2\frac{I}{c}\cos\theta_R = \frac{2}{c}\times\frac{1}{2}\in_0 cE_0^2\times\frac{4}{5}$$

$$P = 0.8\in_0 E_0^2$$

Q69. A solid cylinder of radius R has total charge Q distributed uniformly over its volume. It is rotating about its axis with angular speed ω. The magnitude of the total magnetic moment of the cylinder is

(a) $QR^2\omega$ (b) $\frac{1}{2}QR^2\omega$ (c) $\frac{1}{4}QR^2\omega$ (d) $\frac{1}{8}QR^2\omega$

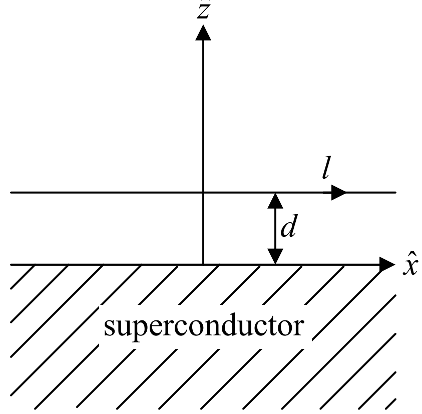
Ans. : (c)

Solution: Magnetic moment due to disc $\mu = \frac{\pi\sigma\omega R^4}{4}$

Due to cylinder $d\mu = \frac{\pi\omega R^4}{4}(\rho dz)$ $(\sigma \to \rho dz)$

$$\mu = \frac{\pi\omega R^4}{4}\int_0^L \frac{Q}{\pi R^2 L}dz = \frac{Q\omega R^4}{4}$$

Q70. An infinitely long wire parallel to the x-axis is kept at $z = d$ and carries a current I in the positive x direction above a superconductor filling the region $z \le 0$ (see figure). The magnetic field $\vec{B}$ inside the superconductor is zero so that the field just outside the superconductor is parallel to its surface. The magnetic field due to this configuration at a point $(x, y, z > 0)$ is



(a) $\left(\frac{\mu_0 I}{2\pi}\right)\frac{-(z-d)\hat{j} + y\hat{k}}{\left[y^2 + (z-d)^2\right]}$

(b) $\left(\frac{\mu_0 I}{2\pi}\right)\left[\frac{-(z-d)\hat{j} + y\hat{k}}{y^2 + (z-d)^2} + \frac{(z+d)\hat{j} - y\hat{k}}{y^2 + (z+d)^2}\right]$

(c) $\left(\frac{\mu_0 I}{2\pi}\right)\left[\frac{-(z-d)\hat{j} + y\hat{k}}{y^2 + (z-d)^2} - \frac{(z+d)\hat{j} - y\hat{k}}{y^2 + (z+d)^2}\right]$

(d) $\left(\frac{\mu_0 I}{2\pi}\right)\left[\frac{y\hat{j}+(z-d)\hat{k}}{y^2+(z-d)^2}+\frac{y\hat{j}-(z+d)\hat{k}}{y^2+(z+d)^2}\right]$

Ans. : (b)

Solution: Verify that $\vec{B}=0$, when $d=0$

Q71. The vector potential inside a long solenoid with n turns per unit length and carrying current I, written in cylindrical coordinates is $\vec{A}(s,\phi,z)=\frac{\mu_0 nI}{2}s\hat{\phi}$. If the term $\frac{\mu_0 nI}{2}s\left(\alpha\cos\phi\hat{\phi}+\beta\sin\phi\hat{s}\right)$, where $\alpha\neq 0,\ \beta\neq 0$ is added to $\vec{A}(S,\phi,z)$, the magnetic field remains the same if

