

(346) :- HAND WRITTEN NOTES:

OF

(1)

# ELECTRONICS & COMMUNICATION ENGINEERING

-: SUBJECT:-

## COMMUNICATION SYSTEM

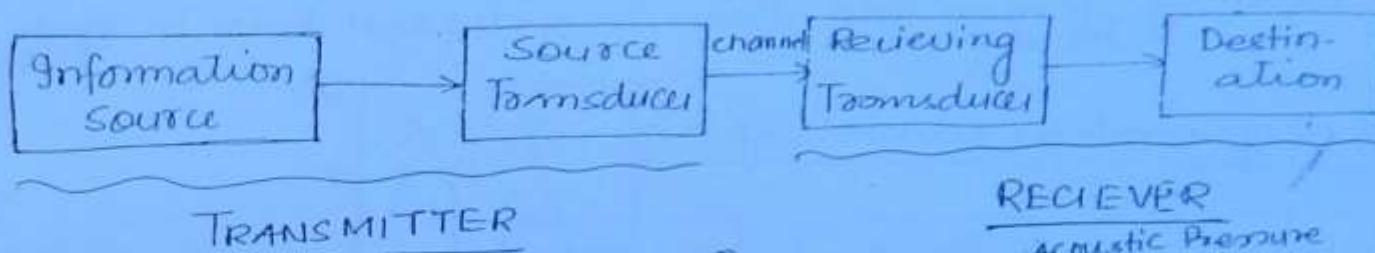


Defn

Communication is the process of transformation of information from source to destination or from transmitter to receiver.

(3)

\* Basic Block diagram of Comm<sup>n</sup> System:- (wired comm<sup>n</sup> system)



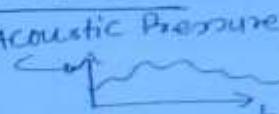
TRANSMITTER

→ Preferable for short dist. Comm<sup>n</sup>.

Note:-

Voice → 300Hz - 3.5KHz { variation of Acoustic Pressure with time }  
Audio → 20Hz - 20KHz { subset }  
Video → 0 to 15MHz { variation of light Intensity with time }

RECEIVER



Information Source:

Information source is the source of information

Source Transducer: Source transducer converts a physical signal to electrical signal equivalent. Eg ~~Micro~~ Mic

channel:

channel is the medium through which signal is transmitted from one place to another.

Note:-

1. wired Comm<sup>n</sup> system, is performed for short distance comm<sup>n</sup>: where the channel will be:-  
a) Co-axial cable  
b) Parallel wire  
c) Twisted pair etc.

for long distance comm<sup>n</sup> wireless comm<sup>n</sup> is preferred where the channel will be free space.

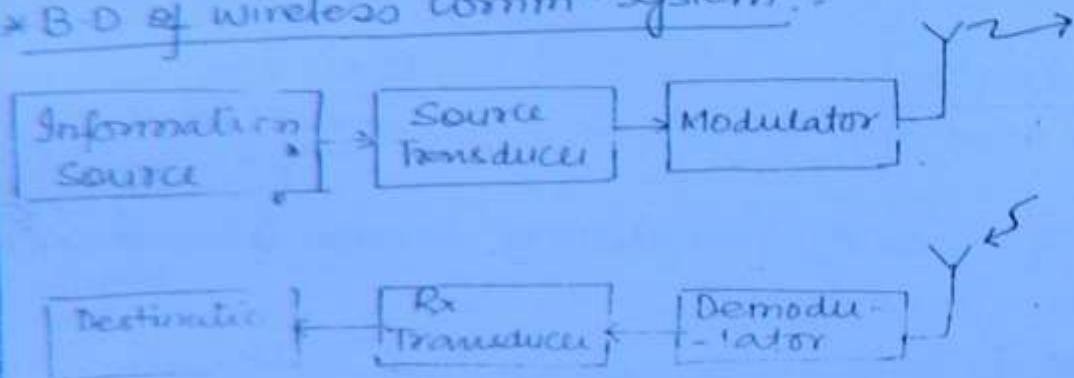
(4)

Review time:

If converts electrical signal as physical signal

Eg. loudspeaker.

\* B-D of wireless comm<sup>n</sup> System:-

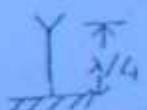


For transmission of a signal to very much long distance through free space, modulation has to be used.

\* Need of Modulation:-

1. Reducing of Antenna height; for faithful transmission, the height of Antenna from the ground should be  $\lambda/4$ .

$$h_t = \lambda/4$$



$$\lambda = \frac{c}{f}$$

Note:- As, only EM wave travel in free space and travels with speed of light. So

$$\lambda = c/f$$

$$\text{So, } h_t = \frac{c}{4f}, h_t \propto \frac{1}{f}$$

Let.  $f = 15\text{ KHz}$ , so,  $h_t = 5\text{ km}$ , which is not possible

Hence frequency has to be increased and hence modulation is introduced.

$$15\text{KHz} \quad \boxed{\text{Modulator}} \quad \xrightarrow{1\text{MHz}} \quad h_t = \frac{c}{4f} = \frac{3 \times 10^8}{4 \times 10^6}$$

(5)

$h_t = 7.5\text{m}$ , practically possible.

Note:

- Transmitting Antenna converts electrical signal as EM signal and will travel with the velocity of light.

So,

$$\boxed{\lambda = c/f}$$

$$\boxed{h_t = c/4f}$$

- Modulation is the process of increasing the frequency of the signal to reduce Antenna height.

$$a) f = 15\text{KHz} \xrightarrow[\text{modulation}]{\text{After}} b) f = 1\text{MHz}$$

$$h_t = \frac{c}{4f}$$

$$\boxed{h_t = 5000\text{mtr}}$$

$$h_t = \frac{c}{4f}$$

$$\boxed{h_t = 75\text{Mtr}}$$

## 2. Multiplexing:

- It is the process of transmission of multiple no. of signal through a single channel at the same time.
- Generally, without modulation, Multiplexing is not possible.
- The lowest possible frequency contained by a signal has to be taken into reference to decide Antenna height.

Note:

- Modulation is used in wired commun' system for multiplexing.

\* FOURIER TRANSFORM: (6) 07/11/2011  
 Fourier Transform is a mathematical tool used to find the frequencies contained by given time domain signal as,

$$x(t) \xrightarrow{\text{F.T.}} X(f)$$

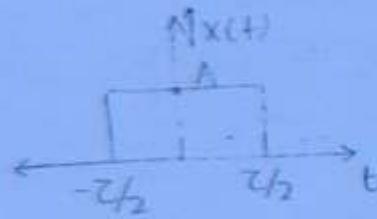
and mathematically,

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

→ Fourier transform is performed by the spectrum Analyser and has 2 modes:

- 1) magnitude plot ( b/w  $|X(f)|$  &  $f$  )
- 2) phase plot ( b/w  $\angle X(f)$  &  $f$  ).

let



$$x(t) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

$$= \int_{-T/2}^{T/2} A e^{-j2\pi ft} dt$$

$$= \frac{A}{j2\pi f} \left[ e^{-j2\pi f t} \right]_{-T/2}^{T/2}$$

$$= \frac{-A}{j2\pi f} \left[ e^{-j2\pi f T/2} - e^{j2\pi f T/2} \right]$$

$$= \frac{-A}{j2\pi f} \left[ e^{-j\pi f T} - e^{j\pi f T} \right]$$

$$= \frac{A}{\pi f} \left[ e^{j\pi f T} - e^{-j\pi f T} \right]$$

$$= \frac{A}{\pi f} \left[ \frac{e^{j\pi f T} - e^{-j\pi f T}}{2j} \right]$$

Note:

1. The Rectangular Pulse covers the frequency from  $+ \infty$  to  $- \infty$  but only the frequency is taken into consideration.

2. Signal B.W is defined as:

$$\boxed{\text{Signal B.W} = \frac{\text{Highest +ve freqn}}{\text{Lowest +ve freqn}}}$$

Now,

In above case,

$$\begin{aligned}\text{Signal B.W} &= +\infty - 0 \\ &= +\infty.\end{aligned}$$

3. Channel Bandwidth is defined as the Range of frequency which the channel allows to pass without any distortion or attenuation.

Generally,

Co-axial cable = 0 to 600 MHz  $\rightarrow$  B.W = 600 MHz

Parallel wire = 0 to 200 MHz  $\rightarrow$  B.W = 200 MHz

Optical Fibre = GHz  $\rightarrow$  B.W = 2n GHz

Also,

$$\boxed{\text{channel } \geq \text{ signal}} \\ \text{B.W} \quad \text{B.W}$$

4. In the above example, the B.W of signal was  $\infty$ . Hence no channel supports its transmission. Hence either has to be performed:

1) channel <sup>to be</sup> designed of B.W =  $\infty \times$

2) signal B.W reduced to some finite value. (Band Limiting)

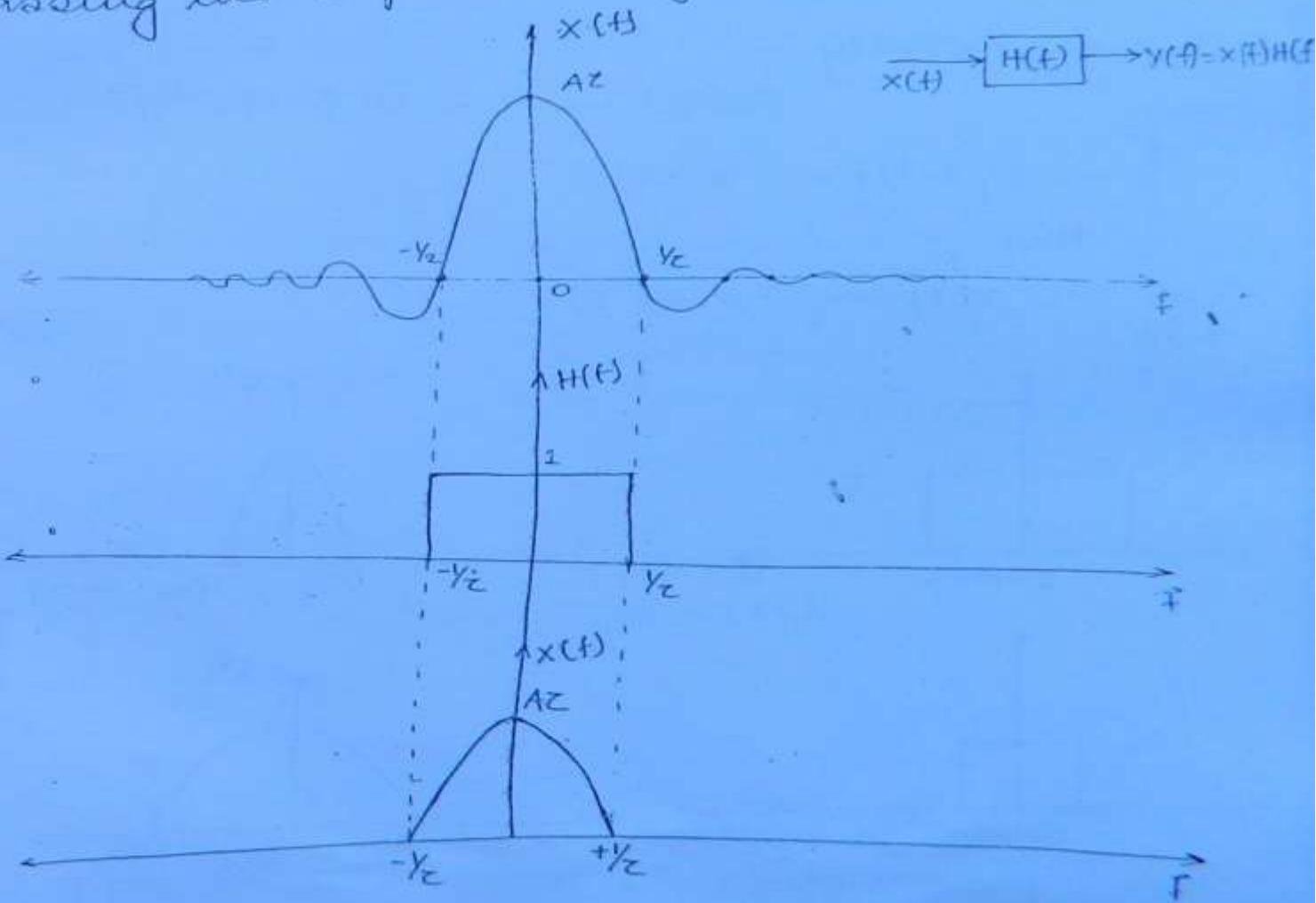
5. For Proper transmission of signal without any loss,

$$\boxed{\text{channel } \geq \text{ signal}} \\ \text{B.W} \quad \text{B.W}$$

Since B.W of above signal is  $\infty$ , a channel having B.W  $\infty$  is needed for transmission, but for practical channel, B.W will be finite. (7)

Note:-

1. The process of decreasing the B.W of a signal from  $\infty$  to some finite value is called as "BAND LIMITING PROCESS".
2. The frequency range from  $-1/\tau_c$  to  $+1/\tau_c$  contains 95 to 99% of the strength of the total signal. Hence we have to eliminate the rest. This is done by passing the signal through a L.P.F.



\*Generally for a signal, most of the strength will be retained by low frequencies, strength of high frequencies go on decreasing and finally becomes zero.

2. For Band Limiting a signal, all the significant frequencies should be retained and insignificant frequencies has to be eliminated. (70)

3. For Band limiting, generally the signal will be passed through proper LPF

NOTE:

To use the channel B.W efficiently, we generally transmit significant frequencies only.

\* Properties of FOURIER TRANSFORM:

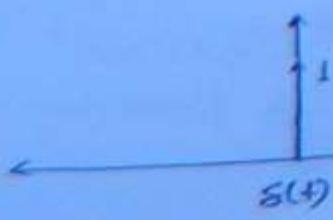
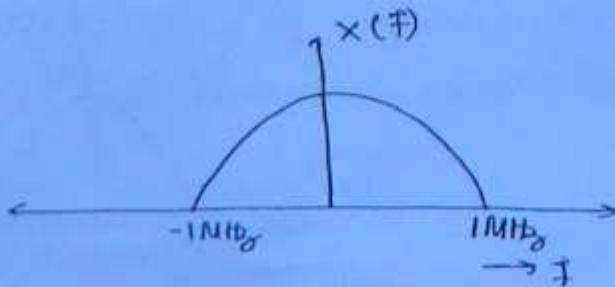
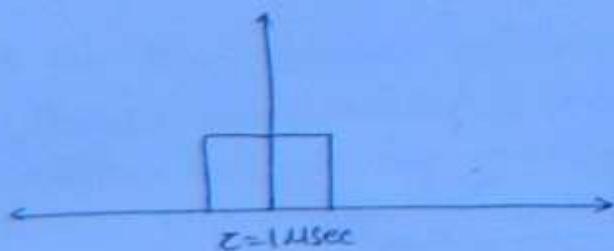
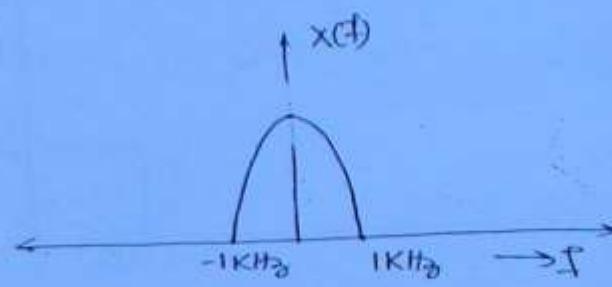
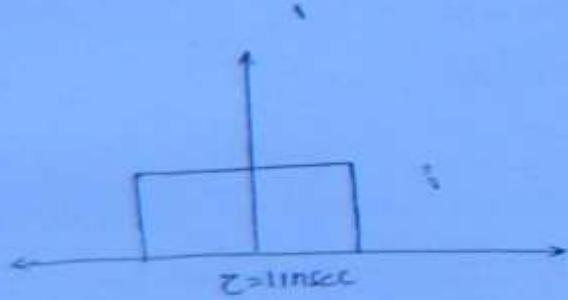
1. Duality Property:-

According to this property:-

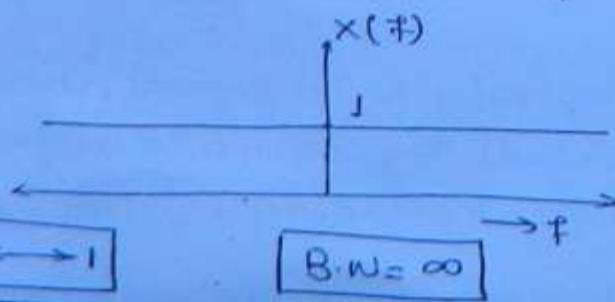
$$\mathcal{F}\{x(t)\} \longleftrightarrow X(f)$$

then,

$$x(t) \longleftrightarrow X(-f)$$



$$s(t) \longleftrightarrow 1$$



Note:

$$1. \left[ \begin{array}{l} \delta(t) = \infty, t=0 \\ = 0; t \neq 0 \end{array} \right] \Rightarrow \left[ \int_{-\infty}^{\infty} \delta(t) dt = 1 \right] \quad (1)$$

$$2. \left[ \begin{array}{l} 2\delta(t) = \infty; t=0 \\ = 0; t \neq 0 \end{array} \right] \Rightarrow \left[ \int_{-\infty}^{\infty} 2\delta(t) dt = 2 \right]$$

Now,

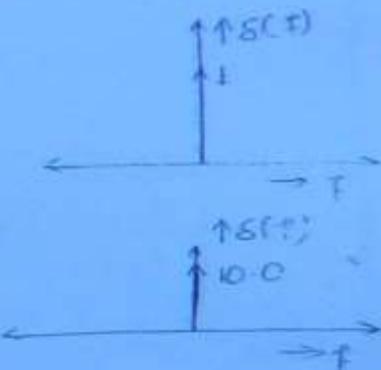
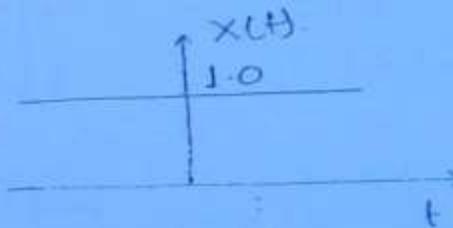
$$\xrightarrow{x(t)} \delta(t) \longleftrightarrow kX(f)$$

by duality property.

$$\xrightarrow{x(t)} 1 \longleftrightarrow \delta(-f) = \delta(f)$$

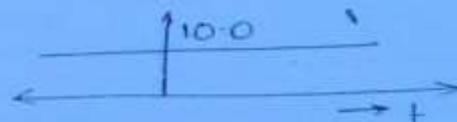
so,

$$1 \longleftrightarrow \delta(f)$$



Also,

$$10 \longleftrightarrow 10\delta(f)$$

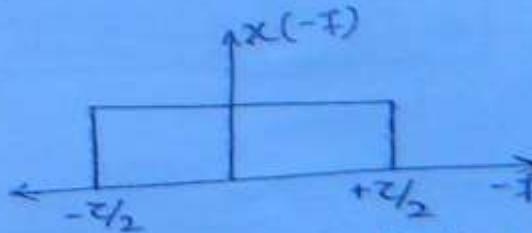


2.

$$\xrightarrow{x(t)} \xrightarrow{F.T} A\pi \sin c(\omega t)$$

$$\text{so, } x(t) \xleftrightarrow{F.T} x(-t).$$

$$\text{so, } A\pi \sin c(\omega t) \longleftrightarrow$$



(It is even function)

(12)

According to this property

$$\text{if } \boxed{x(t) \longleftrightarrow X(f)}$$

then

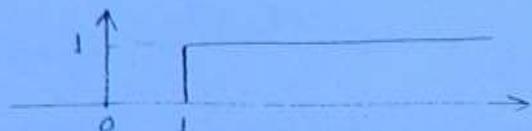
$$\begin{aligned} e^{j2\pi f_0 t} x(t) &\longleftrightarrow X(f - f_0) \\ e^{-j2\pi f_0 t} x(t) &\longleftrightarrow X(f + f_0) \end{aligned}$$

let

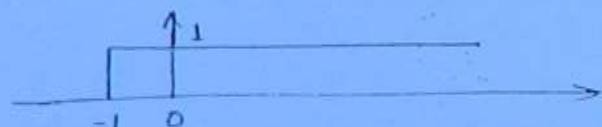
$$\begin{aligned} u(t) &= 1; t \geq 0 \\ &= 0; t < 0 \end{aligned}$$



$$\begin{aligned} u(t-1) &= 1; t \geq 1 \\ &= 0; t < 1 \end{aligned}$$



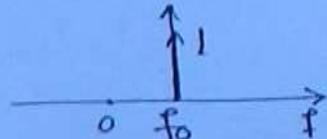
$$\begin{aligned} u(t+1) &= 1; t \geq -1 \\ &= 0; t < -1 \end{aligned}$$

Note:

$$e^{j2\pi f_0 t} \longleftrightarrow ?$$

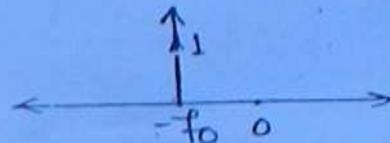
$$\text{As, } \boxed{x(t)} \longleftrightarrow \delta(f) \rightarrow X(f)$$

$$\boxed{e^{j2\pi f_0 t} \cdot 1 \longleftrightarrow \delta(f - f_0)}$$



Also,

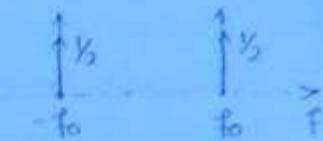
$$\boxed{e^{-j2\pi f_0 t} \longleftrightarrow \delta(f + f_0)}$$



Also,

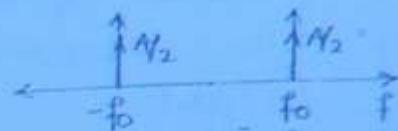
$$\cos 2\pi f_0 t = \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2} \leftrightarrow \frac{1}{2} \{ \delta(f - f_0) + \delta(f + f_0) \}$$

$$1. \text{ (envelope)} \longleftrightarrow \frac{\delta(f-f_0) + \delta(f+f_0)}{2}$$

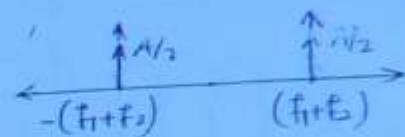


(13)

$$2. A \cos 2\pi f_0 t \longleftrightarrow \frac{A}{2} \left\{ \delta(f-f_0) + \delta(f+f_0) \right\}$$



$$3. A \cos 2\pi (f_1 + f_2)t \longleftrightarrow \frac{A}{2} \left\{ \delta(f-(f_1+f_2)) + \delta(f+(f_1+f_2)) \right\}$$



### 3. Modulation Property:-

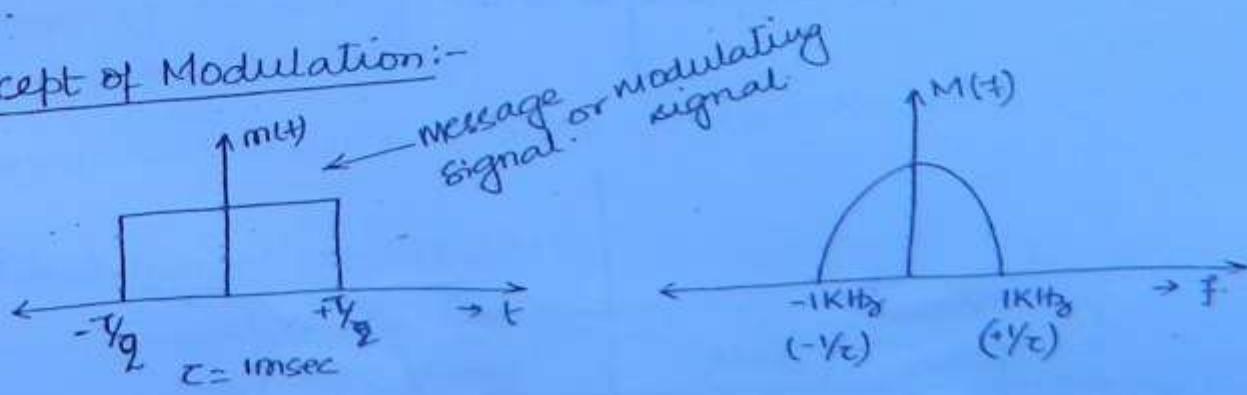
It states that:

If:  $[x(t) \longleftrightarrow X(f)]$

then  $[x(t) \cos 2\pi f_0 t \longleftrightarrow \frac{x(f-f_0) + x(f+f_0)}{2}]$

$$x(t) \left\{ \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2} \right\}$$

### Concept of Modulation:-



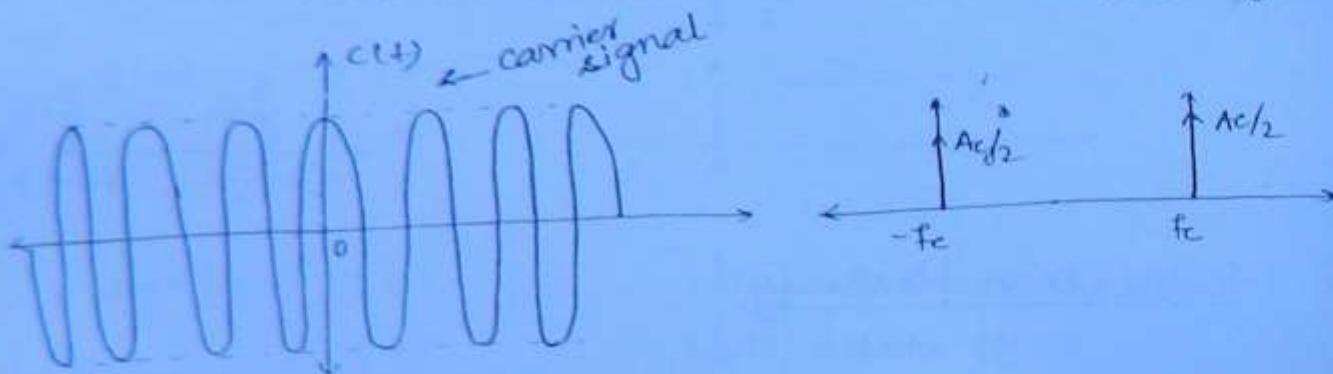
→ If a signal contains all the significant frequency then such signals are called as "BASE BAND SIGNAL".

→ Since Base band signals have significant low frequencies hence they require huge Antenna weights, which is impossible to construct. Hence the process of Modulation is introduced such that the frequency is increased to Reduce Antenna weights.

(14)

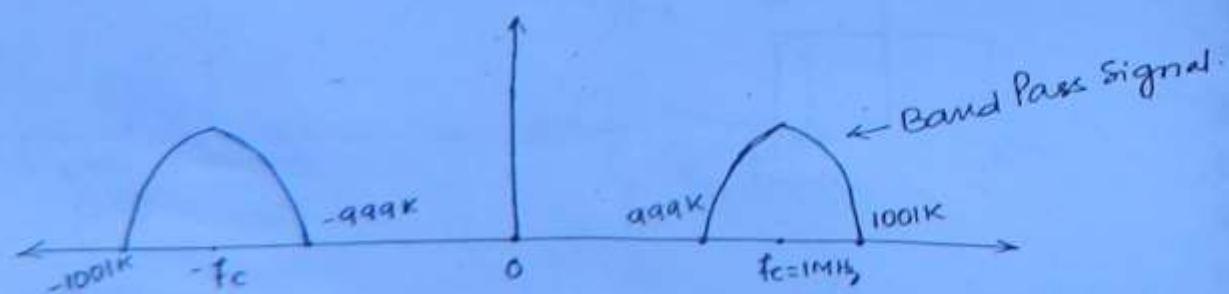
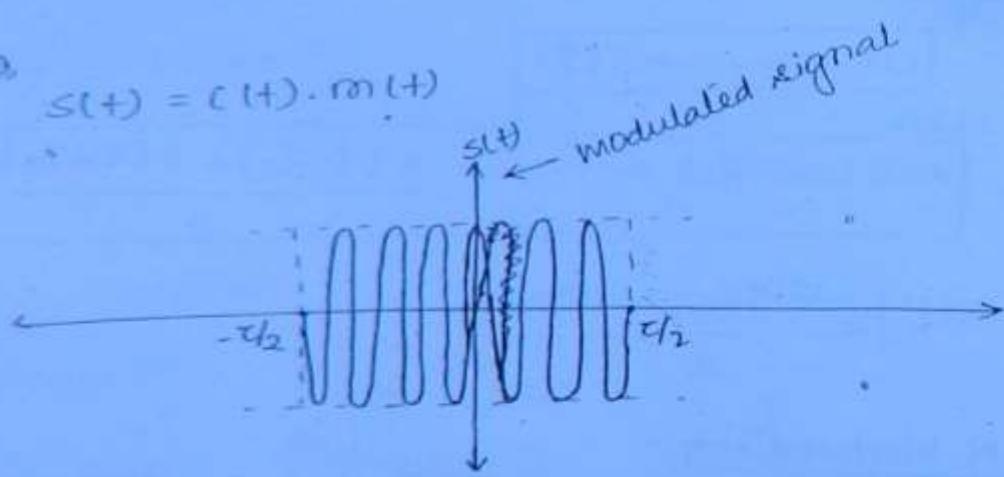
so, we

carrier signal;  $c(t) = A_c \cos(2\pi f_c t)$ ;  $f_c = 1\text{MHz}$   
 $= 1000\text{KHz}$ .



so,

$$s(t) = c(t) \cdot m(t)$$



→ If a signal contain only significant high frequencies then such signal are called BAND PASS SIGNAL.

Note:

A Base Band Signal can't be transmitted faithfully as it requires huge Antenna, but a Band Pass signal can be transmitted faithfully.

(15)

\* By MODULATION:-

- i) the signal is translated from low frequency Region to high frequency Region.
- ii) the Base Band Signal, is converted as Band Pass signal.
- iii) A wide Band Signal, becomes narrow Band signal.

Note:

If:

Highest Frequency (fw) $\ggg$  1  $\leftarrow$  wide Band  
Lowest Frequency (fr)  $\rightarrow$  signal

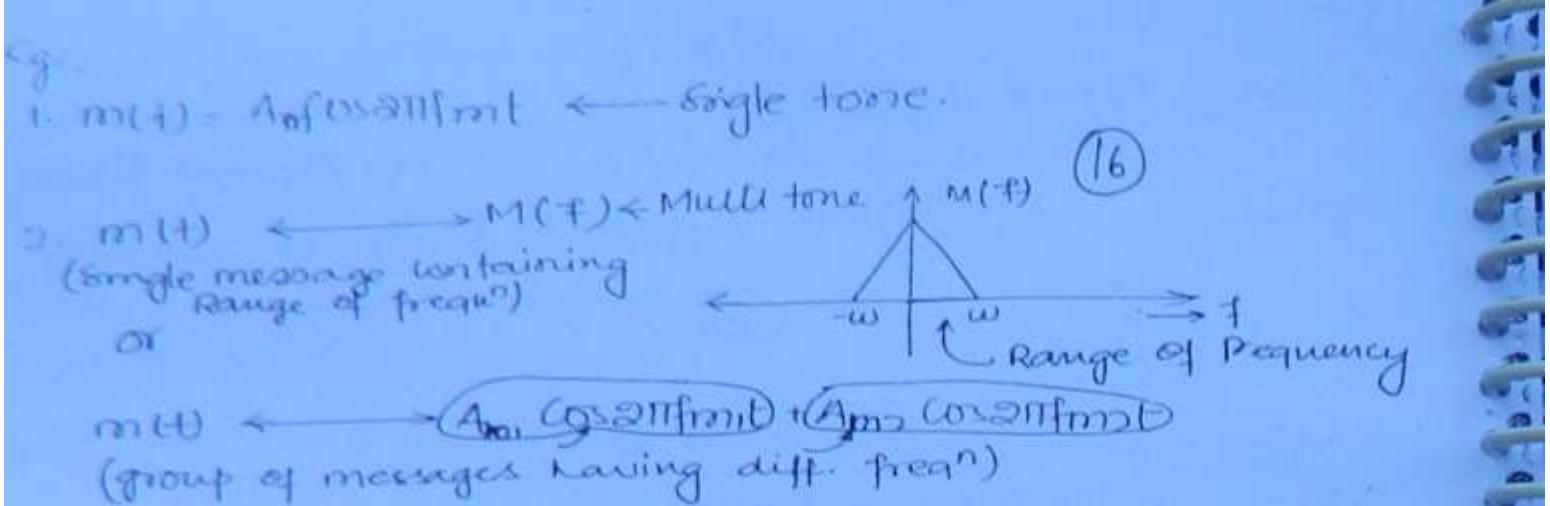
$\frac{\text{Highest frequency}}{\text{Lowest frequency}} \approx 1 \leftarrow \text{Narrow Band}$   
 $\rightarrow$  signal

\* Concept of Demodulation:-

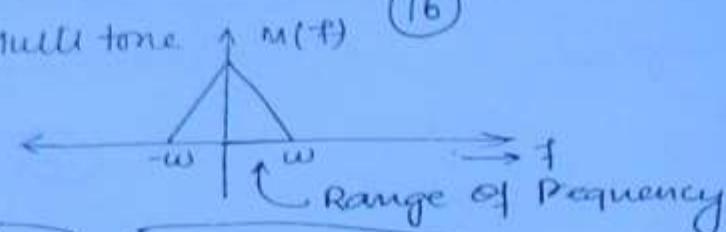
- Process of Receiving back the message signal, from the modulated signal.
- Demodulation will be done at the Receiver.

\* Classification of modulation:



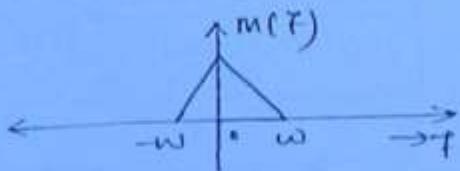


(16)



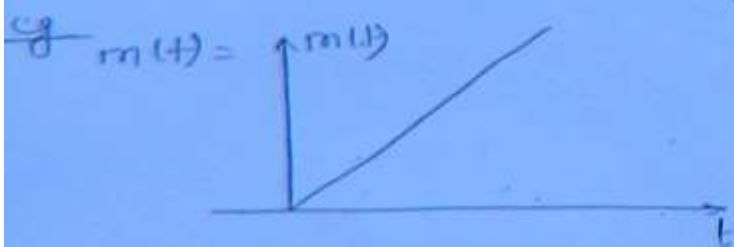
- Sol: basically
- Modulation is of two types:
- i) If the message signal having single freqn then the corresponding frequency is called as **SINGLE TONE MODULATION**.  
 eg  $m(t) = A_m \cos 2\pi f_m t$
  - ii) If the message signal having multiple freqn then the corresponding modulation is called as **MULTI TONE MODULATION**.

eg  $m(t) \longleftrightarrow M(f)$

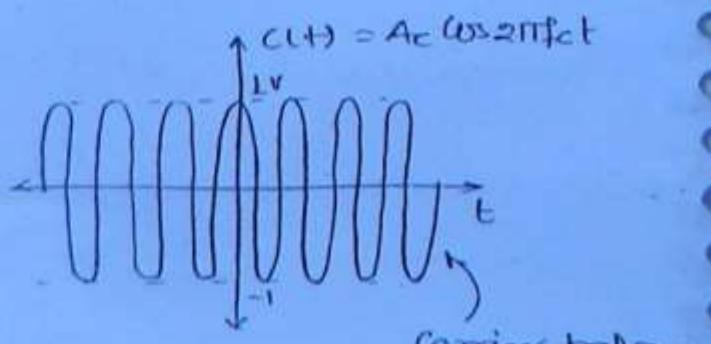
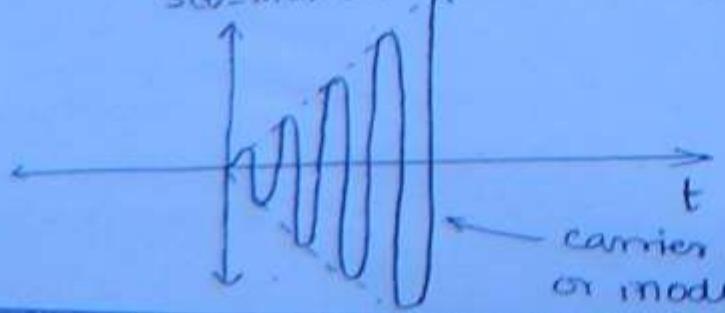


OR

$$m(t) = A_{m1} \cos 2\pi f_{m1} t + A_{m2} \cos 2\pi f_{m2} t$$



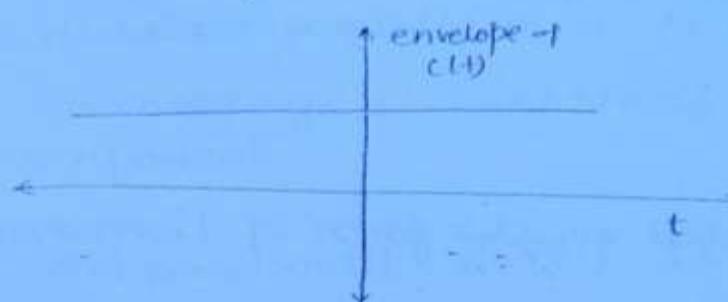
$$s(t) = m(t) \cdot c(t)$$



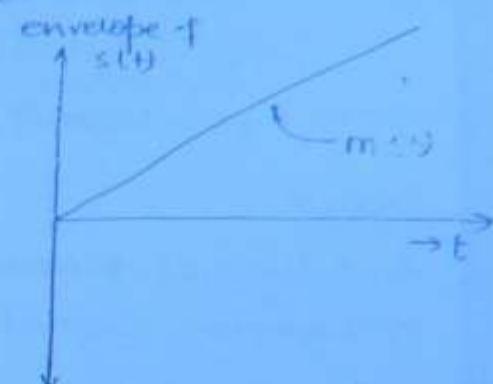
Carrier before  
modulation

carrier after modulation  
or modulated signal.

envelope: A line which touches all the top peak of two signals is called as ENVELOPE.



(17)



Note :-

The above corresponds to Amplitude modulation where the message signal is stored in the form of amplitude variations of carrier after modulation or in the form of envelope of the modulated signal.

### \* MODULATION:-

It is the process in which one of the parameters (Amplitude, frequency or phase) of the carrier signal will be varied linearly in accordance with message signal amplitude variations.

### \* AMPLITUDE MODULATION:-

Defn: It is the process in which Amplitude of the carrier signal will be changed (varied) linearly in accordance with message signal Amplitude Variations.

Assume,

$$m(t) = \text{msg. signal}$$

$$A_c \cos 2\pi f_c t = \text{carrier signal } c(t).$$

### \* General exp of AM signal:-

$$S_{AM}(t) = A_c \{1 + K_A m(t)\} \cos 2\pi f_c t$$

$K_A$  = Amplitude sensitivity of AM modulation.

So,

$$S_{AM}(t) = A_c \cos 2\pi f_c t + A_c K_a M(t) \cos 2\pi f_c t$$

(18)

carrier  
signal

modulated signal

Disadv:

Additional power is wasted in the form of transmission of carrier signal.

Advantage:

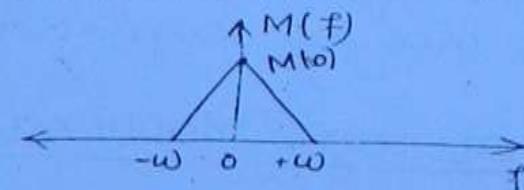
Due to the additional carrier signal, the demodulation of the AM signal becomes easier and cheaper.

Note:-

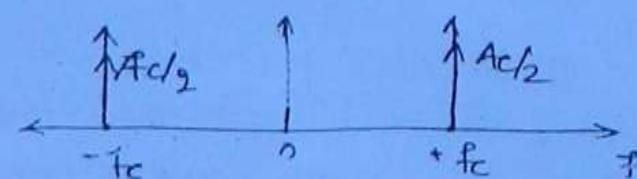
\* AM signal consists of Additional carrier along with the Actual modulated signal.

Because of this additional carrier transmitter power will be wasted but demodulation becomes simple.

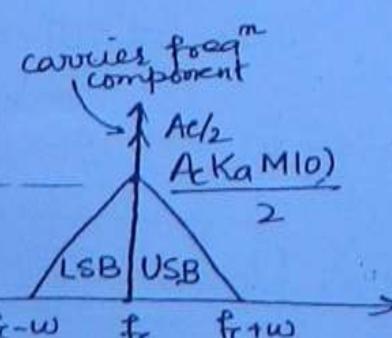
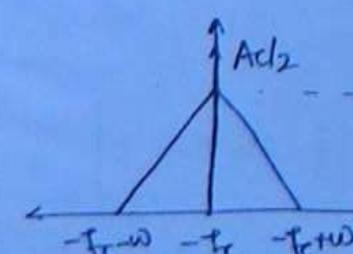
$$m(t) \longleftrightarrow M(f)$$



$$a(t) = A_c \cos 2\pi f_c t \longleftrightarrow$$



$$S_{AM}(t) \longleftrightarrow$$



Note:

The original message signal is contained in the modulated signal ie in the USB & LSB 19

2. No message is contained in the frequency (carrier) component.

$$\begin{aligned} \text{3. AM Bandwidth} &= (f_c + w) - (f_c - w) \\ &= 2w \end{aligned}$$

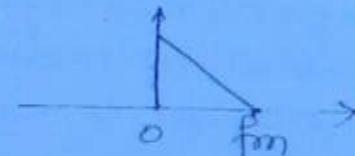
so,

$$\boxed{\text{AM Bandwidth} = 2 \times \text{msg signal B.W}}$$

\* Importance of -ve frequency:

Let only +ve frequency be considered  
Hence,

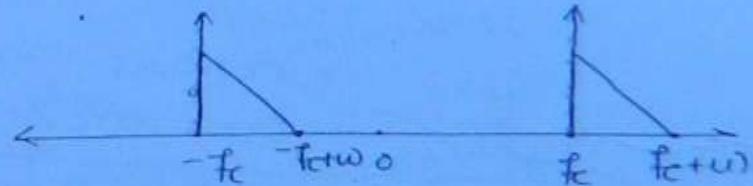
$$m(t) \longleftrightarrow M(f)$$



so,

$$\text{SAM}(t)$$

$$\longleftrightarrow$$



$$\text{B.W} = w$$

originally B.W of AM =  $2w$   
Hence needed the -ve frequency in the discussion

\* AM spectrum consists of:

- 1) Carrier frequency component existing at  $f_c$ .
- 2) USB existing above  $f_c$ .
- 3) LSB existing below  $f_c$ .

Note:

The Actual msg signal will be Retained by only Side Bands

\* The B.W of each of the sideband is equal to  $w$  ie equal to the msg B.W

## \* SINGLE TONE AM:

let,

$$m(t) = A_m \cos 2\pi f_m t$$

f

$$SAM(t) = A_c \{ 1 + K_a m(t) \} \cos 2\pi f_c t$$

(20)

Substituting we get :-

$$SAM(t) = A_c \{ 1 + K_a \cdot A_m \cos 2\pi f_m t \} \cos 2\pi f_c t$$

where,

$$\mu = K_a \cdot A_m = \text{modulation index of AM.}$$

$\mu \times 100\% = \% \text{ of modulation or depth of modulation}$

→ The physical significance of depth of modulation is in the content of message signal that is stored in the carrier signal is called as 'depth of modulation'.

$\mu < 1$ → under modulation	✓ } Generally used.
$\mu = 1$ → critical modulation	
$\mu > 1$ → over modulation	

demodulation of AM signal becomes difficult.

- \* To what extent the carrier signal is modulated by the msg signal is specified by MODULATION INDEX.
- \* over modulation is not preferred because demodulation becomes complex.

so,

$$SAM(t) = A_c \{ 1 + \mu \cos 2\pi f_m t \} \cos 2\pi f_c t \quad \dots \dots (1)$$

on expanding we get :-

$$\text{SAM}(t) = A_c \cos 2\pi f_c t + \frac{A_c \cdot u}{2} \cos 2\pi(f_c + f_m)t + \frac{A_c \cdot u}{2} \cos 2\pi(f_c - f_m)t$$

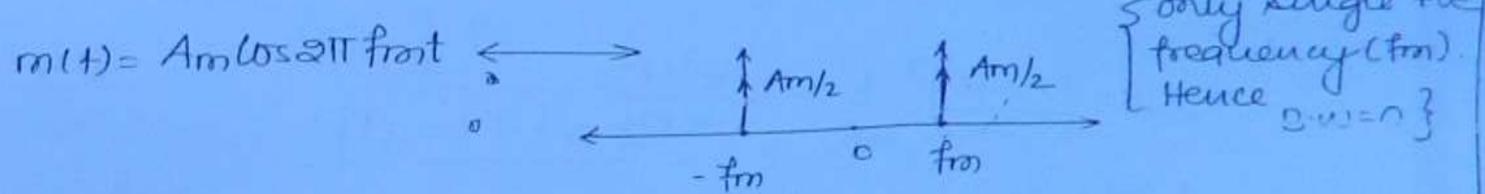
\* Frequency Analysis of Single-tone AM:

(2)

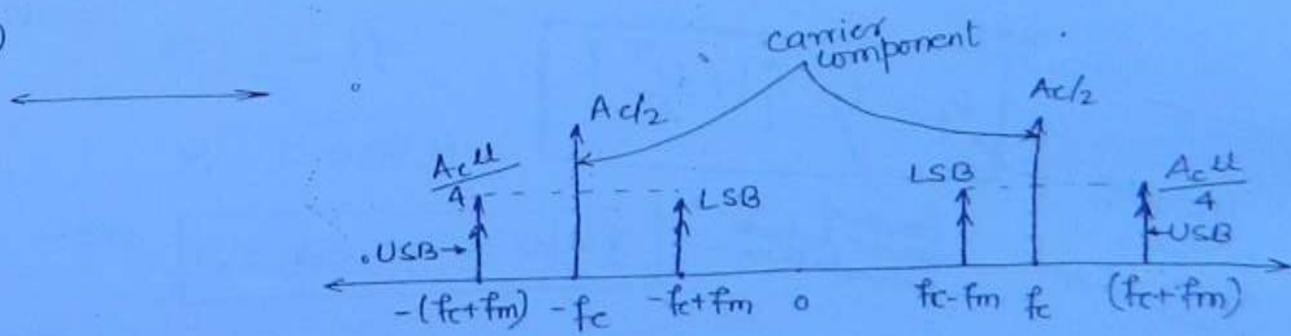
AS,

$$\text{SAM}(t) = A_c \cos 2\pi f_c t + \frac{A_c \cdot u}{2} \cos 2\pi(f_c + f_m)t + \frac{A_c \cdot u}{2} \cos 2\pi(f_c - f_m)t$$

↑  
 carrier component  
 USB  
 LSB.



SAM(t)



So,

$$\text{AM Bandwidth} = (f_c + f_m) - (f_c - f_m) = 2f_m$$

So,  $f_m$  = frequency of msg signal.

So,

$$\text{AM Bandwidth} = 2 \times \text{frequency of msg signal}$$

## \* POWER OF AM SIGNAL:-

(22)

The total power of AM signal is given as:

$$P_t = P_c + P_{USB} + P_{LSB}$$

NOW,

$$P_c = \left(\frac{A_c}{\sqrt{2}}\right)^2 \cdot R = \frac{A_c^2}{2R}$$

$$P_{USB} = \left(\frac{A_c U}{2}\right)^2 \cdot \frac{1}{2R} = \frac{A_c^2 U^2}{8R}$$

$$P_{LSB} = \left(\frac{A_c U}{2}\right)^2 \cdot \frac{1}{2R} = \frac{A_c^2 U^2}{8R}$$

$$\begin{aligned} x(1+) &= V_m & x(1-) &= V_m \cos \omega t \\ \therefore P_{AC} &= \frac{V_m^2}{R} & P_{AC} &= \frac{V_{rms}^2}{R} \\ V_{rms} &= V_m / \sqrt{2}, & P_{AC} &= \frac{V_m^2}{2R} \end{aligned}$$

$$\text{So, } P_t = \frac{A_c^2}{2R} + \frac{A_c^2 U^2}{4R}$$

$$P_t = \frac{A_c^2}{2R} \left\{ 1 + \frac{U^2}{2} \right\}$$

So, \*\*\*

$$P_t = P_c \left\{ 1 + \frac{U^2}{2} \right\}$$

Power of carrier after modulation

Power of carrier before modulation.

NOW,

$$P_t = P_c + \frac{P_c U^2}{2} = P_c + P_{SB}$$

where

$$P_{SB} = \frac{P_c U^2}{2}$$

$$P_{USB} = P_{LSB} = \frac{P_c U^2}{4}$$

\* The Power of carrier is independent of  $U$ .

\*\* The Side Band power depends on  $U$ , and as the  $U$  is increased, the  $P_{SB}$  also increases (Power of the modulated signal & total power of AM signal increases).

$\omega_c = \omega_b$ ; no modulation.

Hence,

$$\boxed{P_t = P_c}$$

(23)

$$\text{as, } SAM(t) = A_c \cos \omega_f t$$

Case 2:

$u=1$ ; total modulation  
(100% modulation)

Hence,

$$P_t = \frac{3}{2} P_c$$

$$\boxed{P_t = 1.5 P_c}$$

$$\text{And, } SAM(t) = A_c \cos 2\pi f_e t + A_c \cos 3\omega_f m t \cos 2\pi f_e t$$

Note: As  $u$  increases from 0 to 1, total AM power is increased by 50%.

Now,

$$P_c = \frac{2}{3} P_t$$

$$P_c = 0.666 P_t$$

$$\text{so, } \boxed{P_c = 66.66\% \text{ of } P_t}$$

Now, as,

$$P_t = P_c + P_{SB}$$

$$P_t = \frac{2}{3} P_t + P_{SB}$$

$$P_{SB} = \frac{1}{3} P_t$$

$$\boxed{P_{SB} = 33.33\% \text{ of } P_t}$$

## CONCLUSION:

1. If  $u=0 \Rightarrow P_c = 100\% \text{ of } Pt \Rightarrow P_{SB} = 0\% \text{ of } Pt$  (24)
2. If  $u=1 \Rightarrow P_c = 66.66\% \text{ of } Pt \Rightarrow P_{SB} = 33.33\% \text{ of } Pt$ .

Note:

In the efficient power distribution case, ie  $u=1$  still  $66.66\% \text{ of } Pt$ ; is wasted in the form of transmission of additional carrier. This is the biggest draw-back of AM.

\* Modulation efficiency ( $\eta$ ):

\* It specifies share of sideband power in total power

Eg  $\eta = 0.1 \rightarrow 10\% \text{ of } P_{SB} \text{ in } Pt$ .

$\eta = 0.3 \rightarrow 30\% \text{ of } P_{SB} \text{ in } Pt$

Mathematically

$$\boxed{\eta = \frac{P_{SB}}{Pt}}$$

so,

$$\eta = \frac{P_c u^2}{P_c \left\{ 1 + \frac{u^2}{2} \right\}}$$

\*\*\*

$$\boxed{\eta = \frac{u^2}{2+u^2}}$$

Case 1:

$$\underline{u=0}$$

for  $u=0$ ;  $\eta=0 \Rightarrow P_{SB} = 0\% \text{ of } Pt$

$$P_c \cancel{P_{RMS}} = 100\% \text{ of } Pt$$

$$\eta = 0.11 \Rightarrow P_{SB} = 11\% \text{ of } P_t$$

$$P_c = 89\% \text{ of } P_t$$

(25)

Case 3: ( $u = 0.707$ )

$$\eta = 0.2 \Rightarrow P_{SB} = 20\% \text{ of } P_t$$

$$P_c = 80\% \text{ of } P_t$$

Case 4: ( $u = 1$ )

$$\eta = 0.33 \Rightarrow P_{SB} = 33.3\% \text{ of } P_t$$

$$P_c = 66.7\% \text{ of } P_t$$

Note:

Let  $P_t = P_c + P_{SB}$

$$50W = 30W + 20W$$

$$60\% \quad 40\%$$

$u \uparrow$

$$60W = 30W + 30W$$

$$50\% \quad 50\%$$

\*  $P_c$  is independent of  $u$  and share of carrier power in total power decreases as  $u$  increases.

\* An unmodulated AM transmitted power is given as 500W. Find AM transmitted power with 100% modulation?

Soln: Given  $P_t = 500W$ ,  $u = 0 \Rightarrow P_c = 500W$  &  $P_{SB} = 0$

~~$$P_c = \frac{3}{2} P_t$$~~

$$u = 1 ; P_t = \cancel{P_c + P_{SB}} = 500W$$

~~$$P_c = \frac{3}{2} \times 500$$~~

$$= 750 \text{ Watt.}$$

$$P_t = P_c \left\{ 1 + \frac{u^2}{2} \right\}$$

$$P_t = 500 \left\{ 1 + \frac{1}{2} \right\} = 750W$$

Note:

Q1. For an AM signal total sideband power is given by 100W with 50% of modulation. Find total AM transmitted power.

(26)

Soln:  $P_{SB} = 100W ; u = \frac{1}{2}$

$$P_t = ?$$

$$P_{SB} = \frac{P_c u^2}{2}$$

$$P_c = \frac{P_{SB} \cdot 2}{u^2} = 100 \times 2 \times 4$$

$$P_c = 800W$$

$$\text{So, } P_t = P_c + P_{SB}$$

$$\boxed{P_t = 900W}$$

Q2. For an AM each of the S.B power is given by 2KW and carrier power is given by 8KW. Find % of modulation.

Soln:  $P_c = 8KW$   
 $P_{SB} = 4KW \quad \{ P_{SB} = P_{U.S.B} + P_{L.S.B} = 2+2 \}$

So,

$$P_{SB} = \frac{P_c u^2}{2}$$

$$u^2 = \frac{P_{SB} \times 2}{P_c}$$

$$u^2 = \frac{4 \times 2}{8}$$

$$\boxed{u = 1} \text{ Ans}$$

100% Modulation

Q3. A carrier of 10V amplitude is amplitude modulated by a message signal of  $4\cos 4\pi \times 10^3 t$  with 50% of modulation.

Antenna Resistance is given by  $5\Omega$ .

1. Find all the parameters of AM

(Q3)

2. Plot AM spectrum and identify the spectral components.

Sol:-

Given:-

$$c_w(t) = 10 \cos 2\pi \times 10^6 t = A_c \cos 2\pi f_c t$$

$$m(t) = 4 \cos 4\pi \times 10^3 t = A_m \cos 2\pi f_m t$$

so,  $A_m = 4 V$ ,  $f_m = 2 \times 10^3 = 2 \text{ kHz}$

$$A_c = 10 V ; f_c = 10^6 = 1 \text{ MHz}$$

so, for single tone modulation

$$B.W = 2f_m$$

$$B.W = 2 \times 2 \text{ K}$$

$$\boxed{B.W = 4 \text{ kHz}} \quad \text{Ans}$$

And,  $u = 0.5$ ;  $R = 5 \Omega$

so,  $\boxed{P_c = \frac{A_c^2}{2R} = \frac{10^2}{2 \times 5} = 10 \text{ W}} \quad \text{Ans}$

$$P_t = P_c \left\{ 1 + \frac{u^2}{2} \right\} = 10 \left\{ 1 + \frac{0.5^2}{2} \right\}$$

$$\boxed{P_t = 11.25 \text{ W}} \quad \text{Ans}$$

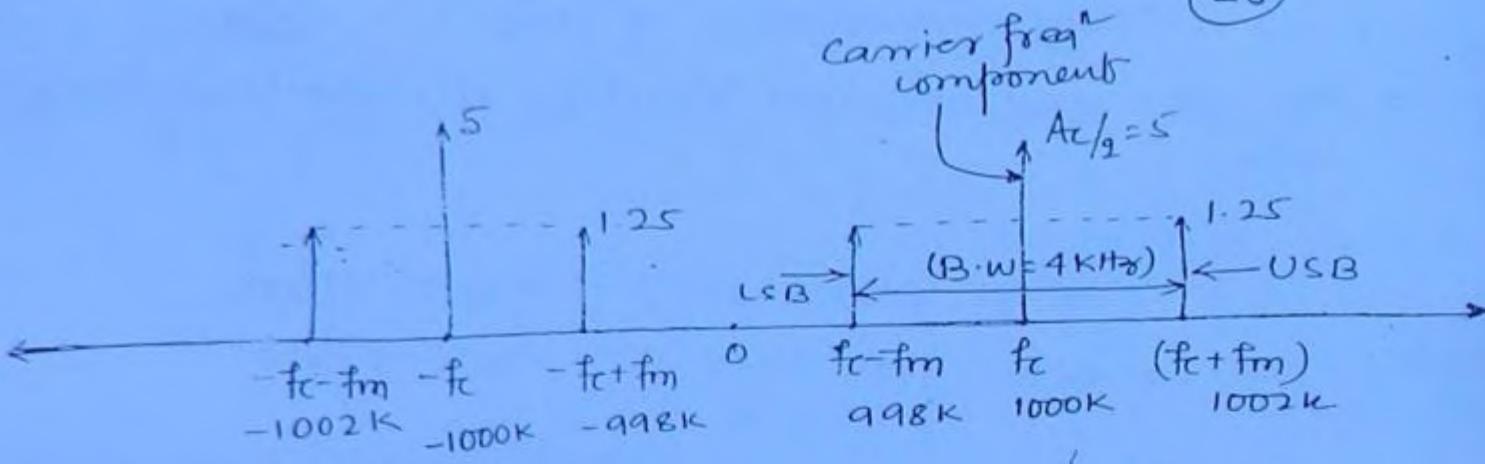
so,  $\boxed{P_{SB} = P_t - P_c = 1.25 \text{ W}} \quad \text{Ans}$

$$\boxed{P_{SSB} = P_{LSB} = P_{SB}/2 = 0.625 \text{ W}} \quad \text{Ans}$$

And,

$$\boxed{\eta = \frac{P_{SB}}{P_t} = \frac{u^2}{2+u^2} = 0.11 = 11\%} \quad \text{Ans}$$

(28)



- Q4. A carrier of  $10\cos 8\pi \times 10^5 t$  is amplitude modulated by a msg signal of  $6\cos \pi \times 10^4 t$ . i) Find all the parameters of AM.  
ii) Plot sideband components & spectrum.

Soln: Given:-

$$c(t) = 10 \cos 8\pi \times 10^5 t = Ac \cos \omega_c t$$

$$Ac = 10; \quad \omega_c = 4 \times 10^5 = 400 \text{ kHz}$$

$$m(t) = 6 \cos \pi \times 10^4 t = Am \cos 2\pi f_m t$$

$$Am = 6; \quad f_m = 5 \text{ kHz}$$

NOW,  
As  $M$  is not given. So, take :-

$$M = \frac{Am}{Ac} = \frac{6}{10} = 0.6$$

NOW,  $P_C = \frac{P_t}{2} \cdot \frac{Ac^2}{2R} = \frac{10^2}{2 \times 1} \quad \left\{ \begin{array}{l} \text{As Antenna Resistance is } \\ \text{not given. Hence } R=1 \end{array} \right.$

$$P_C = 50 \text{ W.}$$

$$P_t = P_C \left\{ 1 + \frac{M^2}{2} \right\}$$

$$P_t = 50 \left\{ 1 + \frac{0.6^2}{2} \right\}$$

$$P_t = 59 \text{ W}$$

AM Band width  $B.W = \omega_{fm}$

$$AM B.W = 2 \times 5K$$

(29)

$$AM B.W = 10K \text{ Ans}$$

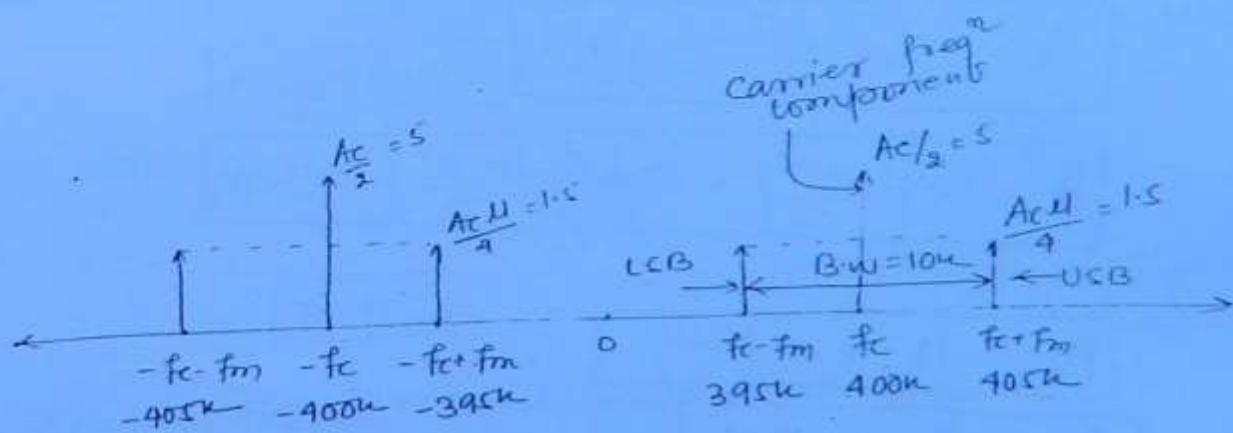
Now,  $P_{SB} = P_t - P_c = 9W \text{ Ans}$

$$P_{USB} = P_{LSB} = 4.5W \text{ Ans}$$

And,  $m = \frac{U^2}{2 + U^2} = \frac{0.6^2}{2 + 0.6^2}$

$$m = 0.15 \text{ Ans}$$

spectrum :



Q5. An AM signal is given by

$$s(t) = 4 \cos 3200\pi t + 10 \cos 4000\pi t + 4 \cos 4800\pi t$$

Find all the parameters of AM and plot the spectrum.

Soln:- As,

$$SAM(t) = A_c \cos 2\pi f_c t + \frac{A_c U}{2} \cos 2\pi(f_c + f_m)t + \frac{A_c U}{2} \cos 2\pi(f_c - f_m)t$$

Comparing we get:-

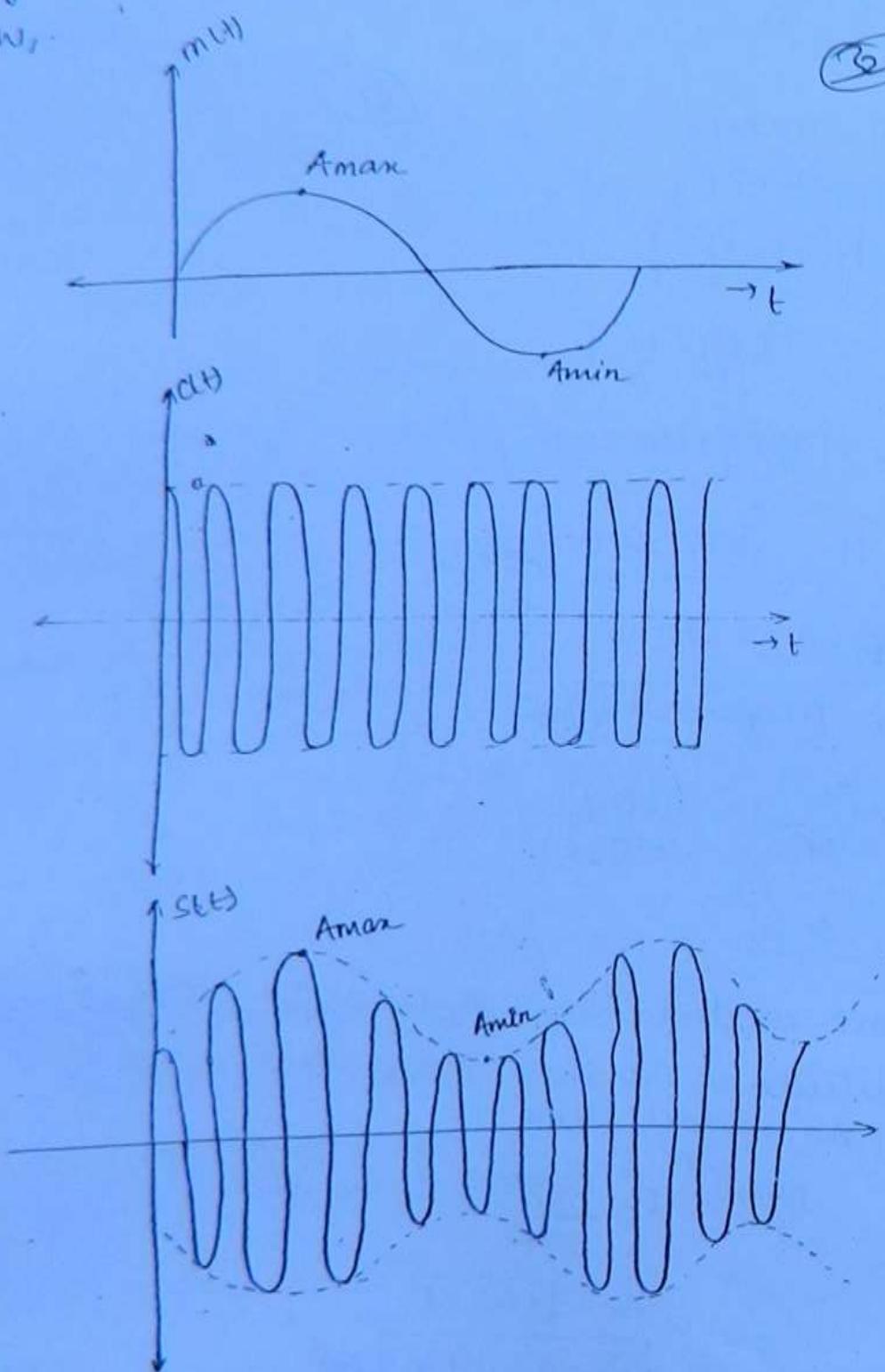
$$SAM(t) = 10 \cos 4000\pi t + 4 \cos 4800\pi t + 4 \cos 3200\pi t$$

{As the value of magnitude remain same for USB & LSB}

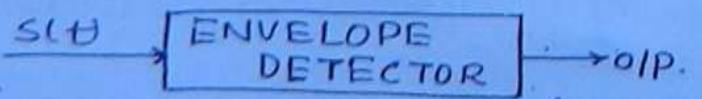
→ The peak amplitude of the carrier after modulation is not const but it varies in accordance to the msg signal.

NOW,

(3)



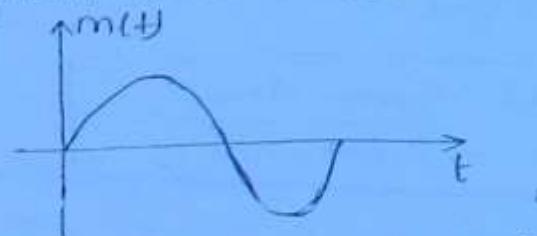
Note :- If the above signal is fed to the input of the envelope detector for its demodulation as in the fig. shown below:-



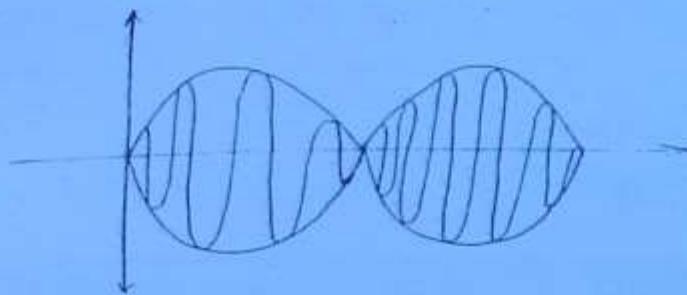
The OIP of the envelope curve is as shown below:



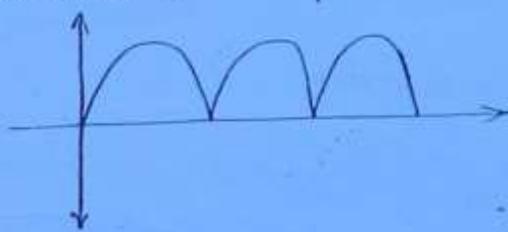
The above signal when shifted by some DC value, the waveform obtained resembles to that of the original signal. The waveform is as shown below:-



Now, if the wave of the AM Transmitted signal is as follows:



And if this above signal is passed through the ED the OIP obtained is as follows:-



← This signal doesn't correspond to the original message signal and by any means it cannot be converted to the original message signal. Hence the above waveform is not the AM transmitted wave.

Note:-

$$\text{Max}^m \text{ peak of AM signal, } A_{\max} = A_c \{1+u\}$$

$\therefore$  max<sup>m</sup> value of the cos $\omega_0 t$  = 1  
 $-1 \leq \cos\omega_0 t \leq 1$

$$\text{Min}^m \text{ peak of AM signal; } A_{\min} = A_c \{1-u\}$$

$$\text{So, } A_{\max} = A_c(1+u) = 12.64(1+0.707) = 21.5 \text{ Volts}$$

$$A_{\min} = A_c(1-u) = 12.64(1-0.707) = 3.7 \text{ Volts}$$

Also,

$$A_{max} + A_{min} = 2A_c$$

So,

$$A_c = \frac{A_{max} + A_{min}}{2}$$

(By)

substituting the value of  $A_c$  in above eq<sup>n</sup> of  $A_{max}$  &  $A_{min}$  we get:

$$M = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

If the peak voltage of AM signal varies from 2 volts to 10V  
Find  $M$ ,  $P_t$  &  $\eta$ ?

Soln: Given:

$$A_{max} = 10 \text{ Volts}$$

$$A_{min} = 2 \text{ Volts}$$

So,

$$M = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{10 - 2}{10 + 2} = \frac{8}{12} = 0.66$$

Ans.

$$A_c = \frac{A_{max} + A_{min}}{2} = \frac{2+10}{2} = 6 \text{ Volts}$$

$$P_c = \frac{A_c^2}{2R} = \frac{6^2}{2 \times 1} = 18 \text{ W}$$

$$P_t = P_c \left\{ 1 + \frac{M^2}{2} \right\}$$

$$P_t = 18 \left\{ 1 + \frac{0.66^2}{2} \right\} = 22 \text{ W}$$

Ans.

$$\eta = \frac{M^2}{2 + M^2} = \frac{0.66^2}{2 + 0.66^2}$$

$$\eta = 18.1 \quad \text{Ans.}$$

## MULTI-TONE AMPLITUDE MODULATION.

Assume,

$$m(t) = A_{m1} \cos 2\pi f_{m1} t + A_{m2} \cos 2\pi f_{m2} t$$

and,

$$SAM(t) = A_c \{ 1 + K_m(t) \} \cos 2\pi f_c t \quad (35)$$

So,

$$SAM(t) = A_c \{ 1 + K_m A_{m1} \cos 2\pi f_{m1} t + K_m A_{m2} \cos 2\pi f_{m2} t \} \cos 2\pi f_c t$$

let,

$$K_m A_{m1} = u_1$$

$$\therefore K_m A_{m2} = u_2$$

so,

$$SAM(t) = A_c \{ 1 + u_1 \cos 2\pi f_{m1} t + u_2 \cos 2\pi f_{m2} t \} \cos 2\pi f_c t$$

on expanding we get:

$$SAM(t) = A_c \cos 2\pi f_c t + A_c u_1 \cos 2\pi f_{m1} t \cos 2\pi f_c t + A_c u_2 \cos 2\pi f_{m2} t \cos 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t + \frac{A_c u_1}{2} \cos 2\pi (f_c + f_{m1}) t + \frac{A_c u_1}{2} \cos 2\pi (f_c - f_{m1}) t$$

carrier frequency component. +  $\frac{A_c u_2}{2} \cos 2\pi (f_c + f_{m2}) t + \frac{A_c u_2}{2} \cos 2\pi (f_c - f_{m2}) t$

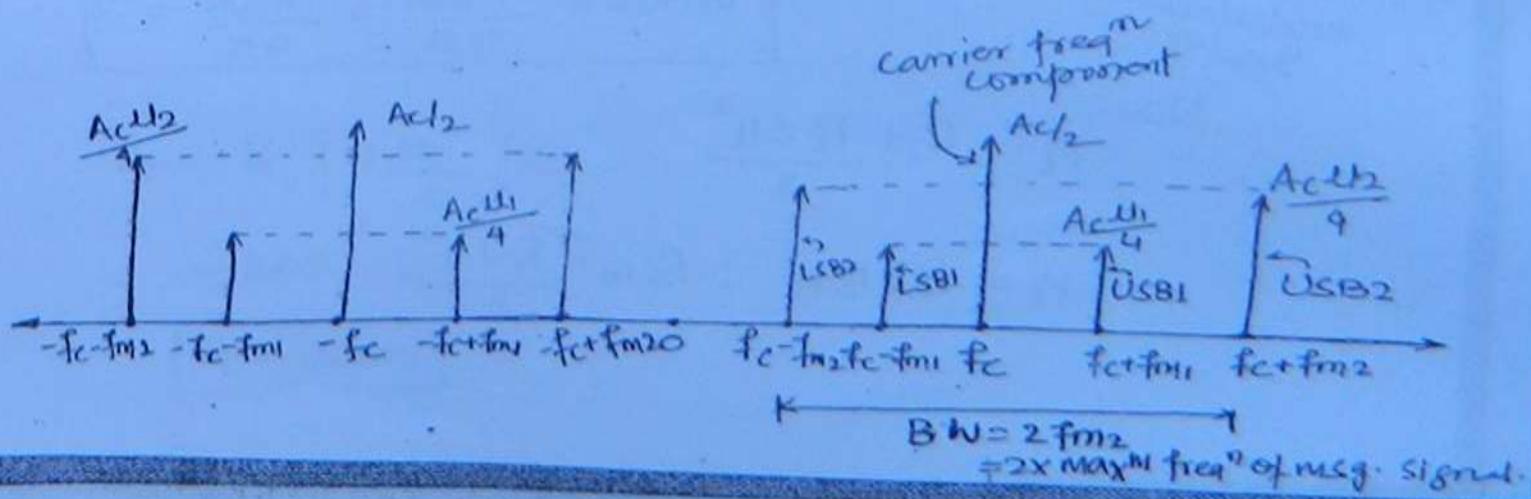
USB1. USB2. LSB1. LSB2.

Spectrum: For plotting spectrum let us assume

$$f_{m2} > f_{m1}$$

$$\therefore u_2 > u_1$$

so,



## \* TOTAL POWER OF AM (MULTI TONE)

The total power ( $P_t$ ) is given as:

$$P_t = P_c + P_{SB}$$

(36)

$$P_t = P_c + P_{USB\text{ (total)}} + P_{LSB\text{ (total)}}$$

$$P_t = P_c + P_{USB1} + P_{USB2} + P_{LSB1} + P_{LSB2}$$

NOW,

$$P_c = \frac{A_c^2}{2R} ; P_{USB1} = \frac{(A_c U_1)^2}{2R} = \frac{A_c^2 U_1^2}{8R} = P_{LSB1}$$

$$P_{USB2} = \frac{(A_c U_2)^2}{2R} = \frac{A_c^2 U_2^2}{8R} = P_{LSB2}$$

So,

$$P_t = \frac{A_c^2}{2R} + \frac{A_c^2 U_1^2}{4R} + \frac{A_c^2 U_2^2}{4R}$$

$$P_t = \frac{A_c^2}{2R} \left\{ 1 + \frac{U_1^2 + U_2^2}{2} \right\}$$

So,

$$P_t = P_c \left\{ 1 + \frac{M_t^2}{2} \right\}$$

where,

$$M_t = \sqrt{U_1^2 + U_2^2}$$

Total modulation index.

Now,

$$P_t = P_c + \frac{P_c M_t^2}{2}$$

$$P_t = P_c + P_{SB} ; P_{SB} = \frac{P_c M_t^2}{2}$$

Now,

$$\eta = \frac{P_c \cdot P_e}{P_t}$$

$$\eta = \frac{\frac{P_c U_t^2}{2}}{\frac{P_c \left\{ 1 + \frac{U_t^2}{2} \right\}}{}}$$

(37)

So, \*\*\*

$$\boxed{\eta = \frac{U_t^2}{2 + U_t^2}}$$

Q1. A carrier of  $20 \cos 4\pi \times 10^6 t$  is amp. modulated by a msg signal of having frequencies 10KHz, 20KHz with modulation indexes 0.6 & 0.8 resp. find all the parameters and plot the spectrum?

Sol<sup>n</sup>: Given:

$$A_c \cos 2\pi f_c t = 20 \cos 4\pi \times 10^6 t$$

$$\text{so, } A_c = 20, \quad f_c = 2000 \text{ KHz}$$

$$f_{m1} = 10 \text{ KHz}, \quad f_{m2} = 20 \text{ KHz}$$

$$U_1 = 0.6, \quad U_2 = 0.8$$

$$\text{so, } U_t = \sqrt{U_1^2 + U_2^2} = \sqrt{0.36 + 0.64} = 1$$

Now,

$$B.W = 2f_{\max}$$

$$= 2 \times 20 \text{ K}$$

$$\boxed{B.W = 40 \text{ KHz}}$$

And

$$\boxed{P_c = \frac{A_c^2}{2R} = \frac{400}{2 \times 1} = 200 \text{ W}}$$

$$P_t = P_c \left\{ 1 + \frac{U_t^2}{2} \right\}$$

$$= 200 \times \frac{3}{2}$$

$$\boxed{P_t = 300 \text{ W}}$$

$$P_{SB} = P_t - P_c$$

$$= 100 \text{ W}$$

$$P_{SCB} = P_{LB} = 50 \text{ W}$$

(38)

$$P_{SCB1} = \frac{A_c U_1^2}{8R} = 18 \text{ W} = P_{LB1}$$

$$P_{SCB2} = 32 \text{ W} = P_{LB2}$$

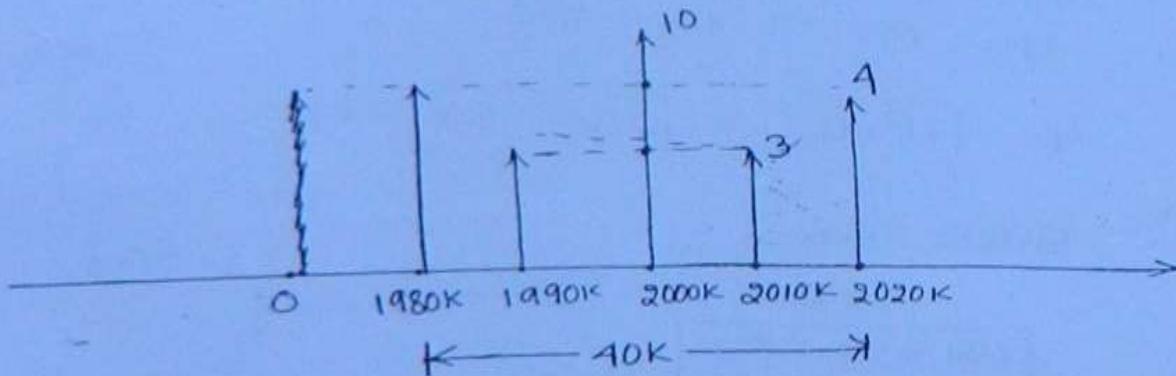
And

$$\eta = \frac{U_t^2}{2 + eU_t^2}$$

$$\eta = \frac{1}{1+2}$$

$$\eta = 0.33 = 33.3\%$$

Spectrum



Q2. An AM signal is given by

$$s(t) = \{20 + 4 \cos 8\pi \times 10^4 t + 8 \cos \pi \times 10^5 t\} \cos \omega c t$$

Find  $U_t$ ,  $P_t$ ,  $B.W$  &  $\eta$ ?

Ans:- As,  $s(t) = 20 \{1 + 4/20 \cos 8\pi \times 10^4 t + 8/20 \cos \pi \times 10^5 t\} \cos \omega c t$

$$SAM(t) = A_c \{1 + u_1 \cos 2\pi f_{m1} t + u_2 \cos 2\pi f_{m2} t\} \cos 2\pi f_c t$$

so,

$$A_c = 20 \text{ V}, \quad u_1 = 4/20, \quad u_2 = 8/20$$

$$f_{m1} = 40 \text{ KHz}; \quad f_{m2} = 50 \text{ KHz}$$

$$\text{So, } A_C U_1 - \frac{1}{2} U_2 = 0.2 ; U_2 = 0.4$$

~~142.01000~~

$$B.W = 2 f_{max}$$

$$\boxed{B.W = 100K} \text{ Ans}$$

(39)

$$\boxed{U_t = \sqrt{0.2^2 + 0.4^2} = 0.44} \text{ Ans}$$

$$\boxed{P_c = \frac{A_c^2}{2R} = \frac{400}{2 \times 1} = 200W} \text{ Ans}$$

$$\begin{aligned} P_t &= P_c \left\{ 1 + \frac{U_t^2}{2} \right\} \\ &= 200 \left\{ 1 + \frac{0.44^2}{2} \right\} \end{aligned}$$

$$\boxed{P_t = 220W} \text{ Ans}$$

$$\text{And, } \eta = \frac{U_t^2}{2 + U_t^2} = \frac{0.44^2}{2 + 0.44^2}$$

$$\boxed{\eta = 0.09 = 9\%} \text{ Ans}$$

## CURRENT RELATIONS IN AM:-

As,

$$P_t = P_c \left\{ 1 + \frac{U_t^2}{2} \right\}.$$

$$I_t^2 R = I_c^2 R \left\{ 1 + \frac{U_t^2}{2} \right\}$$

PSU's  
\*\*\*

$$\boxed{I_t^2 = I_c^2 \left\{ 1 + \frac{U_t^2}{2} \right\}}$$

PSU's  
So, \*\*\*

$$\boxed{I_t = I_c \sqrt{1 + \frac{U_t^2}{2}}}$$

$I_t$  = total Antenna Current.  
 $I_c$  = Carrier current.

$\rightarrow I_c$  is independent of "u"

## VOLTAGE RELATIONS IN AM

As.

$$P_t = P_c \left\{ 1 + \frac{u^2}{2} \right\}$$

(40)

$$\frac{V_t^2}{R} = \frac{V_c^2}{R} \left\{ 1 + \frac{u^2}{2} \right\}$$

$$V_t = V_c \sqrt{1 + \frac{u^2}{2}}$$

- Q1. An unmodulated AM transmitter current is given by 5A  
Find AM transmitter current with 50% of modulation.

Soln: Given:

$$u = 0.5$$

$$I_c = 5A$$

So,

$$I_t = I_c \sqrt{1 + \frac{u^2}{2}}$$

$\left. \begin{array}{l} \text{as } u=0, I_t = I_c = 5A \\ \text{as modulation index changes, } I_c \text{ doesn't change} \end{array} \right\}$

$$I_t = 5 \sqrt{1 + \frac{0.25}{2}}$$

$$I_t = 5 \sqrt{1 + \frac{0.25}{2}}$$

$$I_t = \frac{5.32A}{5.32A}$$

- Q2. An AM transmitter current is given by 10A with 10% of modulation. Find AM transmitter current with 80% of modulation.

Soln: Given,  $I_t = 10A$

$$\therefore \text{as } u = 0.1; I_c = 9.6A \quad \left. \begin{array}{l} 10 = I_c \sqrt{1 + \frac{0.1^2}{2}} \\ I_c = 9.6A \end{array} \right\}$$

So,

$$I_t = I_c \sqrt{1 + \frac{u^2}{2}}$$

$$= 9.6 \sqrt{1 + \frac{0.8^2}{2}} = 9.6 \sqrt{1 + 0.32}$$

$$I_t = 11.05A$$

## ~~CURRENT RELATION FOR MULTI TONE~~

As,

$$P_t = P_c \left[ 1 + \frac{U_t^2}{2} \right]$$

(41)

So,

$$\boxed{I_t = I_c \sqrt{1 + \frac{U_t^2}{2}}} ; \boxed{U_t = \sqrt{U_1^2 + U_2^2}}$$

Q1. An Unmodulated AM transmitter power is given by 10kW when the carrier is modulated by single sinusoidal message signal transmitter power becomes 13.5 kW. Find AM transmitter power if the carrier is simultaneously modulated by 2nd message signals with 60% of modulation.

- a) 12 kW   b) 15 kW   c) 20 kW   d) 22.5 kW

Soln.  $\therefore U = 0$

$$\text{So, } P_t = P_c = 10 \text{ kW}$$

Now,

$$P_t = P_c \left\{ 1 + \frac{U_1^2}{2} \right\}$$

$$\frac{13.5}{10} = 1 + \frac{U_1^2}{2}$$

$$\frac{U_1^2}{2} = 0.35 \Rightarrow U_1^2 = 0.7$$

$$\text{Now, } U_t = \sqrt{U_1^2 + U_2^2} = \sqrt{0.7 + 0.36} \approx 1$$

So,

$$P_t = P_c \left( 1 + \frac{U_t^2}{2} \right)$$

$$P_t = 10 \times \frac{3}{2}$$

$$\boxed{P_t = 15 \text{ kW}} \text{ Ans}$$

Ques An AM transmitters current is given by  $10A$  with carrier is modulated by single sinusoidal message signal with  $40\%$  of modulation. With the carrier is simultaneously modulated by 2nd message signal, AM transmitter current is increased to  $10.5A$ . Find % of modulation due to 2nd message signal?

(42)

Soln:

$$I_t = I_c \sqrt{1 + \frac{U_1^2}{2}}$$

$$10 = I_c \sqrt{1 + 0.08}$$

$$I_c = \frac{10}{\sqrt{1+0.08}}$$

$$I_c = 9.624 A$$

NOW, ~~10.5~~ ~~9.624~~

$$10.5 = 9.624 \sqrt{1 + \frac{U_t^2}{2}}$$

$$\left( \frac{10.5}{9.624} \right)^2 = 1 + \frac{U_t^2}{2}$$

$$\frac{U_t^2}{2} = 0.1903$$

$$U_t = \sqrt{0.3806} = \sqrt{U_1^2 + U_2^2}$$

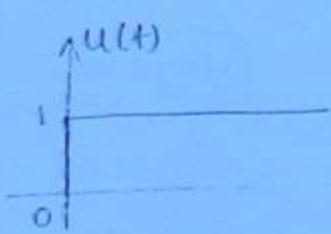
$$U_1^2 + U_2^2 = 0.3806.$$

$$U_2^2 = 0.3806 - 0.16 \Rightarrow U_2 = \sqrt{0.2206}$$

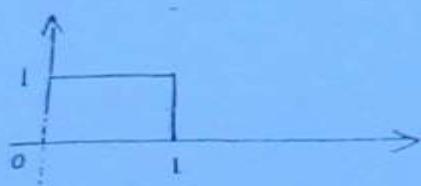
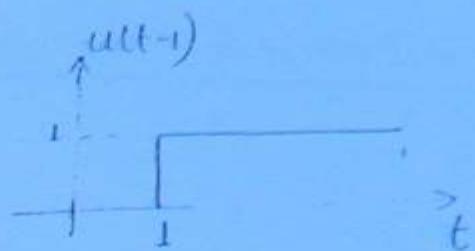
$$U_2 = 0.47 = 17\%$$

Note :-

$$x(t) = u(t) - u(t-1)$$

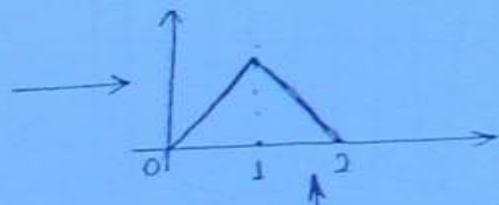
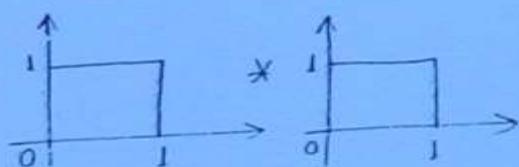


(43)



So,

$$x(t) * x(t)$$



NOW,

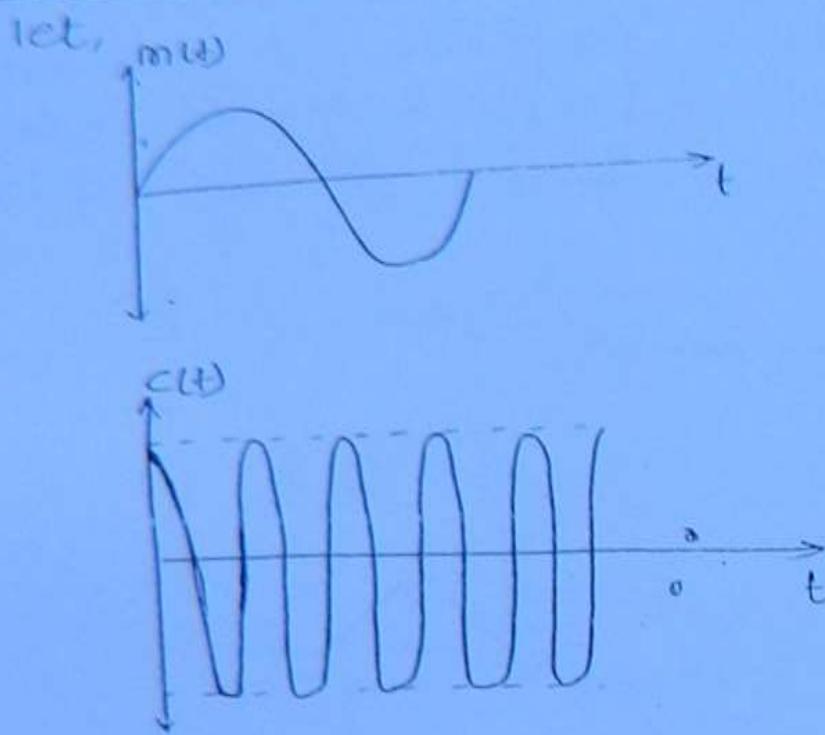
$$\{u(t) - u(t-1)\} * \{u(t) - u(t-1)\}$$

$$= r(t) - r(t-1) - r(t-1) + r(t-2)$$

$$= r(t) - 2r(t-1) + r(t-2)$$

Under, critical and over modulation:-

(44)



Under modulation:-

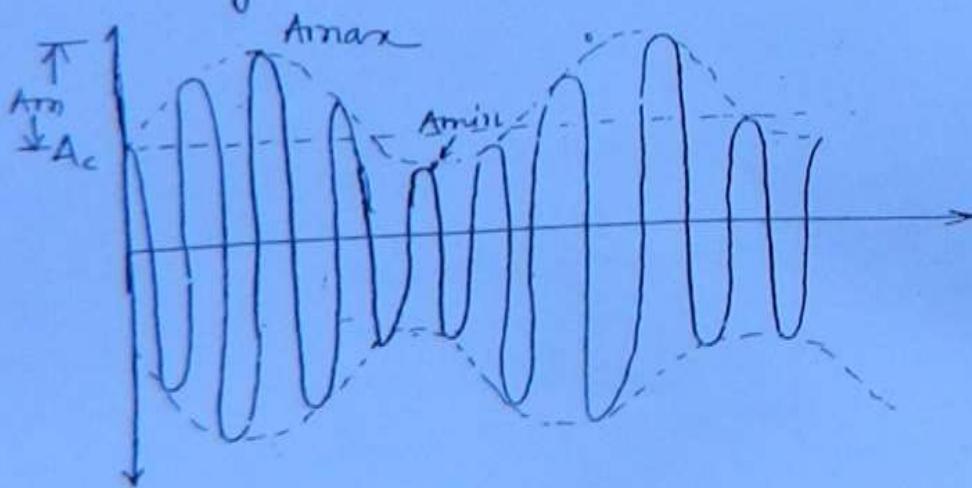
For under modulation

$$\mu < 1$$

$$\mu = \frac{A_m}{A_c} < 1 \Rightarrow A_m < A_c$$

$$A_{\min} = A_c \{1 - \mu\} = +ve$$

So, the waveform is:-



$u < 1$

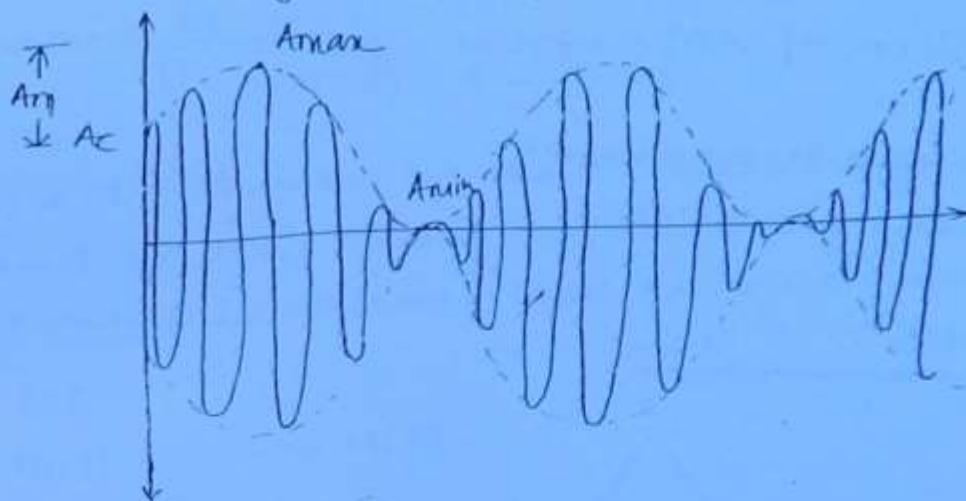
$$\frac{A_m}{A_c} = 1$$

(45)

$$A_m = A_c$$

$$\text{So, } A_{\min} = A_c \{1-u\}^2 = 0$$

So, the waveform is given as:-



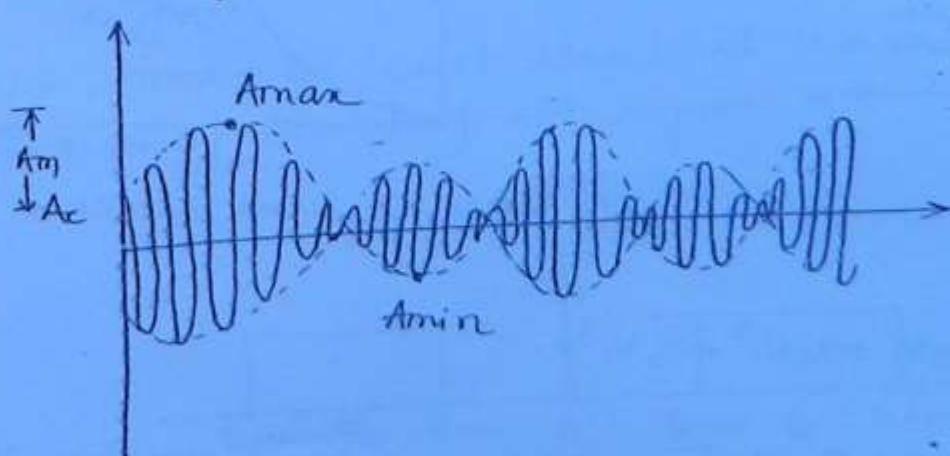
iii) Over modulation:

$u > 1$

$$\frac{A_m}{A_c} > 1 \Rightarrow A_m > A_c$$

$$\text{So, } A_{\min} = A_c \{1-u\}^2 = -\text{ve}$$

So, the waveform is given as:-



### Note:

- \* In under and critical modulation, message signal is preserved in the form of the envelope, so demodulation becomes simple.
- \* In overmodulation, the message signal is not stored in the form of the envelope, so demodulation becomes complex.

(46)

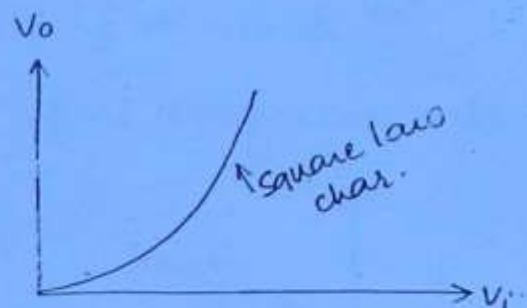
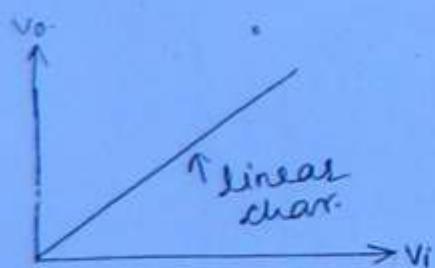
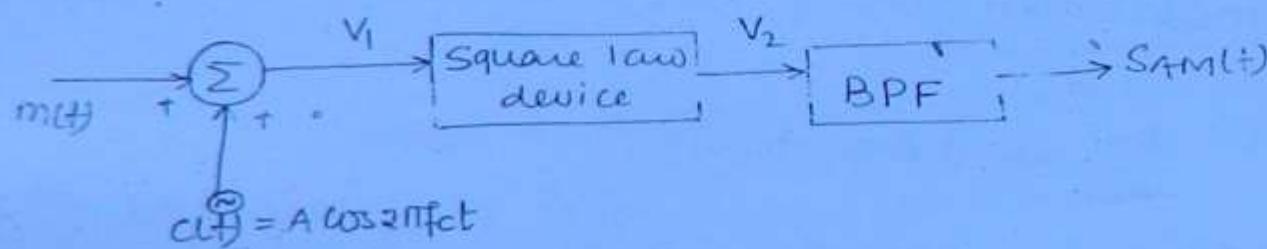
### \* Generation of AM Signal :-

For the generation of AM signals, following modulators are used:-

- 1) Square law modulator.
- 2) Switching modulator

### \* SQUARE-LAW MODULATOR

#### Block diagram:

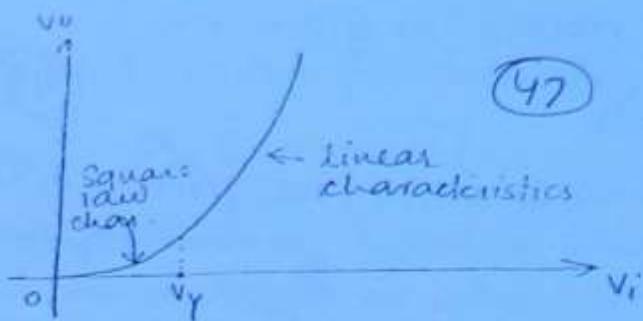


\* The Relation b/w  $V_o$  &  $V_i$  for square law device is given as:-

$$V_o = a_0 V_i + a_1 V_i^2 + a_2 V_i^3 + \dots$$

where,  $a_0, a_1, a_2, \dots \rightarrow$  square law constants

Note



When the applied voltage  $V_i < V_y$  (diode), then the diode exhibits square law characteristic and if  $V_i > V_y$  (diode) it then exhibits linear characteristics.

So,

Input of the diode (square law device) is:-

$$V_I = m(t) + c(t)$$

$$= m(t) + A_c \cos \omega f t$$

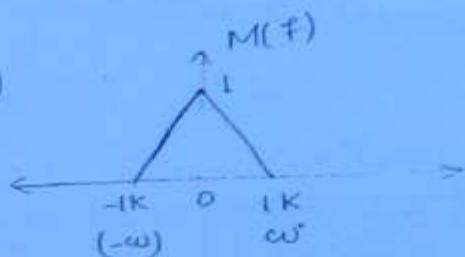
$m(t)$  &  $c(t)$  should be such that the peak voltage of  $V_I$  must be less than cutin voltage of diode, so that diode exhibits square law characteristic.

Now, let

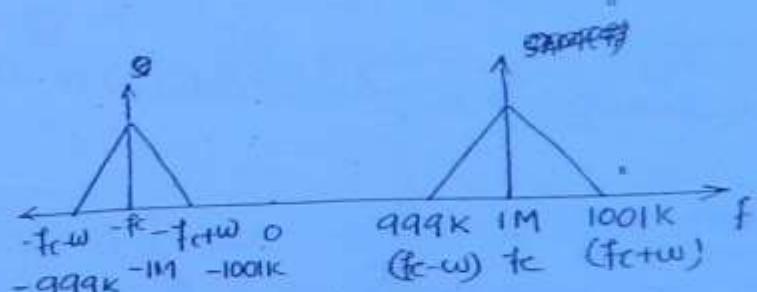
$$m(t) \longleftrightarrow M(f)$$

$$f_c = 1 \text{ MHz}$$

$$= 1000 \text{ K}$$

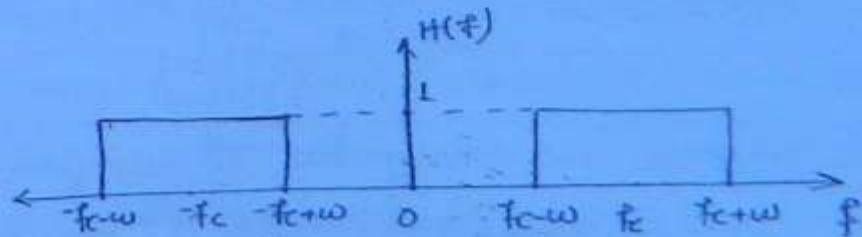


So, SAM(t)



The BPF should be such that it has to allow only the frequency band of AM signal.

So,



The O/P of square law device is given as

(48)

$$(SLD)_{O/P} = V_2 = a_1 V_1 + a_2 V_1^2 \\ = a_1 \{m(t) + A_c \cos 2\pi f_c t\} + a_2 \{m^2(t) + A_c^2 \cos^2 2\pi f_c t \\ + 2A_c m(t) \cos 2\pi f_c t\}$$

Now,

$$(BPF)_{O/P} = a_1 A_c (\cos 2\pi f_c t + 2a_2 A_c m(t)) \cos 2\pi f_c t \quad \left[ m^2(t) = m(t) \times m(t) \right]$$

$$\text{So, } (BPF)_{O/P} = a_1 A_c (\cos 2\pi f_c t + 2a_2 A_c m(t)) \cos 2\pi f_c t$$

$$\begin{aligned} & m(t) \times M(t) \\ & \xrightarrow{-\omega \quad \omega} \times \xrightarrow{-\omega \quad \omega} = \xrightarrow{-2\omega \quad 2\omega} \\ & \cos^2 2\pi f_c t = \frac{1 + \cos 4\pi f_c t}{2} \end{aligned}$$

$$(BPF)_{O/P} = a_1 A_c \left[ 1 + \frac{2a_2}{a_1} m(t) \right] \cos 2\pi f_c t = SAM(t)$$

Comparing with standard AM signal:

$$A_c \{1 + K_a m(t)\} \cos 2\pi f_c t$$

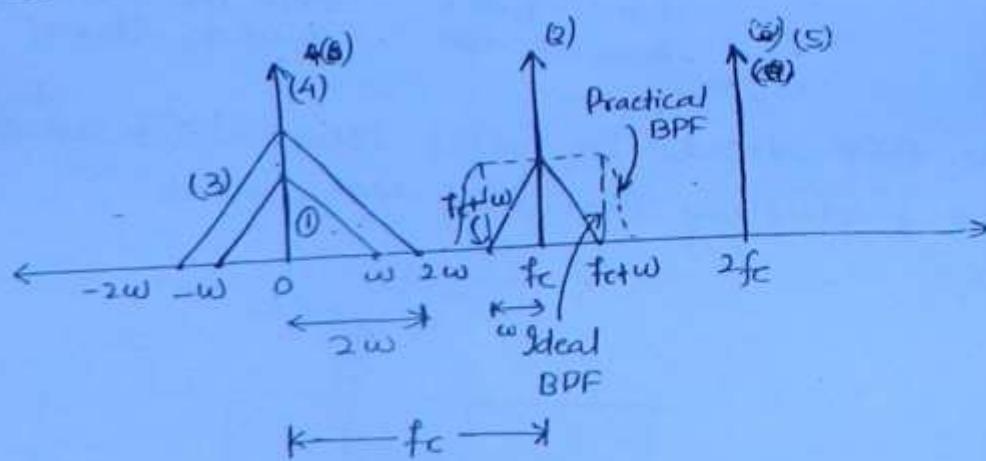
where,

$$A_c' = a_1 A_c$$

$$K_a = \frac{2a_2}{a_1}$$

$$\text{Now, } V_2 = a_1 \{m(t) + A_c \cos 2\pi f_c t\} + a_2 \left\{ A_c^2 \cos^2 2\pi f_c t + m^2(t) + 2A_c m(t) \cos 2\pi f_c t \right\}$$

$$V_2(t) \longleftrightarrow$$



$$f_c \ggg 3\omega$$

← To avoid overlapping of desired and undesired signals.

Note :-

To avoid undesired frequencies, at BPF output

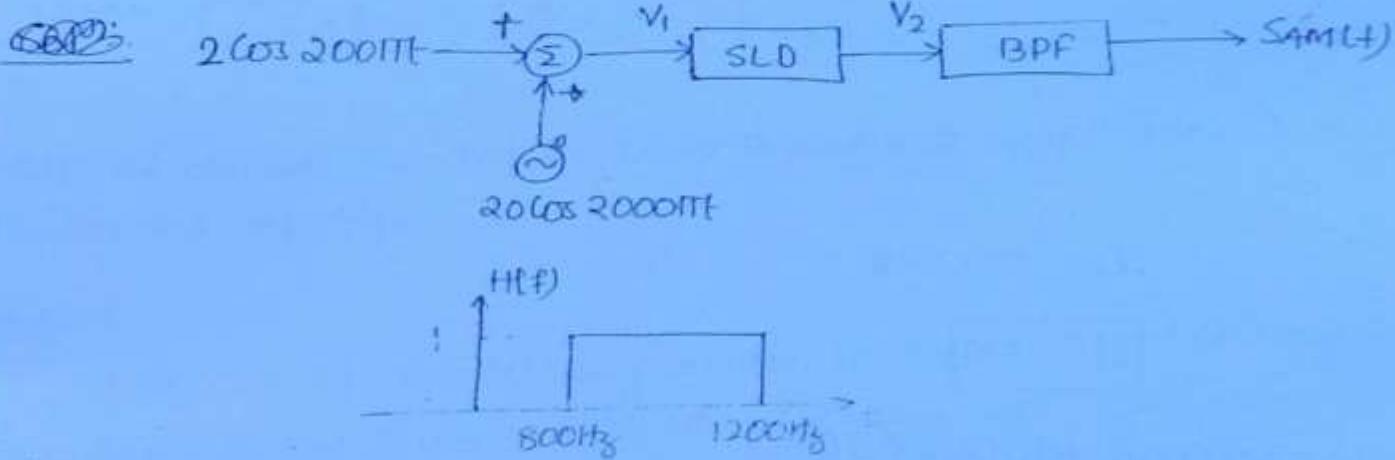
$$f_c \ggg 3\omega$$

(99)

- Q1. For the following Square law modulator, where the square law device is characterised by

$$V_2 = V_1 + 0.1 V_1^2$$

and pass Band of BPF will be from 800Hz to 1200Hz.  
Find all the parameters of Resulting AM signal.



$$\begin{aligned} \text{Soln: } V_2 &= (2\cos 2000\pi t + 20\cos 2000\pi t) + 0.1(2\cos 2000\pi t + 20\cos 2000\pi t)^2 \\ &= 2\cos 2000\pi t + 20\cos 2000\pi t \\ &\quad + 0.1 \left\{ 4\cos^2 2000\pi t + 400\cos^2 2000\pi t + 80\cos 2000\pi t \right. \\ &\quad \left. - \cos 2000\pi t \right\} \end{aligned}$$

$$\begin{aligned} V_2 &= (2\cos 2000\pi t + 20\cos 2000\pi t + 0.4 \frac{\cos^2 2000\pi t}{(1+400\cos^2 2000\pi t/2)} + 40\cos^2 2000\pi t) \\ &\quad + 8\cos 2000\pi t \cos 2000\pi t \\ &\quad \cancel{*} \checkmark (\cos(9200\pi t) + \cos(180\pi t)) \end{aligned}$$

So,

$$(BPF)_{O/P} = SAM(+) = 20\cos 2000\pi t + 8\cos 2000\pi t \cos 2000\pi t$$

$$SAM(+) = 20 \left\{ 1 + 0.4 \cos 2000\pi t \right\} \cos 2000\pi t$$

Above signal is single tone AM. So

$$SAM(+) = A_c \left\{ 1 + 0.4 \cos 2000\pi t \right\} \cos 2000\pi t$$

so,  
 $A_c = 20 \text{ V}$ ;  $U = 0.1$ ;  $f_m = 100 \text{ Hz}$ ,  $f_c = 1000 \text{ Hz}$

(56)

objective

$U = ?$

as,  $U = K_a A_m$

$$\text{and } K_a = \frac{2a_2}{a_1}$$

by comparing with  $V_2 = V_1 + 0.1 V_1^2$   
 $V_2 = a_1 V_1 + a_2 V_1^2$

$$a_1 = 1; a_2 = 0.1$$

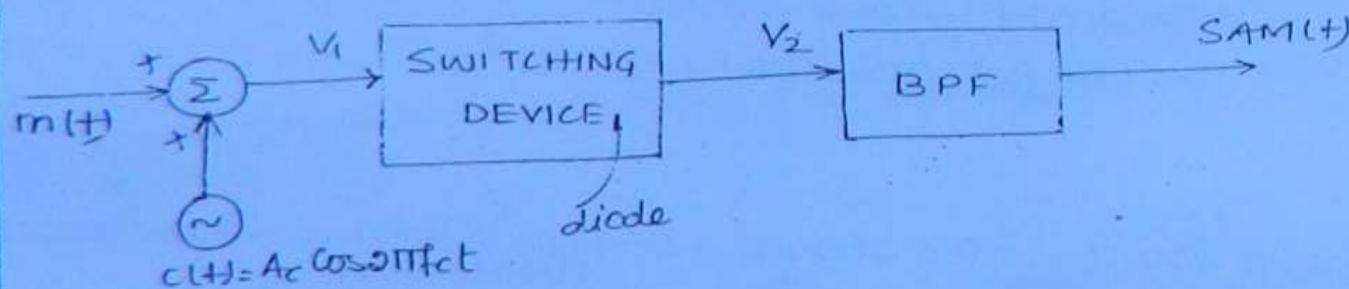
$$\text{so, } K_a = \frac{2 \times 0.1}{1} = 0.2$$

$$U = 0.2 \times 20$$

$$\boxed{U = 0.4}$$

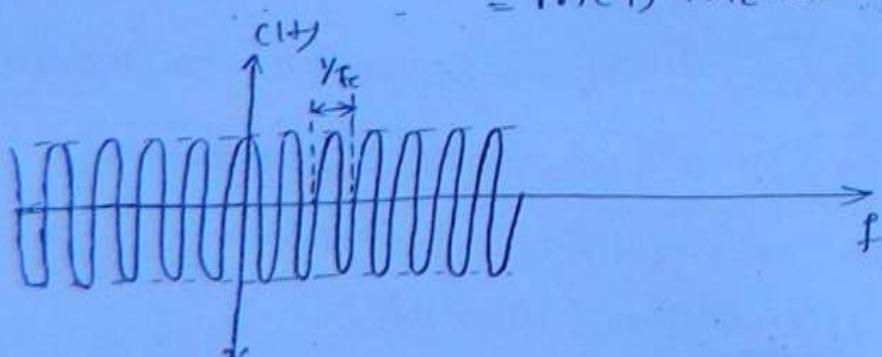
## \*SWITCHING MODULATOR:-

BLOCK diag<sup>m</sup>:



Analysis:-

$$\text{I/P of diode} \Rightarrow V_1 = m(t) + c(t) \\ = m(t) + A_c \cos 2\pi f_c t$$



\* The strength of Practical message signal will be generally of having less strength and carrier signal is generated with high strength so the operation of the diode is mainly controlled by carrier signal. (57)

1. when  $C(t)$  is +ve, diode is forward Biased ie S.C  
then  $V_2 = V_1$

2. when  $C(t)$  is -ve, diode is Reverse Biased ie O.C  
then  $V_2 = 0$

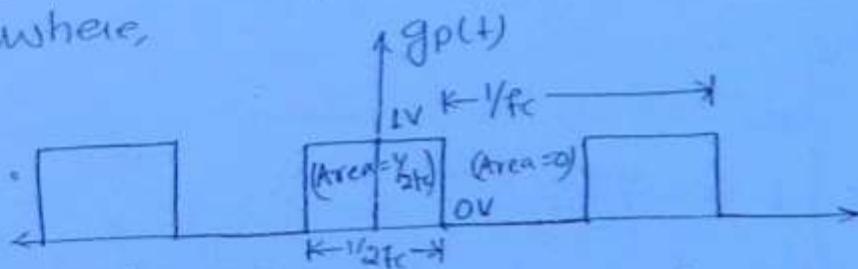
\* O.P of diode switches b/w  $V_1$  and 0 with the time interval of  $1/f_c$ .

Note:

$\therefore$  the O.P of switching device, ie  $V_2$  is switching continuously b/w  $V_1$  & 0, so plotting its spectrum is not easy. Hence to plot its spectrum, let

$$V_2 = V_1 \times g_p(t) \quad \dots \quad (1)$$

where,



The analysis of Periodic signal is done by Fourier series. Hence by Trigonometric FS we have:

$$f(t) = a_0 + \sum_{n=1}^{\infty} \{a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\}; \omega_0 = 2\pi/T \quad \dots \quad (2)$$

$\therefore g_p(t)$  is even, hence  $b_n = 0$ .

so,

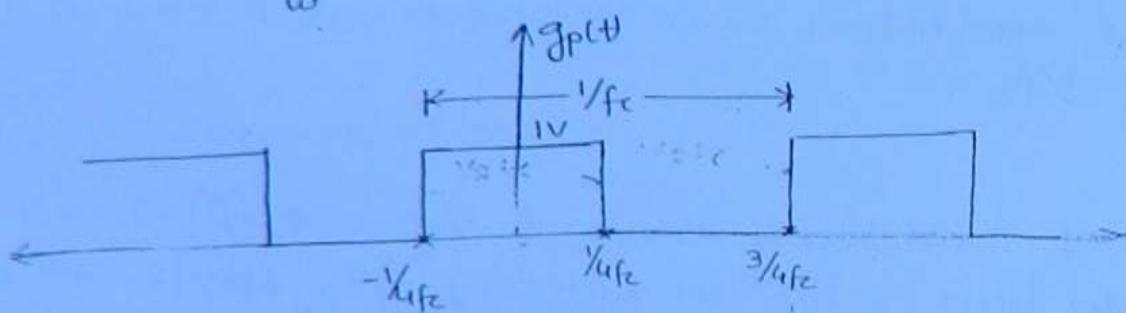
$$g_p(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t$$

$$\text{Now } a_0 = \frac{1}{T} \int_{t_0}^{t_0+T} f(t) dt = \frac{\int f(t)dt \text{ (Area)}}{T}, \quad (52)$$

$$a_0 = \frac{1/2 f_c}{1/f_c}$$

$$\boxed{a_0 = \frac{1}{2}}$$

$\therefore a_n = \frac{2}{T} \int_{t_0}^{t_0+T} f(t) \cos n\omega_0 t dt ; \omega_0 = 2\pi/T = 2\pi f_c$



So,

$$a_n = \frac{2}{1/f_c} \int_{-1/f_c}^{3/4 f_c} g_p(t) \cos 2\pi f_c t dt$$

$$a_n = \frac{2}{1/f_c} \int_{-1/f_c}^{1/4 f_c} 1 \cdot \cos 2\pi n f_c t dt + 0$$

$$= 2f_c \times \left. \frac{\sin 2\pi n f_c t}{-2\pi n f_c} \right|_{-1/f_c}^{1/4 f_c}$$

$$= \frac{1}{n\pi} \left\{ \sin \frac{n\pi}{2} \times \frac{1}{2} - \sin \frac{-n\pi}{2} \times -\frac{1}{2} \right\}$$

$$= \frac{1}{n\pi} \left\{ \sin \frac{n\pi}{2} + \sin \frac{n\pi}{2} \right\}$$

$$A_m = \frac{2}{n\pi} \sin n\pi / 2$$

(53)

So, from equation (2) we get:-

$$g_p(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin n\pi / 2 \cos n\omega_f t$$

and from equation (1) we get:-

$$V_2 = V_1 \cdot g_p(t)$$

$$= (m(t) + A_c \cos 2\pi f_c t) \left[ \frac{1}{2} + \frac{2}{\pi} \cos 2\pi f_c t - \frac{2}{3\pi} \cos 2\pi (3f_c) t + \frac{2}{5\pi} \cos 2\pi (5f_c) t + \dots \right]$$

$\times V_2$  is then passed through a BPF. So, its O/P is given as:-

$$(BPF)_{O/P} = SAM(t) = \left( \frac{2}{\pi} m(t) \cos 2\pi f_c t + \frac{A_c}{2} \cos 2\pi f_c t \right)$$

$$SAM(t) = \frac{A_c}{2} \left\{ 1 + \frac{4}{\pi A_c} m(t) \right\} \cos 2\pi f_c t$$

Comparing with standard equation of AM we get:-

$$A_c' \left\{ 1 + K_a m(t) \right\} \cos 2\pi f_c t$$

so,

$$A_c' = \frac{A_c}{2}$$

\*\*\*

$$\& K_a = \frac{4}{\pi A_c}$$

## \* DEMODULATION OF AM SIGNAL:

(54)

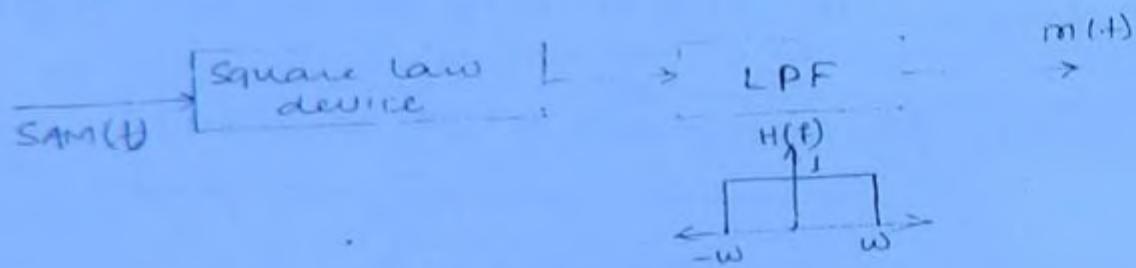
The demodulators that are used for demodulation of AM are:-

- 1) Square law demodulator } LSI
- 2) Envelope detector (diode detector)
- 3) Synchronous detector } any value of  $\phi$ .

\* Synchronous detector is complex and square law demodulator will have some drawbacks. So generally for demodulation of AM; envelope detector will be used.

## \* SQUARE LAW DEMODULATOR:

B-Diagm



As,

$$SAM(t) = A_c \{ 1 + K_a m(t) \} \cos 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t + K_a m(t) \cos 2\pi f_c t \cdot A_c$$

The O/P of the Square law device may be given as:

$$(SLD)_{O/P} \Rightarrow V_2 = a_1 SAM(t) + a_2 \{ SAM(t) \}^2$$

$$= a_1 \{ A_c \cos 2\pi f_c t + K_a m(t) \cos 2\pi f_c t \}$$

$$+ a_2 \{ A_c^2 \cos^2 2\pi f_c t + K_a^2 m^2(t) \cos^2 2\pi f_c t \cdot A_c^2 \\ + 2 K_a A_c m(t) \cos^2 2\pi f_c t \}.$$

*✓ (but blocked by blocking capacitor)*

So,

$$V_2 = [a_1 A_c \cos 2\pi f_c t + a_1 A_c K_a m(t) \cos 2\pi f_c t + \frac{a_2 A_c^2}{2} (1 + \cos 4\pi f_c t) \times \\ \sqrt{\text{but only part of}} \\ + a_2 A_c^2 K_a^2 m^2(t) (1 + \cos 4\pi f_c t) + a_2 \cdot 2 A_c^2 K_a m(t) (1 + \cos 4\pi f_c t)].$$

the signal  $v_2$  is given to LPT and the LPT is given as following:-

$$(\text{LPT})_{\text{LPT}} = \frac{a_2 A c^2 K a^2 m^2(t)}{2} + a_2 \cdot 2 A c^2 K a m(t)$$

unwanted  
signal  
or  
noise

expected  
signal

If  $\frac{S}{N} \gg 1$ ;  $m(t)$  can be reconstructed (almost in the Range of 100 or 1000)

$\frac{S}{N} < 1$ ;  $m(t)$  can't be reconstructed.

Now,

$$\frac{S}{N} = \frac{a_2 A c^2 K a m(t)}{\frac{a_2 A c^2 K a^2 m^2(t)}{2}}$$

$$\frac{S}{N} = \frac{2}{K a m(t)}$$

Now let  $m(t) = A_m \cos 2\pi f_m t$

so,  $\frac{S}{N} = \frac{2}{K a \cdot A_m \cos 2\pi f_m t}$

$$\frac{S}{N} = \frac{2}{A_m \cos 2\pi f_m t}$$

As,  $-1 \leq \cos 2\pi f_m t \leq 1$

so,  $\boxed{\frac{S}{N} = \frac{2}{A_m \pm 1}}$   $\left\{ \cos 2\pi f_m t = 1 \right\}$ .

Now  $\frac{S}{N} \propto \frac{1}{A_m} \Rightarrow \text{To get } \frac{S}{N} = 100 \Rightarrow A_m = 0.02$

For such small value of  $A_m$ ,  $\eta$  will be mostly affected ie low share of sideband power in AM signal.

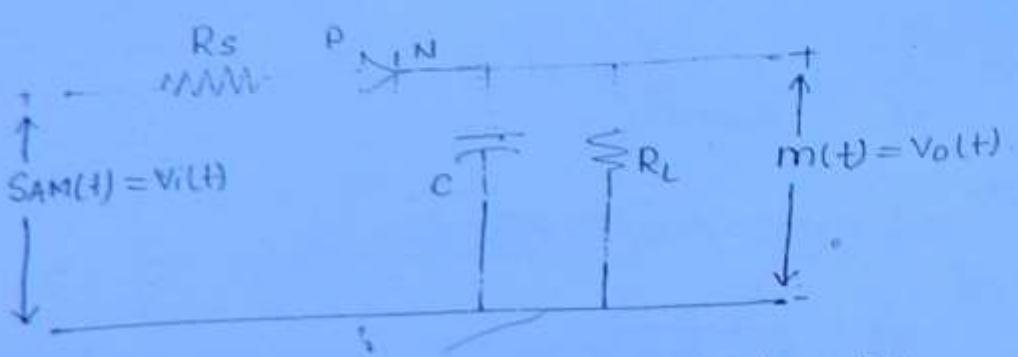
\* Hence in the square law demodulator either of the two i.e.  $\alpha$  or  $m$  has to be sacrificed. It means that if  $\alpha$  is increased  $P_{SB}$  is increased but S/N is reduced and vice versa.

(56)

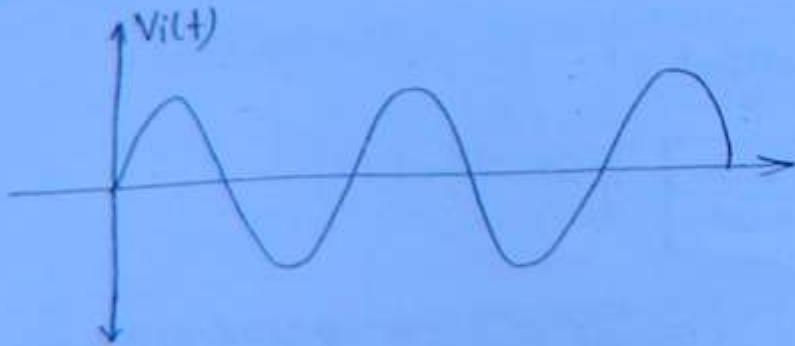
\* To get high value of S/N,  $\alpha$  should be very small. For small values of  $\alpha$ , modulation efficiency will be very much low. But for efficient power distribution,  $m$  should be max<sup>m</sup>. So this method is practically not preferred for AM demodulation.

## \* ENVELOPE DETECTOR:

Ckt diagram:

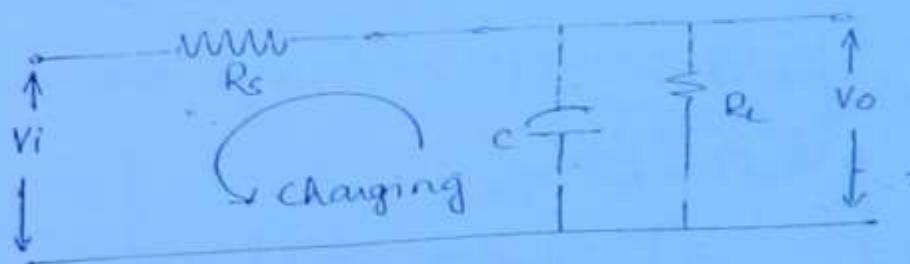


→ Envelope detector is very simple and cheaper.  
→ The +ve envelope of applied signal will be produced at the O/P so it is called as ENVELOPE DETECTOR.

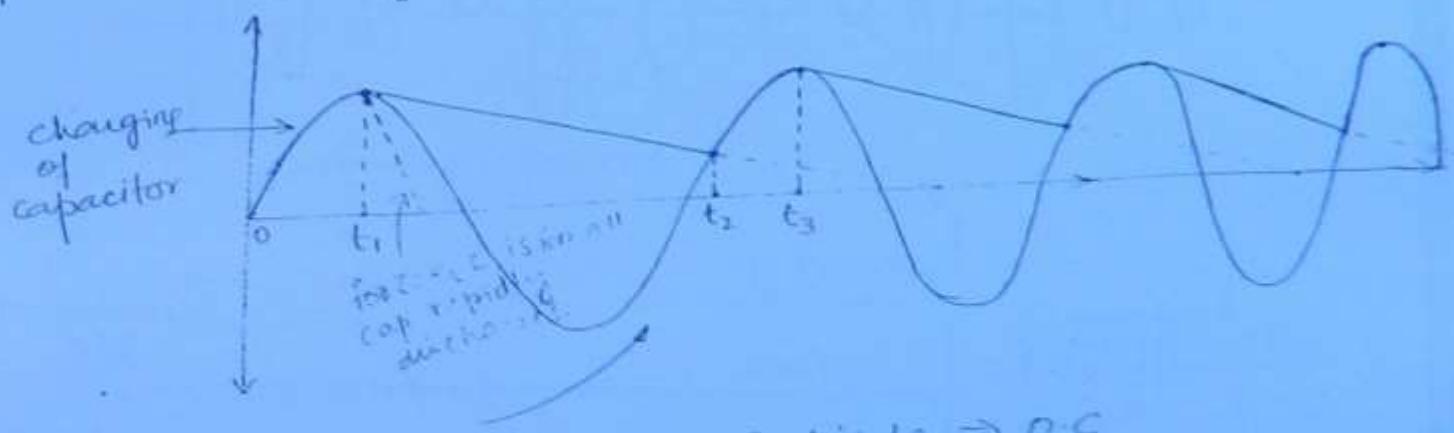


For  $t = 0^+$ ,  $V > N \Rightarrow$  FB diode  $\Rightarrow$  SC

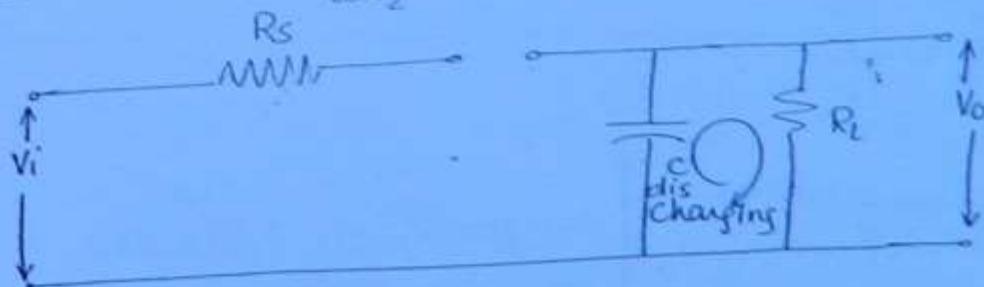
(57)



Note :  
The time const  $= R_s \cdot C$  should be very small. Hence capacitor rapidly charges to input voltage.



Case 2 : For  $t = t_1^+, t_2^+$ ,  $P < N \Rightarrow$  RB diode  $\Rightarrow O \cdot C$



If  $\tau = R_L \cdot C$  is very small, the capacitor rapidly discharges to zero.

If  $\tau = R_L \cdot C$  is very high, the capacitor discharges slowly.

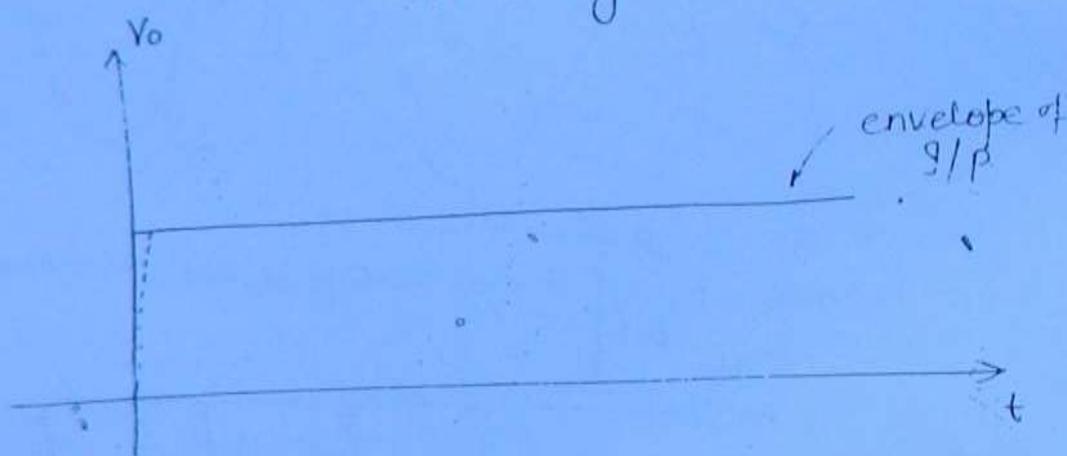
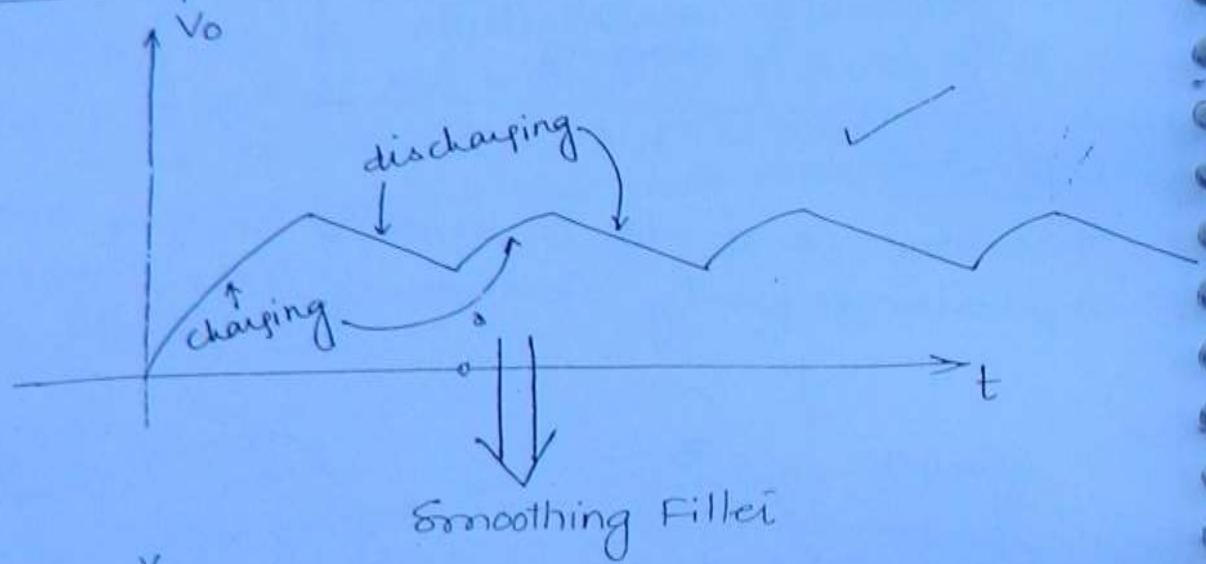
Note :  
 $\tau = R_L \cdot C$ ; should be very high then capacitor discharges very slowly.

Case 3: for  $R = 0$ ,  $C = \infty$   $\rightarrow$  PN diodes = Zener  
i.e. charges & discharges don't work.

The curve shown at back

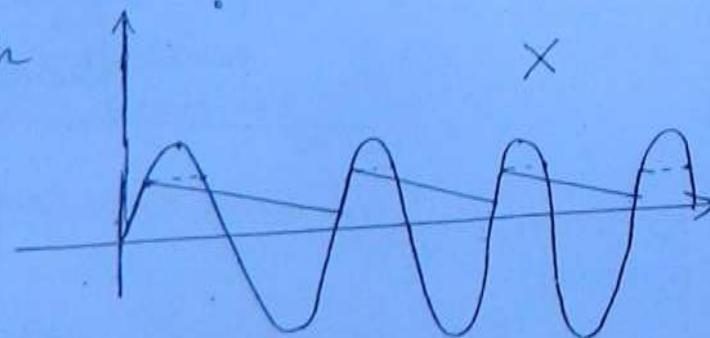
(38)

Voltage across capacitor:



Note:-

If  $R_s \cdot C$  = very high



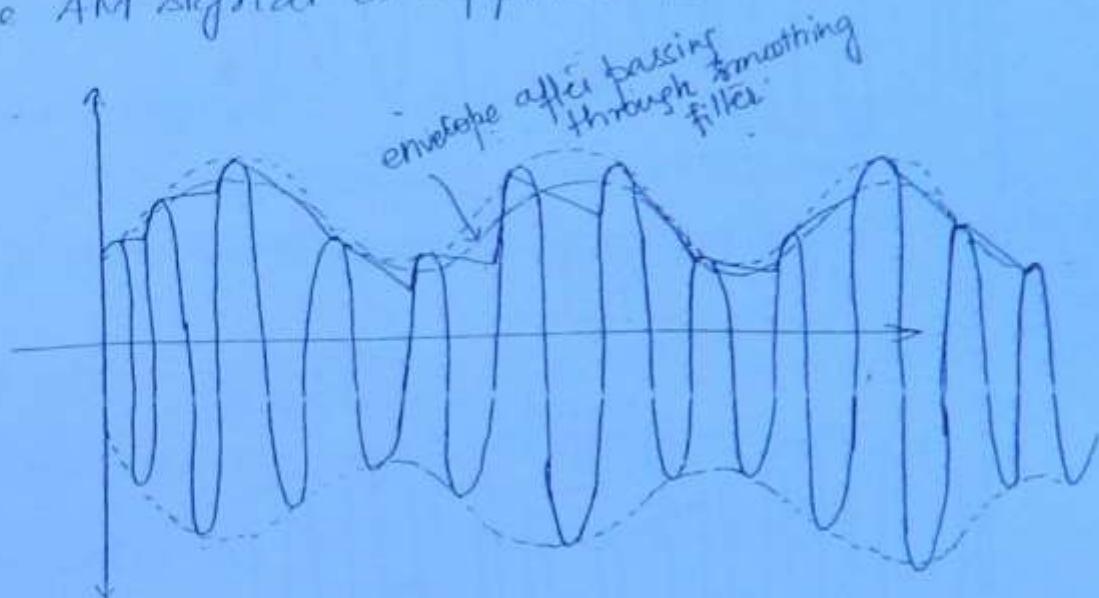
If  $R_s \cdot C$  is very high, even before capacitor voltage reaches to peak voltage of input, diode becomes reverse biased and capacitor will be discharged.

→ To make capacitor voltage to follow the envelope of input  $R_S \cdot C$  should be very small and  $R_C \cdot C$  should be high.

(54)

Note:

If the AM signal is applied to the diode detector then:



Analysis:

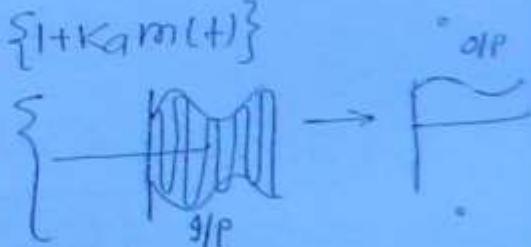
ED G/P

$$1. V_m \cos 2\pi f_m t \rightarrow V_m$$

ED O/P

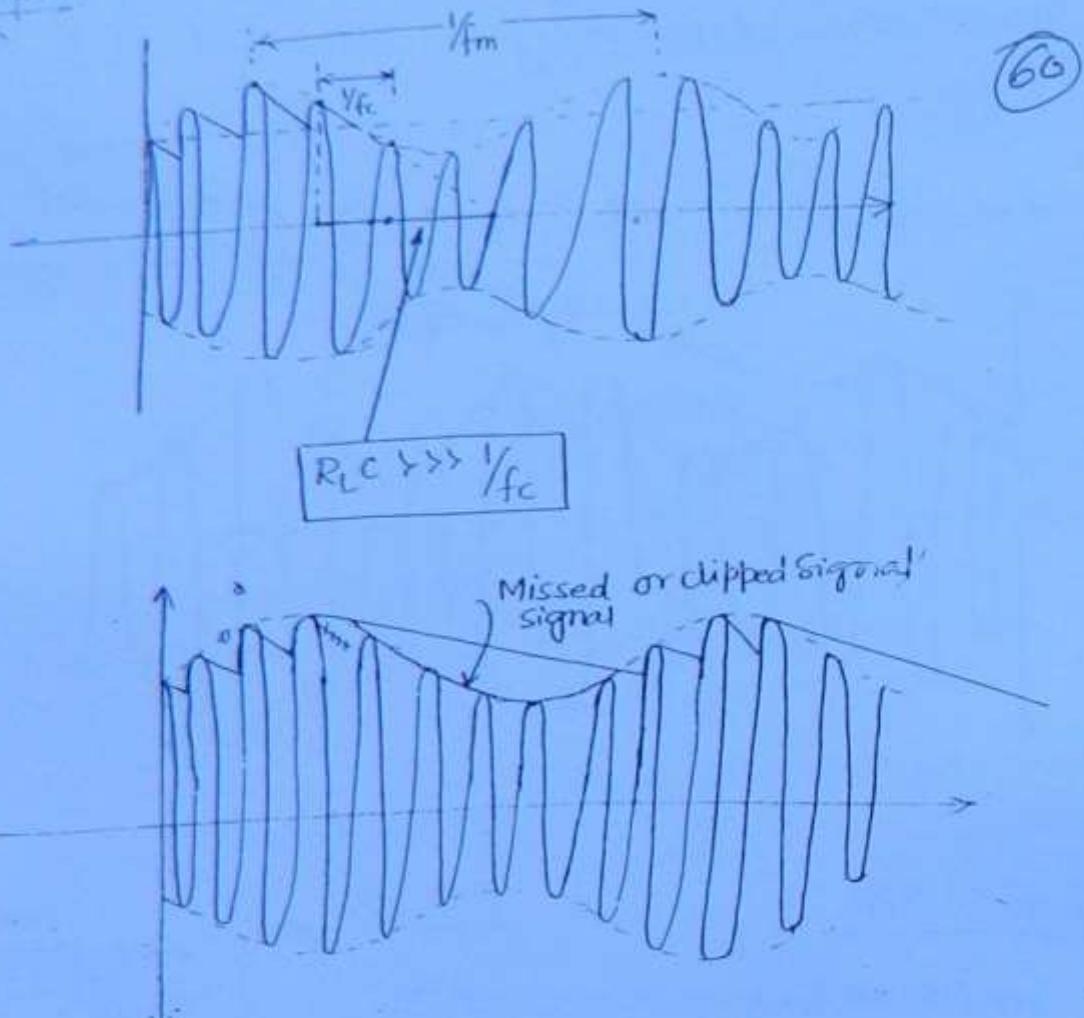


$$2. S(t) = A_c \{1 + K_a m(t)\} \cos 2\pi f_c t \rightarrow A_c \{1 + K_a m(t)\}$$



$$3. A \cos 2\pi f_o t + B \sin 2\pi f_o t \rightarrow \sqrt{A^2 + B^2}$$
$$= \left\{ \sqrt{A^2 + B^2} \times \cos(2\pi f_o t + \Phi) \right\}$$
$$\Phi = \tan^{-1} B/A$$

## Analysis:



(60)

### CASE OF DIAGONAL CLIPPING

To avoid diagonal clipping

$$\boxed{R_L \ggg 1/f_m} \quad \boxed{R_L C \lll 1/f_m}$$

\* Proper choice of  $R_L C \Rightarrow \boxed{\frac{1}{f_c} \lll R_L \cdot C \lll 1/f_m}$

Q1. The carrier signal of 1 MHz is amplitude modulated by message signal of frequency 2 KHz. The proper choice of  $R_L \cdot C$  for envelope detection should be

- a) 1 usec   b) 200 usec   c) 200 nusec   d) 0.5 usec

Sol<sup>n</sup>  $f_r = 1 \text{ MHz} \Rightarrow 1/f_r = 1 \mu\text{sec}$

$$f_m = 2 \text{ K} \Rightarrow \frac{1}{f_m} = 0.5 \text{ msec} = 500 \mu\text{sec}$$

(61)

$$\frac{1}{f_r} \ll R_C \ll \frac{1}{f_m}$$

Q2. An AM signal is given by

$$S(t) = A_c \cos \omega_c t + 2U \cos \omega_m t \cos \omega_m t$$

For proper envelope detection, the min<sup>m</sup> value of  $A_c$  should be:

- a) 0.5 volts b) 1 volt c) 1.1 volt d) 2.5 volt

Sol<sup>n</sup>: Comparing with AM signal:-

$$S(t) = A_c \left\{ 1 + \frac{2}{A_c} \cos \omega_m t \right\} \cos 2\pi f_c t$$

$$SAM(t) = A_c \left\{ 1 + U \cos \omega_m t \right\} \cos 2\pi f_c t$$

So,  $U = \frac{2}{A_c}$

$$U = \frac{2}{A_c}$$

$$\frac{A_m}{A_c} = \frac{2}{A_m} \quad \text{for envelope detection}$$

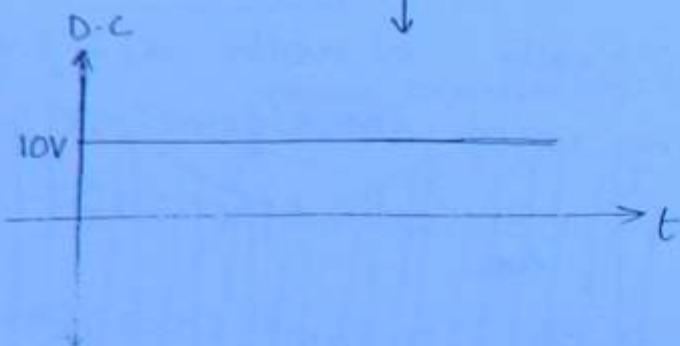
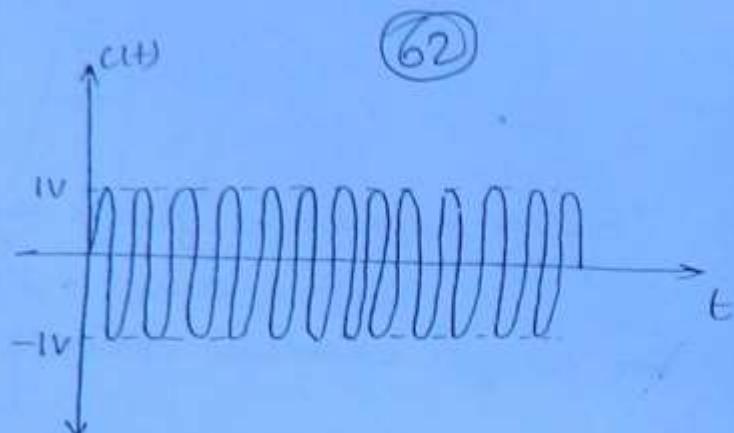
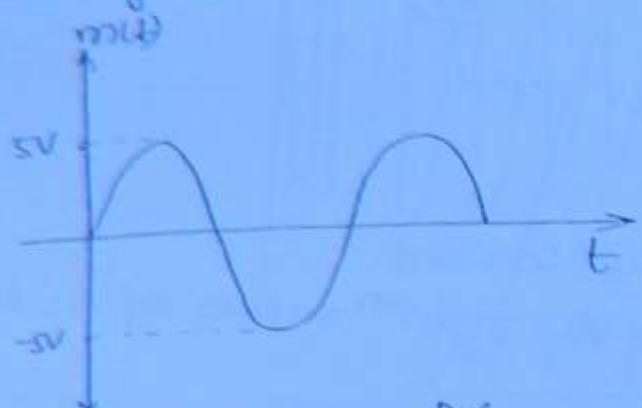
$$U \leq 1$$

$$\Rightarrow \frac{2}{A_c} \leq 1$$

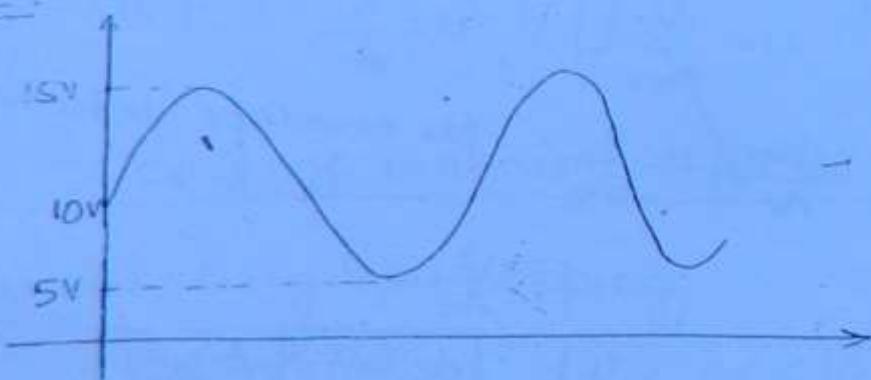
$$\boxed{A_c \geq 2} \text{ Ans.}$$

Hence, the min<sup>m</sup> value of  $A_c$  is 2 volts.

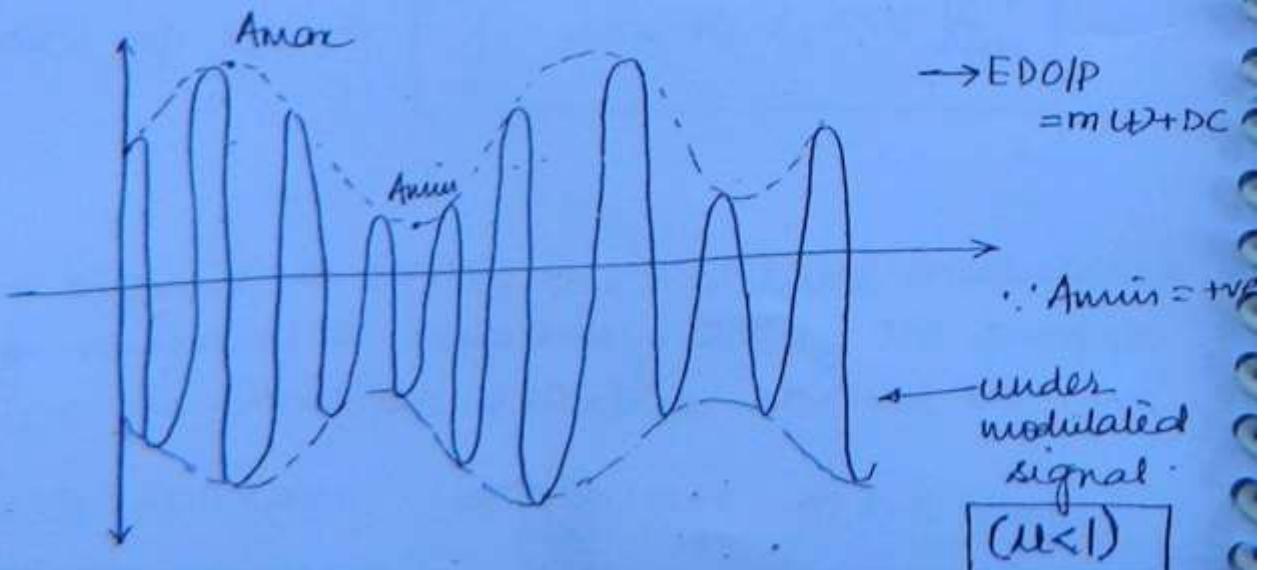
## Analysis:

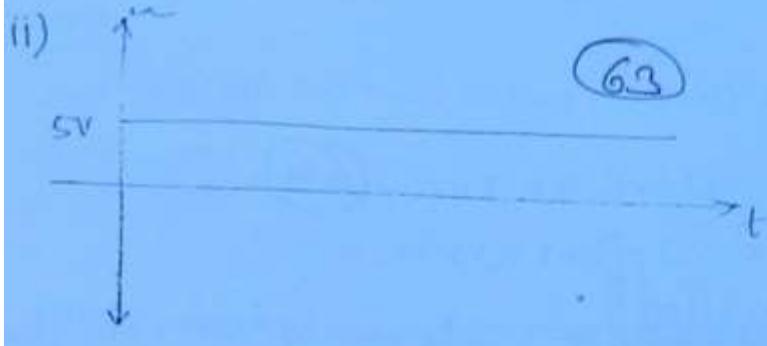


$m(t) + DC$ :

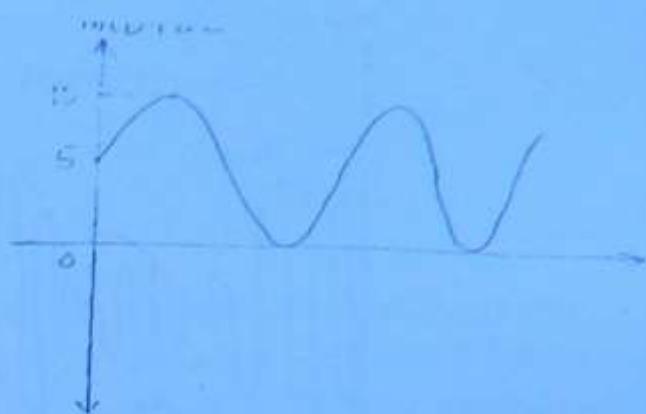


$$\text{Now, } SAM(t) = \{m(t) + DC\}c(t) = \{m(t) + (\text{DC})DC\} \text{ (cosine fact)}$$



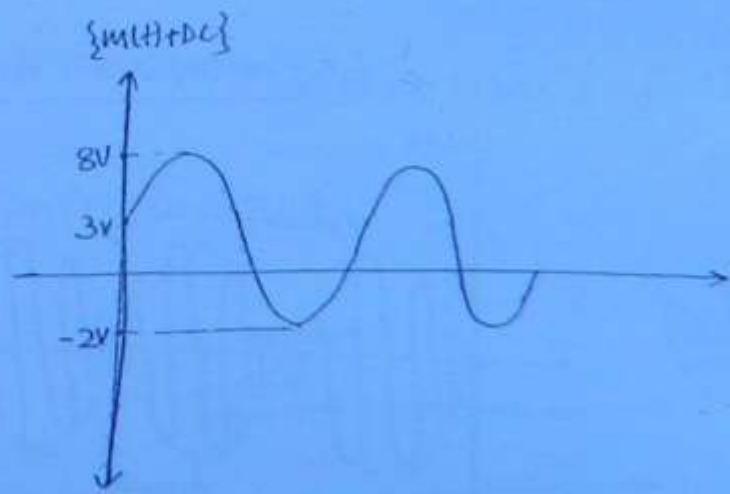
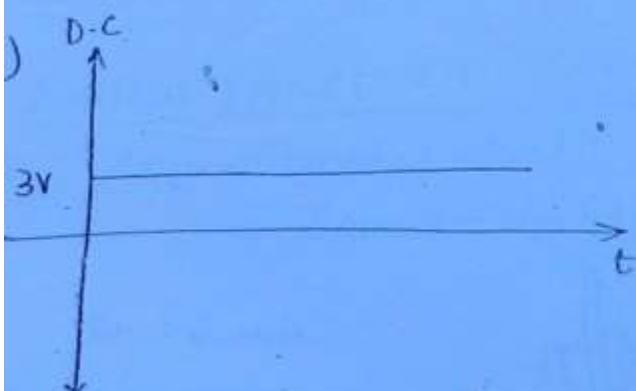
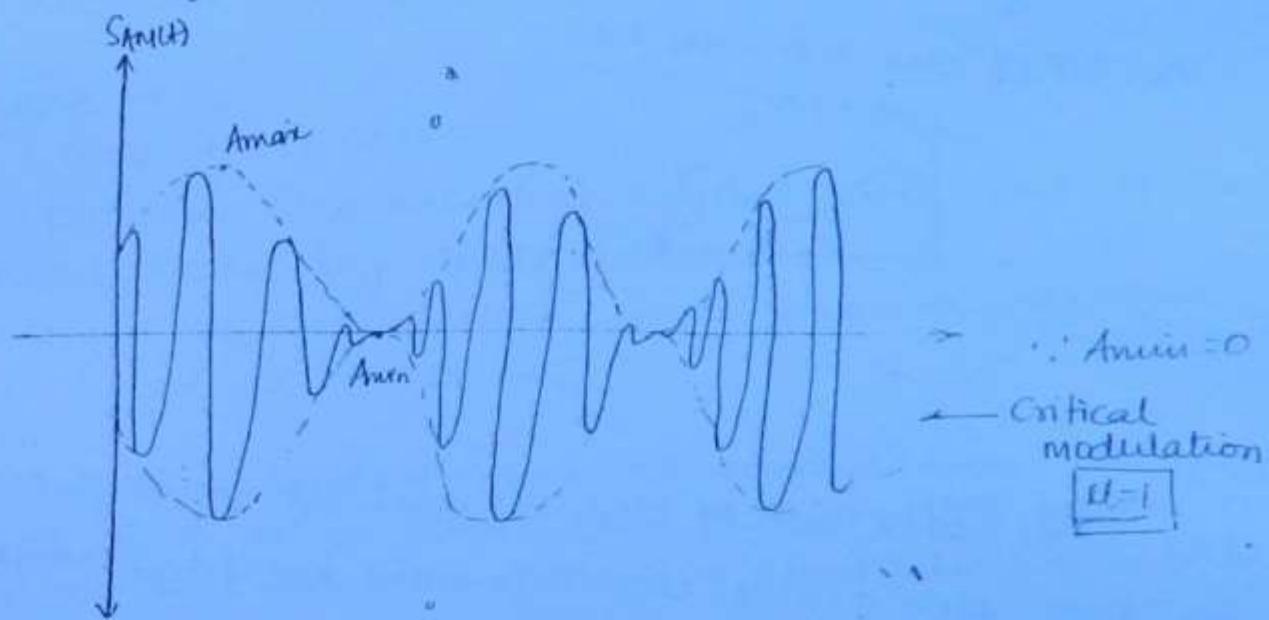


63



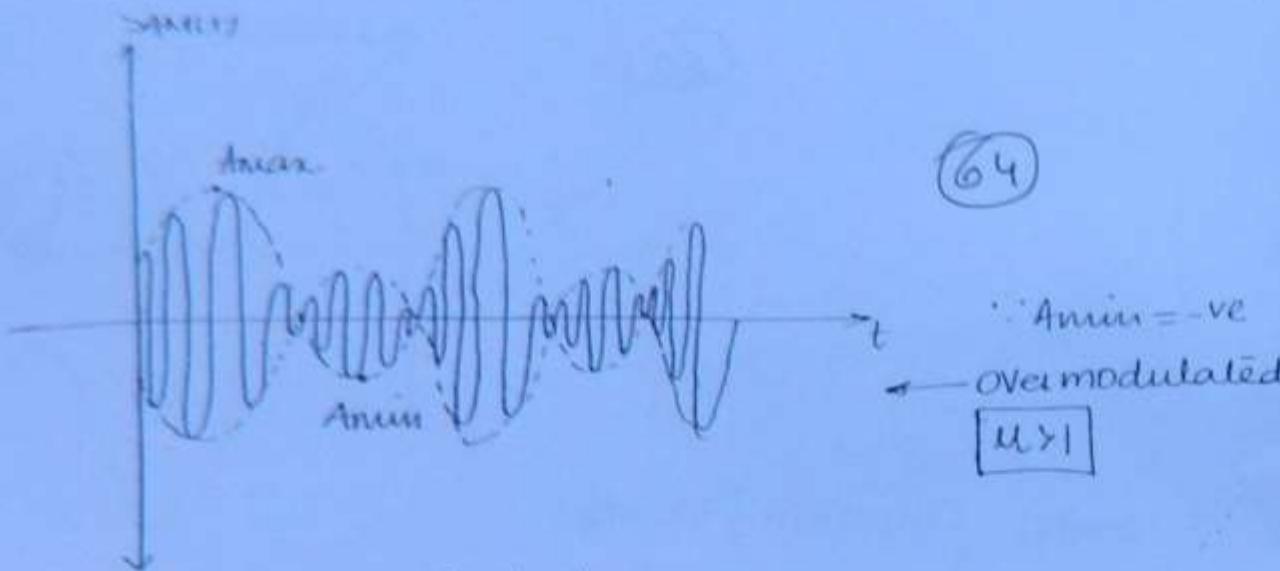
So,

$$SAM(t) = \{m(t) + DC\} \cos \omega_0 t$$

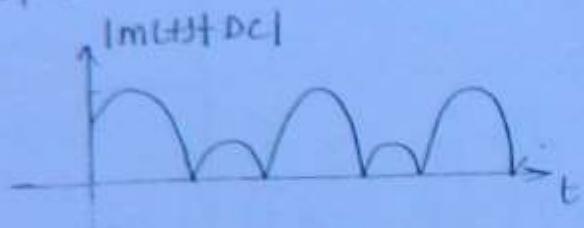


So,

$$SAM(t) = \{m(t) + DC\} \cos \omega_0 t$$



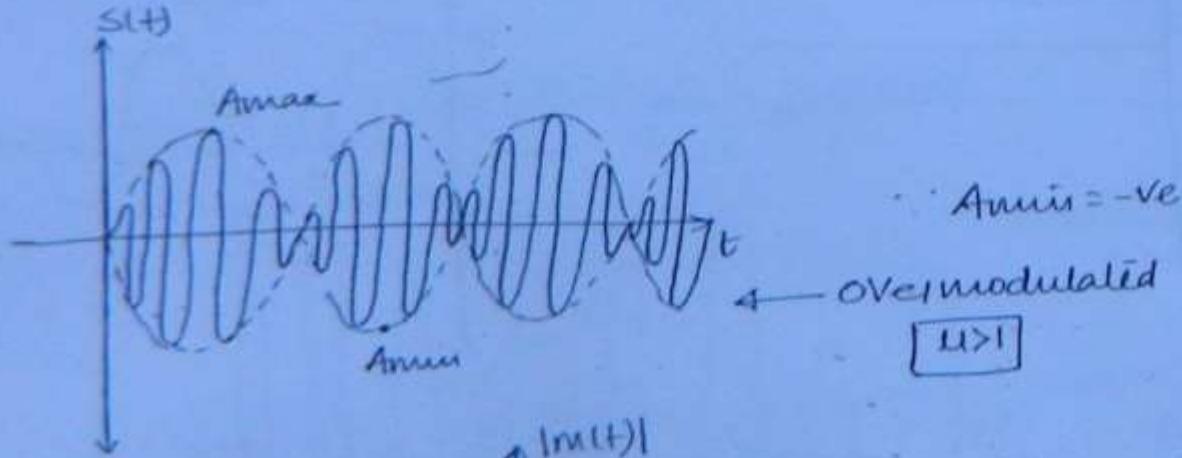
The O/P of the ED will be:-



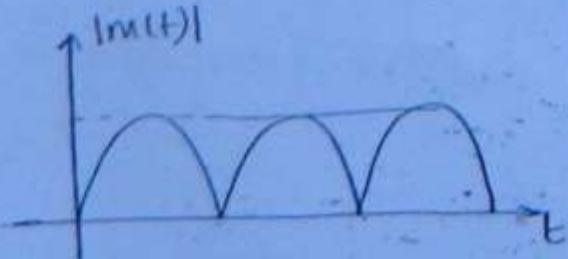
Note:

- For  $u \leq 1$ , the O/P of the ED will be  $Ac \{ 1 + k_a m(t) \}$
- For  $u > 1$ , the O/P of the ED will be  $|A_c \{ 1 + k_a m(t) \}|$

iv.  $s(t) = m(t) \cdot c(t) = m(t) \cos 2\pi f_c t$



O/P of ED is given as:-



NOIC

1. The OIP of ED in case (i) (iii) is totally zero.

and,

$$\begin{aligned} SAM(t) &= \{m(t) + DC\}c(t) \\ &= m(t)c(t) + DC \times c(t) \end{aligned} \quad (65)$$

If the DC term is not added in the modulation of the signal the term  $DC \times c(t)$  would not be obtained. Hence the AM signal obtained would be over modulated and it can't be demodulated back to obtain m(t) by ED. Hence the ED fails in its demodulation.

2. For case iv ie

$$s(t) = m(t) \cdot c(t)$$

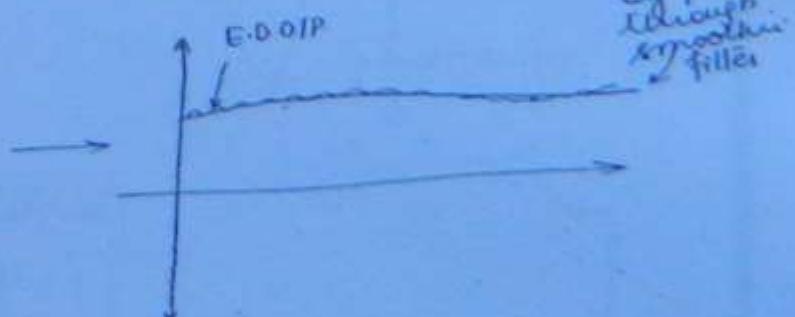
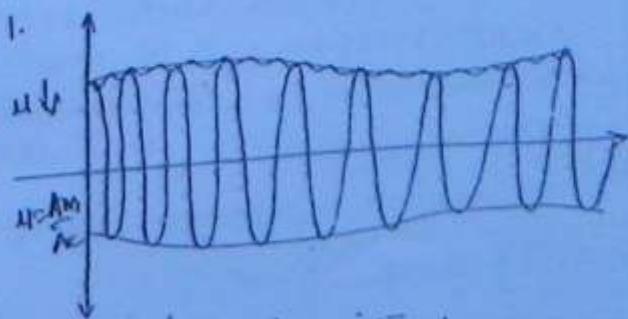
it contains the message signal  $m(t)$ , but it can't be demodulated by diode detector. This scheme of  $s(t) = m(t) \cdot c(t)$  is given a specific name called as DSB-SC.

3. Case V corresponds to DSB-SC modulation. The advantage of DSB-SC is that the Transmitter power will be saved but its drawback is that its Demodulation becomes complex.

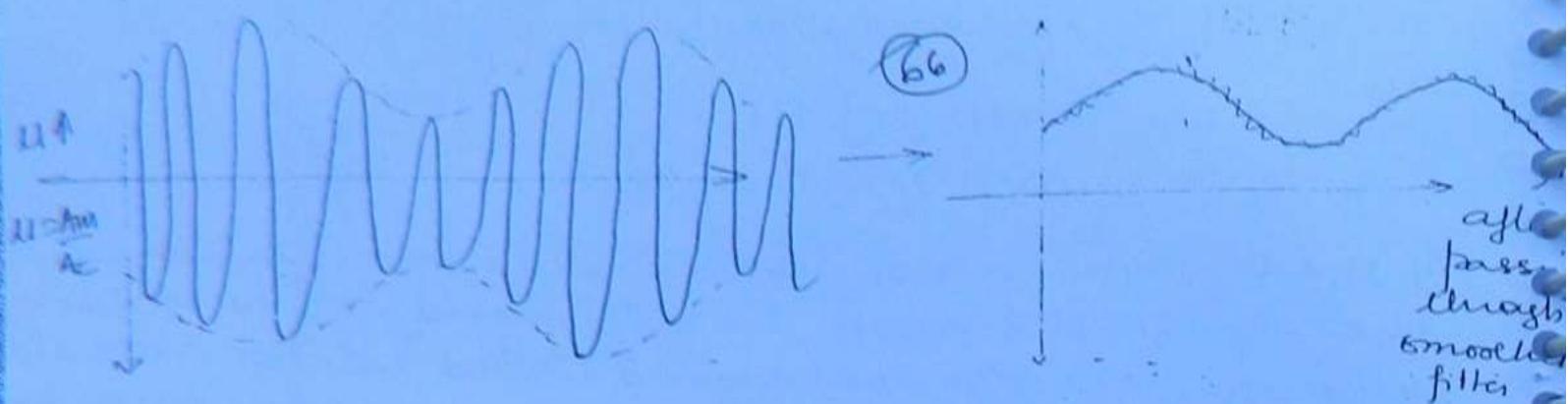
\* IMPORTANCE OF Kalam's sensitivity :-

As, we know that :-

$$SAM(t) = Ac \{1 + K_m(t)\} \cos \omega_m t$$

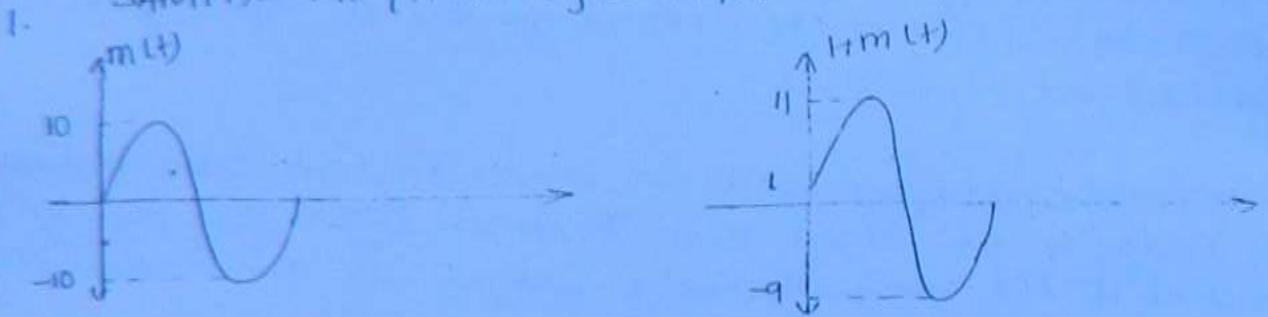


less Amplitude Variations  
(more prone to noise)

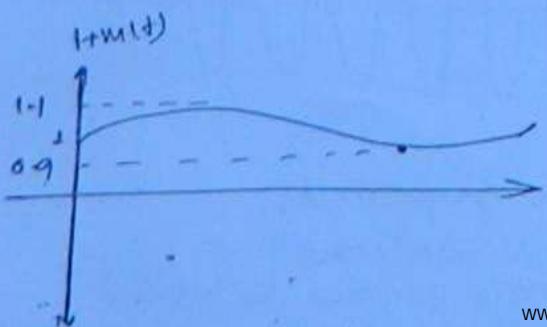
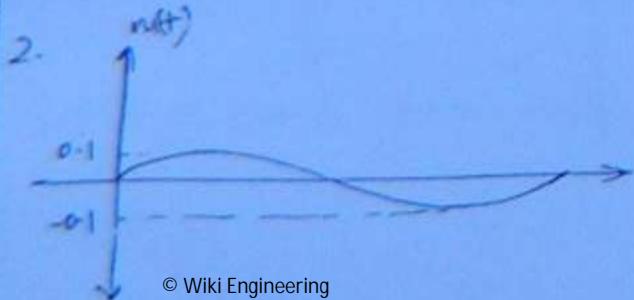
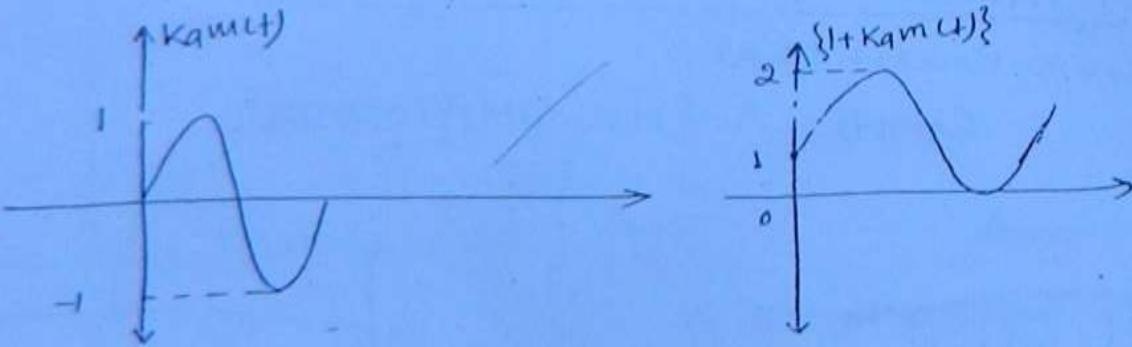


- Note:
1. If  $u$  is high, the effect of Noise is less on the reconstructed message signal.
  2. If  $u$  is low, the effect of Noise is dominant.

NOW,  
 $s_a m(t) = A_c \{1+m(t)\} \cos 2\pi f_c t$

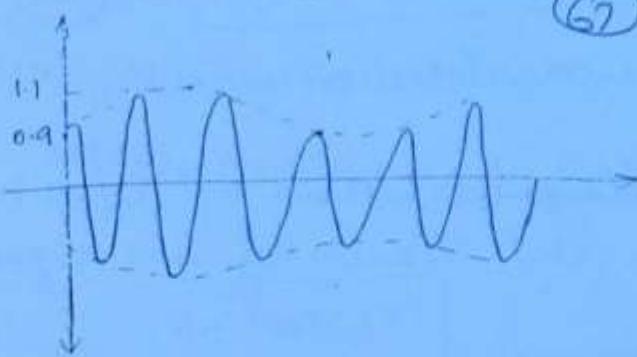


NOW,  $K_a = 1/10 \Rightarrow K_a m(t) \rightarrow$  attenuating message signal.



The AM signal has little amp variations and noise effect of Noise is high

(67)

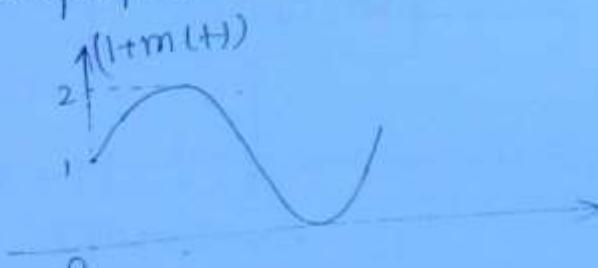
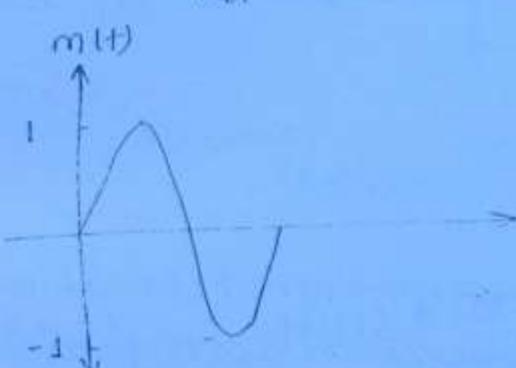


~~X~~ As effect of Noise is high

If dc value of 10 is also hence,  $A_{max} = 10+1 = 11$ ;  $A_{min} = 10-1 = 9$ . Then also amp variations are small and effect of noise is present.

3. As effect of Noise was high, since for small amp. variations. So, the message needs to be amplified.

So,  $K_a = 10 \Rightarrow K_a m(t) = \text{amplified.}$



Note:-

1.  $K_a$  specifies normalization of message signal for proper envelope detection and according to SIMON-HAYKINS,

$$u = K_a \cdot A_m \quad \leftarrow \text{For Square law & switching modulator}$$

Note:- originally,  $u = \frac{A_m}{A_c}$ , corresponds to the value of  $u$  when  $K_a = 1$ .

Now, if the signal needs to amplified or attenuated by  $K_a$ . Then,  $u = K_a \cdot A_m \leftarrow$  value of  $A_c$  is included in  $K_a$  or taken as 1.

2. For envelope detection,

where,

$$K_a = \frac{2A_2}{A_1} \quad \leftarrow \text{Square law modulator}$$

$$K_a = \frac{A_2}{A_1 A_c} \quad \leftarrow \text{switching modulator}$$

$$\max\{K_a m(t)\} \leq 1$$

$$\text{or } \max u \leq 1$$

3. For:

$$\max\{K_m(t)\} > 1 \rightarrow \text{over modulation}$$

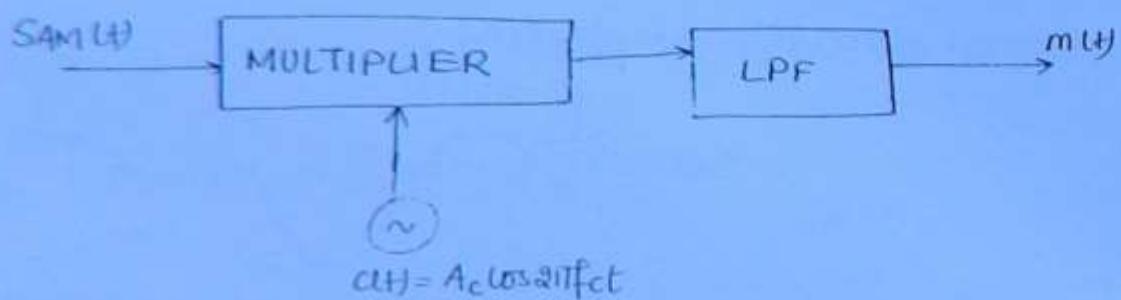
(68)

the O/P of the diode detection will be  $|m(t)|$ .

4. It specifies the voltage variations of <sup>super</sup> ~~of~~ decomposed message signal in the resulting AM signal.

### \*SYNCHRONOUS DETECTION:

#### Block-diagram:



\* For perfect reconstruction of message signal, local oscillator O/P should be perfectly synchronised in both frequency and phase.

\* Frequency synchronisation can be easily achieved but achieving phase synchronisation is very complex & difficult.

\* To achieve phase synchronisation additional complex circuitry has to be used which make the synchronous detector very complex.

#### Analysis:

Case:- I-O(O/P) =  $A_c \cos 2\pi f_c t$  {perfect synchronisation}.

$$\begin{aligned} \text{I-O(O/P)} &= A_c \cos 2\pi f_c t \\ \text{& } S_{AM}(t) &= A_c \{1 + K_m(t)\} \cos 2\pi f_c t \\ &= A_c \cos 2\pi f_c t + A_c K_m(t) \cos 2\pi f_c t \end{aligned}$$

The multiplier O/P is given as:-

$$S_{AM}(t) \times C(t).$$

$$\begin{aligned}
 \text{So, } (\text{Mul})_{\text{o/p}} &= \text{SAM}(t) \times C(t) \\
 &= \{A_c \cos 2\pi f_c t + A_c K_m(t) \cos \omega_m t\} \cdot A_c \cos 2\pi f_c t \\
 &= A_c^2 \cos^2 2\pi f_c t + A_c^2 K_m(t) \cos^2 \omega_m t \\
 &= \frac{A_c^2}{2} (1 + \cos 4\pi f_c t) + \frac{A_c^2 K_m(t)}{2} (1 + \cos 2\pi f_c t).
 \end{aligned} \tag{69}$$

$$\text{So, } (\text{LPF})_{\text{o/p}} = \frac{A_c^2 K_m(t)}{2}$$

- As the AM signal is transmitted for long distances, its ampl gets reduced.
- So, for its Reconstruction, it is passed through Amp<sup>1</sup>

CASE 2 :-

Assume,

$$(\text{L-O})_{\text{o/p}} = \cos \{2\pi f_c t + \Phi\} \leftarrow \{\text{NO phase synchronising}\}$$

$$\begin{aligned}
 \text{So, } (\text{Mul})_{\text{o/p}} &= \text{SAM}(t) \times \cos \{2\pi f_c t + \Phi\} \\
 &= \{A_c \cos 2\pi f_c t + A_c K_m(t) \cos \omega_m t\} \cos \{2\pi f_c t + \Phi\} \\
 &\quad \checkmark \text{(But blocked by black cap. of Amp<sup>1</sup>)} \\
 &= \frac{A_c^2}{2} \cos (4\pi f_c t + \Phi) + \frac{A_c^2}{2} (\cos \Phi + \frac{A_c^2 K_m(t)}{2} \cos (4\pi f_c t + \Phi)) \\
 &\quad + \frac{A_c^2 K_m(t)}{2} \cos \Phi.
 \end{aligned}$$

$$\text{So, } (\text{LPF})_{\text{o/p}} = \frac{A_c^2 K_m(t)}{2} \cos \Phi.$$

- Note :-
1. The message signal can be Reconstruction back, if the value of  $\cos \Phi$  ie  $\Phi$  remains const.
  2. But if the value of  $\Phi$  changes continuously with time, we have to implement an Amp<sup>1</sup> whose gain changes with  $\Phi$  to maintain m(t) at const value, & practically it is impossible to construct.

- \* Also if  $\phi = 90^\circ$   
 $\text{so } \cos\phi = \cos 90^\circ = 0$
- (70)
- So, the OIP at the demodulator is 0, ie no message can be extracted. This is called as "Quadrature NULL EFFECT" (QNE).

Note:-

1. For perfect Reconstruction of message signal  $[\Phi = \text{const}]$
2. To maintain  $\Phi = \text{const}$  additional complex clocking has to be used which makes SD very complex.
3. The demodulation of AM by SD is affected by "QNE".

### \* ADVANTAGES OF AM:-

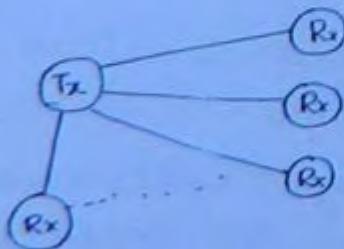
1. Primary Advantage is that the demodulation is simpler.
2. used for long distance comm.

### \* DRAWBACKS OF AM:-

1. Transmitter power is wasted. ( $\max^m m = 33\% \text{ of } P_t = P_{SB}$ )
2. It needs high transmission Bandwidth
3. Highly affected by Noise
4. QNE.

### \* APPLICATIONS OF AM:-

1. It is preferred to be used in Broadcasting.  
 (Point to Multi-point)



Since AM Receiver is simple and cheaper, it is highly preferred in Broadcasting.

## \* DOUBLE SIDE BAND-SUPPRESSED CARRIER (DSB-SC):

Assume, message signal =  $m(t)$

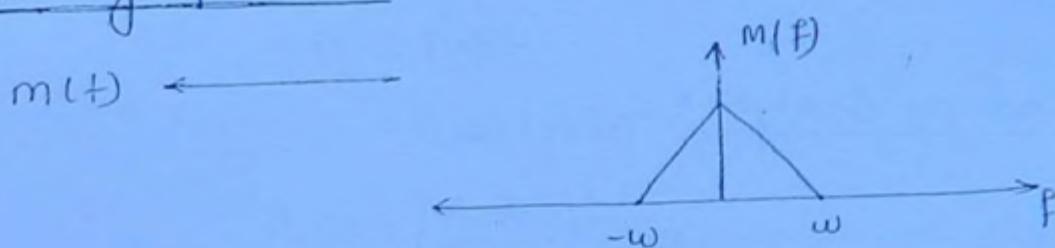
Carrier signal =  $c(t) = A_c \cos 2\pi f_c t$  (71)

## \* General exp. of DSB-SC Signal:

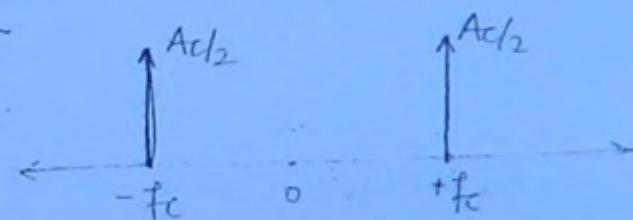
$$S_{DSB}(t) = m(t)c(t)$$

$$S_{DSB}(t) = A_c m(t) \cos 2\pi f_c t$$

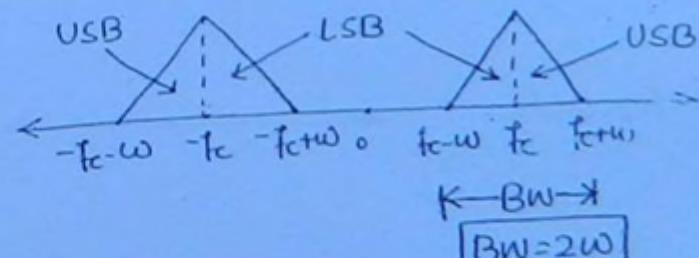
## \* Frequency spectrum:



$$c(t) = A_c \cos 2\pi f_c t$$



$$S_{DSB}(t)$$



Bandwidth = 2x message signal B.W

## \* SINGLE TONE DSB:

Assume,

$$m(t) = A_m \cos 2\pi f_m t$$

(72)

So,

$$s_{DSB}(t) = A_c m(t) \cos 2\pi f_c t$$

$$= A_c \cdot A_m \cos 2\pi f_m t \cdot \cos 2\pi f_c t$$

$$s_{DSB}(t) = \frac{A_c \cdot A_m}{2} \cos 2\pi (f_c + f_m)t + \frac{A_c A_m}{2} \cos 2\pi (f_c - f_m)t$$

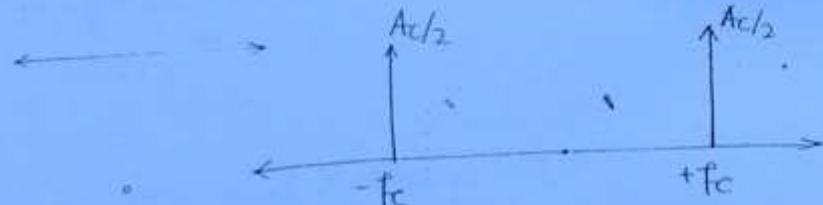
'USB'    'LSB'

Frequency spectrum Analysis:-

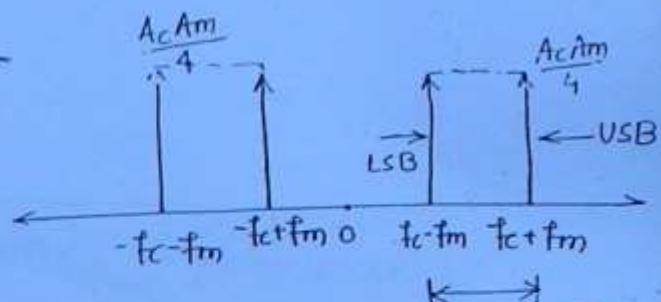
$m(t) \longleftrightarrow$



$$\therefore s = A_c \cos 2\pi f_c t$$



$s_{DSB}(t)$



Band width = 2x message signal frequency

$$[B.W = 2 f_m]$$

Note:

let  $m(t) = 5 \cos \pi t \times 10^3$   
 $c(t) = 10 \cos 2\pi t \times 10^6$

(73)

Then, Peak amp. of the DSB signal =  $\frac{AcAm}{2}$

Same in  
AM also

Peak amp. of the <sup>U</sup> SSB of DSB in spectrum =  $\frac{AcAm}{4}$

x Total Power of DSB-SC Signal:-  
The total power is given as:

$$P_t = P_{SB} \\ = P_{USB} + P_{LSB}$$

$$P_{USB} = \left( \frac{AcAm}{2} \right)^2 / 2R$$

$$P_{LSB} = \frac{Ac^2 Am^2}{8R}$$

So,  $P_t = 2 \cdot \frac{Ac^2 Am^2}{8R}$

$$\boxed{P_t = \frac{Ac^2 Am^2}{4R}}$$

x Modulation efficiency ( $\eta$ ) :-

As,  $\eta = \frac{P_{SB}}{P_t}$

$$\eta = \frac{P_t}{P_t}$$

$$\boxed{\eta = 1 = 100\%}$$

Q1. A carrier of  $20\sqrt{2} \cos \pi \times 10^5 t$  is DSB modulated by a message signal of  $2\sqrt{2} \cos \pi \times 10^3 t$ . Find all the parameters and plot the spectrum?

(74)

Sol<sup>n</sup>: Given:

$$m(t) = 2\sqrt{2} \cos \pi \times 10^3 t$$

$$A_m = 2\sqrt{2}; f_m = \frac{10 \times 10^3}{2} = 0.5 \text{ kHz}$$

$$c(t) = 20\sqrt{2} \cos \pi \times 10^5 t$$

$$\therefore A_c = 20\sqrt{2}; f_c = \frac{10 \times 10^4}{2} = 50 \text{ kHz}$$

NOW,

$$P_t = \frac{A_c^2 A_m^2}{4R} = \frac{800 \times 8^2}{4 \times 1}$$

$$P_t = 1600 \text{ W} \quad \left\{ \text{Normalised power, } R=1 \Omega \right\}$$

Answ.

$$\text{Band width} = 2f_m = 2 \times 0.5 \text{ kHz}$$

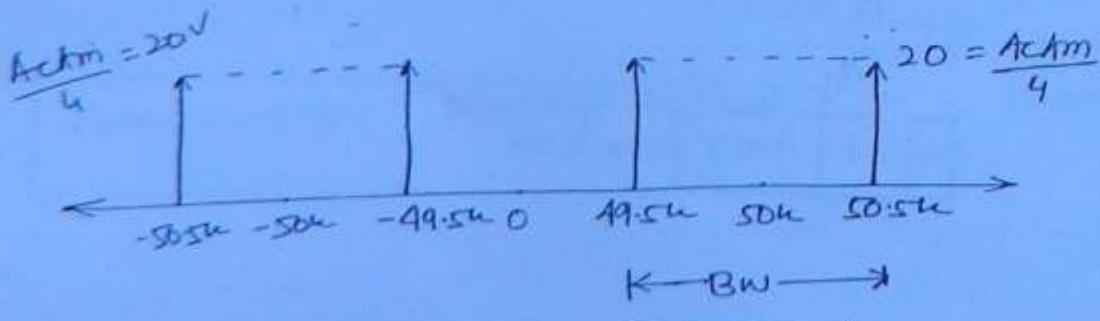
$$B.W = 1 \text{ kHz} \quad \text{Answ}$$

$$2. P_{USB} = P_{LSB} = \frac{P_t}{2} = 800 \text{ W} \quad \text{Answ}$$

$$3. m = 100\%.$$

Answ.

Spectrum:



$$B.W = 1 \text{ kHz}$$

Answ.

## \* MULTI-TONE DSB:

Assume,

$$m(t) = A_{m1} \cos 2\pi f_{m1} t + A_{m2} \cos 2\pi f_{m2} t$$

(75)

$$S_{DSB}(t) = A_c m(t) \cos 2\pi f_c t$$

$$= A_c \{ A_{m1} \cos 2\pi f_{m1} t + A_{m2} \cos 2\pi f_{m2} t \} \cos 2\pi f_c t$$

$$= A_c [A_{m1} \cos 2\pi f_{m1} t \cos 2\pi f_c t + A_{m2} \cos 2\pi f_{m2} t \cos 2\pi f_c t]$$

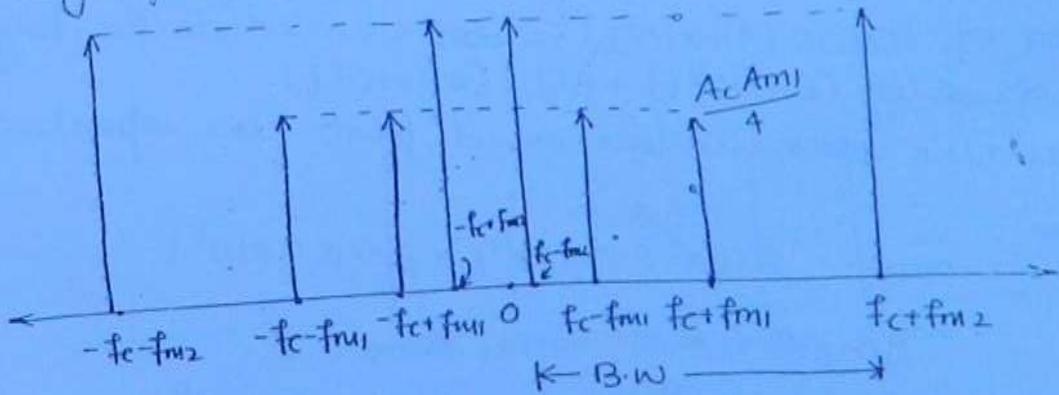
$$S_{DSB}(t) = \frac{A_c A_{m1}}{2} \cos 2\pi (f_c + f_{m1}) t + \frac{A_c A_{m1}}{2} \cos 2\pi (f_c - f_{m1}) t$$

$$+ \frac{A_c A_{m2}}{2} \cos 2\pi (f_c + f_{m2}) t + \frac{A_c A_{m2}}{2} \cos 2\pi (f_c - f_{m2}) t$$

\* For plotting of spectrum.

Let  $f_{m2} > f_{m1}$   
 $A_{m2} > A_{m1}$

## \* Frequency Spectrum:



B.W =  $2 \times \text{Max}^m$  frequency of  
mag. signal.

$$\boxed{B.W = 2 \times f_{max}}$$

## \* Total Power of Multitone DSB:

As,

$$P_f = P_{SB}$$

where,

$$P_{SB} = P_{USB(\text{Total})} + P_{LSB(\text{Total})}$$

$$= P_{USB1} + P_{USB2} + P_{LSB1} + P_{LSB2}$$

Now,

$$P_{USB1} = \frac{\left(\frac{A_c A_m_1}{2}\right)^2}{2R} = \frac{A_c^2 A_m_1^2}{8R} = P_{LSB1}$$

$$P_{USB2} = \frac{\left(\frac{A_c A_m_2}{2}\right)^2}{2R} = \frac{A_c^2 A_m_2^2}{8R} = P_{LSB2}$$

So,

$$P_f = \frac{A_c^2 A_m_1^2}{4R} + \frac{A_c^2 A_m_2^2}{4R}$$

$$\boxed{P_f = \frac{A_c^2 [A_m_1^2 + A_m_2^2]}{4R}}$$

Q1. A carrier of  $10\cos(4\pi \times 10^6 t)$  is DSB modulated by a message signal of  $6\cos(6\pi \times 10^4 t) + 8\cos(8\pi \times 10^4 t)$ . Find all the parameters and plot the spectrum.

Soln: Given

$$m(t) = 6\cos(6\pi \times 10^4 t) + 8\cos(8\pi \times 10^4 t)$$

$$A_m_1 = 6 \quad ; \quad A_m_2 = 8$$

$$f_m_1 = 3 \times 10^4 \text{ Hz} \quad ; \quad f_m_2 = \frac{10 \times 10^4}{2} \\ = 50 \text{ K}$$

$$c(t) = 10\cos(4\pi \times 10^6 t)$$

$$A_c = 10 \quad ; \quad f_c = 2 \text{ MHz} = 2000 \text{ K}$$

So,

$$\boxed{B.W = 2.f_m(\text{max}) = 2 \times 50 \text{ K} = 100 \text{ K}}$$

$$2. P_t = \frac{A_c^2 A_m^2}{4R} + \frac{A_c^2 A_m^2}{4R}$$

(77)

$$P_t = \frac{A_c^2}{4R} \{ A_m^2_1 + A_m^2_2 \}$$

$$= \frac{100}{4} \{ 36 + 64 \}$$

$$\boxed{P_t = 2500 \text{ W}}$$

$$3. P_{USB} = P_{LSB} = 1250 \text{ W}$$

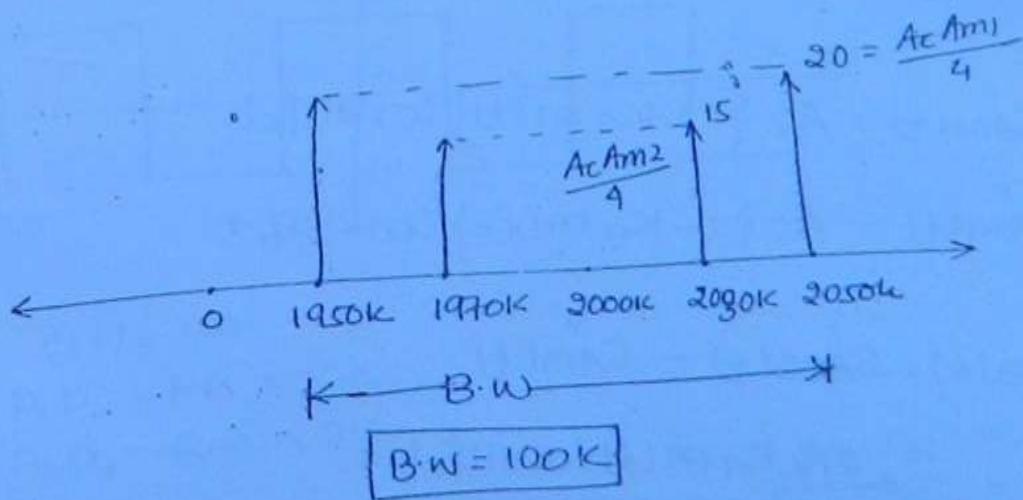
(Total)      (Total)

$$\text{Now, } P_{USB1} = \frac{A_c^2 A_m^2_1}{8R} = P_{LSB1}$$

$$\boxed{P_{USB1} = \frac{25}{100 \times 36} \times 18 = 450 \text{ W}} \\ = P_{LSB1} \quad 8g$$

$$\boxed{P_{USB2} = 800 = P_{LSB2}} =$$

Frequency spectrum:



## \* GENERATION OF DSB-SC SIGNAL:

The Generation methods are:

(78)

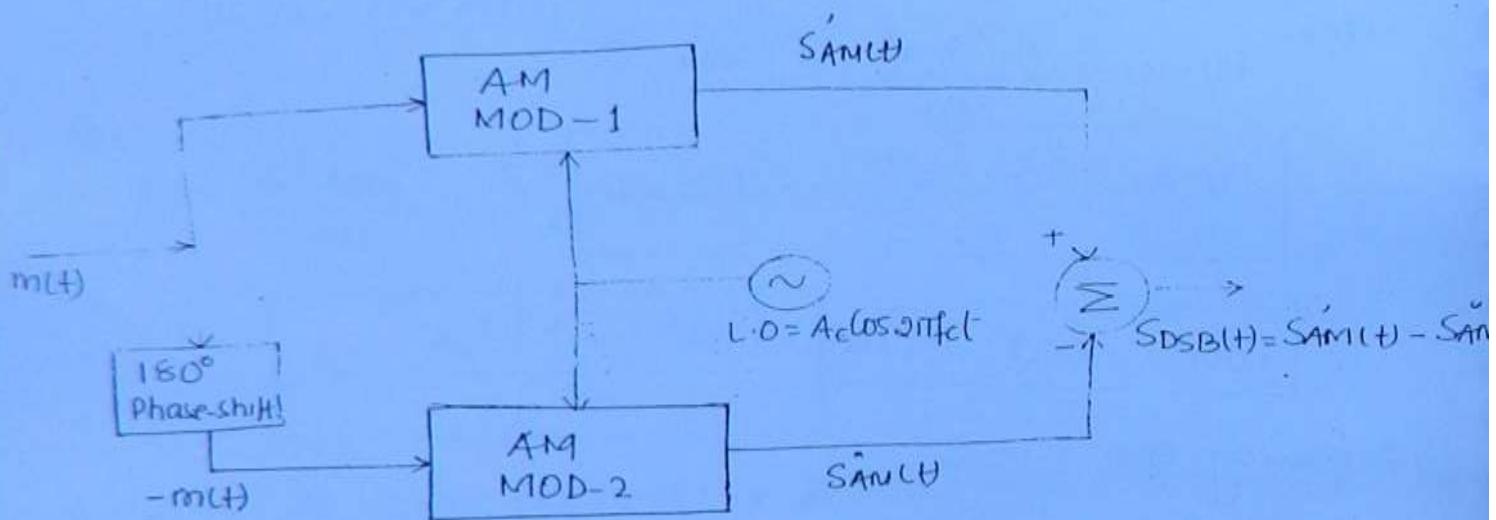
1) Balanced Modulator.

2) Ring modulator.

## \* BALANCED MODULATOR:

1. In this two AM modulators are connected in Balance, to generate DSB signal.

Block diagram:



Now,

$$S'AM(t) = A_c \{ 1 + K_a m(t) \} \cos 2\pi f_c t$$

$$\& S''AM(t) = A_c \{ 1 - K_a m(t) \} \cos 2\pi f_c t$$

So,

$$\begin{aligned} SDSB(t) &= S'AM(t) - S''AM(t) \\ &= 2A_c K_a m(t) \cos 2\pi f_c t \end{aligned}$$

$$SDSB(t) = A'_c m(t) \cos 2\pi f_c t$$

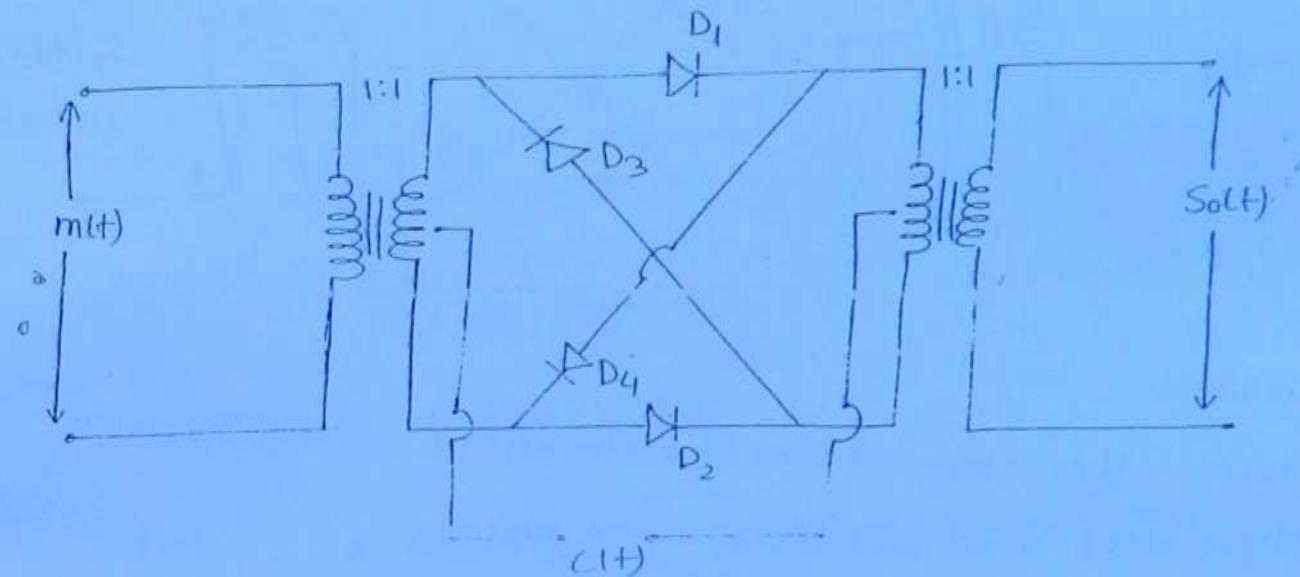
where

$$A'_c = 2A_c K_a$$

\* In this 4 diodes are connected in the form of RING to generate DSB signal

(79)

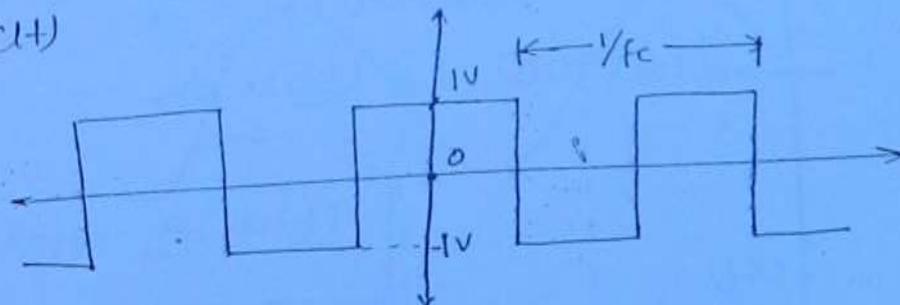
\* Block-Diagram:



Note:

- Assume the diodes are ideal.
- Transformers are centre-tapped and of 1:1 type.

Let,  $c(t+)$

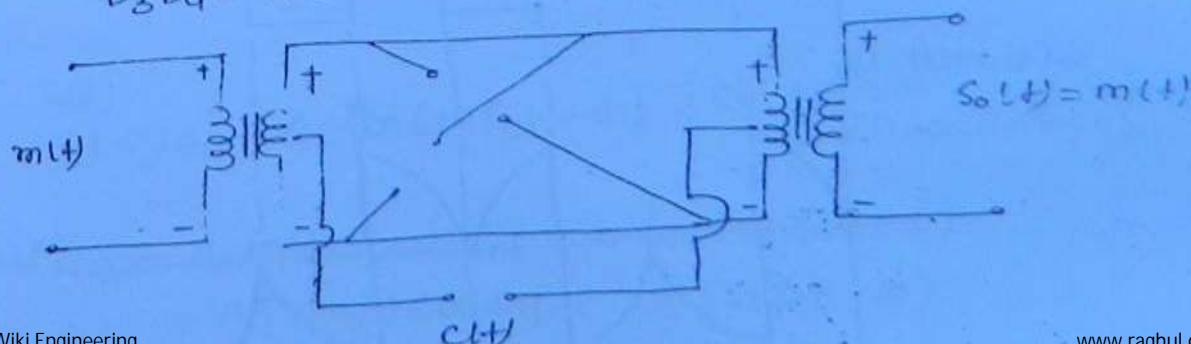


Op^n:

1. When  $c(t+) = +ve$

$$D_1, D_2 = F.B = S \cdot C$$

$$D_3, D_4 = R.B = O \cdot C$$

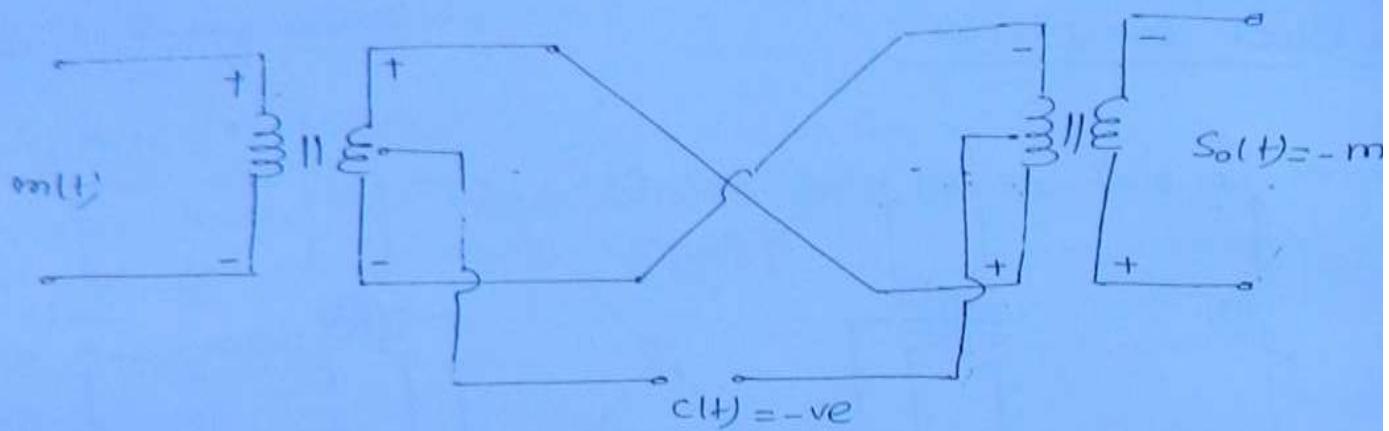


$c(t) = +ve$  : then

$$D_3 D_4 = FB = +ve$$

$$D_1 D_2 = RB = -ve$$

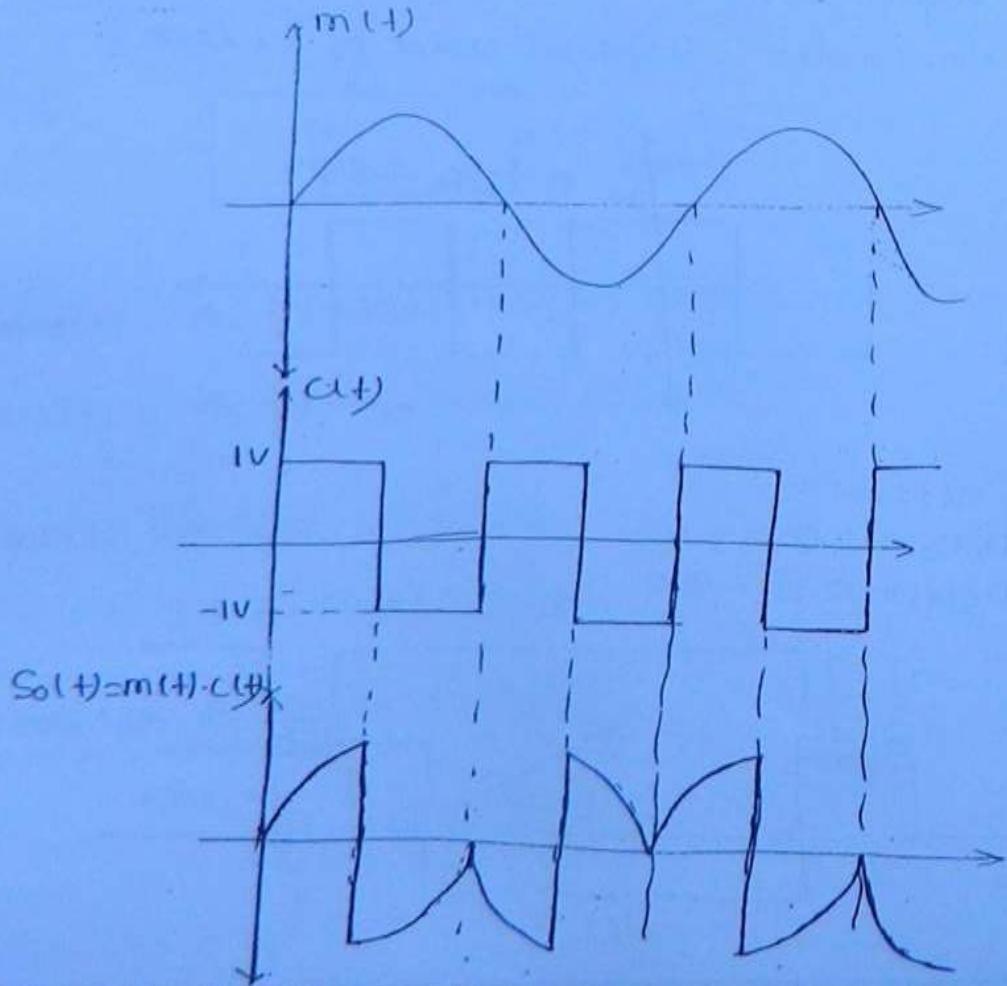
(80)



Conclusion:

1. when  $c(t) = +ve \Rightarrow S_0(t) = +ve m(t)$
- when  $c(t) = -ve \Rightarrow S_0(t) = -ve m(t)$

Ans



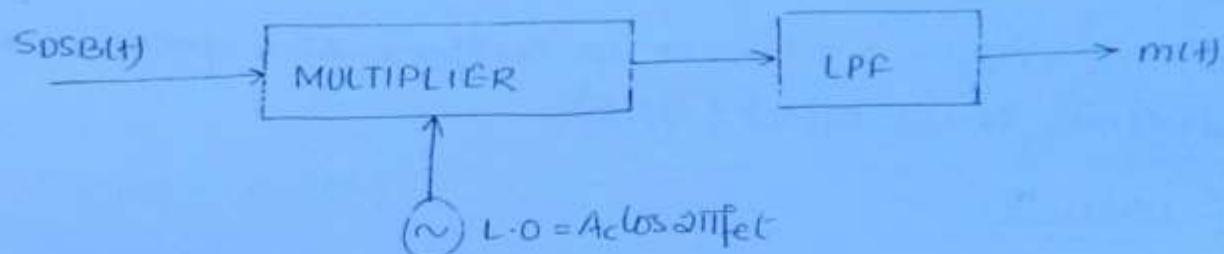
## \* DEMODULATION OF DSB-SIGNAL :-

The demodulation of DSB is done by Synchronous detector.

### \* SYNCHRONOUS-DETECTOR:

(8)

B-Diag<sup>m</sup>



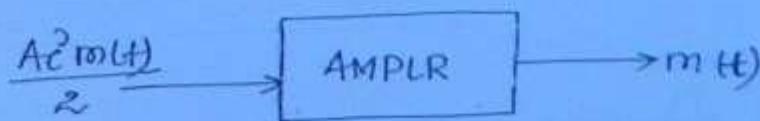
NOW:-

$$SDSB(t) = A_c m(t) \cos 2\pi f_c t$$

Case 1: let L.O. = A\_c cos 2πf\_ct (perfect synchronization)

$$\begin{aligned} (\text{Mul})_{\text{O/P}} &= SDSB(t) \times (\text{L.O.})_{\text{O/P}} \\ &= A_c m(t) \cos 2\pi f_c t \times A_c \cos 2\pi f_c t \\ &= A_c^2 m(t) \cos^2 2\pi f_c t \\ &= \frac{A_c^2 m(t)}{2} \{1 + \cos 4\pi f_c t\} \end{aligned}$$

$$(\text{L.P.F.})_{\text{O/P}} = \frac{A_c^2 m(t)}{2}$$



Case 2:

let,  
 $(\text{L.O.})_{\text{O/P}} = A_c \cos(2\pi f_c t + \Phi)$  {No phase synchronization}

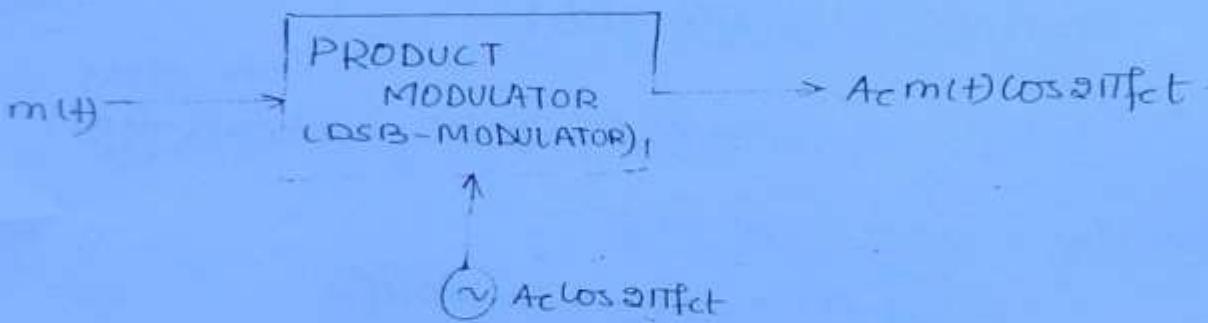
$$\begin{aligned} (\text{Mul})_{\text{O/P}} &= SDSB(t) \cos 2\pi f_c t \times A_c \cos(2\pi f_c t + \Phi) \\ &= A_c^2 m(t) \cos 2\pi f_c t \cdot \cos(2\pi f_c t + \Phi) \end{aligned}$$

$$(MUL)_{O/P} = \frac{A_c^2 m(t)}{2} \cos(\omega_{cif} t + \phi) + \frac{A_c^2 m(t)}{2} \cos \phi$$

(P2)

$$x(LPF)_{O/P} = \frac{A_c^2 m(t)}{2} \cos \phi$$

- \* To maintain the  $\phi$  const additional circuitry has to be utilised.
- \* When  $\phi = 90^\circ$ , O/P = 0, hence it suffers the problem of Quadrature Null effect (QNE).
- \* Product Modulator:



Note:-

- \* Product modulator is the alternate name given to a DSB modulator.

#### ADVANTAGES OF DSB :-

1. Transmitter power will be saved ( $\eta \approx 100\%$ ).
2. used for long distance commn.

#### DRAWBACKS OF DSB :-

1. Demodulation is complex.
2. It needs high transmission Bandwidth.
3. affected by QNE.

#### APPLICATION:

1. It is used in Quadrature carrier Multiplexing

(83)

## X-SINGLE SIDE BAND - SUPPRESSED CARRIER (SSB):-

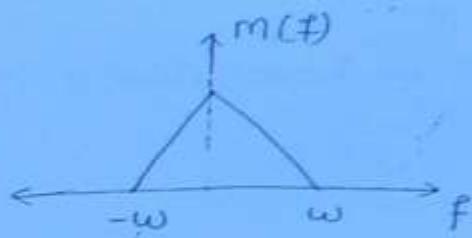
\*The advantage of SSB over AM and DSB is both the transmitter power and BW will be saved.