(a) $\alpha=\beta$ (b) $\alpha=-\beta$ (c) $\alpha=2\beta$ (d) $\alpha=\frac{\beta}{2}$

$$\left[\begin{array}{c}\text{Useful formulae: } \vec{\Delta}t=\frac{\partial t}{\partial S}\hat{S}+\frac{1}{S}\frac{\partial t}{\partial\phi}\hat{\phi}+\frac{\partial t}{\partial z}\hat{z};\\ \vec{\nabla}\times\vec{v}=\left(\frac{1}{s}\frac{\partial v_z}{\partial\phi}-\frac{\partial v_\phi}{\partial z}\right)\hat{s}+\left(\frac{\partial v_s}{\partial z}-\frac{\partial v_z}{\partial s}\hat{\phi}+\frac{1}{s}\left(\frac{\partial(sv_\phi)}{\partial s}-\frac{\partial v_s}{\partial\phi}\right)\hat{z}\right)\end{array}\right]$$

Ans. : (d)

Solution: $\vec{B}=\vec{\nabla}\times\vec{A}=\frac{1}{r}\begin{vmatrix}\hat{r} & r\hat{\phi} & \hat{z}\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z}\\ A_r & rA_\phi & 0\end{vmatrix}=\mu_0 nI\hat{z}$

$$\vec{B}'=\vec{\nabla}\times\vec{A}'=\frac{1}{r}\begin{vmatrix}\hat{r} & r\hat{\phi} & \hat{z}\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z}\\ A_r & rA_\phi & 0\end{vmatrix}=\mu_0 nI\left[(\alpha\cos\phi+1)-\frac{\beta\cos\phi}{2}\right]\hat{z}$$

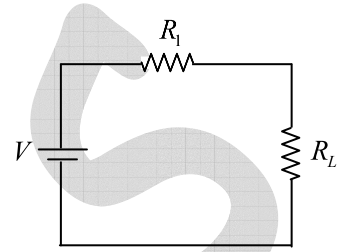
Equate $\vec{B}'=\vec{B}\Rightarrow\left[(\alpha\cos\phi+1)-\frac{\beta\cos\phi}{2}\right]=\mu_0 nI$

$$\Rightarrow\alpha\cos\phi=\frac{\beta}{2}\cos\phi\Rightarrow\alpha=\frac{\beta}{2}$$

Q72. For a given load resistance $R_L = 4.7$ ohm, the power transfer efficiencies $\left(\eta = \frac{P_{load}}{P_{total}}\right)$ of a dc voltage source and a dc current source with internal resistances R_1 and R_2, respectively, are equal. The product $R_1 R_2$ in units of ohm^2 (rounded off to one decimal place) is___________

Ans. : 22.09

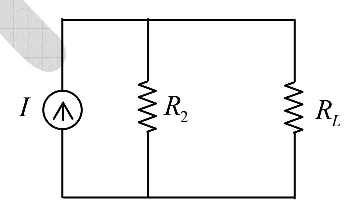
Solution: For dc voltage source



$$P_{total} = \frac{V^2}{R_1 + R_L} \text{ and } P_{R_L} = \left(\frac{V}{R_1 + R_L}\right)^2 R_L$$

$$\eta_{dc\ vol} = \frac{P_{R_L}}{P_{total}} = \frac{R_L}{R_1 + R_L}$$

For dc current source



$$P_{total} = I^2\left(\frac{R_2 R_L}{R_2 + R_L}\right) \text{ and } P_{R_L} = I_L^2 R_L = \left(\frac{R_2 I}{R_2 + R_L}\right)^2 R_L$$

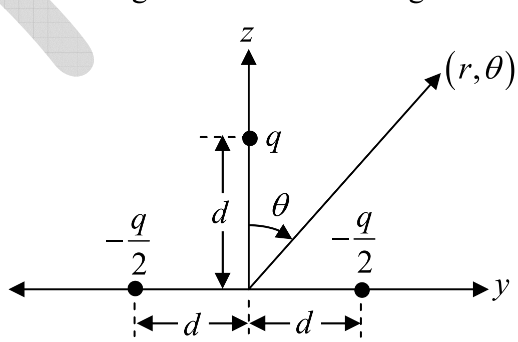
$$\eta_{dc\ curr} = \frac{P_{R_L}}{P_{total}} = \frac{R_2}{R_2 + R_L}$$