Let

$$m(t)$$



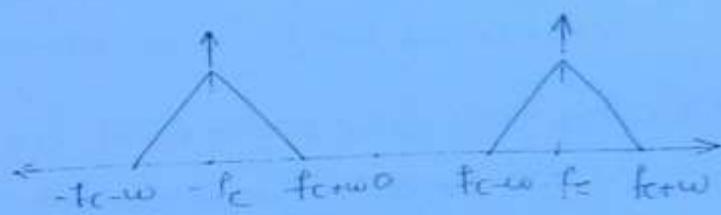
$$c(t) = A_0 \cos 2\pi f_c t$$



$$\text{SAM}(t)$$

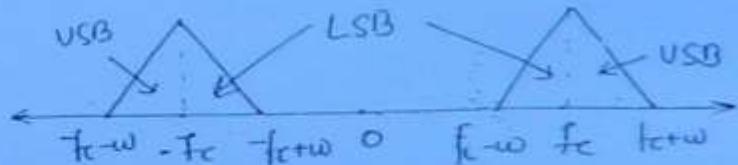


$$\boxed{\text{AM } B \cdot \omega = 2\omega}$$



$$\text{DSB}(t)$$

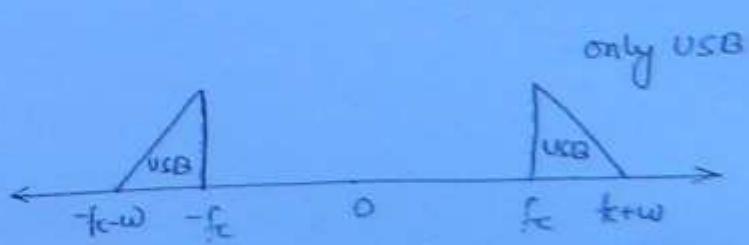
$$\boxed{\text{DSB } B \cdot \omega = 2\omega}$$



$$\text{SSB}(t)$$

$$\boxed{\text{SSB } B \cdot \omega = \omega}$$

Message  
Signal BW



OR

only LSB.



(84)

a

v

w

e

Note:

1. The demodulation of SSB is also done by the SB. The SB gives the message signal as OIP whenever the 2 sidebands (as in DSB) are given as 9IP, 9I abd the m(+) whenever only 1 sideband (as in SSB) is given. Hence SSB is preferred over DSB.

(8)

2. By doing this:

- 1) Power is saved (for emitting 2 sideband as in DSB)
- 2) Band width is saved

Note:

1. In SSB, compared to DSB 50% of the transmitter power and 50% of transmission B.W will be saved.
2. The % of power saved in DSB and SSB compared to AM depends on the modulation index ( $M$ ).

Eg.

## SINGLE TONE SSB:

Let

$$m(t) = A_m \cos \omega_m t$$

$$S.A.M(t) = A_c \{ 1 + m(t) \cos \omega_m t \} \cos \omega_c t$$

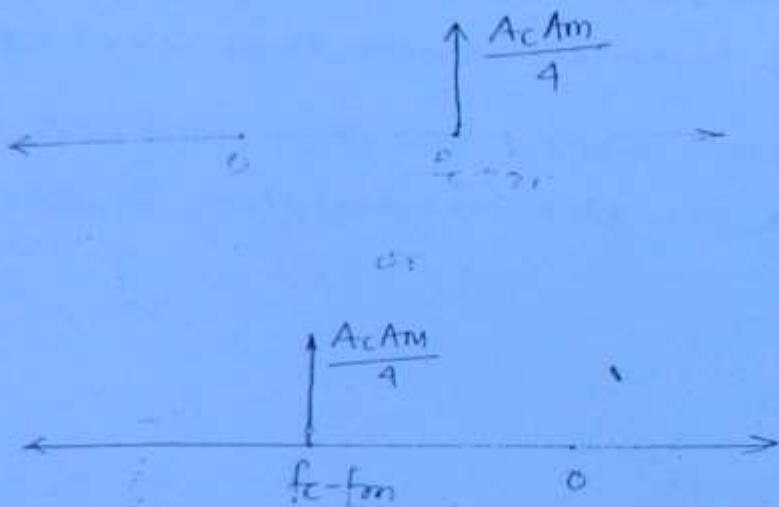
$$S.S.C.B(t) = \frac{A_c A_m}{2} \cos \pi (f_c + f_m)t + \frac{A_c A_m}{2} \cos \pi (f_c - f_m)t$$

(86)

NOW, the corresponding SSB expression is given as:-

$S.S.C.B(t) = \frac{A_c A_m}{2} \cos \pi (f_c \pm f_m)t$	$\rightarrow \text{USB}$
	$\rightarrow \text{LSB}$

spectrum:-



Bandwidth of Single tone SSB:-

$$B.W = 0$$

\* Total Power:-

$$P_t = P_{SSB}$$

$$= P_{USB} \text{ or } P_{LSB}$$

NOW,

$$P_{SSB} = \left( \frac{A_c A_m}{2} \right)^2 / 2R$$

$$P_{USB} = \frac{A_c^2 A_m^2}{8R}$$

$$P_t = \frac{A_c A_m}{8R}$$

x Modulation efficiency (%) :-

(78)

$$\% \eta = \frac{P_{SB}}{P_t} = \frac{P_{SB}}{P_{PSB}} \times 100$$

$$\eta = 100\%$$

x General expression of SSB:-

The General expression of AM is:-

$$SAM(t) = A_c \{ 1 + K_a m(t) \} \cos \omega_c t$$

for DSB is:-

$$SDSB(t) = A_c m(t) \cos \omega_c t$$

The General expression of SSB is given as:-

$$SSSB(t) = \frac{A_c A_m}{2} \cos 2\pi(f_c \pm f_m)t$$

$\rightarrow$  USB  
 $\rightarrow$  LSB

$$SSSB(t) = \frac{A_c A_m}{2} \cos 2\pi f_c t \cos 2\pi f_m t + \frac{A_c A_m}{2} \sin 2\pi f_c t \sin 2\pi f_m t$$

$\rightarrow$  USB  
 $\rightarrow$  LSB

We have;

$$m(t) = A_m \cos 2\pi f_m t$$

and  $90^\circ$  phase shift of  $m(t)$  we get  $\left[ \hat{m}(t) = 90^\circ \text{ phase shift of } m(t) \right]$

$$\hat{m}(t) = A_m \sin 2\pi f_m t$$

where,  $\hat{m}(t) = \text{HILBERT function of } m(t)$

So, substituting in the above equation we get:-

$$SSSB(t) = \frac{A_c m(t) \cos 2\pi f_c t}{2} + \frac{A_c \hat{m}(t) \sin 2\pi f_c t}{2}$$

$\rightarrow$  USB  
 $\rightarrow$  LSB

$\hookrightarrow$  single tone  
SSB signal

% of power saved in DSB and SSB v/s AM

iii,

$$\mu = 0.707$$

Then, after modulation, for AM

$$P_t = 100W$$

$$P_C = 80W \quad P_{DSB} = 20W$$

(88)

For AM:

<u>P<sub>t</sub></u>	<u>P<sub>C</sub></u>	<u>P<sub>DSB</sub></u>	<u>P<sub>SSB</sub></u>
100W	80W	10W	10W

For DSB:-

20W	-	10W	10W
-----	---	-----	-----

For SSB:

10W	-	10W or 10W	
-----	---	------------	--

Note:-

1. In SSB as compared to AM, 90% of transmitter power is saved.
2. In DSB as compared to AM, 80% of transmitter power is saved.
3. In SSB as compared to DSB, 50% of transmitter power is saved.

Note:-

1. The amount of power saved in DSB & SSB compared to AM depends on  $\mu$ .
2. Whatever be the value of  $\mu$  the amount of power saved in SSB compared to DSB is always 50%.

$$P_t = P_C \left\{ 1 + \frac{\mu^2}{2} \right\}$$

$$P_{DSB} = \frac{P_C \mu^2}{2}; P_{SSB} = P_{LSB} = \frac{P_C \mu^2}{4}$$

Now;

% of power saved in DSB compared to AM =  $\frac{\text{Power saved}}{\text{total power}}$

(89)

$$= \frac{P_c}{P_t} = \frac{P_c}{\left(1+U^2\right) P_c}$$

\*\*

$$\% \text{ of power saved in DSB v/s AM} = \frac{2}{2+U^2} = 1-\eta = 1 - \frac{P_{SB}}{P_t}$$

% of power saved in SSB compared to AM =  $\frac{\text{Power saved}}{\text{total power}}$

$$= P_c + \frac{P_c U^2}{4}$$
$$\frac{P_c \left\{ 1 + \frac{U^2}{2} \right\}}{P_c \left\{ 1 + \frac{U^2}{2} \right\}}$$

\*\*

$$\% \text{ of power saved in SSB v/s AM} = \frac{4+U^2}{4+2U^2}$$

Q1. An AM transmitter power is given by 500W. Find the amount of power saved if carrier and one of the SB is suppressed with i)  $U = 0.5$  ?  
ii)  $U = 0.8$  ?

Soln:

Given:  $P_t = 500W$

for  $U=0$ ;  $P_t = P_c = 500W$

so,  $P_t = P_c \left\{ 1 + \frac{U^2}{2} \right\}$

$$= 500 \left\{ 1 + \frac{0.25}{2} \right\}$$

$$= 501.25$$

so, % of power saved in SSB =  $\frac{4+U^2}{4+2U^2}$

i)  $U = 0.5$

% of power saved =  $0.4451 \cdot 100$

Amount of power saved =  $0.4451 \cdot 500$

$$= 222.5W$$

ii)  $U = 0.8$

% of power saved =  $0.8756 \cdot 100$

Amount of power saved =  $0.8756 \cdot 500$

$$= 437.8W$$

## \* Generation of SSB:

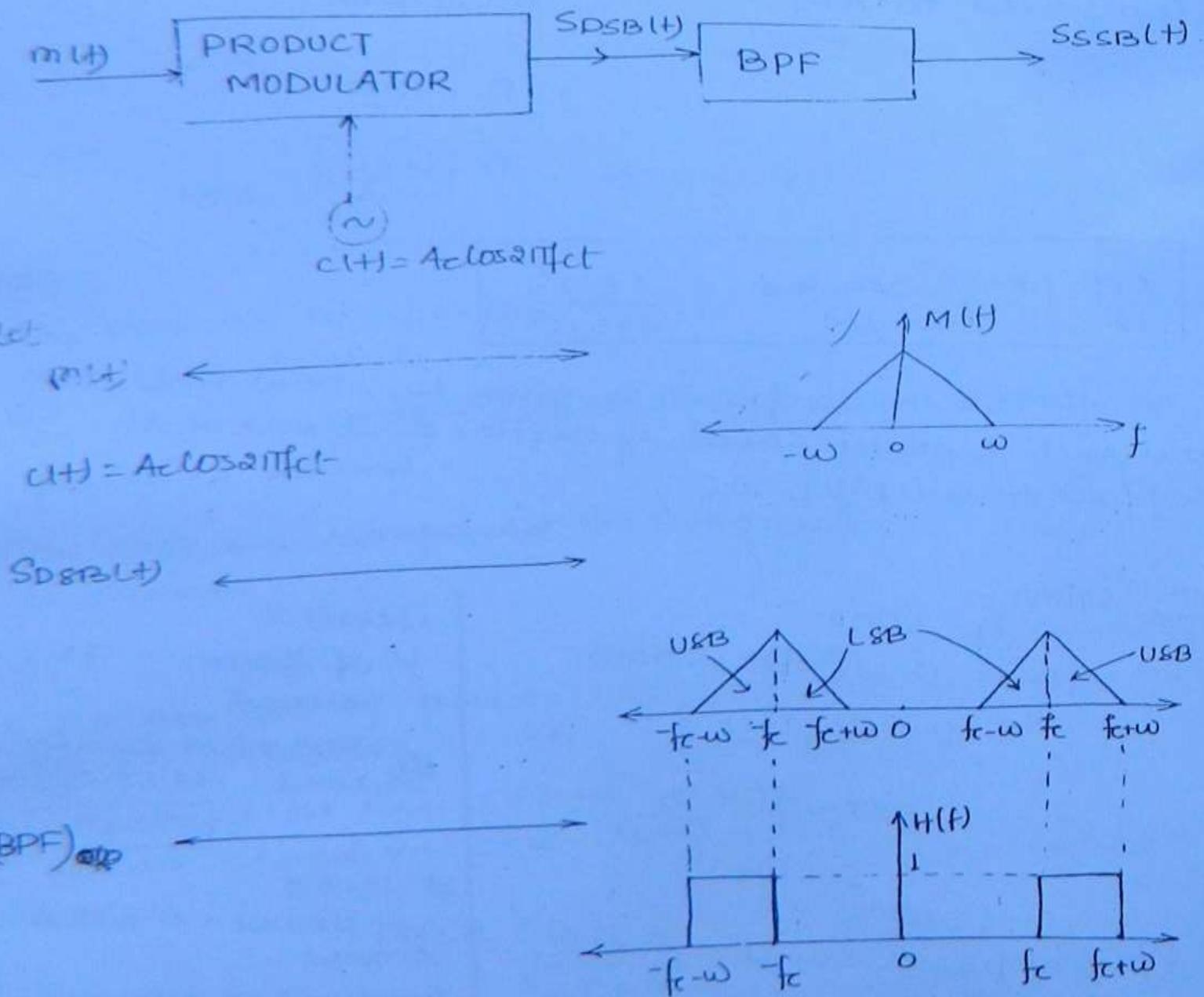
The Generation methods of SSB are: (90)

- Frequency discrimination method
- Phase discrimination method

## \* FREQUENCY DISCRIMINATION METHOD:

In this DSB signal is transmitted through proper Band pass filter to generate SSB.

## \* Block Diagram:

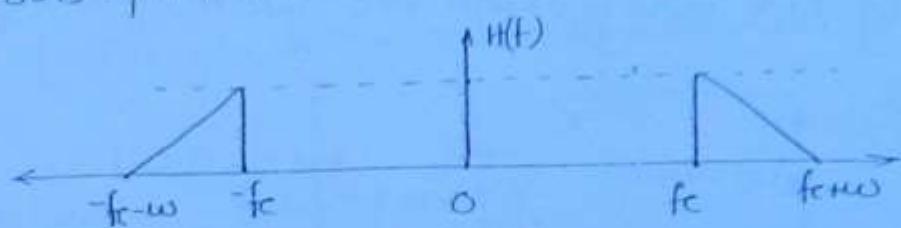


So the BPF; O/P is given as

$$y(t) = H(f) \times I(t)$$

(91)

$(BPF)_{O/P}$  = SSB spectrum

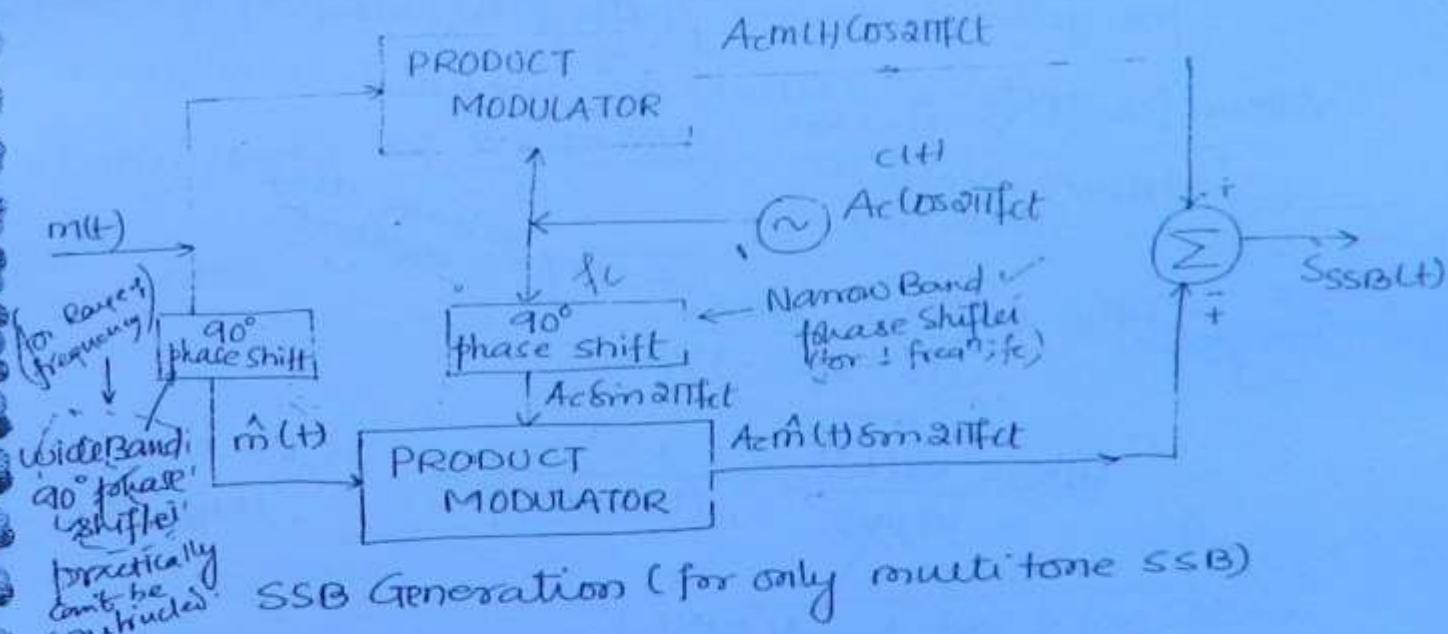


### \* PHASE DISCRIMINATION METHOD:

The General expression of SSB signal is given as

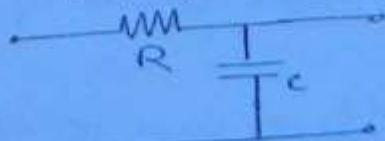
$$SSB(t) = \frac{A_m m(t) \cos \omega_f t}{2} + \frac{A_m \hat{m}(t) \sin \omega_f t}$$

Block diagram:

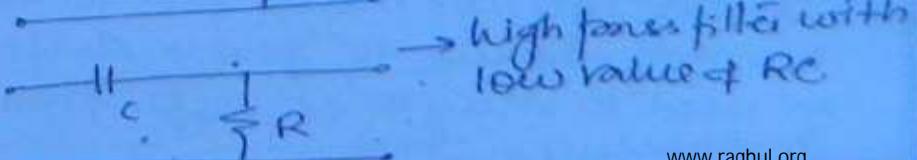


Note:-

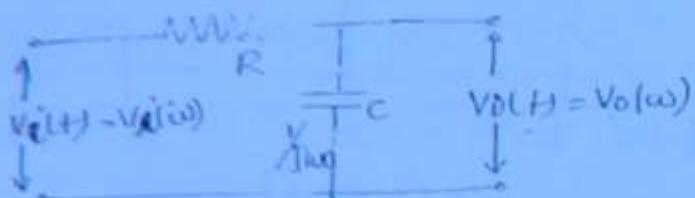
1. Phase shift  $n/w$  is nothing but differentiator
2. Integrator  $\rightarrow$  low pass filter with high RC



3. Differentiator  $\rightarrow$



- Generally for realization of  $90^\circ$  phase shifter differentiators or integrators will be used.
- Q2. as
- the following LPF with high RC works as differentiator.



$$\text{So, } V_o(w) = V_i(w) \cdot H(w)$$

$$|V_o(w)| = |V_i(w)| |H(w)| \leftarrow \text{Magnitude}$$

$$\angle V_o(w) = \angle V_i(w) + \angle H(w) \leftarrow \text{phase.}$$

For getting phase shifted by  $90^\circ$ ,  $\angle H(w) = 90^\circ$

Now, for getting  $\angle H(w) = 90^\circ$ , we have:

$$H(w) = \frac{V_o(w)}{V_i(w)}$$

*By voltage division method*

$$H(w) = \frac{V_i(w) \cdot \frac{1}{jwC}}{(R + \frac{1}{jwC}) V_i(w)}$$

$$H(w) = \frac{V_o(w)}{V_i(w)} = \frac{1}{(1+wRC)}$$

$$\text{So, } \angle H(w) = 0 - \tan^{-1}(\frac{wRC}{1})$$

$$\boxed{\angle H(w) = -\tan^{-1}(wRC)}$$

$\rightarrow$  To obtain  $\angle H(w)$  the value of  $wRC \approx \infty$ , but practically it is not possible.

$\rightarrow$  Let the  $m(t)$  be multitone ie contain range of frequencies. Hence to obtain  $90^\circ$  phase shift for Range of frequency we need to vary  $R & C$  with frequency, which can't be implemented.

Note:  
 x The practical significance of single tone SSB generation  
 chkd is not there; as for transmission, multi tone  
 SSB takes place. Hence there is no practical signifi-  
 cance of single tone SSB/Phase discrimination method.

(93)

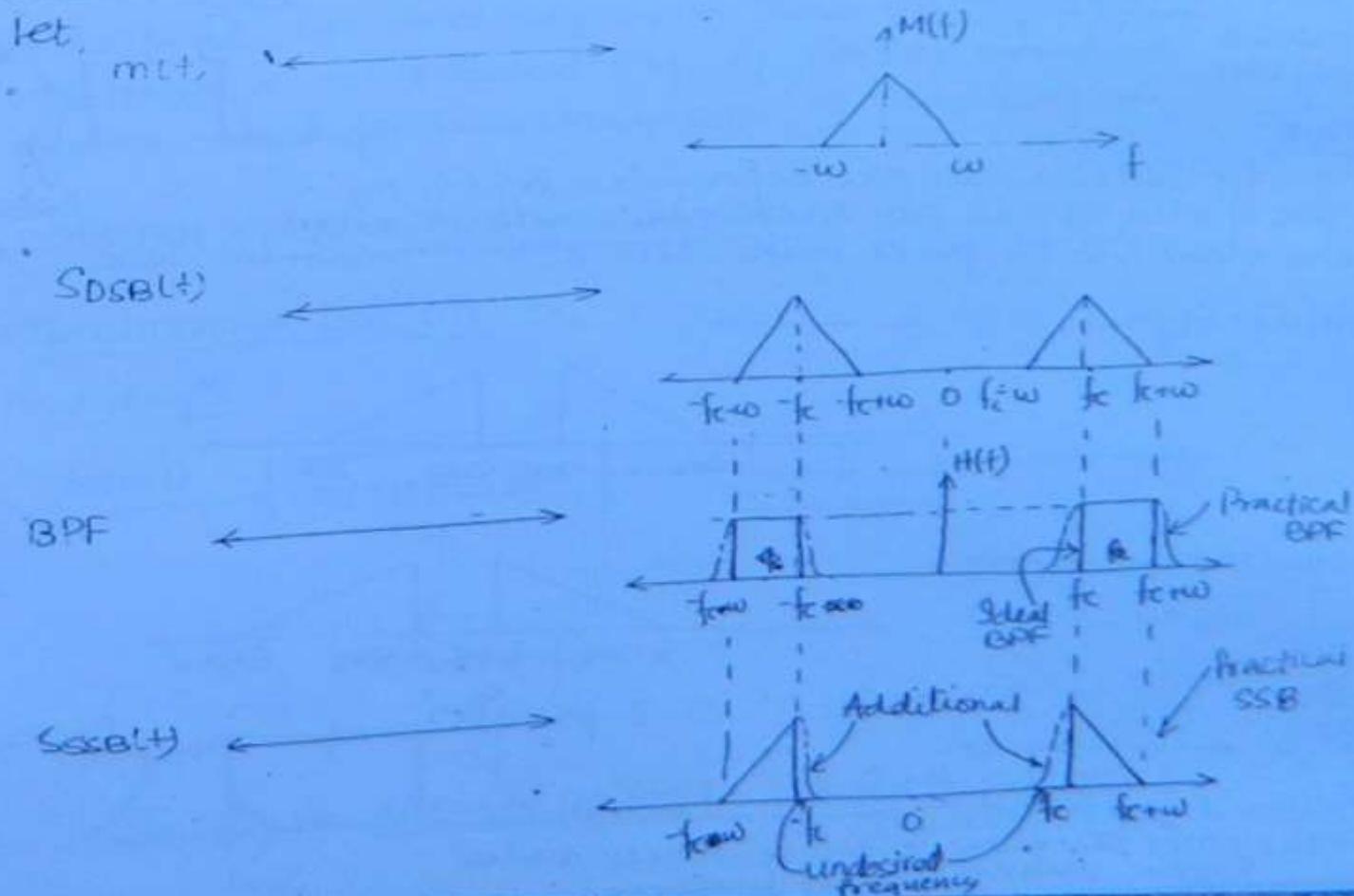
Note:-

$$\angle H(i\omega) = -\tan^{-1}(\omega RC)$$

From above: diff. frequency components of input are  
 experiencing diff. phase shift. So, it is practically  
 not possible to construct wide Band  $90^\circ$  phase  
 shifter. so the above method is failed for the gener-  
 ation of Multi tone SSB signal.

Note: For Generation of Multitone SSB frequency dis-  
 crimination method will be used.

Drawbacks of Frequency discrimination method:

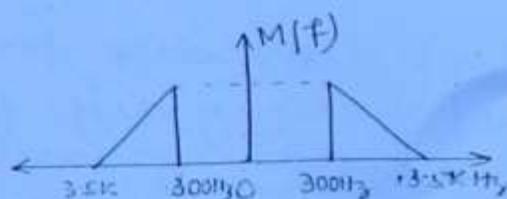


### Note :-

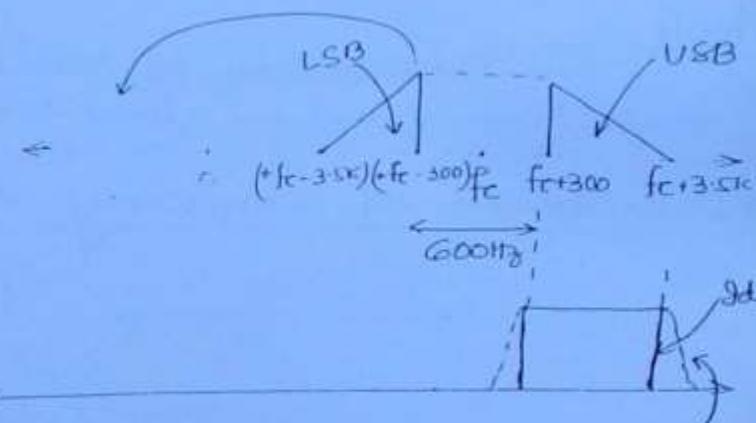
- Since Ideal BPF Cannot be constructed, the resulting SSB signal contains undesired frequencies in addition to actual side bands. (Q)
- Because of above drawback SSB is limited only for voice signal transmission.

### Analysis:-

1. Voice Signal:  $m(t) \longleftrightarrow$



SDSBS(t)  $\longleftrightarrow$



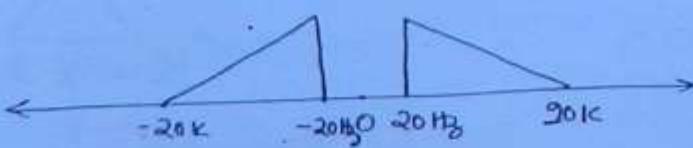
(BPF)  
passing  
only USB

### Note:-

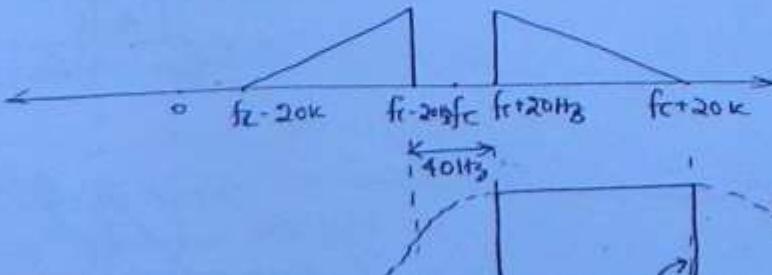
The Sidebands are separated by 600 Hz.  
Also, if the BPF is not ideal, it will not allow the other SB to pass. Hence the transmission can be done.

Practical

2. Audio Signal:  $m(t) \longleftrightarrow$

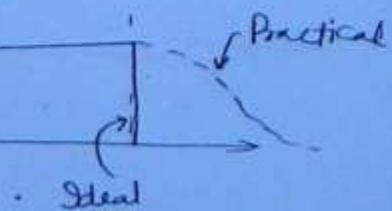


SDSBS(t)  $\longleftrightarrow$

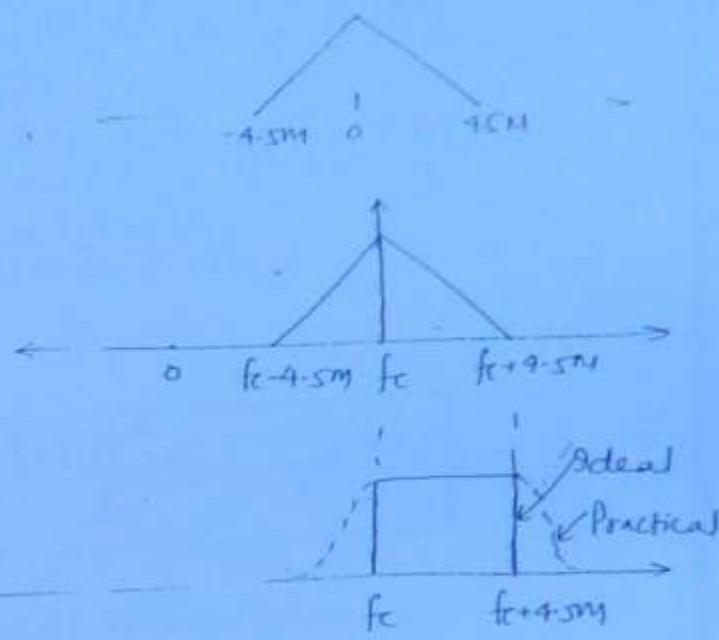


(BPF)  $\longleftrightarrow$

Band gap is 40K Hz  
So, the other SB may interfere. Hence Audio signal transmission is not preferred.



(95)



$\text{Band gap} = 0$   
Hence, the video signal  
can't be transmitted through SSB scheme  
as other SSB may interfere.

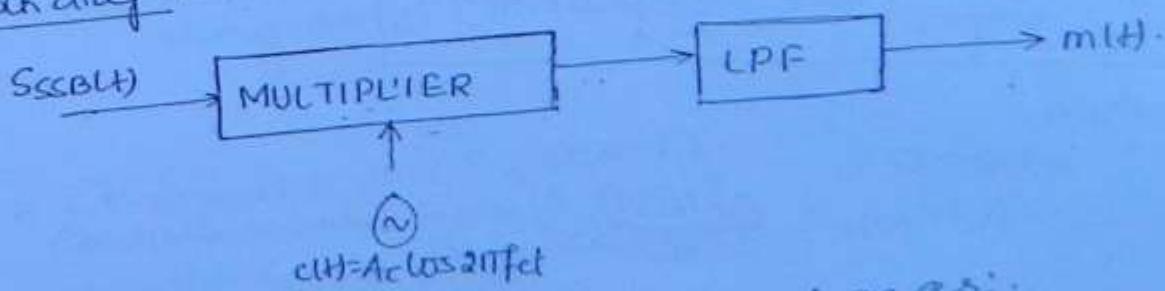
### Conclusion:

- For transmission of a message signal, in the respective DSB spectrum, a wide Band gap should be existing b/w side bands; for voice signal in the corresponding DSB spectrum a wide Band gap of 600Hz exist so SSB can be used for transmission of voice signals.

### \*DEMODULATION OF SSB SIGNAL:-

#### \*Synchronous Detector:

Block diag<sup>m</sup>:



General expression of SSB is given as:-

$$SSB(t) = \frac{A_m m(t) \cos \omega_0 t}{2} - \frac{A_m m(t) \sin \omega_0 t}{2}$$

Case 1:

i)  $(LO)_{OIP} = A_c \cos \omega t \{ \text{Perfect Synchronisation} \}$

ii)  $(Mut)_{OIP} = S_{SSB}(+) \times (LO)_{OIP}$  (96)

$$= \frac{A_c^2 m(t)}{2} \cos^2 \omega t + \frac{A_c^2 \hat{m}(t)}{2} \sin 4\omega t$$

$$\therefore 2 \sin 0.0030 = \sin 2\theta$$

iii)  $(LPF)_{OIP} = \frac{A_c^2 m(t)}{4}$   $\left\{ \cos^2 \omega t = 1 - \cos 4\omega t \right\}$

Case 2:

let,  
i)  $(LO)_{OIP} = A_c \cos(\omega \tau fct + \phi) \{ \text{No phase synchronisation} \}$

ii)  $(Mut)_{OIP} = S_{SSB}(+) \cdot (LO)_{OIP}$

$$= \left\{ \frac{A_c^2 m(t)}{2} \cos \omega \tau fct + \frac{A_c \hat{m}(t)}{2} \sin \omega \tau fct \right\} \left\{ A_c \cos(\omega \tau fct + \phi) \right\}$$

$$= \frac{A_c^2 m(t)}{4} (\cos(\omega \tau fct + \phi)) + \frac{A_c^2 \hat{m}(t)}{4} \cos \phi.$$

$$= \frac{A_c^2 \hat{m}(t)}{4} \sin(\omega \tau fct + \phi) \pm \frac{A_c^2 \hat{m}(t)}{4} \sin \phi.$$

iii)  $(LPF)_{OIP} = \frac{A_c^2 m(t)}{4} \cos \phi \pm \frac{A_c^2 \hat{m}(t)}{4} \sin \phi.$

when,

a)  $\phi = 0$

$$(LPF)_{OIP} = \frac{A_c^2 m(t)}{4}$$

b)  $\phi = 90^\circ$

$$(LPF)_{OIP} = \pm \frac{A_c^2 \hat{m}(t)}{4} \leftarrow \left\{ \begin{array}{l} \because OIP \neq 0; \text{ hence } S_{SSB} \text{ is} \\ \text{not affected by ONE} \end{array} \right\}$$

No ONE

JNUC  
\* Demodulation of SSB is not affected by QNE

\* Advantages of SSB: (97)

- 1) Transmitter power is saved ( $\eta = 100\%$ )
- 2) Transmission Bandwidth is saved.
- 3) NO affect of QNE.

\* Drawbacks of SSB:

- 1) Demodulation is complex.
- 2) Limited only for voice signal transmission.

\* APPLICATION:

- 1) Preferred in voice signal transmission.

Note!

\* Earlier SSB was used for voice signal transmission in local telephone comm<sup>n</sup>.

\* VESTIGIAL SIDEBAND MOD<sup>N</sup> (VSB):-

Video Signal = 0 - 4.5 MHz

$$B.W = 4.5 \text{ MHz}$$

Hence, using AM or DSB modulation =  $2 \times B.W$   
 $= 9 \text{ MHz}$

\* T.V. Signal = 10 MHz (each)  
(including Audio signal)

\* Co-axial cable  $B.W = 600 \text{ MHz}$   
(Broadly used)

→ 60 TV channels can be transmitted.

Let SSB modulation scheme is used.

Hence S.S.B., BW = 4.5 MHz

(98)

Total B.W TV signal  $\approx$  5 MHz  
(including audio signal)

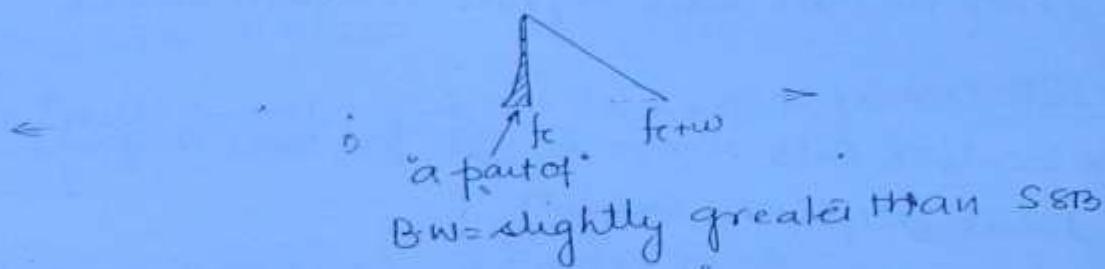
$\Rightarrow$  120 TV channel

$\therefore$  SSB not used for Video signal

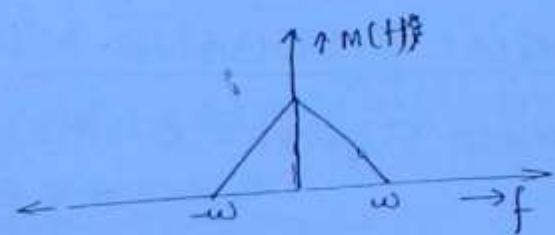
Hence it fails. <sup>though</sup> it has advantage over AM or DSB  
(In BW & Power)

Note:-

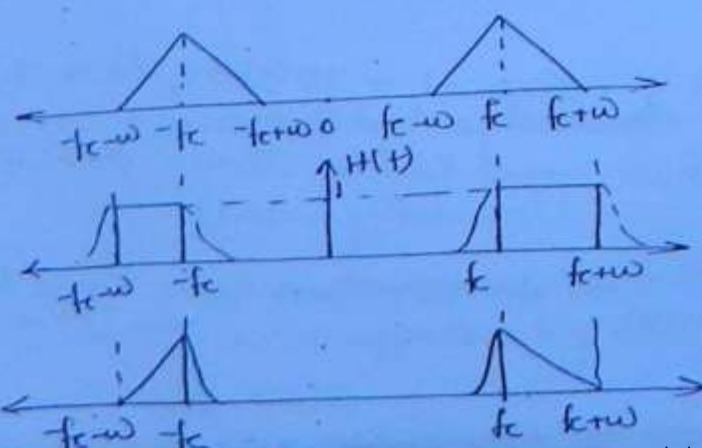
1. VSB provides almost of same Bandwidth as SSB and can be used for the transmission of Video signals
2. 3 steps:



Let SSB  
 $m(t)$



SDSB(L)



(BPF)

SSB(L)

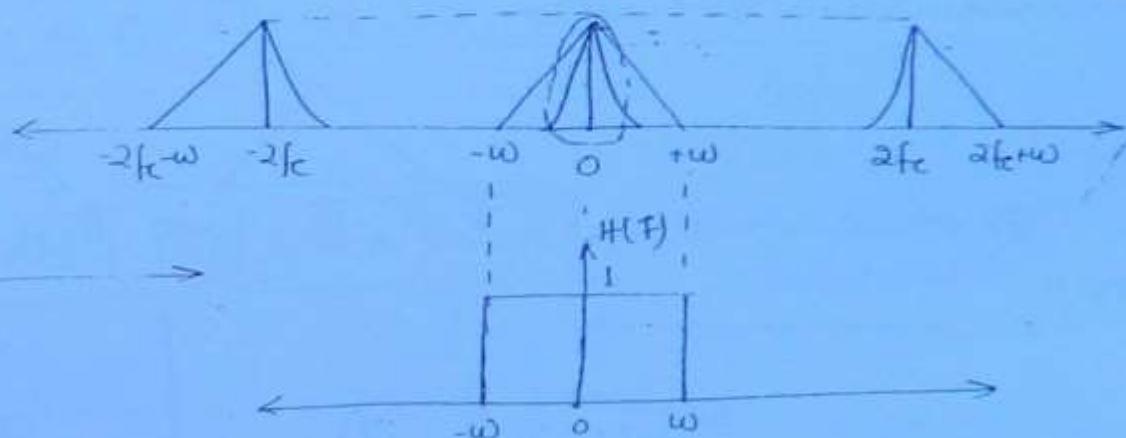
In the RRC or synchronous detector  
Mutual spectrum is given as

(99)

$$S(t) \text{ Correlat.} = \frac{S(t+fc) + S(t-fc)}{2}$$

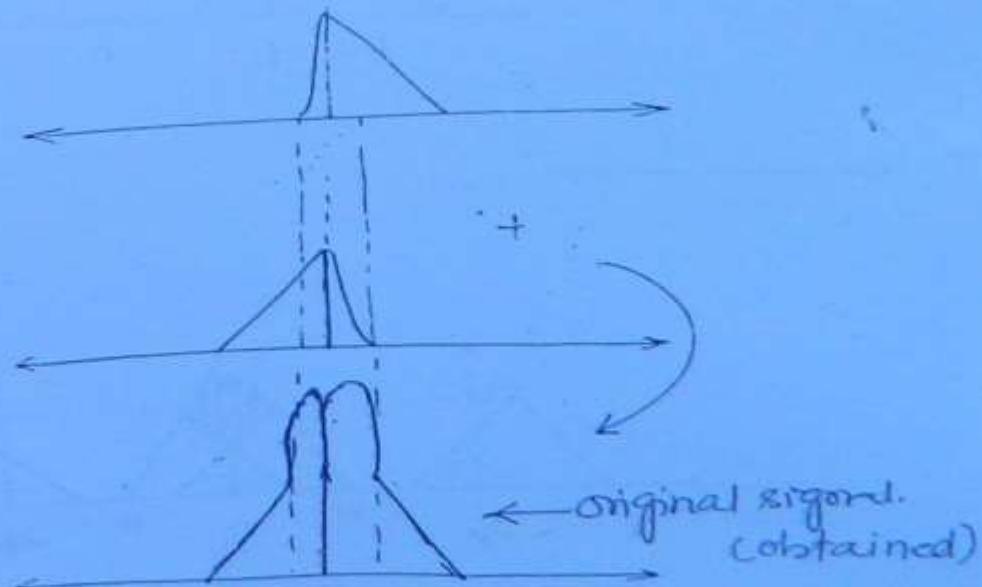
$(1^{\text{st}} \text{ u}) \text{o/p} \leftarrow \rightarrow$

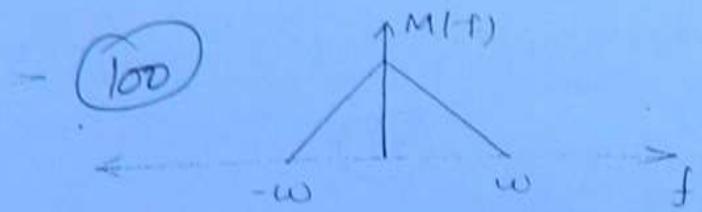
(SSB(+)) shifted left &  
Right by  $fc$ )



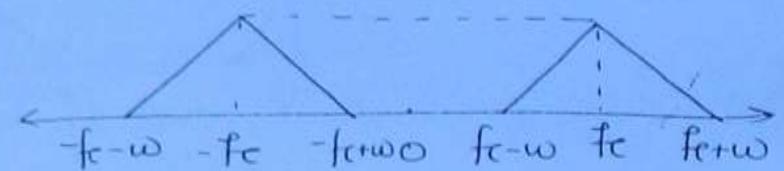
Note:

\* In the reconstructed message signal, the low frequencies near to origin will be interfered by undesired frequencies so that message signal cannot be perfectly reconstructed.



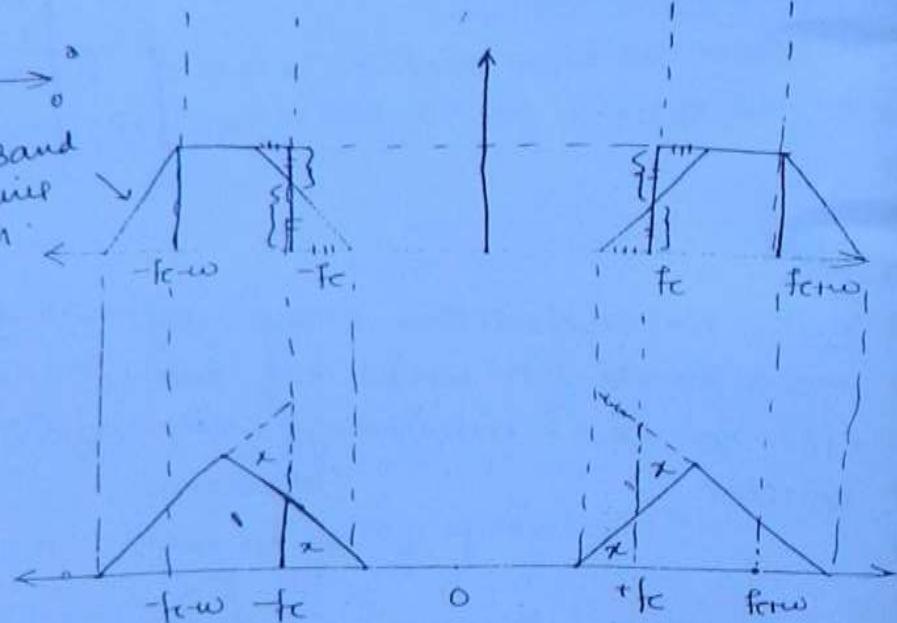


$SDSB(+)$



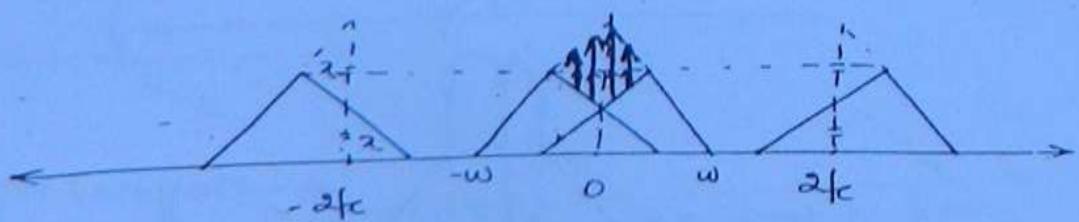
(BPF)

Side Band  
shaping filter



(D.P.)

Mul/o/p



Conclusion:

1. The ~~above~~ DSB signal is passed through sideband shaping filter to generate VSB.

2. H(1) of Side Band shaping filter should be symmetrical about  $f_c$ . 101

3. For demodulation of VSB, synchronous detector will be used.

Analysis of BW & Power with other Modulation schemes:

Bandwidth:-

$$\boxed{\text{AM} \& \text{DSB} > \text{VSB} > \text{SSB}} \leftarrow \text{Comparison of Bandwidth.}$$

Power:-

$$\boxed{\text{AM} > \text{DSB} > \text{VSB} \& \text{SSB}} \leftarrow \text{Comparison of power}$$

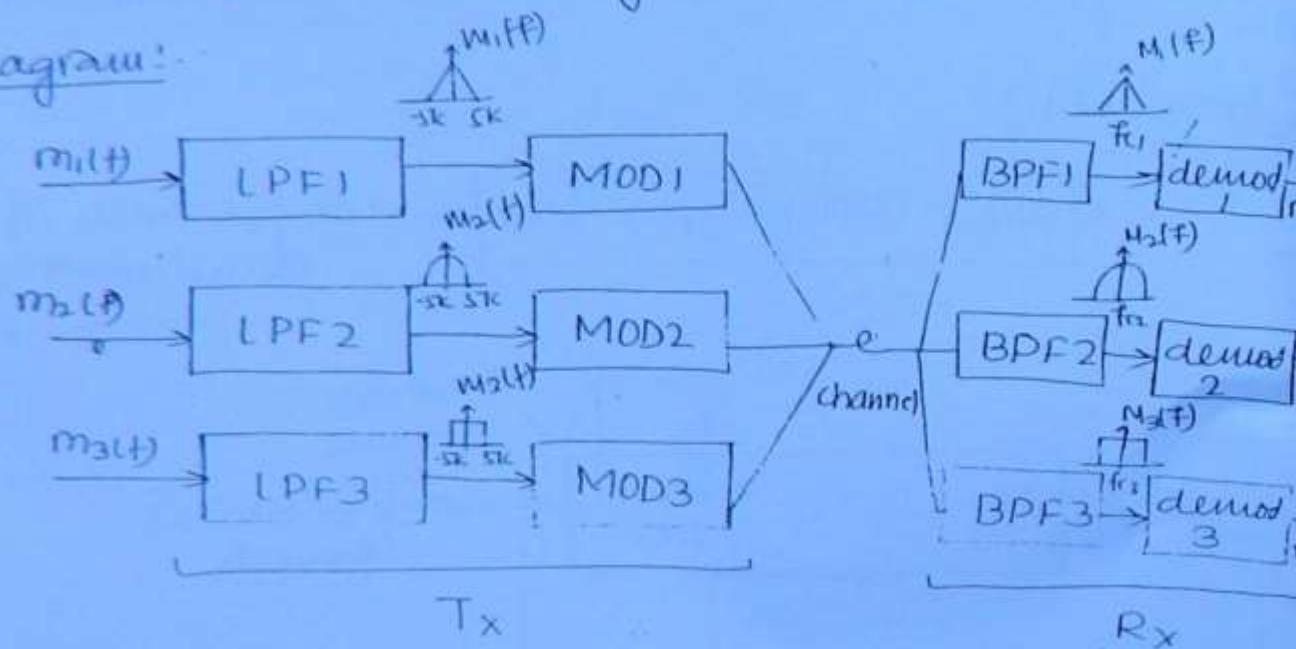
# \*FREQUENCY DIVISION MULTIPLEXING (FDM) :-

(102)

→ used to multiplex continuous signal

\* FDM is used for multiplexing continuous signals.

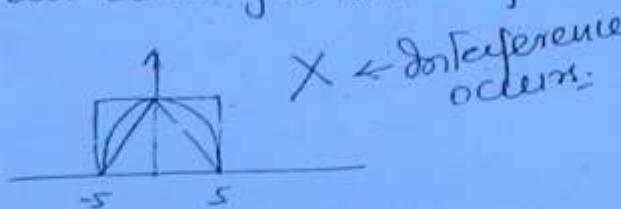
Block diagram:-



\* LPF are used to band limit the signals

Case 1 (no modulators) :-

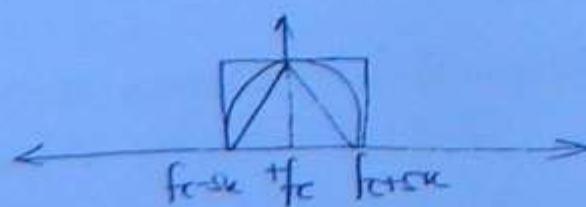
∴ All the messages are occupying same band of frequencies. So, they all will get interfered with each other.



Case 2 (Same carrier frequency) :-

Let same carrier frequency,  $f_c = \text{Acos}(\omega t)$

So,



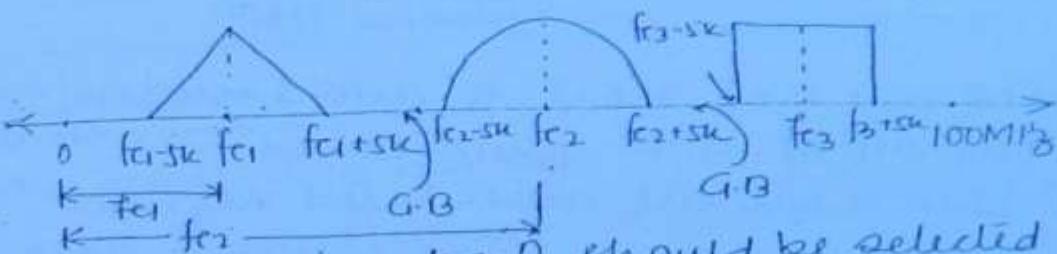
Here also the signals occupy same band of frequency. Hence interference is said to occur.

### Case 3 (diff. carrier frequency) :-

Let the channel B.W be 100MHz (0 to 100MHz)

The limitation is that the carrier frequency should be less than 100MHz

(103)



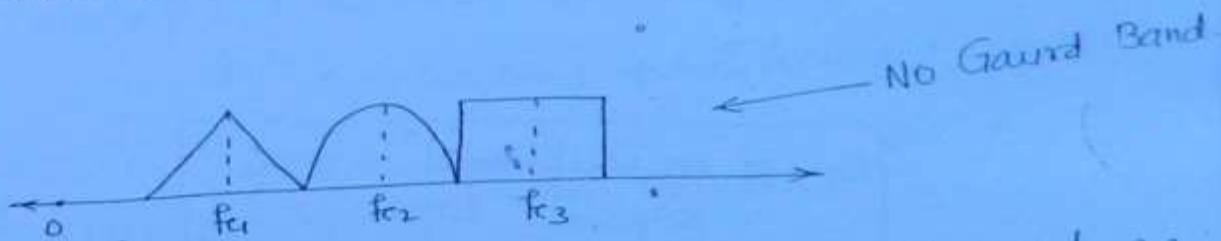
2. Also the carrier freq<sup>n</sup> should be selected such that there exist some spacing b/w spectrums. This spacing is called as 'GUARD BAND'.

So, to avoid interference,

$$f_{c2} > f_{c1} + 10K$$

$$f_{c3} > f_{c2} + 10K$$

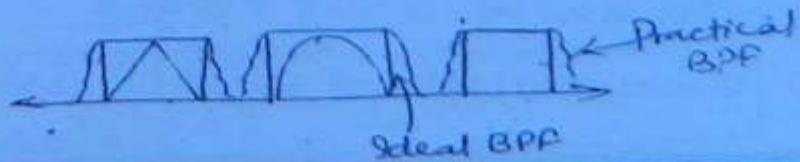
Case 4:  $f_{c2} = f_{c1} + 10K$ ;  $f_{c3} = f_{c2} + 10K$



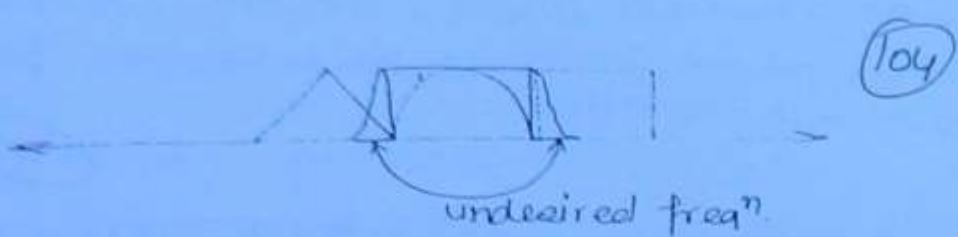
→ B.W is efficiently used, but practically not used as no Guard Band exists. Hence in the Rx section the part of the other msg signal will be extracted.

Note:-

1. when the modulated signal as in Case 3 is given to the BPF (practical) the message signal can be extracted efficiently



2. whereas in case 1, when no G.B exists then.

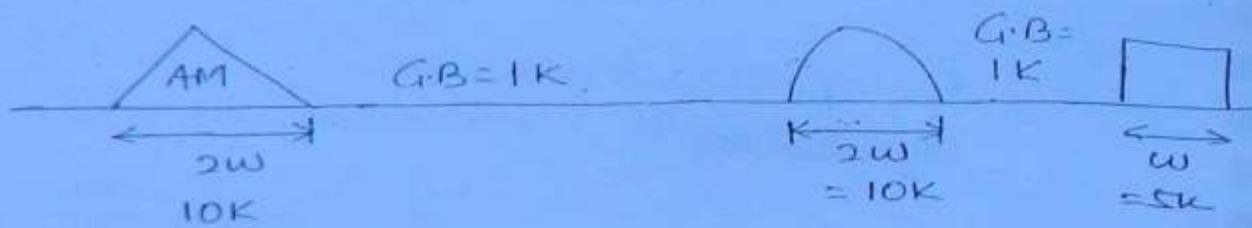


Note :-

Since BPF are not ideal, to avoid undesired frequencies at the OIP of BPF, Guard Band has to be maintained b/w adjacent modulated signals.

Q1) Three msg signals each Band limited to 5KHz are multiplexed using FDM. Guard Band is 1KHz. Find multiplexed signal BW, if the modulation schemes used are AM, DSB & SSB resp.

Soln: For 1<sup>st</sup> msg signal:      2<sup>nd</sup> msg signal      3<sup>rd</sup>



∴ So, B.W of multiplexed signal =  $27\text{K}$

Q2. 10 msg signal, each Band limited to 10K are multiplexed using FDM. Guard Band is 0.5K. find multiplexed signal BW if modulation used is

- a) AM    b) DSB    c) SSB

Soln: For AM

$$\text{B.W} = 2w = 20\text{K}$$

$$\text{So, B.W of 10 msg} = 200\text{K}$$

So, B.W of multiplexed signal =  $200\text{K} + 9 \times 0.5\text{K}$

$$= 204.5\text{K} \text{ (Same for DSB)}$$

For SCD:

$$B.W = 10K$$

$$\text{So, } B.W \text{ of } 10 \text{ mdeg} = 100K$$

(Ans)

$$\text{So, } [B.W \text{ of multiplexed signal} = 100 + 9 \times 0.5 = 104.5 K]$$

Ans

Previous exam Questions:

Q1. An AM signal is given by

$$s(t) = \{1 + m(t)\} \cos \omega_c t$$

$$\text{where, } m(t) = \frac{1}{2} \cos \omega_m t + \frac{1}{2} \sin \omega_m t$$

Find modulation efficiency?

Soln: Given,

$$\begin{aligned} s(t) &= \{1 + m(t)\} \cos \omega_c t \\ &= \left\{1 + \frac{1}{2} \cos \omega_m t + \frac{1}{2} \sin \omega_m t\right\} \cos \omega_c t \end{aligned}$$

Comparing with standard multitone AM:

$$s(t) = A_c \{1 + u_1 \cos \omega_{m1} t + u_2 \sin \omega_{m2} t\} \cos \omega_c t$$

for  $A_m_1 \cos \omega_{m1} t + A_m_2 \sin \omega_{m2} t = m(t)$ .

$$\text{So, for if, } m(t) = A_m_1 \cos \omega_{m1} t + A_m_2 \sin \omega_{m2} t$$

then

$$s(t) = A_c \{1 + u_1 \cos \omega_{m1} t + u_2 \sin \omega_{m2} t\}$$

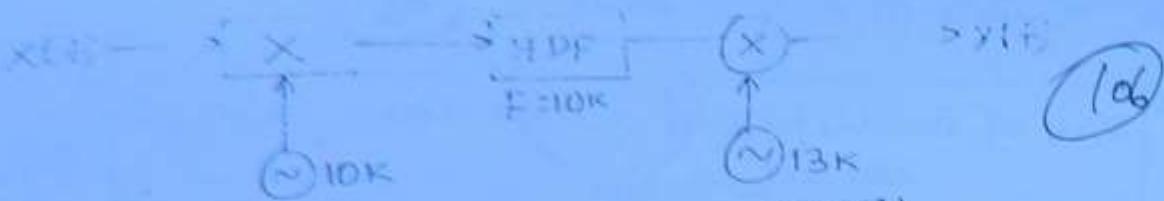
$$\text{So, } u_1 = \frac{1}{2}; u_2 = 0.5$$

$$u_f = \sqrt{0.5^2 + 0.5^2} = \sqrt{2}$$

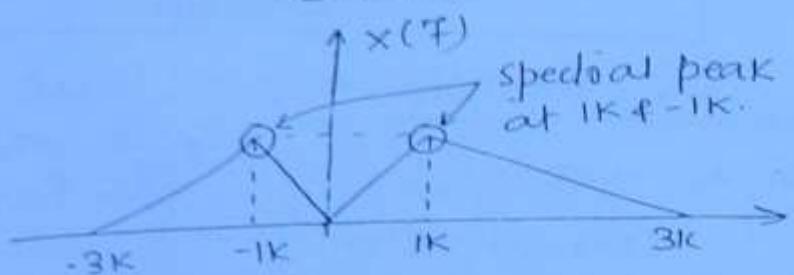
$$\text{So, } \eta = \frac{u_f^2}{2 + u_f^2} = \frac{\frac{1}{2}}{1 + \frac{1}{2}}$$

$$\boxed{\eta = 20\%} \quad \text{Ans}$$

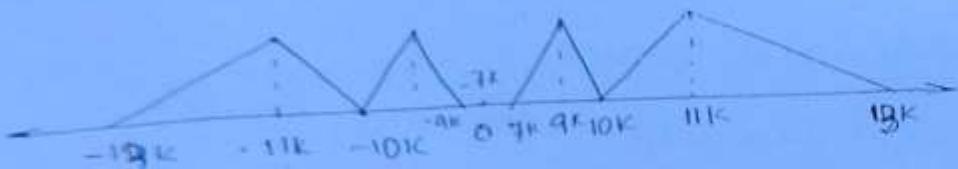
Q) For the following circuit find the frequencies for which spectral peaks will be observed in  $y(t)$



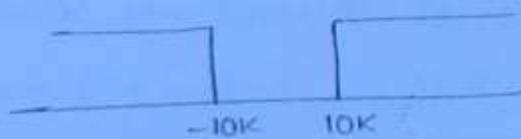
- a) 2K, 24K
- b) 1K, 22K
- c) 1K, 2K, 24K
- d) None



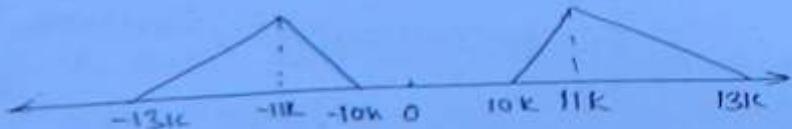
Soln: At O/P of multiplier 1st



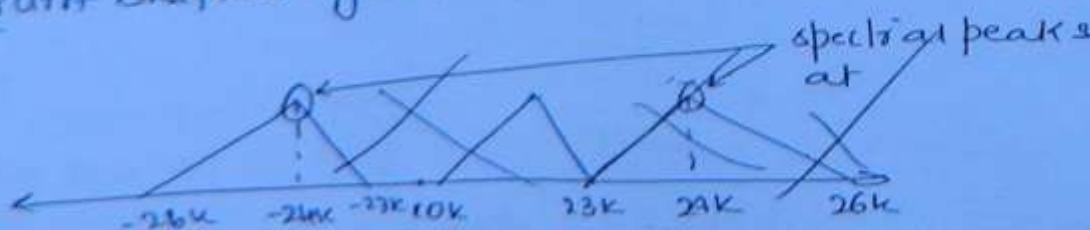
Spectrum of HPF:



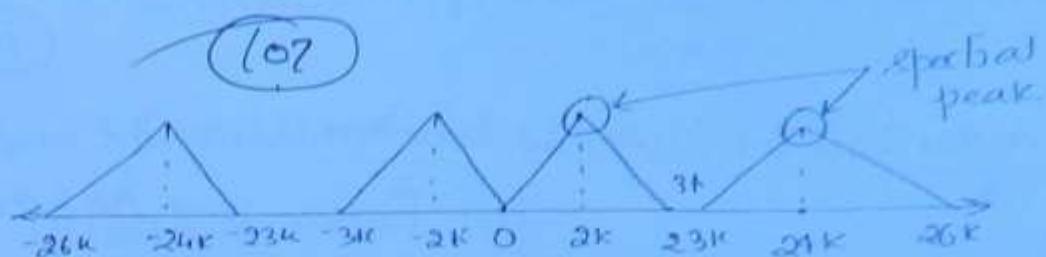
So, O/P of HPF



At O/P of multiplier 2nd:  
spectrum shifted by 13K



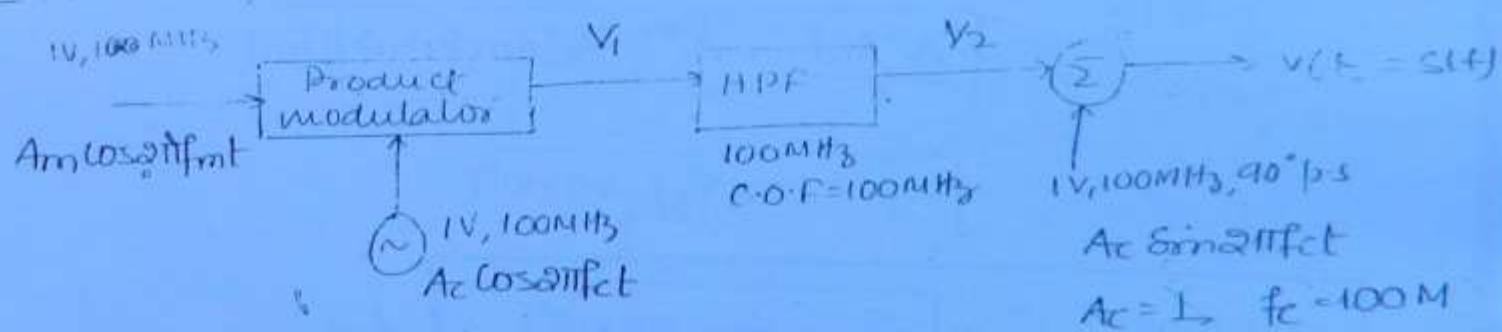
Multiplex 2000 MHz  $\rightarrow$  spectrum analysis



Q3. A sinusoidal carrier of  $IV, 100MHz$ , is product modulated by a sinusoidal msg signal of  $IV, 1MHz$ . The resulting signal is passed through HPF of  $f_{cutoff} = 100MHz$ . Filter o/p is added with sinusoidal signal of  $IV, 100MHz$ ,  $90^\circ$  phase shift. Find the envelope of o/p?

- Soln:
- $\sqrt{\frac{5}{4}} - 6 \sin 2\pi \times 10^6 t$
  - $\sqrt{142} \cos 2\pi \times 10^6 t$
  - $\sqrt{2} - 6 \sin 2\pi \times 10^6 t$
  - (const)

Block diagram



$$\text{So, } Am = 1; f_m = 1MHz \\ Ac = 1; f_c = 100MHz$$

$$\text{So, } V_1 = Am \cos 2\pi f_m t \cdot Ac \cos 2\pi f_c t \\ = \frac{Ac Am}{2} [\cos 2\pi(f_c + f_m)t + \cos 2\pi(f_c - f_m)t] \\ \text{Blocked by HPF}$$

$$\text{So, (HPF) o/p: } V_2 = \frac{Ac Am}{2} \cos 2\pi(f_c + f_m)t$$

$$\text{So, summed o/p } s(t) = \frac{Ac Am}{2} \cos 2\pi(f_c + f_m)t + \sin 2\pi f_m t$$

Now, as

$$A \cos \omega t + B \sin \omega t \xrightarrow{\text{envelope}} \sqrt{A^2 + B^2}$$

(108)

Now,

$$S(t) = \frac{A_c A_m \cos \omega_f t \cos \omega_m t}{2} - \frac{A_c A_m \sin \omega_f t \sin \omega_m t}{2} + A_c \sin \omega_f t$$

$$= \underbrace{\frac{A_c A_m \cos \omega_f t \cos \omega_m t}{2}}_A + \underbrace{\left( A_c - \frac{A_c A_m \sin \omega_m t}{2} \right) \sin \omega_f t}_B$$

$$\text{So, envelope} = \sqrt{A^2 + B^2}$$

$$= \sqrt{\frac{A_c^2 A_m^2}{4} \cos^2 \omega_f t + A_c^2 + \frac{A_c^2 A_m^2}{4} \sin^2 \omega_f t - 2 \frac{A_c^2 A_m}{2} \sin \omega_f t}$$

$$= \sqrt{\frac{A_c^2 A_m^2}{4} + A_c^2 - A_c^2 A_m \sin \omega_f t}$$

$$= \sqrt{\frac{1}{4} + 1 - \sin \omega_f t}$$

$$\boxed{\text{envelope} = \sqrt{\frac{5}{4} - \sin \omega_f t}} \quad \boxed{\text{Ans}}$$

Q9. A non-linear device is characterised by

$$V_o = a V_i + b V_i^3$$

$$\text{where } V_i = m(t) + \cos \omega_f t$$

by considering only DSB terms. find  $\omega_f$  such that resulting DSB signal carrier frequency is  $1 MHz$

$$\text{Soln: As, } V_o = a \{m(t) + \cos \omega_f t\} + b \{m(t) + \cos \omega_f t\}^3$$

$$V_o = a m(t) + a \cos \omega_f t + b \{m^2(t) + 2m(t) \cos^2 \omega_f t + 3m^2(t) \cos \omega_f t\}$$
$$+ 3m(t) \cos^2 \omega_f t + b \{1 + \cos 3\omega_f t\}$$

So, by considering only USB terms we get

$$V_0 = b \cdot \frac{3m(t)}{2} \cos 1\pi f_1 t$$

(109)

$$V_0 = \frac{3b}{2} m(t) \cos 1\pi f_1 t$$

So, by standard derivation

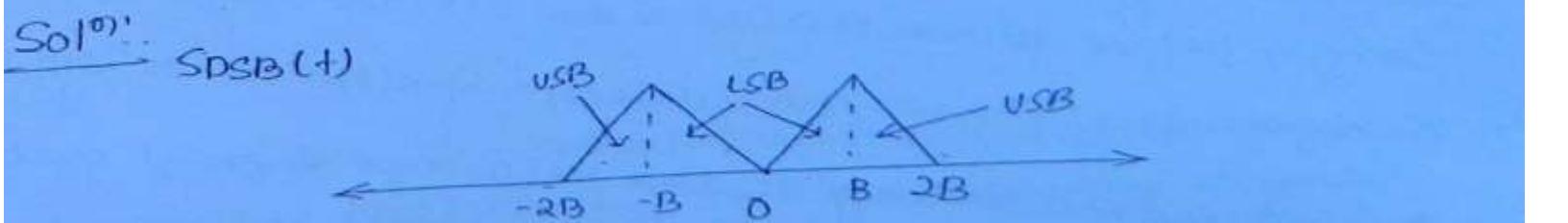
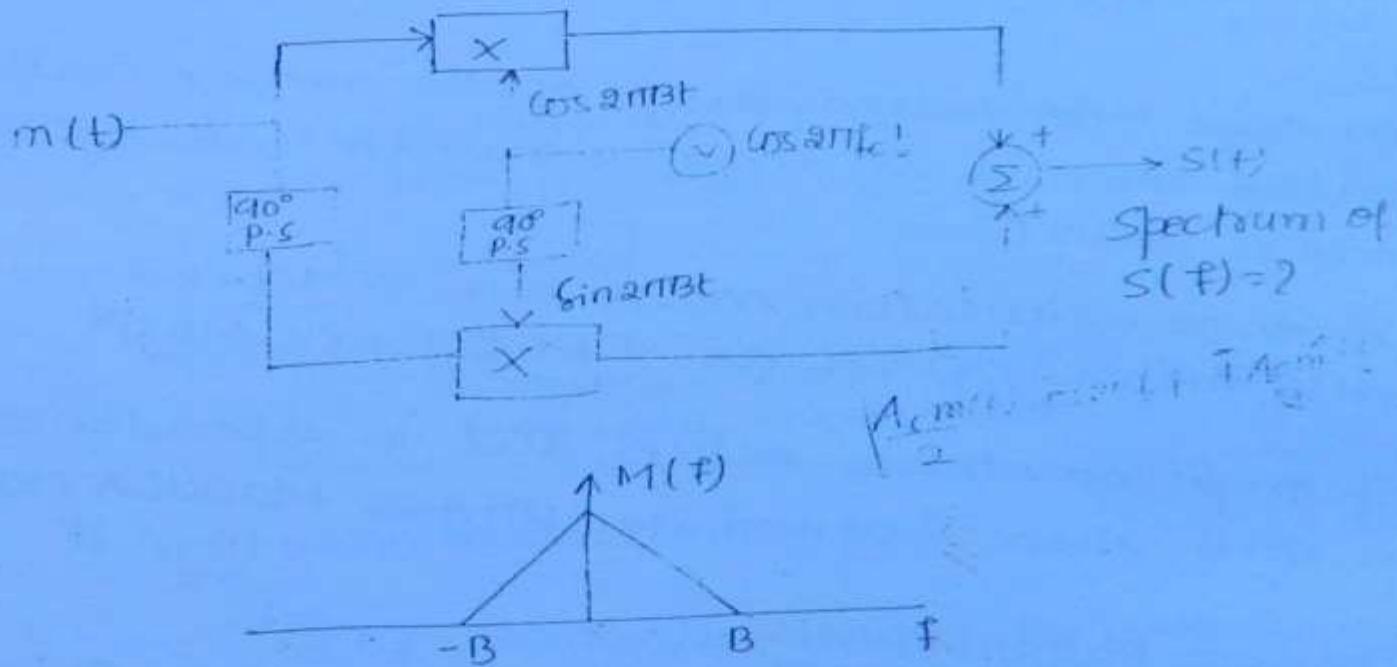
$$V_0 = A_m m(t) \cos 2\pi f_1 t$$

$$\text{So, } f_c = 2f_1$$

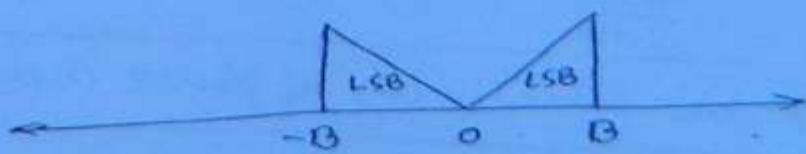
$$f_1 = f_c/2$$

$$f_1 = 0.5 \text{ MHz}$$

Q5



So, spectrum of  $s(t) \leftrightarrow S(f)$



## \* ANGLE - MODULATION.

classified Broadly as:-

(10)

- 1) Frequency Modulation.
- 2) Phase modulation

Now,

i.e.,

$$\text{carrier signal; } c(t) = A_c \cos \{2\pi f_{ct} t + \phi\}$$

Now, i.e.

$$2\pi f_{ct} t + \phi = \theta(t)$$

$$A_c \cos \{\theta(t)\}$$

total Angle

Note:-

1. In Angle modulation, angle of the carrier will be varied linearly in accordance with message signal voltage variations.
2. If Angle modulation occurs due to dependence of  $\theta$  on  $m(t)$ , then it is called as FREQUENCY MOD<sup>N</sup>.
3. If Angle modulation occurs, due to dependence of  $\phi$  on  $m(t)$ ; then it is called as PHASE MODULATION.

## \* PHASE MODULATION:

Carrier before phase modulation =  $c(t) = A_c \cos \{2\pi f_{ct} t\}$

carrier after phase modulation =  $s_p(t) = A_c \cos \{2\pi f_{ct} t + \phi\}$

\*  $\phi$  is varied according to the message signal amp.

so,  $\phi = K_p m(t)$

rad.  
volts  
rad  
volts

$K_p$  = phase sensitivity of phase modulator (rad/volt)

### Note:

1.  $K_p$  specifies the amount of phase change in the carrier for 1 volt change in the message signal  
 if  $m(t) = 0 \Rightarrow \text{no modulation} \Rightarrow \phi = 0$  

### \* FREQUENCY MODULATION:

Assume,

frequency of carrier before modulation =  $f_c$

frequency of carrier after frequency modulation =  $f_i$

Frequency of frequency modulated signal  
= Instantaneous frequency

- \*  $f_i$  is varied in accordance to variations in  $m(t)$   
so, mathematically

$$f_i = f_c + K_f m(t)$$

where,

$K_f$  = frequency sensitivity of (Hz/volt)  
frequency modulation

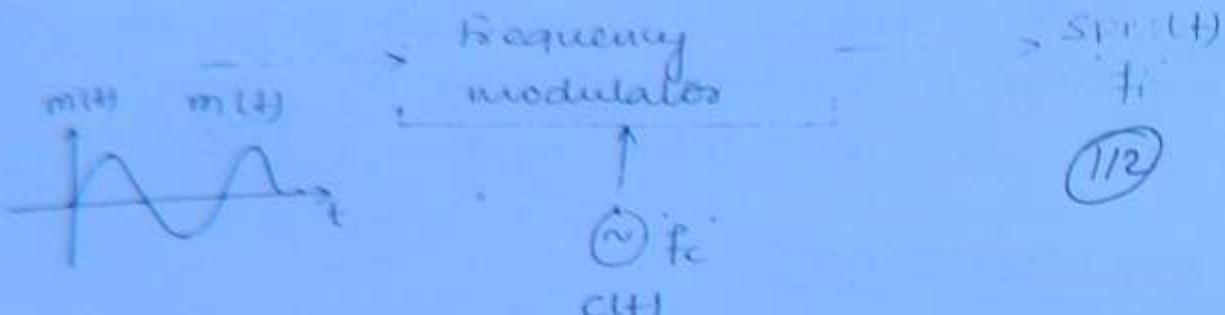
If  $m(t) = 0 \Rightarrow \text{no modulation} \Rightarrow f_i = f_c$

also, let,  $K_f = 25 \text{ Hz/Volt}$   $\leftarrow$  hence for 1 volt change frequency changes by  $25 \text{ Hz}$

### Note:

- $K_f$  specifies the amount of frequency change in the carrier for 1 volt change in the message signal.

$$K_f = 25 \text{ KHz/Volt}$$



Now,

$$\text{as } f_i = f_c + K_f m(t)$$

So,

$$1) m(t) = 0 \Rightarrow f_i = f_c + 0$$

$$2) m(t) = +5V \Rightarrow f_i = f_c + 125K$$

$$3) m(t) = -5V \Rightarrow f_i = f_c - 125K.$$

Conclusion:-

when,

$$1) m(t) = 0 \Rightarrow f_i = f_c.$$

$$2) m(t) = +ve \Rightarrow f_i > f_c.$$

$$3) m(t) = -ve \Rightarrow f_i < f_c.$$

Note:-

1. By changing the amp. of message signal (voltage), variations in frequency is obtained. Hence FM is also called as "Voltage to Frequency Conversion".
2. In FM, message signal voltage variations are converted as carrier signal frequency variation, so is also called as "VOLTAGE TO FREQUENCY CONVERSION".

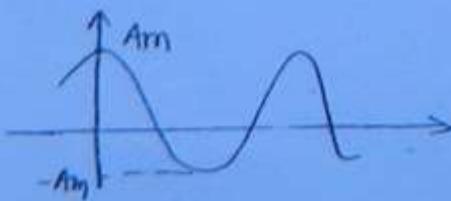
\*Analysis:-

Assume,

$$m(t) = A_m \cos 2\pi f_m t$$

$$f_i = f_c + K_f m(t)$$

↑ frequency



$$\text{max}^m \text{ frequency of FM signal} \rightarrow f_{\text{max}} = f_c + K_f A_m$$

$$\text{min}^m \text{ frequency of FM signal} \rightarrow f_{\text{min}} = f_c - K_f A_m$$

So,

$$\text{max}^m \text{ frequency deviation} = \text{Max}\{K_f m(t)\}$$

(13)

So,

$$\Delta f = K_f A_m$$

So, putting value of  $\Delta f$  in above eqn we get:

$$f_{\text{max}} = f_c + \Delta f$$

$$f_{\text{min}} = f_c - \Delta f$$

$$\text{So, Total frequency swing of FM signal} = f_{\text{max}} - f_{\text{min}} \\ = 2 \Delta f$$

Q1. An unmodulated carrier frequency is given by  $1 \text{ MHz}$ . After frequency modulation, max<sup>m</sup> frequency is given by  $1.4 \text{ MHz}$ . Find  $\Delta f$  and  $f_{\text{min}}$ ?

Sol<sup>m</sup>: Given:

$$\begin{aligned} f_{\text{max}} &= 1.4 \text{ MHz} \\ &= f_c + \Delta f \\ &= 1 + \Delta f = 1.4 \text{ MHz} \end{aligned}$$

$$\Delta f = 0.4 \text{ MHz}$$

Ans

And,

$$\begin{aligned} f_{\text{min}} &= f_c - \Delta f \\ &= 1 - 0.4 \end{aligned}$$

$$f_{\text{min}} = 0.6 \text{ MHz}$$

Ans

Q2: For an FM signal,  $f_{\max}$  is given by  $1.5 \text{ MHz}$ . Total frequency swing is given by  $900 \text{ kHz}$ . Find  $f_c$ ,  $\Delta f$ ,  $f_{\min}$ ?

(114)

Sol<sup>n</sup>: Given

$$\Delta f = \text{Total frequency swing} = 900 \text{ kHz}$$

$$\boxed{\Delta f = 450 \text{ kHz}} \quad \text{Ans}$$

$$\text{Now, } f_{\max} = 1.5 \text{ MHz}$$

$$= f_c + \Delta f$$

$$f_c = f_{\max} - \Delta f$$

$$= 1.5 \times 1000 - 450 \text{ kHz}$$

$$= 1050 \text{ kHz}$$

$$\boxed{f_c = 1.05 \text{ MHz}} \quad \text{Ans}$$

$$\text{And, } f_{\min} = f_c - \Delta f$$

$$= 1.05 \times 1000 - 450$$

$$\boxed{f_{\min} = 600 \text{ kHz}} \quad \text{Ans}$$

Q3: A sinusoidal carrier of  $20 \text{ V}$ ,  $2 \text{ MHz}$  is frequency modulated by a msg signal of  $10 \sin 4\pi \times 10^3 t$ .  $K_f$  is given by  $50 \text{ kHz/volt}$ . Find  $\Delta f$ ;  $f_{\max}$  &  $f_{\min}$ ?

Sol<sup>n</sup>: Given,  $A_c \cos 2\pi f_c t = C(t)$

$$\text{So, } A_c = 20 \text{ V}; f_c = 2 \text{ MHz}$$

~~$$\text{Am } \sin 4\pi \times 10^3 t = m(t)$$~~

$$A_m = 10; f_m = 2 \text{ kHz}$$

$$K_f = 50 \text{ kHz/volt}$$

$$\text{So, } f_{\max} = f_c + K_f m(t)$$

$$= 2000 + 50 \times 10^3 \times 10$$

$$\boxed{f_{\max} = 2500 \text{ kHz}} \quad \text{Ans}$$

$$f_{\min} = f_c - K_f m(t)$$

$$= 2000 - 50 \times 10$$

$$\boxed{f_{\min} = 1500 \text{ kHz}} \quad \text{Ans}$$

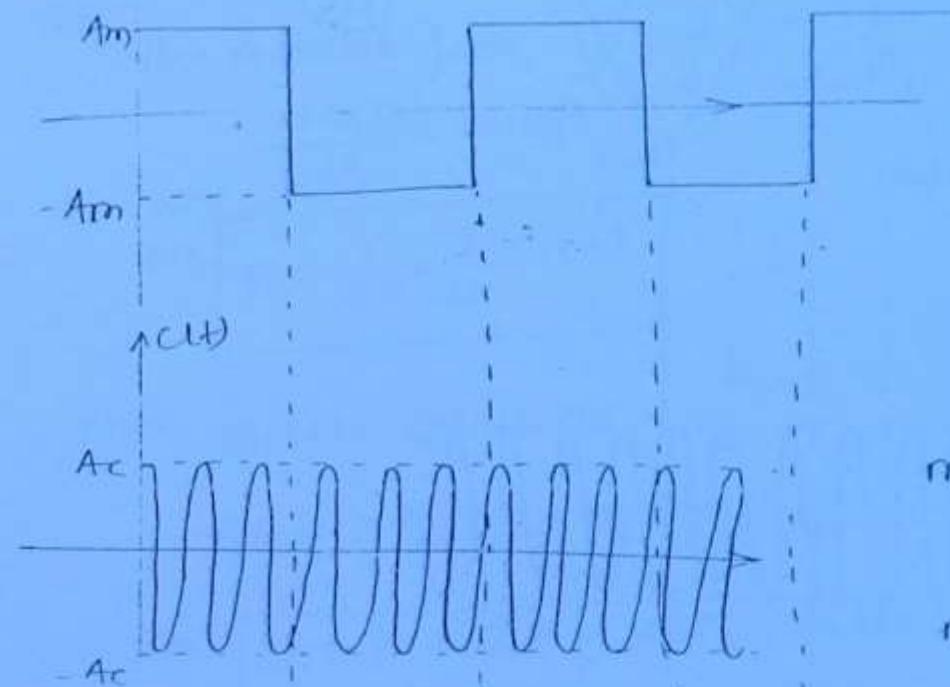
$$\Delta f = f_{\max} - f_{\min}$$

$$\boxed{\Delta f = \frac{1000 \text{ kHz}}{2}}$$

~~Case 1~~

Case 1  $m(t)$

145



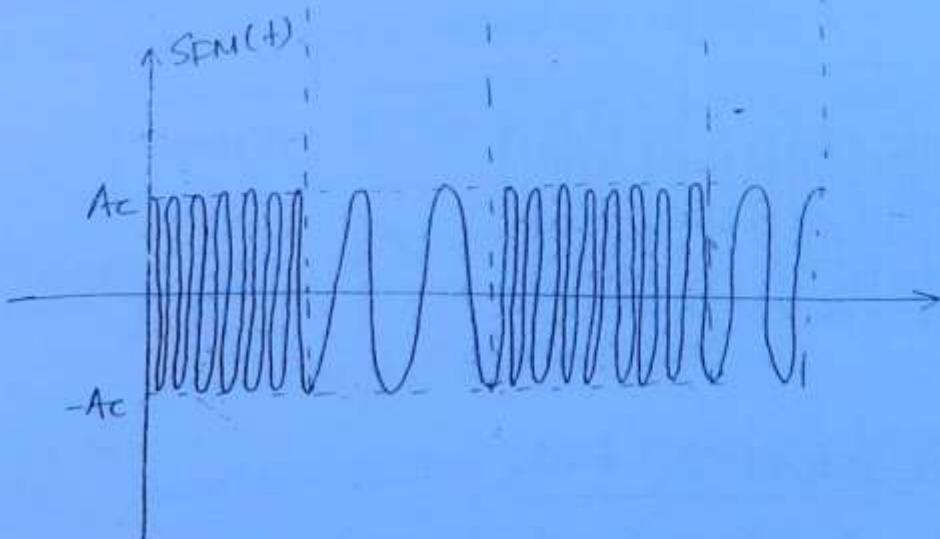
$$f_i = f_c + K_f m(t)$$

$$m(t) = Am \Rightarrow f_i = f_c + K_f Am = f_1$$

$$f_1 > f_c$$

$$m(t) = -Am \Rightarrow f_i = f_c - K_f Am = f_2$$

$$f_2 < f_c$$



~~Case 2~~  
As,  $f_i = f_c + K_f m(t)$

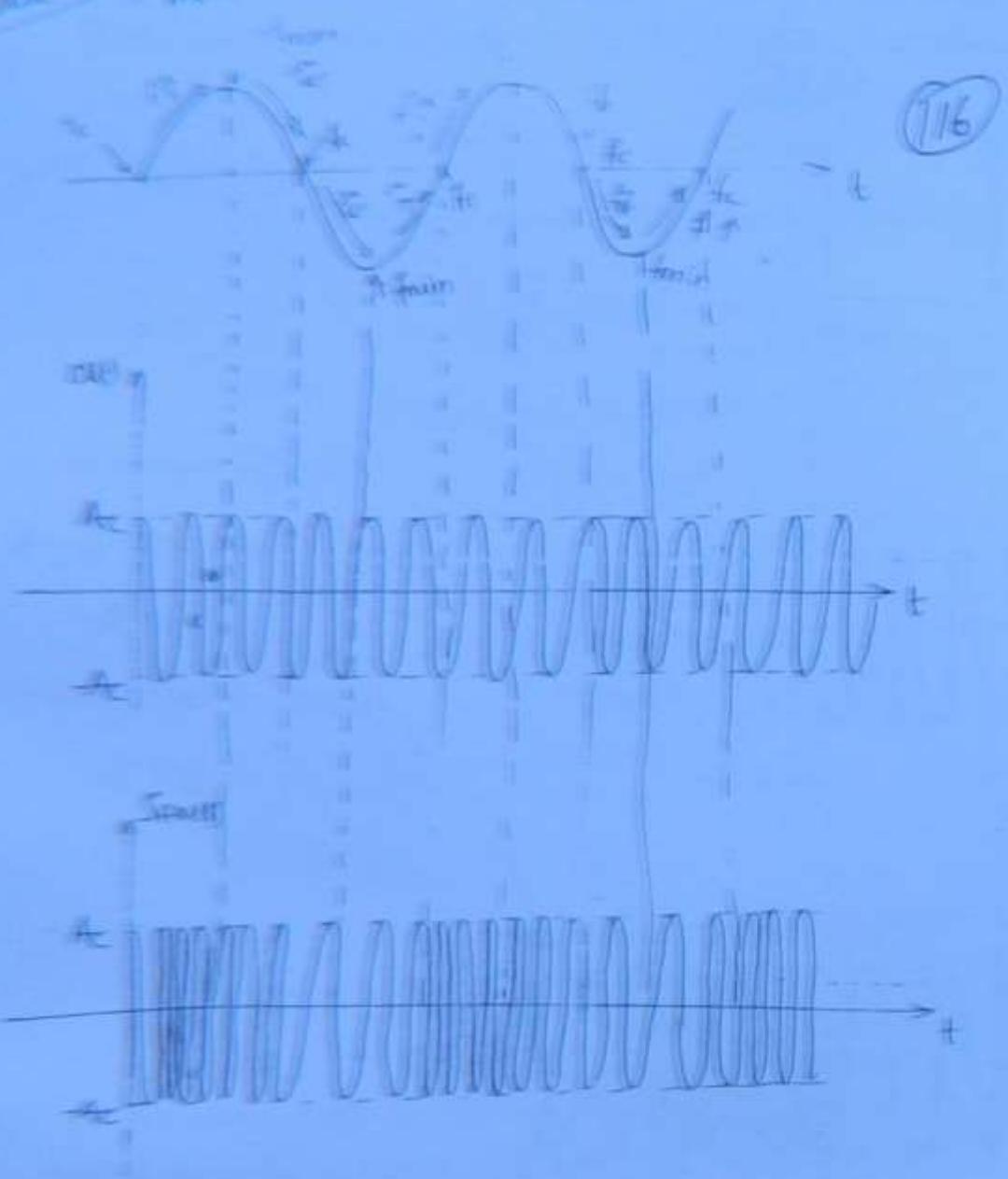
NOW,

$$m(t) = Am \Rightarrow f_i = f_c + K_f Am = f_1$$

$f_1 > f_c$

$$m(t) = -Am \Rightarrow f_i = f_c - K_f Am = f_2$$

$f_2 < f_c$



### GENERAL EXPRESSION OF FM:

1st, Carries Signal =  $C(t) = A_c \cos \{ \omega_c t + \theta(t) \}$   
 $= A_c \cos \{ \theta(t) \}$

where,  
 $\theta(t) = \omega_m t + \phi$  ← no significance of  $\phi$ .

To make  $\phi$  non-significant;

$$\frac{d\theta(t)}{dt} = \omega_m$$

before modulation

=> Universal expression of FM waveform

$$SFMI(t) = A_c \cos \{ \Omega_i t + \int f_m(t) dt \} \quad (i)$$

Instantaneous Angle

So, after modulation

(117)

$$\frac{d\Omega_i(t)}{dt} = 2\pi f_m(t)$$

\*\*\*

$$So, f_m(t) = \frac{1}{2\pi} \frac{d\Omega_i(t)}{dt}$$

$$Now, \Omega_i(t) = 2\pi \int f_m(t) dt$$

$$\Omega_i(t) = 2\pi \int f_c + K_f m(t) dt$$

$$So, \Omega_i(t) = 2\pi f_c t + 2\pi K_f \int m(t) dt$$

Substituting in equation (i) we get -

$$SFMI(t) = A_c \cos \{ 2\pi f_c t + 2\pi K_f \int m(t) dt \}$$

### SINGLE TONE F.M.

Assume,

$$m(t) = Am \cos \omega_m t$$

$$So, SFMI(t) = A_c \cos \{ 2\pi f_c t + 2\pi K_f \int Am \cos \omega_m t dt \}$$

$$So, SFMI(t) = A_c \cos \{ 2\pi f_c t + 2\pi K_f \cdot Am \frac{\sin \omega_m t}{\omega_m} \}$$

Now, let \*\*\*

$$\frac{K_f Am}{\omega_m} = \frac{\Delta f}{f_m} = \beta$$

modulation index  
of FM

Also,

$$\text{modulation index} = \beta = \frac{\text{max frequency deviation}}{\text{message signal freq}}$$

$$SFMT(t) = A_c \cos\{2\pi f_{ct} t + \beta \sin 2\pi f_{mt} t\} \quad \text{Single tone FM}$$

\* Depending upon the value of  $\beta$ , the FM is classified as:-

FM

(118)

$\downarrow$   
NBFM  
( $B \leq 1$ )

$\downarrow$   
WBFM  
( $B > 1$ )

\* Narrow Band FM (NBFM) :-

The single tone FM is given as:-

$$SFMT(t) = A_c \cos\{2\pi f_{ct} t + \beta \sin 2\pi f_{mt} t\} \quad (\beta \text{ small})$$

$$SFMT(t) = A_c \left\{ \cos(2\pi f_{ct} t) \cos(\beta \sin 2\pi f_{mt} t) - \sin(2\pi f_{ct} t) \sin(\beta \sin 2\pi f_{mt} t) \right\}$$

Now, for NBFM ;  $|\beta| \leq 1$  (small value)

So, for small value of  $\theta$

$$\cos \theta \approx 1$$

$$\sin \theta \approx \theta$$

So,

$$SNBFM(t) = A_c \cos 2\pi f_{ct} t - A_c \sin 2\pi f_{ct} t \times \beta \sin 2\pi f_{mt} t$$

So,

$$SNBFM(t) \approx A_c \cos 2\pi f_{ct} t - A_c \beta \sin 2\pi f_{ct} t \sin 2\pi f_{mt} t$$

$$SNBFM(t) = A_c \cos 2\pi f_{ct} t - \frac{A_c \beta}{2} \{ \cos 2\pi(f_c - f_m)t + \cos 2\pi(f_c + f_m)t \}$$

Now, similarity b/w NBFM & AM: comparable; both  $\leq 1$

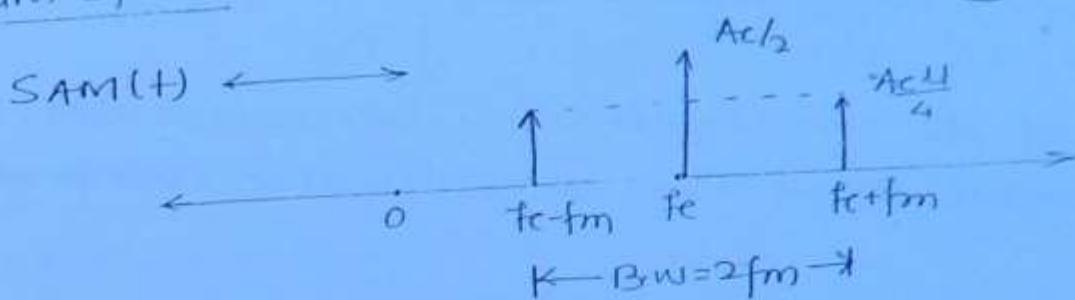
$$SAM(t) = A_c \cos 2\pi f_{ct} t + \frac{A_c \beta}{2} \{ \cos 2\pi(f_c - f_m)t + \cos 2\pi(f_c + f_m)t \}$$

Note:-

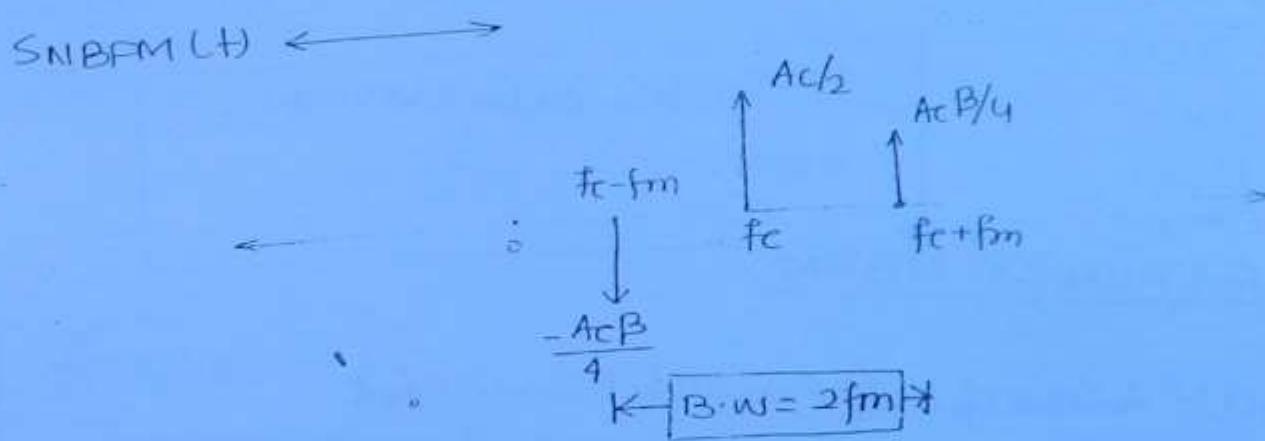
- \* General expression of AM & NBFM are same except  $180^\circ$  phase shift at LSB frequency component.

(119)

Spectrum of AM:



Spectrum of NBFM:



Note:-

- \* Magnitude spectrum of AM & NBFM are same.

Power of NBFM:

$$\text{As, } P_t = P_c + P_{USB} + P_{LSB}$$

$$\text{where, } P_c = \frac{Ac^2}{2R} ; P_{USB} = \left(\frac{AcB}{2}\right)^2 / 2R = \frac{Ac^2 B^2}{8R} = P_{LSB}$$

$$\text{So, } P_t = \frac{Ac^2}{2R} + 2 \times \frac{Ac^2 B^2}{8R} = \frac{Ac^2}{2R} + \frac{Ac^2 B^2}{4R}$$

$$\therefore P_t = \frac{Ac^2}{2R} \left\{ 1 + \frac{B^2}{2} \right\} \Rightarrow P_t = P_c \left\{ 1 + \frac{B^2}{2} \right\}$$

## CONCLUSION

- \* NBFM has much similarity with AM, hence practical significance of NBFM is negligible. (120)
- \* Bandwidth and power requirements of NBFM will be same as AM.
- \* Because of its much similarities with AM, NBFM is given least practical significance compared to WBFM.

Q1. AM and NBFM are having <sup>same</sup> modulation index were added. The resulting signal will be.

a) DSB

b) SSB

✓ SSB ~~with carrier~~ (DSB with carrier).

NOTE:

Graphical (maybe)

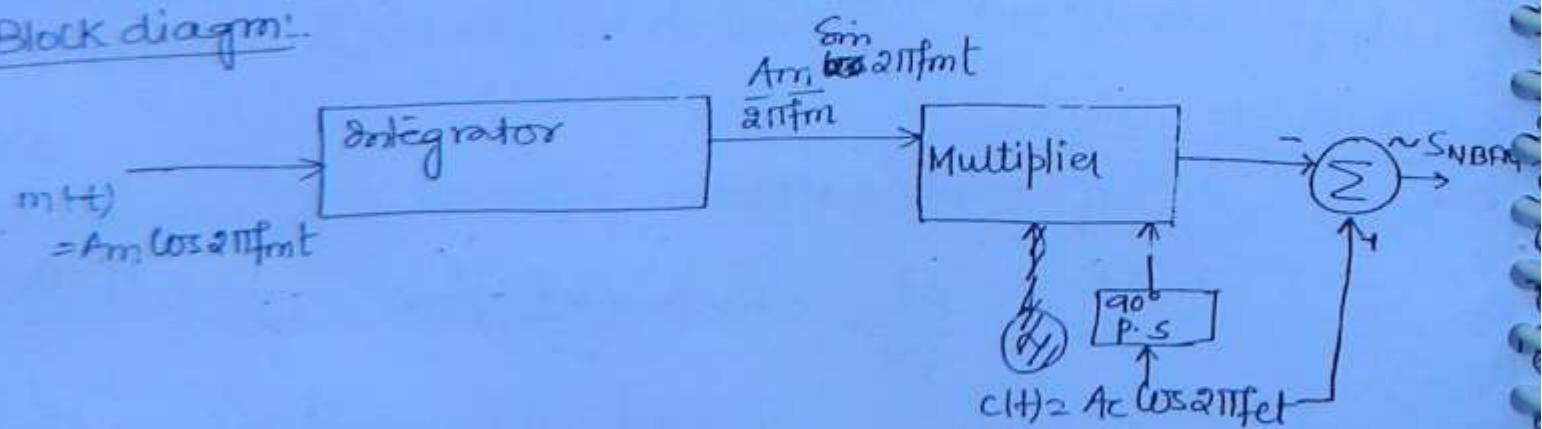
## \*GENERATION OF NBFM!

As,

$$SNBFM(t) = A_c \cos \omega_{IF} t - A_p \sin \omega_{IF} t \cdot \sin \omega_{IF} t$$

$$\cong A_c \cos \omega_{IF} t - \underbrace{A_p \frac{K_f / A_m}{f_m} \sin \omega_{IF} t}_{\text{carrier}} \underbrace{\sin \omega_{IF} t}_{\text{modulated}}$$

Block diagram:



## XWIDE BAND FM (WBFM)

### \* BESSEL FUNCTION:

standard defn is given as:

$$J_n(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(x \sin \theta - n\theta)} d\theta ; \theta = \text{dummy variable}$$

### \* Property of $J_n(x)$ :

i)  $|J_n(x)|$  decreases as  $n \uparrow$  increases

so, 
$$\boxed{J_0(x) > J_1(x) > J_2(x) > J_3(x) \dots}$$

ii)  $J_n(+x) = (-1)^n J_n(x)$

so, 
$$\boxed{\begin{aligned} J_{-n}(x) &= -J_n(x) ; n = \text{odd} \\ &= J_n(x) ; n = \text{even} \end{aligned}}$$

iii) 
$$\sum_{n=-\infty}^{\infty} J_n^2(x) = 1$$

iv)  $J_n(x)$  is a Real quantity.

### \* General expression of WBFM:

General exp. is given as (of single tone):

$$SFM(t) = A_c \cos \{2\pi f_c t + \beta \sin 2\pi f_m t\}$$

$$\text{Now, } \cos \theta = \text{Real} \{e^{j\theta}\}$$

$$\begin{aligned} \text{so, SFM}(t) &= A_c \text{Real} \{e^{j(2\pi f_c t + \beta \sin 2\pi f_m t)}\} \\ &= A_c \text{Re} \{e^{j2\pi f_c t} \cdot e^{j\beta \sin 2\pi f_m t}\} \quad \text{--- (1)} \end{aligned}$$

Fourier Series

Now,  $e^{j\beta \sin 2\pi f_m t}$  is a continuous periodic signal with

$$\boxed{T = 1/f_m}$$

Above signal is periodic. So,  $x(t) = x(t+T)$

(122)

$$e^{j\beta \sin \omega_m t} = e^{j\beta \sin \omega_m (t+T/f_m)} \quad \{ \because T = 1/f_m \}$$

∴ Fourier series is used to find the frequency analysis of continuous periodic signals. So, the exp Fourier series is given as:-

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} ; \omega_0 = \frac{2\pi}{T}$$

where,

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-jn\omega_0 t} dt$$

So,

$$e^{j\beta \sin \omega_m t} = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_m t} \quad (2)$$

Now,  $\frac{1}{2\pi f_m}$

$$c_n = \frac{1}{(1/f_m)} \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} e^{j\beta \sin \omega_m t} \cdot e^{-jn\omega_m t} dt$$

$$= f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} e^{j(\beta \sin \omega_m t - n\omega_m t)} dt$$

Now, as

$$J_n(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(x \sin \theta - n\theta)} d\theta$$

$$\text{So, } \theta = \omega_m t$$

$$d\theta = \omega_m dt$$

$$\text{Now, when } t = -\frac{1}{2f_m} \Rightarrow \theta = -\pi$$

$$t = \frac{1}{2f_m} \Rightarrow \theta = +\pi$$

$$so, C_n = \lim_{\beta \rightarrow \infty} \int_{-\pi}^{\pi} e^{j(\beta \sin \theta - n\theta)} \frac{d\theta}{2\pi f_m} \quad (123)$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin \theta - n\theta)} d\theta = J_n(\beta) \quad \{x = \beta\}$$

so,

$$\boxed{C_n = J_n(\beta)}$$

putting this value in eq<sup>n</sup>(9) we get:

$$\boxed{e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn2\pi f_m t}}$$

substituting this value in eq(11) we get:

$$SFM(t) = SWBFM(t) = A_c \operatorname{Re} \left\{ e^{j2\pi f_c t} \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn2\pi f_m t} \right\}.$$

$$SWBFM(t) = A_c \operatorname{Re} \left\{ \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi(f_c + n f_m)t} \right\}.$$

So, the General exp. of WBFM is given as:-

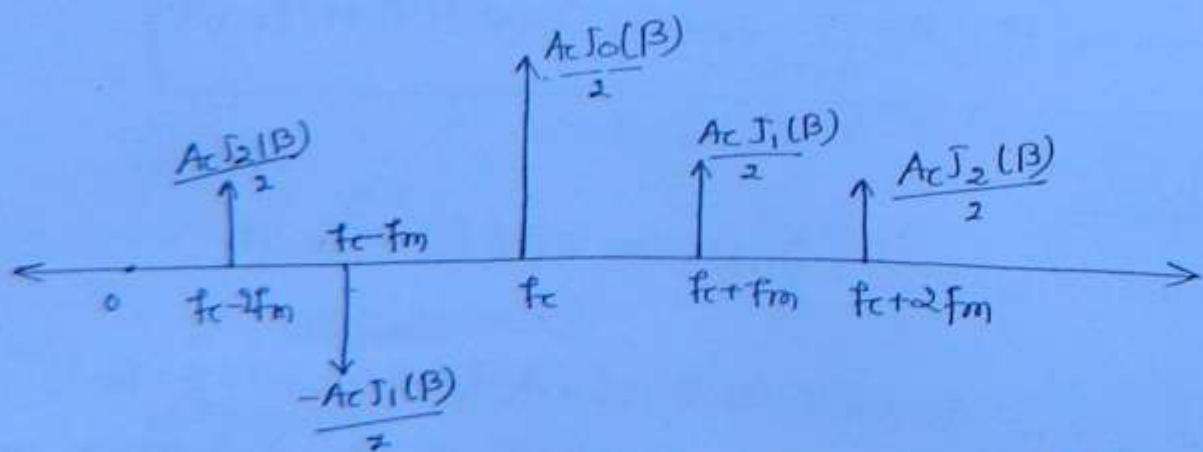
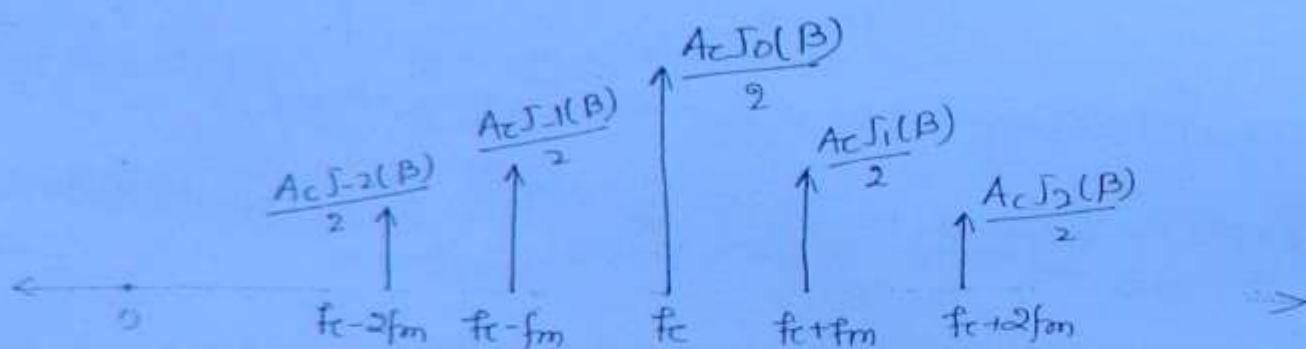
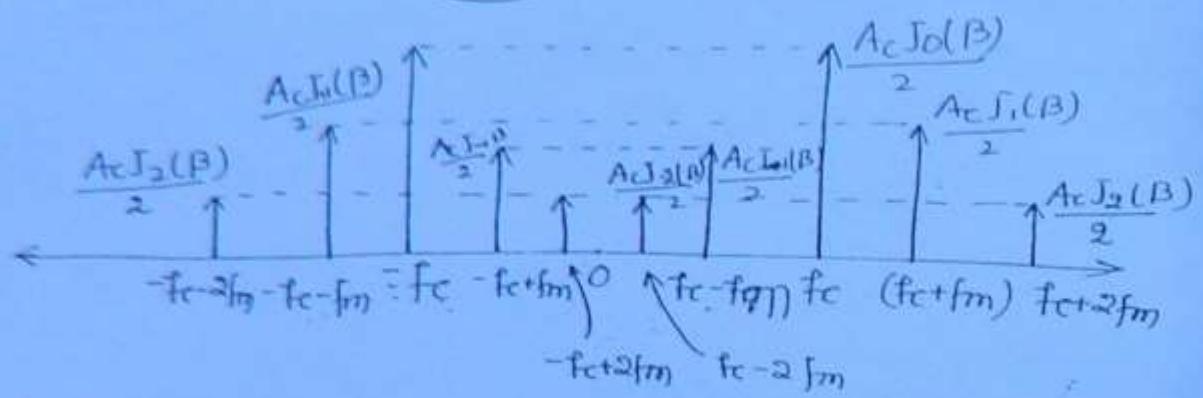
\*\*\*

$$\boxed{SWBFM(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi(f_c + n f_m)t}$$

Analysis:-

$$\begin{aligned} SWBFM(t) &= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi(f_c + n f_m)t \\ &= A_c J_0(\beta) \cos 2\pi f_c t + A_c J_1(\beta) \cos 2\pi(f_c + f_m)t + \\ &\quad A_c J_{-1}(\beta) \cos 2\pi(f_c - f_m)t + A_c J_2(\beta) \cos 2\pi(f_c + 2f_m)t \\ &\quad + A_c J_{-2}(\beta) \cos 2\pi(f_c - 2f_m)t + \dots \end{aligned}$$

124



$$J_0(\alpha) > J_1(\alpha) > J_2(\alpha) > \dots$$

## Conclusion-

- WBFSM consists of carrier frequency component, or no of USBs and  $\infty$  no. of LSBs.
- (125)
- 2) The Actual Bandwidth of WBFSM is  $\infty$ .
- 3) For WBFSM, strength of higher order sidebands go on decreasing and finally becomes zero.
- 4) For WBFSM, lower order sidebands are said to be significant sidebands and higher order sidebands are insignificant.

## \* Power of WBFSM:-

As,

$$P_t = P_c + (P_{USB1} + P_{USB2} + \dots) + (P_{LSB1} + P_{LSB2} + \dots)$$

where,

$$P_c = \frac{A_c^2 J_0^2(B)}{2R} ; P_{USB1} = \frac{A_c^2 J_1^2(B)}{2R} ; P_{LSB1} = \frac{A_c^2 J_{-1}^2(B)}{2R}$$

$$P_{USB2} = \frac{A_c^2 J_2^2(B)}{2R} ; P_{LSB2} = \frac{A_c^2 J_{-2}^2(B)}{2R}$$

So,

$$P_t = \dots + \frac{A_c^2 J_{-2}^2(B)}{2R} + \frac{A_c^2 J_{-1}^2(B)}{2R} + \frac{A_c^2 J_0^2(B)}{2R} + \frac{A_c^2 J_1^2(B)}{2R} + \frac{A_c^2 J_2^2(B)}{2R} + \dots$$

So,

$$P_t = \frac{A_c^2}{2R} \left\{ \dots + J_{-2}^2(B) + J_{-1}^2(B) + J_0^2(B) + J_1^2(B) + J_2^2(B) \right\}$$

$$P_t = \frac{A_c^2}{2R} \left\{ \sum_{n=-\infty}^{\infty} J_n^2(B) \right\}.$$

$P_t = \frac{A_c^2}{2R}$

$$\left[ \because \sum_{n=-\infty}^{\infty} J_n^2(B) = 1 \right]$$

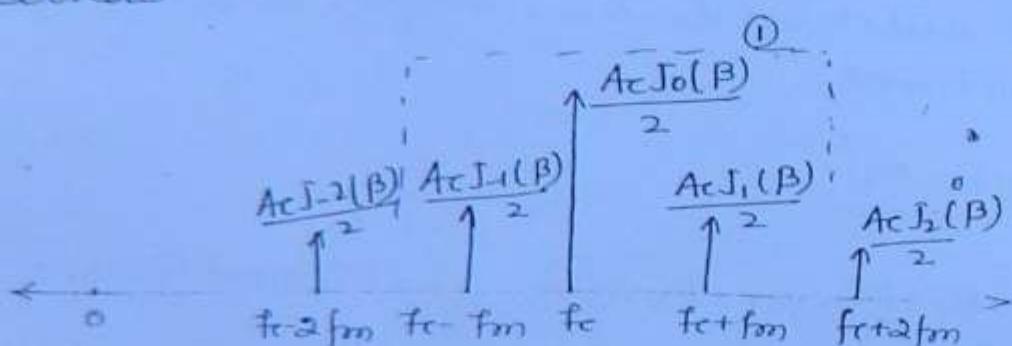
Note :-

1. For WBFM, the power of carrier before modulation same as after modulation.

(126)

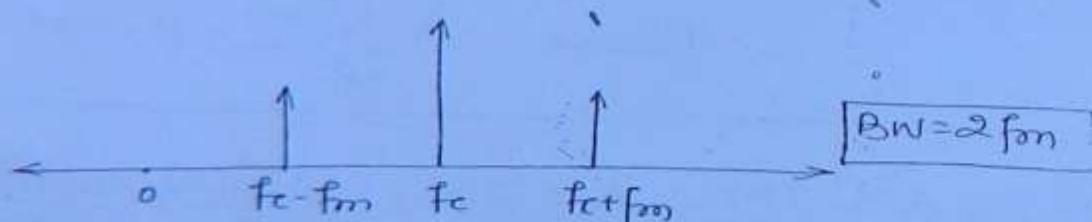
\* Practical Bandwidth of WBFM: { CARSON'S RULE }

\* The Actual B.W of WBFM is  $\infty$ . For transmission of signal, it should be band limited by retaining only significant sidebands and eliminating insignificant sidebands.



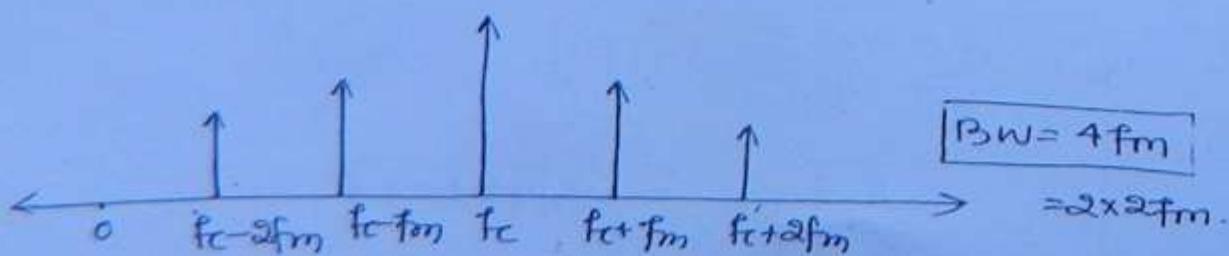
Case 1 (WBFM consist of significant SB's upto 1st order).

After passing through Band limiting we get



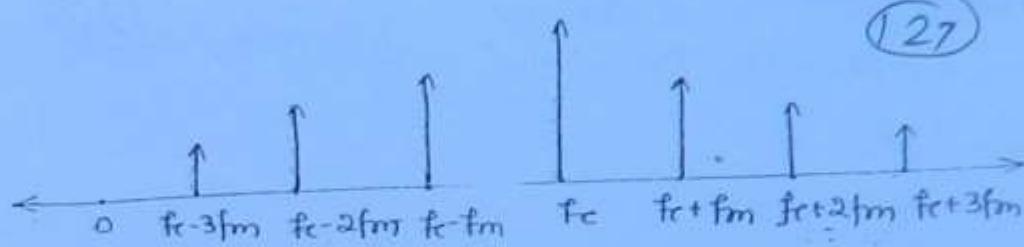
Case 2 (WBFM consist of significant SB's upto 2nd order):-

After Band limiting the signal we get:-



Case 3 (upto 3<sup>rd</sup> order)

After passing through 1<sup>st</sup> and limiting filter we get



(127)

$$\boxed{B \cdot w = 6 \text{ fm} \\ = 3 \times 2 \text{ fm}}$$

### CARSON'S RULE:-

According to Carson, WBFM consist of the significant sidebands upto order "B+1"; when the modulation index is  $\beta$ .

So,

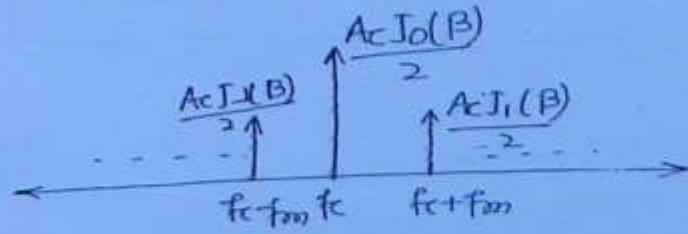
$$\boxed{B \cdot w = (\beta + 1) \times 2 \text{ fm}}$$

Also

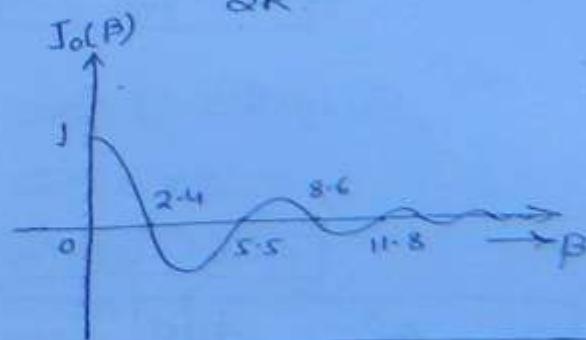
$$\begin{aligned} B \cdot w &= (\beta + 1) \times 2 \text{ fm} \\ &= \left( \frac{\Delta f}{\text{fm}} + 1 \right) \times 2 \text{ fm} \end{aligned}$$

$$\boxed{B \cdot w = 2(\Delta f + \text{fm})}$$

### \* MODULATION EFFICIENCY ( $\eta$ ):-



NOW,  
 $P_c = \frac{Ac^2 J_0^2(\beta)}{2R}$



Standard values.

$$\rightarrow \boxed{J_0(\beta) = 0; \beta = 2.4, 5.5, 8.6, 11.8, \dots}$$

for  $\beta = 2.4, 5.5, 8.6, 11.8$ , hence  $P_c = 0$ , are called as eigen values of  $\beta$ .

Note:  
For above values of  $\beta$ , power taken by carrier prep component will be zero so, that modulation  $\eta$  will become 100%.

(128)

$$J_n(\beta) = 0; \beta = 2.4, 5.5, 8.6, 11.8;$$

$$P_C = 0$$

$$\eta = 100\%$$

Q1. A sinusoidal carrier of 20V, 2MHz is frequency modulated by a sinusoidal msg signal of 10V, 50KHz.

$$K_f = \frac{25 \text{ KHz}}{\text{Volts}}$$

a) find  $\Delta f$ ;  $\beta$ ; Bandwidth & power.

b) repeat above if msg signal amplitude is doubled.

Soln: Given:

$$A_C = 20 \text{ V}; f_c = 2000 \text{ KHz}$$

$$A_m = 10 \text{ V}; f_m = 50 \text{ KHz}$$

$$K_f = \frac{25 \text{ KHz}}{\text{Volts}}$$

$$\text{So, } f_1 = \{f_c + K_f m(t)\} \quad ; \quad f_2 = f_c + K_f m(t) \\ = (2000 + 25 \times 10) \quad \quad \quad = 2000 - 25 \times 10 \\ = 2250 \text{ K} \quad \quad \quad = 1750 \text{ KHz}$$

NOW,

$$\boxed{\beta = \frac{\Delta f}{f_m} = \frac{K_f \cdot A_m}{f_m} = \frac{25 \times 10}{50 \text{ K}} = 5 \text{ (WBFM)}}$$

$$\text{So, } B.W = 2(\beta + 1) f_m$$

$$= 2 \times 6 \times 50$$

$$\boxed{B.W = 600 \text{ K}}$$

$$\boxed{P_t = \frac{A_c^2}{2R} = \frac{400}{2} = 200 \text{ W}}$$

$$2. A_m = 2 \times 10 = 20V$$

$$\text{So, } \Delta f = K_f A_m \uparrow$$

$A_m$  is doubled  $\Delta f$  is doubled.

$$\boxed{\Delta f = 25K \times 20 = 500K}$$

$$\uparrow \beta = \frac{K_f A_m \uparrow}{f_m}$$

$$\boxed{\beta = \frac{25K \times 20}{50} = 10K}$$

$$* B.W = (\beta + 1) \times 2f_m$$

$$= 11 \times 2 \times 50$$

$$\boxed{B.W = 1100K}$$

$$* \boxed{P_t = \frac{A_c^2}{2R} = 200W}$$

Conclusion:

\* when mesg signal amp. is doubled, both  $\Delta f$  and  $\beta$  will also be doubled.

Q2. An FM signal is given by :

$$s(t) = 10 \cos \{2\pi \times 10^6 t + 8 \sin 4\pi \times 10^3 t\}$$

i) Find  $\beta$ ;  $\Delta f$ ; B.W & Power.

ii) Repeat above if mesg. signal frequency is doubled.

Soln: Given,  $s(t) = 10 \cos \{2\pi \times 10^6 t + 8 \sin 4\pi \times 10^3 t\}$

Comparing with :

$$s(t) = A_c \cos \{2\pi f_c t + \beta \sin 2\pi f_m t\}$$

$$\text{So, } A_c = 10V; f_c = 1000K; \boxed{\beta = 8} \quad f_m = 2K$$

NOW,  $\boxed{\Delta f = \beta \times f_m = 16K}$

(129)

$$\begin{aligned} * \text{B.W.} &= 2(\beta+1)f_m \\ &= 2 \times 9 \times 2 \text{K} \\ \boxed{\text{B.W.} = 36 \text{K}} \end{aligned}$$

(130)

$$* P_t = \frac{A_c^2}{2R} = \frac{100}{2} = 50 \text{W}$$

2. Frequency of msg signal doubled. So, new  $f_m$   
 $f_m = 4 \text{ KHz}$

Now,  $\Delta f = K_f \cdot A_m$

$\therefore \Delta f = 16 \text{ KHz} \text{ (no change)}$

Now,  $\beta = \frac{K_f A_m}{f_m \uparrow 2} = 8$  (halved)

Also,

$$\Delta f = \frac{\beta \times f_m}{2} \uparrow 2$$

So,  
 $* \text{B.W.} = 2(\beta+1)f_m$   
 $= 2 \times 8 \times 4 \text{K}$

$$\boxed{\text{B.W.} = 40 \text{K}}$$

$$* P_t = 50 \text{W}$$

Conclusion:

when message signal frequency doubles; "Δf" will not be changed and "β" will halved.

Q3 - A carrier is frequency modulated with max<sup>m</sup> frequency deviation of 16 KHz. Msg. signal freq<sup>n</sup> is given by 1 KHz

a) find β & B.W?

b) Repeat above if message signal amp. is doubled and frequency is reduced to 1 KHz?

Soln:- Given:-

$$\Delta f = 16 \text{ KHz}$$

$$f_m = 1 \text{ KHz}$$

Now,

$$\Delta f = \beta f_m$$

$$\boxed{\beta = 1}$$

$$; \Delta f = \frac{K_f A_m}{f_m}$$

$$\text{Now, } B.W. = 2(\beta+1) f_m$$

$$= 2 \times 5 \times 4$$

$$\boxed{B.W. = 40K}$$

(13)

b) Given:  $\Delta f = 16 \text{ KHz}$  (previous case)

$$f_m = 1 \text{ KHz}$$

$$\text{So, } \Delta f = \frac{K_f A_m}{f_m} \uparrow^2$$

$$\text{So, } \boxed{\Delta f = 32K}$$

$$\text{Now, } \boxed{\beta = \frac{\Delta f}{f_m} = \frac{32}{1} = 32}$$

$$\text{So, } B.W. = 2(\beta+1) f_m$$

$$= 2 \times 33 \times 1 \text{ KHz}$$

$$\boxed{B.W. = 66K}$$

Q4. A carrier is frequency modulated with max<sup>m</sup> frequency deviation of 100K. Find  $\beta$  & B.W if msg signal frequency is  
a) 10 K      b) 1 MHz

Soln: Given:

a)  $\Delta f = 100K$

$$f_m = 10K$$

$$\text{Now, } \Delta f = \beta f_m$$

$$\beta = \frac{\Delta f}{f_m} = 10 (\text{WBFM})$$

$$\text{So, } B.W. = 2 \times (\beta+1) f_m$$

$$= 2 \times 11 \times 10K$$

$$\boxed{B.W. = 220K}$$

b)  $\Delta f = 100K \quad \{ \text{NBFM} \}$

$$f_m = 1000K$$

$$\Delta f = \beta f_m$$

$$\beta = \frac{\Delta f}{f_m} = \frac{100}{1000}$$

$$\boxed{\beta = 0.1}$$

$$B.W. = 2f_m$$

$$= 2 \times 1 M$$

$$\boxed{B.W. = 2M}$$

85 Given:

$$c(t) = 5 \cos 2\pi \times 10^6 t$$

$$m(t) = \cos 4\pi \times 10^3 t$$

(12)

- a)  $c(t)$  &  $m(t)$  are used to generate AM with  $u=0.707$ . Find B.W & power?
- b)  $c(t)$  &  $m(t)$  are used to generate FM ; with max<sup>m</sup> freqn deviation is 3 times B.W of AM. Find the coeff of  $\cos 2\pi \times (0.16 \times 10^3) t$  in F.M expression.
- a) SJ<sub>6</sub>(8)      c) SJ<sub>3</sub>(6)  
b) SJ<sub>8</sub>(6)      d) SJ<sub>3</sub>(8)

Soln: Given,

$$m(t) = \cos 4\pi \times 10^3 t$$

$$A_m = 1; f_m = 2 \times 10^3 \text{ Hz}$$

$$c(t) = 5 \cos 2\pi \times 10^6 t$$

$$A_c = 5; f_c = 0.1000 \text{ KHz}$$

Now,

$$\text{B.W} = 2f_m = 2 \times 2 \text{ KHz}$$

$$\boxed{\text{B.W} = 4 \text{ K}}$$

$$\text{Power} = P_t = P_c \left\{ 1 + \frac{u^2}{2} \right\}$$

$$P_c = \frac{A_c^2}{2R} = \frac{25}{2}$$

$$P_t = \frac{25}{2} \left\{ 1 + \frac{1}{4} \right\} = \frac{25}{2} \times \frac{5}{4}$$

$$\boxed{P_t = \frac{125}{8} \text{ W}}$$

b)  $\Delta f = 3 \times \text{B.W of AM}$

$$= 3 \times 4 \text{ KHz}$$

$$\boxed{\Delta f = 12 \text{ KHz}}$$

$$\text{Now, } B = \frac{\Delta f}{\text{fm}}$$

(133)

$$\boxed{B = \frac{12}{2} = 6} \quad (\text{WBFM}) \quad B = 3 \times 2 \text{ fm}$$

Now,

$$\begin{aligned} \text{WBFM}(t) &= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi (f_c + n \text{ fm}) t \\ &= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi (2016 \times 10^3 t) \end{aligned}$$

Now,  $\cos 2\pi (2016 \times 10^3 t)$ .

$$\text{So, } f_c + n \text{ fm} = \cancel{2016} - 1016$$

$$n \text{ fm} = \cancel{2016} - 1000 - 1016 - 1000$$

$$\text{fm} = \frac{1016 - 1000}{2}$$

$$\boxed{n = 8} \text{ Ans}$$

$$\text{So, Coeff. } \Rightarrow A_c J_8(\beta) = 5 J_8(6) \quad \text{Ans}$$

Q6: A sinusoidal carrier of frequency,  $f_c$  is used for both AM & FM transmitter; max signal freq is given by  $\omega$ . Max freq deviation =  $2 \times \text{B.W.}$  of AM. Find mod index of both AM & FM such that strength of freq component  $f_c + \omega$  is same in both AM & FM spectrum.

$$J_1(10) = 1; J_1(2) = 0.57; J_1(4) = 0.37; J_1(8) = 0.08$$

$$\text{SOLN: } \text{B.W. of AM} = 2 \text{ fm} \quad ; \mu = ? \\ = 10 \text{ kHz} \quad ; \beta = ?$$

$$\Delta f = 2 \times 10 \text{ K} = 20 \text{ K}$$

$$\frac{\Delta f}{\text{fm}} = \boxed{B \Rightarrow B = \frac{20}{5} = 4}$$

$$\text{Now, } \frac{A_c \mu}{A_c} = \frac{A_c J_1(\beta)}{2} \Rightarrow \mu = J_1(4)$$

$$\boxed{\mu = 0.37 \times 2} \Rightarrow \boxed{\mu = 0.74}$$

Strength of  $f_c + \omega$  is same in both AM & FM.

Q1 - A Sinusoidal carrier is frequency modulated by a sinusoidal meg signal of frequency  $\beta$  and amp.  $A_m$ . Conducting an experiment with  $f_m = 1\text{ KHz}$  and increasing  $A_m$  from 0V, it is observed that the strength of the carrier frequency component in the spectrum becomes 0 for the 1st time with  $A_m = 2\text{ V}$ .

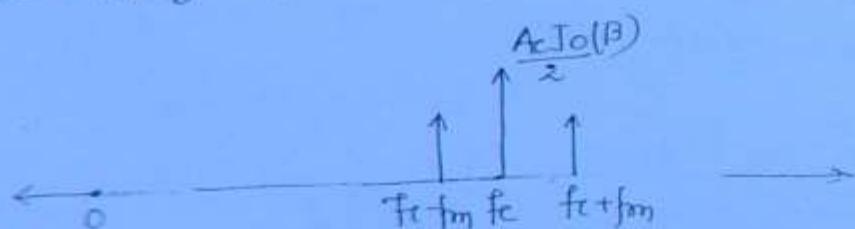
134

Find i)  $K_f$

ii) Find meg signal amplitude for which strength of carrier frequency component becomes zero for the 2nd time?

Soln: Strength of frequency component =  $\frac{A_c J_0(\beta)}{2}$

$$f_m = 1\text{ KHz} ; A_m = 0\text{ V}$$



$$\text{Now, } J_0(\beta) = 0 ; \beta = 2.4, 5.5, 8.6, 11.8$$

$$\text{Now, } \beta_1 = \frac{K_f A_m}{f_m}$$

$$\text{Now, } 2.4 = \frac{K_f \times 2}{1\text{ K}}$$

$$K_f = 1.2\text{ KHz/Volt} \text{ Ans}$$

Now,

$$\beta_2 = \frac{K_f A_m}{f_m}$$

$$5.5 = \frac{1.2 \times 10^3 \times A_m}{1 \times 10^3}$$

$$A_m = 4.58\text{ V} \text{ Ans}$$

Q8. An unmodulated FM transmitter power is given by 100W; with modulation; it is observed that strength of 1st order side band is 0. Find

- the power of carrier frequency component
- Total ~~second~~ side Band power (15)
- total Second order Side Band power.

$$\begin{array}{lll} J_0(2\cdot4) = 0 & ; \quad J_1(2\cdot4) = 0\cdot3 & ; \quad J_2(2\cdot4) = -0\cdot28 \\ J_0(3\cdot8) = 0\cdot4 & ; \quad J_1(3\cdot8) = 0 & ; \quad J_2(3\cdot8) = 0\cdot2 \\ J_0(5\cdot1) = -0\cdot36 & ; \quad J_1(5\cdot1) = -0\cdot23 & ; \quad J_2(5\cdot1) = 0 \end{array}$$

Soln. Given:-  $P_t = 100W = Ac^2/2R = Ac = \sqrt{200}$

$$1st \text{ order SideBand} = \frac{Ac J_1(\beta)}{2} = 0$$

$$J_1(\beta) = 0 = J_1(3\cdot8)$$

$$\boxed{\beta = 3\cdot8}$$

NOW,  
 $\frac{Ac J_0(\beta)}{2}$  = Carrier frequency component

$$\text{Power} = \frac{Ac^2 J_0^2(\beta)}{2}$$

$$= \frac{200 \times J_0^2(3\cdot8)}{2} = \frac{200 \times 0\cdot16}{2}$$

$$\boxed{\text{Power} = 16 \text{ W}} \text{ Ans.}$$

NOW,

$$ii) \text{ Total power} = \frac{Ac^2}{2R} = 100W = P_t$$

$$so, \boxed{\text{Total side Band power} = 100 - 16 \\ = 84W} \text{ Ans.}$$

$$\begin{aligned} iii) \text{ second order power} &= \frac{Ac^2 J_2^2(\beta)}{2R} + \frac{Ac^2 J_{-2}^2(\beta)}{2R} \\ &= \frac{Ac^2}{2} \left\{ J_2^2(\beta) + J_{-2}^2(\beta) \right\} \\ &= Ac^2 \left\{ J_2^2(\beta) \right\} = 200 \times J_2^2(3\cdot8) \end{aligned}$$

$$P_{\text{bandwidth}} = 2 \times 8 \times 0.09$$

(136)

$$P_{\text{bandwidth}} = 8W \quad [\text{Ans}]$$

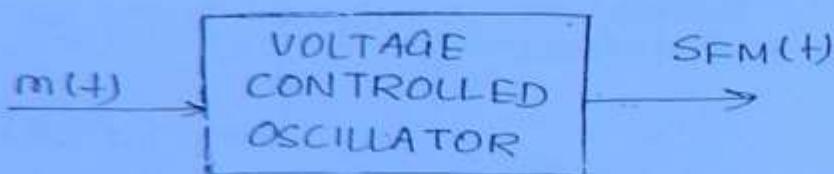
## \* GENERATION OF FM:

Generation of FM done by:-

- 1) Direct Method.
- 2) Indirect Method or Armstrong Method.

## \* DIRECT METHOD:

$\text{FM} \leftrightarrow V_{CO}$

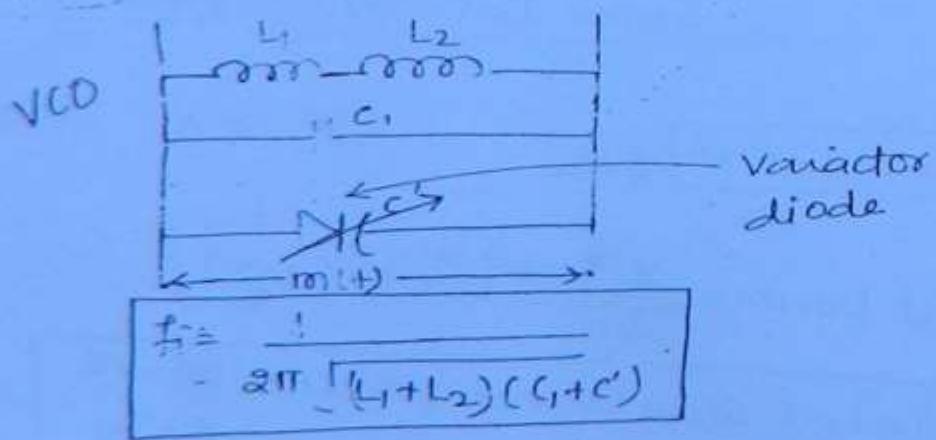


for direct method :-

Hartley oscillator

$$\Rightarrow f_i = \frac{1}{2\pi \sqrt{(L_1+L_2)(C_1+C_2)}}$$

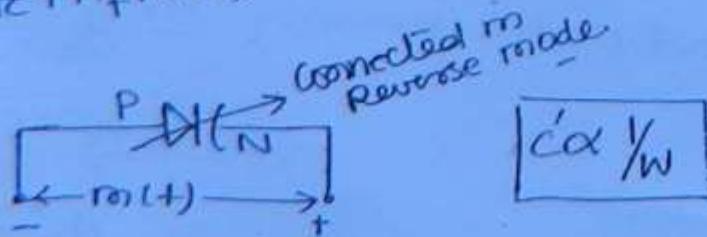
tank circuit VCO :-



$$\text{As, } f_i = f_C K_f m(t)$$

Case 1 :-

$$m(t) = +ve$$



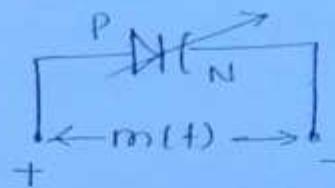
when connected in the Reverse mode

hence

$$[\omega \uparrow \rightarrow c \downarrow \rightarrow f_i \uparrow]$$

(131)

Case 2  $m(t) = -ve$



when connected in the Forward mode

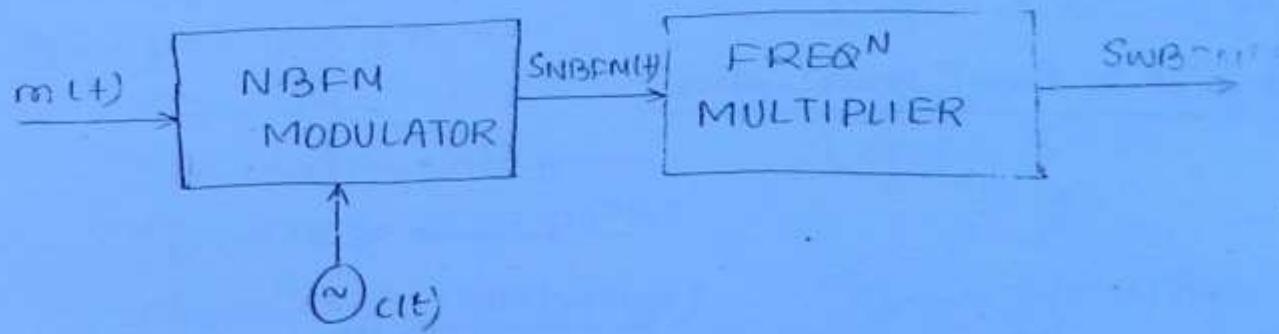
$$[\omega \downarrow \rightarrow c \uparrow \rightarrow f_i \downarrow]$$

Hence, the frequency is varied in accordance to the mod. signal voltage variations.

\* INDIRECT METHOD (ARMSTRONG METHOD)

\* In this method WBFM is generated from NBFM.

Block diagram:

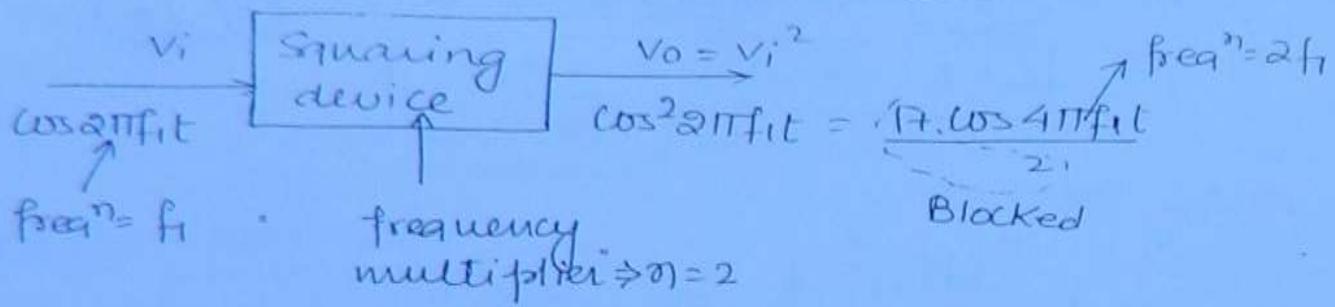


\* Frequency multiplier is nothing but square law device followed by proper Band pass filter

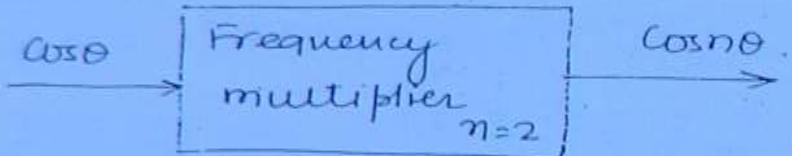
$$V_i \rightarrow \text{SLD} \rightarrow V_2 = a_1 V_i + a_2 V_i^2 + a_3 V_i^3 + \dots$$

\* Pass Band of BPF is such that it allows the  $V_i^{1/2}$  of the Square law device

(138)



Note :-



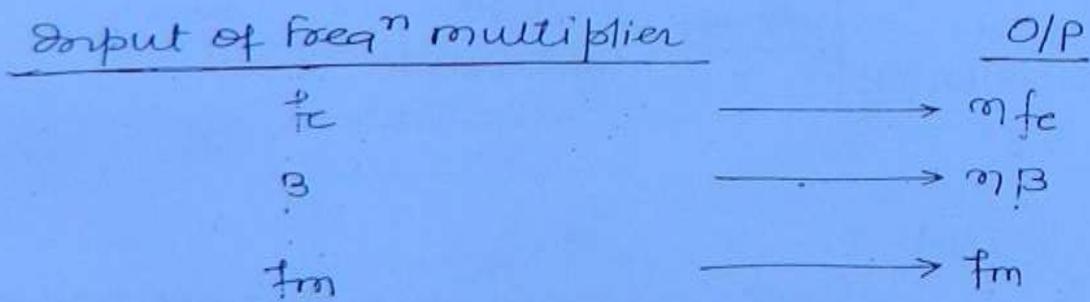
\* The General exp of single tone FM is

$$S_{NBFM}(t) = A_c \cos \left[ \underbrace{2\pi f_c t + \beta \sin 2\pi f_m t}_{\theta} \right]; \beta \leq 1$$

$$(Freq^n mul)_{O/P} = S_{NBFM} = A_c \cos \left[ 2n\pi f_c t + (m\beta) \sin 2\pi f_m t \right]$$

it should such that

$$m\beta > 1$$



$$\uparrow \Delta f = \uparrow \beta f_m$$

so, when  $\beta$  is increased by  $n$

Hence,  $\Delta f$  is increased by  $n$

so,

$$\text{now } \Delta f = n \Delta f$$

Q1 An FM is given by:-

(T39)

$$S(t) = 10 \cos \{2\pi \times 10^6 t + 0.2 \sin 2\pi \times 2 \times 10^3 t\}$$

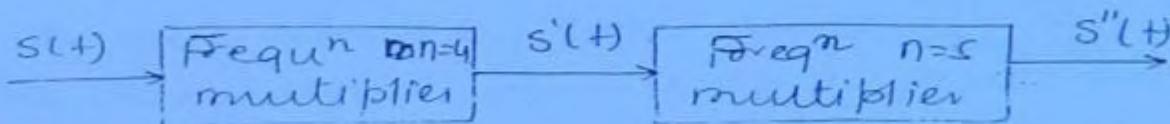
It is passed through cascaded frequency multiplier of having multiplying const of 4 and 5 Respt. Find all the parameters of FM signal at the O/P of each of the multiplier?

Sol): Given:-

$$S(t) = 10 \cos \{2\pi \times 10^6 t + 0.2 \sin 4\pi \times 10^3 t\}.$$

Comparing with standard eqn:-

$$S(t) = A_c \cos \{ \cos 2\pi f_c t + \beta \sin 2\pi f_m t \}.$$



$$A_c = 10 \text{ V}; \beta = 0.2$$

$$f_c = 1 \text{ MHz}; f_m = 2 \text{ kHz}$$

$$\Delta f = \beta f_m \\ = 0.4 \text{ kHz}$$

After passing: (n=4)

$$\checkmark A_c = 10 \text{ V}$$

$$\checkmark f_c = 4 \times 1 = 4 \text{ MHz}$$

$$\checkmark \beta = 4 \times 0.2 = 0.8 \text{ (NBFM)}$$

$$\checkmark f_m = 2 \text{ kHz} \text{ (no change)}$$

$$\checkmark \Delta f = 4 \times 0.4 (\beta \times f_m) \\ = 1.6 \text{ K} \checkmark$$

$$\checkmark B \cdot W = 2 f_m \\ = 4 \text{ K}$$

$$\checkmark P_f = \frac{A_c^2}{2R} \left\{ 1 + \frac{\beta^2}{2} \right\}$$

$$= \frac{100}{2} \left\{ 1 + \frac{0.64}{2} \right\}$$

$$P_f = 66 \text{ W} \checkmark$$

After Passing (n=5):

$$\checkmark A_c = 10 \text{ V}$$

$$\checkmark f_c = 5 \times 4 \text{ M} = 20 \text{ MHz}$$

$$\checkmark \beta = 0.8 \times 5 = 4 \text{ (NBFM)}$$

$$\checkmark f_m = 2 \text{ K}$$

$$\checkmark \Delta f = 5 \times 1.6 \text{ K} \\ = 8 \text{ K}$$

$$\checkmark B \cdot W = 2(B+1) f_m \\ = 2 \times 5 \times 2 \text{ K} \\ = 20 \text{ K}$$

$$\checkmark P_f = \frac{A_c^2}{2R} = 50 \text{ W}$$

# \* Demodulation of FM:

(140)

10-11-2009

- i) Slope detector.
- ii) Balanced slope detector.

## \* Phase discrimination method:

- i) Foster Seeley detector.
- ii) Ratio method.
- iii) PLL method. (mostly used). ( $\text{FES} \leftarrow \text{subj.}$   
 $\text{obj.}$ )

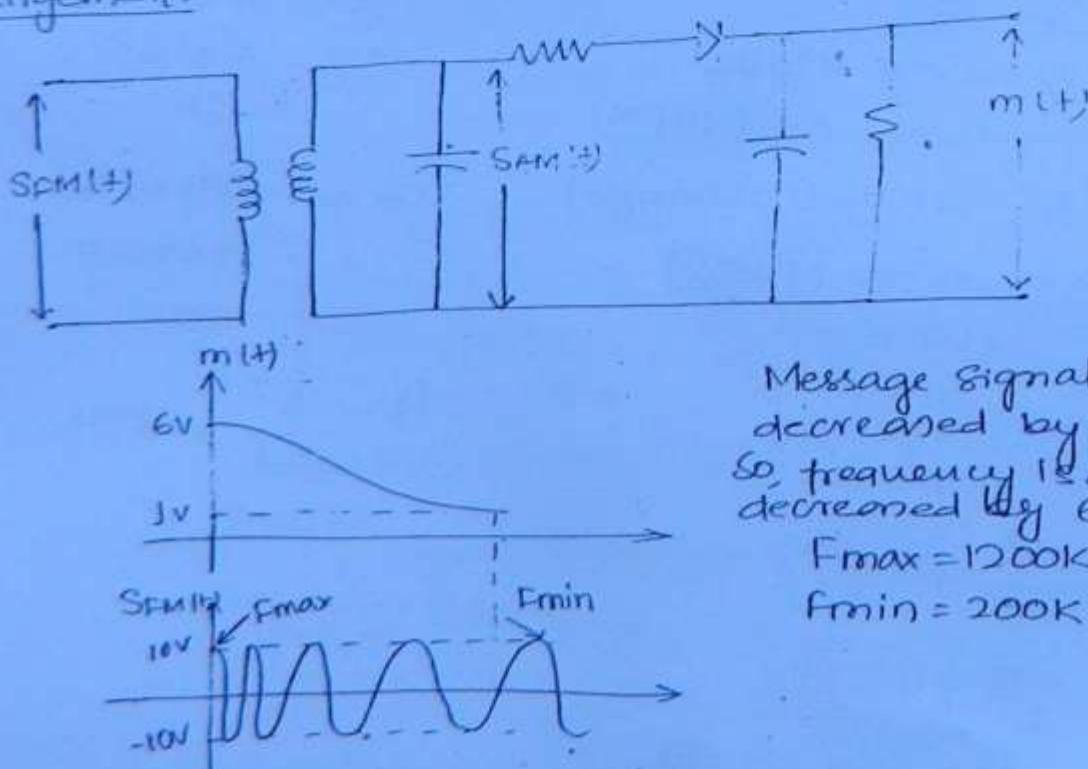
\* Every other demodulator except PLL, needs  $m(t)$  for their constr<sup>n</sup>, so for FM demodulation; generally PLL method will be used.

### i) Slope detector:

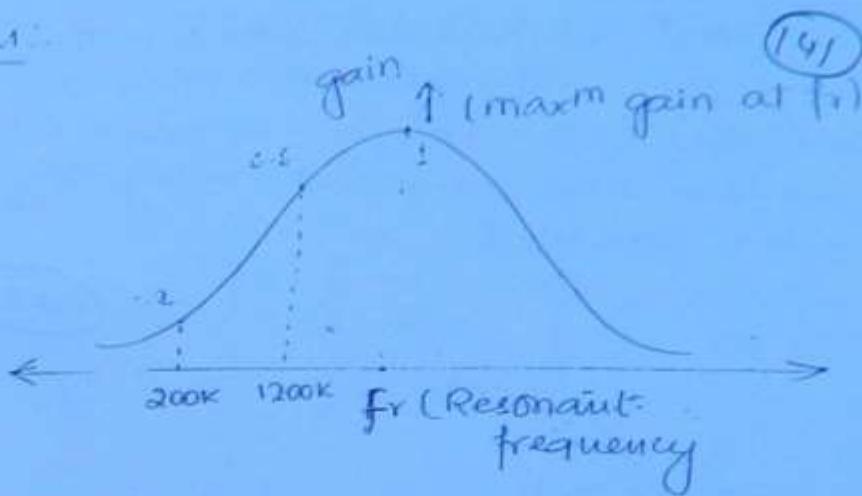
Block diagram:



Ckt arrangement:



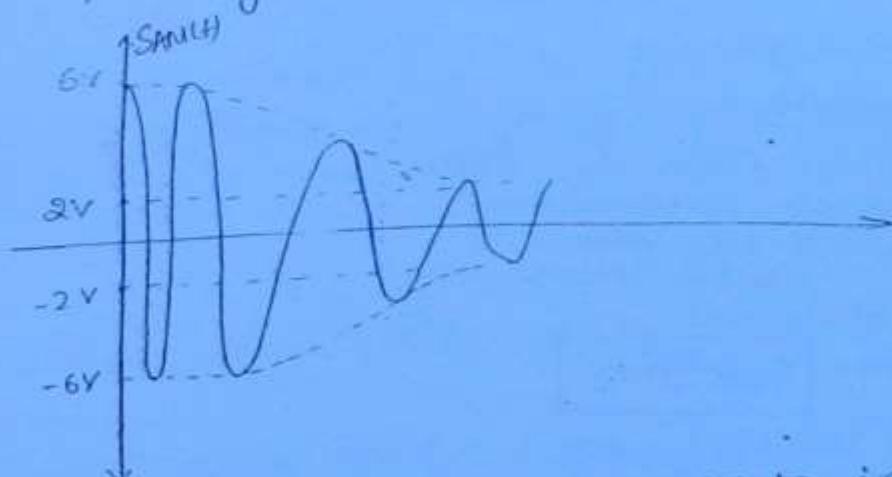
## Tuned Amp char



\* The  $f_r$  should be such that  $f_r > f_{max}$   
(of Tuned Amp)

\* The gain offered by Tuned amp<sup>r</sup> at 1200K is 0.6; so the resulting signal peak amp. is decreased from 10V to 6V and for  $f_{min} = 200K$ ; the peak amp. is 2V

so, o/p is given as:



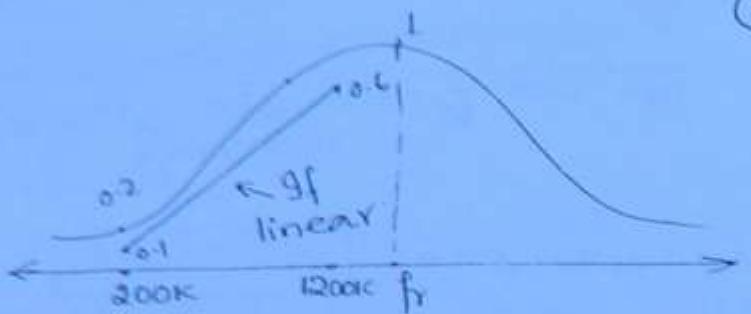
so, the O/P of envelope detector is



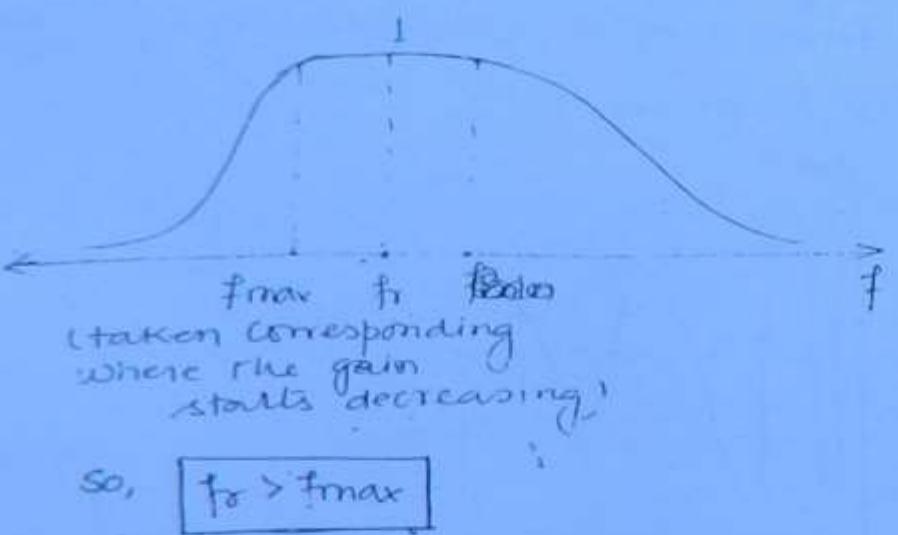
The above doesn't resemble to mit but has some deviation and error is obtained.

- \* The gain freq<sup>n</sup> characteristics was linear i.e. the same amount of gain is obtained as the frequency decreases. The the msg. signal O/P of ED would be same as the original msg. signal and no error was present.

(142)



- \* To avoid above a practical tuned ampl<sup>i</sup> is constructed of gain frequency char. as following:



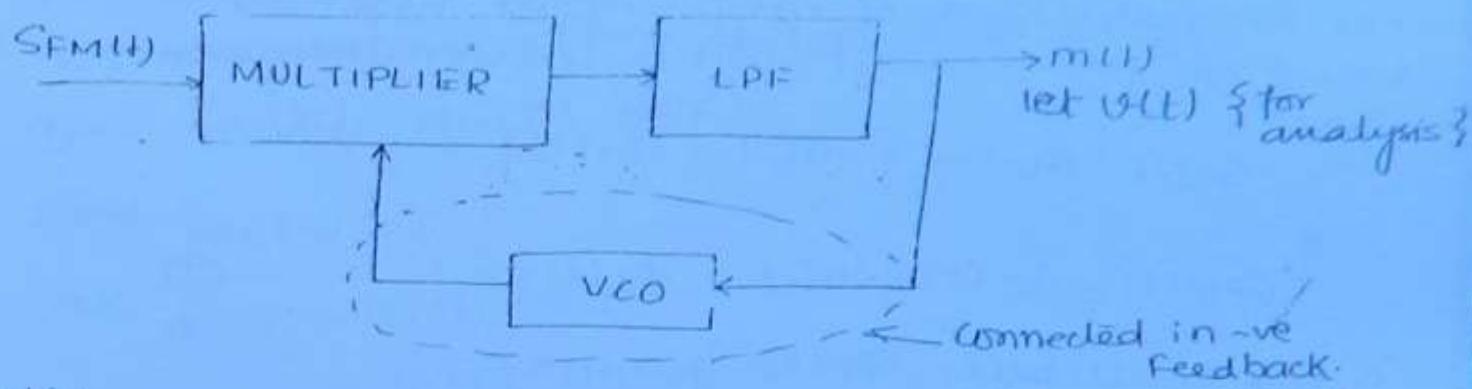
- Note :-
1. Since gain frequency char. of Tuned Ampl<sup>i</sup> is nonlinear in nature, some amount of non-linearity will be introduced in frequency to voltage conversion. So, the Reconstructed msg. Signal is not perfectly corresponds to transmitted msg. Signal and is called as SLOPE ERROR.

2. In Balanced slope detector, 2 slope detectors are connected in balanced to decrease the slope error, to the min<sup>m</sup> possible extent.

## \* PLL ME THOD:

Block diagram:

(143)



Note:

1. The frequency of FM signal changes continuously w.r.t msg. signal voltage variations so to maintain freqn synchronisation L.O of synchronous detector is replaced by VCO.
2. For the VCO, the msg. signal is taken as input, so that VCO o/p frequency changes continuously wrt m(t) voltage variations, frequency synchronisation can be achieved.

\* As,

$$SPM(t) = A_c \cos \left\{ 2\pi f_i t + \frac{2\pi K_f \int m(t) dt}{\Phi_1} \right\}$$

$$\& (VCO)_{o/p} = A_v \cos \left\{ 2\pi f_o t + \frac{2\pi K_v \int v(t) dt}{\Phi_2} \right\}$$

For perfect reconstruction of msg. signal,

- i)  $f_i$  should be made equal to  $f_o$  ie

$$f_i = f_o = f_c$$

Then PLL is said to be working in the LOCK MODE.

- ii)  $\Phi_1(t)$  should be made equal to  $\Phi_2(t)$

$$\Phi_1(t) = \Phi_2(t)$$

Then PLL said to be working in the CAPTURE MODE.

NOTE

Because of no feedback, within no time  $f_1$  will be made equal to  $f_2$  and PLL will go to LOCK MODE.

(144)

For reconstruction of msg signal VCO output should have  $90^\circ$  phase shift w.r.t transmitter carrier; so that

$$SV(t) = Av \sin \{ 2\pi f_{ct} + \Phi_2(t) \} \leftarrow \text{VCO O/P} \quad \text{--- (1)}$$

$$+ SPM(t) = Ac \cos \{ 2\pi f_{ct} + \Phi_1(t) \} \quad \text{--- (2)}$$

So the multiplier O/P is given as:-

$$SV(t) \times SPM(t) = \frac{AcAv}{2} \left\{ \sin(4\pi f_{ct} + \Phi_1(t) + \Phi_2(t)) + \sin(\Phi_1(t) - \Phi_2(t)) \right\}$$

$$= \frac{AcAv}{2} \sin(4\pi f_{ct} + \Phi_1(t) + \Phi_2(t)) - \frac{AcAv}{2} \sin(\Phi_1(t) - \Phi_2(t))$$

$$\text{i.e. } \Phi_1(t) - \Phi_2(t) \approx \Delta t \leftarrow \text{phase error}$$

For PLL, the come will be taken to make  $\Phi_1(t)$  very close to  $\Phi_2(t)$  so that  $\Delta t$  will be very small.

So,

$$SV(t) \cdot SPM(t) = \frac{AcAv}{2} \sin(4\pi f_{ct} + \Phi_1(t) + \Phi_2(t)) \xrightarrow[\text{(Mul) O/P.}]{\substack{\text{no effect} \\ \text{as blocked by} \\ \text{LPF}}} - \frac{AcAv}{2} \sin(\Delta t) \xrightarrow[\text{(Mul) O/P.}]{\substack{\text{no effect} \\ \text{as blocked by} \\ \text{LPF}}} \sin(\Delta t)$$

The 1st term in the above exp corresponds to very high frequencies, so it is not allowed by the LPF and since  $\Delta t$  is very small.

$$\sin(\Delta t) \approx \Delta t$$

$$\text{So, } (\text{Mul})_{\text{OIP}} \approx - \frac{AcAv}{2} \Delta t$$

$$(\text{Mul})_{\text{BIP}} \approx - \Delta t$$

This is given to the LPF.

$$\text{So, } v(t) = \Phi_e(t) \times h(t)$$

So, taking Fourier transform on both sides we get:

$$v(f) = \Phi_e(f) \cdot H(f) \quad \dots \dots \dots (A)$$

$$\text{Now, } \Phi_e(f) = ?$$

$$\begin{aligned} \text{as, } \Phi_e(t) &= \Phi_1(t) - \Phi_2(t) \\ &= \Phi_1(t) - 2\pi K_V \int v(t) dt, \end{aligned}$$

where,  $v(t) \stackrel{e}{=} \Phi_e(t) \times h(t)$ .

$$\frac{d\Phi_e(t)}{dt} = \frac{d\Phi_1(t)}{dt} - 2\pi K_V \cdot v(t)$$

$$\frac{d\Phi_e(t)}{dt} = \frac{d}{dt} \Phi_1(t) - 2\pi K_V \left\{ \Phi_e(t) \times h(t) \right\}.$$

taking F.T. on both sides

$$j2\pi f \Phi_e(f) = j2\pi f \Phi_1(f) - 2\pi K_V \left\{ \Phi_e(f) \cdot h(f) \right\}$$

$\therefore \frac{d}{dt} \Phi(t) \leftrightarrow j\omega \Phi(f)$

$$\Phi_e(f) \left\{ jf + K_V H(f) \right\} = jf \Phi_1(f)$$

$$\boxed{\Phi_e(f) = \frac{jf \Phi_1(f)}{\{jf + K_V H(f)\}}}$$

Also,

$$\Phi_e(f) = \frac{\Phi_1(f)}{1 + \frac{K_V H(f)}{jf}}$$

Now, when [pass Band gain of LPF =  $\infty$ ]  
 so,  $\boxed{\Phi_e(f) = 0 \Rightarrow \Phi_e(t) \Rightarrow \Phi_1(t) = \Phi_2(t)}$

But practically it is not possible, so the pass Band gain of LPF is made high.

- Note
- If the pass band gain of LPF is  $\infty$  ie  $H(f) = \infty$  (14b),  
then,  $\Phi_1(f) = 0$   
but for a practical LPF,  $H(f)$  will be finite
  - By making  $H(f) = \text{very large}$ ;  $\Phi_1(f)$  will be made very small quantity.

NOW, substituting value of  $\Phi_1(f)$  in eqn(A) we get

so,

$$V(f) = \frac{\Phi_1(f)}{1 + \frac{K\omega}{jf} H(f)} \cdot H(f)$$

$H(f)$  is very large

$$\text{so, } \frac{1 + \frac{K\omega}{jf} H(f)}{H(f)} \approx \frac{K\omega}{jf} H(f)$$

so,

$$V(f) = \frac{\Phi_1(f)}{\frac{K\omega}{jf} H(f)} \cdot H(f)$$

$$V(f) = \frac{jf \Phi_1(f)}{K\omega}$$

$$V(f) = \frac{jf \Phi_1(f)}{K\omega} \times \frac{2\pi}{2\pi}$$

By inverse F-T we get

$$v(t) = \frac{1}{2\pi K\omega} \cdot \frac{d}{dt} \Phi_1(f)$$

$$= \frac{1}{2\pi K\omega} \cdot \frac{d}{dt} \left\{ \int_{-\infty}^t K_f \int m(t) dt \right\}$$

So,

$$V(t) = \frac{K_F}{K_V} m(t)$$

(147)

Now,

Note:

1. If  $S_V(t) = A_C \cos \{2\pi f_{cl} t + \Phi_2(t)\}$   
 $S_{FM}(t) = A_C \cos \{2\pi f_{cl} t + \Phi_1(t)\}$

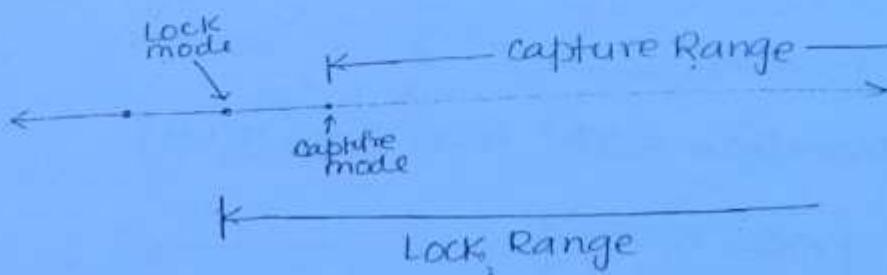
So,  $(MUI)_{O/P} = \frac{A_C A_V}{2} \cos \{4\pi f_{cl} t + \Phi_1(t) + \Phi_2(t)\}$   
+  $\frac{A_C A_V}{2} \cos \{\Phi_2 - \Phi_1(t)\}$ .

$$\Phi_e(t) = \Phi_1(t) - \Phi_2(t) \approx 0$$

$$\text{So, } \cos^2 \{\Phi_e(t)\} \approx 1$$

So, Input of LPF = 1  
Hence msg signal could not be obtained.

2.



So,

$$L \cdot R \geq C \cdot R$$

Note:-

1. For PLL; -ve feedback is Responsible for LOCK MODE.
2. LPF is responsible for CAPTURE MODE of PLL.
3. The frequencies produced by VCO corresponds to LOCK MODE of PLL.

4. For PLL :

$$L \cdot R \geq C \cdot R$$

c) P.M. demodulation is based on two differentiation process

(148)

## \* PHASE MODULATION:

\* Carrier before phase modulation =  $A_c \cos \{2\pi f_c t\}$ .

\* Carrier after phase modulation =  $SPM(t) = A_c \cos \{2\pi f_c t + \phi\}$   
(Phase modulated signal)

where,

$$\phi = K_p m(t)$$

$K_p$  = Phase sensitivity of phase modulator (rad/volt).

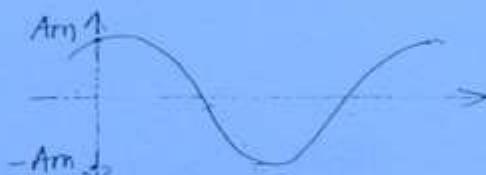
Phase deviation

So,

$$SPM(t) = A_c \cos \{2\pi f_c t + K_p m(t)\}$$

General exp. of phase modulation

Let,  $m(t) = A_m \cos \{2\pi f_m t\}$ .



So,

max<sup>m</sup> phase deviation =  $\Delta\phi = \max \{K_p m(t)\}$

$$\therefore \Delta\phi = K_p A_m$$

So,

$$SPM(t) = A_c \cos \{2\pi f_c t + K_p \cdot A_m \cos \{2\pi f_m t\}\}$$

Let,  $K_p \cdot A_m = \beta$  = modulation index of P.M.

NOW, Corresponding to F.M we have,

$$\beta = \frac{K_p A_m}{f_m} = \frac{\Delta f}{f_m}$$

$$M.I = \frac{\text{Max}^m \text{ freqn deviation}}{\text{mrg. Signal freqn.}}$$

And, corresponding to PM we have:

$$\beta = K_p A_m = \Delta\phi$$

149

M.I = max<sup>m</sup> phase deviation

Note: For phase modulation,  $\beta$  &  $\Delta\phi$  are independent of mod. signal frequency variations

Now, let  $\beta = K_p A_m$

so,

$$S_{PM}(t) = A_c \cos \{2\pi f_c t + \beta \cos \omega_m t\}$$

$$\& S_{FM}(t) = A_c \cos \{2\pi f_c t + \beta \sin \omega_m t\}$$

- Note:
- General expressions of FM & PM are same except  $90^\circ$  phase shift at mod. frequency component.
  - The magnitude spectrum of PM will be same as FM so that B.W and power requirements of PM & FM will be the same.

\* B.W of PM (WBFM):

$$B.W = 2(\beta + 1) f_m \Rightarrow 2(\Delta\phi + 1) f_m = B.W$$

\* Power of PM:

$$P_t = \frac{A_c^2}{2R}$$

Q) A phase modulated signal is given by:

$$SpM(t) = 10 \cos\{2\pi \times 10^6 t + 6 \sin 6\pi \times 10^3 t\}$$

(18b)

i) Find all parameters of PM.

ii) Repeat above if msg signal freq is doubled

Soln:  $SpM(t) = 10 \cos\{2\pi \times 10^6 t + \beta \sin 6\pi \times 10^3 t\}$ .

$$SpM(t) = A_c \cos\{\omega_c t + \beta \sin 2\pi f_m t\}$$

$$A_c = 10$$

i)  $f_c = 1 \text{ MHz} = 1000 \text{ K}$

$$f_m = 3 \text{ K}$$

$$\beta = 6 \text{ rad} = \Delta\phi$$

So,  $B.W = 2(B+1)f_m$  ;  $\boxed{\beta = \Delta\phi = 6 \text{ rad.}}$  Ans.

$$B.W = 42 \text{ K}$$

Ans.

\*  $P_t = \frac{A_c^2}{2R} = \frac{100}{2} = 50 \text{ W}$  Ans.

Now, ii) let  $f_m = 6 \text{ K}$

$$\boxed{\beta = \Delta\phi = K_p A_m = 6 \text{ rad.}}$$
 Ans.

$$\text{So, } B.W = 2(B+1)f_m^2 \\ = 2 \times 7 \times 6$$

$$B.W = 84 \text{ K}$$

Ans.

\*  $P_t = \frac{A_c^2}{2R} = \frac{100}{2} = 50 \text{ W.}$  Ans.

Note:-  
For PM; as the msg signal frequency doubles the corresponding B.W will also be doubled.

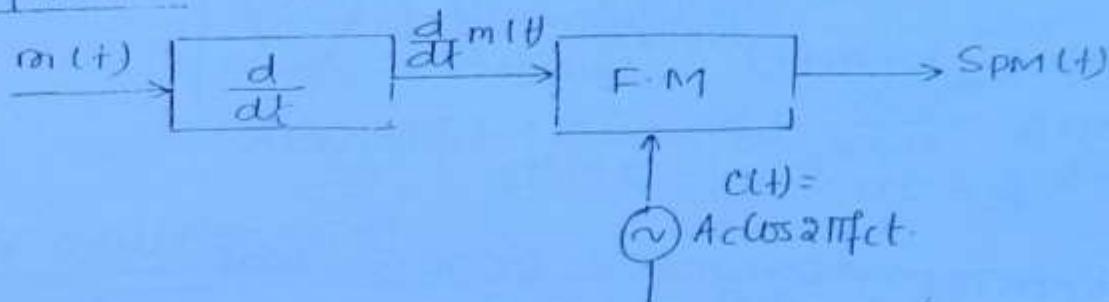
## Generation of FM from PM & PM from FM

As,

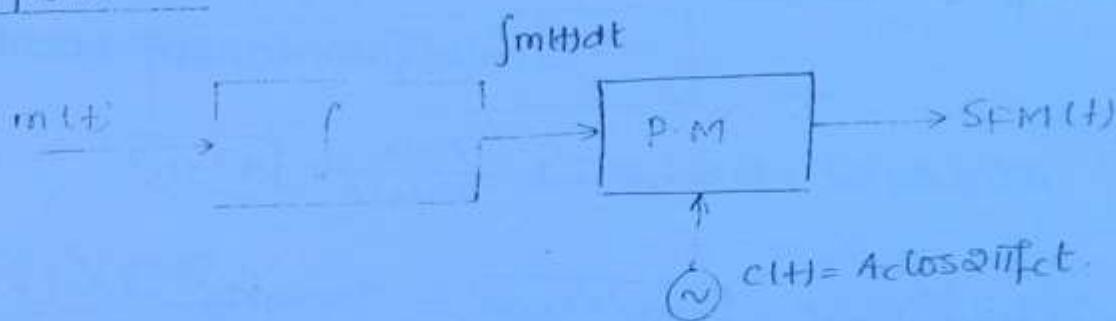
$$SPM(t) = A \cos \{2\pi f_c t + k_p m(t)\} \quad (15)$$

$$\therefore SPM(t) = A_c \cos \{2\pi f_c t + 2\pi k_p \int m(t) dt\}$$

PM from FM:



PM from PM:



Note:-

- Phase modulation of  $m(t)$  is nothing but frequency modulation of  $d/dt m(t)$ .

Q1: An Angle modulated signal is given by

$$S(t) = \cos \{2\pi (10^6 \times 2t + 30 \sin 150t + 40 \cos 150t)\}$$

Find max<sup>m</sup> freq<sup>n</sup> deviation & max<sup>m</sup> phase deviation?

SOP: Given:

$$S(t) = \cos \underbrace{2\pi \{2 \times 10^6 t + 30 \sin 150t + 40 \cos 150t\}}_{\Phi(t)}$$

$\therefore$  Angle modulated signal, so  $\omega$  is

$$\Phi(t) = 2\pi \{2 \times 10^6 t + 30 \sin 150t + 40 \cos 150t\}$$

For FM:

$$\text{So, } f_i = \frac{1}{2\pi} \cdot \frac{d}{dt} \Phi(t)$$

$$f_i = \frac{1}{2\pi} \cdot \frac{d}{dt} \left\{ 2\pi (2 \times 10^6 t + 30 \sin 150t + 40 \cos 150t) \right\}$$

(152)

$$= \frac{1}{2\pi} \left\{ 2\pi (2 \times 10^6 + 4500 \cos 150t - 6000 \sin 150t) \right\}$$

Now,

$$f_i = f_c + K_{fm}(t)$$

frequency deviation

$$\text{so, } f_c = 2 \times 10^6 \text{ Hz} = 2 \text{ MHz}$$

$$+ K_{fm}(t) = 4500 \cos 150t - 6000 \sin 150t$$

$$\text{so max freq^n deviation, } \Delta f = \max \{ K_{fm}(t) \}$$

$$= \max \{ 4500 \cos 150t - 6000 \sin 150t \}$$

$$\text{as, } A \cos 2\pi f_{ct} + B \sin 2\pi f_{ct} \xrightarrow[\text{value}]{\max} \sqrt{A^2 + B^2}$$

$$\text{so, } \Delta f = \sqrt{4500^2 + 6000^2} = 7500 \text{ Hz}$$

$$\boxed{\Delta f = 7.5 \text{ kHz}} \text{ Ans.}$$

FST PM

As,  $S_{pm}(t) = A_c \cos \{ 2\pi f_{ct} + K_{pm}(t) \}$

phase deviation

$$\text{Now, } S(t) = A_c \cos 2\pi \{ 2 \times 10^6 t + 30 \sin 150t + 40 \cos 150t \}$$

$$= A_c \cos \{ 4\pi \times 10^6 t + 60\pi \sin 150t + 80\pi \cos 150t \}$$

$$\text{so, } 2\pi f_c = 4\pi \times 10^6$$

$$K_{pm}(t) = 60\pi \sin 150t + 80\pi \cos 150t$$

$$\Delta\phi = \max \{ K_{pm}(t) \} = \sqrt{(60\pi)^2 + (80\pi)^2}$$

$$\boxed{\Delta\phi = 100\pi \text{ rad.}} \text{ Ans.}$$

W5W  
Pg: 16 / 6 21

Now,  $\hat{m}(t) = 10\cos \omega_0 t + 5\sin 3000\pi t + 10\sin 2000\pi t$

Now,  $A \cos 2\pi f_0 t + B \cos 2\pi f_0 t \xrightarrow[\text{value.}]{\max^m} (A+B)$

$A \sin 2\pi f_0 t + B \sin 2\pi f_0 t \xrightarrow[\text{value.}]{\max^m} (A+B) \sin 2\pi f_0 t$

\* For multitone FM:

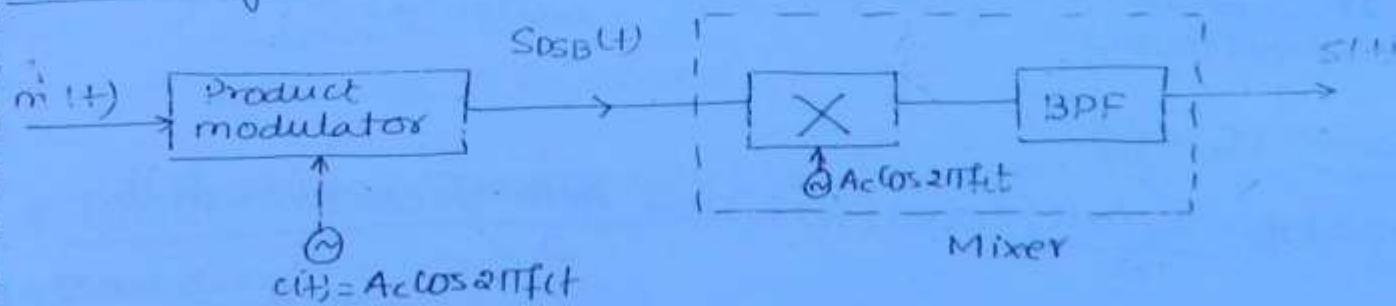
$$\text{Mod}^n \text{ Index} = \text{deviation Ratio} = \frac{\Delta f}{f_{\max}}$$

$$B.W. = 2(B+1) f_{\max}$$

\* MIXER:

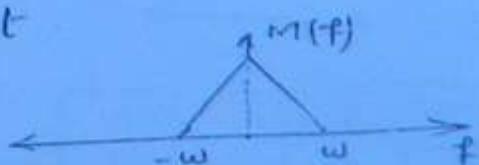
Mixer is used for frequency translation of Modulated signal.

Block diagram:

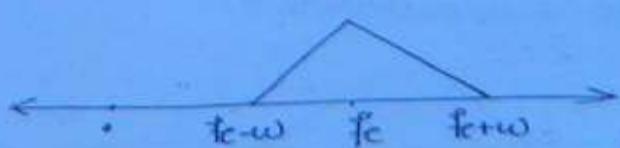


$$S_{SSB}(t) = A c m(t) \cos 2\pi f_0 t$$

Let  $m(t) \longleftrightarrow M(f)$



$$S_{SSB}(t) \longleftrightarrow$$



$$(MUL)_{O/P} = SDSI(1+)(LO)_{O/P}$$

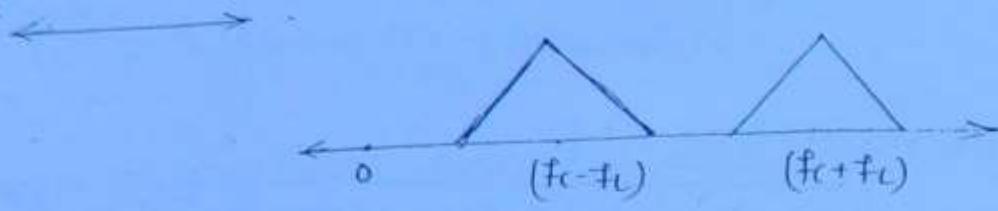
(154)

$$= A_{cm}(4) \cos 2\pi f_c t \times \cos 2\pi f_l t$$

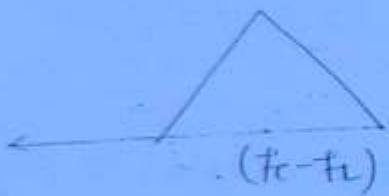
Case i ( $f_c > f_l$ )

$$(MUL)_{O/P} = \frac{A_{cm}(4)}{2} \left\{ \cos 2\pi(f_c + f_l)t + \cos 2\pi(f_c - f_l)t \right\}$$

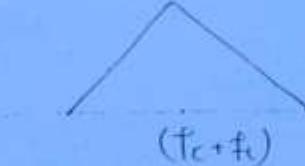
$(MUL)_{O/P}$



So,  $(Mixer)_{O/P}$  (depending upon Pass Band of filter)



when B.P.F O/P is this,  
it is called as DOWN-  
CONVERSION

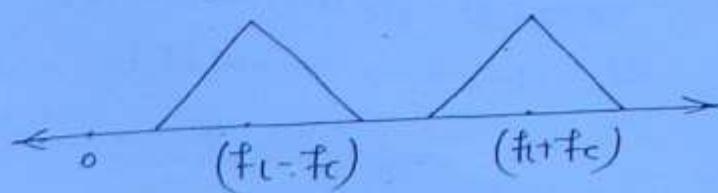


when B.P.F allows the  
 $(f_c + f_l)$  component; it is  
called as UP-CONVERSION

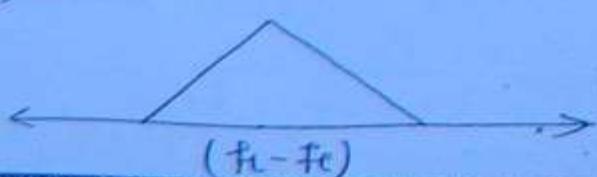
Case ii: ( $f_c < f_l$ )

$$(MUL)_{O/P} = \frac{A_{cm}(4)}{2} \cos 2\pi(f_l + f_c)t + \frac{A_{cm}(4)}{2} \cos 2\pi(f_l - f_c)t$$

$(MUL)_{O/P}$

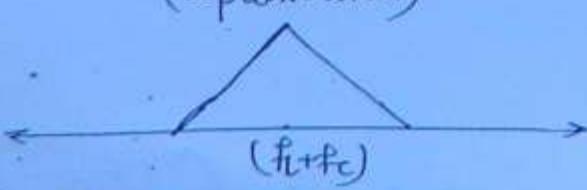


$(Mixer)_{O/P}$  (downconversion)



OR

(upconversion)



Note :-

1. Down conversion will give subtraction of the frequencies  
and upconversion gives sum of the frequencies
- ~~Date : 11-11-11  
X-PSU's~~



### \* RECEIVERS :-

- Tuned Radio Frequency (TRF).
- Super heterodyne (SHD).

Depending on modulation scheme :-

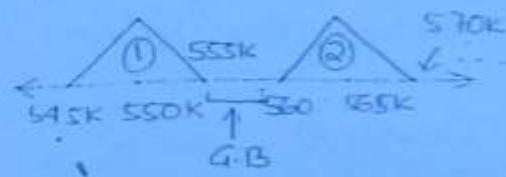
- 1) AM Receiver.
- 2) FM Receiver.

### AM standards :-

According to federation committee of commn(FCC) :-

Carrier freq<sup>n</sup> = 550K to 1650K.

A.M. B.W = 10K.



### FM standards :-

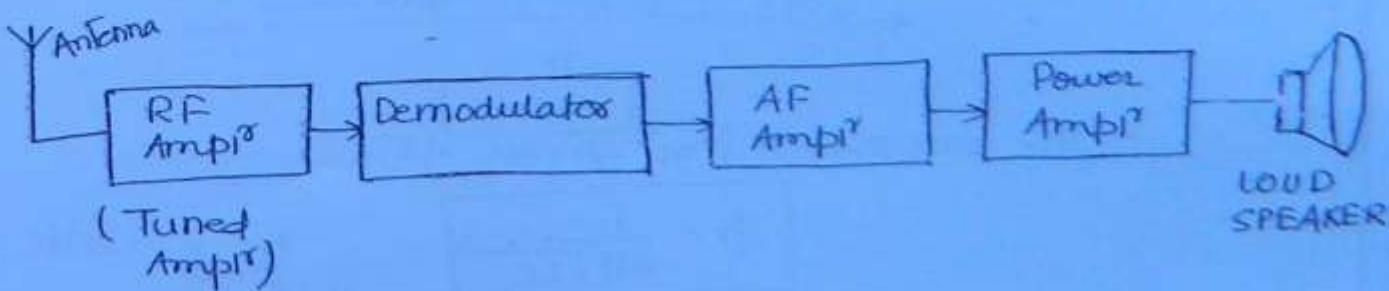
According to FCC :-

Carrier Freq<sup>n</sup> = 88 to 108 MHz.

FM B.W = 200K.

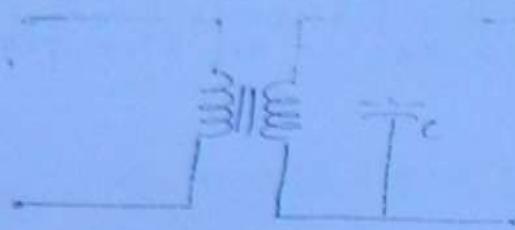
### \* TUNED RADIO FREQUENCY (TRF) Receiver :-

#### \* Block diagram :-

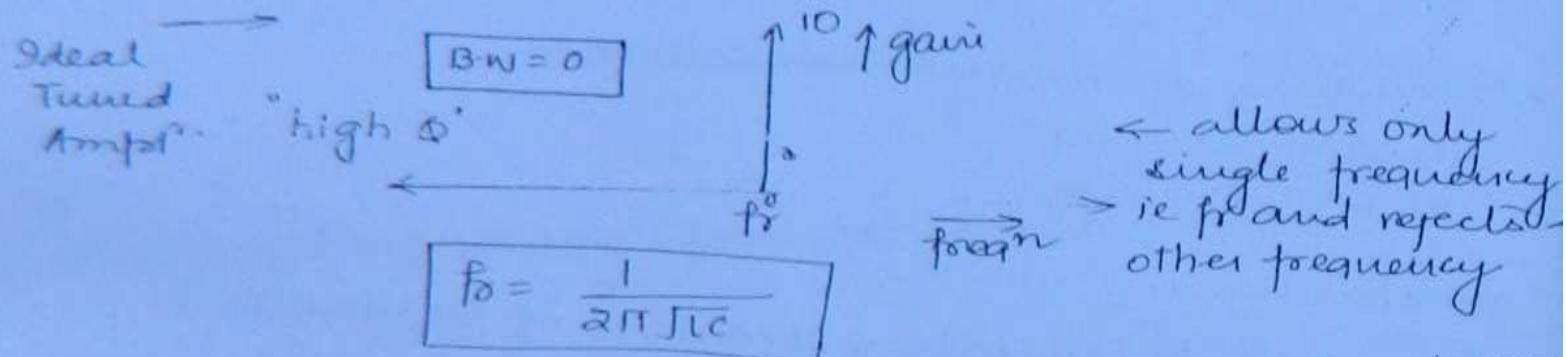


## Tuned Ckt (of Amp):

(156)

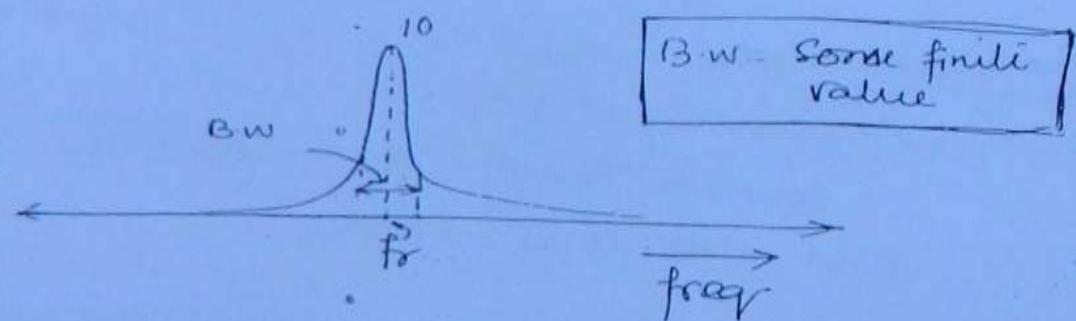


## Gain Frequency Char. of Tuned Ampl.

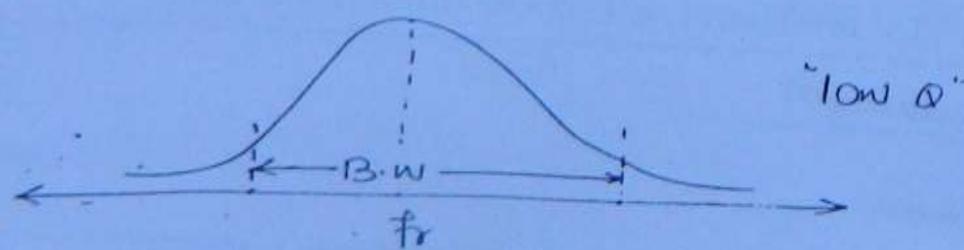


\* The Range of frequencies which the Tuned amp, w/o any Attenuation is called as BW of Tuned Amp.

Practical Tuned Amp's  
(highly Qualitative)



Practical Tuned Amp  
(General case)



The Resonant freq given as:-

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Quality factor given as:-

$$Q = \frac{1}{2\pi} \sqrt{\frac{L}{C}}$$

Note:

1. Quality factor is also said as the sharpness of the GAIN-FREQN CHAR.

sharpness high = Q high,  
sharpness low = Q low.

(151)

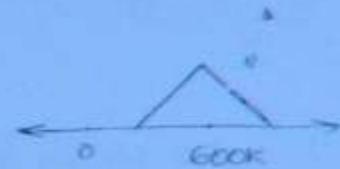
2. Also, B.W is inversely proportional to Q.

$$\boxed{B.W \propto 1/Q}$$
$$B.W = f_r/Q$$

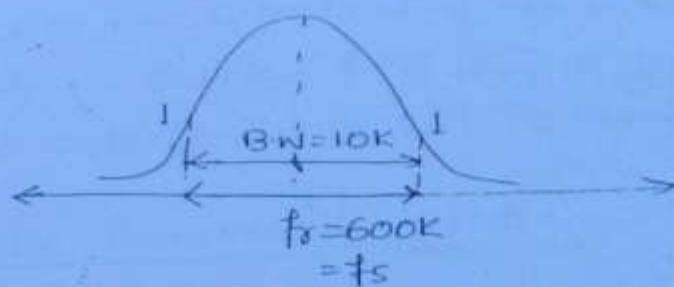
\* Opt of TRF Rx:

Case 1:-

1) Assume Rx is tuned to 600K station  
ie  $f_r = 600K$



2) Assume Tuned Ampl should be



3)  $f_r$  should be equal to 600K;  $f_r = 600K$

4) To get B.W of 10K; Q should be

$$B.W = \frac{f_r}{Q}$$

$$Q = f_r/B.W = \frac{600K}{10K}$$

$$\boxed{Q = 60}$$

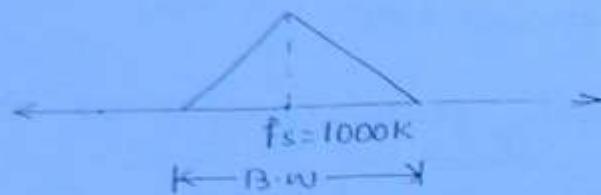
5) Now,  $f_r = \frac{1}{2\pi\sqrt{LC}} = 600K$

$$Q = \frac{1}{2\pi} \times \sqrt{\frac{1}{LC}} = 60$$

CASE 2 -

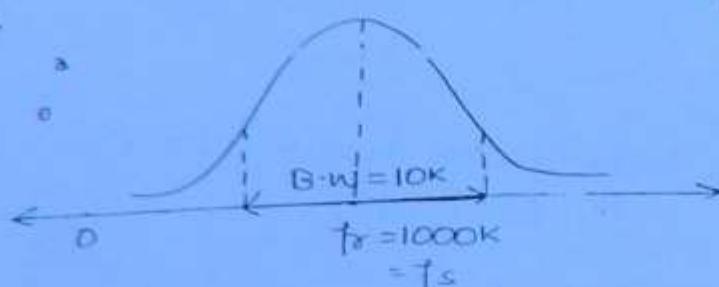
1. Assume the BW is limited to 1000K station  
ie  $f_s = 1000K$ .

(158)



2. Tuned Ampl<sup>r</sup> should be as:-

$$f_0 = 1000K$$
$$B.W = 10K$$



3. To get BW of 10K; Q should be

$$Q = 100$$

4. As.  $f_0 = \frac{1}{2\pi\sqrt{LC}}$ ;  $\alpha = \frac{1}{2\pi} \sqrt{\frac{L}{C}}$

for changing the frequency, capacitance is varied.  
such that  $f_0 = 1000K$

but correspondingly due to diff formulae of  $\alpha$  &  $f_0$ ,  
the value of  $Q$  is not exactly 100.

so, let for  $f_0 = 1000K \Rightarrow Q = 120$

so,

$$B.W = \frac{f_0}{Q} = \frac{1000}{120}$$

$$B.W = 8.3K$$

Hence, some BW of Tuned Ampl<sup>r</sup> is lost and the msg signal correspondingly can't be reconstructed.

## Conclusion:

(159)

1. If B.W of T.Ampl<sup>r</sup>: 100K  $\Rightarrow$  m(H) can be perfectly Reconstructed.
  2. If B.W of Tuned Ampl<sup>r</sup> < 10K  $\Rightarrow$  m(H) can't be Reconstructed perfectly & freq<sup>n</sup> attenuated.
  3. If B.W of T.Ampl<sup>r</sup> > 10K  $\Rightarrow$  m(H) can't be Reconstructed perfectly & some unwanted freq<sup>n</sup> are allowed.
- \* Due to the above discussion, the selectivity of the TRF Rx is poor.

## Note:-

1. For TRF Rx, as the Tuning changes B.W of the Tuned Ampl<sup>r</sup> will be changed accordingly.
2. For excellence in selectivity, to what may be the station, Rx is Tuned for B.W of Tuned Ampl<sup>r</sup> should be 10K only. So selectivity of TRF Rx is worst.

## Characteristic Parameters of Rx:

1. Selectivity: It is the ability of the Rx to allow only the desired freq<sup>n</sup> components and rejecting undesired frequency components.  
It mainly depends on characteristics of Tuned Ampl<sup>r</sup>.
2. Sensitivity: It specifies the min<sup>m</sup> strength of the signal to be maintained at the Rx input; to produce faithful output.

$\times$  IF Ampl<sup>2</sup> = Tuned Ampl<sup>2</sup>  
always tuned to 455 K  
and  $B \cdot W = 10K$

(16)

so,  $\Omega = \frac{f_r}{B \cdot W} = 45.5$

455 K

$\times$  Mixer is designed to work as a downconverter and  
f<sub>c</sub> is preferred to take as greater than f<sub>s</sub> ( $f_c > f_s$ ); so  
that Mixer OIP is equal to:

$$f_L - f_s = 455 \text{ K} = \text{IF} = \frac{\text{doubt intermediate}}{\text{freq}}$$

Op<sup>n</sup>:

case 1:

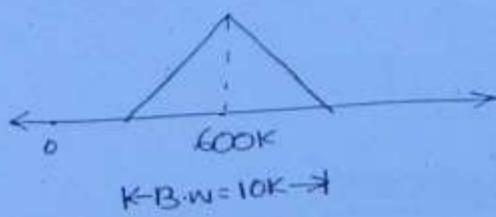
1. Assume Rx is tuned to 600 K Hz station.



2. ~~I.F. amplifier~~ tuning

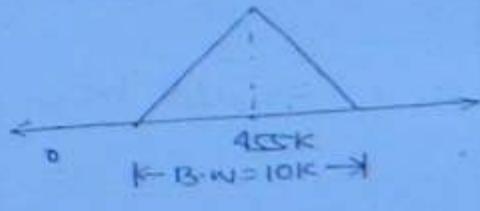


3.



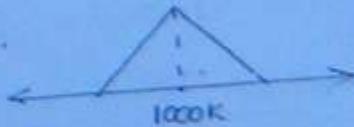
mixer

$$f_L = 1455 \text{ K}$$

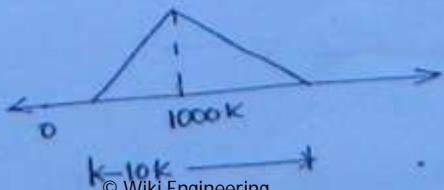


case 2:

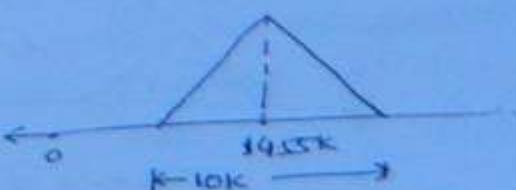
1. Assume Rx is tuned to 1000 K Hz station.



2.



mixer  
 $f_c = 1455 \text{ K}$

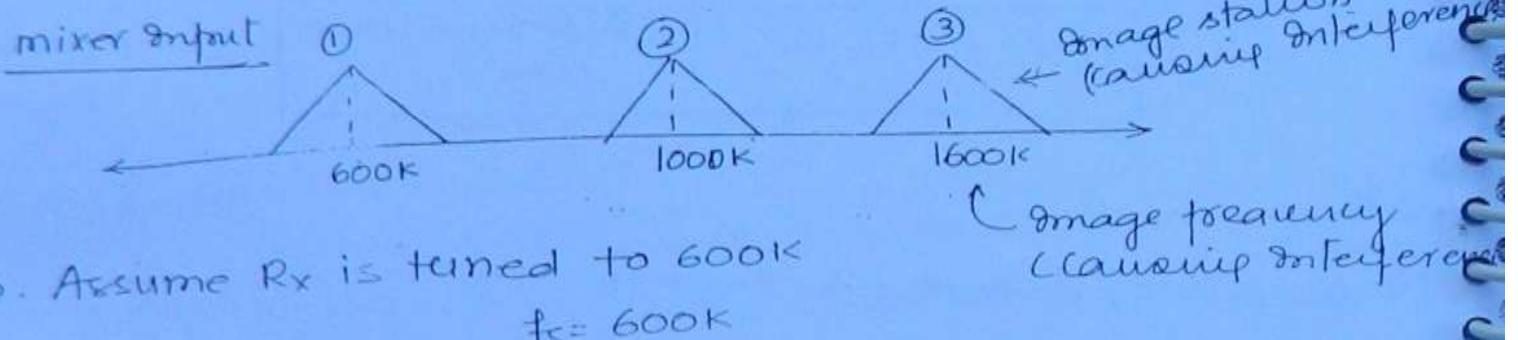


- Note:
1. In TRX Rx, tuning is achieved by changing 'f<sub>t</sub>' of the tuned Amp.
  2. In SHD Rx; tuning is achieved by changing "f<sub>c</sub>" of the local oscillator.

(T62)

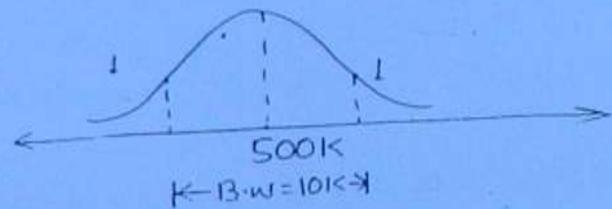
### \* Image Frequency:

Let the messages spectrum be:



2. Assume Rx is tuned to 600K  
 $f_s = 600K$

3. Let desired intermediate frequency, IF = 500K

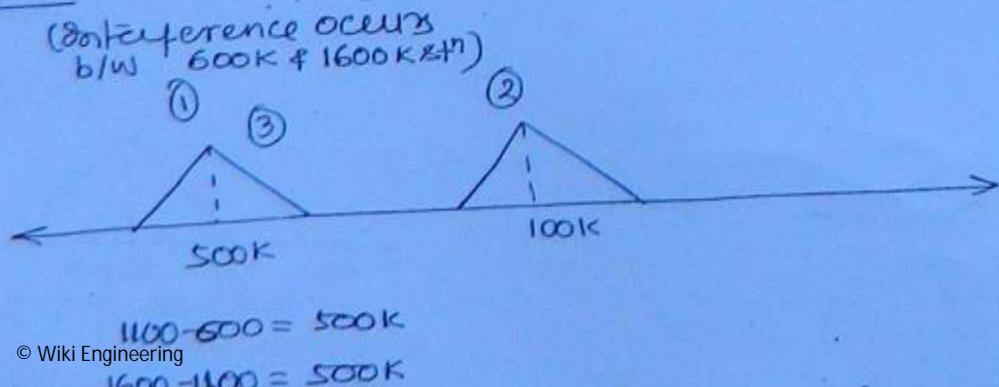


4. f<sub>L</sub> should be,  $f_L - f_s = I.F$

$$f_L = I.F + f_s = 500K + 600K$$

$$\boxed{f_L = 1100K}$$

### Mixed O/P:



$$1100 - 600 = 500K$$

$$1600 - 1100 = 500K$$

x 1600K station is causing interference to 600K station  
so 1600K is said to be image frequency of 600K

Now,

(163)

Image freq<sup>n</sup>,  $f_{si} = f_s + 2I \cdot F$

Let,

$$I \cdot F = 600K$$

$$f_s = 600K$$

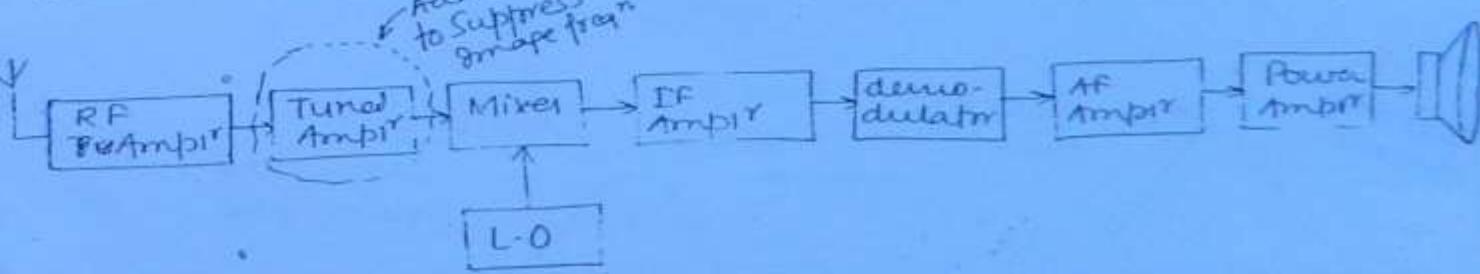
$$f_{si} = 600 + 1000$$

$$\boxed{f_{si} = 1600K}$$

### \* Suppression of Image frequency:-

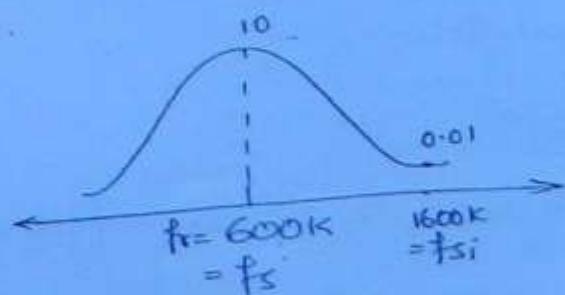
1. Since, desired station and image station are occupying same frequency band at Mixer output, so for comfortable reconstruction of desired signal image frequency should be suppressed.

2. To suppress image frequency, a tuned Amplifier will be used before mixer.



### "Practical SHD"

#### x Gain-Freq<sup>n</sup> char. of Tuned Amplif<sup>r</sup>:

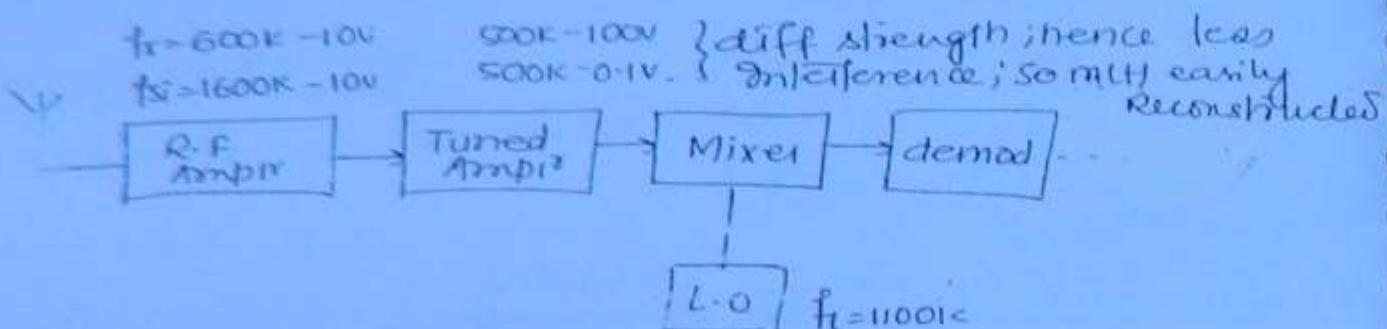
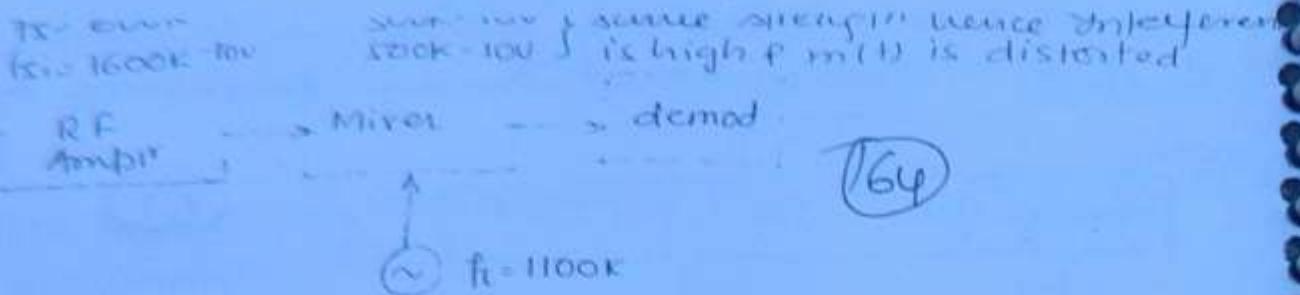


So, OIP of Tuned Amplif<sup>r</sup>;  $f_s = 600K \rightarrow 100V$

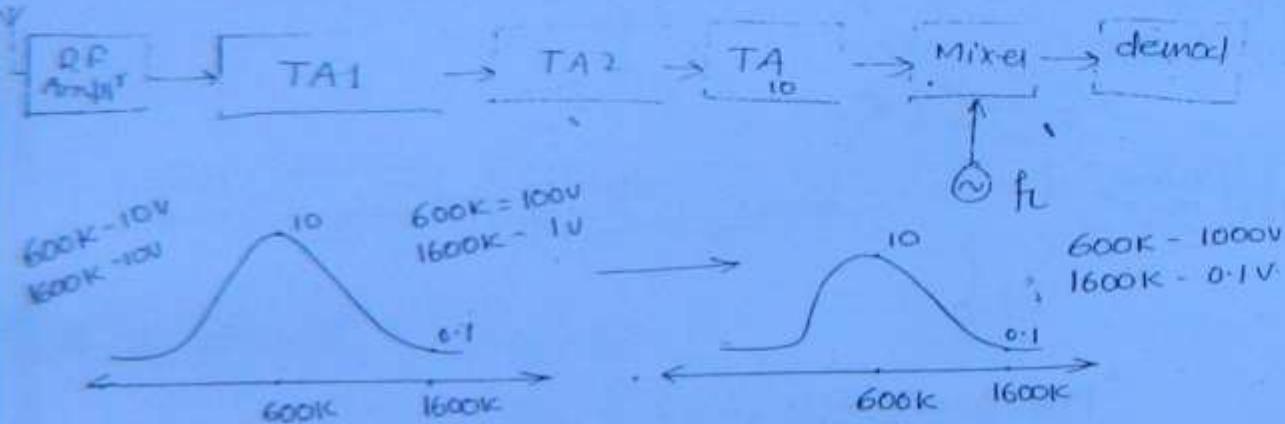
$$f_{si} = 1600K \rightarrow 0.1V$$

$$f_s = 600K; f_{si} = 1600K$$

The Resonant freq<sup>n</sup> of this Amplif<sup>r</sup> is not fixed as that of IF Amplif<sup>r</sup>. But it can be raised in accordance to the desired stn.



\* For much separation of image freqn; Cascaded Tuned Amplif will be used at the output of Mixer.  
So,



Note:

\* For getting optimum suppression, Cascaded Tuned Amplif should have same characteristics.

\* Image Rejection Ratio (IRR):

It specifies effectiveness of tuned Amplif in suppressing image frequency.

OR

It specifies how many times, image frequency is attenuated wrt desired frequency.

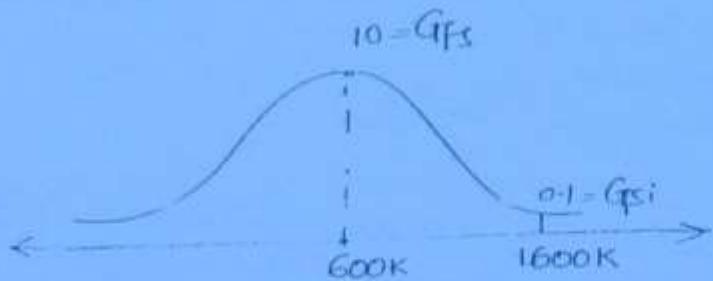
Mathematically,

$$\text{IRR} = \frac{\text{Gain offered by TA to fs}}{\text{Gain offered by TA to fsi}}$$

(165)

or  $\alpha = \text{IRR} = \frac{G_{fs}}{G_{fsi}}$

So,



$$\alpha = \frac{G_{fs}}{G_{fsi}} = 10/1$$

$$\alpha = 100$$

If Tuned Amplifier are connected in cascade then,

$$\text{effective IRR} = \alpha_1 \cdot \alpha_2 \cdot \alpha_3$$

so, from above discussion

$$\alpha_1 = 100 ; \alpha_2 = 100$$

$$\therefore \text{effective } \alpha = 100 \times 100 \\ = 10000$$

Known

\* If  $G_{fs}$  &  $G_{fsi}$  are not, then;

$$\alpha = \sqrt{1 + P^2 Q^2}$$

where,  $Q = \text{Quality factor of TA}$

$$P = \frac{fsi}{fs} - \frac{fi}{fsi}$$

\* If 2 tuned Amplifier of having diff. char are connected in CASCADE, then

$$\alpha = \sqrt{P_1^2 Q_1^2 + 1} \times \sqrt{1 + P_2^2 Q_2^2}$$

\* If tuned Amp's having same char. then:

$$\alpha = \sqrt{1 + P^2 Q^2} \times \sqrt{1 + P^2 Q^2}$$
$$= 1 + P^2 Q^2$$

(186)

Q1. A Rx is tuned to 600 KHz, I.F. = 450 KHz  
Find  $f_L$  &  $f_{SI}$ .

Soln: Given:  $f_S = 600\text{K}$

$$f_I = 450\text{K}$$

So,  $f_I = f_L + f_S$

$$f_L = f_I + f_S = 600 + 450$$

$$f_L = 1050\text{K} \quad \underline{\text{Ans}}$$

And,  $f_{SI} = f_S + 2f_I$   
 $= 600\text{K} + 900\text{K}$

$$f_{SI} = 1500\text{K} \quad \underline{\text{Ans}}$$

Q2. A Rx is tuned to 500K; local osc. freqn is given by 1050 K. Find i) I.F. and  $f_{SI}$   
ii) Find IRR, if  $Q = 50$ .

Soln: Given:

$$f_S = 500\text{K}$$

$$f_L = 1050\text{K}$$

$$f_I = f_L - f_S$$
$$= 1050 - 500$$

$$f_I = 550\text{K}$$

So,  $f_{SI} = f_S + 2f_I$   
 $= 500 + 1100\text{K}$

$$f_{SI} = 1600\text{K}$$

Now,  $P = \frac{f_{SI}}{f_S} - \frac{f_S}{f_{SI}} = \frac{1600}{500} - \frac{500}{1600}$   
 $P = 2.8$

$$\text{Q. } \alpha = \sqrt{1 + P^2 Q^2} \\ = \sqrt{1 + (2.8)^2} \times 100$$

(127)

$$\boxed{\alpha = 144.3} \text{ Ans.}$$

- Q3. A Rx is tuned to 750K; Corresponding storage freqn is given by 1750K. find i) f<sub>t</sub> & I.F  
 ii) find IRR if a tuned ampl'r of having Q = 50 & 70 are connected in cascade

Sol<sup>n</sup>: Given: f<sub>s</sub> = 750K

$$f_{si} = 1750K$$

$$\text{so, } f_{si} = f_s + 2f_f$$

$$f_f = \frac{1750 - 750}{2} = 500K$$

$$\text{Now, } f_i = f_t - f_s$$

$$500 = f_t - 750K$$

$$f_t = 1250K$$

$$\text{Now, } p = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{1750}{750} - \frac{750}{1750} = 1.9$$

$$\text{Now, } \alpha = \sqrt{1 + P^2 Q_1^2} \times \sqrt{1 + P^2 Q_2^2} \\ = \sqrt{1 + 1.9^2 \times 2500} \times \sqrt{1 + 1.9^2 \times 4900}$$

$$\boxed{\alpha = 12636.05}$$

- Q4. A Rx is tuned to 1MHz, E.F = 455 KHz and Q = 100.  
 i) Find IRR  
 ii) Find IRR if the Rx is tuned to 25MHz

Sol<sup>n</sup>: Given, f<sub>s</sub> = 1000K  
 I.F = 455K

$$f_{si} = f_s + 2f_f$$

$$= 1000 + 910 \text{ K}$$

(16)

$$f_{si} = 1910 \text{ K}$$

$$\text{So, } P = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{1910 \text{ K}}{1000} - \frac{1000}{1910}$$

$$P = 1.386$$

$$\begin{aligned}\text{So, } \alpha &= \sqrt{1+P^2 Q^2} \\ &= \sqrt{1+386^2 \times 10000} \\ \boxed{\alpha = 138.6}\end{aligned}$$

$$\text{Now, } f_s = 25 \text{ MHz} = 25000 \text{ K}$$

$$\begin{aligned}\text{So, } f_{si} &= f_s + 2f_f \\ &= 25000 + 910 \\ &= 25910 \text{ K}\end{aligned}$$

$$P = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{25910}{25000} - \frac{25000}{25910}$$

$$P = 0.072$$

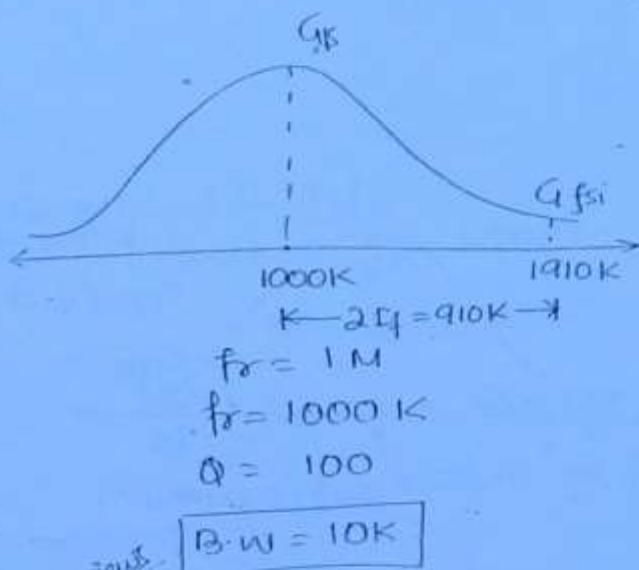
$$\begin{aligned}\text{So, } \alpha &= \sqrt{1+P^2 Q^2} \\ &= \sqrt{1+0.072^2 \times 10000} \\ \boxed{\alpha = 7.26}\end{aligned}$$

Note :-  
1. In the 2nd case, when  $f_s = 25 \text{ MHz}$ ;  $\alpha$  is very small i.e.