Since $\eta_{dc\ vol} = \eta_{dc\ curr}$

$$\Rightarrow \frac{R_L}{R_1 + R_L} = \frac{R_2}{R_2 + R_L} \Rightarrow R_L\left(R_2 + R_L\right) = R_2\left(R_1 + R_L\right) \Rightarrow R_1 R_2 = R_L^2$$

$$\Rightarrow R_1 R_2 = (4.7)^2 = 22.09\ \Omega^2$$

Q73. Consider a system of three charges as shown in the figure below:



For $r = 10\ m; \theta = 60$ degrees; $q = 10^{-6}$ Coulomb, and $d = 10^{-3}\ m$, the electric dipole potential in volts (rounded off to three decimal places) at a point (r, θ) is ________

[Use: $\frac{1}{4\pi \in_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$]

Ans. : 0.045

Solution: Monopole moment $= -\frac{q}{2} - \frac{q}{2} + q = 0$

$$\vec{p} = -\frac{q}{2} \times (-d\hat{y}) - \frac{q}{2}(d\hat{y}) + q(d\hat{z})$$

$$\vec{p} = qd\hat{z}$$

$$V(r,\theta) = \frac{1}{4\pi \in_0} \frac{\vec{p} \cdot r}{r^2} = \frac{1}{4\pi \in_0} \frac{qd \cos\theta}{r^2}$$

$$V(r,\theta) = 9 \times 10^9 \times \frac{10^{-6} \times 10^{-3} \times \cos 60^0}{(10)^2}$$

$$= 9 \times 10^9 \times \frac{10^{-9}}{2 \times 100} = 0.045$$

Q74. The electric field of an electromagnetic wave is given by $\vec{E} = 3\sin(kz - \omega t)\hat{x} + 4\cos(kz - \omega t)\hat{y}$. The wave is

(a) linearly polarized at an angle $\tan^{-1}\left(\frac{4}{3}\right)$ from the x - axis

(b) linearly polarized at an angle $\tan^{-1}\left(\frac{3}{4}\right)$ from the x - axis

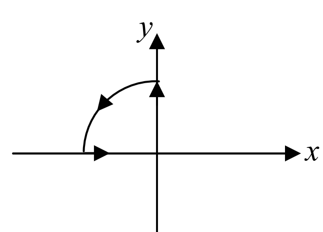
(c) elliptically polarized in clockwise direction when seen travelling towards the observer

(d) elliptically polarized in counter-clockwise direction when seen travelling towards the observer

Ans. : (d)

Solution: At $z = 0$, $E_x = -3\sin\omega t$, $E_y = 4\cos\omega t$

At $\omega t = 0$, $E_x = 0$, $E_y = 4$



At $\omega t = \frac{\pi}{2}, \; E_x = -3, \; E_y = 0$

Q75. In a set of N successive polarizers, the m^{th} polarizer makes an angle $\left(\frac{m\pi}{2N}\right)$ with the vertical. A vertically polarized light beam of intensity I_0 is incident on two such sets with $N = N_1$ and $N = N_2$, where $N_2 > N_1$. Let the intensity of light beams coming out be $I(N_1)$ and $I(N_2)$, respectively. Which of the following statements is correct about the two outgoing beams?

(a) $I(N_2) > I(N_1)$; the polarization in each case is vertical

(b) $I(N_2) < I(N_1)$; the polarization in each case is vertical

(c) $I(N_2) > I(N_1)$; the polarization in each case is horizontal

(d) $I(N_2) < I(N_1)$; the polarization in each case is horizontal

Ans. : (c)

Solution: $I(N_1) = I_0\left[\cos\left(\frac{n/2}{N_1}\right)\right]^{2N_1}, \; I(N_2) = I_0\left[\cos\left(\frac{n/2}{N_2}\right)\right]^{2N_2}$

$I(N_2) > I(N_1)$

For last polarization, pass axis will be horizontal.

Ex: $N_1 = 5$

$I(5) = I_0\left[\cos\left(18^*\right)\right]^{10} = 0.605\, I_0$

$N_2 = 10$

$I(10) = I_0\left[\cos\left(9^*\right)\right]^{20} = 0.780\, I_0$

$I(10) > I(5)$