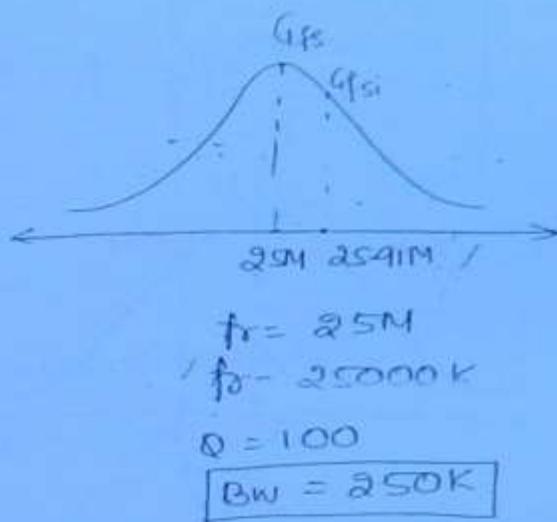
2. For comfortable reconstr'n of signal  $\alpha$  has to be increased.

~~30, f<sub>T</sub>~~  
Case 1 (when f<sub>S</sub> = 1M):

169



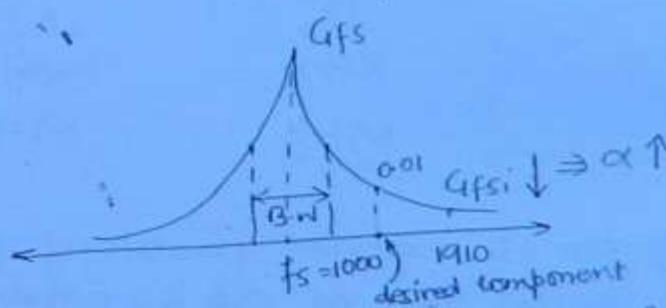
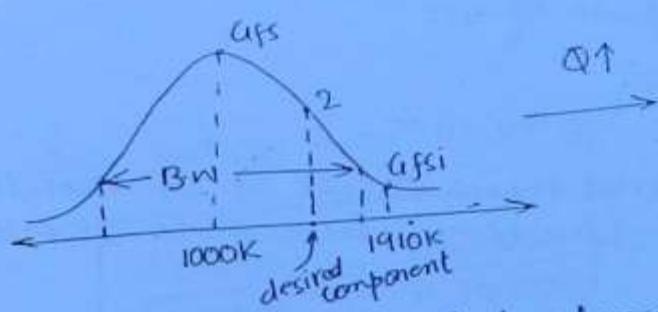
Case 2 (when f<sub>S</sub> = 2.5M)



\* Measures to increase  $\alpha'$

$$\text{As } \alpha' = \sqrt{1 + P^2 Q^2}$$

i) by increasing Q:



Practically it is not preferred, as the B.W decreases, hence the selectivity of the Rx is affected.

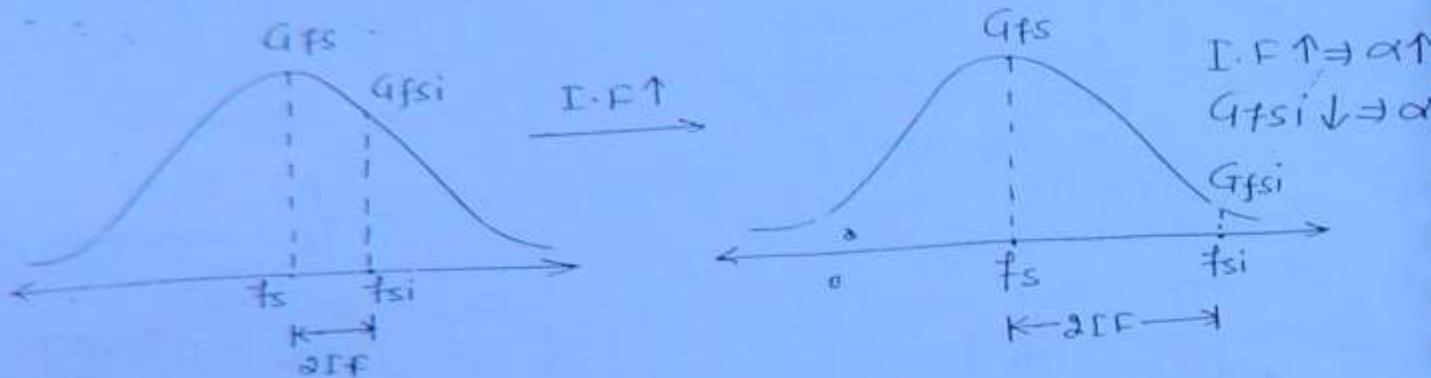
Hence, it may be concluded that some of the desired frequency components will be attenuated. Hence, the desired mag. signal (audio signal) cannot be perfectly reconstructed.

Note: The above is not suggested because B.W of the tuned Amplifier will be very much decreased and it affects the selectivity of the RX.

$$\text{As, } \frac{f_{si}}{f_s} = \frac{f_c}{f_{ci}}$$

(170)

Hence,  $f_{si} = f_s + 2IF \uparrow$



Conclusion:

To get high value of IRR; both  $f_c$  and  $IF$  should be in the same order.

$\frac{f_s}{KHz}$	$\frac{IF}{KHz}$
MHz	KHz
MHz	MHz

- Q. For the above problem, find new value of IF required to get IRR of 138.6% when the Rx is tuned to 25 MHz?

Soln: Given,  $\alpha = 138.6$   
 $f_s = 25000 K$

$$\text{So, } \alpha = \sqrt{1+P^2Q^2} \Rightarrow 138.6 \\ 1+P^2Q^2 = (138.6)^2$$

$$PQ = 138.6$$

$$P = 1.385$$

$$\text{Now, } \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = 1.385$$

$$\frac{f_{si}^2 - f_s^2}{f_s \times f_{si}} = 1.385$$

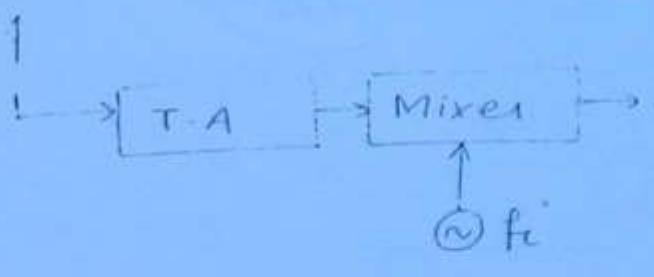
$$\Rightarrow \frac{f_{si}^2 - 625}{f_{si}} = 34.65$$

$$f_{si}^2 - 34.65f_{si} - 625 = 0$$

$$f_{si} = 47.74 M$$

$$\Rightarrow f_s + 2IF = 47.74$$

$$IF = 11.37 M$$



(171)

Case 1: ( $f_L > f_s$ )

Assume  $I_F = 500K$

$$I_F = f_L - f_s = 500K$$

NOW, for AM, Range is  $f_s \rightarrow 500K$  to  $1650K$

i. when  $f_s = 550K \Rightarrow f_L = 1050K$

$$\text{so, } f_L = \frac{1}{2\pi\sqrt{(L_1+L_2)}C} \Rightarrow 1050 = \frac{1}{2\pi\sqrt{(L_1+L_2)}C_{(\max)}} \quad \dots(1)$$

2. when,  $f_s = 1650K \Rightarrow f_L = 2150K$

$$\text{so, } 2150 = \frac{1}{2\pi\sqrt{(L_1+L_2)}C_{(\min)}} \quad \dots(2)$$

So, eqn(2)  $\div$  eqn(1) we get:-

$$\frac{C_{\max}}{C_{\min}} \approx \left( \frac{2150}{1050} \right)^2 \approx 4$$

Case 2: ( $f_L < f_s$ )

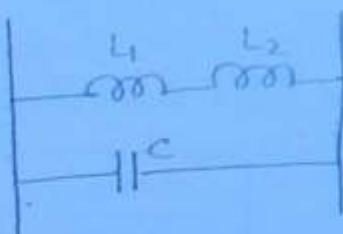
let  $I_F = 500K$

$$I_F = f_s - f_L = 500K$$

i.  $f_s = 550K \Rightarrow f_L = 50K$

$$\text{so, } 50K = \frac{1}{2\pi\sqrt{(L_1+L_2)}C_{\min}} \quad \dots(1)$$

$$50K = \frac{1}{2\pi\sqrt{(L_1+L_2)}C_{\max}} \quad \dots(2)$$



$$f = \frac{1}{2\pi\sqrt{(L_1+L_2)}C}$$

$$2. \text{ when } f_S = 1650 \text{ K } \Rightarrow f_L = 1150 \text{ K}$$

(172)

(Q)

$$\text{so, } 1150 \text{ K} = \frac{1}{2\pi \sqrt{(L_1+L_2) C_{\text{main}}}}$$

$\text{eq}^n(2) \div \text{eq}^n(1)$  we get

$$\frac{C_{\text{max}}}{C_{\text{min}}} \approx \left(\frac{1150}{50}\right)^2 \approx 500$$

Conclusion:

Tuning of capacitors will be easy for  $f_L > f_S$ . so, it is preferred.

- \* AM - For long distance
- FM - For short distance

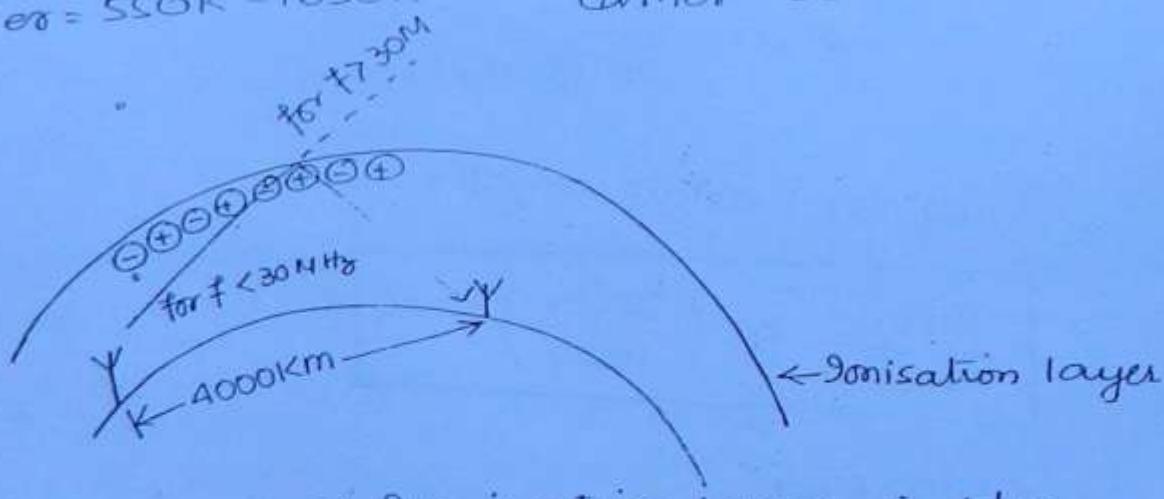
Discussion :-

As, for AM:-

$$\text{Carrier} = 550 \text{ K} - 1650 \text{ K}$$

For FM

Carrier - 88 M to 108 M.

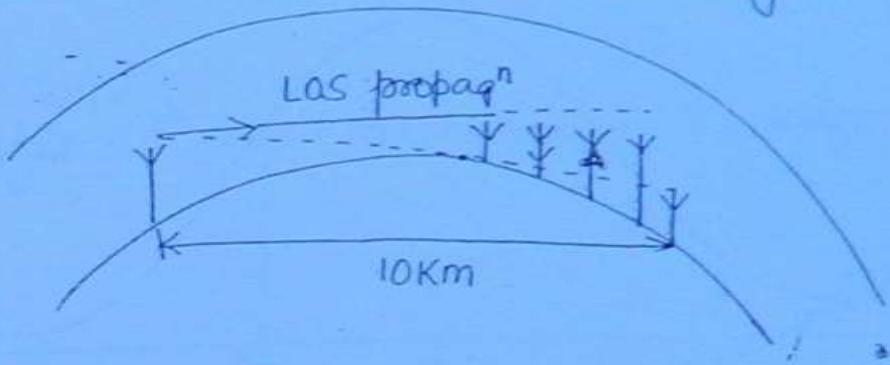


- \* Due to the property of ionisation layer, if the freqn  $< 30 \text{ M}$ , it will reflect back the signal.
- \* But if  $f > 30 \text{ M}$ , escapes the layer and never comes back.
- \* If the distance b/w xmitter and Rx is 4000 km (upto) it is called as single HOP propagation.

\* If for distance  $> 1000$  km, transceivers are used to regenerate the signal, and is called as Ionospheric layer propagation.

(173)

For FM: For FM, ~~the~~ line of sight propagation used. The max<sup>m</sup> distance that may be covered by FM is 10 Km.



\* If LOS propagation is not maintained, then the FM signal is blocked by curvature of Earth and Signal is lost.

\* Freq<sup>n</sup> Reuse Technique can be implemented.

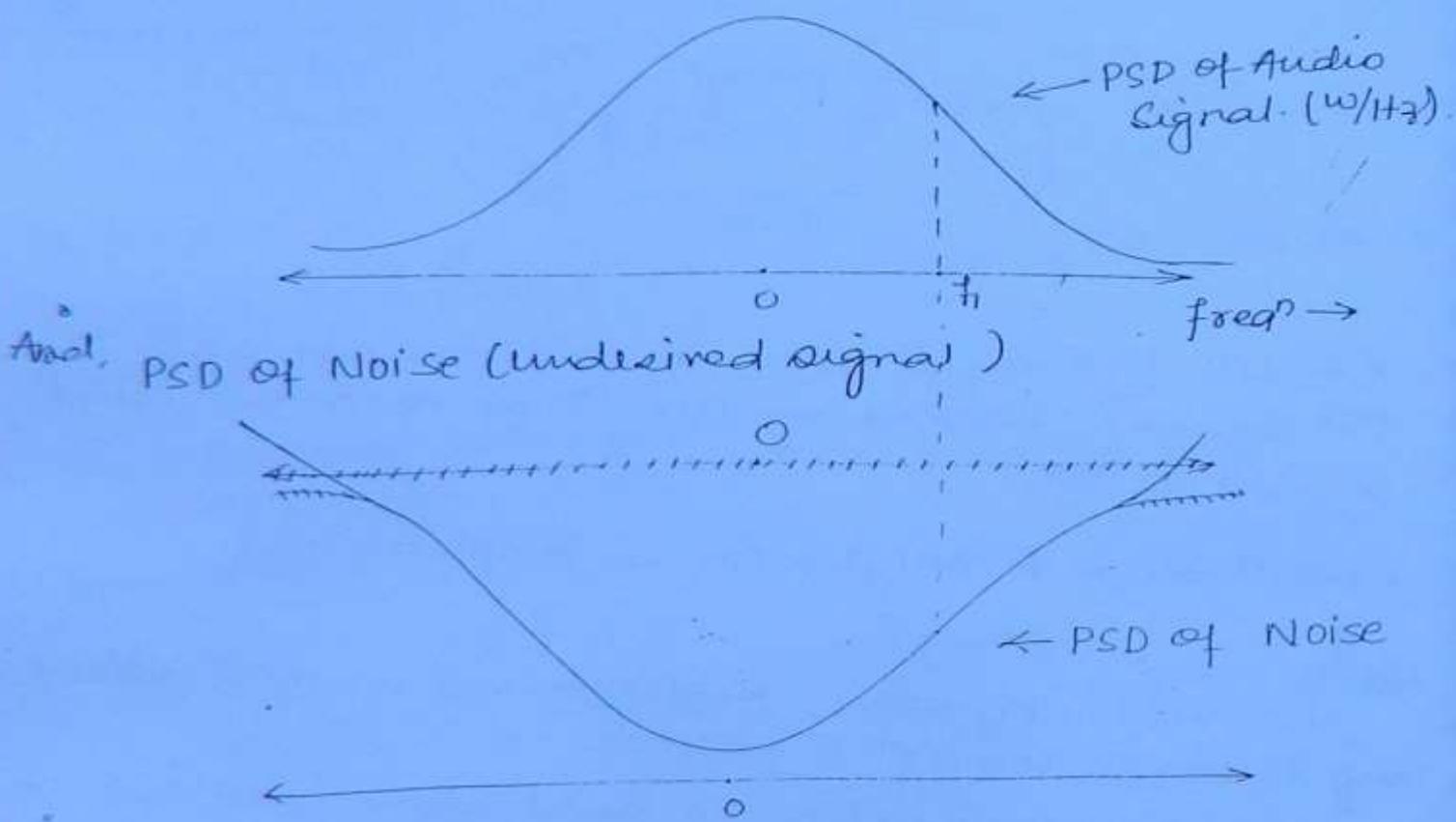
Note:-

1. For AM; Sky wave propagation is used where long distance comm<sup>n</sup> is possible.
2. In FM; LOS propagation is used, where short dist. Comm<sup>n</sup> is possible.
3. Freq<sup>n</sup> Reuse concept can be effectively implemented in FM transmission.

## X PRE-EMPHASIS & DE-EMPHASIS:

(74)

As we know that, the PSD of Audio Signal is given by:



Anal. PSD of Noise (Undesired signal)

### Analysis:

- \* upto  $f_1$ ;  $\frac{S}{N} > 1$ ; these freq<sup>n</sup> components can be reproduced at the Rx output.
- \* Above  $f_1$ ;  $S/N < 1$ ; these high freq<sup>n</sup> components cannot be reproduced at the Rx output
- \* Hence, for high freq<sup>n</sup>, :
  - either S power has to be increased
  - decreasing N power (but it is Indeterministic).
- \* So, artificially increasing the signal strength at high freq<sup>n</sup> is called as PRE-EMPHASIS.

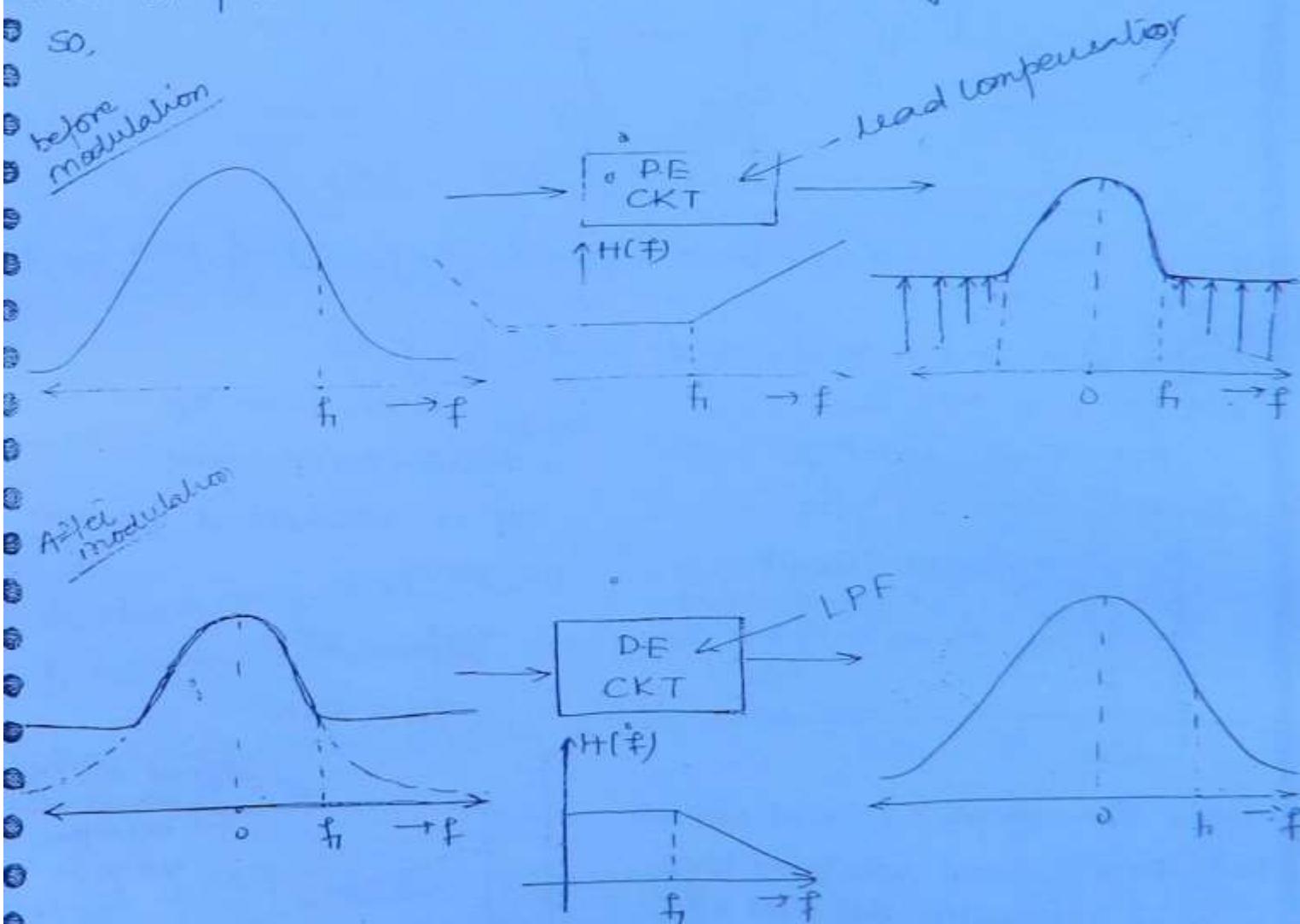
## \* PRE-EMPHASIS:-

\* To improve fidelity S/N of high freqn components of the Audio signal has to be increased.

(P13)

\* The process of increasing the strength of high freqn components of Audio signal is called as PRE-EMPHASIS

\* Pre-Emphasis will be done in Tx before modulation so,



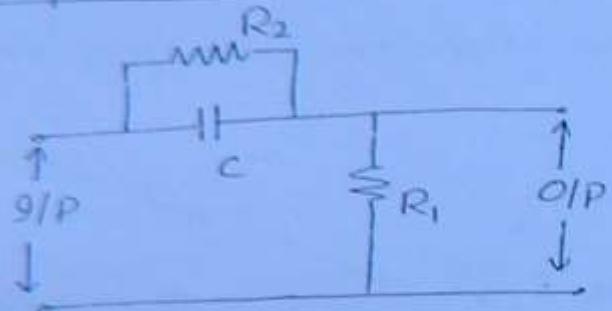
## \* DE-EMPHASIS:-

\* It is the process of decreasing the strength of high freqn components of Audio signal.

\* De-emphasis will be done in the Rx after demodulation.

(176)

### x Lead compensator:

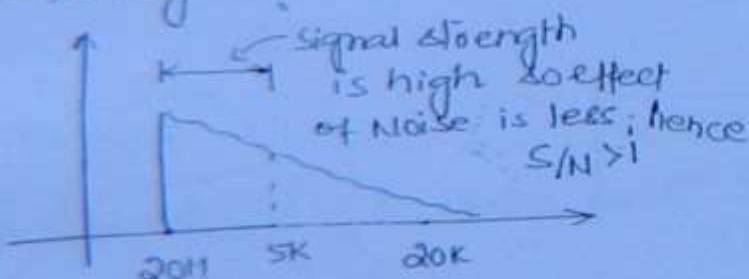


Q. why Pre-emphasis & De-emphasis needed in FM but not in AM?

As for AM, the standard AM B.W is 10 KHz

$$\text{AM B.W} = 10 \text{ KHz}$$

$$\text{So, Negg B.W} = 5 \text{ KHz}$$



Hence, Pre-emphasis not needed. But, freqn above 5K has less signal strength, so  $S/N < 1$ .

Now, for voice signal range is 300Hz to 3.5K, hence the  $S/N < 1$ . So, it can be transmitted very easily w/o any effect of noise.

Note:-

On AM transmission, low freqn of audio signal is only considered for transmission, so, PE & DE are not used.

AS, for F.M

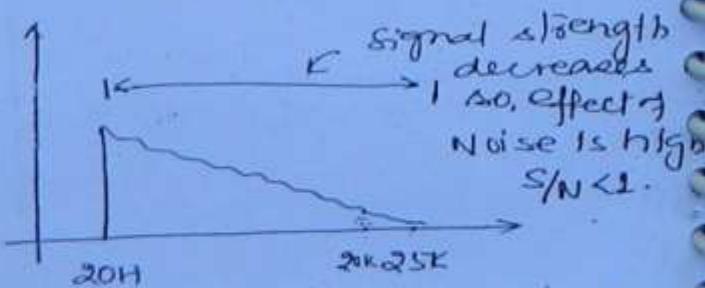
$$\text{B.W} = 200 \text{ KHz}$$

$$2(\Delta f + f_m) = 200 \text{ K}$$

if i.e. standard for FM is 70KHz.

$$2\Delta f_{\text{fm}} = 50 \text{ K}$$

$$f_m = 25 \text{ K}$$



Hence, Pre-emphasis needed to increase  $S/N$ .

\* In FM transmission, the width of the signal also will be considered for transmission, and for high freq S/N < 1 so PE & DE are required.

(177)

### \* F.M RECEIVERS:

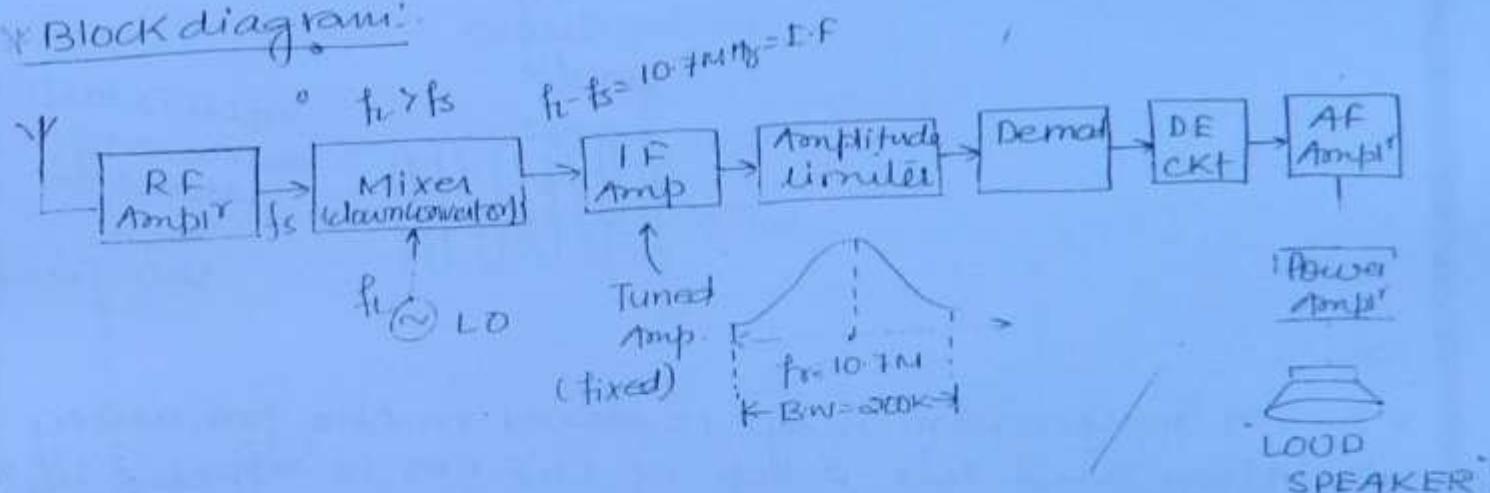
For FM, the standards are:

carrier freq = 88 MHz to 108 MHz.

F.M BW = 200 kHz.

I.F = 10.7 MHz.

### \* BLOCK diagram:

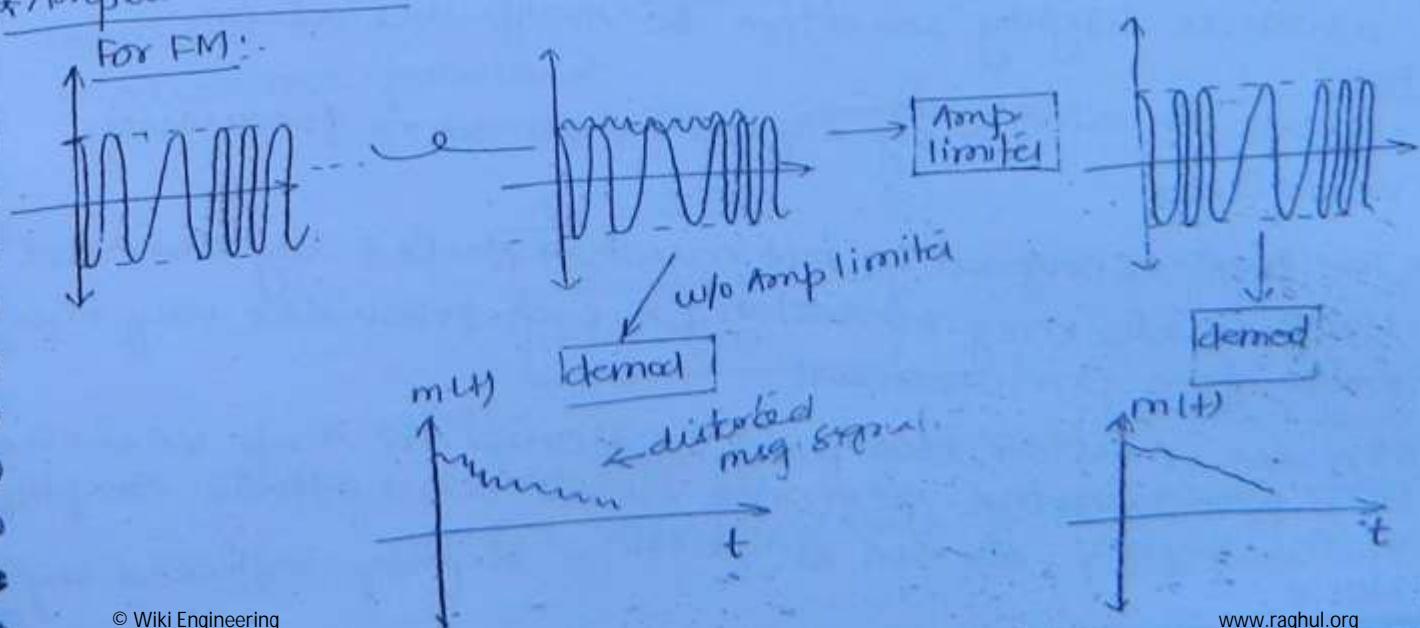


Note:

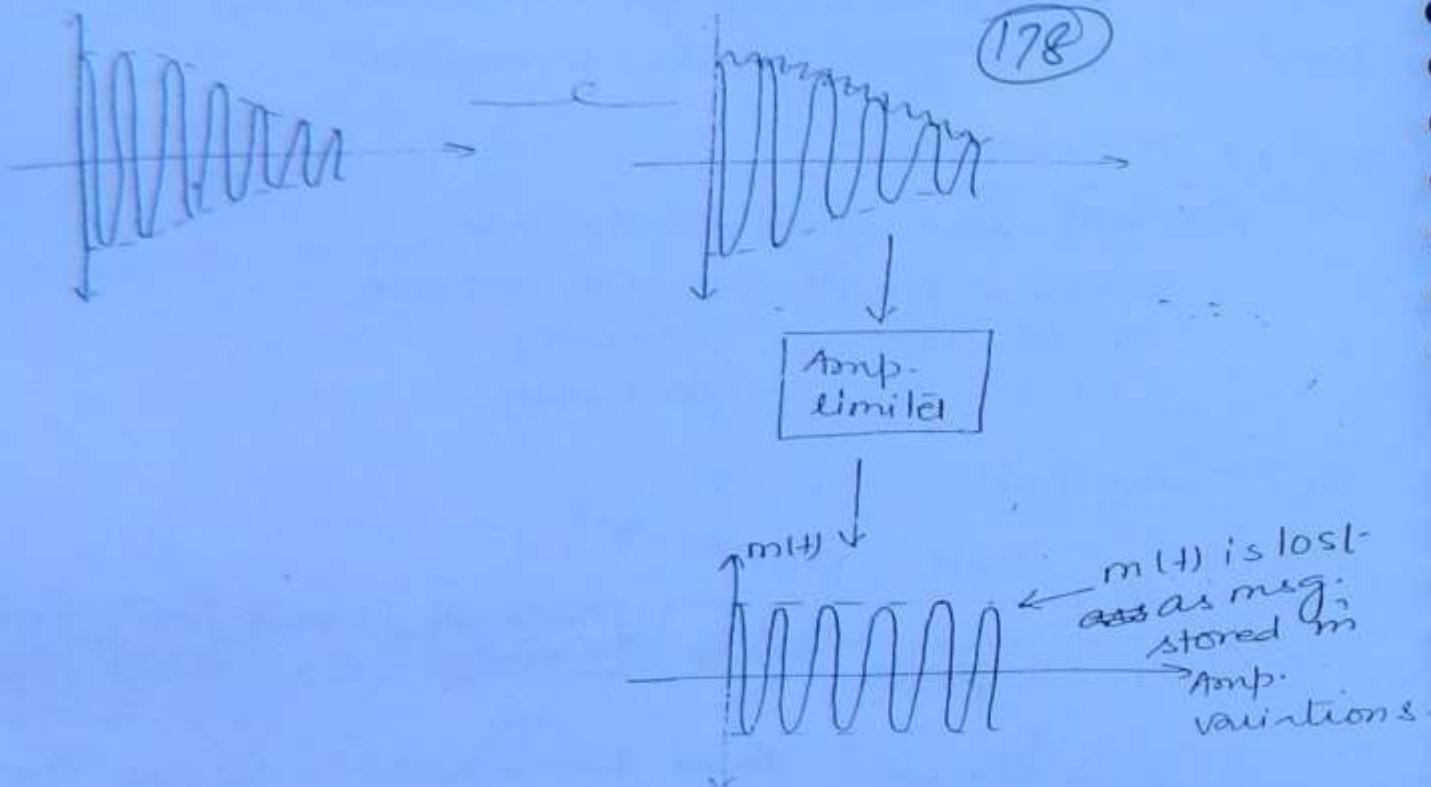
As the tuning of the Rx (i.e.  $f_s$ ) changes, correspondingly  $f_i$  has to be changed such that:  $f_i - f_s = \text{I.F.} = 10.7 \text{ MHz}$

\* Amplitude limiter: → gives a const amplitude

For FM:



\* In Amp. limiter is used in AM, then



Note:-

\* In FM transmission, msg is stored in the frequency variation and the amp. of the FM is affected by noise during channel transmission.

\* The msg can be reconstructed, as msg stored in freqn variation, but the amp. limiter is used to remove the variation of Amp. in channel transmission. This is done to overcome the drawback of demodulation which is highly sensitive to Amp. variation.

Note:-

\* In FM transmission, msg signal is stored in frequency variations.

\* The freqn of signal is not much affected by channel noise. So, the msg transmission in FM provides very much noise-free environment.

\* In AM transmission, the msg is stored in Amp. variation and most of the channel noise also affects Amplitude of the signal, so AM transmission is highly affected by noise.

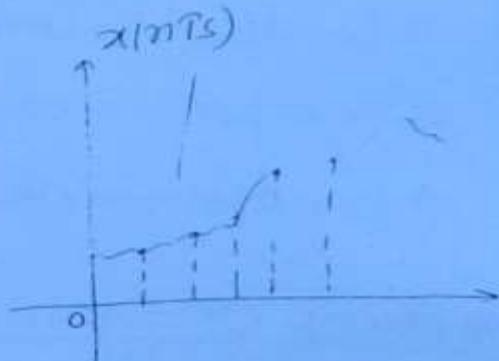
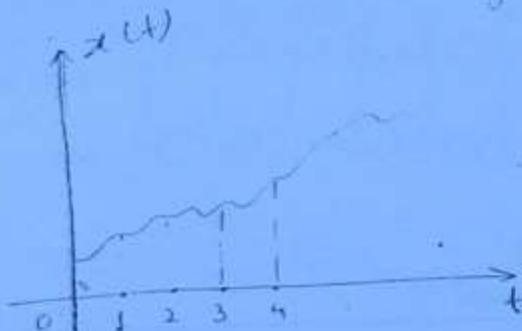
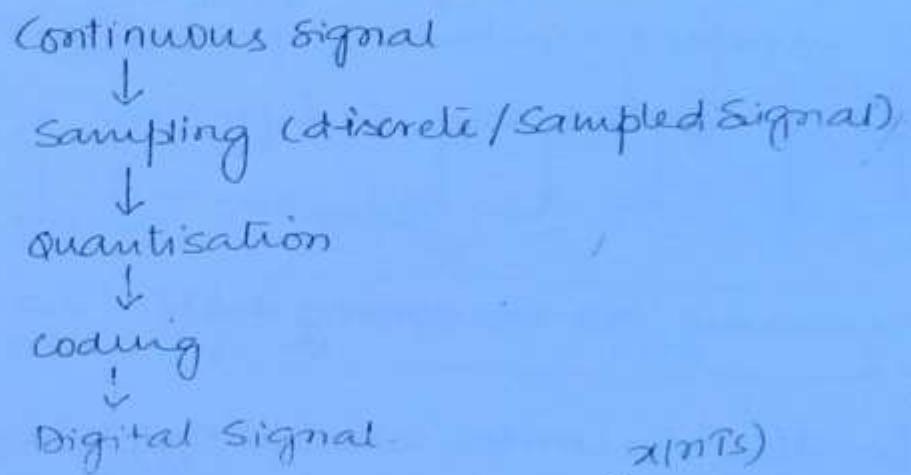
# - DIGITAL COMMUNICATION -

why Digital comm??

(119)

→ It provides highly Noise free environment, hence widely used.

x Process to convert continuous to digital signal:



Sampling Theorem:

A continuous signal Band limited to  $f_m$  Hz can be converted to Sampled signal evnt. without any information loss provided

$$f_s \geq 2f_m \text{ samples/sec}$$

$$\frac{1}{T_s} \geq 2f_m$$

$$T_s \leq \frac{1}{2f_m} \text{ sec}$$

where,

$T_s$  = Sampling interval (sec)

$f_s$  = Sampling Rate or Sampling freq<sup>n</sup>  
(samples/sec).

if fm = 1kHz  
then  $f_s > 2000$  samples/sec

(18)

or  $T_s \leq 0.5$  ms

Note: A continuous signal perfectly reconstructed from its sampled equivalent provided;

$f_s > 2f_m$  sample/sec.

or  $T_s \leq 1/2f_m$  sec

\* Depending on sampling rate, sampling is divided as:-

1)  $f_s < 2f_m \rightarrow$  under sampling. (Causes Aliasing)

2)  $f_s = 2f_m \rightarrow$  critical sampling. (Ideal cases)

3)  $f_s > 2f_m \rightarrow$  over sampling (Practical cases).

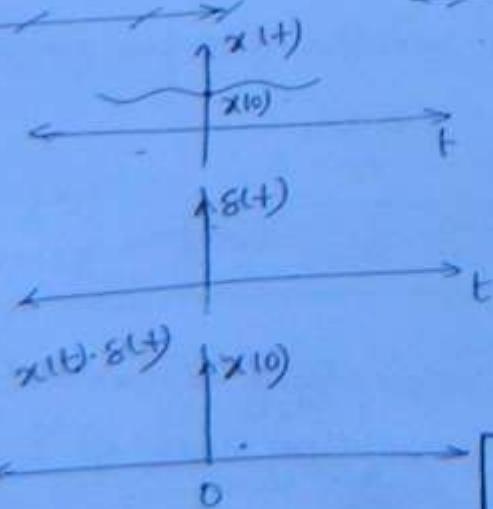
### \* PROOF OF SAMPLING THEOREM:-

Id,

$$\sum_{n=-\infty}^{\infty} \delta(t-nT_s) \xrightarrow{FT} f_s \sum_{n=-\infty}^{\infty} \delta(f-nf_s)$$



Note



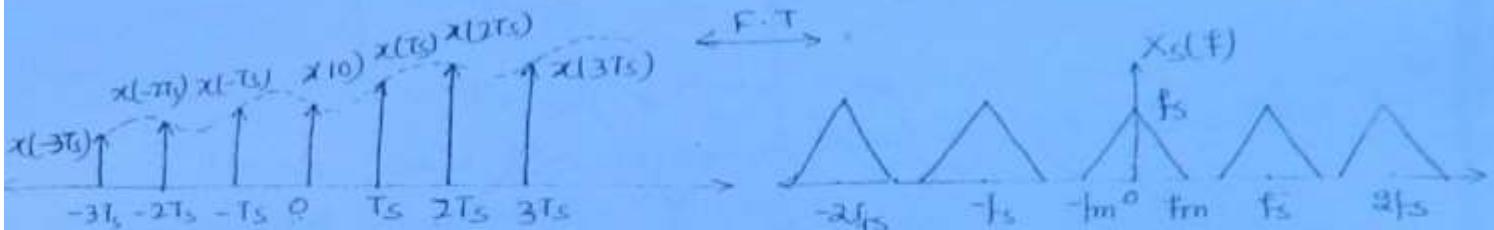
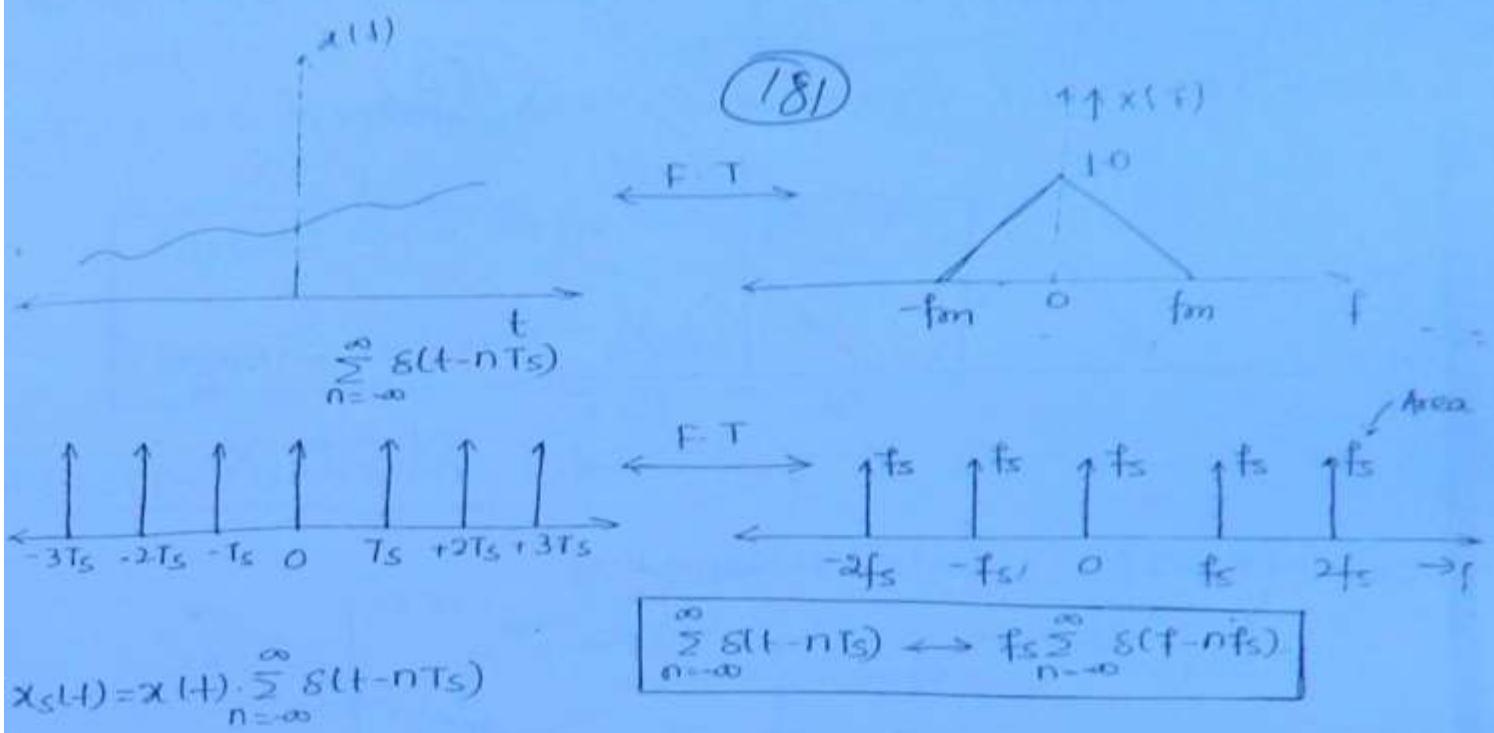
$$x(t) \cdot \delta(t) = \infty; t=0 \\ = 0; t \neq 0$$

$$x(t) \cdot \delta(t) = x(0) \cdot \delta(t)$$

$$x(t) \cdot \delta(t-t_0) = x(t_0) \cdot \delta(t-t_0)$$

discrete train

$$\sum_{n=-\infty}^{\infty} \delta(t-nT_s) = \dots + \delta(t+T_s) + \delta(t) + \dots + \delta(t-T_s) + \dots$$



NOW,  $x_s(t) = x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t-nTs)$

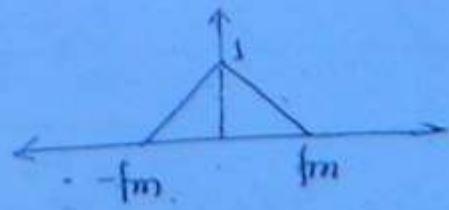
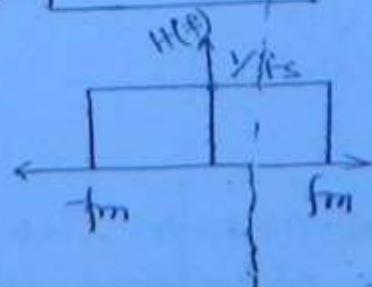
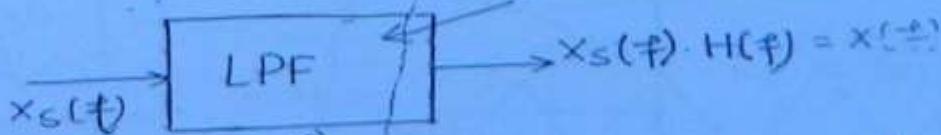
$$X_s(f) = X(f) * \sum_{n=-\infty}^{\infty} \delta(f-nfs)$$

$$X_s(f) = \sum_{n=-\infty}^{\infty} x(f-nfs)$$

Time domain multiplication;  
freq domain convolution  
 $X(f) * \delta(f) = X(f)$   
 $x(t) * \delta(t-t_0) = x(t-t_0)$

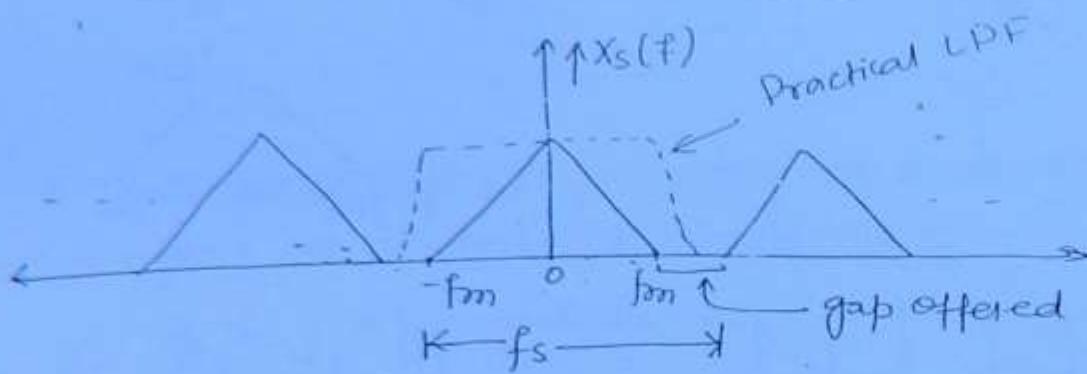
$$X_s(f) = \dots + x(f+fs) + x(f) + x(f+fs).$$

Reconstruction filter



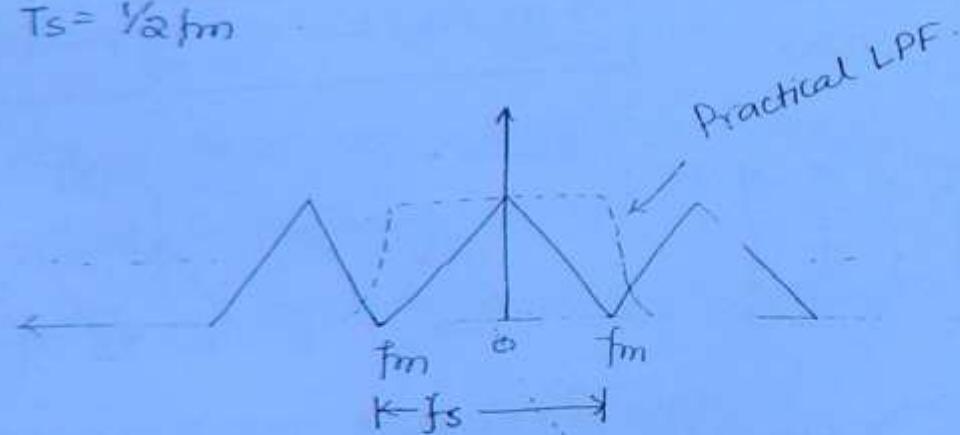
case1 ( $f_s > 2f_m$ ): over sampling

(182)



case2 ( $f_s = 2f_m$ ): critical sampling

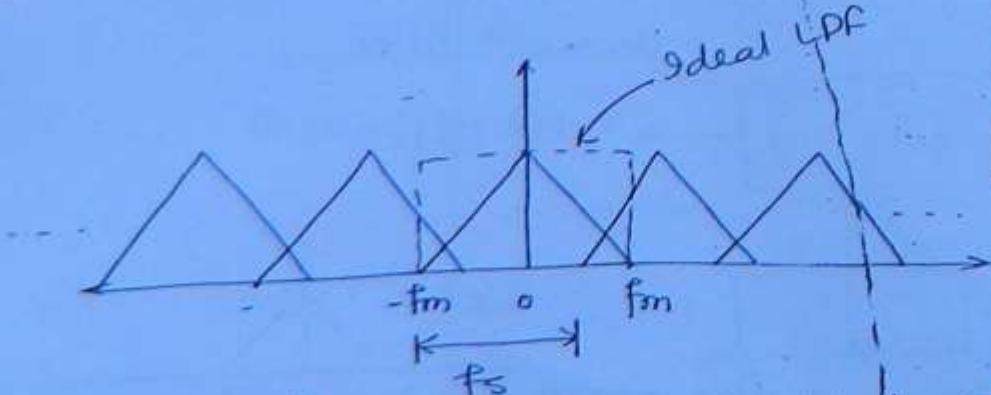
$$T_s = \frac{1}{2}f_m$$



Since Practical LPF is available so, the o/p obtained is as shown above.

Hence, the critical sampling is preferred for Ideal Sampling

case3: ( $f_s < 2f_m$ ): under sampling → Causes ALIASING

 $f_s < 2f_m$   
or  
 $T_s > \frac{1}{2}f_m \text{ sec}$ 

using the Ideal LPF, we cannot obtain the original msg and undesired frequencies are obtained. Hence not preferable.

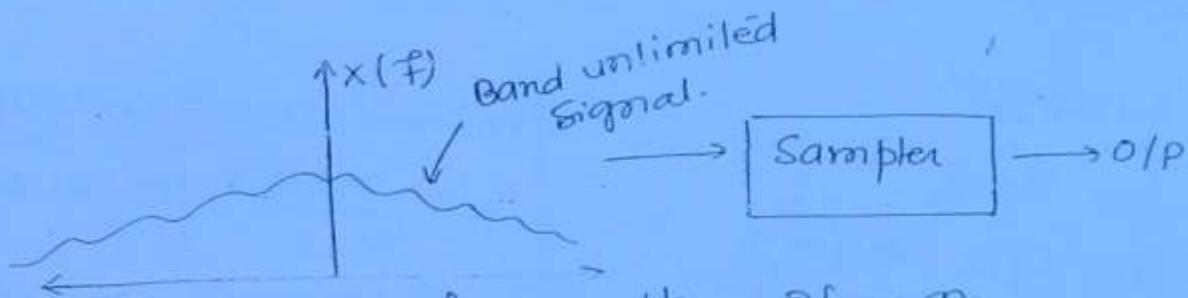
∴ Hence min. sampling rate and max. sampling interval to be maintained to avoid ALIASING are called as "NYQUIST RATE" & "NYQUIST INTERVAL" Respect

$$\text{Nyquist Rate} = 2 f_m \text{ samples/sec.}$$

$$\text{Nyquist interval} = \frac{1}{2 f_m} \text{ sec}$$

(183)

### \*ANTI-ALIASING FILTER:



$$f_m = \infty ; \text{ then } 2f_m = \infty$$

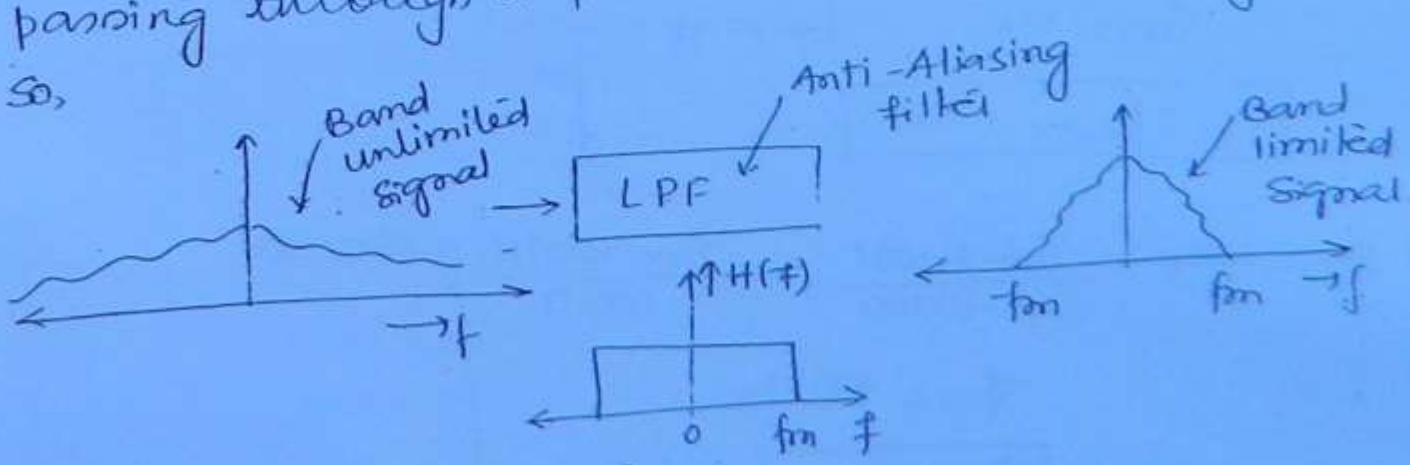
Hence sampler should take  $\infty$  samples/sec to avoid aliasing

But for practical sampler  
 $f_s$  = finite value

$$\text{So, } f_s < 2f_m$$

To avoid this we have to Band-limit the signal, i.e passing through a filter (Called Anti-Aliasing filter)

So,



Q1 Find Nyquist Rate of the following

- i)  $10\sin 8\pi \times 10^3 t$
- ii)  $6\sin 4\pi \times 10^3 t + 8\cos 12\pi \times 10^3 t$
- iii)  $\sin 4\pi \times 10^3 t \cdot \cos 12\pi \times 10^3 t$
- iv)  $\sin^2 4\pi \times 10^3 t \cdot \cos 12\pi \times 10^3 t$

- v)  $\text{Sinc} 100t$
- vi)  $\text{Sinc}^2 100t$
- vii)  $\text{Sinc} 400t \cdot \text{Sinc} 600t$
- viii)  $\text{Sinc} 400t \times \text{Sinc} 600t$

(184)

Sol:

i)  $10\sin 8\pi \times 10^3 t$

$$f_m = 4\text{KHz}$$

$$\boxed{f_s = 2f_m = 8\text{KHz}}$$

ii)  $6\sin 4\pi \times 10^3 t + 8\cos 12\pi \times 10^3 t$

$$f_{m1} = 2\text{K} \quad f_{m2} = 6\text{K}$$

$$\boxed{f_s = 2f_{\max} = 12\text{K}}$$

iii)  $\sin 4\pi \times 10^3 t \cdot \cos 12\pi \times 10^3 t$

$$= \frac{1}{2} \{ \sin 16\pi \times 10^3 t - \sin 8\pi \times 10^3 t \}$$

$$f_{m1} = 8\text{K} \quad f_{m2} = 4\text{K}$$

$$\boxed{f_s = 2f_{\max} = 16\text{K}}$$

v)  $\text{Sinc} 100t = \frac{\sin \pi 100t}{\pi \times 100} = \frac{\sin 100\pi t}{100\pi} \quad \left. \begin{array}{l} \text{see next page} \\ \text{for } \text{Sinc} 100t \end{array} \right\}$

$$f_m = 50\text{Hz}$$

$$\boxed{f_s = 100\text{Hz}}$$

sinc function = rectangular - no noise (no aliasing property)

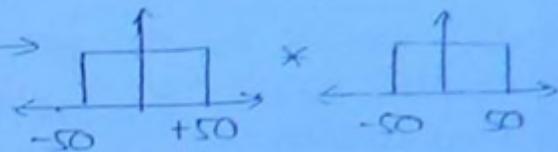
$$\tau = 100$$

(18s)

$$f_{\max} = 50 \text{ Hz}$$

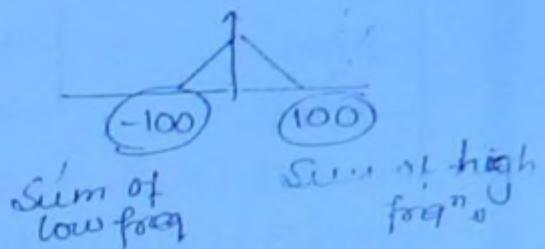
$$\text{so } f_s = 100 \text{ Hz}$$

vi)  $\text{sinc}^2\{100t\} = \text{sinc}(100t) \cdot \text{sinc}(100t) \leftrightarrow$

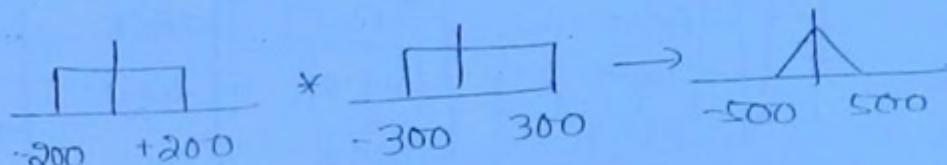


$$f_{\max} = 100$$

$$f_s = N \cdot R = 200 \text{ Hz}$$



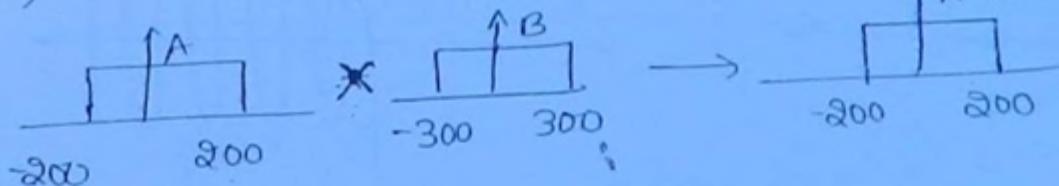
vii)  $\text{sinc}(400t) \cdot \text{sinc}(600t)$



$$f_{\max} = 500 \text{ Hz}$$

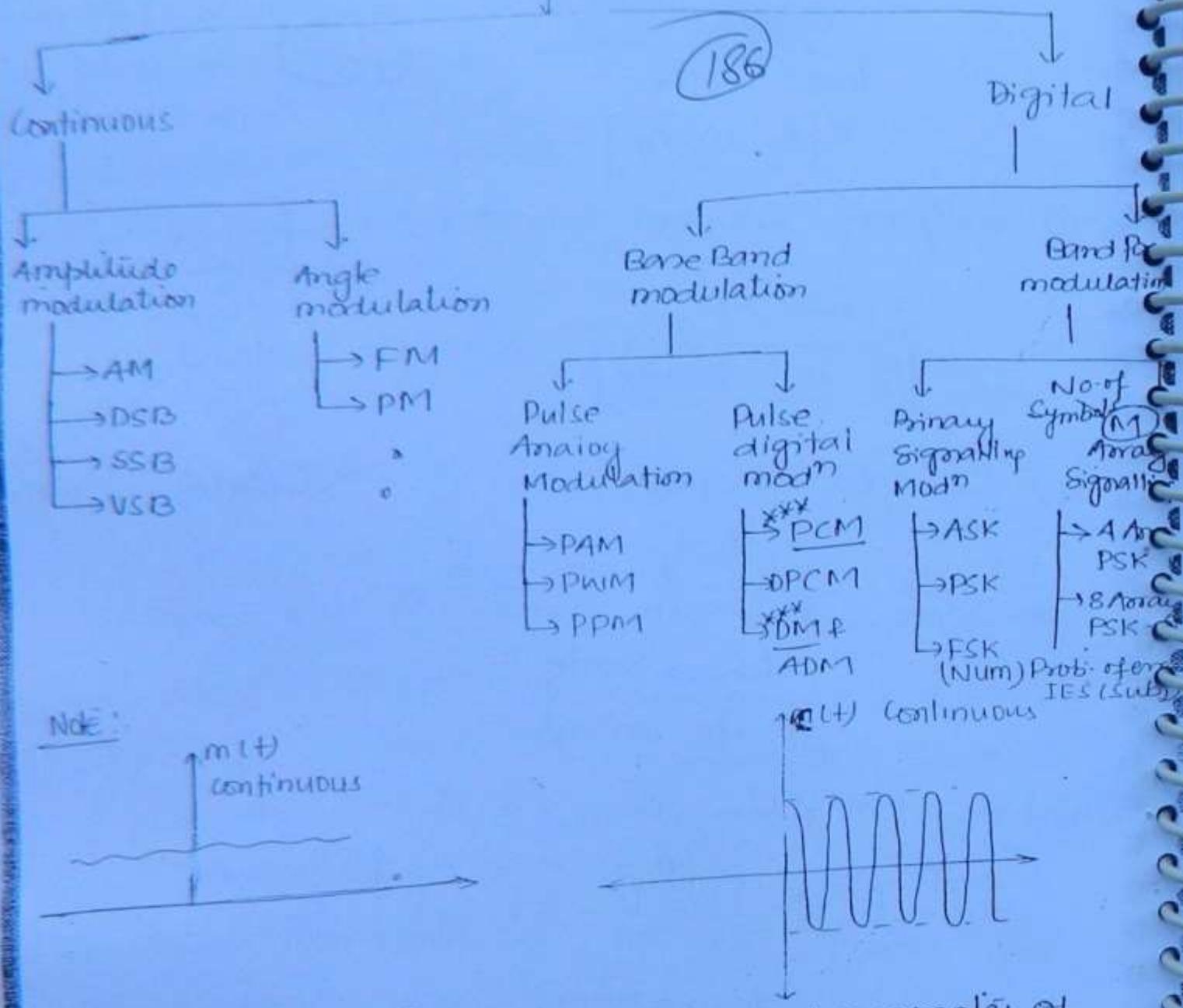
$$f_s = N \cdot R = 1000 \text{ Hz}$$

viii)  $\text{sinc}(400t) * \text{sinc}(600t) \rightarrow \text{sinc}$



$$f_{\max} = 200 \text{ Hz}$$

$$f_s = 100 \text{ Hz}$$



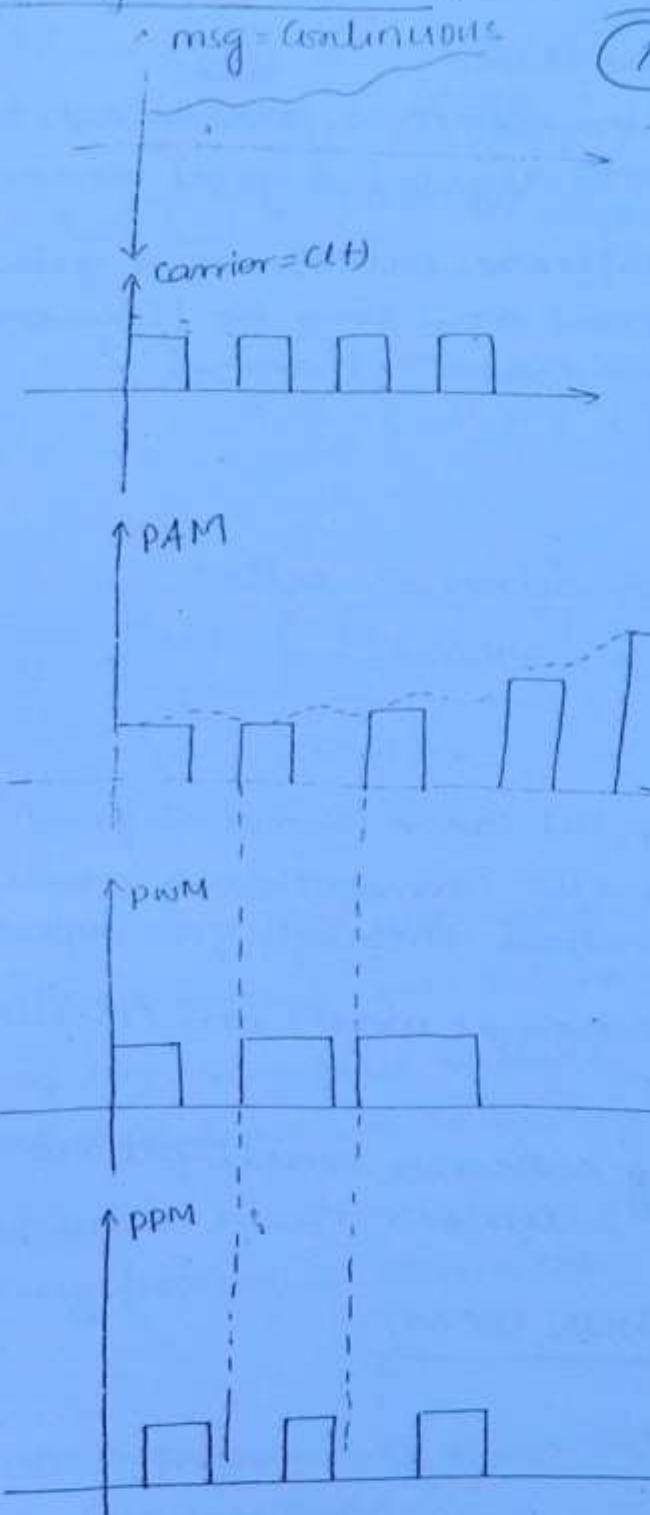
In continuous modulation, one of the parameters of continuous signal will be varied continuously in accordance with msg signal voltage variations.

Note: directly

1. Base Band signals will be transmitted through wired channel. so, it is also called as BASE BAND channel or low pass channel.

2. Band Pass signal will be transmitted through free space, so free space is also called as Band Pass

## Pulse Analog modulation x:-



(187)

→ Transmitted through wired channel and as the amplitude is continuous and hence the effect of noise is high.

→ PAM, PWM & PPM are also transmitted through wired n/w but the amp variation is not continuous. so effect of noise is less

Note:-

In Pulse modulation, Pulse by pulse transmission is exploited where each of the pulse corresponds to Base Band signal. So these modulation schemes are called as BASE BAND Modulation.

Note: (Pulse digital modulation). (188)  
1. Pulse digital modulation schemes are used to convert a continuous signal as, digital signal equivalent.

2. The electrical signal representation of digital corresponds to a Base Band signal and can be transmitted directly through Base Band channel.

### \* Band Pass Modulation:

\* Band Pass modulation schemes like:

- i) ASK
- ii) PSK
- iii) FSK

are used to convert digital Base Band signal as Band Pass signal and the corresponding modulated signal can be transmitted through free space.

\* In Primary signalling scheme (mod<sup>n</sup>) one (1) bit is transmitted at a time.

\* In M-ARRAY signalling schemes multiple no. of bits will be transmitted at a time.

### \* PULSE CODE MODULATION (PCM):

#### \* PCM transmission System:

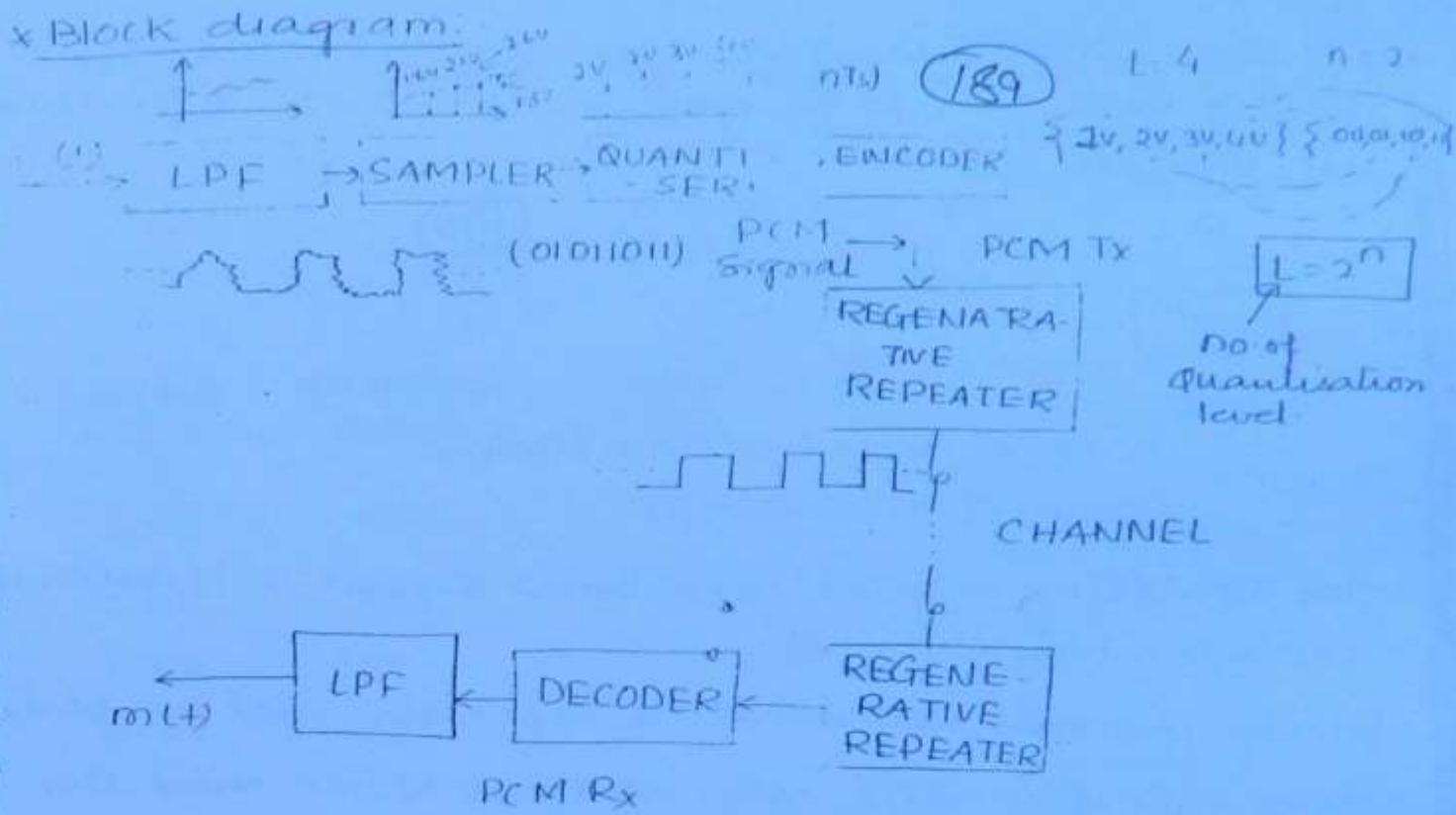
##### \* Function of LDF:

- i) Band limit the signal
- ii) avoids the "ALIASING"

##### \* Function of RR (Regenerative Repeater)

- i) to eliminate the channel noise which distorts the amp. of the signal

- ii) to regenerate the fresh copy of the original signal by removing the effect of noise on its amplitude.



Note :

- \* LPF before sampler will work as "ANTI ALIASING" filter
- \* The filtered signal will be oversampled by the sampler
- \* Each of the sampled values will be rounded off to nearest quantisation level by the "QUANTISER"
- \* Quantisation mechanism is irreversible, but encoding + decoding process is reversible.

Note :

- \* ENCODER represents each of the quantised level by a unique binary code.

x Importance of Quantiser:-

$$f_m = 1 \text{ MHz}$$

$$f_s = 2 \times 10^6 \text{ samples}$$

so, over a period of time a large no. of sample has to be taken and to encode such large no. of sample is not possible. Hence, Quantiser is needed.

Note:  
\* As there is no Quantiser, numerous no. of Unique Binary codes has to be generated which is practically not possible

(190)

\*  $L = 2^n$

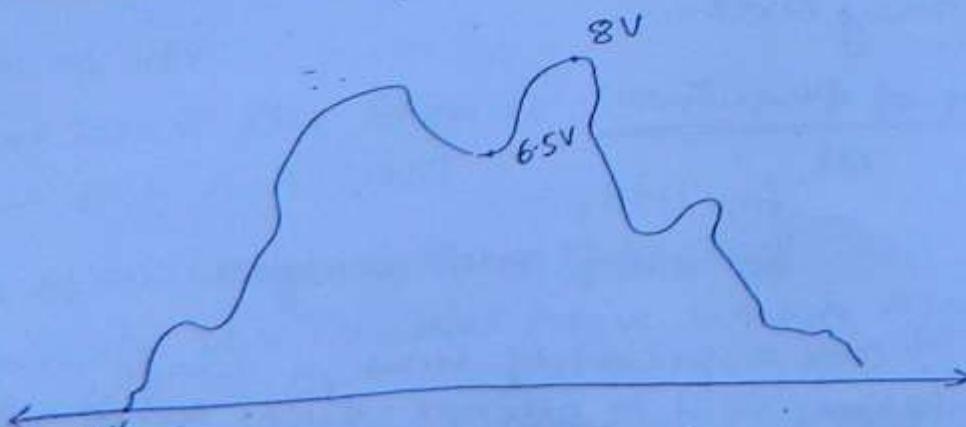
where,

$L$  = no. of Quantisation levels.

$n$  = no. of bits/sample.

- \* The resulting digital base band signal is transmitted through wired channel.
- \* In the channel, Regenerative Repeaters will be placed.
- \* These RR eliminates total channel Noise and the fresh copy of transmitted Signal will be regenerated.
- \* Decoder will do the Reverse of Encoder.
- \* The O/P of decoder will be the quantised Signal & is given to Reconstruction filter (LPF).
- \* In the Reconstructed Signal, finite amount of the quantisation error will be permanently retained.
- \* Quantisation process is irreversible, and ENCODING process is reversible.

### QUANTISATION PROCESS:



→ 2 bit encoder

n = 2 bits/sample

L = 4

(191)

+ 8V

Q = 0V → 11

Q = 4V → 10

Q = 8V, 12V, 16V, 20V

Q = 2V → 01

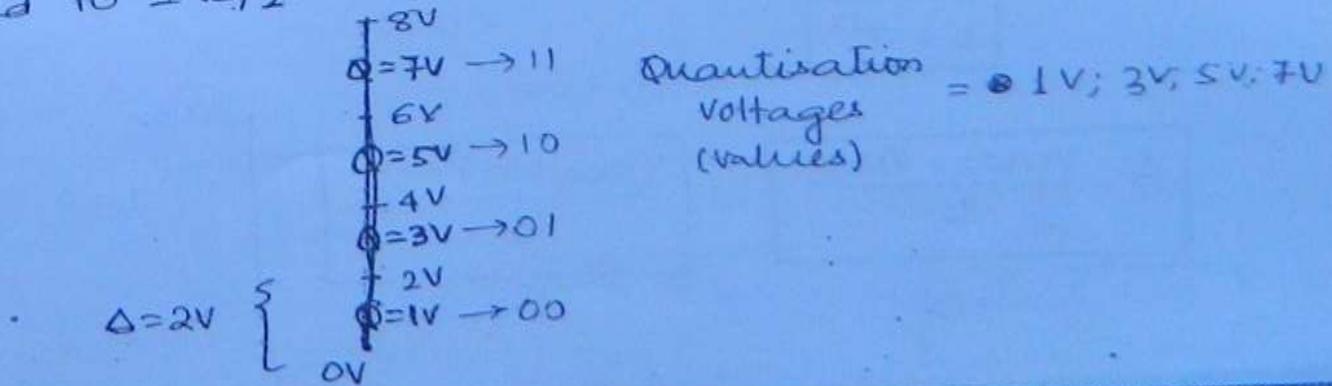
Q = 0V → 00

<u>Sampled value</u>	<u>Quantized value</u>	<u>Encoder O/P</u>	$\Delta e = S \cdot V - Q \cdot V$
0.5V	0V	00	0.5V
2.6V	2V	01	0.6V
6.8V	6V	11	0.8V
7.9V	6V	11	1.9V

$$[\Delta e]_{\max} = \Delta$$

Note:-

- The total dynamic range of the system is divided into L equal no. of steps.
- The Bottom of each step is assumed as Quantisation level
- The sampled value in each specific step will be Rouned off to bottom of the step.
- In this quantisation process;  $[\Delta e]_{\max} = \Delta$ .
- In the following quantisation process  $[\Delta e]_{\max}$  is decreased to  $\pm \Delta/2$ .



<u>SV</u>	<u>Q.V</u>	<u>Encoder Q/P</u>	<u>Qe: SV Q.V</u>
0V	IV	00	0.2
2.2V	3V	01	0.8
4.4V	7V	11	0.9
7V	7V	11	0

Note:

if

$$SV = 0V; \text{ then } Q.V = 1V \Rightarrow Q_e = 1V$$

$$SV = 8V; \text{ then } Q.V = 7V \Rightarrow Q_e = 1V$$

\* The total dynamic Range of the signal is divided into  $L$  equal no. of steps.

\* The middle of each step will be assumed as the quantisation level.

\* The sampled value in a specific step will be rounded off to middle of the step or to the nearest quantisation level.

\* In this process,

$$[Q_e]_{\max} = \pm \Delta/2$$

### \* BASIC FORMULA'S IN PCM SYSTEM!

Let,

$n$  = no. of bits per sample

$L$  = No. of Quantisation level.

$$L = 2^n$$

$$\Delta = \frac{V_{\max} - V_{\min}}{L}$$

$$\Rightarrow \frac{V_{\text{Peak to Peak}}}{2}$$

$V_{\max}$

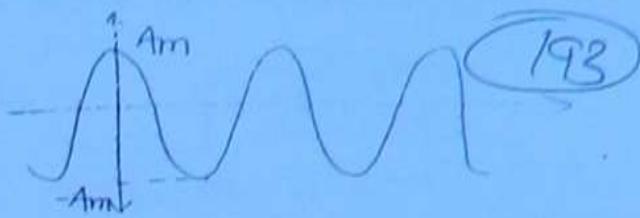
$L = 4$ .

$V_{\min}$

(iii) Amplitude Quantization

So,

$$\Delta = \frac{2Am}{2^n} = \frac{2Am}{2^n}$$



\* Quantized Error:  $Qe = \text{Sampled value} - \text{Quantized value}$

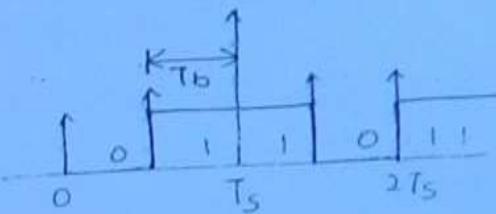
So,

$$[Qe]_{\max} = \pm \Delta/2$$

, let  $n = 2$  bits/sample

\* Bit Duration,  $T_b$ :  $T_s/2$

$$T_s = 2T_b$$



So, for  $n$  no. of bits:

$$\text{Bit duration, } T_b = T_s/n$$

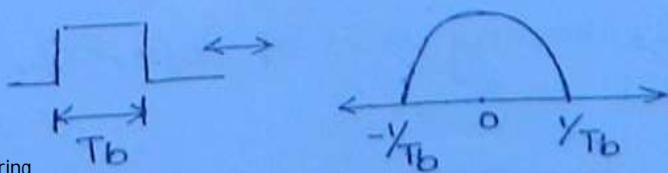
\* Bit Rate;  $R_b = \text{bits/sec} = \frac{\text{bits}}{\text{sample}} \times \frac{\text{sample}}{\text{sec}}$

$$R_b = n f_s$$

or  $R_b = \frac{n}{T_s} = \frac{1}{T_b}$

\* Max<sup>m</sup> Transmitter B.W:-

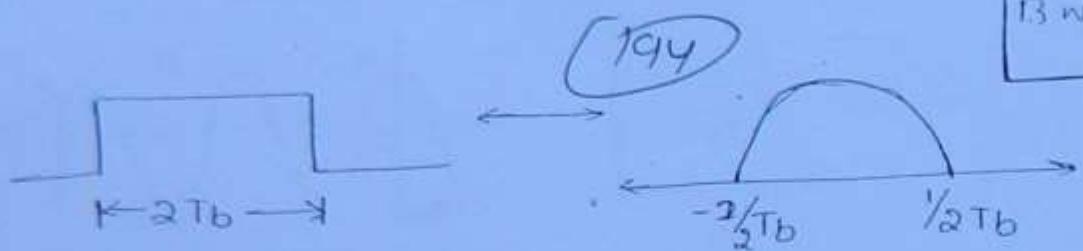
one bit is transmitted at a time



$$B.W = \frac{1}{T_b} = 0$$

$$B.W = \frac{1}{T_b} = R_b$$

\* 2 bits are transmitted at a time



$$B.W = \frac{1}{2T_b} = \frac{R_b}{2}$$

\* 3 bits are transmitted at a time



$$B.W = \frac{1}{3T_b} = \frac{R_b}{3}$$

Note:

\* As the time domain signal width decreases, then the frequency domain signal width decreases.

So,

$$[B.W]_{\max} = R_b$$

\*\*\* Transmission B.W or Min Tx B.W:

\* In practical cases, min<sup>m</sup> 2 bits are transmitted in a bunch, hence the channel B.W required is  $R_b/2$ .

Note:

$$\text{Min}^m \text{ Tx. B.W} = R_b/2$$

\* In this, if the channel has B.W of  $R_b/2$ ; then it will allow the signals having B.W of  $R_b/3, R_b/4, \dots, R_b/n$ . Hence the min B.W Required is  $R_b/2$ .

Q. If the signal of transmission is monaural known to be PCM system. Find all the parameters of the PCM system?

(195)

Soln: Given,

"bits reqd. to xmit 1 sample" so,  $m(t) = 10 \cos 8\pi \times 10^3 t \Rightarrow A_m = 10V$   
 $\rightarrow n = 4 \text{ bits/sample. } f_m = 4 \text{ KHz}$

$L = 2^n = 16$  quantisation levels.

NOW,

i) step size:  $\Delta = \frac{V_{\max} - V_{\min}}{L}$

$$\Delta = \frac{20 \text{ V}}{16} \text{ Ans.}$$

Sampling Rate is not mentioned. so let it be equal to Nyquist Rate.

So, ii)  $f_s = N \cdot R = 2f_m = 8 \text{ K Ans.}$

iii)  $[\Delta e]_{\max} = \pm \Delta / 2 = \pm \frac{20}{30} \text{ volt} \text{ Ans.}$

iv)  $R_b = n f_s$   
 $= 4 \times 8$

$$R_b = 32 \text{ K Ans.}$$

v)  $T_b = 1/R_b = \frac{1}{32 \text{ K}} = \frac{1}{32} \text{ msec Ans.}$

vi)  $[B \cdot W]_{\max} = R_b = 32 \text{ K Ans.}$

vii)  $B \cdot W = \frac{R_b}{2} = 16 \text{ K. Ans.}$

Q2. A sinusoidal message signal of peak voltage 20V and having freq of 5KHz is transmitted through 256 level PCM system. Sampling rate is 25% higher than N.R. Find all the parameters?

(196)

Soln: Given,  $A_m = 20V$ ;  $f_m = 5KHz$

L = 256 levels;  $f_s = N \cdot R + 25\% \text{ of } N \cdot R$

$$\text{So, } L = 2^n = 256$$

$$2^n = 256 = 2^8$$

$$\boxed{n = 8}$$

$$\text{Now, } N \cdot R = 2 f_m = 10 KHz$$

$$\text{So, } f_s = 10 K + \frac{25}{100} \times 10000$$

$$\boxed{f_s = 12.5 K}$$

$$\text{So, i) } \Delta = \frac{V_{P-P}}{L} = \frac{2 \times 20}{256} = \frac{40}{256} \text{ Volts} = 0.15625 \text{ V}$$

$$\text{ii) } [Qe]_{max} = \pm \Delta/2 = \pm 90/256 \text{ Volts}$$

$$\text{iii) } R_b = n f_s \\ = 100 \text{ Kbps}$$

$$\text{iv) } T_b = 1/R_b = 0.01 \text{ ms}$$

$$\text{v) } [BW]_{max} = R_b = 100 K$$

$$\text{vi) } B.W = R_b/2 = 50 K$$

Q3. A msg signal of  $800 \sin 2\pi \times 10^4 t$  is transmitted through a PCM system. Sampling Rate is 50% higher than N.R. Max Quantization error can be almost of 0.1% of peak amj of msg. signal. Find all parameters?

Ques.

$$m(t) = 8 \cos(2\pi \times 10^6 t)$$

$$A_m = 8; f_m = 10 \text{ KHz}$$

197

$$\text{So, } [\Phi_e]_{\max} \leq \frac{\Delta}{2} = \frac{0.1}{100} \times 8 = 0.008 \quad \left\{ \because \Delta = 0.1\% \text{ of } A_m \right\}$$

Now,

$$\pm \Delta \geq 0.008$$

$$\pm \frac{V_p - V_n}{L} \geq 0.008$$

$$\text{So, } L \leq \frac{16 \times 1000}{0.008} \Rightarrow 2^n \leq 1000$$

$$n = 10 \quad \text{Ans}$$

$$1 < 1000 \leq 2^n \Rightarrow n = 10$$

$$\text{Now, } f_s = 1.5 \times N \cdot R \\ = 1.5 \times 20 \text{ K}$$

$$f_s = 30 \text{ K} \quad \text{Ans}$$

$$[\Phi_e]_{\max} = \pm \frac{\Delta}{2} = \frac{8}{1024 \times 2} \\ = \frac{16}{2048} \text{ volts}$$

ii)  $R_b = n f_s$

$$R_b = 300 \text{ K} \quad \text{Ans}$$

iii)  $T_b = \frac{1}{R_b} = \frac{1}{300} \text{ ms}$

iv)  $[\text{B.W}]_{\max} = R_b = 300 \text{ K}$

$$\text{B.W} = R_b / 2 = 150 \text{ K} \quad \text{Ans}$$

Ques. A sinusoidal msg signal is transmitted through PCM system where  $[\Phi_e]_{\max}$  can be atmost of  $2^{-1}$  of Peak to peak amp. of msg signal. Find no. of bits per sample reqd?

Soln:  $[\Phi_e]_{\max} \leq \frac{\Delta}{2} = 2^{-1} \text{ of } 2A_m$

~~$$\frac{1}{2} \left\{ \frac{8A_m}{2^n} \right\} \leq 2A_m \times \frac{2^{-1}}{100} \Rightarrow 82.225$$~~

$$\frac{1}{2} \left\{ \frac{8A_m}{2^n} \right\} \leq 2A_m \times \frac{2^{-1}}{100} \Rightarrow \frac{1}{2}n \leq \frac{1}{25} \Rightarrow 2^n \geq 25 \Rightarrow n = 5$$

Ques A sinusoidal msg signal of  $4.5mV \times 10^{-3} t$  is transmitted through 8 level PCM system. Sampling rate is 5 times N.R.

(198)

- Find all the parameters?
- Given sampled values -3.2V, -2.8V, -0.1V, 1.5V, 3.9V  
Find corresponding Quantiser and Encoder O/P?
- Plot the Quantiser characteristics?

Soln: Given,  $m(t) = 4.5mV \times 10^3 t$

$$A_m = 4V; f_m = 2KHz$$

i)  $L = 8 \Rightarrow n = 3 \text{ bits/sample}$

$$f_s = 5 \times 2f_m$$

$$f_s = 20KHz$$

$$2) \Delta = \frac{V_{PP}}{2^3} = \frac{8}{8} \text{ volts} = 1 \text{ volt}$$

$$3) [Q_e]_{max} = \pm \Delta/2 = \pm \frac{1}{2} \text{ volt} = \pm 0.5 \text{ volt}$$

$$4) R_b = n f_s = 3 \times 20K = 60K \text{ bps}$$

$$5) T_b = \frac{1}{R_b} = \frac{1}{60} \text{ ms}$$

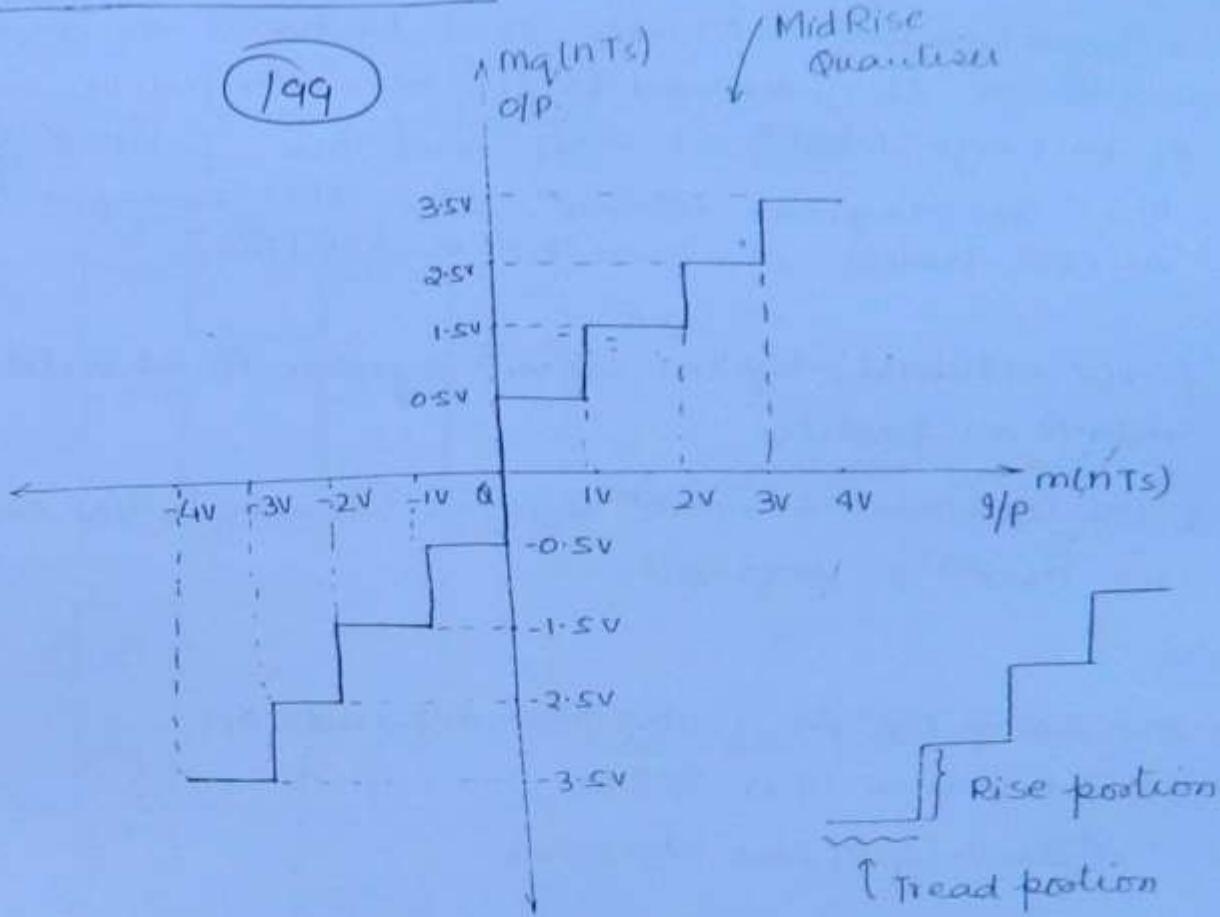
$$6) [B.W]_{max} = R_b = 60K$$

$$B.W = R_b/2 = 30K$$

ii) Now, the sampled values are:-

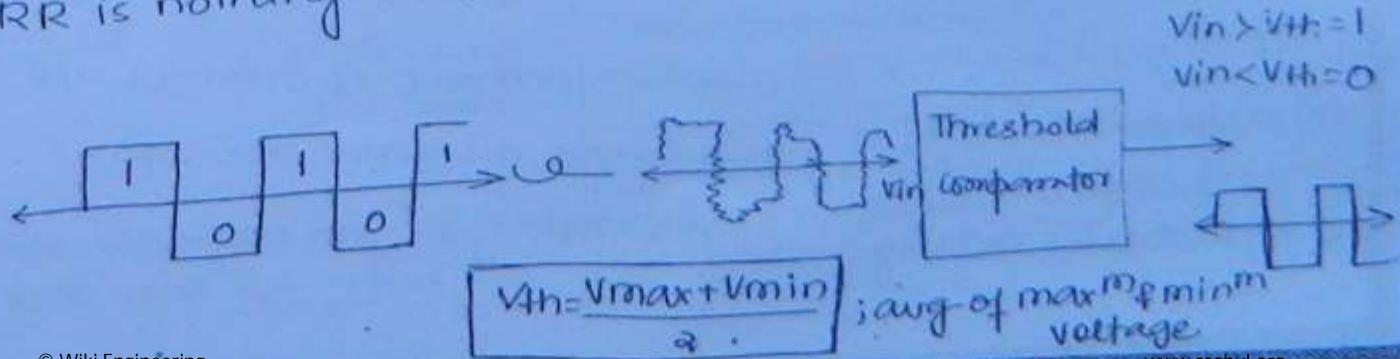
	<u>S.V</u>	<u>O.V</u>	<u>Encoder O/P</u>	<u>Q.e = S.V</u>
111 $\leftarrow 3.5V$	-3.2V	-3.5V	000	0.3V
110 $\leftarrow 2.5V$	-2.8V	-2.5V	001	-0.3V
101 $\leftarrow 1.5V$	-2V	-1.5V	011	0.4V
100 $\leftarrow 0.5V$	-1V	-0.5V	101	0V
011 $\leftarrow -0.5V$	0V	0.5V	111	0.4V
010 $\leftarrow -1.5V$	-2V	1.5V		
001 $\leftarrow -2.5V$	-3V	2.5V		
000 $\leftarrow -3.5V$	-4V	3.5V		

### iii) Quantiser characteristics



Note :-

- \* If the origin lies in the Rise (middle) position, then Quantiser called as Mid Rise Quantiser.
- \* If the origin lies in the middle of Tread portion, then Quantiser is called as Mid-Tread Quantiser (NO importance).
- \* RE-GENERATIVE REPEATER (RR): (no significance in obj ES-subj)
  - \* Digital Transmission provides very much Noise free environment due to Regenerative Repeater.
  - \* RR is nothing but threshold comparator.



### Note

- \* Regenerator/repeater can't be used in Analog comm as bcos the signal  $\{s(t)\}$  takes infinite amount of voltage levels at diff instants of time. ~~(RR)~~
- But in digital comm, the  $s(t)$  sweeps  $+5V$  or  $-5V$ , hence it can be used (RR).

1. For efficient digital comm system  $P_e$  should be as min<sup>m</sup> as possible.
2. For continuous comm system (Analog), S/N should be as max<sup>m</sup> as possible.

### Note:

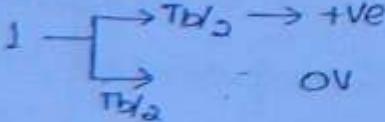
1. The no. of RR in a channel depends on
  - i) distance b/w Tx & Rx
  - ii) Quality of the channel

### \* Electrical Representation of Primary Signals:

The electrical representation schemes available are:

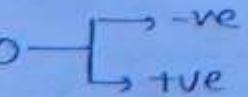
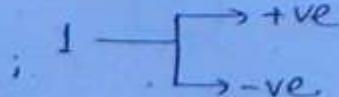
i) ON-OFF       $0 \rightarrow 0V$   
                       $1 \rightarrow +ve$

ii) NRZ       $0 \rightarrow -ve$   
                       $1 \rightarrow +ve$

iii) RZ       $0 \rightarrow 0V$   
  
 $1 \rightarrow +ve$

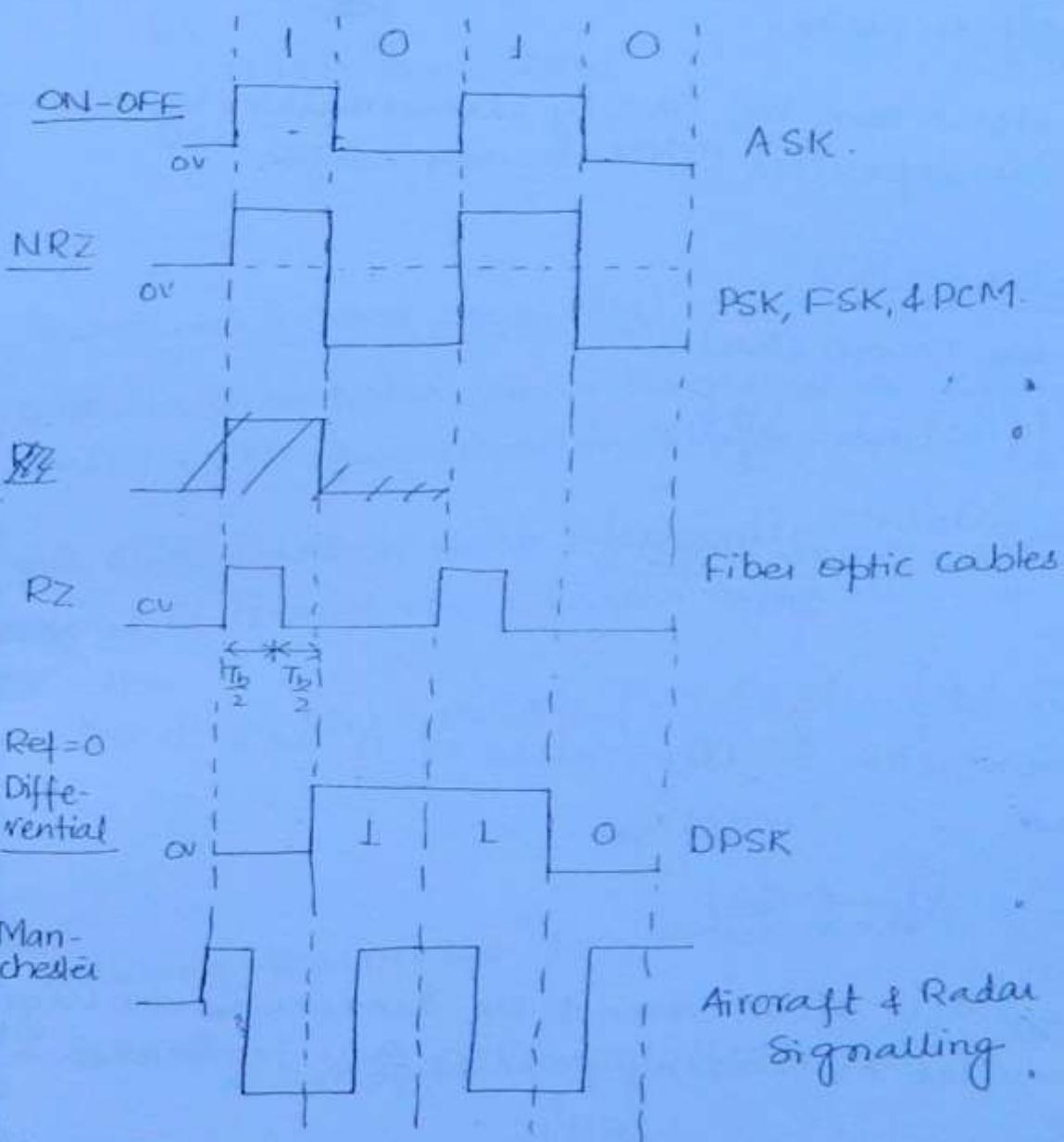
$0 \rightarrow$  complement of previous O/P

iv) differential coding       $1 \rightarrow$  same as previous O/P.

v) Manchester coding       $0 \rightarrow$    
 $1 \rightarrow$  

## \*Analysis of Coding Techniques

(420)



Note:

d.E.S

For Generation of RZ, X-NOR gate is used.



- (0, 0) → 1
- (0, 1) → 0
- (1, 0) → 0
- (1, 1) → 1

Whenever the next bit is 0, the O/P corresponds to the complement of the other bit and for 1, the same bit will be the O/P.

\* For Generation of differential encoded signal, X-gate will be used.

## \* Noise in PCM System

Noise in PCM system is classified as:

1) Channel Noise

2) Quantisation Noise.

(D02)

\* Channel noise can be easily eliminated by using Regenerative Repeater (RR).

## \* Quantisation Noise!

As we know that

$$\therefore [Qe]_{\max} = \frac{\Delta \downarrow}{2}$$

$$\text{Now, } \Delta = \frac{V_{\max} - V_{\min}}{2^n \uparrow}$$

### Note:

1. To decrease the  $\Delta$ , the value of  $n$  has to be increased.

$$\text{ie } n \uparrow \rightarrow \Delta \downarrow \rightarrow Qe \downarrow$$

2. But the value of  $n$  cannot be increased to high values, because increasing  $n$ , the B.W is increased.

$$\text{ie } n \uparrow \rightarrow B.W \propto \frac{n f_s}{2}$$

Hence, the value of  $n$  should be such that :-

1) B.W is not high

2)  $Qe$  is min<sup>m</sup>

\* In PCM system as the  $Qe \downarrow$ , correspondingly B.W requirements will be decreased.  
This is the drawback of PCM system.

\* For a PCM system, it is required to ensure that both of DC and D.C requirements will be satisfied

\* Signal to Quantisation Noise power Ratio (SQNR) :-

Let,

$$m(t) = A_m \cos 2\pi f_c t$$

(203)

$$\text{So, Signal power, } S = \frac{A_m^2}{2R}$$

Quantisation Noise power (Nq) :-

Quantisation Noise power is called as the power associated with Quantisation Noise.

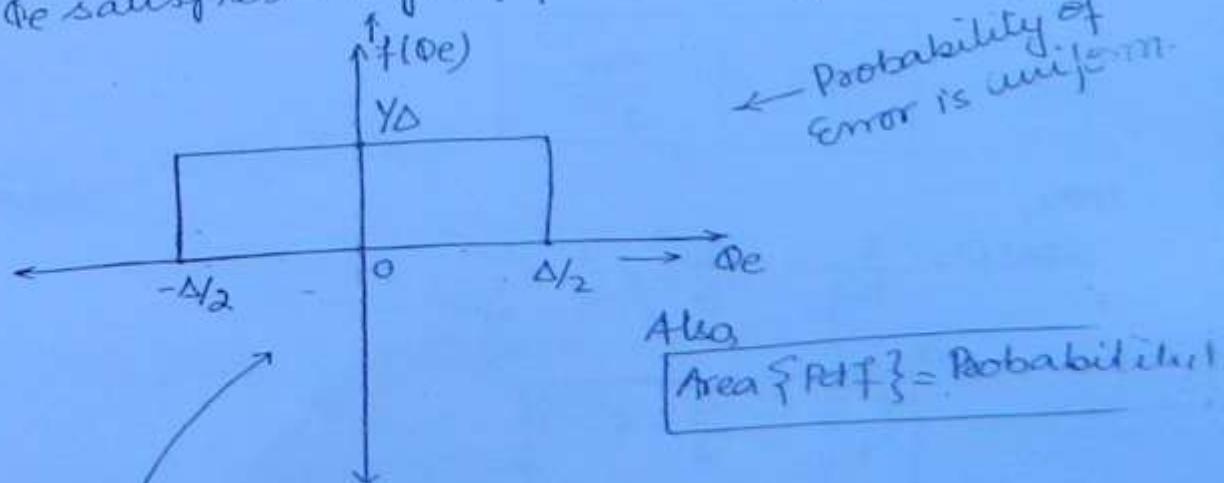
\* As, Quantisation Noise is undeterministic in nature.  
so it is treated as Random variable.

$$\text{Now, } N_q = \text{Power}\{\Delta e\} = m_s \text{ of }\{\Delta e\} = E\{\Delta e^2\} = \int \Delta e^2 \cdot f(\Delta e) d\Delta e$$

↑  
Expectation.  
↑  
probability density function of  $\Delta e$ .

Following Assumption has to be made:-

Assume,  $\Delta e$  satisfies uniform probability density funcn.



And,

$$\text{Area } \{F(\Delta e)\} = 1$$

So,

$$Nq = \int_{-\Delta/2}^{\Delta/2} \alpha e^2 \cdot \frac{1}{\Delta} \cdot A \Delta e$$

(204)

$$= \frac{1}{\Delta} \cdot \frac{\alpha e^3}{3} \Big|_{-\Delta/2}^{\Delta/2}$$

$$= \frac{1}{3\Delta} \left\{ \frac{\Delta^3}{8} + \frac{\Delta^3}{8} \right\}$$

$$= \frac{1}{24\Delta} \times 2\Delta^3$$

$$\boxed{Nq = \frac{\Delta^2}{12}}$$

For,  $m(t) = Am \cos 2\pi f_m t$

$$\text{So, } \Delta = \frac{2Am}{2^n}$$

$$\text{So, } Nq = \frac{1}{12} \left\{ \frac{2Am}{2^n} \right\}^n$$

$$= \frac{1}{12} \times \frac{4Am^2}{2^{2n}}$$

$$\boxed{Nq = \frac{1}{3} \cdot \frac{Am^2}{2^{2n}}}$$

Now,

$$\text{SQNR} = \frac{s}{Nq} = \frac{Am^2}{2} \Big/ \frac{1}{3} \cdot \frac{Am^2}{2^{2n}}$$

~~\*\*\*~~

$$\boxed{\uparrow \frac{s}{Nq} = \frac{3}{2} \cdot 2^{2n}}$$

Note:-

As  $n \uparrow \rightarrow L \uparrow \rightarrow \Delta \downarrow \rightarrow \alpha e \downarrow \rightarrow Nq \downarrow \rightarrow S/Nq \uparrow$   
But  $n \uparrow \rightarrow B.W \uparrow$ .

$$1. \text{ For } n=1 \Rightarrow (\frac{S}{Nq})_1 = \frac{3}{2} \cdot 2^{2^{1-1}}$$

$$2. \text{ For } n=2 \Rightarrow (\frac{S}{Nq})_2 = \frac{3}{2} \cdot 2^{2^{2-2}}$$

$$3. \text{ For } n=3 \Rightarrow (\frac{S}{Nq})_3 = \frac{3}{2} \cdot 2^{2^{3-3}}$$

(205)

Now,

$$\left(\frac{S}{Nq}\right)_2 = 2^{2^1} \cdot (\frac{S}{Nq})_1$$

$$\left(\frac{S}{Nq}\right)_3 = 2^{2^2} \cdot (\frac{S}{Nq})_2$$

$$\left(\frac{S}{Nq}\right)_3 = 2^{2^2} \cdot (\frac{S}{Nq})_1$$

Conclusion:

As  $n$  changes to  $(n+k)$  hence  $(\frac{S}{Nq})$  increases by  $2^{2^K}$  times

\* As the no of bits / samples increases from  $n$  to  $(n+k)$   
 $(\frac{S}{Nq})$  is increased by  $2^{2^K}$  times

$$n \rightarrow (n+k) \Rightarrow (\frac{S}{Nq})_{n+k} \rightarrow 2^{2^K} (\frac{S}{Nq})_n$$

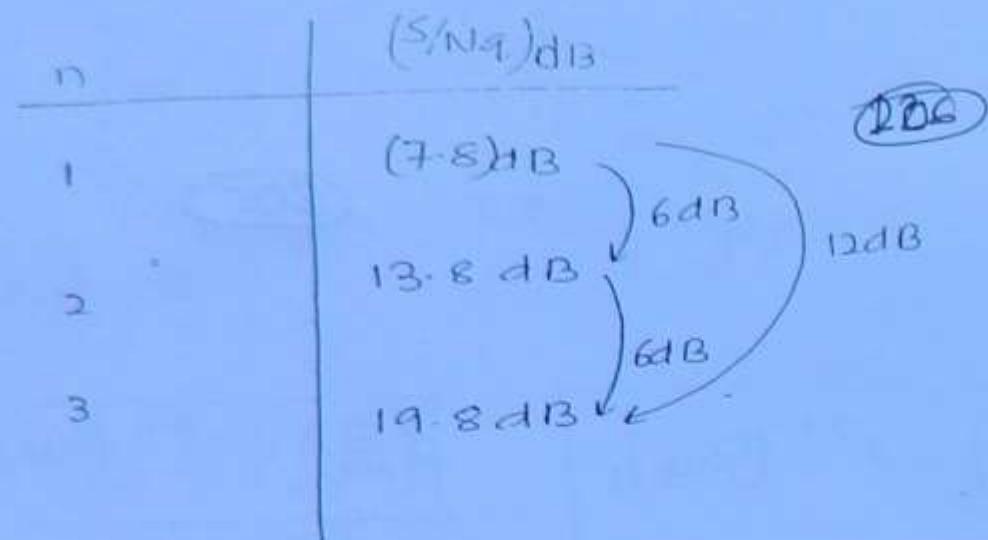
\*  $(\frac{S}{Nq})_{dB} = ?$

$$\begin{aligned} \left(\frac{S}{Nq}\right)_{dB} &= 10 \log_{10} \left(\frac{S}{Nq}\right) \\ &= 10 \log_{10} \left\{ \frac{3}{2} \cdot 2^{2^n} \right\} \\ &= 10 \log_{10} \frac{3}{2} + 10 \log_{10} 2^{2^n} \end{aligned}$$

$$\left(\frac{S}{Nq}\right)_{dB} = 1.76 + 6.02n$$

$$\left(\frac{S}{Nq}\right)_{dB} \approx (1.8 + 6n) dB$$

Note



As, the no. of bits /sample increased from  $n$  to  $(n+K)$  the S/NR is increased by  $6K$  dB.

$$n \rightarrow n+K \Rightarrow \left( \frac{S}{Nq} \right)_{dB} = 6K \cdot \left( \frac{S}{Nq} \right)_{dB}$$

Q1. A msg signal of  $88m 8\pi \times 10^3 t$  is to be transmitted through PCM System. Sampling Rate is 50% higher than NR & min<sup>m</sup> S/NR should be 22 dB. Find

i) transmission B.W

ii)  $(S/NR)_{dB} = ?$

Soln: Given,  $m(t) = 88m 8\pi \times 10^3 t$

$$A_m = 8; f_m = 4 K$$

$$\text{so, } N \cdot R = 2f_m = 8 K$$

$$\begin{aligned} \text{so, } f_s &= 1.5 NR \\ &= 1.5 \times 8 K \end{aligned}$$

$$f_s = 12 K$$

$$\text{Now, } (S/NR) = \frac{3}{2} \cdot 2^{2n}$$

$$\text{For } (S/NR) = \text{min} \Rightarrow 22 \leq \frac{3}{2} \cdot 2^{1.8+6n}$$

$$\frac{3}{2} \cdot 2^{1.8} = 10.8$$

$$6n \geq 20.2$$

$$n \geq 3.36 \approx 4 \text{ bits}$$

Now,

$$B.W = \frac{nfc}{9}$$

$$= \frac{4 \times 12 \times 10^3}{2}$$

(20)

$B.W = 24 \text{ kHz}$  Ans.

Now,  $(SQNR) = 1.8 + 6n \text{ dB}$   
 $= 1.8 + 24$

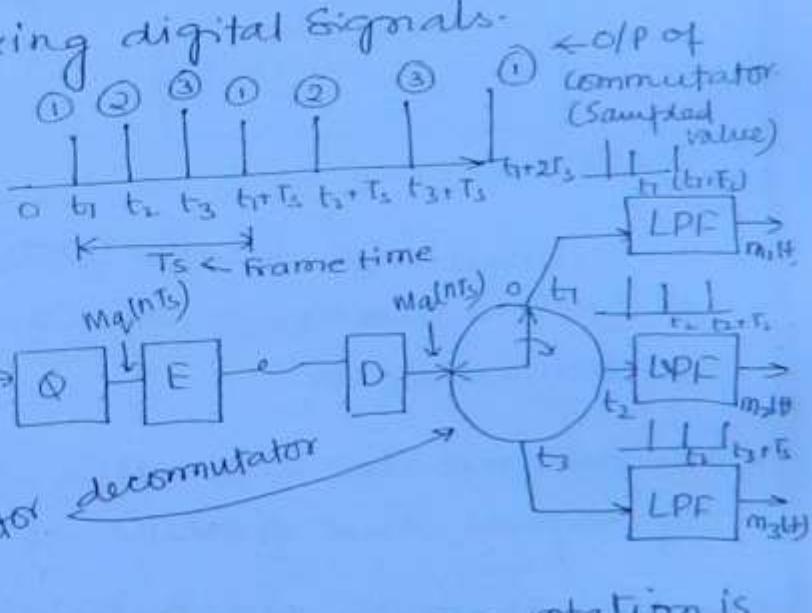
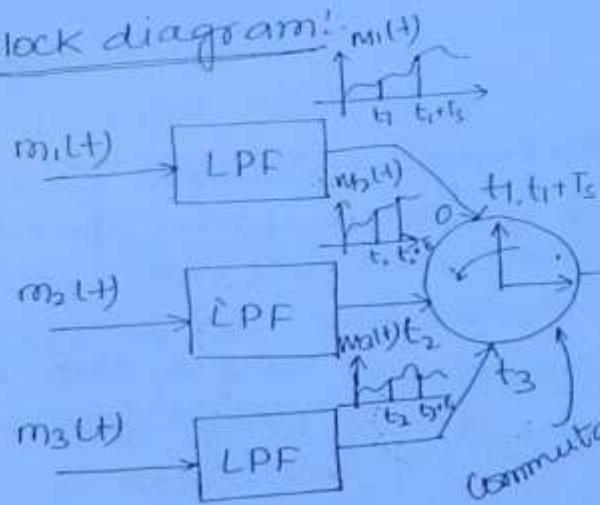
$(SQNR) = 25.8 \text{ dB}$  Ans.

\*PSUS

## \* TIME DIVISION MULTIPLEXING:-

TDM is used for multiplexing digital signals.

Block diagram:



\* Time taken by commutator to complete one rotation is called as Frame time,  $T_s$ .

\* LPF will work as Anti-Aliasing filters.

\* Commutator is a Rotating switch which rotates in the Anti-clockwise direction with uniform speed.

\* The time taken by commutator to make one complete rotation is called as Frame time and it is specified as  $T_s$  (Sampling interval).

- \* For proper op": speed synchronisation can be maintained b/w commutator & dc commutator
- \* To ensure speed synchronisation, small amount of additional bits will be transmitted at the end of each frame time and these are called as the synchronisation bits.

(209)

So,

$$(nN+a)T_b = T_s$$

$$T_b = \frac{T_s}{(nN+a)}$$

$$R_b = (nN+a)f_s$$

Note:

1. In FDM; carrier frequencies will be unique for the msg signals to be multiplexed.
2. In TDM; the time slots are unique for the msg signals to be multiplexed.
3. The time slot of  $t_1 + KT_s$ ;  $K = 0, 1, 2, \dots$  is dedicated for  $m_1(t)$   
 $t_2 + KT_s$ ;  $K = 0, 1, 2, \dots = m_2(t)$
4. In FDM; sharing of channel B.W will be done whereas in TDM; no sharing of B.W.
5. FDM is complex than TDM.
6. The effect of noise will be high in FDM as compared to TDM.

Q1 Five signals are multiplexed using TDM. No of quantization levels used are 256. Find transmission B.W of system.

Soln: Given:

$$N = 5$$

$$f_m = 5 \text{ KHz}$$

$$n = 8$$

L = 256 levels.

(256)

$$N \cdot R = 2 f_m$$

$$f_s = N \cdot R = 10 \text{ K}$$

$$B.W = \frac{R_b}{2} = \frac{n N f_s}{2} = \frac{8 \times 5 \times 10}{2}$$

$$B.W = 200 \text{ K Hz}$$

Q2: 10 signals each band limited to 2K are multiplexed using TDM. Time taken by commutator to make 1 complete rotation is 125ms. Find Bit Rate of the Tr if 5 bit encoder is used.

Soln: Given, N = 10

$$f_m = 2 \text{ K}$$

$$T_S = 125 \text{ ms} = \frac{1}{f_s} \Rightarrow f_s = 8 \text{ K}$$

$$n = 5 \text{ bits/sample}$$

$$f_s = 2 f_m = N R = 8 \text{ K}$$

$$\text{So, } \cancel{\text{Bit Rate}} = R_b = n N f_s = 5 \times 10 \times 8 \text{ K} \\ = 400 \text{ Kbps.}$$

$$\boxed{\text{Bit Rate, } R_b = 400 \text{ Kbps}}$$

Q3: 10 sinusoidal msg signal, each having freq of 10 KHz are multiplexed using TDM. Sampling Rati is 257. High than N. Max<sup>m</sup> Qe can be atmost of 1% of Peak to Peak

amp. of msg signal  $\rightarrow$  no. of synchronization bits were transmitted at the end of each frame find Bit Rate of the Tx?

Soln: Given:

(2/1)

$$N = 10$$

$$f_m = 10 \text{ K}$$

$$f_s = 1.25 \text{ NR}$$

$[\Delta e]_{\max} = 1/2 \text{ of Peak to Peak Amplitude}$

$$a = 5$$

$$\text{Now, } f_s = 1.25 \times 2 f_m$$

$$= 1.25 \times 20$$

$$f_s = 25 \text{ K}$$

Now,  $[\Delta e]_{\max} \leq 1/2 \text{ of } 2 \text{ Am.}$

$$\frac{\Delta}{2} \leq 0.02 \text{ Am}$$

$$\frac{1}{2} \left\{ \frac{2 \text{ Am}}{2^n} \right\} \leq 0.02 \text{ Am}$$

$$2^n \geq 50 \Rightarrow n = 6$$

Now,

$$R_b = (nN + a) f_s$$

$$= (10 \times 6 + 5) 25 \text{ K}$$

$$R_b = 1625 \text{ Kbps. Ans.}$$

(ES Subj  
only)

### \* DIFFERENTIAL PULSE CODE MODULATION (DPCM) :-

$$\text{As, } [\Delta e]_{\max} = \frac{\Delta}{2}$$

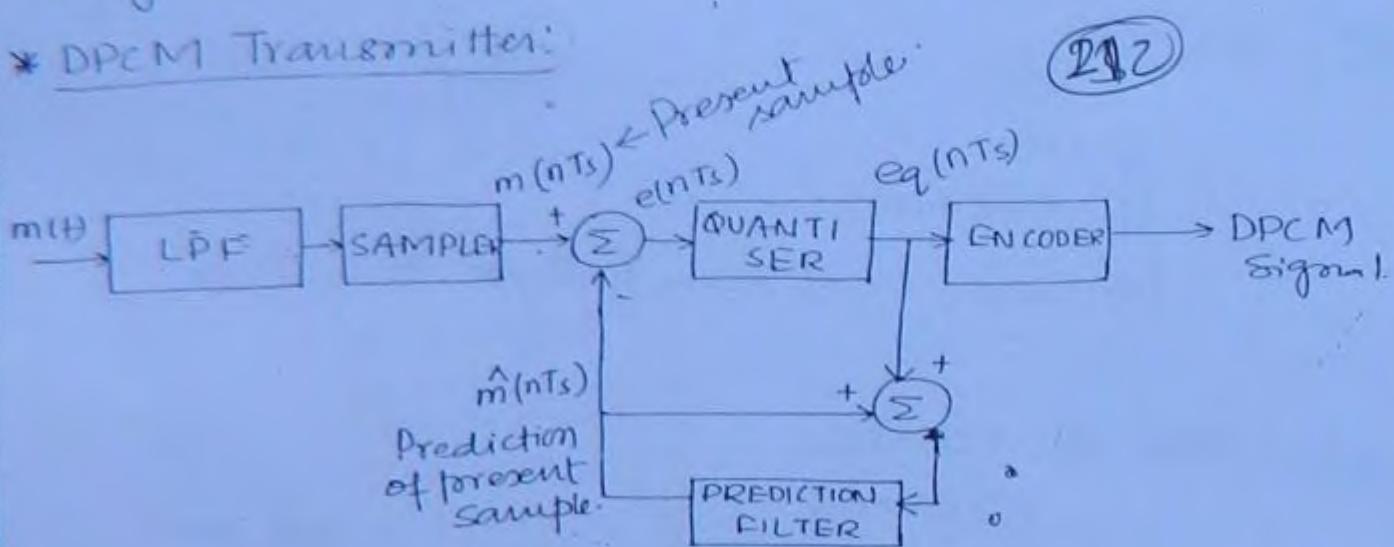
$$\Delta = \frac{V_{\max} - V_{\min}}{2^n}$$

$\times 2^n$  DPCM, the dynamic Range of the signal, without any effect on B.W., then  $\Delta e$  is Reduced.

is altered

\* In DPCM the dynamic range of signal to be quantized will be decreased such that w/o affecting the BW, processing requirement,  $\Delta e$  will be decreased.

### \* DPCM Transmitter:



Prediction Filter: (ie Past Behaviour of Signal)

\* By Analysing to some finite sustantile, nearby future values will be predicted by the Filter.

\* It has numerous no. of delay elements

\* It has huge storage capacity and has complex logical circuitry

### Op<sup>n</sup> of DPCM Tx:

$$(\text{Summer})_{\text{Op}} = e(nTs) = m(nTs) - \hat{m}(nTs) \quad \dots \quad (1)$$

where,  
 $e(nTs)$  = Prediction error

\* Prediction filter will be very precision CKT, so that dynamic Range of  $e(nTs)$  will be very small so that corresponding  $\Delta e$  will also be small.

Now,

$$q(nTs) = e(nTs) - eq(nTs) \quad \dots \quad (2)$$

$\xrightarrow{\text{quantiz. error}}$   
 Prediction Filter =  $\hat{m}(nTs) + eq(nTs)$ .  
 S/P

from (2) we have  $eq(nTs) = e(nTs) - q(nTs)$ . so,

$$\text{filter S/I P} = m(nTs) + e(n) - q(nTs)$$

$$= m(nTs) - q(nTs) \quad \{ \text{from eq(1)} \}$$

quantised  
value of  
 $m(nTs)$

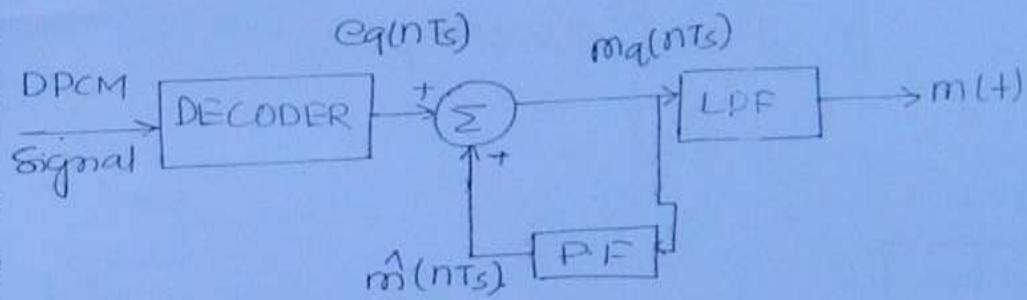
$$= m_q(nTs) \quad \{ \text{s.v.} - q.v = q.e \}$$

(213)

\* All the quantised values will be stored in the Prediction filter.

\* By analysing previous quantised values of the given signal, prediction filter predicts present sample of the given signal.

### \* DPCM RECIEVER:



$$\begin{aligned} (\text{LPF})_{\text{S/I}} &= eq(nTs) + \hat{m}(nTs) \\ &= m_q(nTs) \end{aligned}$$

Note:

\* In the reconstructed msg signal, finite amount of quantisation error will be retained, which is very small compared to PCM transmission.

\* DPCM is complex than PCM.

## \* DELTA MODULATION: (also called as 1 bit DPCM)

\* Delta Modulation needs very much minm Tx B.W compared to PCM and DPCM. 914

As  $J.B.W = \frac{f_s n f_s}{2}$ ; for DM  $\Rightarrow n = 1$  bit/sample.

To decrease B.W,  $f_s$  can't be decreased as to avoid the cause of undersampling {  $f_s > 2f_m$  }.

oversample to be maintained.

Note:

\* In Delta Modulation Tx B.W Required will be decreased to ; minm possible extent by selecting lowest possible value of  $n$ : ie 1 bit/sample.

$$n = 1 \text{ bit/sample}$$

As.  $R_b = n f_s$

and in DM;  $n=1$

$$\text{so, } R_b = f_s$$

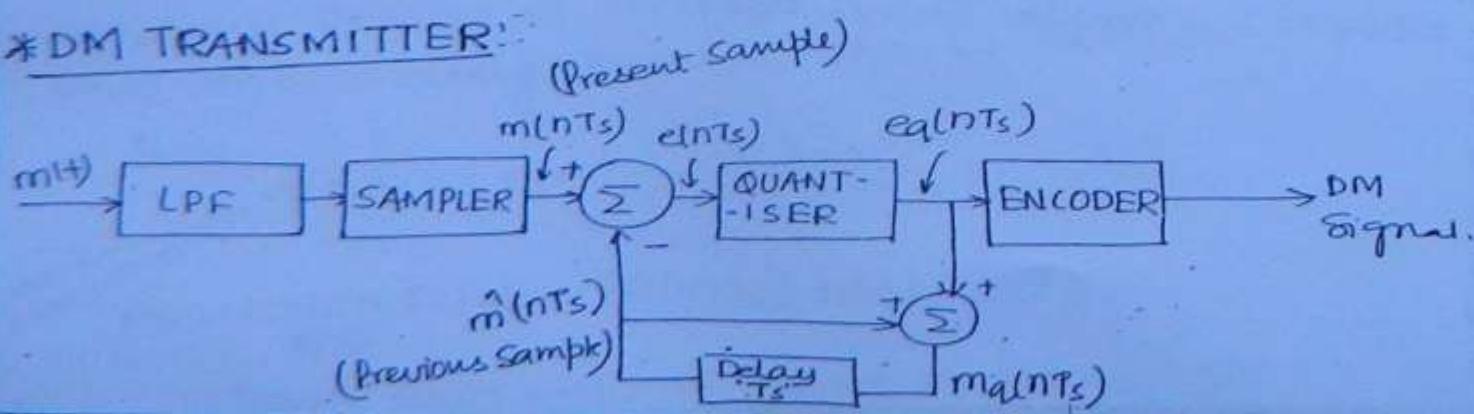
\* In DM;

$$\boxed{\text{Sampling Rate} = \text{Bit Rate} = \text{Pulse Rate}}$$

\* As; in DM;  $n=1$

$$\text{so, } L = 2 \rightarrow \begin{cases} +\Delta (\text{step size}) \neq V_{max} - V_{min}/L & \xrightarrow{\text{encoder O/P}} 1 \\ -\Delta (\text{step size}) \neq V_{max} - V_{min}/L & \xrightarrow{\text{encoder O/P}} 0 \end{cases}$$

\* DM TRANSMITTER:



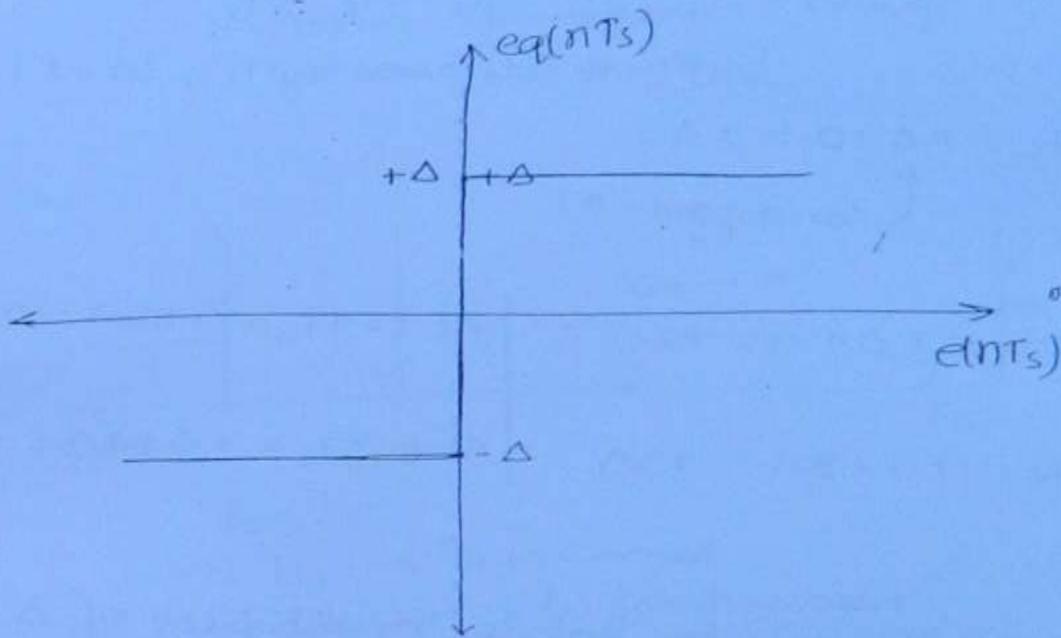
## G/P of quantiser

$$e(nT_s) = m(nT_s) - \hat{m}(nT_s)$$

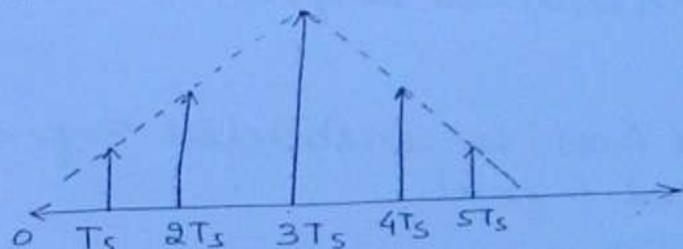
Present sample      Previous sample

(28)

## Quantiser characteristics:-



Let,

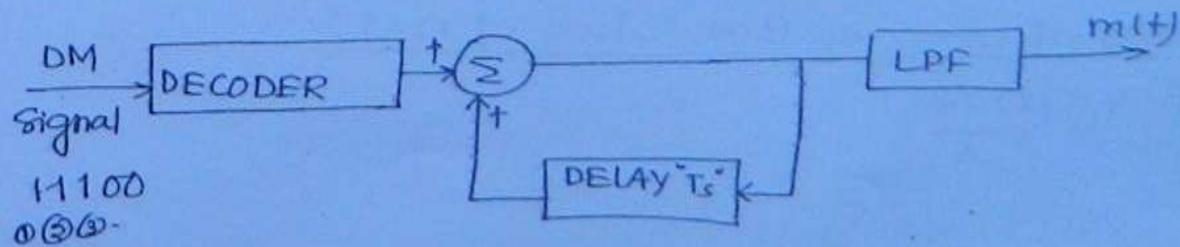


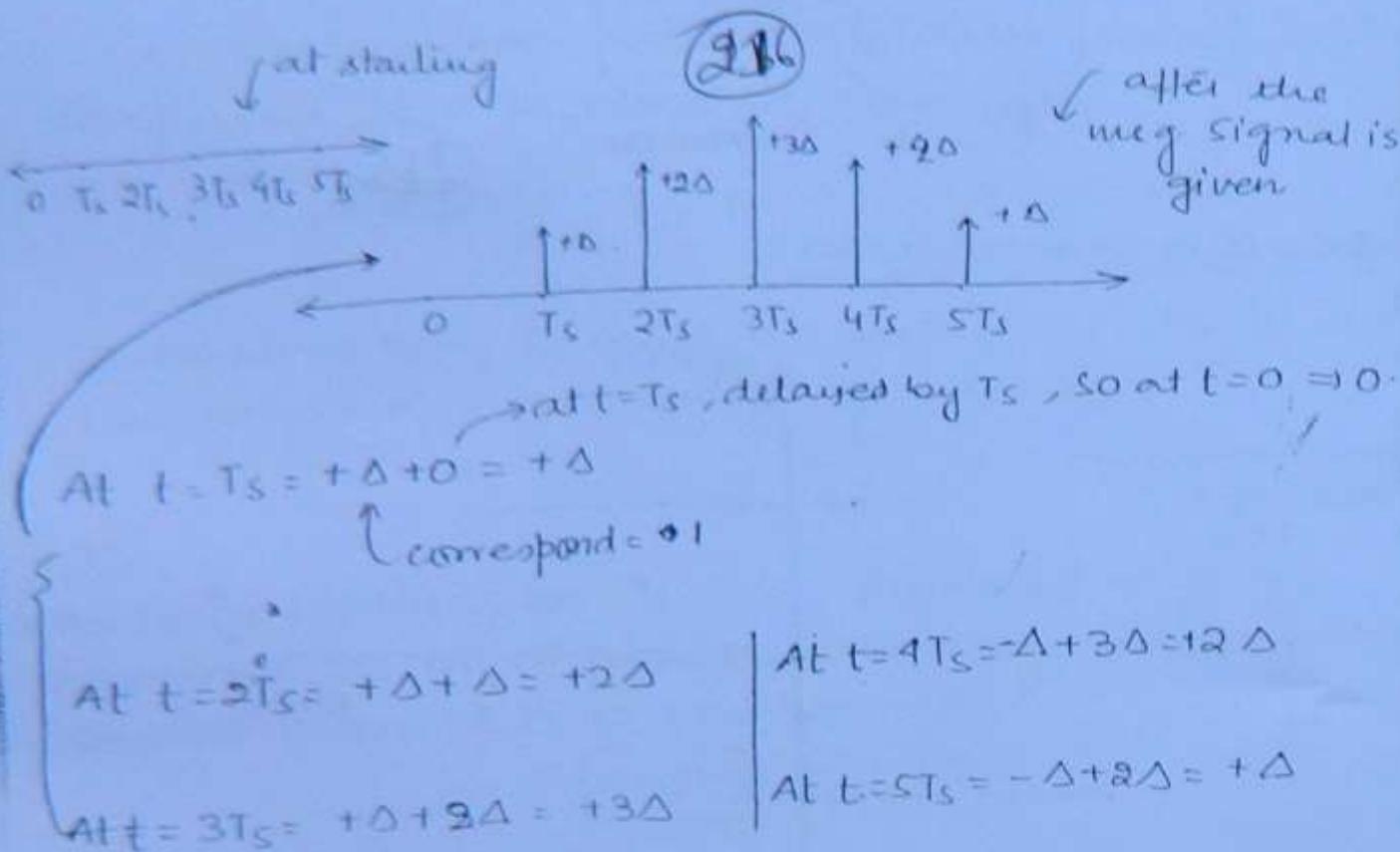
$e(nT_s)$  +ve +ve +ve -ve -ve {Present Sample - Previous Sample}

$eq(nT_s)$  +Δ +Δ +Δ -Δ -Δ

DM Signal 1 1 1 0 0

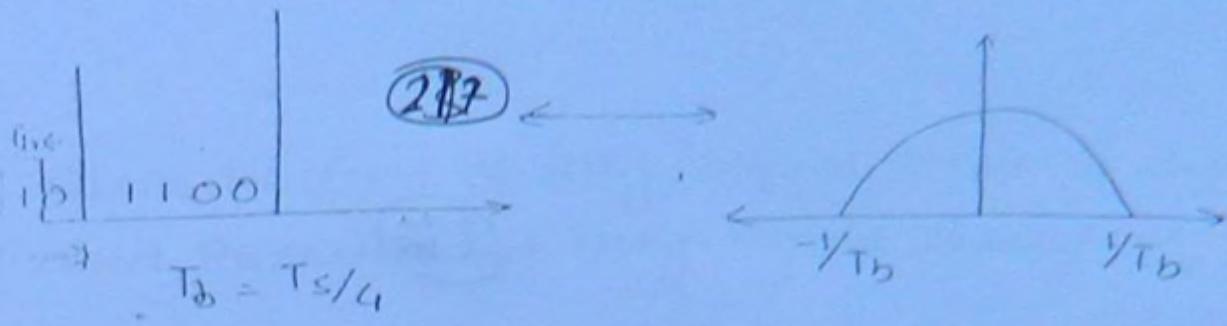
## \* DM RECEIVER:-





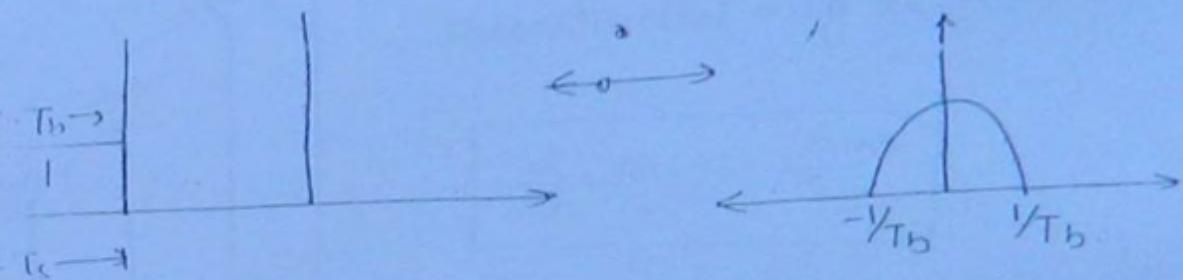
Note:

1. The S/I P of LPF will be integer multiples of  $\Delta$ .
2. The pattern of the Reconstructed signal mainly depends on choice of  $\Delta$ .
3. For optimum step size ( $\Delta_{opt}$ ) reconstructed signal will be same as Transmitted signal.
4. If  $\Delta < \Delta_{opt} \rightarrow$  Slope Overload error occurs.  
 $\Delta > \Delta_{opt} \rightarrow$  Granular error occurs.
5. To overcome SOE; step size has to be increased.
6. To overcome GE; step size has to be decreased.
- X Physical significance of B.W is less if  $n=1$ :
- \* As  $B.W = \frac{1}{2} n f_s$



as  $T_b$  is small.  
hence spectrum is large.  
so large B.W reqd.

et :



$T_b = T_s$   
as  $T_b$  is increased  
spectrum gets compressed  
hence B.W reqd is less.

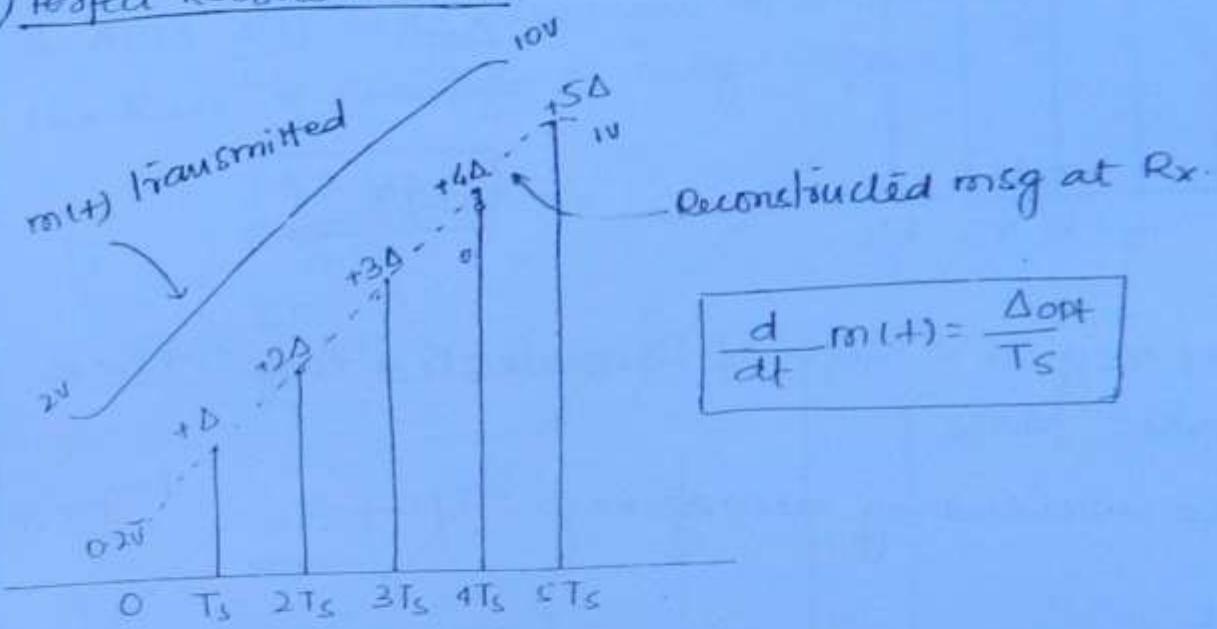
918

Note :-

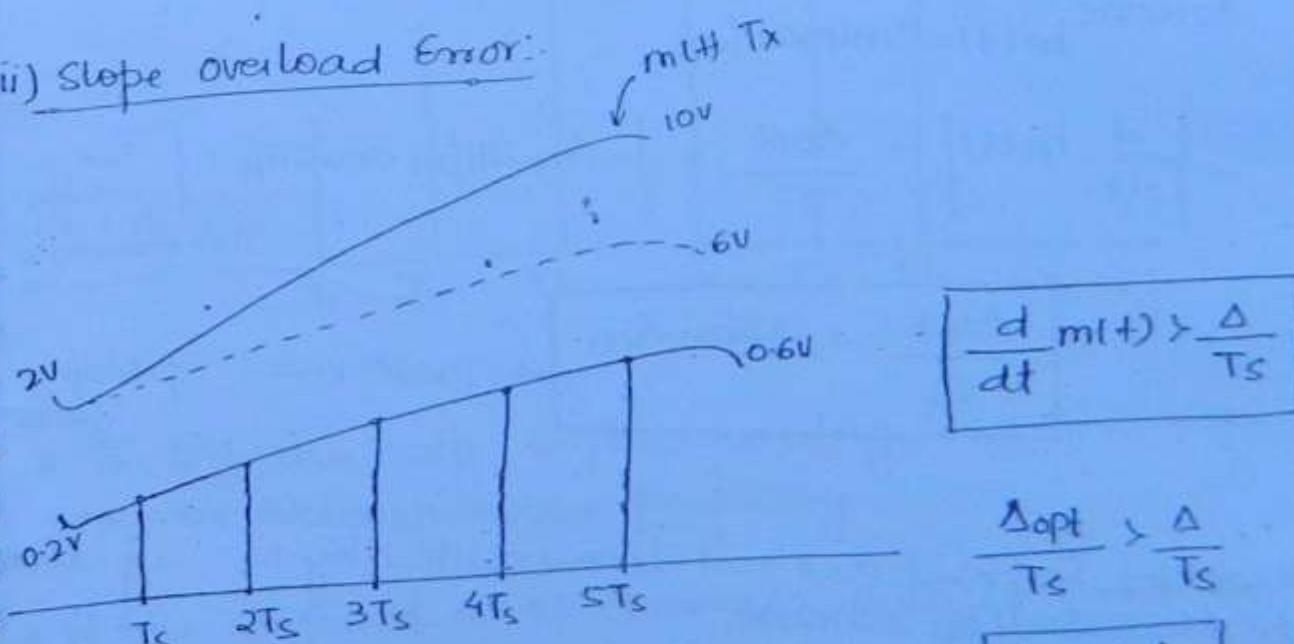
\* For proper reconstruction of msg. Signal slope of the transmitted and reconstructed msg. signal should be same.

(279)

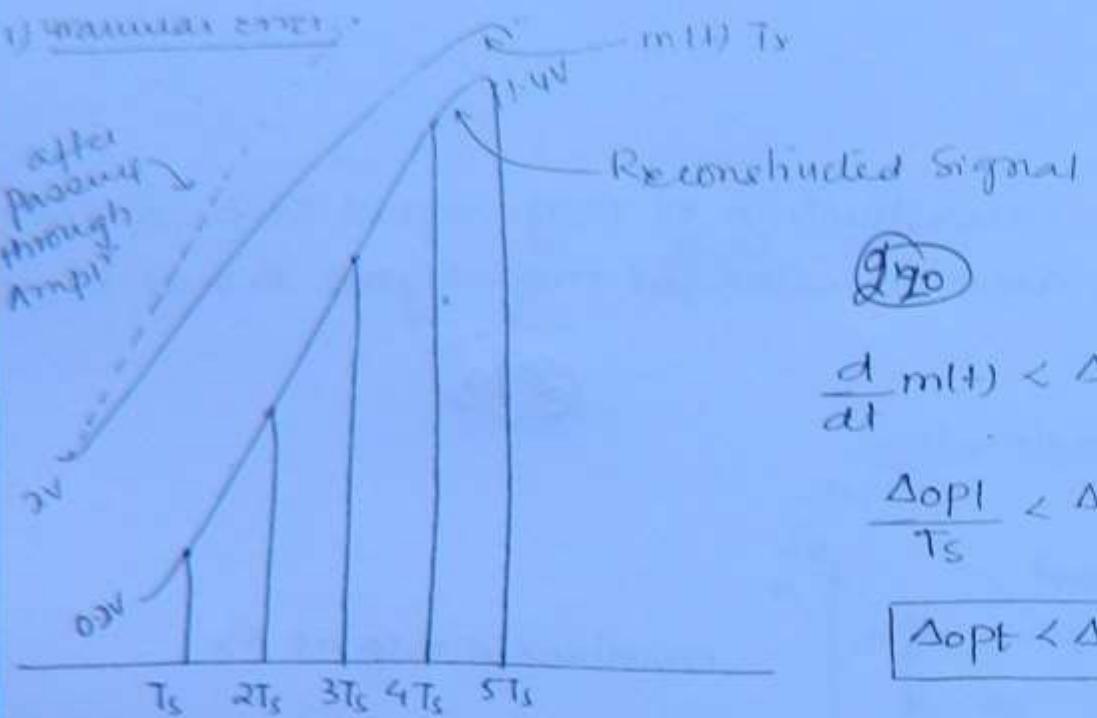
i) Perfect Reconstruction :-



ii) Slope Overload Error :-



Slope of Tx Signal  $>$  Slope of Reconstructed Signal  
then SOE takes place.  
\* can be avoided by increasing step size.



$$\frac{d}{dt} m(t) < \Delta/T_s$$

$$\frac{\Delta_{opt}}{T_s} < \Delta/T_s$$

$$\boxed{\Delta_{opt} < \Delta}$$

- \* Slope of msg Tx < Slope of Reconstructed msg : then G-E takes place
- \* can be avoided by decreasing step size.

### Analysis:

Assume,  $m(t) = Am \sin(2\pi f_m t)$

$$\left| \frac{d}{dt} m(t) \right| = \frac{\Delta_{opt}}{T_s} = \left| -Am 2\pi f_m \sin(2\pi f_m t) \right|$$

$$\boxed{\frac{\Delta_{opt}}{T_s} = 2\pi f_m \cdot Am} \quad \leftarrow \text{max value of slope}$$

### Note:-

- \* Sinusoidal signals are not transmitted by using the Delta Modulation scheme.

- \* For slope overload error to occur.

$$\frac{\Delta_{opt}}{T_s} > \frac{\Delta}{T_s} \Rightarrow \boxed{\frac{\Delta}{T_s} < 2\pi f_m \cdot Am}$$

\* For Unimodular noise we have

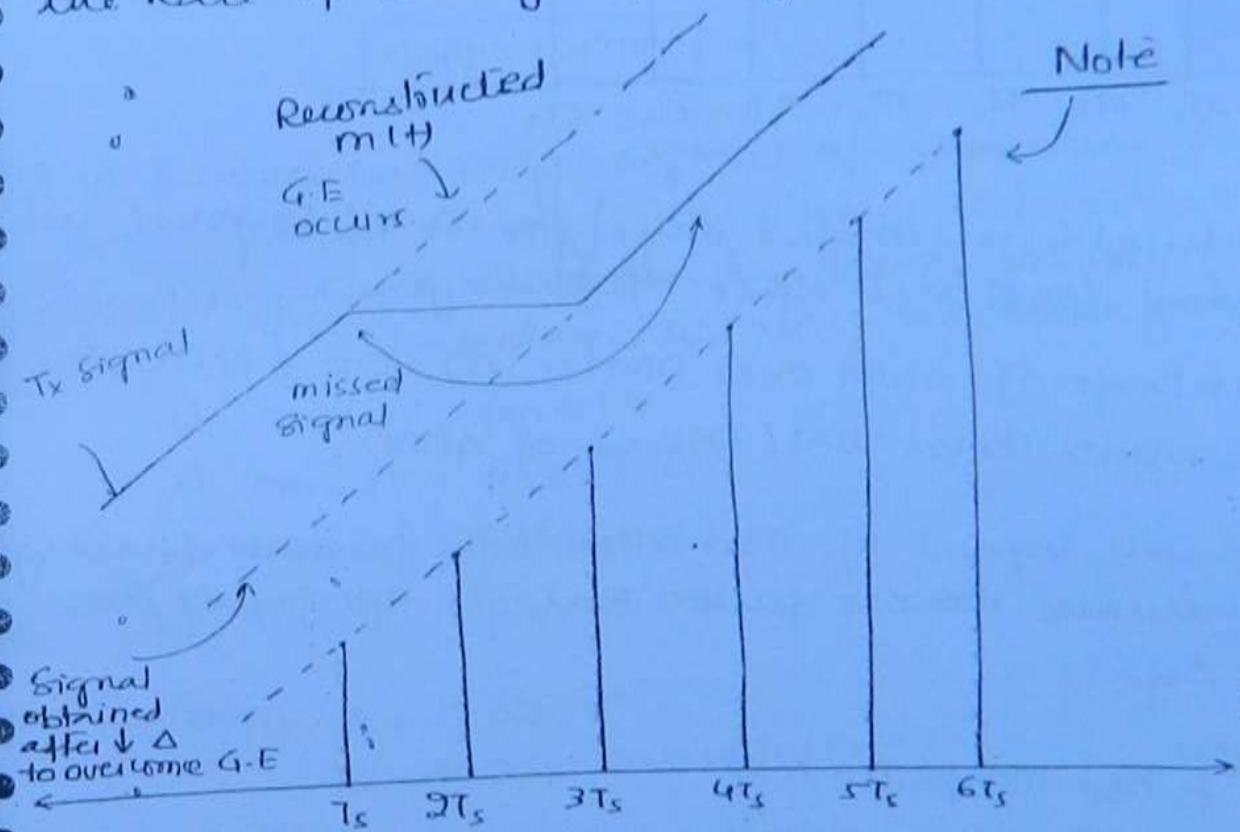
$$\frac{\Delta_{opt}}{T_s} < \Delta/T_s$$

$$\left[ \frac{\Delta}{T_s} \rightarrow \text{Unif. Am}\right]$$

(22)

## \* ADAPTIVE DELTA MODULATION:

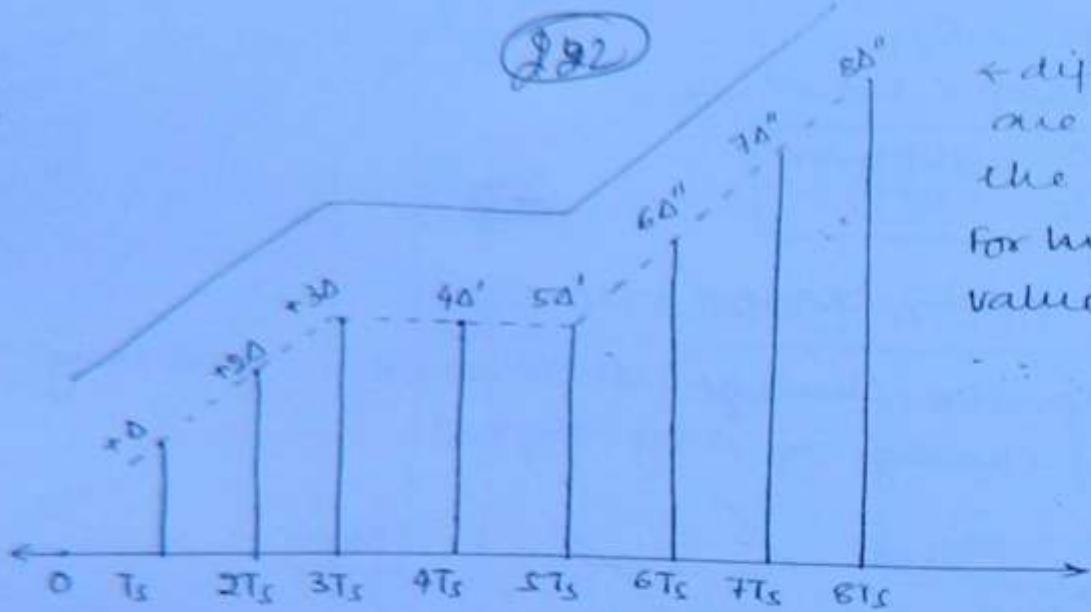
In ADM, step size changes continuously according to the Rate of change of msg. signal.



Note:

- \* In DM, the msg. signal having constant slope can only be Reconstructed perfectly
- \* If the slope of m(t) changes, then it can't be Reconstructed back as shown in above fig
- \* To overcome above, ADM is used.

But in ADM:



different step sizes  
are used to linearize  
the Tx mit).

For high slope, high  
value of  $\Delta$

Note:

\* Delta Modulator is limited only for Tx of Signal which are having constant Rate of change.

\* The Advantage of ADM over DM is no SOE + GE.

\* ADM is complex than DM ( Disad of ADM)

Q) A continuous Signal of  $8\sin 8\pi \times 10^3 t$  is passed through Delta Modulator whose pulse Rate is 4000pulse/sec  
Find the  $\Delta_{opt}$ ?

Soln: Given

$$f_s = 4000 \text{ pulse/sec}$$

$$m(t) = 8\sin 8\pi \times 10^3 t$$

$$A_m = 8V; f_m = 4KHz$$

$$\frac{\Delta_{opt}}{T_s} = 2\pi f_m A_m$$

for DM, pulse Rate = Sampling Rate

$$f_s = 1/T_s = 1000 \text{ samples/sec}$$

$$\Delta_{opt} \times 1000 = 2\pi \times 4K \times 8$$

$$\boxed{\Delta_{opt} = 16\pi \text{ Volts}}$$

Q2 A msg signal of  $m(t) = 10t$  is transmitted over channel  
bit rate is 1 Kbps. Find  $\Delta_{opt}$

Soln: Given.  $m(t) = 10t$

$$R_b = 1 \text{ Kbps}$$

(Q23)

Now, Since given  $m(t)$  is not sinusoidal so,

$$\frac{\Delta_{opt}}{T_s} = 2\pi f_m A_m \quad \frac{d m(t)}{dt} = \frac{\Delta_{opt}}{T_s}$$

$$\frac{d}{dt} 10t = \Delta_{opt} \times 1000 \quad \left\{ \because R_b = \frac{1}{T_s} \right\}$$

$$\boxed{\Delta_{opt} = 10 \text{ mV}}$$

Q3 A sinusoidal msg signal of frequency,  $f_m$ , amplitude  $A_m$  is passed through DM whose step size is  $0.628 \text{ V}$ . Sampling Rate is given by 40000 samples/sec. For which of the following, DM will be slope overloaded

- a)  $A_m = 3 \text{ V}$ ;  $f_m = 1 \text{ K}$
- b)  $A_m = 2 \text{ V}$ ;  $f_m = 1.5 \text{ K}$
- c)  $A_m = 2 \text{ V}$ ;  $f_m = 2.5 \text{ K}$
- d)  $A_m = 1 \text{ V}$ ;  $f_m = 2.5 \text{ K}$

Soln: Given.  $\Delta = 0.628 \text{ V}$

$$f_s = 40000 \text{ sample/sec}$$

Now,

$$\frac{\Delta}{T_s} = 0.628 \times 40000 = 25120$$

Now,

$$\frac{\Delta_{opt}}{T_s} = 2\pi f_m A_m$$

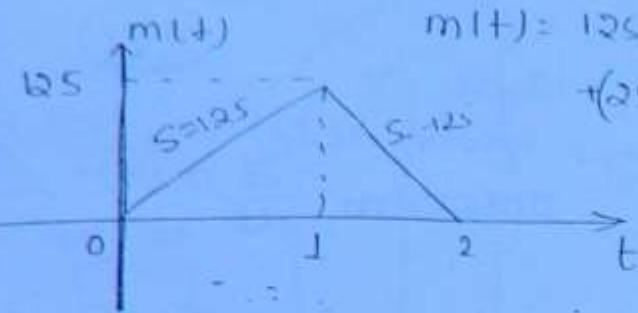
For slope overload error

$$\frac{\Delta}{T_s} < \left( \frac{\Delta_{opt}}{T_s} = 2\pi f_m A_m \right)$$

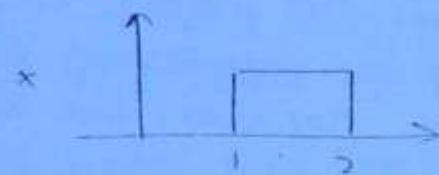
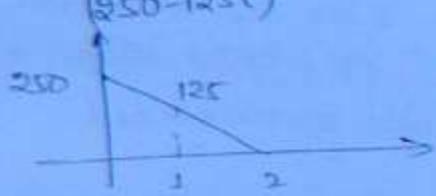
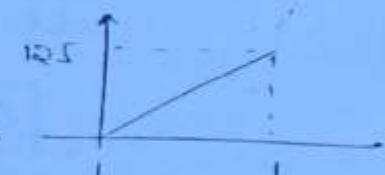
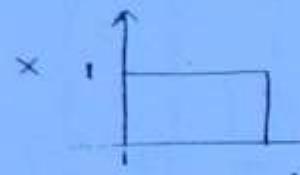
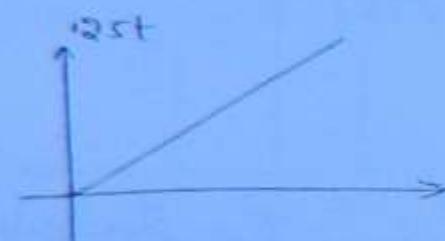
So, for  $A_m = 2 \text{ V}$ ;  $f_m = 2.5 \text{ K}$ ,  $2\pi f_m A_m = 31415.925$   
Hence SOE occurs.

Q4 For following msg signal, find  $\Delta_{opt}$  with  $f_s = 1$  1000 samples/sec.

(Q4)



$$m(t) = 125[u(t) - u(t-1)]^2 + (250 - 125t)[u(t-1) - u(t-2)]$$



Now, since signal is not sinusoidal, so

$$\frac{dm(t)}{dt} = \frac{\Delta_{opt}}{T_s}$$

$$125 = \Delta_{opt} \times 1000$$

$$\boxed{\Delta_{opt} = 125 \text{ mV}}$$

Q5 A msg Signal of Peak to Peak voltage 1.536 volts is passed through PCM system of having 128 quantisation level. Find Quantisation Noise power.

Soln: Given:

$$V_{P-P} = 1.536 \text{ V}$$

$$L = 128 = 2^n \Rightarrow n = 7$$

NOW,

$$(N_q) = \frac{\Delta^2}{12} = \frac{1}{12} \left\{ \frac{1.536}{128} \right\}^2$$

$$\boxed{N_q = 12 \mu \text{W}}$$

Q6 For a message power of  $1 \text{ mW}$ , compute  $n$  to get min<sup>m</sup> SNR of 1000 a) 2 b) 3 c) 4 d) 5

Sol<sup>m</sup>: Given,

$$\text{SNR} \geq 1000$$

(225)

$$\frac{3}{2} \cdot 2^{2n} \geq 1000$$

$$2^{2n} \geq \frac{2000}{3}$$

$$2^{2n} \geq 666.66$$

$$n \geq 5$$

$$\boxed{n=5} \quad \text{Ans.}$$

Q7. A msg signal band limited to 4K is transmitted through 256 level PCM system. Find Tx B.W of the system  
a) 16 K b) 32 K c) 64 K d) 128 K

Sol<sup>m</sup>: Given:  $f_{\text{an}} = 4\text{K}$

$$L = 256 \Rightarrow n = 8$$

$$\text{Now, } f_s = 2f_{\text{an}} = 8\text{K}$$

$$\text{So, } B.W = \frac{n f_s}{2} = \frac{8 \times 8\text{K}}{2}$$

$$\boxed{B.W = 32\text{K}}$$

Q8. A msg signal sampled at 8K is transmitted through 512 level PCM system.  $(\text{SNR})_{\text{dB}} = ?$   
 $R_b = ?$

Sol<sup>m</sup>: Given,  $f_{\text{an}} = 8\text{K} \Rightarrow f_{\text{an}} = 4\text{K}$   
 $L = 512 \Rightarrow n = 9$

$$\text{So, } (\text{SNR})_{\text{dB}} = 1.8 + 6n = 1.8 + 54$$

$$\boxed{(\text{SNR})_{\text{dB}} = 55.8\text{dB}}$$

$$R_b = \frac{n f_s}{2}$$

$$R_b = 9 \times 8\text{K}$$

$$\boxed{R_b = 72\text{Kb/s}}$$

Q9 A msg signal whose peak Amp is 5V and bias is -3V is transmitted through PCM system of having step size of 1V. Find  $(SONR)_{dB} = ?$

Soln:-  $(SONR)_{dB} = 1.8 + 6n \quad (296)$

Given,  
 $V_{max} = 5V$ ,  
 $V_{min} = -3V$   
 $\Delta = 1V$

$$\Delta = \frac{V_{max} - V_{min}}{2^n} = 1V$$

$$\frac{5+3}{2^n} = 1 \Rightarrow 2^n = 8$$

$$n = 3$$

$$\text{So, } SONR = 1.8 + 18$$

$$(SONR)_{dB} = 19.8 \text{ dB}$$

Q10 For a PCM system as the no. of Quantisation level increases from ~~2~~ 2 to 8, then Tx B.W. Reqd will be

- a) Increased by 4 times.
- b) Tripled.
- c) doubled.
- d) No change.

Soln: Quantisation level  $L \rightarrow 2 \text{ to } 8$   
 $n \rightarrow 1 \text{ to } 3$

$$(B.W)_{n=1} = n f_s / 2 = f_s / 2$$

$$(B.W)_{n=3} = n f_s / 2 = 3f_s / 2$$

Hence Tripled

Q11 for a PCM system of having Bit Rate of  $10^8 \text{ bits/sec}$  No. of Quantisation levels are given by 256. Find the maxm freq<sup>n</sup> of the signal allowed by the PCM system?

Soln: Given:

$$R_b = 10^8 \text{ bits/sec}$$

$$L = 256 \Rightarrow n = 8$$

Now,

$$B.W. = nfs/2 = R_b/2$$

227

$$\frac{10^8}{2} = \frac{8 \times 2 f_m}{2}$$

$$f_m = \frac{10^8}{16}$$

$$f_m = 6.25 \text{ MHz}$$

Q12. A msg signal of  $m(t) = 6\cos 2000\pi t + 2\cos 4000\pi t$  is passed through DM whose slope & pulse rate is 5000 pulses/min. Find, min<sup>m</sup> value of  $\Delta$  required to overcome slope overload error.

Soln:  $m(t) = 6\cos 2000\pi t + 2\cos 4000\pi t$

$$Am_1, \cos 2\pi f_m t + Am_2, \cos 2\pi f_m t$$

$$Am_1 = 6 \quad Am_2 = 2$$

$$f_m_1 = 1000 \text{ Hz} \quad f_m_2 = 2000 \text{ Hz}$$

$$\frac{\Delta_1}{T_s} = 2\pi f_m_1 Am_1$$

$$\frac{\Delta_2}{T_s} = 2\pi f_m_2 Am_2$$

$$\frac{\Delta_1}{T_s} = 2\pi \times 1000 \times 6$$

$$\Delta_1 \times 5000 = 2\pi \times 2000 \times 2$$

$$\Delta_1 \times 5000 = 2\pi \times 6000$$

$$\Delta_2 = \frac{8\pi}{5}$$

$$\Delta_1 = \frac{12\pi}{5} = 7.52 \text{ V}$$

$$\Delta_2 = 5.03 \text{ V}$$

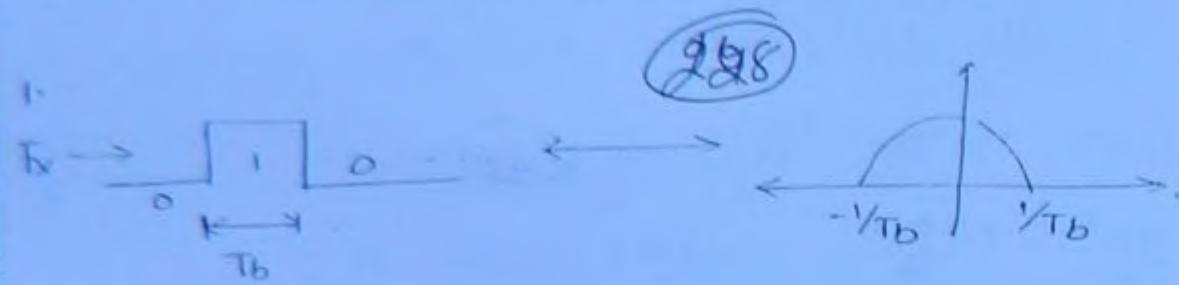
So, min<sup>m</sup> step size Required to overcome SOE is given by  $\max \{\Delta_1, \Delta_2\}$ .

So,

$$\Delta = 7.52 \text{ V}$$

min<sup>m</sup> step size Reqd. to overcome GE is  $\min \{\Delta_1, \Delta_2\} = 5.03 \text{ V}$

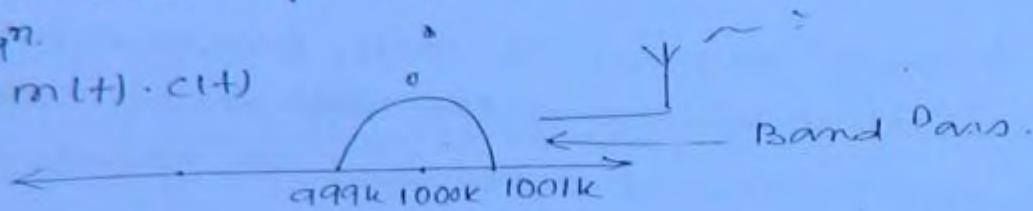
## \* Band Pass Delta Transmission



i) when  $T_b = 1 \text{ msec} \Rightarrow 1/T_b = 1 \text{ K} = f_{bi}$

then  $f_c = 1 \text{ M}$  is used and the resulting is Band Pass signal as it donot contain msg in low freqn.

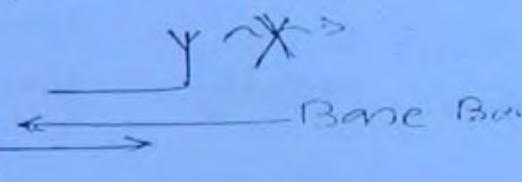
$$s(t) = m(t) \cdot c(t)$$



ii) when  $T_b = 1 \mu\text{sec} \Rightarrow 1/T_b = 1 \text{ M} \Rightarrow f_{bi}$

and  $f_c = 1 \text{ M}$  is used, then the spectrum

$$s(t) = c(t)m(t)$$



This is still Base Band signal as it contains significant low frequencies

Hence,  $f_c$  should be in order of 10M.

Note:-

- \* By using ASK, PSK & FSK, a digital Base band signal is converted as Band Pass Signal and can be transmitted through free space.
- \* In these modulation schemes, one of the modulator of carrier signal switches b/w 2 specific values as the msg. signal switching b/w 2 specific voltages. So these modulation schemes are called

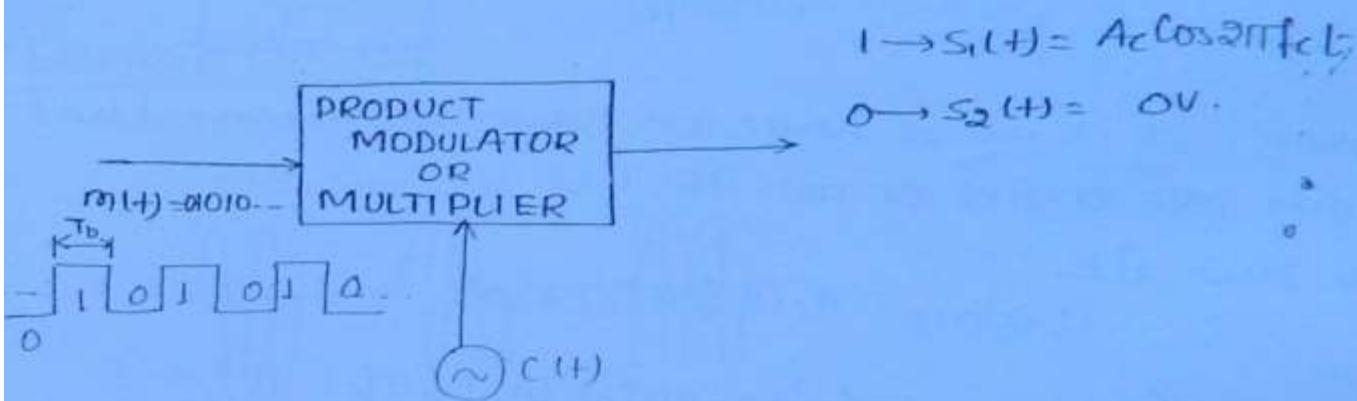
as information, 0 0 0

## \* Amplitude Shift Keying (ASK)

In this modulation, binary 1 is represented by presence of carrier and binary 0 by absence of carrier

(229)

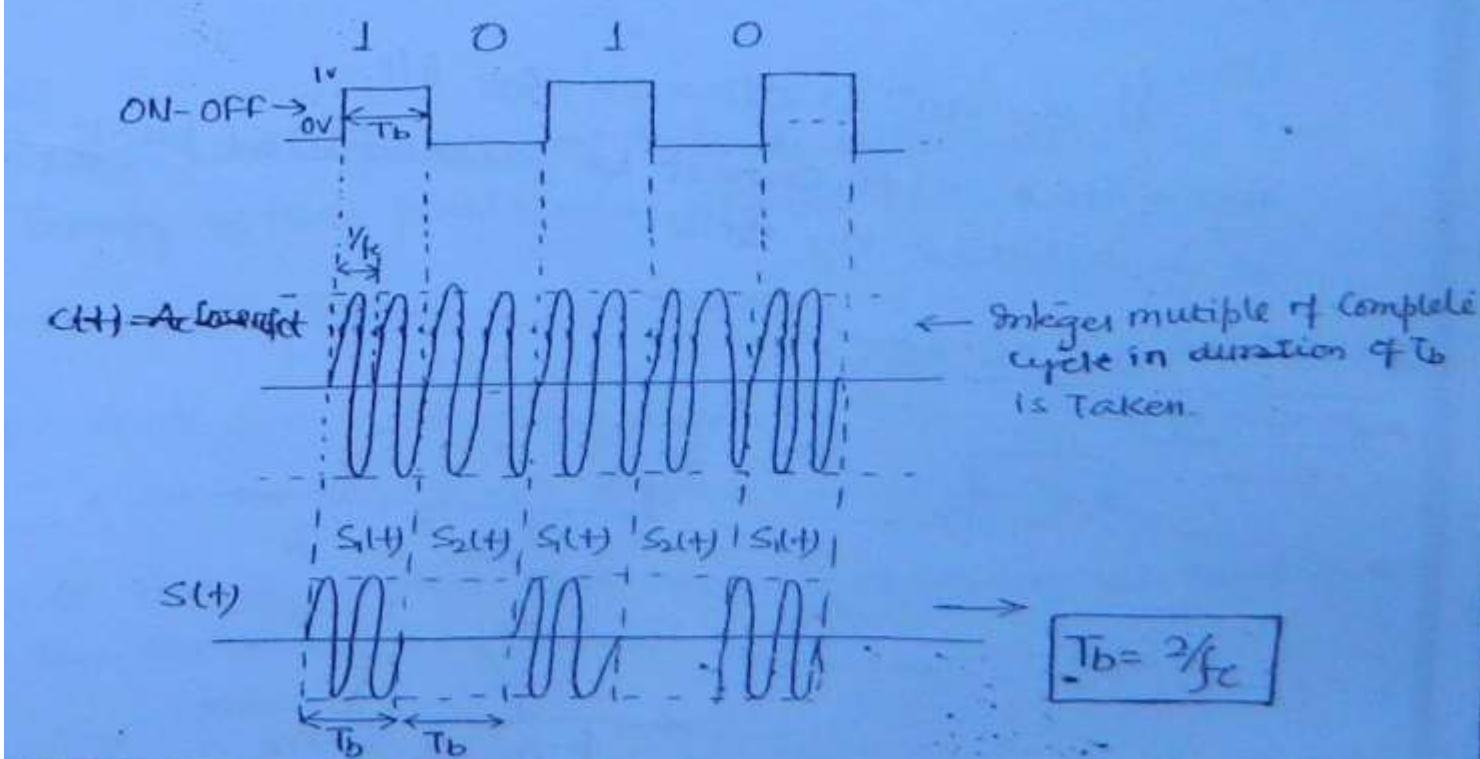
### ASK Transmitter:



Electrical signal representation technique is ON-OFF coding, for which

$$\begin{aligned} 1 &\rightarrow +ve \\ 0 &\rightarrow -ve \end{aligned}$$

### Graphical Representation:



## \* ASK RECEIVER

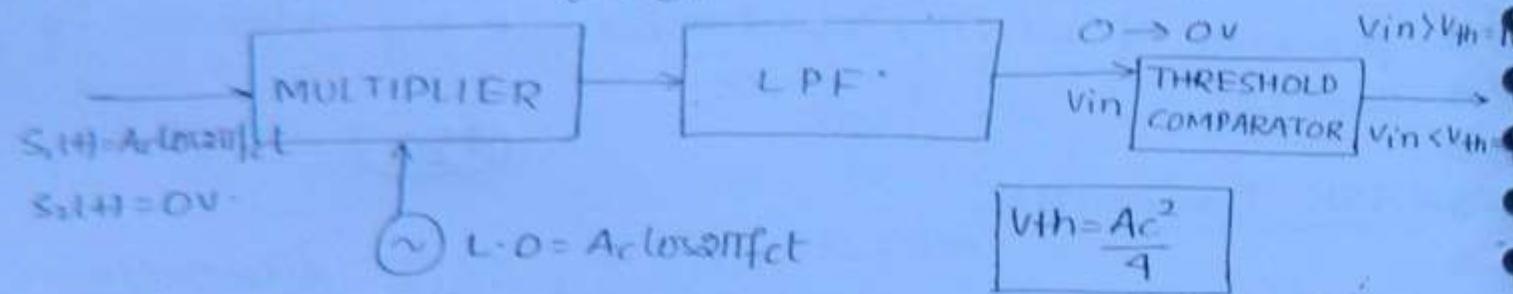
Q50

\* For demodulation of ASK, SD will be used.

$\rightarrow$  Ac component

$O \rightarrow OV$

$$I \rightarrow \frac{Ac^2}{2}$$



Since, SD is used hence we have to observe that either QNE occurs or not

for this let,

$$(I \cdot O)_{O/P} = Ac \cos(2\pi f_{ct} + \phi)$$

$$S_0 \underset{O}{(MUL)} \underset{\leftarrow 1}{(O/P)} = Ac^2 \cos 2\pi f_{ct} \cos(2\pi f_{ct} + \phi) \leftarrow 1$$

$\leftarrow 0$

$$(LPF)_{O/P} = \frac{Ac^2}{2} \cos \phi \leftarrow 1$$

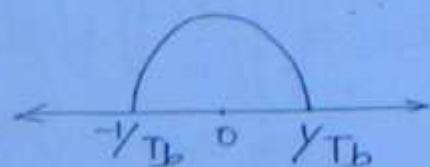
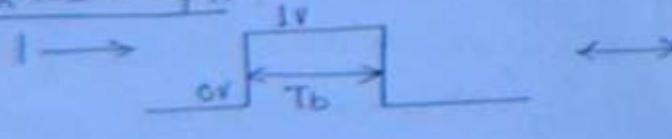
$O \leftarrow 0$

NOW, if  $\phi = 90^\circ \Rightarrow O/P = OV$  for  $I/P = 1$

Hence, the m(t) cannot be reconstructed back and it is affected by QNE.

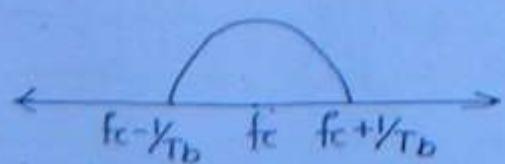
## \* TRANSMISSION BW OF ASK

Transmission of I:



$S(t)$

$\longleftrightarrow$



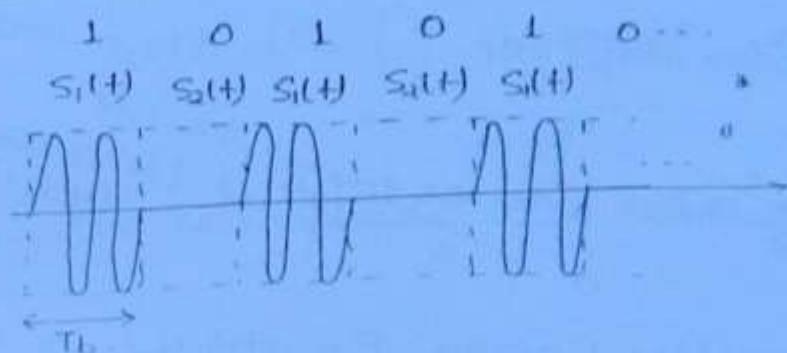
$$S_b(t) = 0$$

(23)

So, ASK BW =  $(f_c + \frac{1}{T_b}) - (f_c - \frac{1}{T_b})$   
=  $\frac{2}{T_b}$

ASK BW =  $2R_b$

### \* ENERGY PER BIT:



NOW,

$$\text{As } E = \int_{-\infty}^{\infty} x^2(t) dt$$

$$\text{& } P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x^2(t)| dt = \frac{E}{T} \Big|_{T \rightarrow \infty} \left\{ \begin{array}{l} \text{when } \\ T = \infty \end{array} \right\}.$$

Note:-

1. when the signal is of infinite duration, then the Energy of that particular signal is also  $\infty$ .
2. for finite signals, the energy is also finite.
3.  $0 < E < \infty \rightarrow$  Energy signal (Power=0).  
 $0 < P < \infty \rightarrow$  Power signal (Energy= $\infty$ ).
4. An Energy signal can never be Power signal and also the vice-versa is correct.

\* All periodic signals are Power Signals and its Energy

\* Transmission of 1:

238

$$\begin{aligned} E_b &= \int_0^{T_b} S_1^2(t) dt \\ &= \int_0^{T_b} (A_c \cos 2\pi fct)^2 dt \\ &= \int_0^{T_b} A_c^2 \cos^2 2\pi fct dt \\ &= \int_0^{\frac{T_b}{2}} \frac{A_c^2}{2} dt + \int_{\frac{T_b}{2}}^{T_b} \frac{A_c^2 \cos 2\pi fct}{2} dt \xrightarrow{\text{Area}=0} \text{(since complete cycles)} \end{aligned}$$

To Save Transmitter Energy,  $E_b$  should be small.

So,

$$E_b = \frac{A_c^2 T_b}{2} \Rightarrow A_c = \sqrt{\frac{2 E_b}{T_b}}$$

\* Transmission of 0:

$$E_b = \int_0^{T_b} S_2^2(t) dt$$

$$E_b = 0$$

\* CONSTELLATION DIAGRAM:-

$$1 \rightarrow S_1(t) = A_c \cos 2\pi fct = \sqrt{\frac{2 E_b}{T_b}} \cos 2\pi fct$$

$$0 \rightarrow S_2(t) = 0$$

As,

$$E_b = \int_0^{T_b} S_1^2(t) dt = \int_0^{T_b} \left\{ \sqrt{\frac{2 E_b}{T_b}} \cos 2\pi fct \right\}^2 dt$$
$$E_b = E_b \int_0^{T_b} \left\{ \sqrt{\frac{2}{T_b}} \cos 2\pi fct \right\}^2 dt$$

$$\int_0^T \left| \frac{2}{T_b} \cos 2\pi f_c t \right|^2 dt = 1$$

Normalized Function

So,

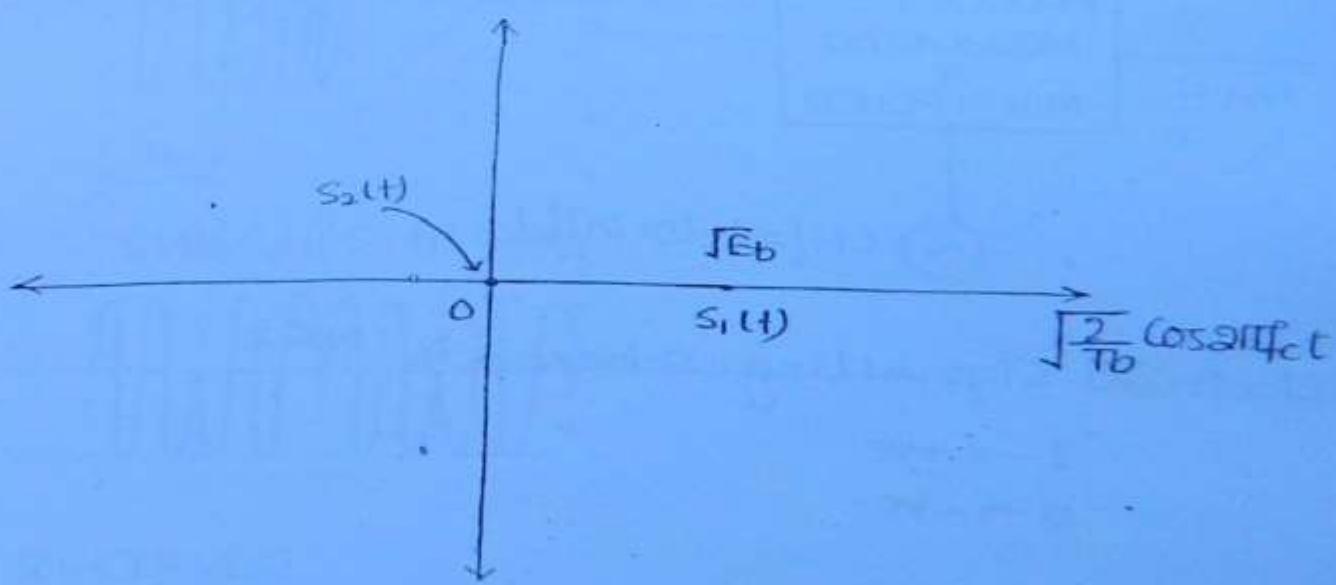
$$\boxed{\text{Energy } \left\{ \int \frac{2}{T_b} \cos 2\pi f_c t \right\} = 1}$$
239

\* In constellation diagram the func<sup>n</sup> whose energy is equal to 1 is said to be as Normalised function

~~Answer~~

Now,  $s_1(t) = \sqrt{E_b} \cdot \underbrace{\frac{2}{T_b} \cos 2\pi f_c t}_{f(t)} ; s_2(t) = 0$

\* Each axis corresponds to a Normalised function



In constellation diag<sup>n</sup> the reference axes corresponds to Normalised functions.

\* Conclusion:

- distance of  $s_1(t)$  from the origin =  $\sqrt{E_b}$

$$\boxed{\text{Energy } \{ s_1(t) \} = (\sqrt{E_b})^2 = E_b.} \quad \{ \text{Square of distance} \}$$

\* Distance of  $s_1(t)$  from origin = 0

$$\boxed{\text{Energy } \{s_1(t)\} = 0}$$

(168)

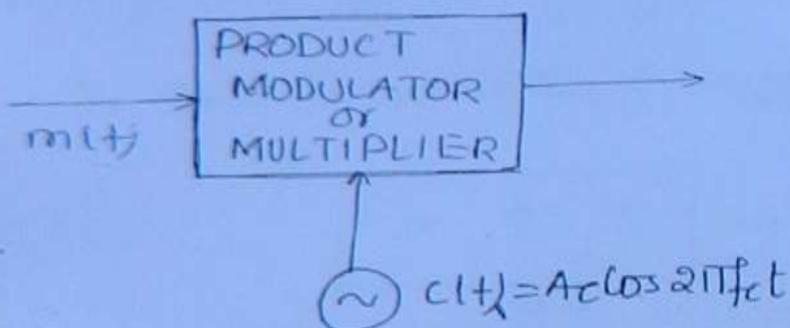
(234)

\* distance b/w signalling points  $d_{12} = \sqrt{E_b}$

### \* PHASE SHIFT KEYING (PSK):

\* In PSK, Binary 1 is represented by Actual carrier and Binary 0 by  $180^\circ$  phase shift of carrier

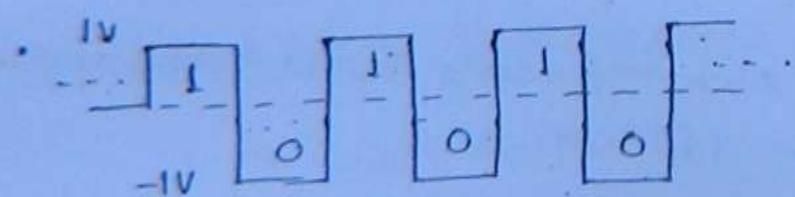
### \* PSK TRANSMITTER:-



\* Electrical Signalling scheme is NRZ.

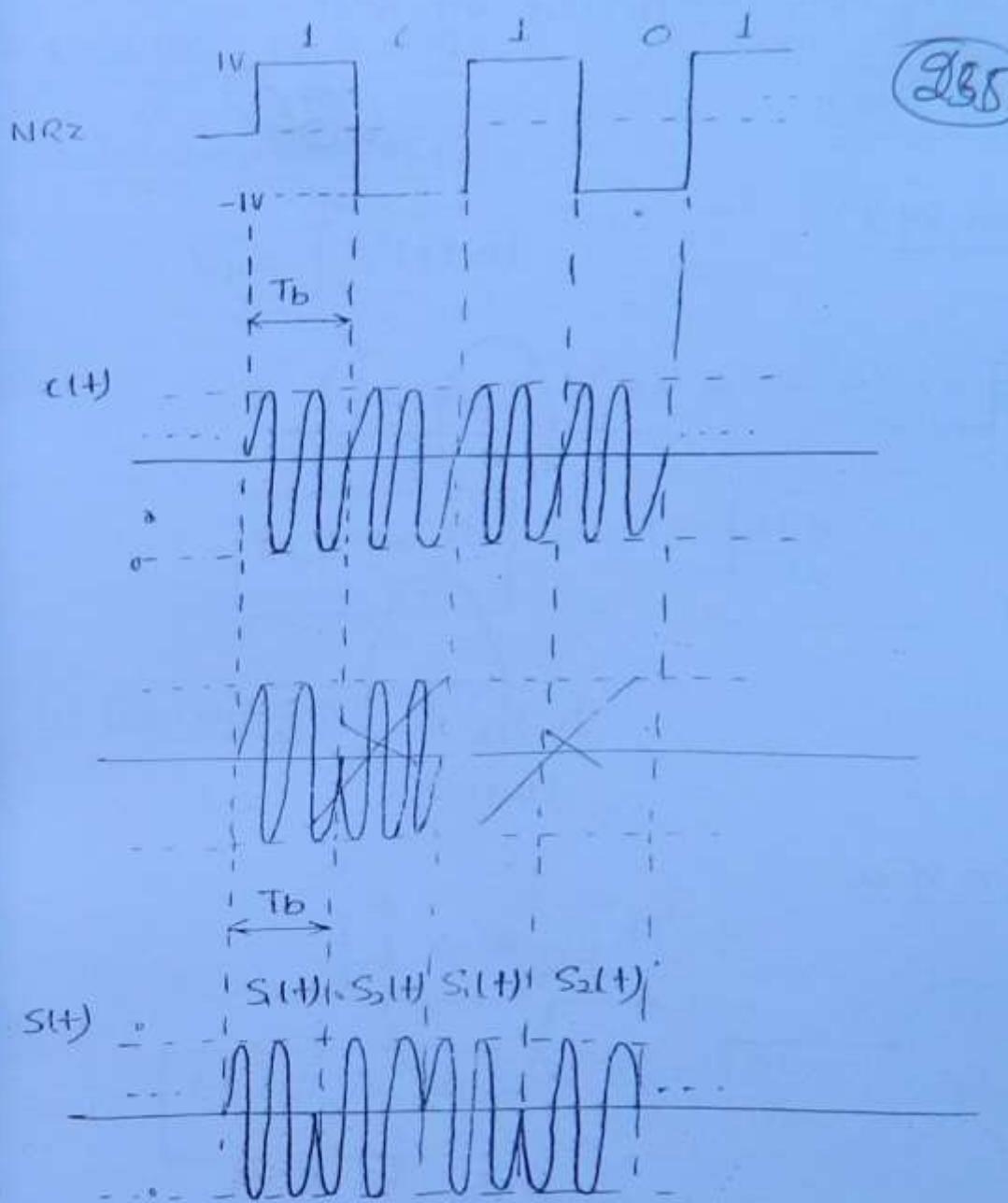
1 → +ve

0 → -ve



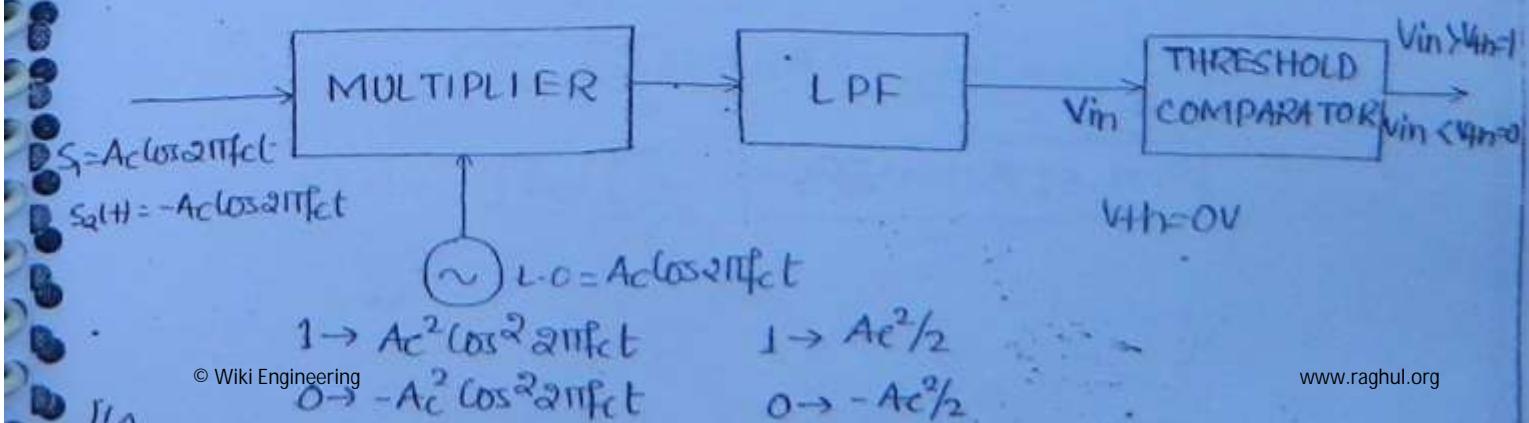
$$1 \rightarrow s_1(t) = Ac \cos 2\pi f_c t$$

$$0 \rightarrow s_2(t) = -Ac \cos 2\pi f_c t = Ac \cos \{2\pi f_c t + 180^\circ\}$$



### \* PSK RECEIVER:

\* For demodulation of PSK, SD will be used.



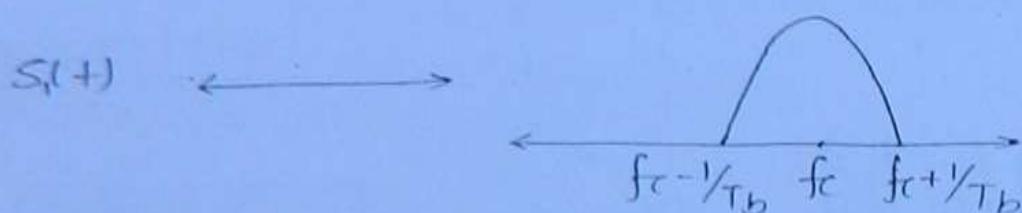
Note:

\* Demodulation of PSK is affected by QNE.

\* Bandwidth of PSK:

286

a) Transmission of 1:



b) Transmission of 0:



So,

$$\text{B.W of PSK} = (f_c + 1/T_b) - (f_c - 1/T_b)$$

$$= 2/T_b = 2R_b$$

$$\boxed{\text{B.W of PSK} = 2R_b}$$

NOTE

X-channel power requirements of ASK, PSK will be the same.

\* ENERGY PER BIT:

(169) (237)

a) Transmission of 1:

$$E_b = \int_0^{T_b} s_1^2(t) dt$$

$$= \int_0^{T_b} \{A_c \cos 2\pi f_c t\}^2 dt$$

$$\boxed{E_b = \frac{A_c^2 T_b}{2}} \Rightarrow A_c = \sqrt{\frac{2 E_b}{T_b}}$$

b) Transmission of 0:

$$E_b = \int_0^{T_b} s_2^2(t) dt$$

$$= \int_0^{T_b} (-A_c \cos 2\pi f_c t)^2 dt$$

$$\boxed{E_b = \frac{A_c^2 T_b}{2}} \Rightarrow A_c = \sqrt{\frac{2 E_b}{T_b}}$$

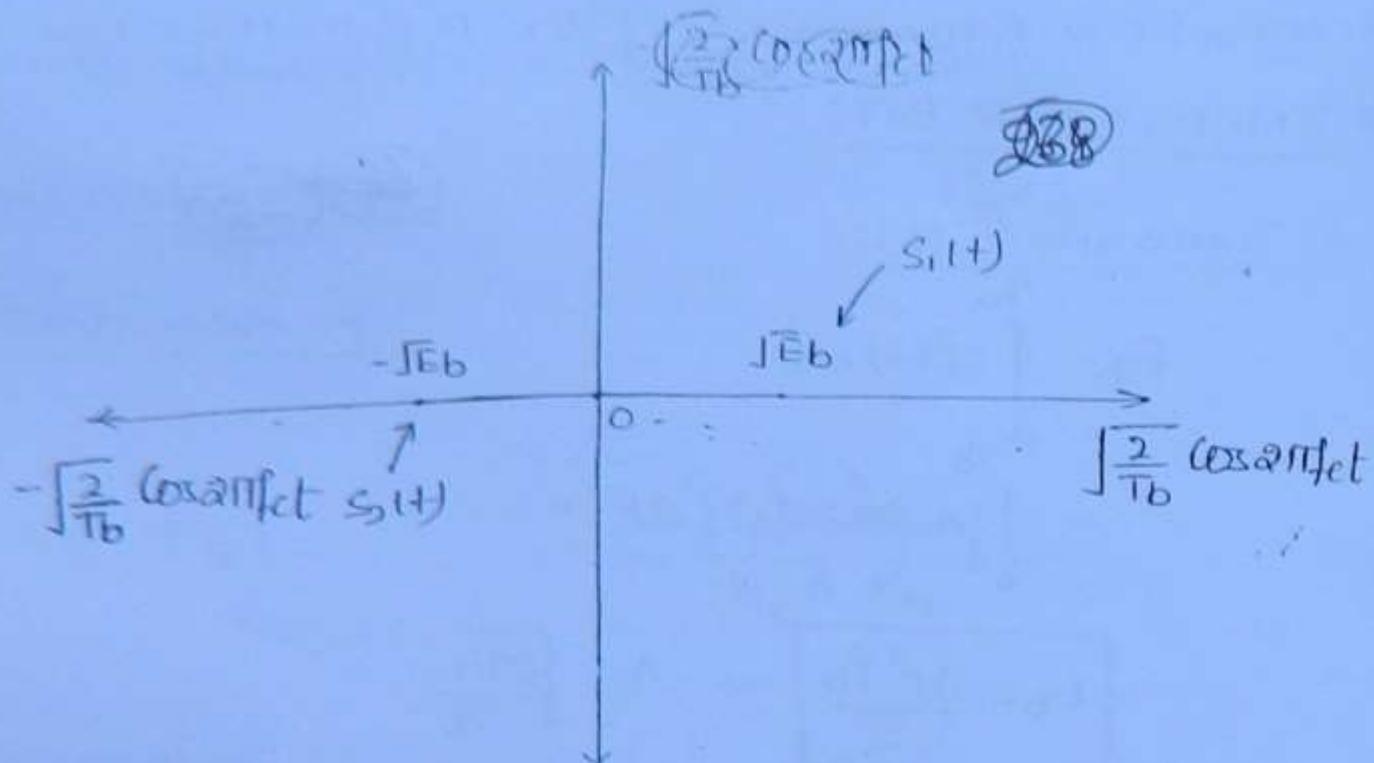
\* CONSTELLATION DIAGRAM:-

$$1 \rightarrow s_1(t) = A_c \cos 2\pi f_c t = \sqrt{2 E_b / T_b} \cos 2\pi f_c t$$

$$0 \rightarrow s_2(t) = -A_c \cos 2\pi f_c t = -\sqrt{2 E_b / T_b} \cos 2\pi f_c t$$

$$\text{So, } s_1(t) = \sqrt{E_b} \cdot \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$$

$$s_2(t) = -\sqrt{E_b} \cdot \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$$



So,

$$\text{Energy } \{S_1(t)\} = (\sqrt{E_b})^2 = E_b$$

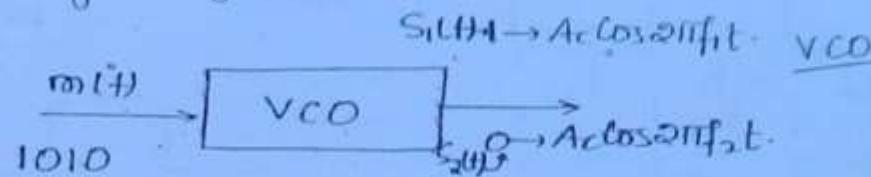
$$\text{Energy } \{S_2(t)\} = (-\sqrt{E_b})^2 = E_b$$

\* distance b/w signalling points ,  $d_{12} = 2\sqrt{E_b}$

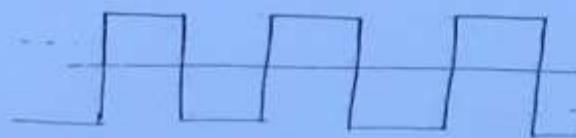
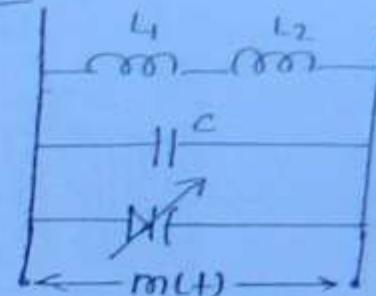
## FREQUENCY SHIFT KEYING (FSK)

In this Binary 1 is represented by high freq carrier & Binary 0 by low freq carrier.

SBG



$$\text{NRZ} \Rightarrow 1 = +\text{ve} \\ 0 = -\text{ve}$$

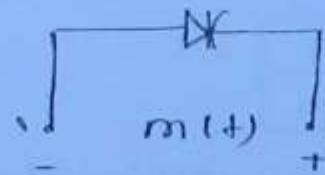


$$f_i = \frac{1}{2\pi \sqrt{L_1 + L_2}(C + C')}$$

### i) Tx of 1:

$$m(t) \text{ is } +\text{ve}$$

then



Varactor diode connected in Reverse mode.

And, as  $C' \propto 1/w$

so, in R-B width of depletion layer is high  
hence  $C'$  is less

so,  $f_i$  is high.

$$\boxed{\text{R-B} \uparrow \rightarrow C' \downarrow \rightarrow f_i \uparrow}$$

### ii) Tx of 0:

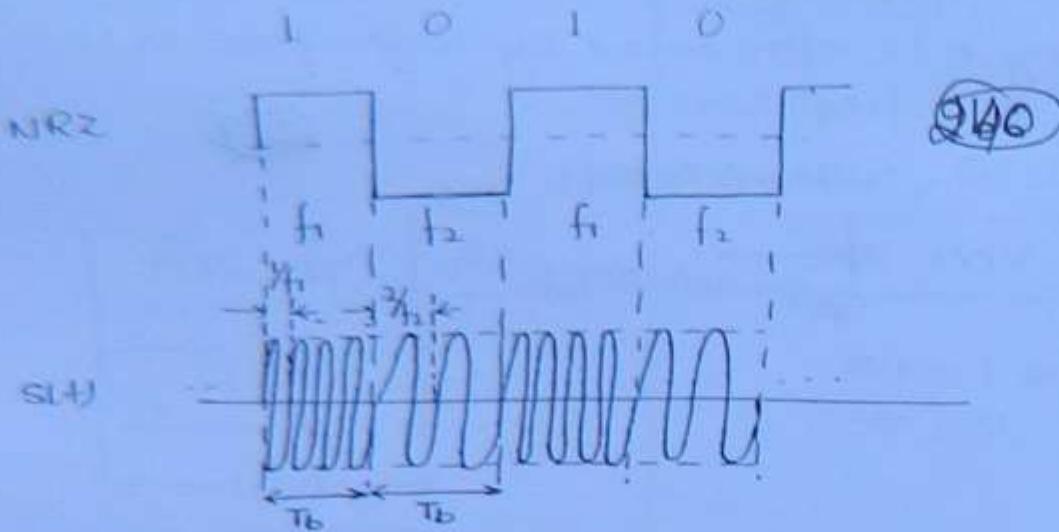
$$m(t) \text{ is } -\text{ve}$$

$$\Rightarrow \boxed{f_i \ggg f_2}$$

$$\boxed{\text{F-B} \uparrow \rightarrow C' \uparrow \rightarrow f_i \downarrow}$$

As  $f_i > f_2$ , then also both the frequencies should be in the Range of MHz.

## \* (Graphical Interpretation):



$$T_b = 1/f_1 \quad ; \quad T_b = 2/f_2$$

So, in general,

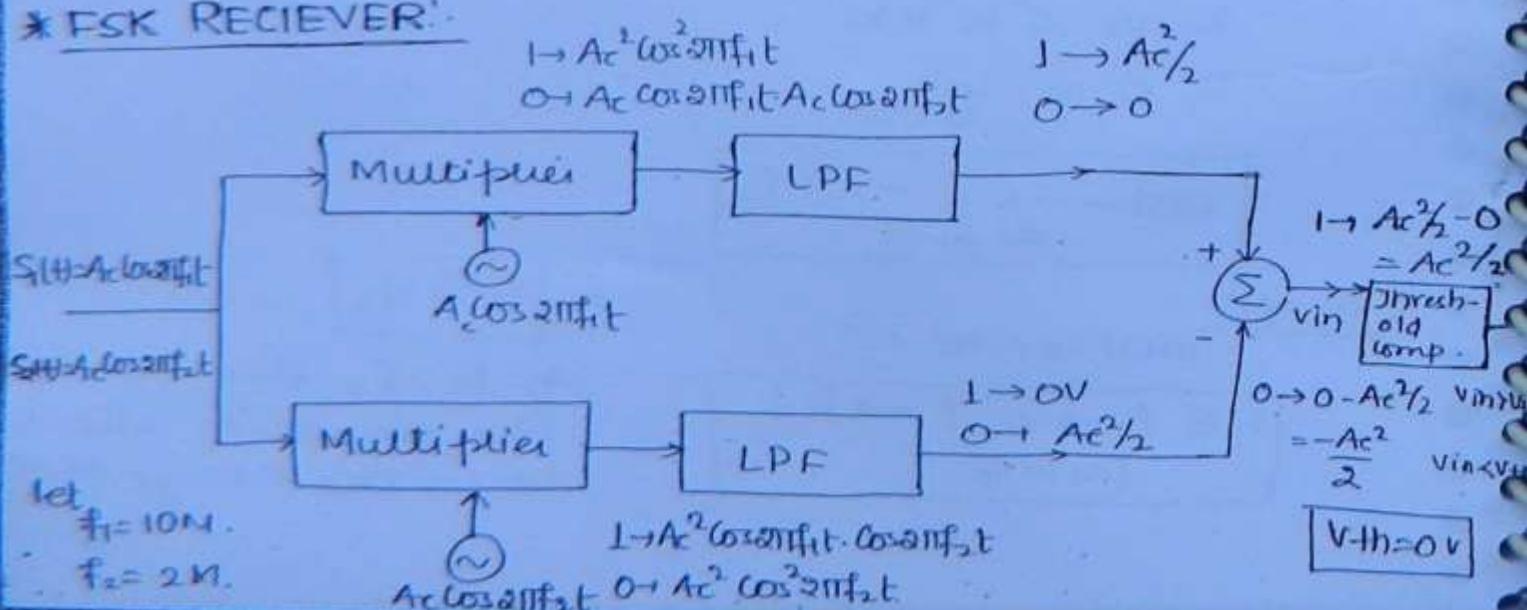
$$T_b = \frac{n_1}{f_1} \quad ; \quad T_b = \frac{n_2}{f_2}$$

$$f_1 = \frac{n_1}{T_b} \quad ; \quad f_2 = \frac{n_2}{T_b}$$

$f_1$  &  $f_2$  should be integer multiple of the Bit Rate

$$\text{i.e. } f_1 = n_1 T_b \quad ; \quad f_2 = n_2 T_b$$

## \* FSK RECEIVER:



INFORMATION

- \* The demodulation of FSK is affected by QNH.

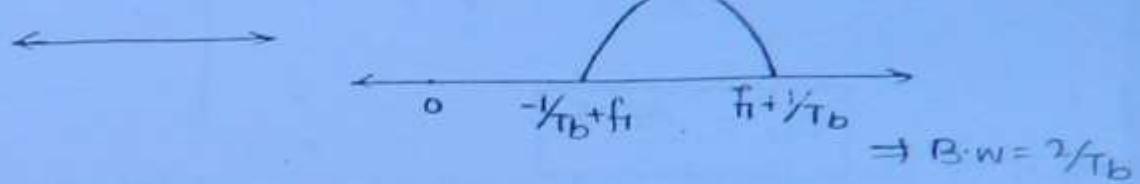
Transmission B.W.

(21/07)

1. Tx of FSK:



$s_1(t)$



2. Tx of O:



So,

the  $FSK \text{ B.W.} = (f_1 + \gamma_{Tb}) - (f_2 - \gamma_{Tb})$  {highest +ve freq - lowest +ve freq}.

$$B.W-FSK = f_1 - f_2 + 2R_b$$

Note:

\* FSK needs high Transmission B.W compared to ASK and PSK. (drawback of FSK).

## \* Energy per bit:

### \* Tx of 1:

$$\begin{aligned}
 E_b &= \int_0^{T_b} s_1^2(t) dt \\
 &= \int_0^{T_b} (A_c \cos \omega_{\text{RF}} t)^2 dt \\
 &= \int_0^{T_b} \frac{A_c^2}{2} dt + \int_0^{T_b} \frac{A_c^2}{2} \cos^2 \omega_{\text{RF}} t dt \quad \text{at } \xi: \text{ complete cycle} = \text{Area} = 0
 \end{aligned}$$

$$E_b = \frac{A_c^2 T_b}{2}$$

### \* Tx of 0:

$$\begin{aligned}
 E_b &= \int_0^{T_b} s_0^2(t) dt \\
 &= \int_0^{T_b} (A_c \cos \omega_{\text{RF}} t)^2 dt \\
 &= \int_0^{T_b} \frac{A_c^2}{2} dt + \int_0^{T_b} \frac{A_c^2}{2} \cos^2 \omega_{\text{RF}} t dt \quad \xi: \text{complete cycle} \\
 &\quad - \text{Area} = 0
 \end{aligned}$$

$$E_b = \frac{A_c^2 T_b}{2}$$

Note:-

\* Transmitter Energy Requirements of PSK and FSK will be the same.

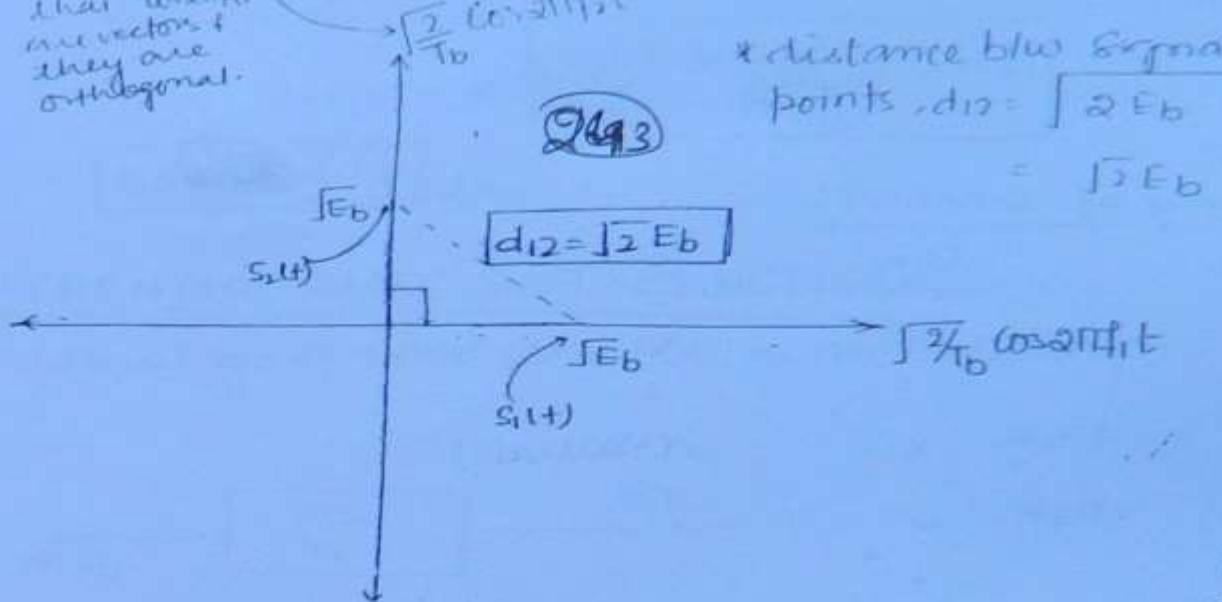
## \* CONSTELLATION DIAGRAM:

$$1 \rightarrow s_1(t) = A_c \cos \omega_{\text{RF}} t := \sqrt{\frac{2 E_b}{T_b}} \cos \omega_{\text{RF}} t$$

$$0 \rightarrow s_0(t) = A_c \cos \omega_{\text{RF}} t := \sqrt{\frac{2 E_b}{T_b}} \cos \omega_{\text{RF}} t$$

So,  $s_1(t) = \sqrt{E_b} \cdot \sqrt{\frac{2}{T_b}} \cos \omega_{\text{RF}} t$ . } in terms of Normalised  
 $s_0(t) = \sqrt{E_b} \cdot \sqrt{\frac{2}{T_b}} \cos \omega_{\text{RF}} t$ . } functions.

By now  
that cos $\omega t$  & cos $\frac{\pi}{T_b} t$   
are vectors &  
they are  
orthogonal.



\* distance b/w signalling points,  $d_{12} = \sqrt{2 E_b}$   
 $= \sqrt{2 E_b}$

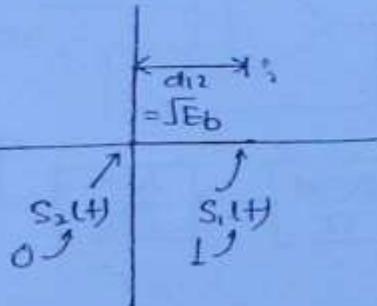
we can't locate cos $\omega t$  as all the axis corresponds to freq.

\*  $\sqrt{\frac{2}{T_b}} \cos\omega t$  and  $\sqrt{\frac{2}{T_b}} \cos\frac{\pi}{T_b} t$  are orthogonal functions in the interval  $(0, T_b)$ .

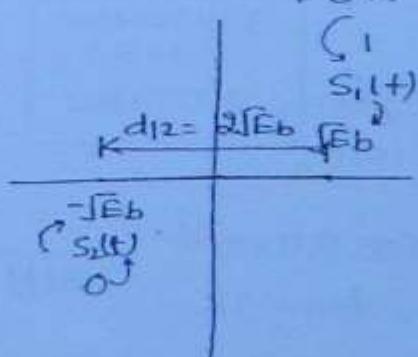
By interpreting these functions as vectors, the phase angle b/w resulting vectors will be  $90^\circ$ .

Conclusion:

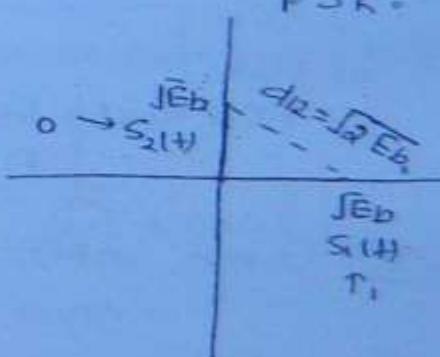
ASK



PSK



FSK



In a constellation diagram, if the dist. b/w signalling points is less; then Pe will be more, and vice versa.

Pe depends upon the dist b/w signalling pts.

So

$$\boxed{\text{Pe : ASK} > \text{FSK} > \text{PSK}}$$

## Comparison of B.W.

B.W :-  $\frac{1}{4}$  ASK < FSK  
PSK

Q16)

### Usage of Schemes:

	<u>B.W</u>	<u>Pe</u>
ASK	✓	X
FSK	X	✓(Modulate)
PSK	✓	✓

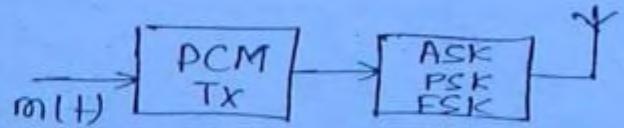
### Note:

PSK is much preferred signalling scheme compared to ASK and FSK

Q1 A msg signal of  $8\cos 8\pi \times 10^3 t$  is given to 10 bit PCM system. The resulting PCM signal is transmitted through free space, by using Band Pass modulation scheme. Find the Tx signal B.W if modulation scheme is

- a) ASK
- b) PSK
- c) PSK with  $F_H = 2\text{MHz}$ ,  
 $F_L = 1\text{MHz}$ .

Soln: Given,  $m(t) = 8\cos 8\pi \times 10^3 t$   
 $A_m = 8$ ;  $f_m = 4\text{K}$ .



$$n = 10$$

∴ Sampling Rate is not given

$$\text{So, } f_S = \text{NR} = 2f_m = 8\text{K}$$

$$\text{So, } R_b = n f_S = 10 \times 8\text{K} \\ = 80\text{ Kbps.}$$

So, For ASK,  $B.W = 2R_b = 160\text{K}$

For PSK  $B.W = 160\text{K}$

For FSK,

B.W. =  $\frac{f_2 - f_1}{f_1 + f_2}$

$$B.W. = \frac{(2-1)M}{M+160K} = 160K$$

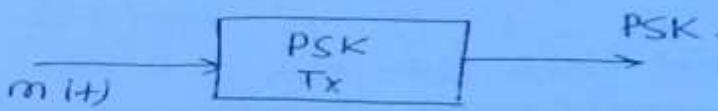
$$B.W. = 1.16M \quad \text{Ans}$$

Q75

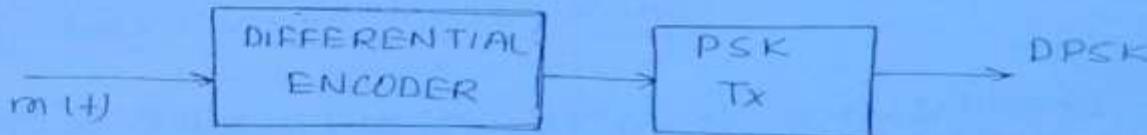
### \* DIFFERENTIAL PHASE SHIFT KEYING (DPSK):

The advantage of DPSK over PSK is no QNE.

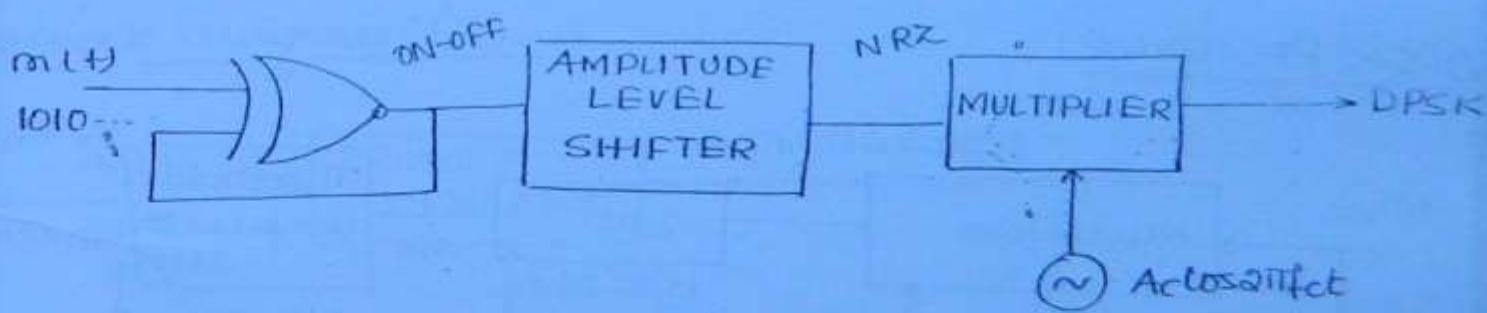
PSK:



DPSK:



\* Functional circuitry:



m(t) 1 0 1 0



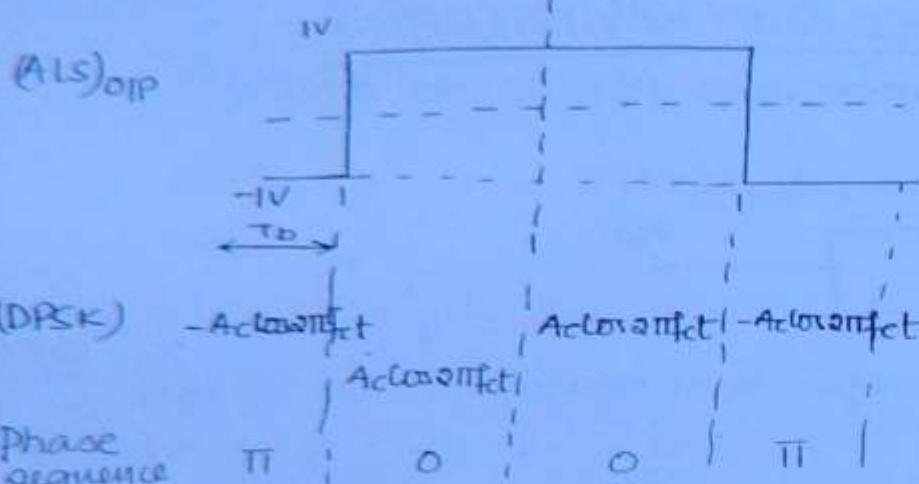
Note:

The O/P of Differential encoder is in ON-OFF form & the input of PSK Tx should be NRZ form. Hence it needs to be converted. It is done by the Amplitude level shifter.

$m(t)$       1      0      1      0



Q196

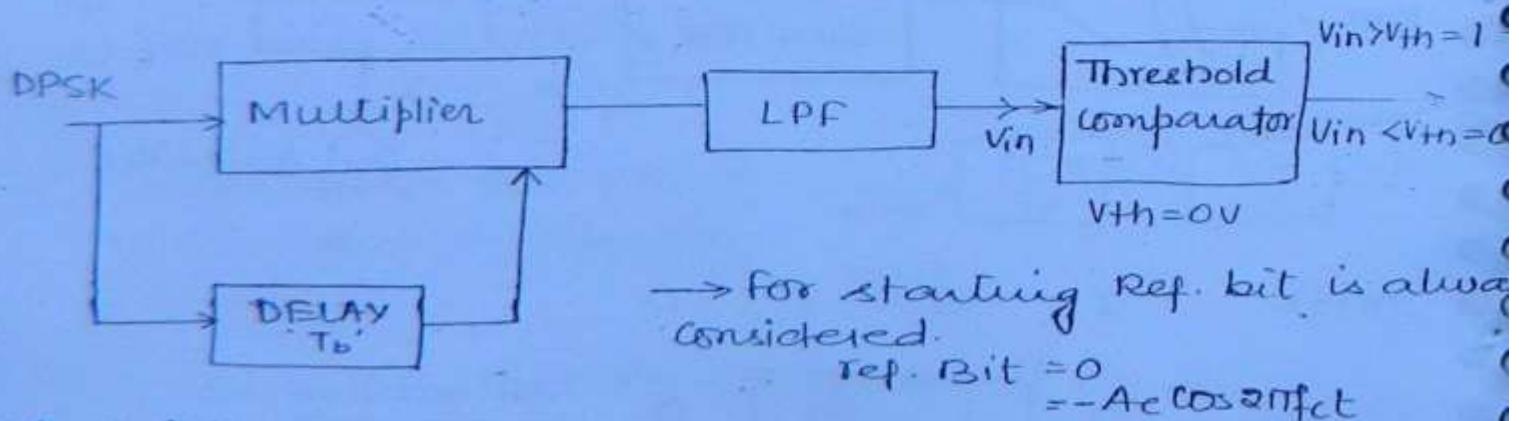


Amplitude level shifter

Note:

$m(t)$  will be given and the ref bit will also be given and the phase sequence of resulting DPSK will be asked.

### DPSK RECEIVER

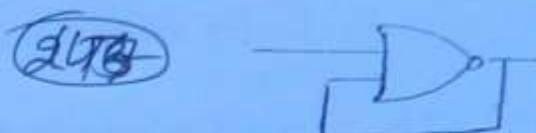


\*Analysis:

DPSK $\rightarrow$	-Ac cos 2 $\pi f_c t$	Ac cos 2 $\pi f_c t$	Ac cos 2 $\pi f_c t$	-Ac cos 2 $\pi f_c t$
$(Mul)_{O/P} \rightarrow$	$Ac^2 \cos^2 2\pi f_c t$	$-Ac^2 \cos^2 2\pi f_c t$	$Ac^2 \cos^2 2\pi f_c t$	$-Ac^2 \cos^2 2\pi f_c t$
$(LPF)_{O/P} \rightarrow$	$Ac^2/2$	$-Ac^2/2$	$Ac^2/2$	$-Ac^2/2$
Final O/P $\rightarrow$	1	0	1	0

v) A binary original sequence is transmitted, reference bit is 1. Find phase sequence of the resulting DPSK signal

- a) 01100
- b) 10111
- c) 00100
- d) 11101



$$\text{Soln: } m_1(t) = 0 \quad 1 \quad 0 \quad 0$$

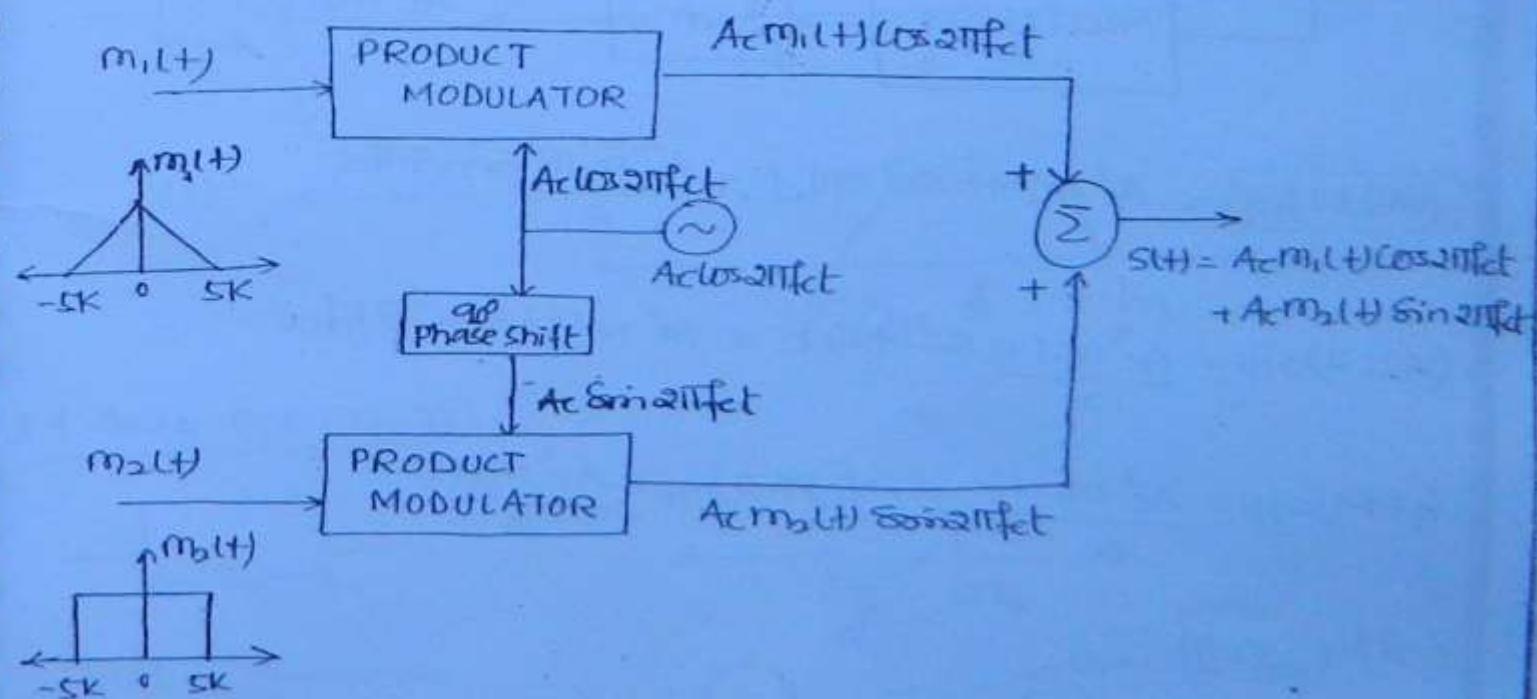
$$(\text{D}\cdot\text{E})_{\text{DPSK}} \quad 0 \quad 0 \quad 1 \quad 0 \\ \text{ref} = 1 \quad -A\cos \quad -A\cos \quad A\cos \quad A\cos$$

$\phi$ sequence	$\pi$	$\pi^0$	0	$\pi$
				Ans.

### \* Quadrature Carrier Multiplexing:

\* By using this 2 signals will be multiplexed, where the corresponding carriers will have same freq<sup>n</sup>, and having  $90^\circ$  phase shift b/w them.

### \* Block diagram (Tx):

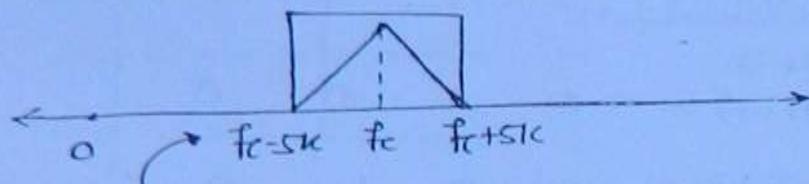


$$M_1(t) = A_c m_1(t) \cos \omega_{c1} t \longleftrightarrow \frac{A_c}{2} \left\{ m_1(f - f_0) + m_1(f + f_0) \right\}$$

$$M_2(t) = A_c m_2(t) \sin \omega_{c2} t \longleftrightarrow \frac{A_c}{2j} \left\{ m_2(f - f_0) - m_2(f + f_0) \right\}$$

So,  
s(t)  $\longleftrightarrow$

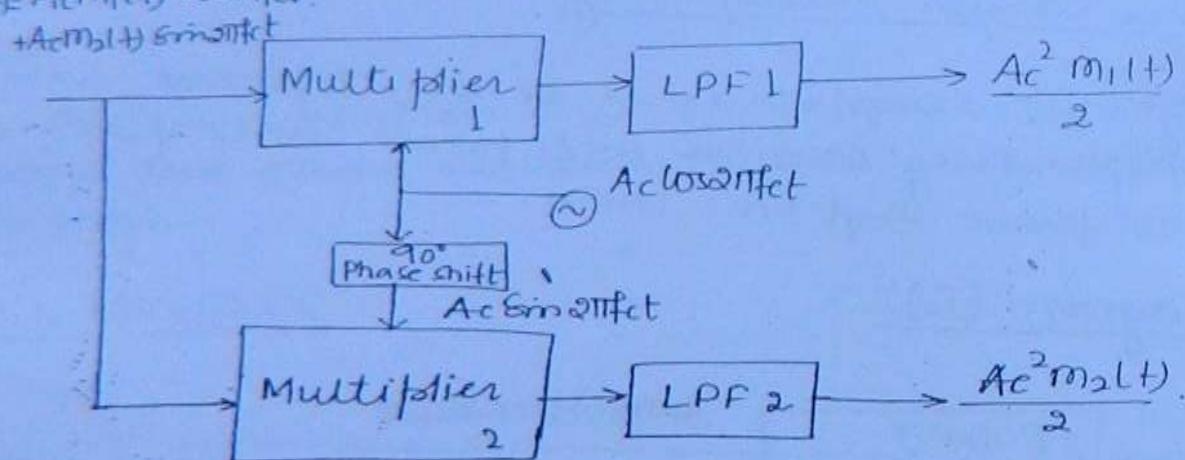
247 248



No interference since carriers are quadrature to each other.

### \*Received Block diagram:

SIN:  $A_c m_1(t) \cos \omega_{c1} t$



$$(MUL1)_{O/P} = A_c^2 m_1(t) \cos^2 \omega_{c1} t + \frac{A_c^2 m_2(t) \sin 4\pi fct}{2}$$

$$(MUL2)_{O/P} = \frac{A_c^2 m_1(t) \sin 4\pi fct}{2} + A_c^2 m_2(t) \sin^2 2\pi fct$$

$$(LPF1)_{O/P} = \frac{A_c^2 m_1(t)}{2}; (LPF2)_{O/P} = \frac{A_c^2 m_2(t)}{2}$$

## \* M-ARRAY SIGNALLING :-

\* In ASK, PSK & FSK, one bit is transmitted at a time  
ie  $N=1$

No. of symbols possible  $\Rightarrow M = 2 \Rightarrow 0, 1$

(Q9)

\* 2 No. of symbols are possible, hence ASK, PSK & FSK are called as binary signalling schemes or 2 array signalling schemes.

\* ASK  $\rightarrow$  BASK

PSK  $\rightarrow$  BPSK

FSK  $\rightarrow$  BFSK

\* For 4 Array PSK:

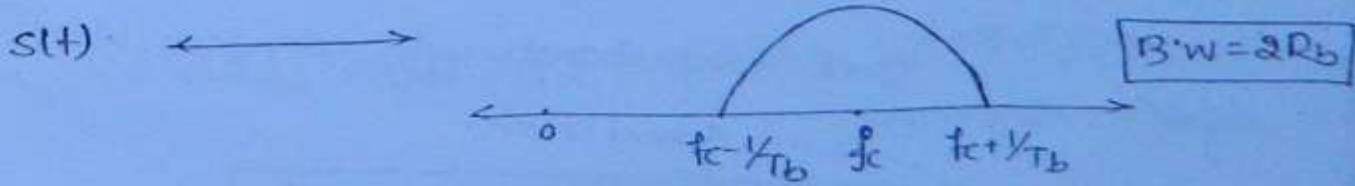
$$M = 4$$

$$N = 2 \cdot \left\{ 2^{\frac{D}{2} - 1} \right\}$$

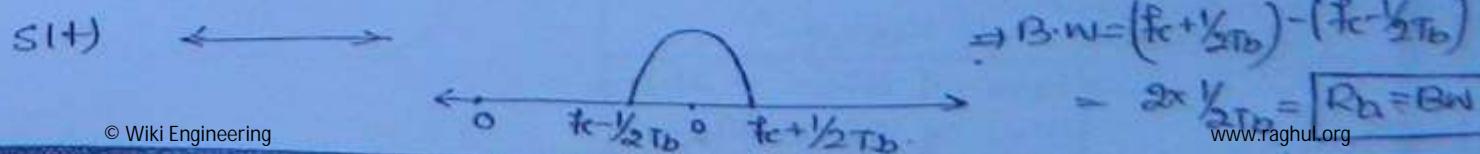
Hence 2 bits are transmitted at a time ( $N=2$ )

so, no. of symbols possible are  $M=4$

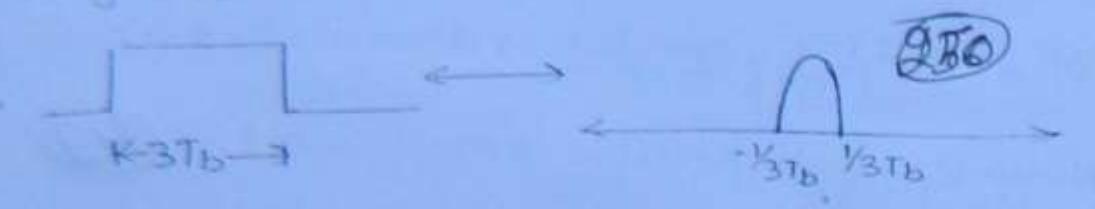
\* 2-Array PSK ( $N=1$ ):



\* 4 Array PSK ( $N=2$ ): -



### \* N Array PSK



$$B \cdot W = \frac{2}{3} T_b$$

$$\boxed{B \cdot W = 0.6 T_b}$$

#### \* Conclusion:

\* By increasing the No. of bits to be transmitted in specific time instant, then the B.W decreases.

$$\boxed{N \uparrow \rightarrow B \cdot W \downarrow}$$

\* If increasing N to very high value, the complexity of Tx & Rx increases.

\* As, the no. of bits to be transmitted in specific time instant increases, transmission B.W required will be decreases.

But Correspondingly complexity of the system increases.

$\omega_c$  = Actual carrier.

\* Phase shift in M-Array PSK =  $\phi = 2\pi/M$

a) for  $M=2 \Rightarrow \phi = \pi \Rightarrow S_1(t) = A_c \cos \omega_c t \quad \square \pi$   
 $0, S_2(t) = -A_c \cos \omega_c t \quad \square \pi$

b) for  $M=4 \Rightarrow \phi = \pi/2$   
 4-Array PSK = QPSK.  $\Rightarrow$

Quadrature  
phase shift keying

$\rightarrow$  B.W of BPSK =  $2R_b$

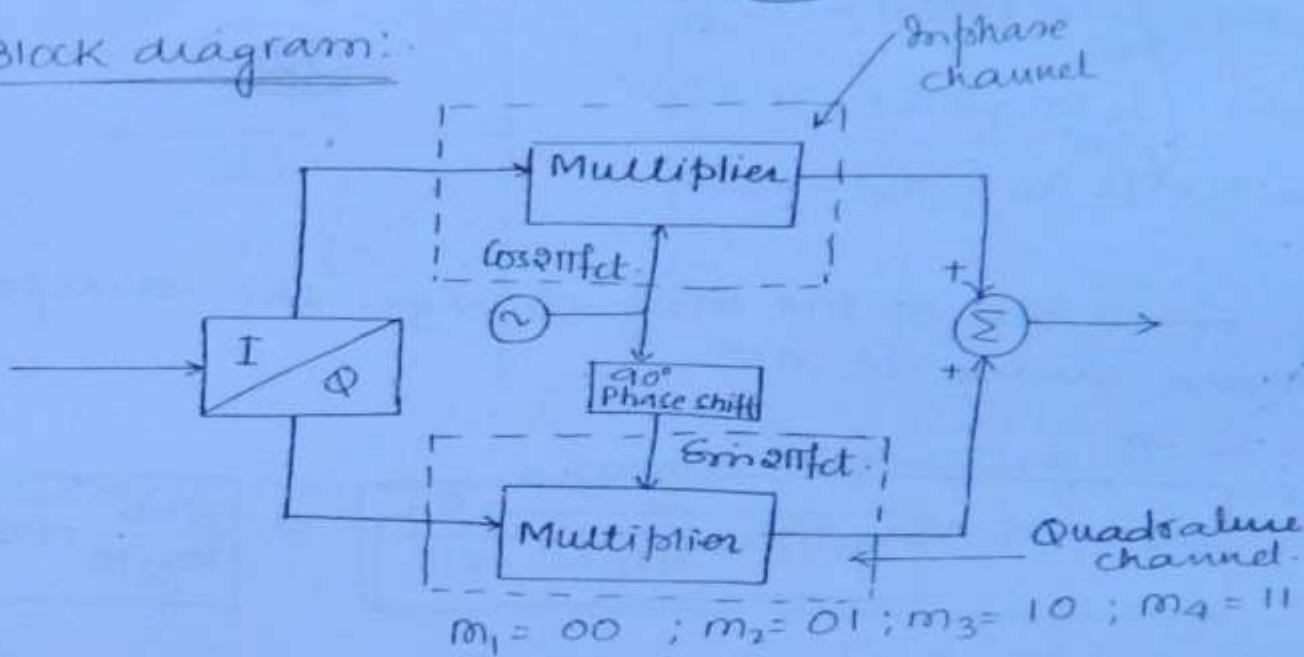
$\boxed{\text{B.W of QPSK} = R_b}$

## ~~FQPSK Transmitter~~

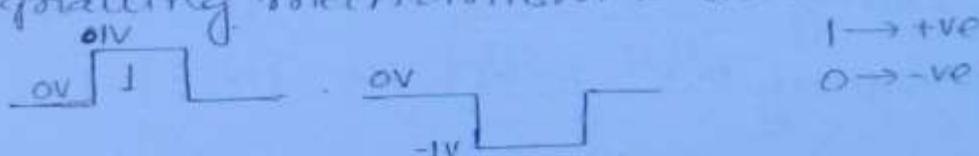
QPSK is 4 Amay PSK

251

\* Block diagram:



\* NRZ signalling mechanism is used



$$00 \rightarrow s_1(t) = -\cos 2\pi f t - \sin 2\pi f t$$

$$01 \rightarrow s_2(t) = -\cos 2\pi f t + \sin 2\pi f t$$

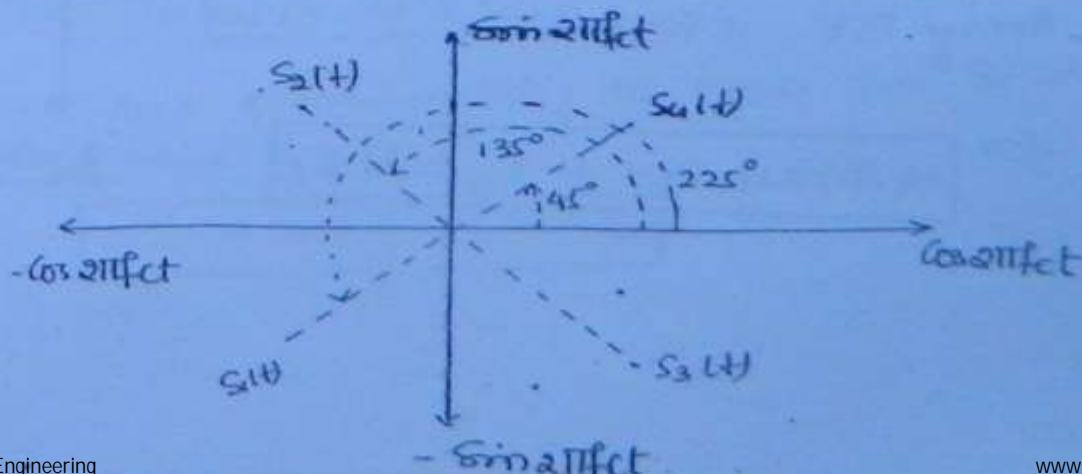
$$10 \rightarrow s_3(t) = \cos 2\pi f t - \sin 2\pi f t$$

$$11 \rightarrow s_4(t) = \cos 2\pi f t + \sin 2\pi f t$$

Now, as

$$A \cos 2\pi f t + B \sin 2\pi f t = \sqrt{A^2 + B^2} \cos \{2\pi f t + \Phi\}$$

$$\Phi = \tan^{-1}(B/A)$$



$$\begin{aligned}
 S_1(t) &= \sqrt{2} \cos \{2\pi fct + 0^\circ\} \\
 S_2(t) &= \sqrt{2} \cos \{2\pi fct + 90^\circ\} \\
 S_3(t) &= \sqrt{2} \cos \{2\pi fct - 45^\circ\} \\
 S_4(t) &= \sqrt{2} \cos \{2\pi fct - 90^\circ\}
 \end{aligned}$$

258

- \* As single bit of the msg changes the corresponding phase change is  $90^\circ$ .

Note:

$$\text{B.W of M-Array PSK} = \frac{2}{NT_b} = \frac{2R_b}{N} \Rightarrow \frac{2R_b}{\log_2 M} = \text{BW}$$

1. For 2 Array PSK, BW =  $2R_b \cdot \{N=1\}$

2. For 4 Array PSK, BW =  $R_b \cdot \{N=2\}$

Only Es  
No Gels  
No PSW

As.  $M = 2^N$

\* CONSTELLATION DIAGRAM:

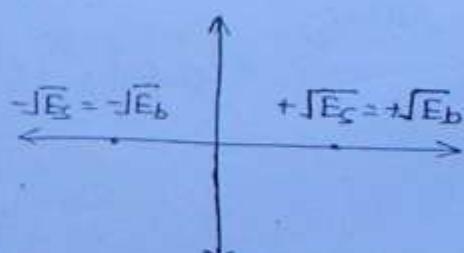
\* For M Array PSK; distance of each of the signalling pt from the origin =  $\sqrt{E_s}$ .  $\leftarrow$  Symbol energy

\* For 2 Array PSK  $\Rightarrow E_s = E_b$

\* For 4 Array PSK  $\Rightarrow E_s = 2E_b$

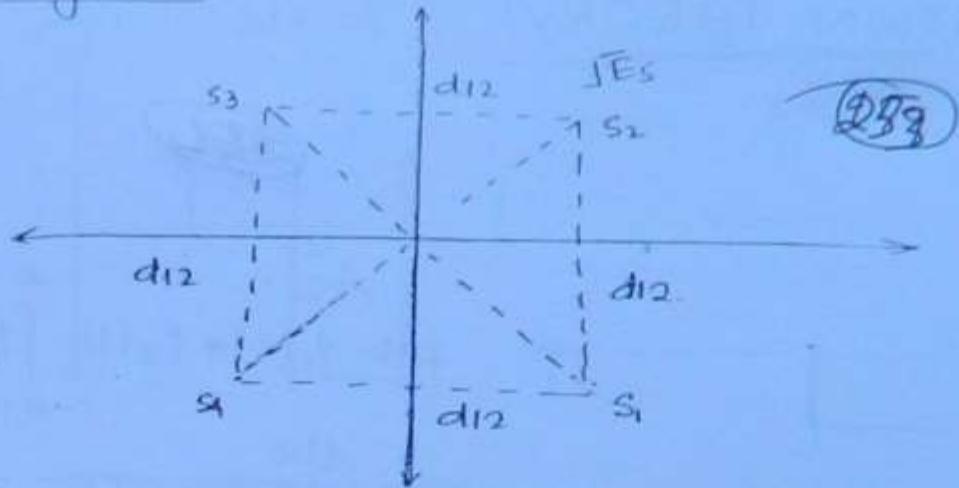
\* So,

for  $M$  Array PSK  $\Rightarrow E_s = M E_b$



"2-Array PSK"

#### 4 Array PSK:



\* distance b/w two adjacent signalling pts is

$$d_{12} = 2\sqrt{E_s} \sin \frac{\pi}{M}$$

\* For 2 Array PSK ;  $d_{12} = 2\sqrt{E_b} \cdot \sin \frac{\pi}{2}$

$$d_{12} = 2\sqrt{E_b}$$

\* For 4 Array PSK ;  $d_{12} = 2\sqrt{E_s} \sin \frac{\pi}{M}$

$$= 2\sqrt{2E_b} \cdot \sin \frac{\pi}{4}$$

$$= 2\sqrt{2E_b} \cdot \frac{1}{\sqrt{2}}$$

$$d_{12} = 2\sqrt{E_b}$$

#### Conclusion:

\* Since, the distance of adjacent signalling pts is the same, ie  $d_{12} = 2\sqrt{E_b}$

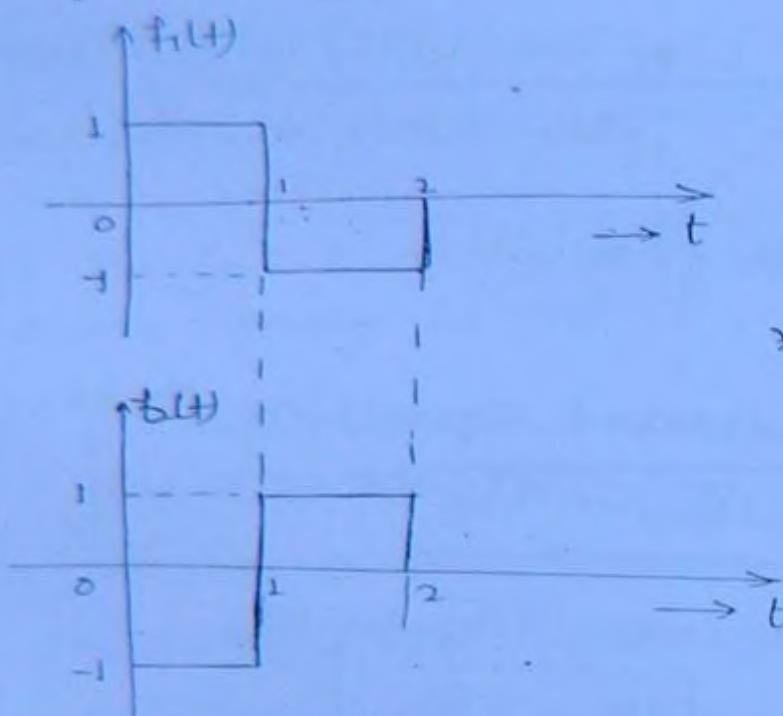
Hence, the probability of error for BPSK and QPSK are same

$$P_e(\text{BPSK}) = P_e(\text{QPSK})$$

# \* INFORMATION THEORY

\* Analysis:

(25)



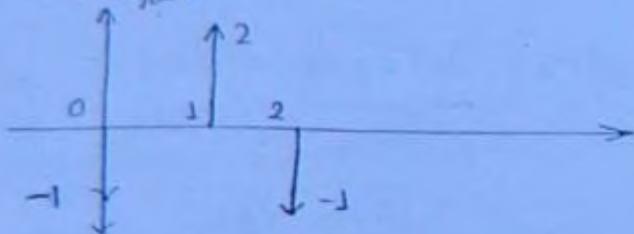
$$\text{As, } f(t) = f_1(t) \times f_2(t) = \int_{-\infty}^{\infty} f_1(\tau) \times f_2(t-\tau) d\tau$$

Also,

$$\frac{d}{dt} \{f_1(t) \times f_2(t)\} = f_1(t) \times \frac{d}{dt} f_2(t)$$

$$\text{So, } f_1(t) \times f_2(t) = \int_{-\infty}^t f_1(\tau) \times \frac{d}{dt} f_2(t-\tau) d\tau$$

Now,  $\frac{d}{dt} f_2(t)$



Note:

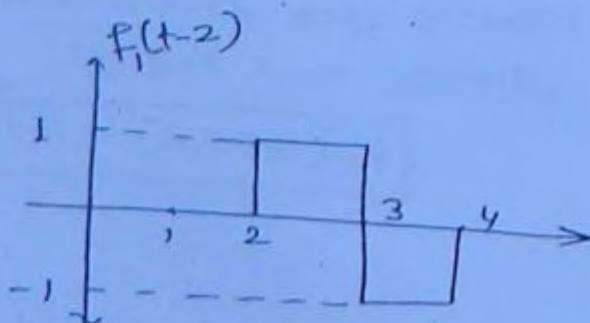
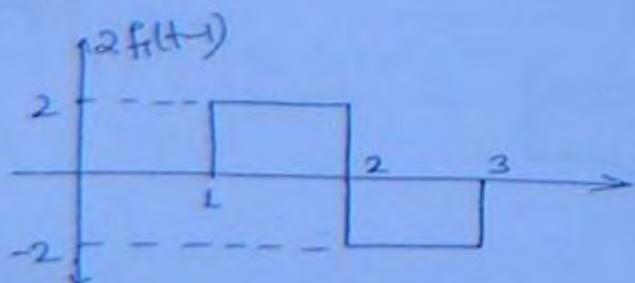
$$\frac{d}{dt} u(t) = \begin{cases} \infty, & t=0 \\ 0, & t \neq 0 \end{cases}$$

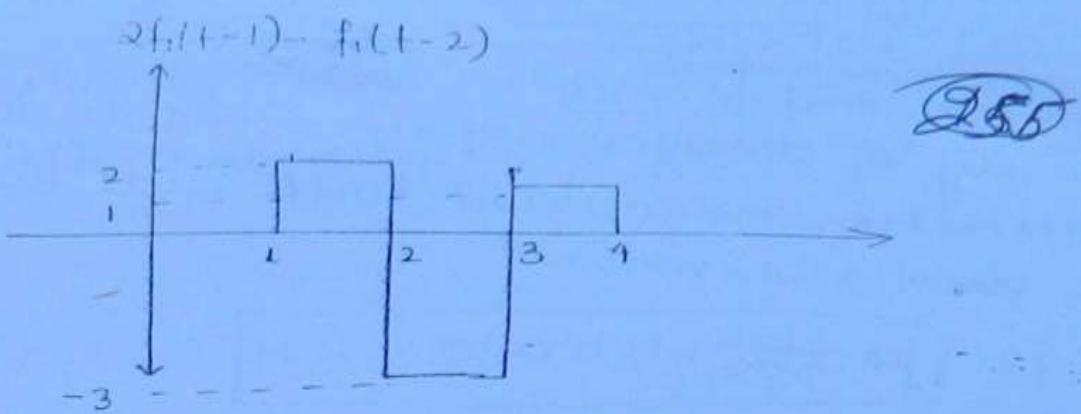
$$\text{So, } \frac{d}{dt} u(t) = -\delta(t) + 2\delta(t-1) - \delta(t-2)$$

$$\text{So, } f_1(t) \times \frac{d}{dt} \{f_2(t)\} = f_1(t) \times \{-\delta(t) + 2\delta(t-1) - \delta(t-2)\}$$

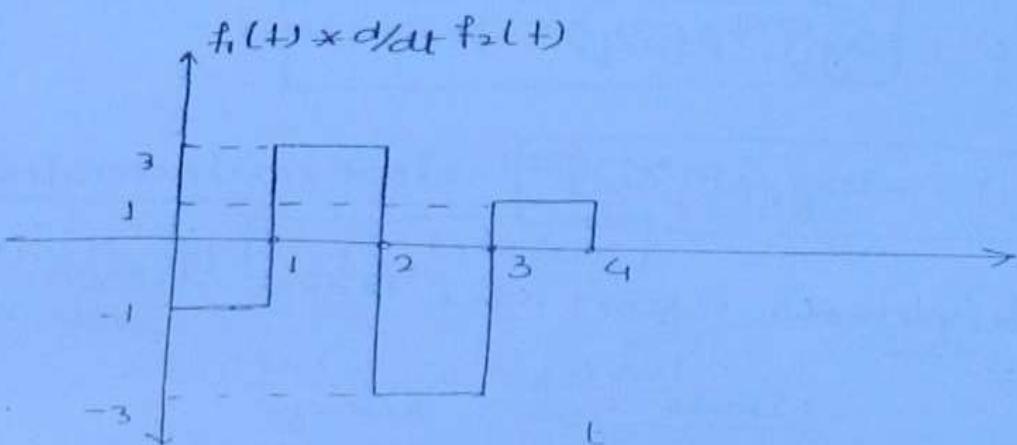
$$f_1(t) \times \frac{d}{dt} f_2(t) = -f_1(t) + 2f_1(t-1) - f_1(t-2)$$

Now,





Q50

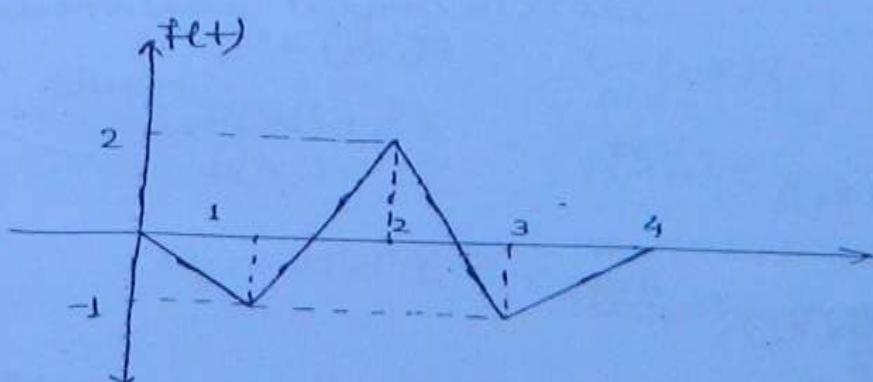


Now,  $f(t) = \int_{-\infty}^t f_1(t) \times d/dt f_2(t) dt = \text{Area of the fun}$

i) At  $t=0.1 \Rightarrow \text{Area} = 0.1$

At  $t=0.2 \Rightarrow \text{Area} = -0.2$

$t=1 \Rightarrow \text{Area} = -1$ .



Note:

$$\begin{cases} f(t) = f_1 \otimes f_2 \\ A_1 \cdot A_2 = A_1 * A_2 \leftarrow \text{Areas.} \end{cases}$$

## \* INFORMATION THEORY

256

- \* Information means importance.
- \* If the probability of occurrence of an event is less then the information associated with that event will be more and vice-versa.

$$I(x_i) \propto \frac{1}{P(x_i)}$$

$$I(x_i) = \log_b (1/P(x_i))$$

or  $I(x_i) = -\log_b P(x_i)$

- \* Units of  $I(x_i)$  depends upon the base chosen  
ie

b	Units
2	bit
e	nat
10	decit

- Q1. A source is generating 3 possible symbols with probabilities of  $\frac{1}{4}$ ,  $\frac{1}{2}$ ,  $\frac{1}{4}$ . Find the information associated with each of the symbol.

Soln:- Given,  $P(x_1) = \frac{1}{4}$ ;  $P(x_2) = \frac{1}{2}$ ;  $P(x_3) = \frac{1}{4}$

$$I(x_1) = \log_2 \left( \frac{1}{P(x_1)} \right) = \log_2 4 = +2 \text{ bits}$$

$$I(x_2) = \log_2 \left( \frac{1}{P(x_2)} \right) = \log_2 2 = 1 \text{ bit}$$

$$I(x_3) = \log_2 \left( \frac{1}{P(x_3)} \right) = \log_2 4 = 2 \text{ bits.}$$

Note:-

The prob. of occurrence of  $x_2$  is high so that information associated with  $x_2$  will be less.

## Average Information or Entropy :-

\* Units of  $H$  is bits/symbol

\* Mathematically, it is given as:-

$$H = \sum_i P(x_i) \cdot P(x_i)$$

$$H = \sum_i P(x_i) \log_2 \frac{1}{P(x_i)}$$

$$H = -\sum_i P(x_i) \log_2 P(x_i)$$

## Information Rate (R) :-

\* Units of  $R$  is bits/sec

\* Now,

$$R = \frac{\text{bits}}{\text{Symbol}} \times \frac{\text{Symbol}}{\text{sec}}$$

$$\text{Efficiency} = \gamma = \frac{H}{L}$$

avg codeword length = L

$$R \leq L \leq R+1$$

So,  $R = H \times \gamma \Rightarrow \text{Information} = \frac{\text{Symbol}}{\text{Rate}} \times \frac{\text{Entropy}}{\text{Rate}}$

Q. A source is generating 4 possible symbols with the probabilities of  $\frac{1}{8}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}$ .

Find Entropy and Information Rate if the source is generating 1 symbol/msec.

Soln: Given,

$$P(x_1) = \frac{1}{8} ; P(x_3) = \frac{1}{4}$$

$$P(x_2) = \frac{1}{8} ; P(x_4) = \frac{1}{2}$$

$$\text{Symbol Rate} = 1 \text{symbol/msec}$$

$$\gamma = 1000 \text{ symbol/sec}$$

Now,

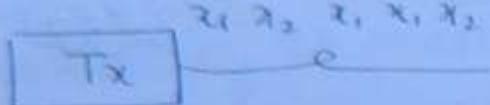
$$H = \sum_{i=1}^4 P(x_i) \log_2 \frac{1}{P(x_i)} = \frac{1}{8} \log_2 8 + \frac{1}{8} \log_2 8 + \frac{1}{4} \log_2 4 + \frac{1}{2} \log_2 2$$

$$H = 1.75 \text{ bits/symbol}$$

$$\text{So, } R = 1.75 \times 1000 = 1.75 \text{ Kbps}$$

\* Entropy is measure of uncertainty

x Analysis:



250

Case 1:

$$P(x_1) = P(x_2) = \frac{1}{2}$$

$$H = \sum_{i=1}^2 P(x_i) \log_2 \frac{1}{P(x_i)}$$

$$= \frac{1}{2} \log_2 2 + \frac{1}{2} \log_2 2$$

$$H = 1 \text{ bits/symbol} = H_{\max}$$

when the prob are equal.

Case 2:

$$P(x_1) = 1; P(x_2) = 0$$

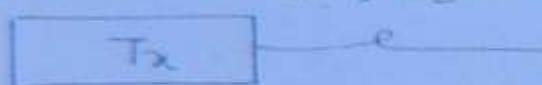
$$H = -\sum_{i=1}^2 P(x_i) \log_2 P(x_i)$$

$$= 0 + 0$$

$$H = 0 \text{ bits/symbol} = H_{\min}$$

when the prob. of one is 1 & other 0.

2.



Case 1:

$$P(x_1) = P(x_2) = P(x_3) = \frac{1}{3}$$

$$H = \sum_{i=1}^3 P(x_i) \log_2 \frac{1}{P(x_i)}$$

$$H_{\max} = \frac{1}{3} \log_2 3 + \frac{1}{3} \log_2 3 + \frac{1}{3} \log_2 3$$

$$H_{\max} = \log_2 3 \text{ bits/symbol}$$

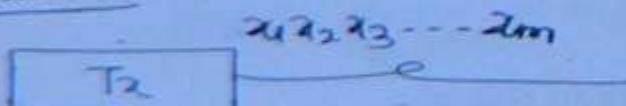
Case 2:

$$P(x_1) = 1; P(x_2) = 0 = P(x_3)$$

$$H_{\min} = \sum_{i=1}^3 P(x_i) \log_2 \frac{1}{P(x_i)}$$

$$H_{\min} = 0 \text{ bits/symbol}$$

Conclusion:



$$P(x_1) = P(x_2) = P(x_3) = \dots = P(x_m) = \frac{1}{M}$$

$$H_{\max} = \log_2 M \text{ bits/symbol}$$

$$; H_{\min} = 0 \text{ bits/symbol}$$

\*~~X10~~ if all the symbols are having equal prob. of occurrence then Entropy will be maxm.

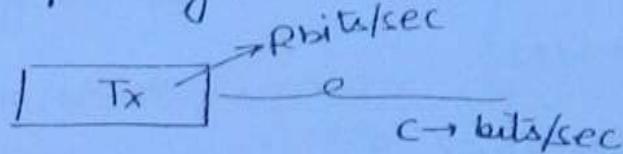
### \* CHANNEL CAPACITY:

(185)

(259)

\* It specifies the no of bits allowed by the channel in 1 sec

\* Channel capacity;  $c = \text{bits/sec}$



Hence,

$$C > R \leftarrow \text{No information loss}$$

### \* SHANON - HARTLEY LAW:

It gives the Relation b/w channel capacity (C) and its Bandwidth (B-W)

Mathematically,

$$C = B \log_2 (1 + S/N)$$

Normal S/N (not in dB).

$$(S/N)_{dB} = 10 \log_{10} (S/N)$$

where,

C = Channel capacity (bits/sec)

B = Channel B-W (Hz).

S = Signal power expected at channel o/p.

N = Noise power.

$(S/N)_{dB}$

$(S/N)$

1) 10 dB

10

2) 20 dB

100

3) 15 dB

$10^{1.5}$

Q) For a channel of  $B \cdot W = 4 \text{ KHz}$   
 $(S/N) = 15 \text{ dB}$

Find the channel capacity

280

Soln: As  $(S/N) = 15 \text{ dB}$

$$\text{so, } (S/N) = 10^{1.5} = 31.6$$

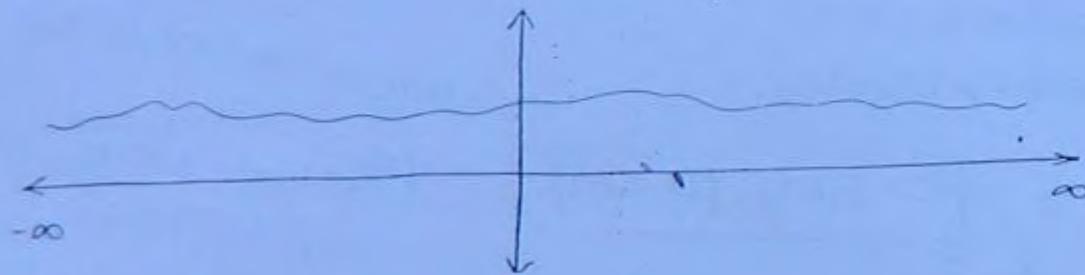
$$\text{so, } C = B \log_2 \{1 + S/N\}$$

$$= 4 \log_2 \{1 + 31.6\}$$

$$C = 20.1 \text{ Kbps. Ans}$$

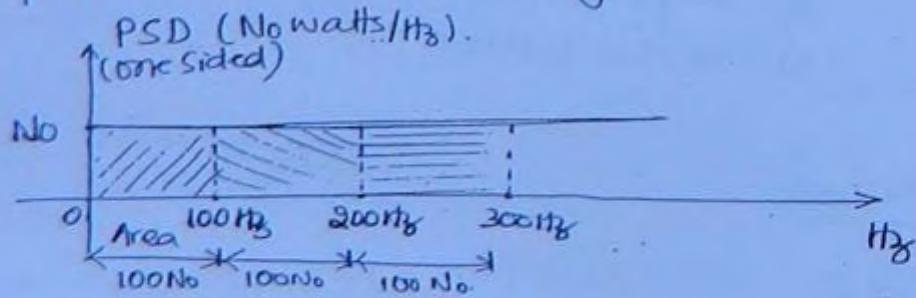
\* Capacity of AWGN (additive white Gaussian Noise) channel :-

\* White Noise has the frequency spectrum as following



It covers the all frequency component

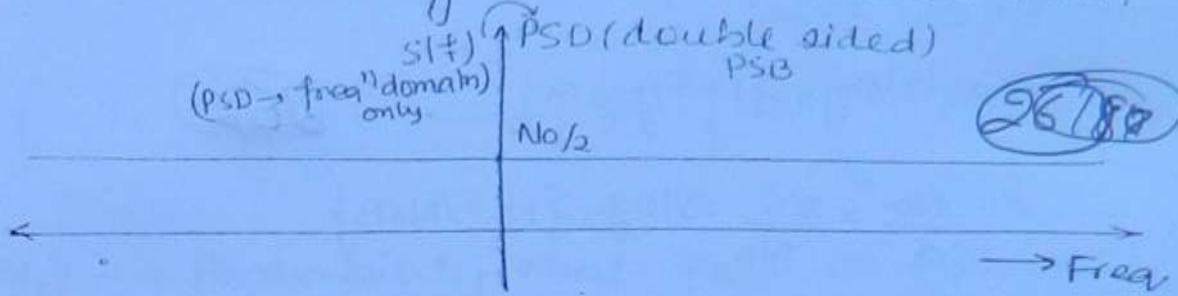
\* The PSD of the white noise is given as:-



\* Freq'n bw 0 to 100Hz will be affected by 100No watt of power

\* Freq'n bw 100 to 200Hz will be affected by 100No watt of power

\* If the -ve frequency is also considered then;



\* Regarding white Noise, its power is given as:

$$N(\text{watts}) = \frac{\text{watts}}{\text{Hz}} \times \text{Hz}$$

$$\boxed{N = N_0 \times B} \text{ watts}$$

\* Default power spectral density is one sided PSD.

Note:

\* Each of the frequency component transmitted through a channel is affected by same amount of white Noise power.

\* The channel BW is given as:

$$\uparrow C = B \log_2 \{ 1 + S/N \}.$$

(linear)

so, for a AWGN channel,

$$\uparrow C = B \log_2 \{ 1 + S/N_0 B \}.$$

(non linear)

Conclusion:

\* For AWGN channel as  $B \rightarrow \infty$ .  
channel capacity becomes.

$$\boxed{C_{\infty} = 1.44 S/N_0}$$

Proof:

As we know that :

$$C = B \log_2 \{ 1 + S/N_0 B \}$$

26B

$$C = \frac{N_0}{S} \times \frac{BS}{N_0} B \log_2 \{ 1 + S/N_0 B \}$$

$$\text{Let } \frac{N_0 B}{S} = \infty x$$

as  $B \rightarrow \infty \Rightarrow x \rightarrow \infty$ .

So,

$$C_{\infty} = \frac{S}{N_0} \underset{x \rightarrow \infty}{\lim} x \log_2 (1 + \frac{1}{x})$$

$$C_{\infty} = \frac{S}{N_0} \log_e 2$$

~~V. imp  
XXX~~

$$C_{\infty} = 1.44 S/N_0$$

Q. For AWGN of having BW 4 KHz, two sided Noise PSD is given by :-  $10^{-12}$  watt/Hz. Find the channel capacity required to get signal power of 0.1 mw at the O/P of the channel.

Soln:- Given, BW = 4 KHz

$$\frac{N_0}{2} = 10^{-12} \Rightarrow N_0 = 2 \times 10^{-12}$$

$$\begin{aligned} \text{So, } N_0 B &= 2 \times 10^{-12} \times 4 \times 10^3 \\ &= 8 \times 10^{-9} \text{ watt.} \end{aligned}$$

Now,

$$C = B \log_2 \{ 1 + S/N_0 B \}$$

$$= 4 \times 10^3 \log_2 \left\{ 1 + \frac{0.1 \times 10^{-3}}{8 \times 10^{-9}} \right\}$$

$$C = 54.44 \text{ Kbps}$$

Ans

$$C = 54.44 \text{ Kbps.}$$

Ans

# \* CHANNEL TRANSITION MATRIX OR CHANNEL MATRIX



$$\begin{matrix} x_1 & 0 \\ x_2 & 1 \end{matrix} \left| \begin{matrix} x & 0 \\ y & 1 \end{matrix} \right| \begin{matrix} 0 & y_1 \\ 1 & y_2 \end{matrix}$$

$P(0/x_i) \rightarrow$  Probability that  $x_i$  is Rx as 0.

$P(1/x_i) \rightarrow$  Prob. that  $x_i$  is Rx as 1.



$$\text{So, } [P(Y/x)] = \begin{bmatrix} P(Y_1/x_1) & P(Y_2/x_1) & \dots & P(Y_n/x_1) \\ P(Y_1/x_2) & P(Y_2/x_2) & \dots & P(Y_n/x_2) \\ \vdots & \vdots & \ddots & \vdots \\ P(Y_1/x_m) & P(Y_2/x_m) & \dots & P(Y_n/x_m) \end{bmatrix}_{m \times n}$$

$$\text{So, } \sum_{j=1}^n P(Y_j/x_i) = 1 \quad : \text{for any value of } i$$

$$P(Y_1/x_1) + P(Y_2/x_1) + \dots + P(Y_n/x_1) = 1$$

\* Sum of the elements in each Row of channel transition matrix will be equal to 1.

$$\times \quad P[x] = [P(x_1) \quad P(x_2) \quad \dots \quad P(x_m)]_{1 \times m}$$

Transmit  
matrix

$$\begin{bmatrix} P(y) \end{bmatrix}, \begin{bmatrix} P(y_1) & P(y_2) & \dots & P(y_n) \end{bmatrix}_{1 \times n}$$

output matrix

So,

$$\begin{bmatrix} P(y) \end{bmatrix}_{1 \times n} = \begin{bmatrix} P(x) \end{bmatrix}_{1 \times m} \begin{bmatrix} P(y/x) \end{bmatrix}_{m \times n}$$
Ques

\* Channel Matrix =  $\begin{bmatrix} P(x,y) \end{bmatrix} = \begin{bmatrix} P(x_1, y_1) & P(x_1, y_2) & \dots & P(x_1, y_n) \\ P(x_2, y_1) & P(x_2, y_2) & \dots & P(x_2, y_n) \\ \vdots & \vdots & \ddots & \vdots \\ P(x_m, y_1) & P(x_m, y_2) & \dots & P(x_m, y_n) \end{bmatrix}_{m \times n}$

Note:

$P(x_i, y_j)$  = Probability that when  $x_i$  is generated, and to be received as  $y_j$ .

Now,  $\begin{bmatrix} P(x,y) \end{bmatrix}_{m \times n} = \begin{bmatrix} P(x) \end{bmatrix}_{m \times m} \text{diagonal} \times \begin{bmatrix} P(y/x) \end{bmatrix}_{m \times n}$

;  $\begin{bmatrix} P(x) \end{bmatrix}_{m \times m} \text{diagonal} = \begin{bmatrix} P(x_1) & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & P(x_2) & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & P(x_3) & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \dots & \dots & P(x_m) \end{bmatrix}_{m \times m}$

\* Binary Symmetric Channel:

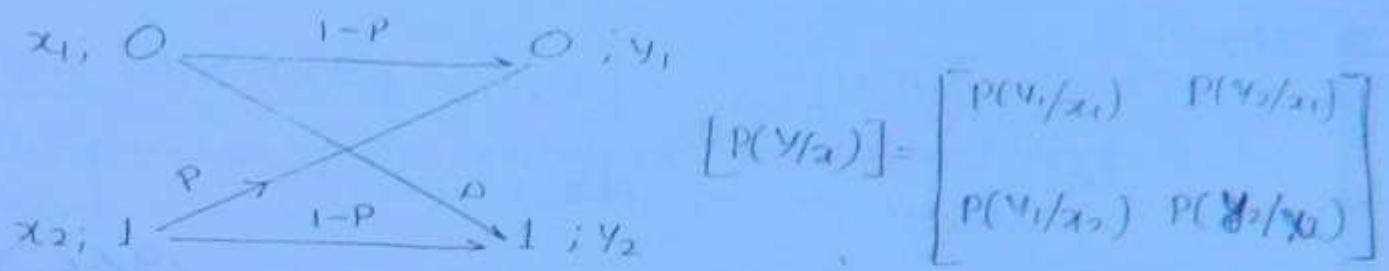
\* Tx generating 0 & 1

\* Rx receiving 0 & 1

\* So, for Binary Symmetric channel.

$$P(y_0) = P(0/1)$$

$$P_{e0} = P_{e1}$$



$$[P(Y/x)] = \begin{bmatrix} P(Y_1/x_1) & P(Y_2/x_1) \\ P(Y_1/x_2) & P(Y_2/x_2) \end{bmatrix}$$

So  $[P(Y/x)] = \begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}$

(265)

### \*CONDITIONAL ENTROPY:

\* It specifies uncertainty about Receiver w.r.t Transmitter.

\* Mathematically,

$$H(Y/x) = - \sum_{i=1}^m \sum_{j=1}^n P(x_i, y_j) \log_2 P(y_j/x_i)$$

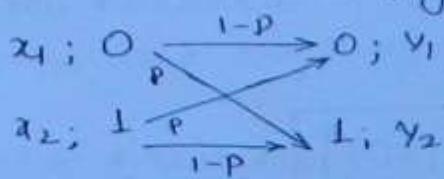
Let,

1)  $P(Y_1/x_1) = 0.9$ ;  $P(Y_2/x_1) = 0.1$ ;  $P(Y_1/x_2) = 1$ ;  $P(Y_2/x_2) = 0$ .  
~~H(Y/x) less~~  $\leftarrow$  Since  $P(Y/x)$  is high.

2)  $P(Y_1/x_1) = 0.4$ ;  $P(Y_2/x_1) = 0.6$ ;  $P(Y_1/x_2) = 0.5$ ;  $P(Y_2/x_2) = 0.5$ .  
~~H(Y/x) high~~  $\leftarrow$  Since  $P(Y/x)$  is low.

$\Rightarrow H(Y/x)$  = height of uncertainty for Rx w.r.t Tx.

Q. Find conditional entropy for Binary Symmetric channel.



Sol<sup>n</sup>:

$$\text{So, } H(Y/x) = - \sum_{i=1}^2 \sum_{j=1}^2 P(x_i, y_j) \log_2 P(y_j/x_i).$$

Now,  $P(Y/x) = \begin{bmatrix} P(Y_1/x_1) & P(Y_2/x_1) \\ P(Y_1/x_2) & P(Y_2/x_2) \end{bmatrix} = \begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}.$

NOW,

$$P(x,y) = [P(x)]_{\text{diag}} \cdot [P(y/x)]$$

$$\text{let, } P(x_1) = \alpha ; \quad P(x_2) = 1 - \alpha$$

$$\text{so, } [P(x)] = [\alpha \quad 1-\alpha]$$

(Ques)

$$[P(x)]_{\text{diag}} = [\alpha \quad 0 \quad 0 \quad 1-\alpha]$$

So,

$$P(x,y) = [\alpha \quad 0 \quad 0 \quad 1-\alpha] \begin{bmatrix} 1-P & P \\ P & 1-P \end{bmatrix}$$

$$P(x,y) = \begin{bmatrix} \alpha(1-P) & \alpha P \\ (1-\alpha)P & (1-\alpha)(1-P) \end{bmatrix} = \begin{bmatrix} P(x_1, y_1) & P(x_1, y_2) \\ P(x_2, y_1) & P(x_2, y_2) \end{bmatrix}$$

NOW,

$$H(y/x) = - \sum_{i=1}^2 \sum_{j=1}^2 P(x_i, y_j) \times P \log_2 P(y_j/x_i)$$

$$= - \left\{ P(x_1, y_1) \log_2 P(y_1/x_1) + P(x_1, y_2) \log_2 P(y_2/x_1) \right. \\ \left. + P(x_2, y_1) \log_2 P(y_1/x_2) + P(x_2, y_2) \log_2 P(y_2/x_2) \right\}$$

So,

$$H(y/x) = - \left\{ \alpha(1-P) \log_2 (1-P) + \alpha P \log_2 P + \right. \\ \left. (1-\alpha)P \log_2 P + (1-\alpha)(1-P) \log_2 (1-P) \right\}$$

$$H(y/x) = - \left\{ P \log_2 P + (1-P) \log_2 (1-P) \right\}$$

So,

$$H(y/x) = P \log_2 P + (1-P) \log_2 (1-P)$$

$$\text{Entropy } I(x_i, y) = H(y) - H(y/x)$$

## \* RANDOM VARIABLES:

\* It is the process of Assigning no. to the outcome of an experiment.

(267)

\* Let, 2 coins are tossed, hence the outcomes are:

$$\{H\ H, \ H\ T, \ T\ H, \ T\ T\} = [S]$$

All these outcomes are taken under the variable called as Sample space variable.

\* Under some specific condition, the sample space variable is transformed into Random variable.

S	X (Random variable) Correspond to no. of Heads
H H	2
H T	1
T H	1
T T	0

\* When, the Random variable takes the discrete variable then it is called as (discrete Random variable).

\* For a variable to be considered as Random variable, the criteria is that the variable should be deterministic in nature.

Note:

1. A Random Variable 'X' is defined as it specifying no. of heads in the exp. of tossing a coin twice.

So,

Sample variable  
 $\{S\}$

Random variable,  $X = \{x\}$

H H	2
H T	1
T H	1
T T	0

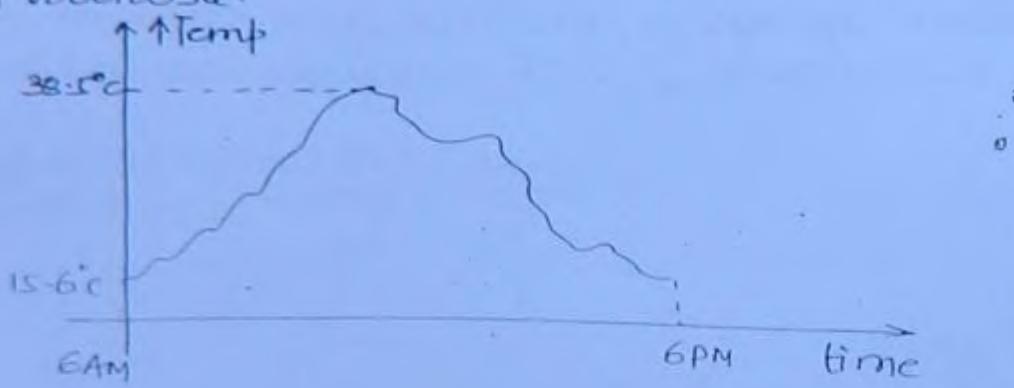
2. If Random variable takes discrete set of values then it is called as Discrete Random variable.

3. The above is discrete Random variable.

268  
198

4. If Random variable takes continuous set of values then it is called as Continuous Random variable.

5. Random variable which is specifying temp. in a Room from 6 AM to 6 PM corresponds to Continuous Random variable.



### \*PROBABILITY MASS FUNCTION:

It specifies probability of a Random variable taking each of its possible values.

Q. Plot Probability Mass Function for a Random variable which is specifying no. of heads in the event of Tossing a coin twice.

SOL<sup>n</sup>:  $P_x(x_i) = P(X=x_i)$ .

S
HH
HT
TH
TT

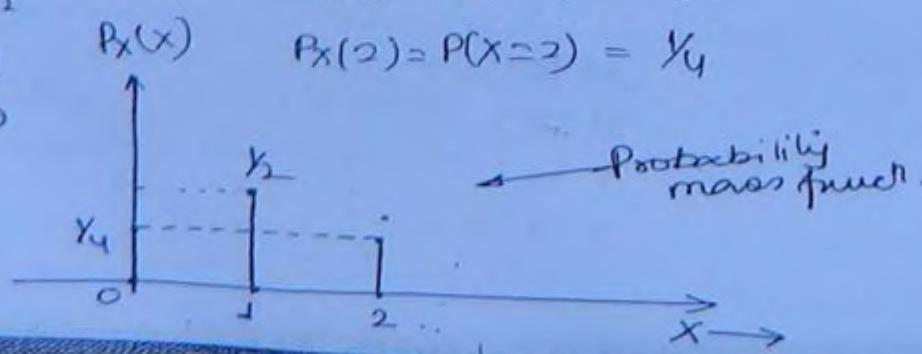
$$x = \{x_1, x_2, x_3, x_4\}$$

2  
1  
1  
0

$$\text{So, } P_x(0) = P(X=0) = y_4$$

$$P_x(1) = P(X=1) = y_2$$

$$P_x(2) = P(X=2) = y_1$$



Properties of Prob. Mass Function

1)  $0 \leq P_X(x_i) \leq 1$

2)  $\sum_i P_X(x_i) = 1$

269

Note:-

Prob. Mass Func<sup>n</sup> (PMF) is used to specify discrete Random Variable.

### \*CUMULATIVE PROBABILITY DISTRIBUTION FUNCTION(CDF):-

\* Standard notation is given as  $F_X(x) = P(X \leq x)$

\* It specifies Probability of Random variable (X) taking the values upto 'x'.

& Construct CDF for the above discrete Random variable.

Soln:

S	X
HH	2
HT	1
TH	1
TT	0

Now,

$x = \{x\} =$	0	1	2
$P_X(x) =$	$y_1$	$y_2$	$y_3$

Now,  $F_X(-1) = P(X \leq -1) = 0$ .

$F_X(0) = P(X \leq 0) = y_1$ .

$F_X(0.5) = P(X \leq 0.5) = y_1$ .

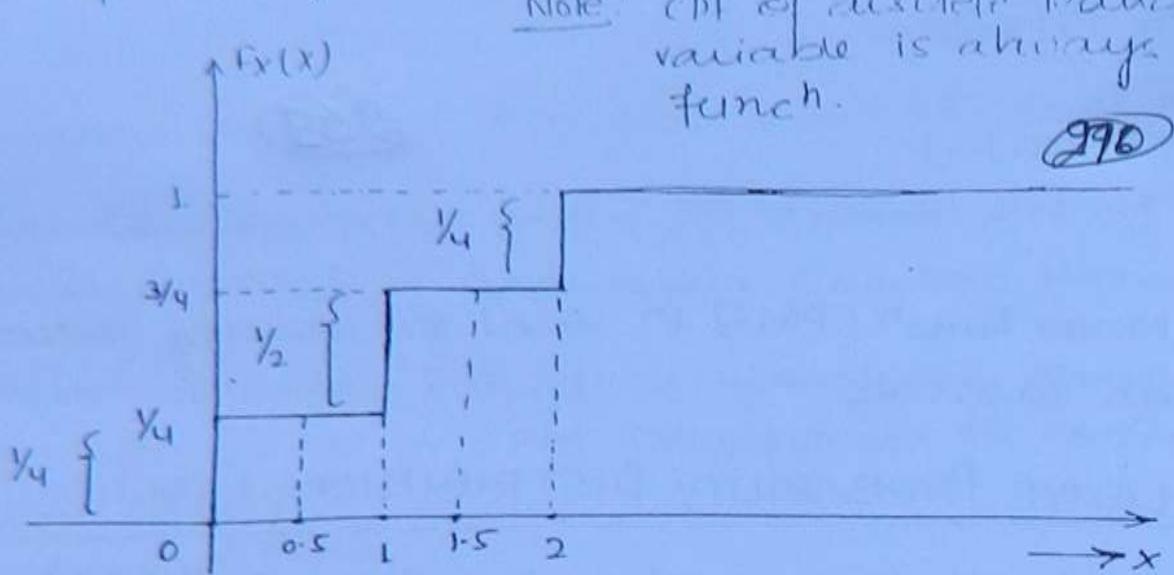
$F_X(1) = P(X \leq 1) = y_1 + y_2 = \frac{3}{4}, \{P_X(0) + P_X(1)\}$ .

$F_X(1.9) = P(X \leq 1.9) = \frac{3}{4}$ .

$F_X(2) = P(X \leq 2) = 1$ .

$F_X(10) = P(X \leq 10) = 1$ .

so the plot is given as.



$$P(X=1) = \frac{1}{2} \quad \{\text{Jump offered.}\}$$

$$P(X=2) = \frac{1}{4} \quad \{\text{Jump offered.}\}$$

Also,

$$F_X(x) = \frac{1}{4}u(x) + \frac{1}{2}u(x-1) + \frac{1}{4}u(x-2)$$

Note:

$F_X(x)$  of a discrete Random variable will be a staircase function.

$X = \{x_i\}$	-1	0	1	2
$P_X(x_i)$	0.3	0.2K	0.4	0.1

(27) (21)

i) K = ?

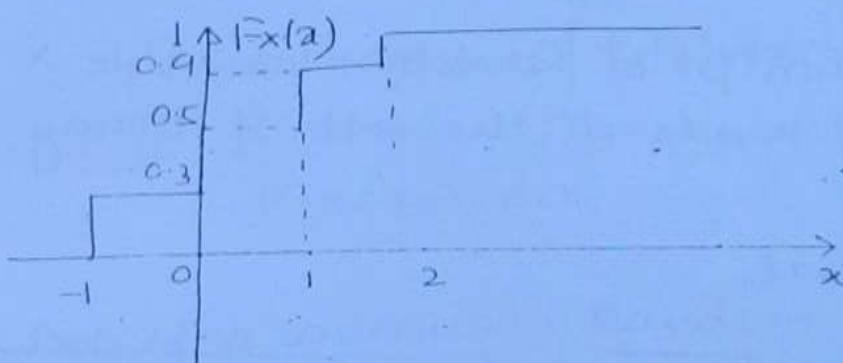
ii) Plot  $F_X(x)$

Sol: As  $\sum_{i=1} P_X(x_i) = 1$

$$0.3 + 0.2K + 0.4 + 0.1 = 1$$

$$\boxed{K=1} \quad \underline{\text{Ans}}$$

ii)



### \* Properties of $F_X(x)$ :

i)  $F_X(-\infty) = P(X \leq -\infty) = 0$ .

ii)  $F_X(\infty) = P(X \leq \infty) = 1$

iii)  $P(X \leq x) = F_X(x)$ .

iv)  $P(x_1 \leq X \leq x_2) = F_X(x_2) - F_X(x_1)$   
 $P(x \leq x_2) - P(x \leq x_1)$ .

v)  $P(X > x) = 1 - F_X(x)$ .

### \* PROBABILITY DENSITY FUNCTION (PDF):

\* Denoted by  $f_X(x)$ .

\* It is generally used to specify continuous Random Variable.

\* The Relation b/w PDF and Distribution function is given as:

$$f_x(x) = \frac{d}{dx} F_x(x)$$

(298)

$$F_x(x) = \int_{-\infty}^x f_x(a) da = P(X \leq x)$$

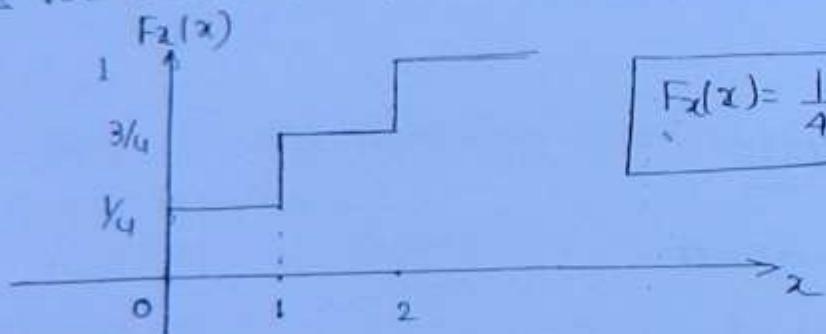
Now,

$$P(2 \leq x \leq 2) = \int_{2}^{2} f_x(a) da$$

$$P(X \geq x) = \int_x^{\infty} f_x(a) da.$$

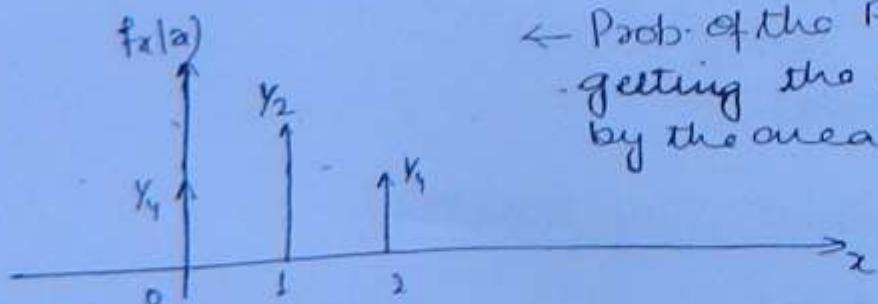
Q. Plot density function for a Random variable  $X$  which is specifying no. of heads in the event of tossing a coin twice.

Soln: As we know that,



$$F_x(x) = \frac{1}{4} u(x) + \frac{1}{2} u(x-1) + \frac{1}{4} u(x-2)$$

$$\text{So, } f_x(x) = \frac{d}{dx} F_x(x)$$

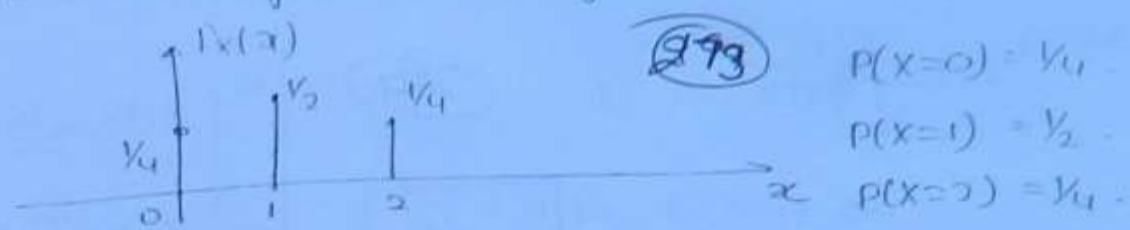


← Prob. of the Random variable getting the prob. can be given by the area of the PDF.

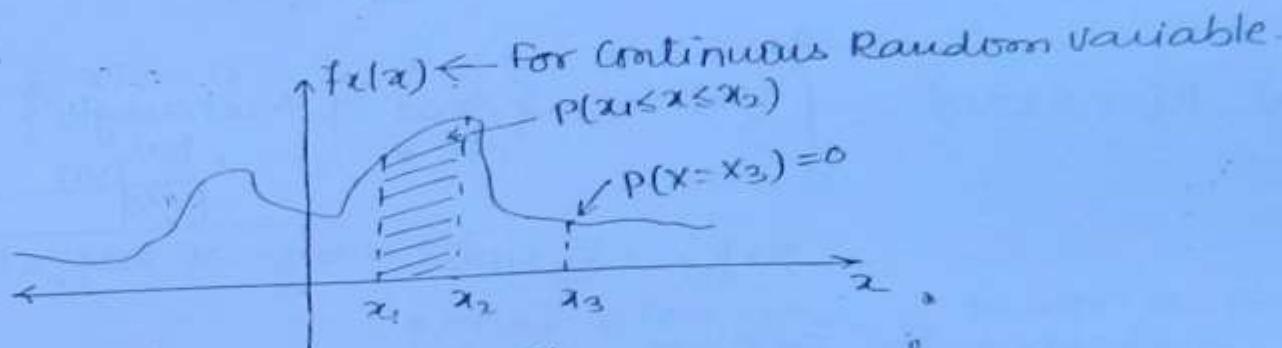
$$\text{So, } f_x(x) = y_1 \delta(x) + \frac{1}{2} y_2 \delta(x-1) + \frac{1}{4} y_3 \delta(x-2)$$

Conclusion: Density function of discrete Random variable will be in terms of delta function.

The Prob. Mass function is given as.



Note:



$$P(x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} f(x) dx = P(x_1 < x \leq x_2)$$

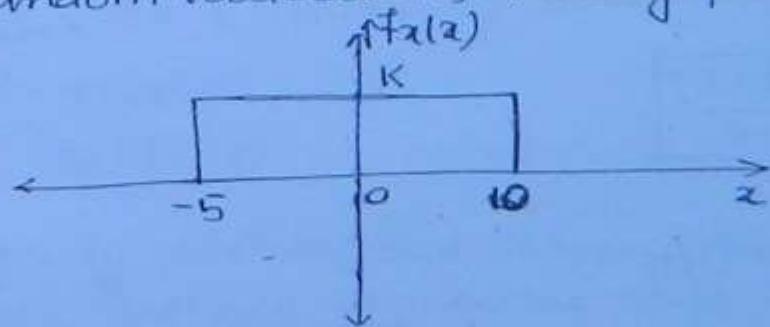
$$P(x=x_3) = 0$$

\* The Prob. of a continuous Random variable taking a specific single value will be zero.

Now,

$$P(-\infty \leq x \leq \infty) = \int_{-\infty}^{\infty} f(x) dx = 1$$

Q. For a Random variable X; density funcn was given below



- Find K value.
- $P(-5 \leq x \leq 10)$ .
- $P(-5 \leq x \leq 5)$ .
- Plot  $f(x)$ .

Sol<sup>no</sup> 9) Corresponds to the Continuous Random Variable.

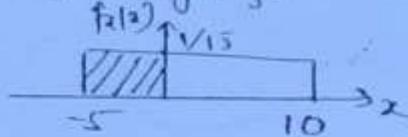
Now, as  $\int_{-\infty}^{\infty} f_x(x) dx = 1$ , Ans

so,  $15K = 1 \Rightarrow K = \frac{1}{15}$

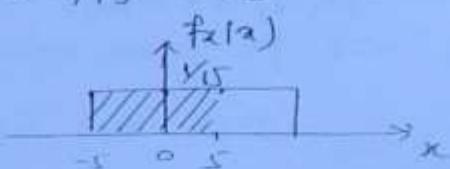
Also,  
$$\int_{-5}^{10} K dx = Kx \Big|_{-5}^{10} = 1$$
  
 $15 \cdot K = 1 \Rightarrow K = \frac{1}{15}$

b)  $P(-5 \leq x \leq 0) = \int_{-5}^0 f(x) dx \quad \{ \text{Area of Rectangle?} \}$

$= 5 \times \frac{1}{15} = \frac{1}{3}$  units



c)  $P(-5 \leq x \leq 5) = \int_{-5}^5 f_x(x) dx = 10 \times \frac{1}{15} = \frac{2}{3}$  units



d)  $F_x(x) = \int_{-\infty}^x f_x(x) dx$

$= \int_{-\infty}^x \frac{1}{15} dx = \frac{1}{15} \times x \Big|_{-\infty}^x$

$$F_x(x) = \frac{(x+5)}{15}$$

Now,  $F_x(x) = \frac{(x+5)}{15}$

So,  $F_x(-10) = 0 \quad \{ \because F_x(x) = P(X \leq x) \}$

So,  $F_x(x) = \int_{-\infty}^{-10} f_x(x) dx = 0$ .

$$F_X(15) = \int_{-\infty}^{15} f(x) dx = 1$$

(295)

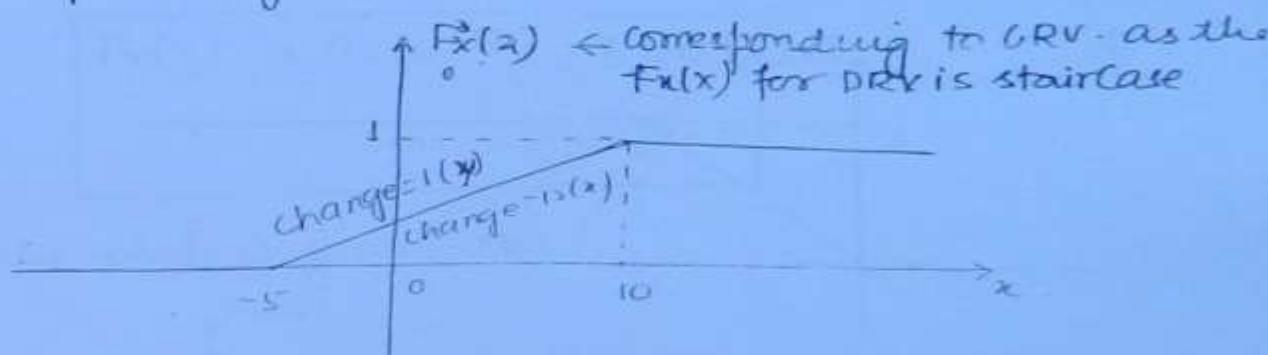
Conclusion:-

$$F_X(x) = \frac{x+5}{15}; -5 \leq x \leq 10$$

$$F_X(x) = 0; x < -5$$

$$1; x > 10$$

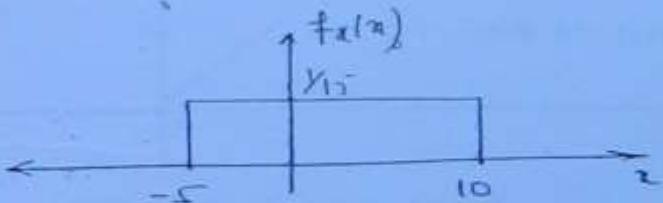
So, the plot is given as:-



Now,

$$\frac{d}{dx} F_X(x) = f_X(x) \quad \left\{ \because \frac{d}{dx} = \text{slope} \right\}$$

so,

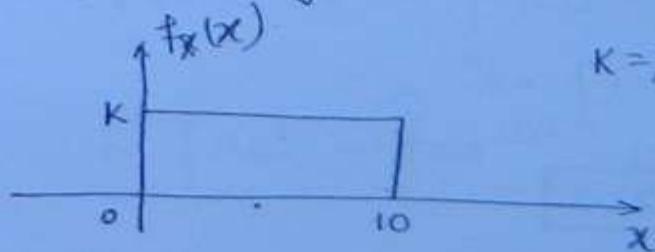


Q2. A continuous R.V  $x$ ; uniformly distributed in the interval [0 to 10]. Plot, a)  $f_X(x)$ .

- b)  $P(F_X(x))$

Soln:- uniformly distributed means that the prob. of variable taking values at diff. instant is equal.

So,



$$K = y_{10}; \text{ since Area} = 1.$$

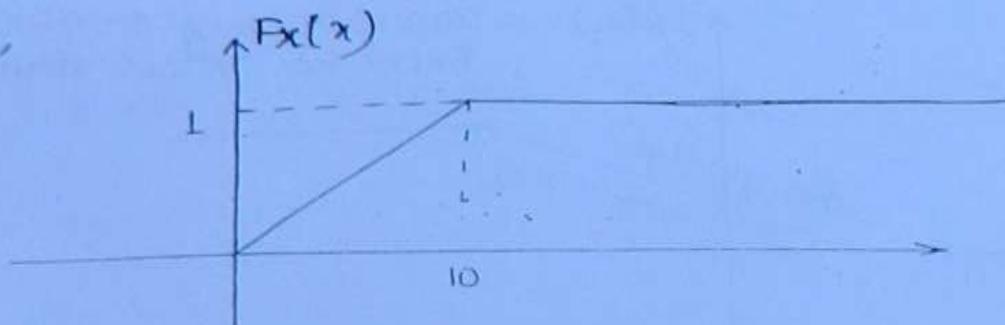
$$\begin{aligned}
 F_X(x) &= \int_{-\infty}^x f_X(x) dx \\
 &= \int_0^x f_X(x) dx \\
 &= \int_0^x \frac{1}{10} dx
 \end{aligned}$$

(298)

$$\begin{cases}
 F_X(x) = 0 & ; x < 0 \\
 = \frac{x}{10}, & 0 \leq x \leq 10 \\
 = 1 & ; x > 10
 \end{cases}$$

$$F_X(x) = \frac{x}{10}$$

So,



Q. For a CRV; given

$$f_X(x) = ae^{-bx}; x \geq 0$$

i) Find Relation b/w a & b.

ii) Plot  $F_X(x)$ .

Soln: As,  $\int_{-\infty}^{\infty} f_X(x) dx = 1$

$$\int_0^{\infty} f_X(x) dx = 1 = \int_0^{\infty} ae^{-bx} dx = 1$$

$$= -\frac{a}{b} e^{-bx} \Big|_0^{\infty} = 1$$

$$= -\frac{a}{b} \left[ 0 - 1 \right] = 1$$

$$\Rightarrow a = b$$

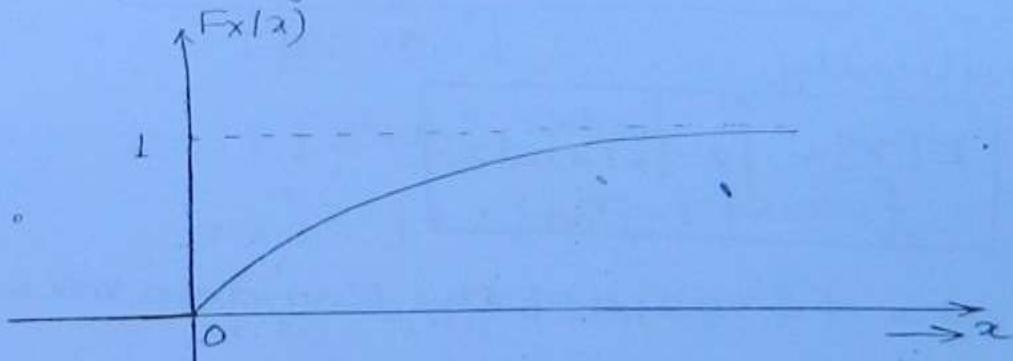
$$\text{So, } f_X(x) = ae^{-ax}; x \geq 0$$

277

$$\begin{aligned} \text{Now, } F_X(x) &= \int_{-\infty}^x f_X(x) dx \\ &= \int_0^x ae^{-ax} dx \\ &= -\frac{a}{a} e^{-ax} \Big|_0^x \\ &= -1 \{e^{-ax} - 1\} \end{aligned}$$

$$\boxed{\begin{aligned} F_X(x) &= (1 - e^{-ax}); x \geq 0 \\ &0 \quad ; x < 0 \end{aligned}}$$

So, the plot is given as:



Q4. For a CRV, x. Given that

$$f_X(x) = ae^{-bx}$$

Find the Relation b/w a & b.

Soln:- Given that,

$$f_X(x) = ae^{-bx}$$

$$\text{as, } \int_{-\infty}^{\infty} f_X(x) dx = 1 = \int_{-\infty}^0 ae^{-b(-x)} dx + \int_0^{\infty} ae^{-bx} dx = 1$$

$$= \int_{-\infty}^0 ae^{bx} dx + \int_0^{\infty} ae^{-bx} dx = 1$$

$$\frac{a}{b} e^{ax} \Big|_{-\infty}^{\infty} - \frac{a}{b} e^{-bx} \Big|_0^{\infty} = 1$$

(278)

$$= \frac{a}{b} \{1 - 0\} - \frac{a}{b} \{0 - 1\} = 1$$

$$= a/b + a/b = 1$$

$$2a = b \quad \text{Ans}$$

So,  $F_x(x) = ae^{-2ax|x|}$

## Statistical Averages of Random Variable:

### a) MEAN:

$$\text{Mean } [x] = \text{Expectation, } E[x] = \bar{x} = m_1$$

Mathematically,

$$E[x] = \int_{-\infty}^{\infty} x \cdot f_x(x) dx$$

Mean is the d.c value of the Random variable.

### b) MEAN SQUARE VALUE (MSQ):

$$\text{msg}[x] = E[x^2] = \bar{x^2} = m_2$$

Mathematically,

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f_x(x) dx$$

$x^2$  gives the total power of the Random variable.

### c) VARIANCE ( $\sigma^2$ )

298

as

$$E[K] = K$$

$$E[K] = \int_{-\infty}^{\infty} K \cdot f_X(x) dx = K \cdot$$

and,  $E[Kx] = \int_{-\infty}^{\infty} K \cdot x \cdot f_X(x) dx$

$$E[Kx] = K E[x]$$

And,  $E[x_1 + x_2] = E[x_1] + E[x_2]$

So, the variance is defined as:-

$$\begin{aligned}\sigma^2 &= E[(x - \bar{x})^2] \\ &= E[(x - m_1)^2] \\ &= E[x^2 + m_1^2 - 2xm_1] \\ &= E[x^2] + E[m_1^2] - E[2xm_1] \\ &= m_2 + m_1^2 - 2m_1 E[x] \\ &= m_2 + m_1^2 - 2m_1 \cancel{m_1} \\ &= m_2 + m_1^2 - 2m_1^2\end{aligned}$$

$$\sigma^2 = m_2 - m_1^2$$

So,  $\sigma^2 = \text{total power} - \text{d.c power}$

$$\sigma^2 = \text{A.C power of Random variable}$$

#### d) STANDARD DEVIATION:

As

$\sigma^2$ : Variance

(280)

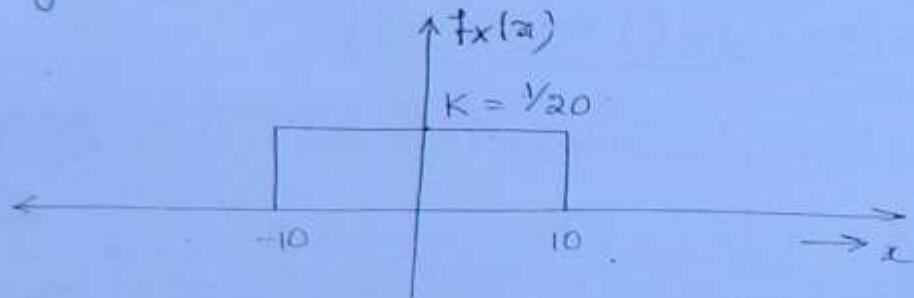
So

$$\sigma = \sqrt{\text{Variance}}$$

$\sigma$  = A C component of Random variable.

- Q. A continuous Random variable is uniformly distributed in the interval (-10, 10). Find all of its statistical averages.

SOP:-



$$\begin{aligned}
 \text{a) Mean, } m_1 &= \int_{-\infty}^{\infty} x f_x(x) dx \\
 &= \int_{-10}^{10} x \cdot \frac{1}{20} dx \\
 &= \frac{1}{20} \int_{-10}^{10} x dx = \frac{1}{20} \times \frac{x^2}{2} \Big|_{-10}^{10} \\
 &= \frac{1}{40} \times 0
 \end{aligned}$$

$$m_1 = 0$$

Conclusion:-

If the density function is symmetric about the vertical axis passing through the origin, then the Mean value of the <sup>R.V</sup> function is 0.

b) Mean square value,  $m_2 = \int_{-\infty}^{\infty} x^2 f_x(x) dx$

**(28)**

$$= \int_{-10}^{10} x^2 \frac{1}{20} dx$$

$$= \frac{x^3}{3} \Big|_{-10}^{10} \times \frac{1}{20}$$

$$= \frac{1}{60} \times 2000$$

$m_2 = 33.33$

Ans

c) Variance,  $\sigma^2 = m_2 - m_1^2$

$$= 33.33 - 0$$

$\sigma^2 = 33.33$

Ans

$\left\{ \begin{array}{l} \therefore \sigma^2 = m_2 - m_1^2 \\ \therefore m_1 = 0 \\ \text{so, } \boxed{\sigma^2 = m_2} \end{array} \right. ***$

d) Standard deviation =  $\sqrt{\text{Variance}}$

$$= \sqrt{33.33}$$

$\sigma = 5.77$

Ans

Q2. For a CRV,  $X$ ; given

$$f_x(x) = \lambda e^{-\lambda x}; x \geq 0$$

Find all of its statistical averages.

SOL: i) Mean =  $\int_{-\infty}^{\infty} x f_x(x) dx =$

$$= \int_0^{\infty} x \cdot \lambda e^{-\lambda x} dx$$

$$= \lambda \int_0^{\infty} x \cdot \frac{e^{-\lambda x}}{-x} dx \quad \left\{ \because \int u v du = u \int v du - \int u' v du \right.$$

$$= \lambda \left[ x \cdot \frac{e^{-\lambda x}}{-\lambda} \Big|_0^{\infty} - \int_0^{\infty} 1 \cdot \frac{e^{-\lambda x}}{-\lambda} dx \right]$$

$$= \lambda \left\{ -\frac{1}{\lambda} (0 - 0) + \frac{1}{\lambda^2} [e^{-\infty} - e^0] \right\} = \boxed{m_1 = \frac{1}{\lambda}}$$

2) Mean Square value,

$$m_{sq} = \int_{-\infty}^{\infty} x^2 f_x(x) dx$$

281 282

$$\boxed{m_{sq} = \frac{2}{\lambda^2}} \text{ Ans}$$

3)  $\sigma^2 = m_2 - m_1^2$

$$= \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2$$

$$\boxed{\sigma^2 = \frac{1}{\lambda^2}} \text{ Ans}$$

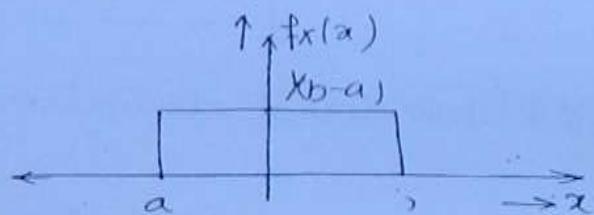
4) Standard deviation =  $\sqrt{\sigma^2}$

$$\boxed{S.D = \frac{1}{\lambda}} \text{ Ans}$$

### \* UNIFORM PROBABILITY DENSITY FUNCTION:

It is defined as:

$$f_x(x) = \frac{1}{(b-a)} ; a \leq x \leq b$$



Q-A CRV; is possessing uniform density function specified above. Find all of its statistical averages.

Sol<sup>n</sup>: As, we know that,

$$\begin{aligned} \text{Mean} &= \int_{-\infty}^{\infty} x f_x(x) dx \\ &= \int_a^b x \cdot \frac{1}{(b-a)} dx \\ &= \frac{1}{(b-a)} \int_a^b x dx \\ &= \frac{1}{(b-a)} \frac{x^2}{2} \Big|_a^b \\ &= \frac{1}{(b-a)} \left[ \frac{b^2 - a^2}{2} \right] \Rightarrow \boxed{m_1 = \frac{b+a}{2}} \end{aligned}$$

2) Mean Square Value,  $m_{sq}$  =  $\int_{-\infty}^{\infty} x^2 f_x(x) dx$

(283)

$$= \int_a^b x^2 \frac{1}{(b-a)} dx$$

$$= \frac{1}{3(b-a)} \left[ x^3 \right]_a^b$$

$$= \frac{b^3 - a^3}{3(b-a)}$$

$m_{sq} = \frac{b^2 + a^2 + ab}{3}$

3) Now, variance,  $\sigma^2 = m_2 - m_1^2$

$$= \frac{b^2 + a^2 + ab}{3} - \frac{(a+b)^2}{4}$$

$\sigma^2 = \frac{(a-b)^2}{12}$

1) Standard deviation =  $\sqrt{\sigma}$

$S.D = \frac{(a-b)}{2\sqrt{3}}$

### \* GAUSSIAN DENSITY FUNCTION:

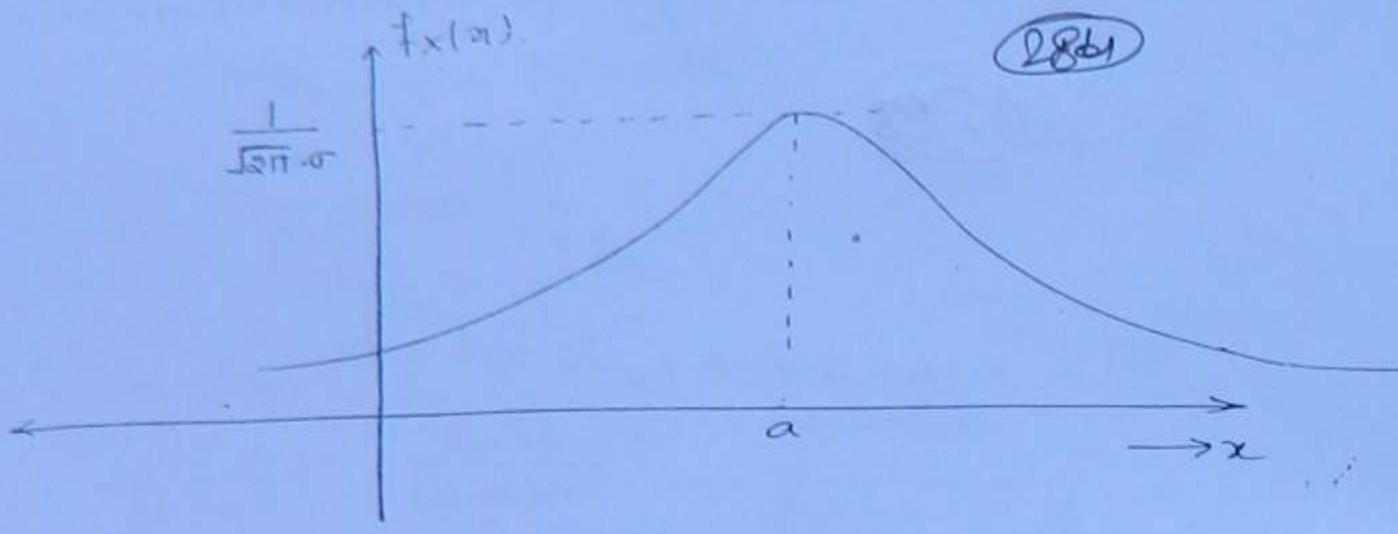
It is given as:-

$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-a)^2}{2\sigma^2}}$

For,  $x=a$

$f_x(x)$  will be max<sup>m.</sup>

Hence the plot is given as:-



2861

$$\begin{aligned}
 a) \text{ Mean} - m_1 &= \int_{-\infty}^{\infty} x \cdot f_x(x) dx \\
 &= \int_{-\infty}^{\infty} x \cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-a)^2}{2\sigma^2}} dx
 \end{aligned}$$

After Analysis,

$$m_1 = a$$

$$\begin{aligned}
 b) \text{ Mean Square value, } msq &= \int_{-\infty}^{\infty} x^2 f_x(x) dx \\
 &\boxed{msq = \sigma^2 + a^2}
 \end{aligned}$$

$$\begin{aligned}
 c) \text{ Variance, } \sigma^2 &= m_2 - m_1^2 \\
 &= \sigma^2 + a^2 - a^2
 \end{aligned}$$

$$\boxed{\text{Variance} = \sigma^2}$$

d) Standard deviation

$$\boxed{SD = \sigma}$$

### x Analysis:

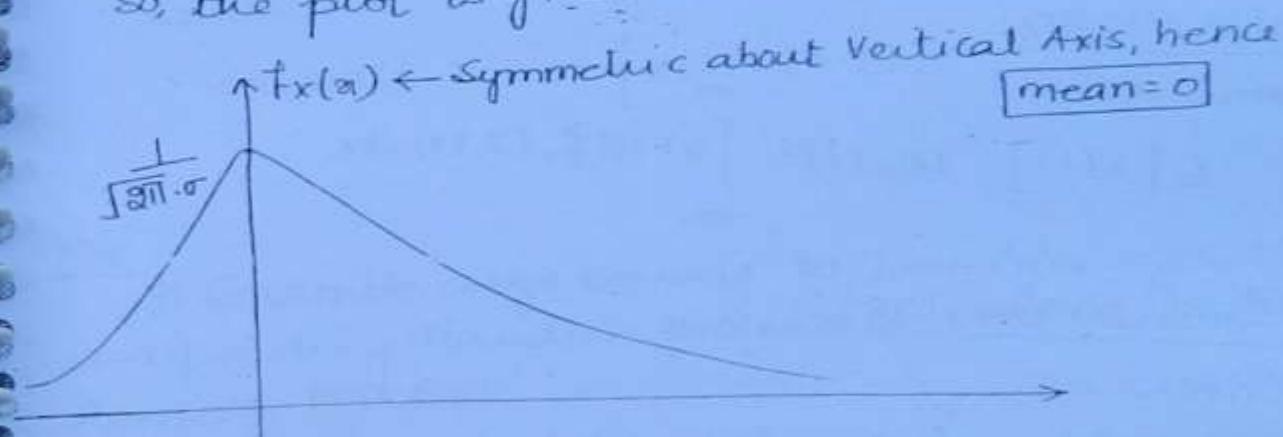
if mean =  $\alpha = M_1 = 0$

So,  $\alpha = 0$

(285)

$$\text{Then, } f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-x^2/2\sigma^2} \quad \left| \text{ max at } x=0 \right.$$

so, the plot is given as:-



### Note:

\* while Noise possesses Gaussian density function, so it is also called as the Gaussian Noise.

only ES sub Numerical

### RANDOM PROCESS:-

\* Random variable as a function of time is called as the RANDOM PROCESS.

#### RANDOM VARIABLE

1. denoted as  $x$

2.  $F_x(a) = P(x \leq a)$

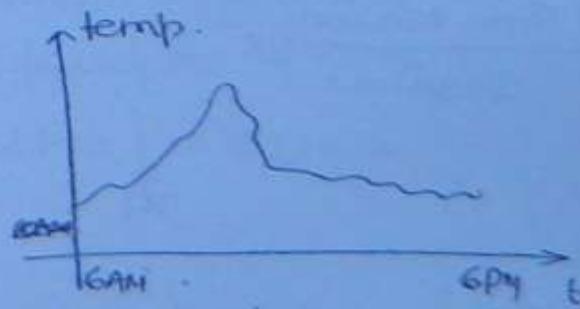
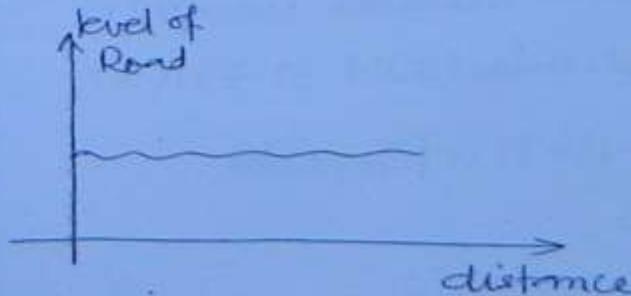
3.  $f_x(a) = \frac{d}{dx} F_x(a)$

#### RANDOM PROCESS

1. denoted as  $x(t)$ .

2.  $F_x(x, t) = P(x(t) \leq x)$

3.  $f_x(a, t) = \frac{d}{da} F_x(a, t)$ .



## \*Statistical Averages of Random Process

### 1) ENSEMBLE AVERAGES:-

(286)

The Averages calculated on a group of similar Random processes are called as Ensemble Avg.

\*depends on density function

#### a) Ensemble mean:-

Given as:-

$$E[x(t)] = m_1(t) = \int_{-\infty}^{\infty} x(t) f_x(x, t) dx$$

#### b) Mean Square Value, msq:-

Given as:-

$$m_2(t) = \int_{-\infty}^{\infty} x^2(t) f_x(x, t) dx$$

#### c) Auto Correlation Function:-

Given as:-

$$*** R(\tau) = E[x(t) x(t-\tau)]$$

### 2) TIME AVERAGES:-

\*The statistical averages computed on a Random process on a time basis is called as Time Averages

\*\*\* Time Averages are independent of density function

#### a) Time Mean:-

Given as:-

$$\langle x(t) \rangle = \int_{t_1}^{t_2} x(t) dt$$

b) Mean square value, m<sub>x</sub>:

Given as:

$$\langle x^2(t) \rangle = \int_{t_1}^{t_2} x^2(t) dt$$

(287)

c) Auto Correlation Function :-

Given as:

$$\langle x(t)x(t-\tau) \rangle = \int_{t_1}^{t_2} x(t)x(t-\tau) dt$$

Note:-

If Ensemble avg equals to Time Averages, then the corresponding Random process is said to be ERGODIC RANDOM PROCESS.

\* If only means are same, then it is said to be ERGODIC IN MEAN/MSQ/AUTOCORRELATION.

\* STRICT SENSE STATIONARY RANDOM PROCESS:

\* If Prob. density function of Random Process is independent of time, then it is said to be "SSSRP".

So,

$$f_x(x, t) = f_x(x, t + \Delta t)$$

\* WIDE SENSE STATIONARY RANDOM PROCESS:

\* A Random process is said to be WSSRP, if it satisfies the following

- Mean should be constant, independent of t.
- ACF i.e  $R(\tau)$  should be function of only  $\tau$ .

$$R(\tau) = E[x(t)x(t-\tau)].$$

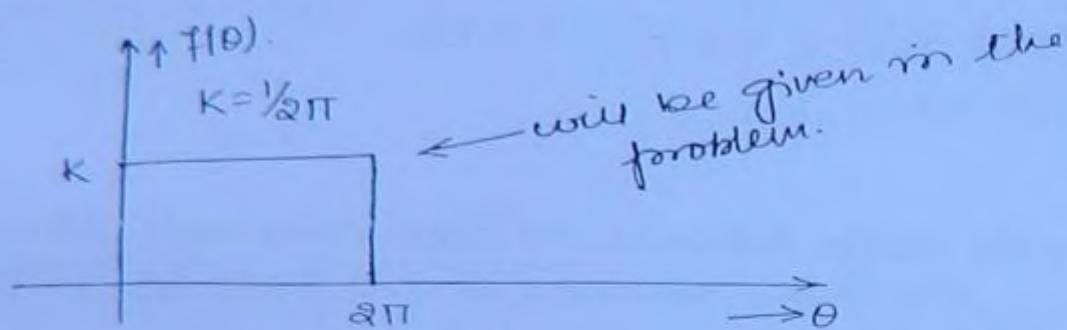
Q A Random process is given by  
 $x(t) = A \cos \{ \omega_0 t + \theta \}$

(288)

where  $A$  &  $\omega_0$  are constants and  $\theta$  is random variable; which is uniformly distributed in the interval  $(0, 2\pi)$ .

Find whether the given RP is WSS or not?

Sol:



$$\text{Now, mean of Function } m_1(t) = \int_{-\infty}^{\infty} x(t) \cdot f(\theta) d\theta$$

$$= \int_0^{2\pi} A \cos \{ \omega_0 t + \theta \} \cdot \frac{1}{2}\pi d\theta$$

$$= \frac{1}{2\pi} \int_0^{2\pi} A \cos \{ \omega_0 t + \theta \} d\theta$$

$\therefore \theta$  is random variable, all other parameters are considered as constant including  $t$

$$m_1(t) = \frac{1}{2\pi} \int_0^{2\pi} A \cos \omega_0 t \cos \theta d\theta - \frac{1}{2\pi} \int_0^{2\pi} A \sin \omega_0 t \sin \theta d\theta$$

$$= \frac{A \cos \omega_0 t}{2\pi} \int_0^{2\pi} \cos \theta d\theta - \frac{A \sin \omega_0 t}{2\pi} \int_0^{2\pi} \sin \theta d\theta$$

So,

$$m_1(t) = 0 = \text{constant}$$

$$\text{Also, } ACF = R(\tau) = E[x(t)x(t-\tau)]$$

$$= E[A \cos(\omega_0 t + \theta) \cdot A \cos \{ \omega_0 (t-\tau) + \theta \}]$$

$$= E[A \cos(\omega_0 t + \theta) \cdot A \cos(\omega_0 t - \omega_0 \tau + \theta)]$$

$$R(\tau) = \left[ \frac{A^2}{2} \cos(2\omega_0 t - \omega_0 \tau - 2\theta) + \frac{A^2}{2} \cos \omega_0 \tau \right]$$

(28)

$$R(\tau) = E\left[\frac{A^2}{2} \cos(2\omega_0 t - \omega_0 \tau + 2\theta)\right] + E\left[\frac{A^2}{2} \cos \omega_0 \tau\right].$$

Now As  $E[K] = K$

$$\text{so, } E\left[\frac{A^2}{2} \cos \omega_0 \tau\right] = A^2/2 \cos \omega_0 \tau \quad \begin{matrix} \text{S. Integration done} \\ \text{w.r.t } \theta \end{matrix}$$

Now,

$$E\left[\frac{A^2}{2} \cos(2\omega_0 t - \omega_0 \tau + 2\theta)\right] = \int_0^{2\pi} \frac{A^2}{2} \cos(\underbrace{2\omega_0 t - \omega_0 \tau}_{A} + \underbrace{2\theta}_{B}) \cdot \frac{1}{2\pi} d\theta$$

$$\Rightarrow E\left[\frac{A^2}{2} \cos(2\omega_0 t - \omega_0 \tau + 2\theta)\right] = 0.$$

So,

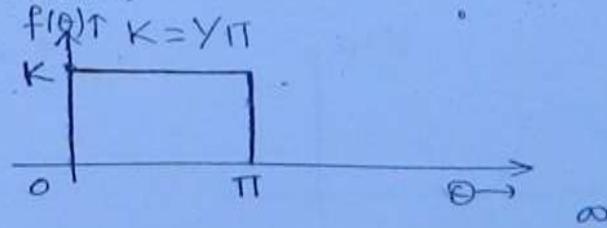
$$\text{ACF} = R(\tau) = \frac{A^2}{2} \cos \omega_0 \tau$$

Hence, it is function of only  $\tau$ .

Hence, the given function is ~~WSSRP Ans~~

Q. Repeat Above if  $\theta$  is uniformly distributed in the interval  $(0, \pi)$ .

Sol:



Now mean of func<sup>n</sup> =  $m_1(t) = \int_{-\infty}^{\infty} x(t) f(\theta) d\theta$

$$\begin{aligned} &= \int_0^{\pi} \frac{1}{\pi} A \cos(\omega_0 t + \theta) d\theta \\ &= \frac{A}{\pi} \int_0^{\pi} \cos(\omega_0 t + \theta) d\theta \end{aligned}$$

$$m(t) = \frac{A}{\pi} \int_0^{\pi} [\cos(\omega t - \sin \omega t \sin \theta)] d\theta$$

(290)

$$= \frac{A \cos \omega t}{\pi} \int_0^{\pi} \sin \theta d\theta - \frac{A \sin \omega t}{\pi} \int_0^{\pi} \sin \theta d\theta$$

$$= \frac{A \cos \omega t \sin \theta}{\pi} \Big|_0^{\pi} - \frac{A \sin \omega t \times -\cos \theta}{\pi} \Big|_0^{\pi}$$

$$= \frac{A \cos \omega t \times (0 - 0)}{\pi} + \frac{A \sin \omega t}{\pi} (\cos \pi - \cos 0)$$

$$= \frac{A \sin \omega t}{\pi} (-1 - 1)$$

$$m(t) = \boxed{-\frac{2A \sin \omega t}{\pi}}$$

$m(t)$  is not constant, but is a function of  $t$ .  
Hence given RP is not WSS RP.

## CONVOLUTION AND CORRELATION

(29)

- Convolution is used to find the response of the system
- Correlation is used to find the similarity b/w the signals.

Now,

Mathematically, convolution is given as:-

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) x_2(t-\tau) d\tau$$

\* Correlation means correlation of 2 functions.

Hence

x-correlation is mathematically given as:-

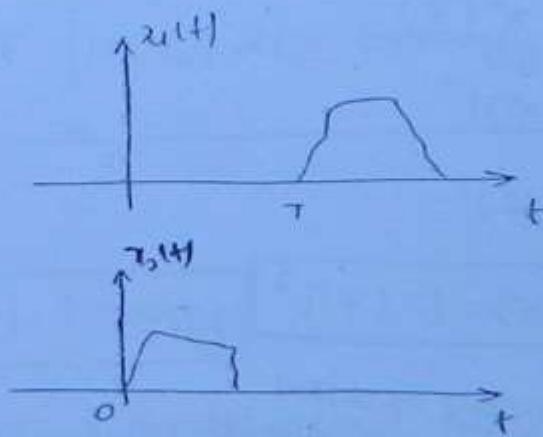
$$R_{12}(\tau) = \int_{-\infty}^{\infty} x_1(t) x_2(t-\tau) dt$$

NOW, for the 2 signals to be equal, we have!

$$\int_{-\infty}^{\infty} x_1(t) \cdot x_2(t) dt = X(t)$$

If the value of  $X(t)=0$ , hence, the signals are atmost dissimilar.

But let



The value of  $\int_{-\infty}^{\infty} x_1(t) \cdot x_2(t) dt$  is zero

In this case, but some similarity is present. Hence we delay the 2nd signal by  $\tau$  and for the value of  $\tau$  for which

max<sup>m</sup> area is overlapped, it is said to be the similarity value. ie

$$R_{12} = \int_{-\infty}^{\infty} x_1(t) x_2(t-\tau) dt$$

Searching/Scanning parameter.

Now,

$$R_{12}(\tau) = \int_{-\infty}^{\infty} x_1(t) x_2(t-\tau) dt$$

(29)

$$R_{12}(\tau) = x_1(t) \times x_2(-t) / t \rightarrow \tau$$

\* Auto correlation of a signal means finding the similarity of a signal with its shifted version.

Hence,

Mathematically,

$$ACF[x(t)] = R(\tau) = \int_{-\infty}^{\infty} x(t) x(t+\tau) dt$$

$$\text{So, } R(\tau) = x(t) \times x(t)$$

So, by Fourier transform we get:

$$R(\tau) \longleftrightarrow x(f) \cdot x(-f)$$

Note:

Now, if  $x(t)$  is Real, then

$$x^*(f) = x(-f)$$

$$\text{So, } R(\tau) \longleftrightarrow x(f) \cdot x^*(f)$$

$$R(\tau) \longleftrightarrow |x(f)|^2$$

$$R(\tau) \longleftrightarrow S(f)$$

where,  $S(f) = |x(f)|^2$

Energy spectral density of  $x(t)$ .

So,  
Conclusion:

$$\text{Fourier transform of ACF} = \text{ESD}$$

And,

$$\text{IFT} \left[ S(f) \right] = R(\tau)$$

293

Now,

$$\int_{-\infty}^{\infty} s(f) e^{j2\pi f \tau} df = R(\tau)$$

put  $\tau = 0$ .

So,  $\int_{-\infty}^{\infty} s(f) df = R(0) \quad \dots (1)$

Auto-correlation function at origin  
will give the Area of ESD.

Note!

&  $R(\tau)$  is max<sup>m</sup> at  $\tau = 0$ .

and as the value of  $\tau$  is decreasing, the similarity is decreasing and  $R(\tau)$  is decreasing

Now,

$$R(\tau) = \int_{-\infty}^{\infty} x(t) x(t-\tau) dt$$

at  $\tau = 0$

$$R(0) = \int_{-\infty}^{\infty} x^2(t) dt = \text{Energy}[x(t)] \quad \dots (2)$$

So, from (1) & (2) we get:

$$\int_{-\infty}^{\infty} s(f) df = \int_{-\infty}^{\infty} |x(f)|^2 df = \text{Energy}[x(t)]$$

PARSEVAL'S  
THEOREM.

Conclusion:

$$\text{Area[ESD]} = \text{Energy}$$

The above discussion is valid only for the Energy Signal.  
If the Signal is Power Signal, we have to generalise the discussion by Average Auto-correlation Function.

## \* AVG AUTO-CORRELATION FUNCTION

let

Periodic signal by  $x_T(t)$ .

(296)

- All periodic signals are Power signals; but Reverse is not true.

Now,

$$R(\tau) = \int_{-\infty}^{\infty} x_T(t) \cdot x_T(t-\tau) dt$$

at  $\tau = 0$

$$R(0) = \int_{-\infty}^{\infty} x_T^2(t) dt = \text{Energy of } x_T(t) \quad \left. \begin{array}{l} \text{It fails at value} \\ \infty \end{array} \right\} \text{of ACF} = \infty$$

- Auto-correlation function is max<sup>im</sup> at  $\tau = 0$ , but the value should be some finite value.

- But for above case, at  $\tau = 0$ ,  $R(0) = \infty$ .

Hence, the formula for ACF of periodic signal fails for the analysis of Power/Periodic signals.

- ACF shouldn't be  $\infty$ . so, it is failed for Power signals.
- For Power signals, avg' auto correlation func<sup>n</sup> will be defined.

So,

$$\tilde{R}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x_T(t) x_T(t-\tau) dt$$

$$\tilde{R}(0) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x_T^2(t) dt = \text{Power of } x_T(t) \quad \text{--- (3)}$$

taking F-T on both sides we get :-

$$\tilde{R}(\tau) \Leftrightarrow \lim_{T \rightarrow \infty} \frac{1}{T} |x_T(f)|^2$$

$$\tilde{R}(\tau) \Leftrightarrow \lim_{T \rightarrow \infty} \frac{1}{T} \underbrace{\int_{-T/2}^{T/2} x_T(t) dt}_{E_S} \rightarrow E_S$$

$\therefore$  Energy of Power Signal over entire range =  $\infty$ .  
But for 1 period, E = finite value

$$So, \tilde{R}(\tau) \longleftrightarrow S(f)$$

Now,

$$IFT[S(f)] = \tilde{R}(\tau)$$

(295)

$$So, \int_{-\infty}^{\infty} S(f) e^{j2\pi f \tau} df = \tilde{R}(\tau)$$

$$\tau \rightarrow 0$$

$$\left[ \int_{-\infty}^{\infty} S(f) df = \tilde{R}(0) \right] \dots (4)$$

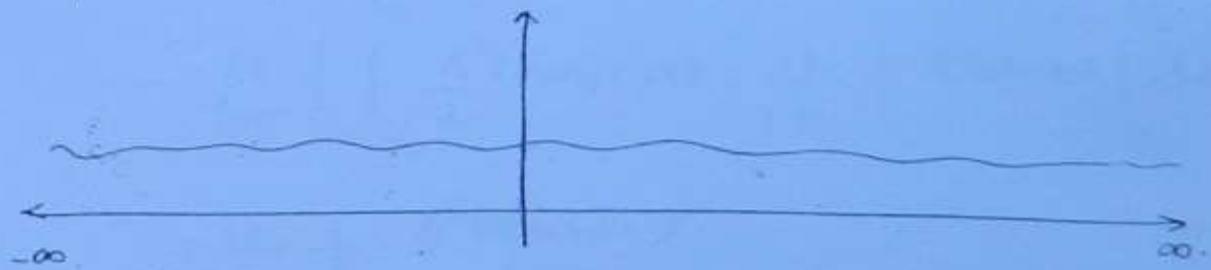
Conclusion :-

So, from eqn (3) & (4) we get:-

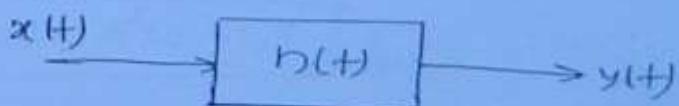
\*\*\* Area [PSD] = Power

Note :-

while Noise corresponds to power signal; as it occupies the spectrum from  $-\infty$  to  $+\infty$ .



Note :-



$$Y(f) = H(f) \cdot X(f)$$

$$|Y(f)|^2 = |H(f)|^2 |X(f)|^2$$

(ESD)\_{O/P} = |H(f)|^2 \cdot (ESD)\_{S/P}

ANSWER

$$|Y(t)|^2 = |H(t)|^2 |x(t)|^2$$

296

$$\frac{|Y(t)|^2}{T} = \frac{|H(t)|^2 |x(t)|^2}{T}$$

$$\lim_{T \rightarrow \infty} \frac{|Y(t)|^2}{T} = |H(t)|^2 \lim_{T \rightarrow \infty} \frac{|x(t)|^2}{T}$$

$$(P_{SD})_{OLP} = |H(t)|^2 (P_{SD})g/P$$

Q1. Given,

$$\chi(t) = A \cos(\omega_0 t)$$

find

- i) ACF
  - ii) PSD
  - iii) Power

Sol<sup>n</sup>:  $\therefore x(t)$  is periodic fun<sup>n</sup>

Hence, Avg. ACF has to be calculated

50

$$\begin{aligned}
 & \text{So, } R(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) x(t-\tau) dt \\
 &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} A \cos \omega_0 t A \cos (\omega_0 t - \omega_0 \tau) dt \\
 &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{A^2}{2} \cos(2\omega_0 t - \omega_0 \tau) dt + \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \cos \omega_0 \tau dt \cdot \frac{A^2}{2} \\
 &\quad \because \text{Periodic} \\
 &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{A^2}{2} \cos \omega_0 \tau dt = \lim_{T \rightarrow \infty} \frac{1}{T} \cdot \frac{A^2}{2} \cos \omega_0 \tau \int dt \\
 &= \lim_{T \rightarrow \infty} \frac{1}{T} \cdot \frac{A^2}{2} \cos \omega_0 \tau \cdot T
 \end{aligned}$$

$$R(z) = A^2 \cos \omega_0 z$$

~~Nole l.~~

$$\left. \begin{array}{l} A \cos(\omega_0 t + \phi) \\ \text{or} \\ A \sin(\omega_0 t + \phi) \end{array} \right\} \rightarrow R(z) = \frac{A^2}{2} \cos \omega_0 z$$

$$b) S(f) = FT[R(\tau)]$$

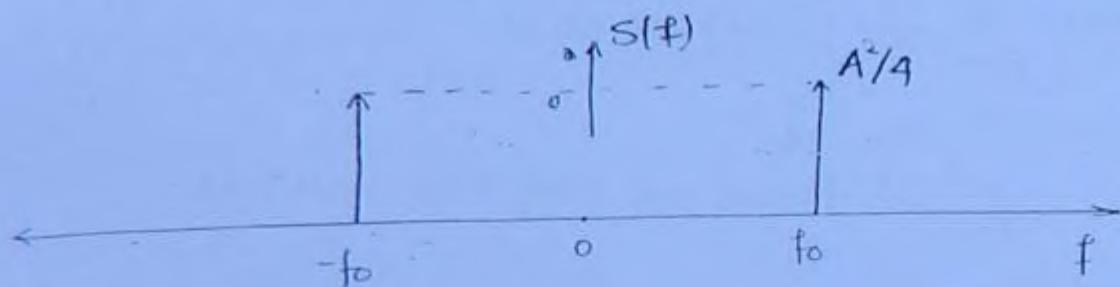
$$= FT\left[\frac{A^2}{2} \cos 2\pi f_0 \tau\right]$$

(298)

$$= \frac{A^2}{2} \left[ \frac{S(f+f_0) + S(f-f_0)}{2} \right]$$

$$\text{So, } S(f) = \frac{A^2}{4} \left\{ S(f+f_0) + S(f-f_0) \right\}. \quad \text{Ans}$$

Plot :



$$c) \text{Power} = \text{Area}[S(f)]$$

$$= \frac{A^2}{4} + \frac{A^2}{4}$$

$$\text{So, Power} = \frac{A^2}{2} \quad \text{Ans}$$

$$\text{Also, Power} = R(\tau)|_{\tau=0}$$

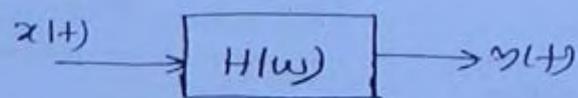
$$= \frac{A^2}{2} \cos 2\pi f_0 \tau \Big|_{\tau=0}$$

$$\text{Power} = R(0) = \frac{A^2}{2} \quad \text{Ans}$$

Q2. A signal of  $x(t) = e^{-2t} u(t)$ , is passed through a system, given by  $H(\omega) = \frac{1}{(j\omega+4)}$ . Find Energy spectral density of O/P of the system?

$$\text{Sol: } (\text{ESD})_{\text{O/P}} = (\text{ESD})_{\text{I/P}} \cdot |H(\omega)|^2$$

$$\text{Now, } |Y(\omega)|^2 = |X(\omega)|^2 \cdot |H(\omega)|^2$$



$$\text{Now, } |X(\omega)| = \frac{1}{\sqrt{\omega^2+4}} \quad \left\{ \because e^{-at} u(t) \longleftrightarrow \frac{1}{s+a} \right\}$$

$$|X(\omega)| = \frac{1}{\sqrt{\omega^2+4}} \Rightarrow |X(\omega)|^2 = \frac{1}{(\omega^2+4)}$$

$$|H(\omega)| = \frac{1}{\sqrt{\omega^2 + 16}}$$

$$|H(\omega)|^2 = \frac{1}{\omega^2 + 16}$$

(29)

So,  $|Y(\omega)|^2 = \frac{1}{(\omega^2 + 4)(\omega^2 + 16)}$  ← PSD of O/P.  
Ans

Q3. A Random variable, of having Auto correlation function

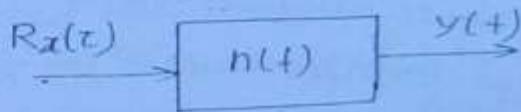
$$R(\tau) = e^{-\sigma|\tau|}$$

is passed through a system, whose Impulse Response is given by  $h(t) = \frac{1}{2}e^{-\alpha|t|} u(t)$ .

Find PSD of the O/P signal?

Soln. Given,  $R(\tau) = e^{-\sigma|\tau|}$

$$h(t) = \frac{1}{2}e^{-\alpha|t|} u(t)$$



Now, as

$$(PSD)_{O/P} = (PSD)_{I/P} \cdot |H(\omega)|^2$$

$$\begin{aligned} (PSD)_{I/P} &= F.T \text{ of } R_x(t) \\ &= F.T [e^{-\sigma|\tau|}] \quad \left\{ e^{-\alpha|t|} \leftrightarrow \frac{2\alpha}{\alpha^2 + \omega^2} \right\}. \end{aligned}$$

$$(PSD)_{I/P} = \frac{2\sigma}{\sigma^2 + \omega^2}$$

$$\text{Now, } H(\omega) \longleftrightarrow \frac{1}{2} \left\{ \frac{1}{\omega + j\omega} \right\}.$$

$$|H(\omega)|^2 \longleftrightarrow \frac{1}{4} \times \frac{1}{(\omega^2 + \omega^2)}$$

So,

$$(PSD)_{O/P} = \frac{2\sigma}{(\sigma^2 + \omega^2)} \times \frac{1}{4} \times \frac{1}{(\omega^2 + \omega^2)}$$

$$(PSD)_{O/P} = \frac{\sigma \omega^2}{2(\sigma^2 + \omega^2)(\omega^2 + \omega^2)}$$

## \* NOISE IN ANALOG COMMUNICATION

(300)

\* Noise may be classified as:

- 1) Internal Noise (within the system)-
- 2) External Noise (external source).

→ Automobile Noise.  
→ Atmospheric Noise.  
→ Solar Noise, etc.

→ Shot Noise  
→ Thermal Noise  
→ Flicker Noise etc

\* Thermal Noise is also called as:-

- a) white Noise.
- b) Gaussian Noise.
- c) Johnson Noise.

### \* Thermal Noise:

- \* Due to Thermal Agitation, atoms in the electrical components will gain energy, moves in Random fashion and collide with each other; heat will be generated. This corresponds to Thermal Noise.
- \* Each of the frequency component is emitted through a comm<sup>n</sup> system will be affected by Thermal Noise, so it is also called as "WHITE NOISE".

\* Thermal Noise Power is given mathematically as:-

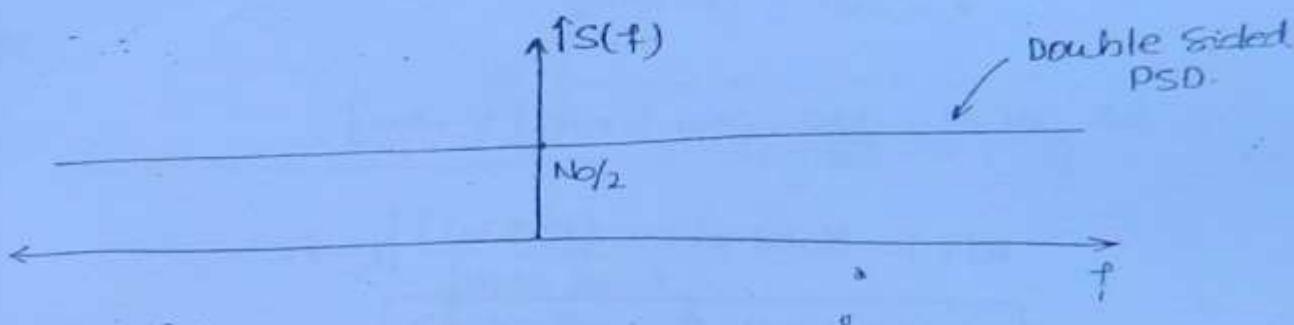
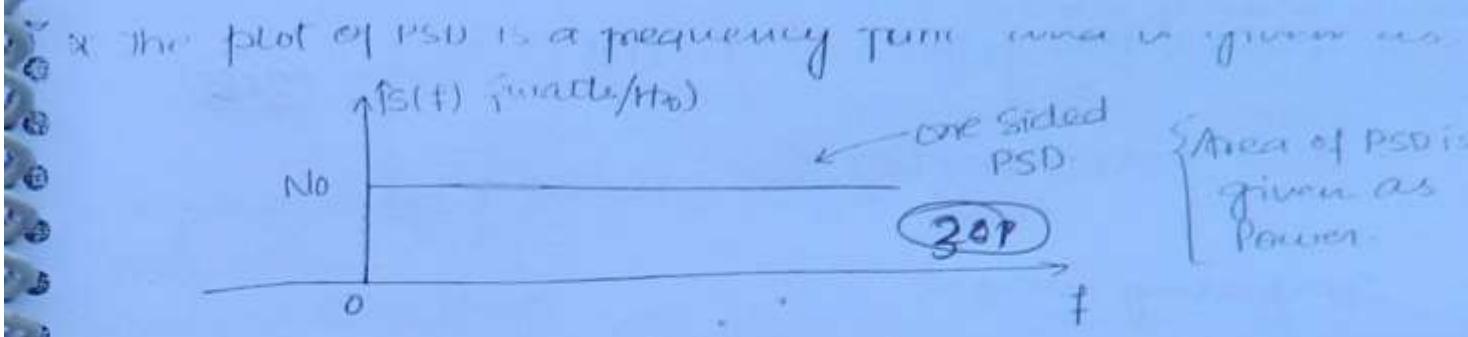
$$N_c = K T B \text{ watt}$$

watts      where  $K = \text{Boltzmann const} = 1.38 \times 10^{-23} \text{ joules/Kelvin}$

$T = \text{Absolute Temp } ^\circ\text{K.}$

$B = \text{Bandwidth.}$

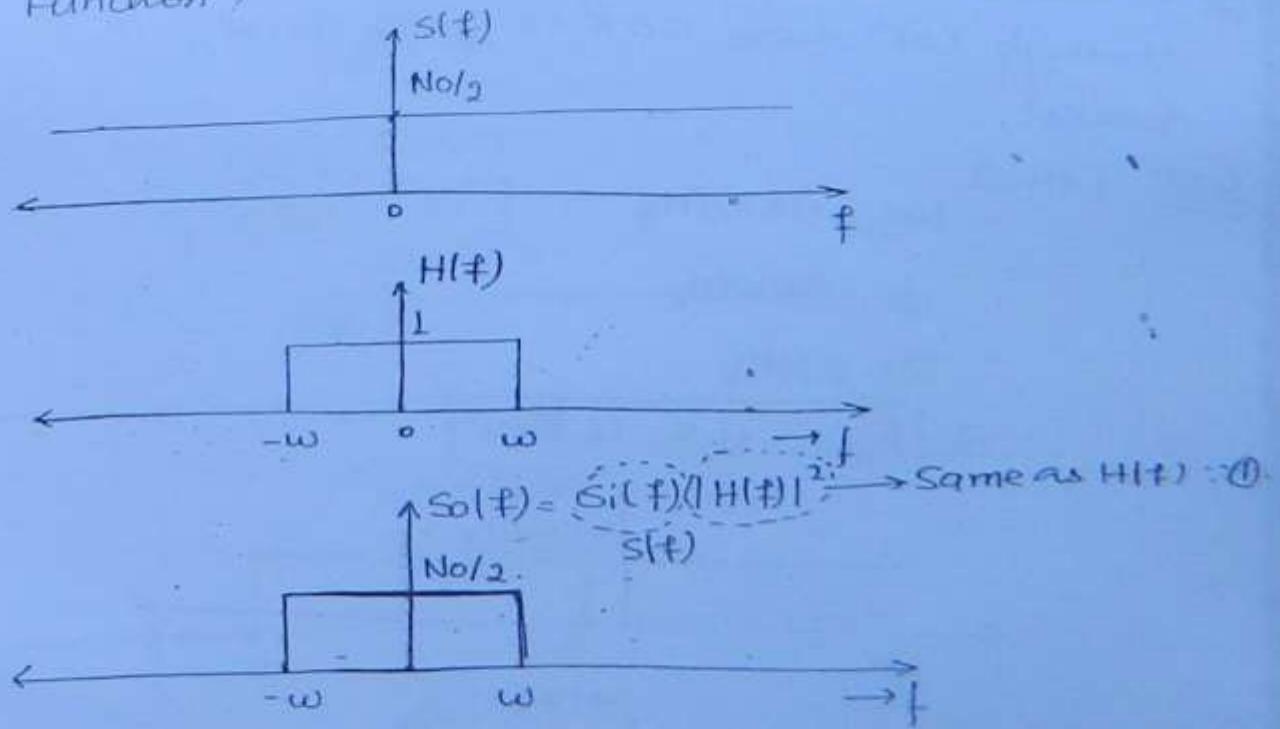
\*  $K T = N_o = \text{Power spectral density (const.)}$



Previous Papers :-

- (Q) A white Noise of having 2 Sided PSD  $No/2$  watts/Hz is passed through a LPF whose cut-off frequency ( $f_c$ ) is  $\omega$  Hz. Find O/P white Noise power and corresponding Auto-correlation function?

Soln:



$$\text{So, O/P Noise power } No = \text{Area of } S_o(f) \\ = 2\omega \times No/2$$

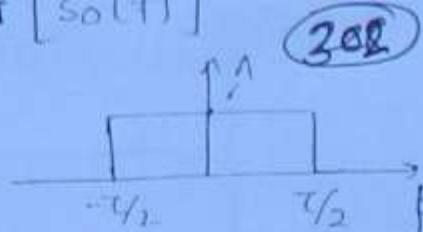
$$\text{O/P Noise power} = No\omega \text{ watts}$$

Auto-correlation function  $R(\tau) = \text{IFT} [S(f)]$

Q2

Now,

$$A \tau \sin(\omega\tau) \longleftrightarrow$$



Comparing we get

$$\omega = 2\pi f; A = N_0/2$$

$$\text{So, } R(\tau) = \frac{N_0}{2} \cdot 2\pi f \sin[\tau \cdot 2\pi f]$$

$$R(\tau) = N_0 \sin[\tau \cdot 2\pi f] \Big|_{f \rightarrow \omega}$$

$$R(\tau) = N_0 \sin[2\pi\omega\tau] \quad \text{Ans}$$

Now,

$$R(0) = N_0 \omega = \text{Power} \quad \text{Ans}$$

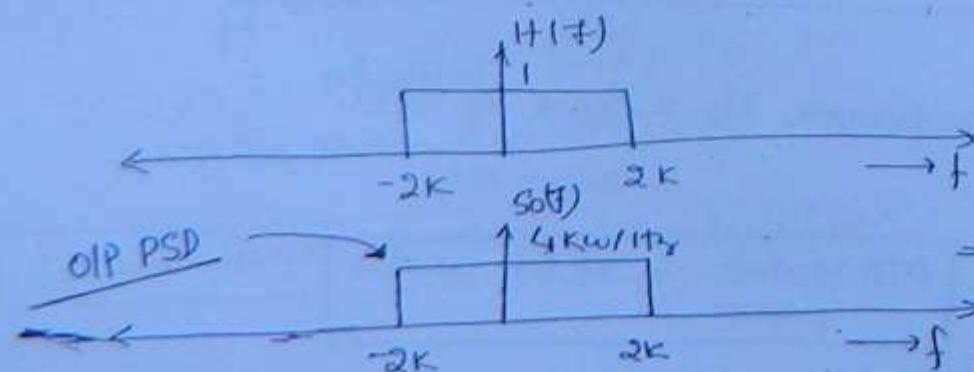
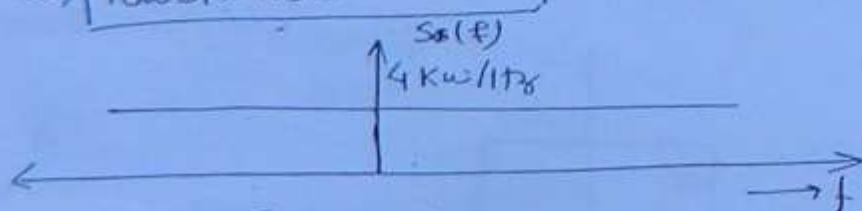
Q2. A white noise of having 2 sided PSD  $4 \text{ kW/Hz}$  is passed through LPF whose C.O.F is  $2 \text{ kHz}$ . Find O/P white Noise power?

Sol: Given,  $N_0/2 = 4 \text{ kW/Hz}$

$$N_0 = 8 \text{ kW/Hz}$$

$$B = 2 \text{ kHz}$$

$$\text{So, Power} = N_0 B = 16 \text{ kW}$$



$$\Rightarrow No = \text{Area}[S_o(f)] = \frac{4 \text{ kW} \times 2K}{4K}$$

$$No = 4 \text{ kW} \quad \text{Ans}$$

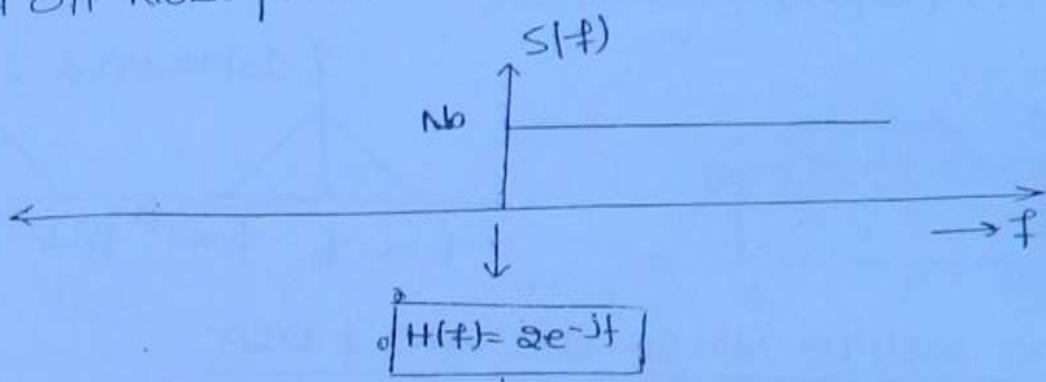
Q3 while noise of having PSD  $S(f) = N_0$  is passed through a system specified by  $H(f) = 2e^{-jf}$

303

The resulting is passed through LPF whose  $C_0(f)$  is  $B \text{ Hz}$ .

Find O/P Noise power.

$S_o(f)$ :



PSD at O/P of the System  $\rightarrow S(f) \cdot |H(f)|^2 = S_o(f)$

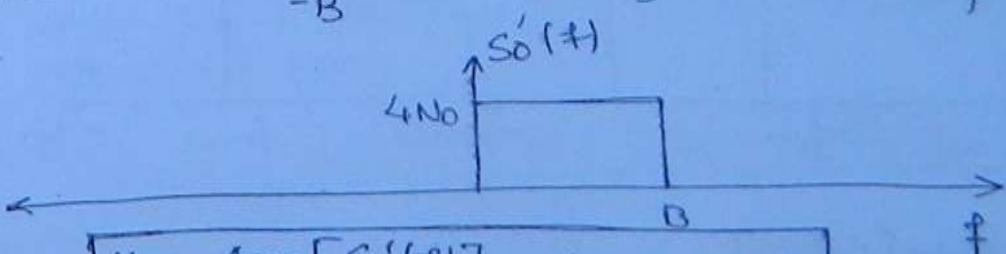
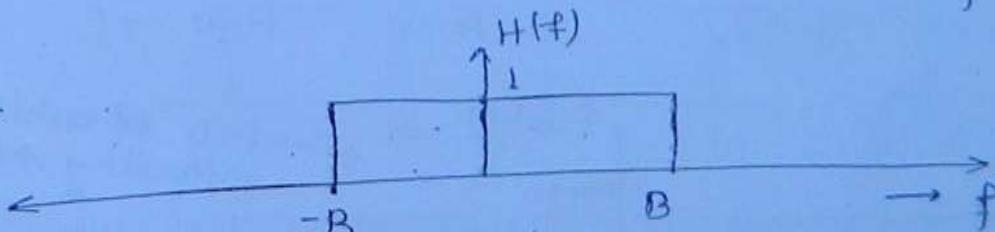
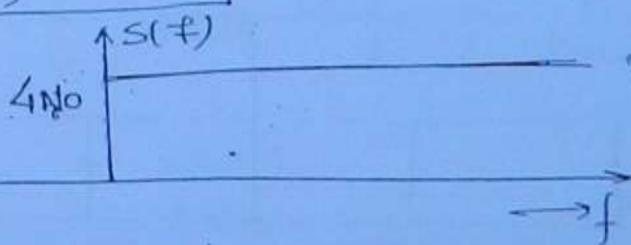
$$H(f) = 2e^{-jf}$$

$$|H(f)| = 2|e^{-jf}|$$

$$|H(f)|^2 = 4$$

$$\text{So, } S_o(f) = 4 \cdot S(f)$$

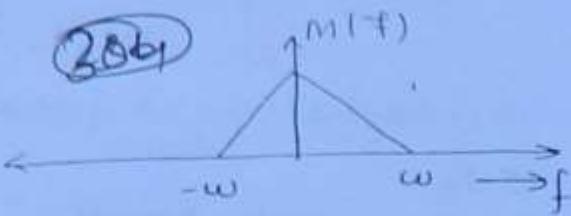
$$S_o(f) = 4N_0$$



$$N_0 = \text{Area}[S'_o(f)] = 4N_0B \text{ watts}$$

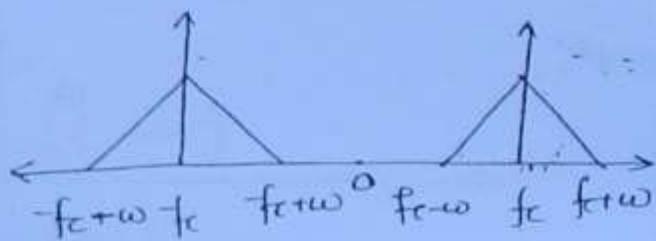
## \* Analysis (Effect of Noise on AM, DSB, SSB)

Assume  $m(t) \longleftrightarrow M(f)$



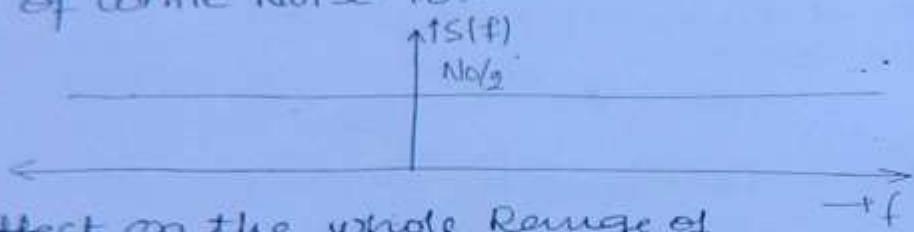
$c(t) = A \cos \omega t$

SAM(f)



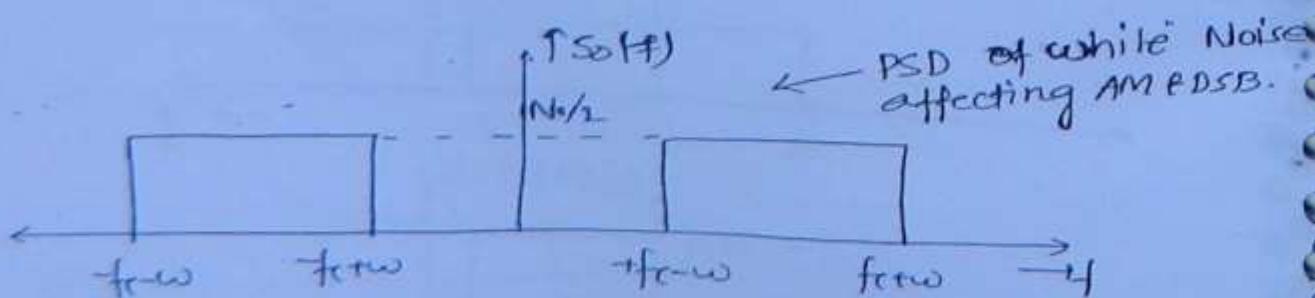
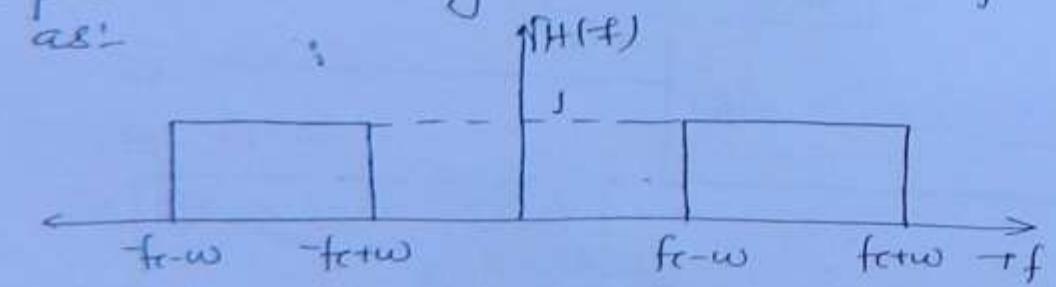
## \* EFFECT OF WHITE NOISE ON AM & DSB:

The PSD of white noise is:-



It has effect on the whole Range of frequency

\* To visualize the effect on the freq<sup>n</sup> components of AM & DSB we pass this through a BPF. So the spectrum is given as:-



$$\text{So, } N = \text{Area}[S(f)]$$

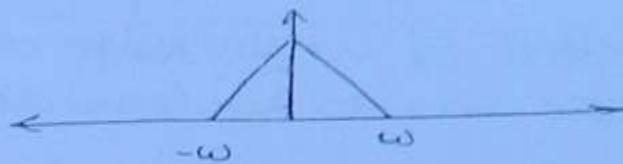
$$= 2w \times No/2 \times 2 \quad [ \text{Two Sided} ]$$

So, Noise power,  $N = \text{Now with}$

### EFFECT OF WHITE NOISE ON SSB!

(305)

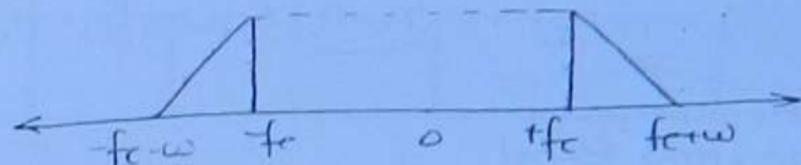
Let  $m(t) \longleftrightarrow$



$c(t) \rightarrow \text{Accosampt}$

So,

$\text{SSB}(t) \longleftrightarrow$



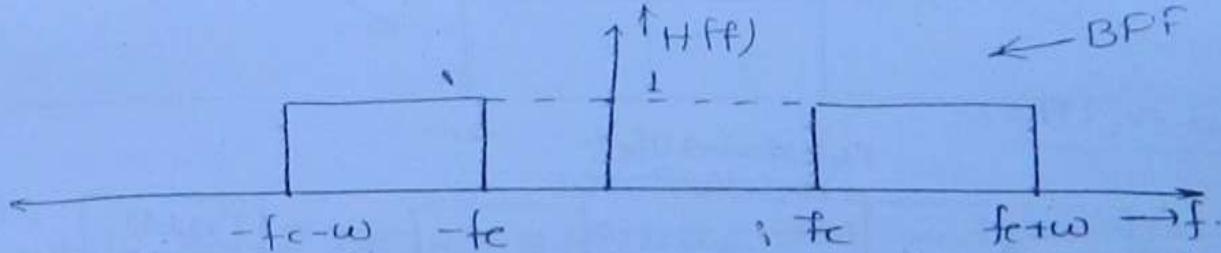
$T(f)$

$No/2$



$H(f)$

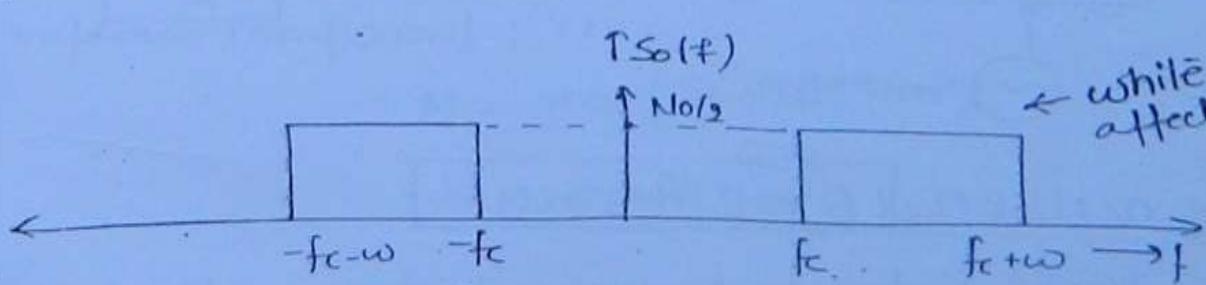
$BPF$



$T_S(t)$

$No/2$

white Noise power  
affecting SSB.



So, white Noise Power =  $N = \omega \times No/2 + \omega \times No/2$   
affecting SSB

$N = \text{Now with}$

## NARROW BAND NOISE

(3B6)

\* When white noise is passed through BPF, the resulting is said to be Narrow Band Noise.

\* To find effect of white noise on AM, DSB and SSB, Narrow Band Noise has to be considered.

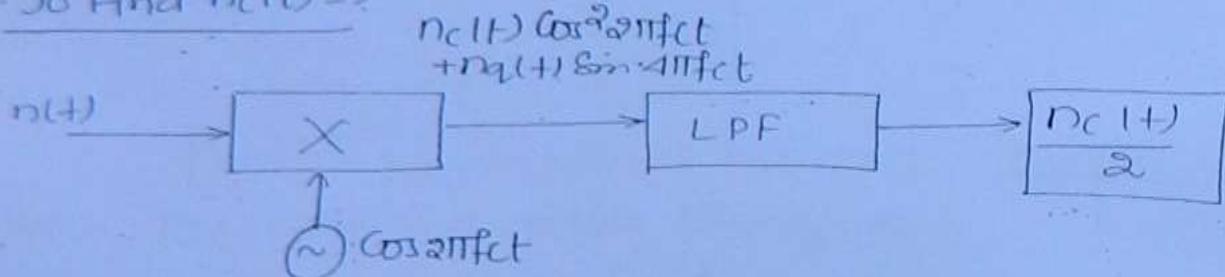
Analysis:

$$n(t) = \underbrace{n_c(t)}_{\text{in phase component}} \cos \omega t + \underbrace{n_q(t)}_{\text{quadrature component}} \sin \omega t$$

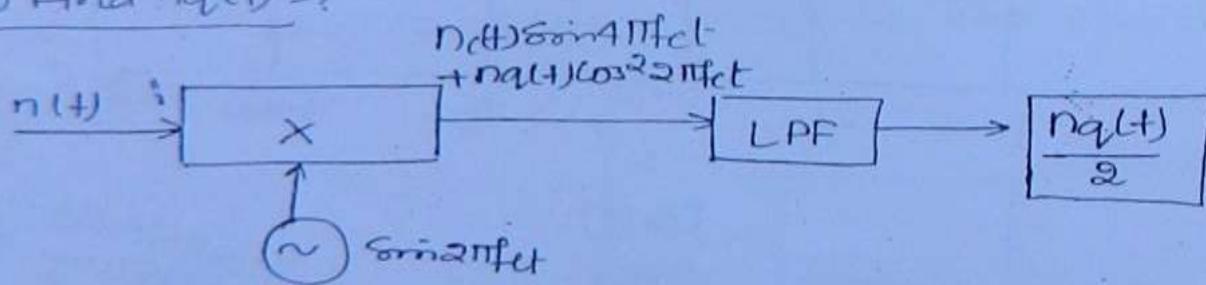
time domain  
Narrow Band  
Noise

$$n(t) = \text{IFT}[s_0(f)]$$

1. To find  $n_c(t) = ?$



2. To find  $n_q(t) = ?$



PSD.

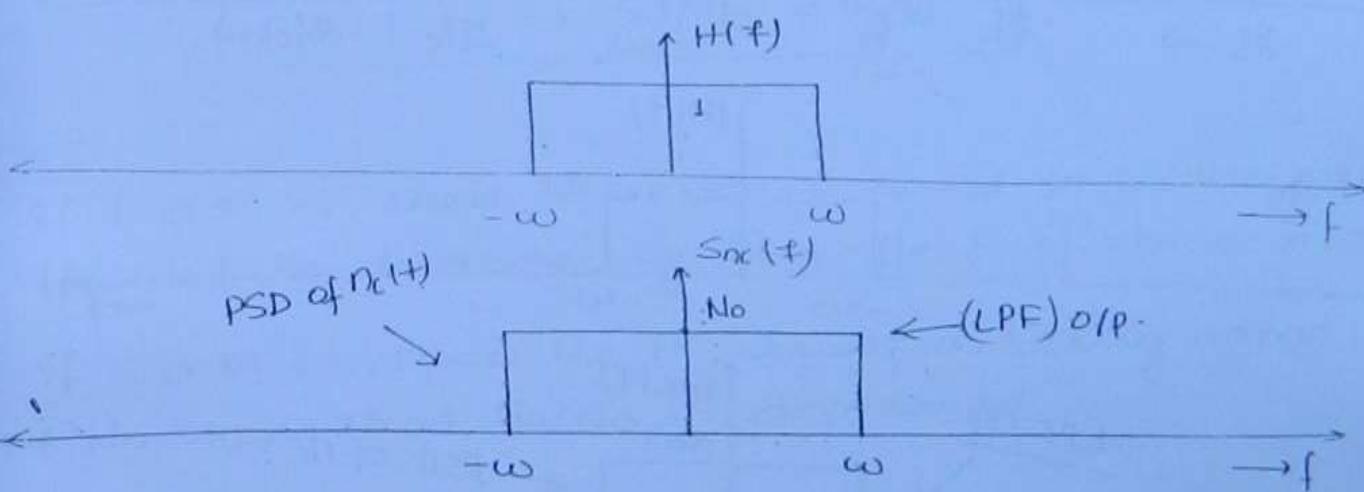
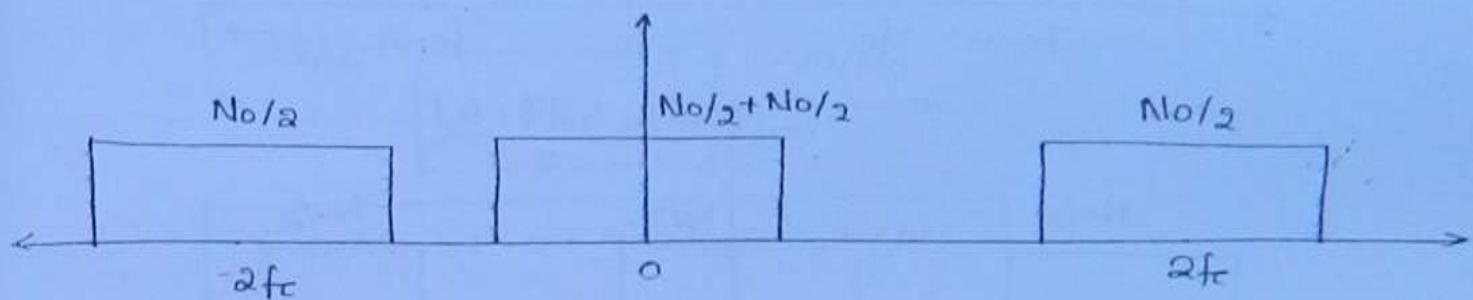
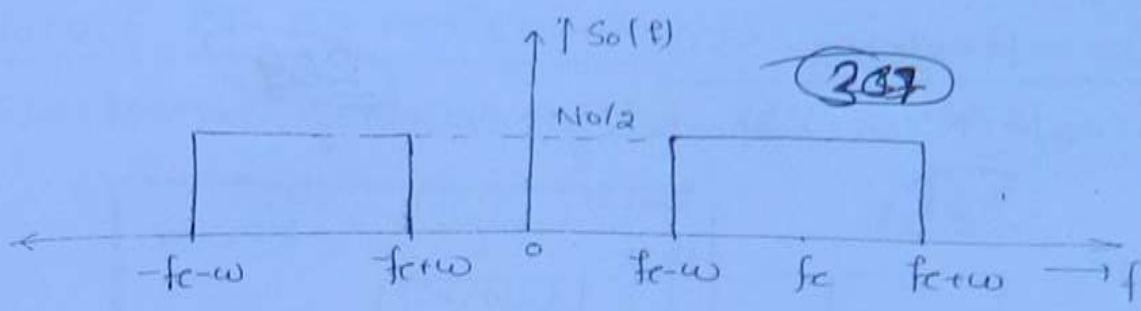
\* Effect of  $n_c(t)$  &  $n_q(t)$  on AM, DSB:

1) by  $n_c(t)$  :-

$$n(t) \longleftrightarrow s_0(f)$$

when multiplied by  $\cos \omega t$ , the  $s_0(f)$  is shifted left & right by amount  $f_c$

$$\text{so (O/P)mul} \longrightarrow \frac{s_0(t+f_c) + s_0(t-f_c)}{2}$$



So, while Noise power affecting AM and DSB due to its inphase component;  $N$

$$N = \text{Area}[S_{nc}(f)]$$

$$\boxed{N = 2N_0\omega \text{ watts}}$$

\* Due to the whole component of white Noise is also  
 $N = 2N_0\omega$ .

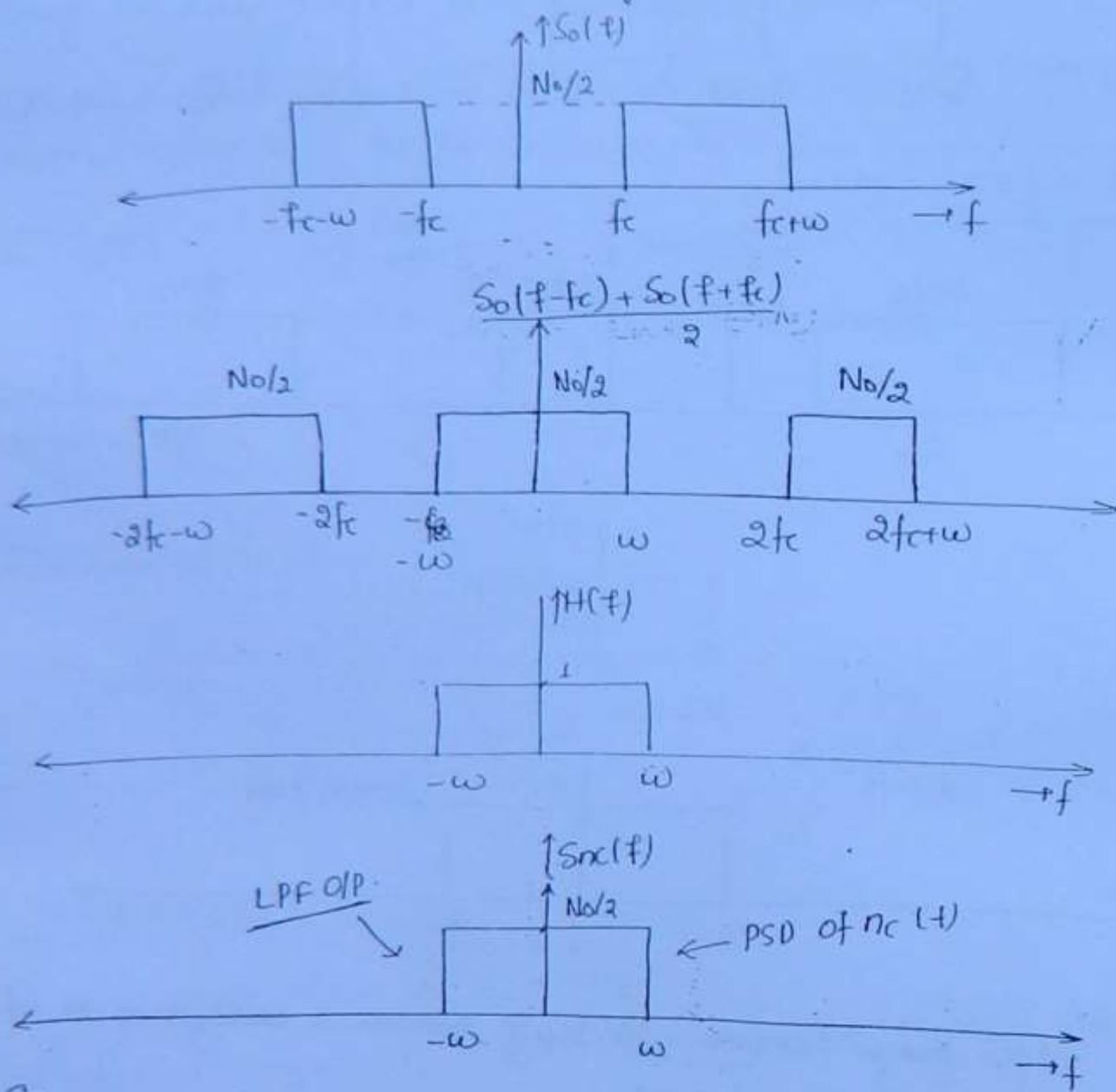
Conclusion:

The effect of white Noise on AM and DSB is only due to its inphase components, so that the effect of quadrature component will be null (i.e. 0).

$\times$  PSD of M(H) affecting SSB

300

$\times$  PSD of white Noise affecting SSB is given as



So, Power = Area [ $S_{nc}(t)$ ]

$$\boxed{\text{Power} = N_0 w}$$

$\times$  Due to Actual white Noise,  $\boxed{\text{Power} > N_0 w}$

Conclusion:

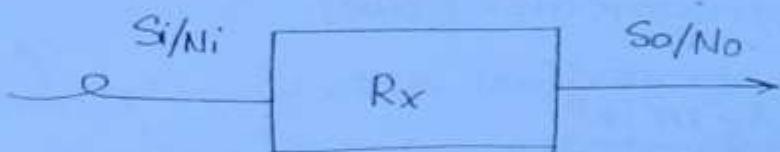
The effect of Quadrature Component of white Noise on SSB will be NULL.

## \*FIGURE OF MERIT (F.O.M)

Mathematically F.O.M is given as:

$$F.O.M = \frac{(S_o/N_o)}{(S_i/N_i)}$$

(303)



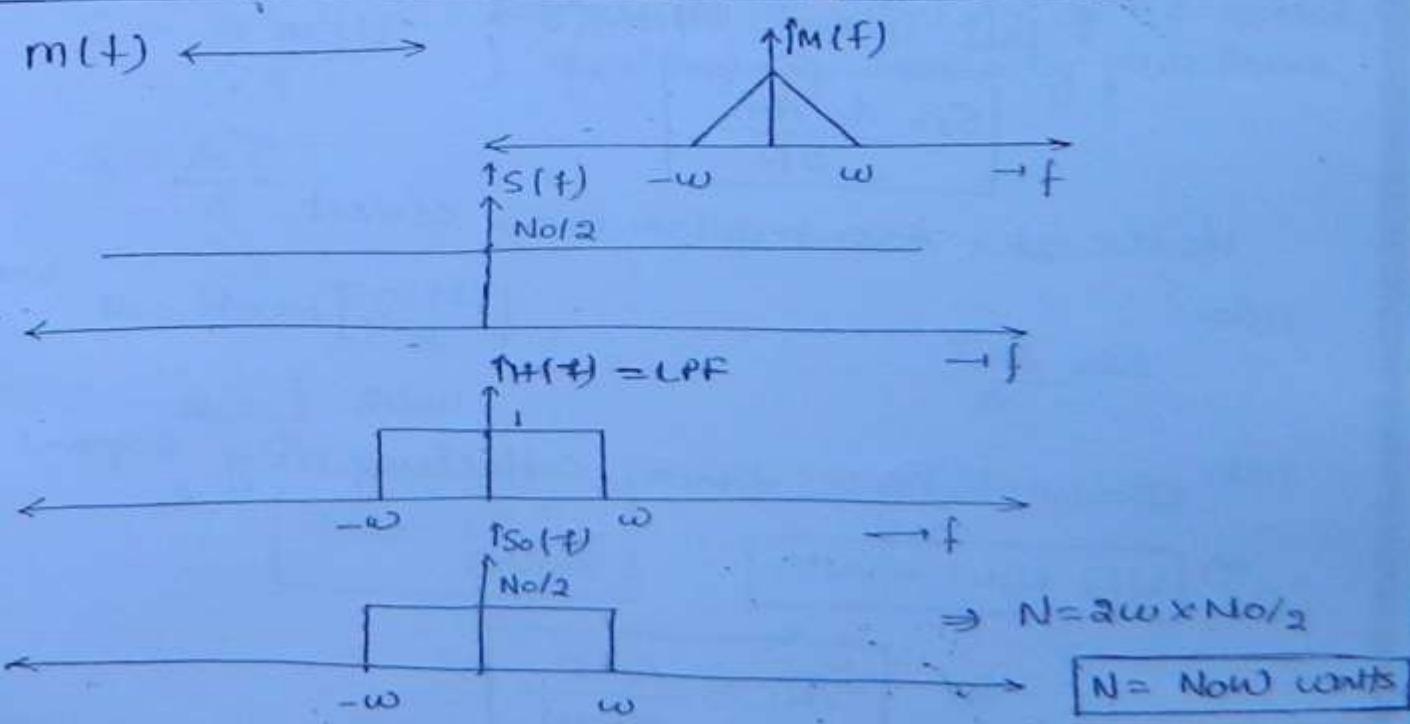
$$F.O.M > 1 \Rightarrow S_o/N_o > S_i/N_i$$

$$F.O.M < 1 \Rightarrow S_o/N_o < S_i/N_i$$

Note :

1. If  $F.O.M > 1$ , then Receiver is said to be very much efficient in decreasing the effect of Noise alone.
2. If  $F.O.M < 1$ , then Rx is itself adding some amount of Noise, so that  $S_o/N_o$  is decreased.

## \*WHITE NOISE POWER AFFECTING MSG SIGNAL.



## \* FIGURE OF MERIT OF DSB RV

As,  $S_{DSB}(t) = A_c m(t) \cos 2\pi f_c t$

Power of signal,  $S_i = \frac{A_c^2 m^2(t)}{2R}$

(310)

as  $m(t)$  is changing, hence, it gives the instantaneous power.

So,

$$S_i = \frac{A_c^2 m^2(t)}{2}$$

Let,  $m^2(t) = \text{instantaneous power of } m(t) = P$

So,  $S_i = \frac{A_c^2 P}{2} \quad \dots \dots \quad (A)$

Now, let  $m(t) = A_m \cos 2\pi f_m t$

So,  $S_{DSB} = A_c A_m \cos 2\pi f_m t \cos 2\pi f_c t$

So, Power (DSB) =  $\frac{A_c^2 A_m^2}{4R}$

Now,  $P \text{ of } m(t) = A_m^2 / 2R$

& put in eqn (A) we get:-

$$\boxed{S_i = \frac{A_c^2 A_m^2}{2R}}$$

Hence, the assumption was correct.

Now,

$$S_i = \frac{A_c^2 P}{2}$$

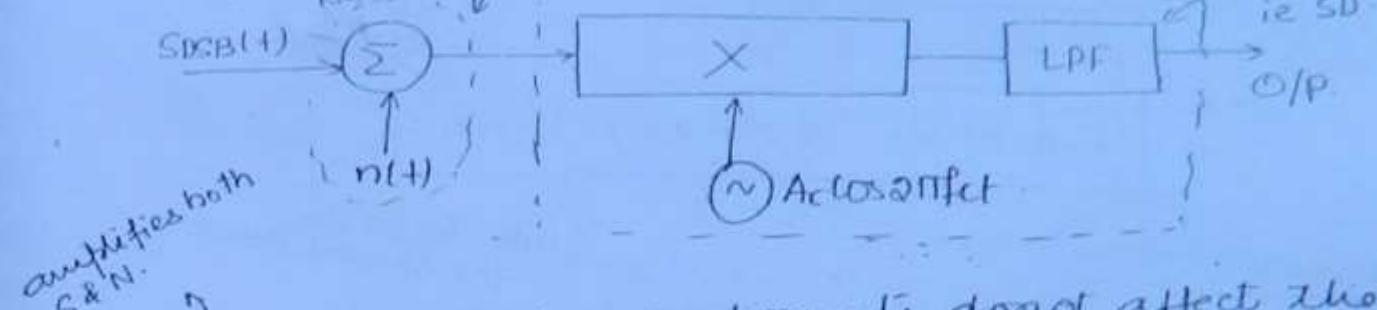
Let,  $N_i = \text{white noise power affecting msg signal}$

So,  $\boxed{N_i = N_o W \text{ watts.}}$

So,  $\boxed{S_i/N_i = \frac{A_c^2 P}{2N_o W}}$

• SSB(+) is transmitted using narrow band noise source

Noise is added in the channel  
channel noise considered  $\rightarrow (S_0/N_0)$



× The amplifier and other components do not affect the (S/N) but only demodulator affect the (S/N). Hence, it is taken into consideration only

$$S_0 \text{ (mul) } O/P = \{S(t) + n(t)\} \text{ Cosωt}$$

$$= \{A_{cm}(t) \cos \omega t + n_c(t) \cos \omega t + n_q(t) \sin \omega t\}$$

· Cosωt

And,

$$(LPF)_{O/P} = \underbrace{\left(\frac{A_{cm}(t)}{2}\right)}_{\text{Signal}} + \underbrace{\left(\frac{n_c(t)}{2}\right)}_{\text{Noise}}$$

So,  $S_0 = \text{Power} \left[ \frac{A_{cm}(t)}{2} \right]$

$$S_0 = \frac{A_{cm}^2(t)}{4}$$

$\therefore A_{cm}(t)$  is either AC or DC is not specified. So, squaring gives power

$$S_0 = \frac{A_{cm}^2 P}{4}$$

And,  $N_0 = \text{Power} \left[ \frac{n_c(t)}{2} \right]$

$$N_0 = \frac{1}{4} \cdot 2N_0W$$

So,

$S_0/N_0 = \frac{A_{cm}^2 P}{2N_0W}$
--------------------------------------

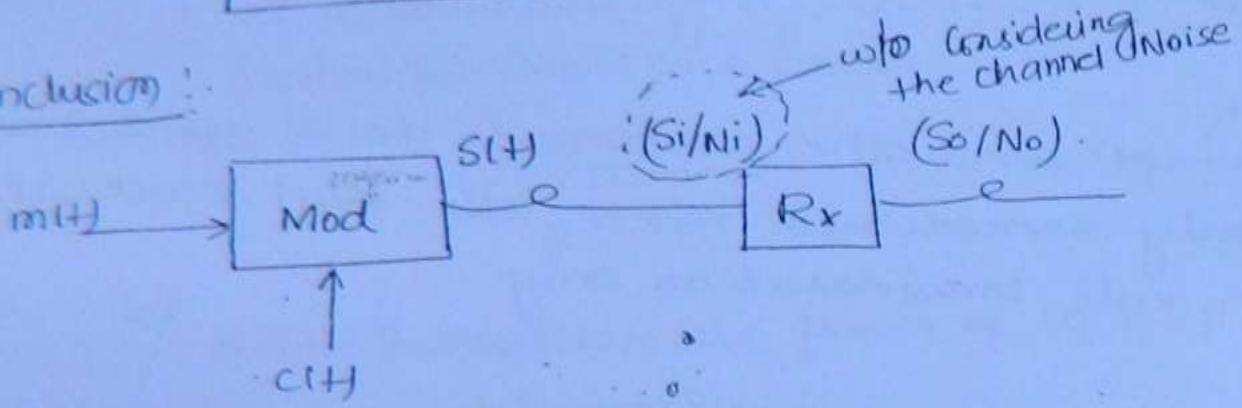
Now,

$$F.O.M = \frac{(S_o/N_o)}{(S_i/N_i)}$$

8.12

$$F.O.M = 1 \Rightarrow (S_o/N_o) = (S_i/N_i)$$

Conclusion:



\*  $(S_i/N_i)$  is calculated by considering the effect of Noise on the msg signal. But  $(S_i/N_i)$  is to be calculated at the Rx by considering the effect of Noise on the modulated signal. But  $(S_o/N_o) = (S_i/N_i)$ ; hence it may be concluded that the demodulator is eliminating the effect of channel noise.

\* Synchronous detector is working efficiently in nullifying the white noise affecting DSB signal in the channel.

\* F.O.M of SSB Rx:

General exp of SSB is given as:

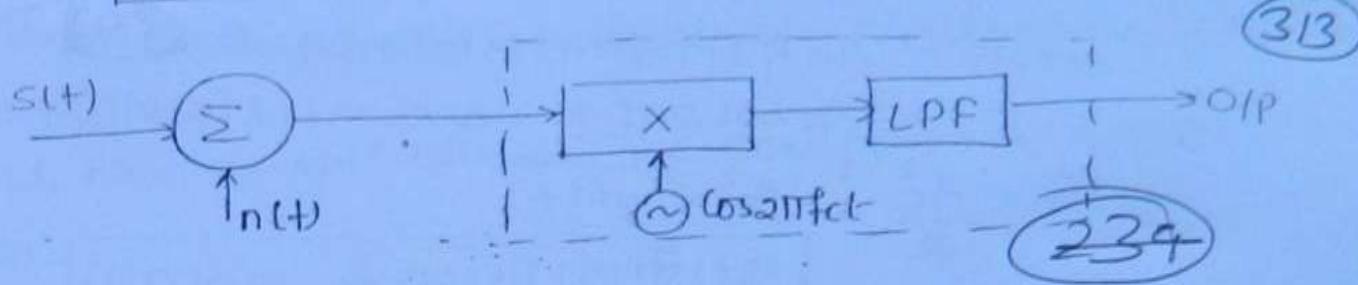
$$S(t+) = \frac{A_c m(t) \cos 2\pi f_c t}{2} + \frac{\hat{A}_c m(t)}{2} \sin 2\pi f_c t$$

$$S_i = \frac{S_{DSB}}{2} \left\{ \text{half of DSB-power} \right\}$$

$$S_i = \frac{A_c^2 P}{4}$$

$N_i$ : Noise power affecting msg signal

$$N_i = N_{\text{now}} \text{ watt} \Rightarrow S_i/N_i = A_c^2 P / 4 N_{\text{now}}$$



$$(m.u)_{\text{O/P}} = \{s(t) + n(t)\} \cos 2\pi fct$$

$$\begin{aligned} &= \left\{ \frac{A_c m(t)}{2} \cos 2\pi fct + \frac{A_c \hat{m}(t)}{2} \sin 2\pi fct + n(t) \cos 2\pi fct \right. \\ &\quad \left. + n_q(t) \sin 2\pi fct \right\} \cos 2\pi fct \end{aligned}$$

$\downarrow$  Signal

$$(LPF)_{\text{O/P}} = \left\{ \frac{A_c m(t)}{4} + \frac{n_c(t)}{2} \right\} \leftarrow \text{Noise}$$

Now,

$$S_o = \text{Power} \left\{ \frac{A_c m(t)}{4} \right\}$$

$$S_o = \frac{A_c^2 m^2(t)}{16}$$

$$S_o = \frac{A_c^2 P}{16}$$

$$N_o = \text{Power} \left\{ \frac{n_c(t)}{2} \right\}$$

$$= \frac{1}{4} n_c^2(t)$$

$$N_o = \frac{1}{4} \times N_{\text{now}}$$

$$N_o = N_{\text{now}}/4$$

$$S_o, \quad \left( \frac{S_o}{N_o} \right) = \frac{A_c^2 P}{4 N_{\text{now}}}$$

$$\therefore \left( \frac{S_o}{N_o} \right) = (S_i/N_i)$$

$$S_o, \quad F.O.M = 1$$

## \* FORM OF AM Rx:

General exp of AM signal is given as:

$$\begin{aligned} S_{AM}(t) &= A_c \{ 1 + K_a m(t) \} \cos 2\pi f_c t \\ \text{So, } S_i &= \frac{A_c^2}{2} + \frac{A_c^2 K_a^2 m^2(t)}{2} \end{aligned}$$

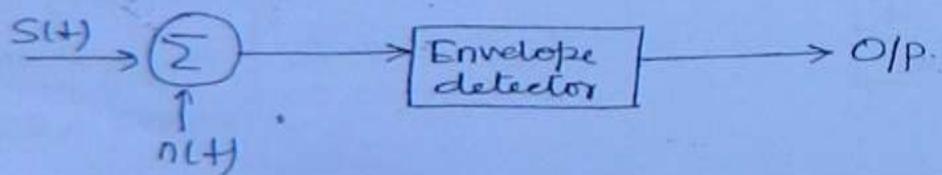
$$S_i = \frac{A_c^2}{2} + \frac{A_c^2 K_a^2 P}{2}$$

$$S_i = \frac{A_c^2}{2} (1 + K_a^2 P)$$

$N_i$  = Noise power affecting msg

$$N_i = N_{now} \text{ watts}$$

$$\text{So, } \left( \frac{S_i}{N_i} \right) = \frac{A_c^2}{2 N_{now}} (1 + K_a^2 P)$$



$$\text{Now, } (ED)y_p = \{ S(t) + n(t) \}$$

$$= \{ A_c \cos 2\pi f_c t + A_c K_a m(t) \cos 2\pi f_c t + n_c(t) \cos 2\pi f_c t + n_q(t) \sin 2\pi f_c t \}$$

Now, as

$$A_c \cos 2\pi f_c t + B \sin 2\pi f_c t \xrightarrow{\text{E/D O/P}} \sqrt{A^2 + B^2}$$

$$(ED)y_p = \underbrace{\{ A_c \cos 2\pi f_c t + A_c K_a m(t) \cos 2\pi f_c t + n_c(t) \cos 2\pi f_c t \}}_A + \underbrace{\{ n_q(t) \sin 2\pi f_c t \}}_B$$

$$\text{So, } (ED)_{OIP} = \sqrt{\{A_c K_a m(t) + n_c(t)\}^2 - \{n_o(t)\}^2}$$

(215)

As we know that the effect of quadrature component is 0.

And, the amp<sup>r</sup> blocks D.C components also

$$\text{So, } (ED)_{OIP} = \sqrt{A_c K_a m(t) + n_c(t)}$$

Signal                      Noise

Now,

$$S_o = \text{Power} \{ A_c K_a m(t) \} = A_c^2 K_a^2 m^2(t) = A_c^2 K_a^2 P$$

$$N_o = \text{Power} \{ n_c(t) \} = 2 N_o \omega$$

So,

$$\frac{S_o}{N_o} = \frac{A_c^2 K_a^2 P}{2 N_o \omega}$$

Then,

$$\begin{aligned} F.O.M &= \frac{(S_o/N_o)}{(S_i/N_i)} \\ &= \frac{A_c^2 K_a^2 P \times 2 N_o \omega}{2 N_o \omega \times A_c^2 (1 + K_a^2 P)} \end{aligned}$$

$$F.O.M = \frac{K_a^2 P}{(1 + K_a^2 P)}$$

let,

$$m(t) = A_m \cos 2\pi f_m t$$

$$\text{then Power} \{ m(t) \} = P = \frac{A_m^2}{2}$$

putting in above we get:-

$$\begin{aligned} F.O.M &= \frac{K_a^2 \frac{A_m^2}{2}}{(1 + K_a^2 \frac{A_m^2}{2})} \end{aligned}$$

$$F.O.M. = \frac{(K_2 A_m)}{\{2 + (K_2 A_m)^2\}}$$

8/64

$$F.O.M. = \frac{u^2}{2+u^2}$$

\* Also,  $\eta = \frac{u^2}{2+u^2} = F.O.M.$

Now,  $u = 0.5 \Rightarrow \eta = 0.11$

$u = 0.707 \Rightarrow \eta = 0.2$

$u = 1 \Rightarrow \eta = \frac{1}{3} = 0.33$

Conclusion:

FOM  $\uparrow$  as  $u \uparrow$

$$(F.O.M.)_{max} = \frac{1}{3} \text{ for } u=1$$

so,

$$\left(\frac{S_o}{N_o}\right) = \frac{1}{3} \left(\frac{S_i}{N_i}\right)$$

Note:

1. The performance of Envelope detector against channel noise is poor.

No IMP

\* F.O.M OF FM Receiver:

The F.O.M of FM Receiver is given by:-

$$F.O.M. = \frac{3 K_f^2 P}{w^2}$$

where,

$K_f$  = freq<sup>n</sup> sensitivity of FM mod

P = Power of m(t)

w = msg B.W

Let  $m(t) = A_m \cos 2\pi f_m t$

$$P = \frac{A_m^2}{2} \cdot 4\pi^2 f_m^2$$

(245) (317)

$$\text{So, F.O.M} = \frac{3K_f^2 \cdot A_m^2 / 2}{f_m^2}$$

$$\text{F.O.M} = \frac{3}{2} \left\{ \frac{K_f A_m}{f_m} \right\}^2$$

$$= \frac{3}{2} \left\{ \frac{\Delta f}{f_m} \right\}^2$$

$$\boxed{\text{F.O.M} = \frac{3}{2} \beta^2}$$

\* For NBFM:

For NBFM;  $\beta \leq 1$  (small)

$$\beta_{\max} = 1$$

$$\text{So, } \boxed{\text{F.O.M} = 3/2 = 1.5}$$

\* Let  $\beta = 0.5 = \frac{1}{2}$

$$\boxed{\text{F.O.M} = \frac{3}{2} \times \frac{1}{4} = 3/8 = 0.375}$$

Conclusion:

FOM of NBFM is small.

\* For WBFM:

For WBFM;  $\beta > 1$  (high)  ~~$\beta_{\min} = 1$~~ .

$$\text{FOM} = \frac{3}{2} \cdot \beta^2$$

As  $\beta \uparrow \quad \text{FOM} \uparrow$

but  $\beta \uparrow \rightarrow \uparrow B \cdot w = 2(\beta + 1)f_m$

Conclusion: So, Generally, the value of  $\beta$  is Restricted to 10

$$\boxed{\beta = 10, \text{ FOM} = 150}$$

WBFM is preferred over NBFM because of its high FOM.

Q1 For an FM, given

$$(S/N)_{O/P} = 30 \text{ dB}, (S/N)_{I/P} = 20 \text{ dB}$$

218

Find the value of  $B$ .

$$\text{Soln: } (S/N)_O = 30 \text{ dB} \Rightarrow 10 \log_{10} (S/N)_O = 30$$

$$(S/N)_{O/P} = 10^3 = 1000$$

$$(S/N)_I = 20 \text{ dB} \Rightarrow 10 \log_{10} (S/N)_I = 20$$

$$(S/N)_I = 100$$

So,

$$\frac{(S/N)_O}{(S/N)_I} = 10 = \frac{3}{2} B^2$$

$$B = \sqrt{\frac{20}{3}}$$

$$B = 2.58$$

Q2 A video signal of having BW of 10MHz power of 1mW is transmitted through a channel. Power loss in the channel is given by 40dB.

Noise PSD is given by  $10^{-20} \text{ watts/Hz}$ .

Find S/N at the I/F of the Receiver.

Soln: Given, BW = 10MHz.

Power = 1mW.

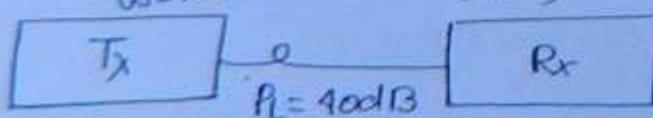
Power loss in channel = 40dB.

Noise PSD; No  $= 10^{-20} \text{ watts/Hz}$

$$P_t = 1 \text{ mW}$$

$$W = 10 \text{ MHz}$$

$$(S_i/N_i) = ?$$



As we know that,

$$N_i = N_0 u$$

$$= 10^{-20} \times 100 \times 10^6$$

$$N_i = 10^{-12} \text{ watts}$$

If there is no path loss  $\Rightarrow P_t = S_i$

But Path loss = 10dB (P.L.)

(245) (319)

$$\text{So, } S_i = (P_t - P_L)$$

$$(S_i)_{\text{dB}} = (P_t)_{\text{dB}} - (P_L)_{\text{dB}}$$

$$\therefore 10 \log_{10}(S_i) = 10 \log_{10}(P_t) - 10 \log_{10}(P_L)$$

$$\Rightarrow 10 \log_{10}(S_i) = 10 \log(P_t/P_L)$$

$$S_i = (P_t/P_L)$$

$$S_i = \frac{1 \times 10^{-3}}{(10^4)}$$

$$S_i = 10^{-7} \text{ watts}$$

$$\text{Then, } (S_i/N_i) = \frac{(10^{-7})}{(10^{-12})}$$

$$(S_i/N_i) = 10^5$$

$$\boxed{\left(\frac{S_i}{N_i}\right)_{\text{dB}} = 50 \text{ dB}}$$

- Q3 Audio signal band limited to 15KHz is transmitted through a channel after modulation. Power loss in the channel is given by 50dB. 2 sided Noise PSD is  $10^{-10} \text{ watts/Hz}$ . Find transmitted power required to get  $(S/N)_{\text{O/P}}$  of 10dB if the modulation scheme used is:-

- DSB.
- AM with  $M=1$ .
- FM with  $B=5$ .

Sol<sup>n</sup> Given,

B.W., w = 15KHz

P<sub>L</sub> = 50dB

$$\frac{N_0}{2} = 10^{-10} \text{ mW/Hz/1Bz}$$

$$(S/N)_{O/P} = 40 \text{ dB}$$

(246) (320)

a) For DSB

$$(S/N)_{O/P} = (S/N)_{I/P}$$

$$\text{So, } (S_i/N_i) = \left( \frac{S_i}{N_0 w} \right) = 40 \text{ dB}$$

$$\left( \frac{S_i}{(2 \times 10^{-10} \times 15 \times 10^3)} \right) = 10000$$

$$S_i = 0.03$$

$$\text{Now, } S_i = \frac{P_t}{P_L} = 0.03$$

$$\text{So, } P_t = (0.03 \times P_L) \\ = 0.03 \times 1 \times 10^5$$

$$\boxed{P_t = 3 \text{ kW}} \quad \underline{\text{Ans}}$$

b) For AM :

$$F.O.M = \frac{U^2}{2+U^2} = \frac{1}{3}$$

$$\text{So, } \left( \frac{S}{N} \right)_{O/P} = \frac{1}{3} (S_i/N_i)$$

$$\text{So, } \left( \frac{S_i}{N_i} \right) = 3 \times \cancel{40000} \times 10^4$$

$$S_i = 3 \times 10^4 \times 2 \times 10^{-10} \times 15 \times 10^3$$

$$\frac{P_t}{P_L} = 0.09$$

$$P_t = 0.07 \times 10^{4.5}$$

$$\boxed{P_t = 9 \text{ kW}} \quad \underline{\text{Ans}}$$

(22)

2) For FM:

$$FOM = \frac{3}{2} \beta^2$$

$$\text{for } \beta = 5$$

$$FOM = 37.5$$

$$\text{So, } (S_0/N_0) = 37.5 \times (Si/Ni)$$

$$(Si/Ni) = \frac{10^4}{37.5}$$

$$Si = \frac{10^4}{37.5} \times 2 \times 10^{-10} \times 15 \times 10^3$$

$$\frac{P_t}{P_L} = \frac{1}{1250}$$

$$\boxed{P_t = 80 \text{ W}} \quad \underline{\text{Ans}}$$

## X NOISE & DIGITAL COMMUNICATION:

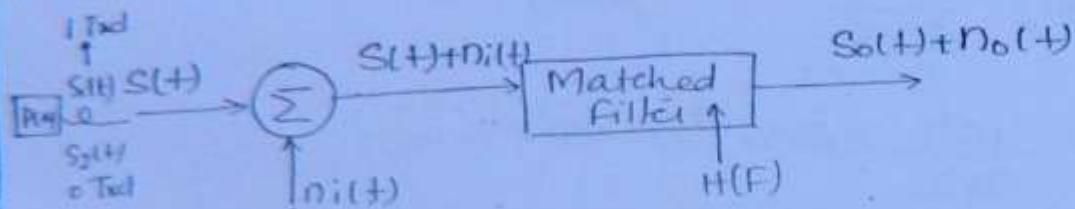
(322)

### \* MATCHED FILTER

- \* The importance of Matched Filter is to reduce the Prob. of Error.

Note:-

- \* Matched filter is used for digital Rx before threshold comparator
- \* It increases S/N<sup>↑</sup> ratio so that Pef will be decreased.



$$(\text{SNR})_{\text{O/P}} = \frac{|S_o(t)|^2}{N_o}, \quad N_o = \text{O/P Noise power.}$$

- \* Matched filter increases the SNR, so we have to calculate the characteristic of MF ie H(F).

- \* Let,  $n(t)$  = white noise possessing Gaussian density function with 'o' mean, and having 2 sided PSD of  $N_0/2w$ .

NOW,  $S_o(t) = S(t) * h(t)$

where,

$S(t)$  = Input Signal Power

$h(t)$  = Impulse Response of MF.

Taking FT we get:-

$$S_o(f) = S(f) \cdot H(f).$$

$$\begin{aligned} f \quad S_o(t) &= \text{IFT}\{S_o(f)\} \\ &= \int_{-\infty}^{\infty} S(f) \cdot e^{j2\pi ft} df \end{aligned}$$

$$S(f) = \int_{-\infty}^{\infty} S(f) H(f) e^{j2\pi f t} df$$

(32)

Now,  $N_o$ : O/P noise power

$$(Noise PSD)_{OIP} = (Noise PSD)_{YP} |H(f)|^2$$

$$S_o(f) = \frac{N_o}{2} |H(f)|^2$$

So, OIP noise power  $\{N_o\}$  = Area  $[S_o(f)]$ .

$$N_o = \int_{-\infty}^{\infty} S_o(f) df$$

$$N_o = \int_{-\infty}^{\infty} \frac{N_o}{2} |H(f)|^2 df$$

$x(SNR)$  at a specific time instant of  $t=T$  is given by:

$$\text{Then } (SNR)_{OIP} = \frac{|S_o(T)|^2}{N_o}$$

$$\text{So, } \left(\frac{S}{N}\right)_o = \frac{\left| \int_{-\infty}^{\infty} S(f) H(f) e^{j2\pi f T} df \right|^2}{N_o}$$

$\times (S/N)$  depends upon  $H(f)$ . Hence for  $H(f)$  value the  $(S/N)_o$  reaches max<sup>m</sup> has to be calculated.

Now, according to Schwartz's inequality

$$\left( \int_{-\infty}^{\infty} S(f) H(f) df \right) \left| \int_{-\infty}^{\infty} x_1(f) x_2(f) df \right|^2 \leq \int_{-\infty}^{\infty} |x_1(f)|^2 df \cdot \int_{-\infty}^{\infty} |x_2(f)|^2 df$$

So, applying above to  $(S/N)_o$  we get:

$$\left| \int_{-\infty}^{\infty} \frac{S(f)}{x_1} H(f) e^{j2\pi f T} df \right|^2 \leq \int_{-\infty}^{\infty} |S(f)|^2 df$$

$$\left| \int_{-\infty}^{\infty} \frac{H(f)}{N_0} \frac{s(f) e^{j2\pi f T}}{x_2} df \right|^2 \leq \int_{-\infty}^{\infty} |H(f)|^2 df \cdot \int_{-\infty}^{\infty} |s(f) e^{j2\pi f T}|^2 df$$

$$\left| \int_{-\infty}^{\infty} H(f) s(f) e^{j2\pi f T} df \right|^2 \leq \int_{-\infty}^{\infty} |H(f)|^2 df \cdot \int_{-\infty}^{\infty} |s(f)|^2 df$$

$\therefore |e^{j2\pi f T}|^2 = 1$

Provided,  $H(f) = s^*(f) e^{-j2\pi f T}$

The equality relation holds good.

$$\left( \frac{S}{N} \right)_0 = \frac{\left| \int_{-\infty}^{\infty} s(f) H(f) e^{j2\pi f T} df \right|^2}{\int_{-\infty}^{\infty} \frac{N_0}{2} |H(f)|^2 df}$$

$$\left( \frac{S}{N} \right)_0 \leq \frac{\int_{-\infty}^{\infty} |H(f)|^2 df \cdot \int_{-\infty}^{\infty} |s(f)|^2 df}{\int_{-\infty}^{\infty} \frac{N_0}{2} |H(f)|^2 df}$$

provided,  $H(f) = s^*(f) e^{-j2\pi f T}$ , the equality relation holds good.

$$\text{So, } \left( \frac{S}{N} \right)_{0 \max} = \frac{\int_{-\infty}^{\infty} |H(f)|^2 df \cdot \int_{-\infty}^{\infty} |s(f)|^2 df}{\int_{-\infty}^{\infty} \frac{N_0}{2} |H(f)|^2 df}$$

ESD of  $s(f)$

$$\left( \frac{S}{N} \right)_{0 \max} = \frac{\int_{-\infty}^{\infty} |s(f)|^2 df}{\int_{-\infty}^{\infty} |s(f)|^2 df / (\frac{N_0}{2})}$$

$$\left(\frac{S}{N}\right)_{\text{max}} = E(N_0/2) \quad (325)$$

where, E - Energy of s(t)

So,

$$\left(\frac{S}{N}\right)_{\text{max}} = (2E/N_0)$$

$$\left(\frac{S}{N}\right) = \frac{E}{N_0}$$

S → mapped to Energy of Input Signal

N → mapped to Input PSD of Input n(t)

Note:

For max ( $S/N$ ) is corresponding to the Ratio of Input Signal Energy and Output Noise PSD.

Ans

$$h(t) = \text{IFT}[H(f)]$$

$$= \int_{-\infty}^{\infty} H(f) e^{j2\pi f t} df$$

$$= \int_{-\infty}^{\infty} S^*(f) e^{-j2\pi f T} e^{j2\pi f t} df$$

Now, if s(t) is real, then  $S^*(f) = S(-f)$ .

$$\text{So, } h(t) = \int_{-\infty}^{\infty} S(-f) e^{-j2\pi f T} \cdot e^{j2\pi f t} df$$

$$\text{let } -f \rightarrow f$$

$$h(t) = \int_{-\infty}^{\infty} S(f) e^{j2\pi f T} \cdot e^{-j2\pi f t} (-df)$$

$$\therefore - \int_a^b = \int_b^a$$

$$\text{So, } h(t) = \int_{-\infty}^{\infty} S(f) e^{j2\pi f T} \cdot e^{-j2\pi f t} df$$

$$h(t) = \int_{-\infty}^{\infty} s(\tau) e^{j\omega_0 t(T-\tau)} d\tau$$

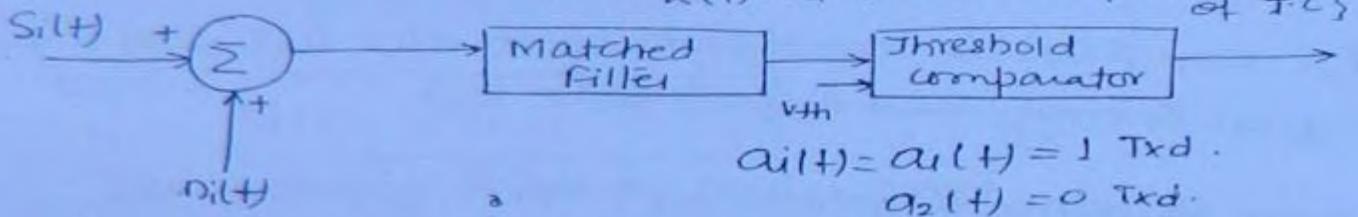
(326)

So,

$$h(t) = s(T-t)$$

### \* Probability Error of digital signalling schemes:

$z(t) = a_i(t) + n_o(t)$   $\xrightarrow{\{z \rightarrow \text{input voltage}} \text{input of T.C}\}$



$$a_1(t) = a_i(t) = 1 \text{ Txd.}$$

$$a_2(t) = 0 \text{ Txd.}$$

$s_1(t) = s_i(t)^0 \rightarrow \text{binary } 1 \text{ was Txd.}$

$s_2(t) \rightarrow \text{binary } 0 \text{ was Txd}$

\* Assume,  $n_i(t)$  corresponds to white Noise of having 2 sided PSD  $N_0/2$ , and possessing Gaussian density func' with 0 mean.

Case 1 :-

Assume no signal component was transmitted by the Tx. ( $a_i(t)=0$ ).

So,  $z(t) = n_o(t)$

$z = n_o$

So,  $E[z] = E[n_o]$

$E[z] = 0$   $\left. \begin{array}{l} \text{mean of } n_i \text{ is } 0, \text{ hence mean of } n_o \text{ is} \\ \text{also } 0. \end{array} \right\}$

So,  $f(z) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z-\mu)^2}{2\sigma^2}}$   $\left. \begin{array}{l} \text{Gaussian} \\ \text{density funcn}. \end{array} \right\}$

$\leftarrow \text{variance of } z$   
 $= \text{AC power.}$

Now,  $z = n_o$

$\text{variance}[z] = \text{variance}[n_o]$

$\text{AC power}(z) = \text{AC power}(n_o)$ .

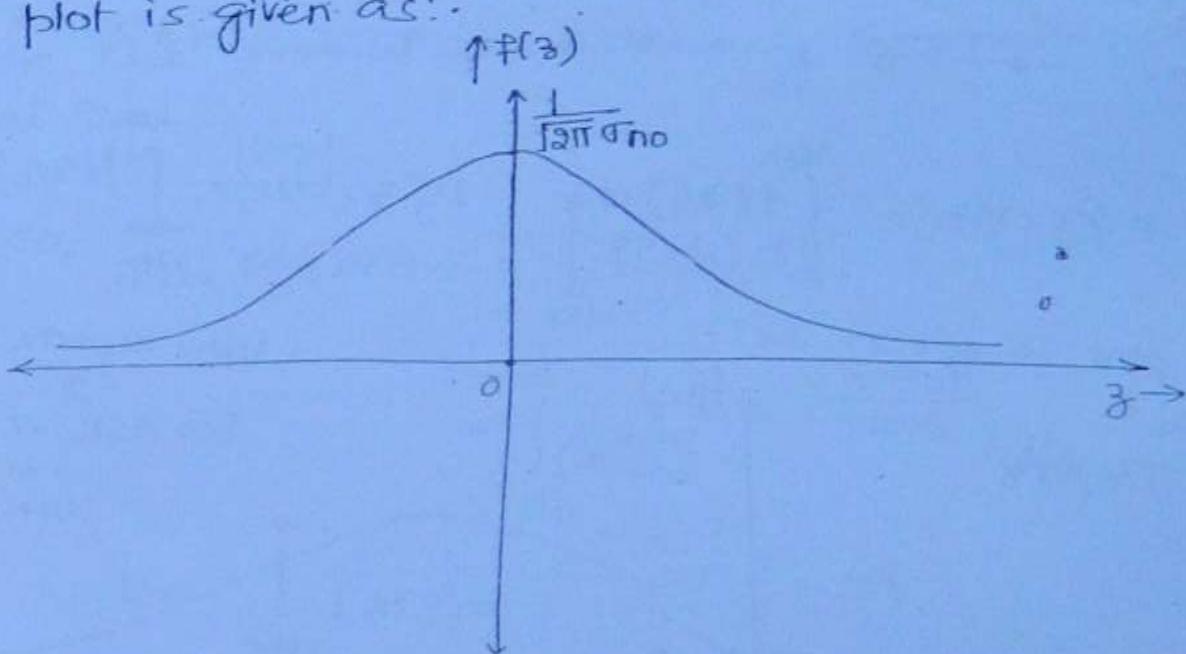
so, the  $\sigma = \sigma_{no} = \text{variance of noise}$

$$f(z) = \frac{1}{\sqrt{2\pi}\sigma_{no}} e^{-\frac{(z-\mu)^2}{2\sigma_{no}^2}}$$

(327)

$$\text{So, } f(z) = \frac{1}{\sqrt{2\pi}\sigma_{no}} \cdot e^{-\frac{z^2}{2\sigma_{no}^2}} \left\{ \begin{array}{l} \text{mean = 0} \\ \sigma = 0 \end{array} \right\}.$$

The plot is given as:-



\* As the Noise is white Noise, so the strength of noise is very small. So,

$$P(\text{no small}) = \text{high.}$$

$$P(\text{no large}) = \text{low.}$$

$$\left. \begin{array}{l} P(X \leq z) = \int_{-\infty}^z F(x) dx \\ \end{array} \right\}$$

Case 2:-

Assume binary 1 was transmitted.

$$\text{So, } z = a_1(t) + n_0(t)$$

$$z = a_1 + n_0$$

$$E[z] = E[a_1 + n_0] = E[a_1] + E[n_0].$$

$$E[z] = E[a_1] = a_1$$

$$E[z] = a_1$$

The density function is given by

$$f(\beta|1) = \frac{1}{\sqrt{2\pi\sigma_{n_0}^2}} e^{-\frac{(\beta-\alpha_1)^2}{2\sigma_{n_0}^2}}$$

328

$\left. \begin{array}{l} \text{So } \alpha_1 \text{ is power} \\ \text{and } \alpha_2 \text{ is dc term} \\ \text{so A.C power of } \alpha_1 = 0 \\ \text{A.C power of } n_0 = \sigma_{n_0}^2 \end{array} \right\}$

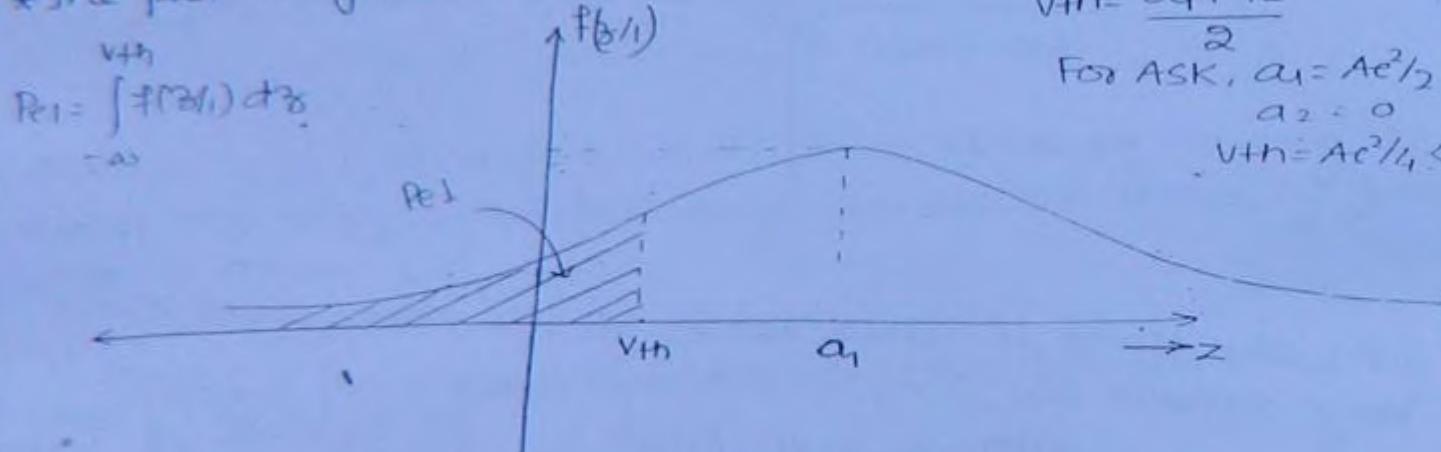
Now,

$P_{e1} = \text{Prob. of Rx } 0 \text{ when } 1 \text{ was Txd.} = P\{\beta < V_{th}\}$

$$P_{e0} = \xrightarrow{\quad 1 \quad} 0 \xrightarrow{\quad u \quad} = P\{\beta > V_{th}\}$$

So,  $P\{\beta < V_{th}\} = \int_{-\infty}^{V_{th}} f(\beta|1) d\beta ; P\{\beta > V_{th}\} = \int_{V_{th}}^{\infty} f(\beta|0) d\beta.$

\* The plot is given as:-



$$V_{th} = \frac{\alpha_1 + \alpha_2}{2}$$

For ASK,  $\alpha_1 = Ae^2/2$

$$\alpha_2 = 0$$

$$V_{th} = Ae^2/4 < A_F$$

Case 3: when Primary '0' was transmitted.

$$Z = \alpha_2 + n_0$$

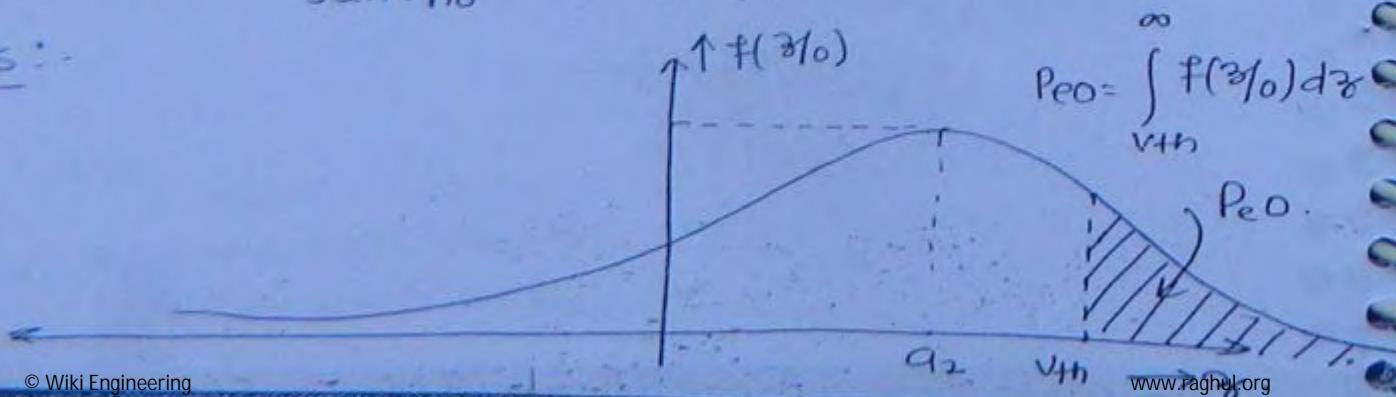
$$E[Z] = E[\alpha_2] + 0$$

$$E[Z] = \alpha_2$$

So,  $f(\beta|0) = \frac{1}{\sqrt{2\pi\sigma_{n_0}^2}} e^{-\frac{(\beta-\alpha_2)^2}{2\sigma_{n_0}^2}}$

$$V_{th} = Ae^2/4 > \alpha_2 = 0$$

Plot is:-



Note:

1. when Binary 1 was transmitted, no error occurs if  $Z > V_{TH}$

Then

$$P_{e1} = P(Z < V_{TH})$$

(329)

2. when Binary 0 was transmitted, no error occurs if  $Z < V_{TH}$

$$P_{e0} = P(Z > V_{TH}).$$

\* Assume the channel was binary symmetric channel so that,

$$P_{e1} = P_{e0}$$

$$\text{So, } P_{e0} = P(Z > V_{TH}) = \int_{V_{TH}}^{\infty} f(z) dz.$$

$$= \int_{V_{TH}}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}} \cdot e^{-\frac{(z-a_2)^2}{2\sigma_n^2}} \cdot dz$$

$$P_{e0} = \int_{V_{TH}}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}} \cdot e^{-\frac{(z-a_2)^2}{2\sigma_n^2}} \cdot dz$$

$$P_{e0} = \frac{1}{\sqrt{2\pi\sigma_n^2}} \int_{V_{TH}}^{\infty} e^{-\frac{(z-a_2)^2}{2\sigma_n^2}} \cdot dz$$

Now, as error func<sup>n</sup> is given as:

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-y^2/2} dy.$$

$$\text{Now, let, } \frac{z-a_2}{\sigma_n} = y$$

$$z-a_2 = \sigma_n y.$$

$$dz = \sigma_n dy$$

$$\begin{cases} z=0 \Rightarrow y=\infty \\ z=V_{TH} \Rightarrow y=\left(\frac{V_{TH}-a_2}{\sigma_n}\right) \end{cases}$$

$$= \frac{\frac{a_1+a_2-a_2}{2}}{\sigma_n}$$

$$= \frac{(a_1-a_2)/2}{\sigma_n}$$

$$\therefore y = (a_1-a_2)/2\sigma_n$$

$$P_e = P_{e1} P_{e2} = \frac{1}{\sqrt{2\pi}\sigma_{no}} \int e^{-y^2/2} dy \text{ multiply}$$

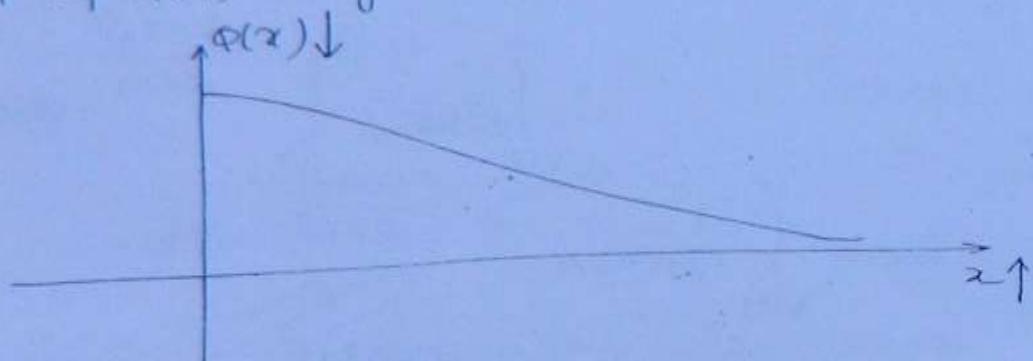
$$\frac{(a_1-a_2)}{2\sigma_{no}}$$

(33)

$$P_e = \frac{1}{\sqrt{2\pi}} \int_{a_1-a_2}^{\infty} e^{-y^2/2} dy$$

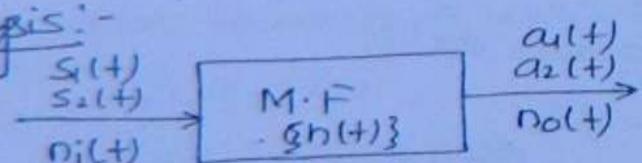
$$P_e = Q \left\{ \frac{a_1-a_2}{2\sigma_{no}} \right\} \quad \left[ \because Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-y^2/2} dy \right].$$

\* The plot of  $Q(x)$  is given as:-



$$\text{Now, } P_e = Q \left\{ \frac{(a_1-a_2)^2}{4\sigma_{no}^2} \right\}$$

\* Analysis:-



$$\text{Now } \frac{(a_1-a_2)^2}{\sigma_{no}^2} = \frac{\text{diff signal Power}}{\text{total Noise power}}$$

$$\therefore \sigma^2 = m_s - m_s^2 \xrightarrow{10} \text{mean} \\ \text{Total power}$$

$$\times \frac{S(t)}{n(t)} \xrightarrow{\text{M-P}} \frac{S_o(t)}{n_o(t)} \quad \left( \frac{S}{N} \right)_o = \frac{|S_o(t)|^2}{N_o} = E/(N\sigma_2^2)$$

when,  $n(t) = S(T-t)$ .

$$\text{Now, } \frac{(a_1 - a_2)^2}{\sigma_{n_0}^2} \propto \text{Ed}/(N_0/2) ; \text{ Ed = Diff. Signal Energy}$$

ie energy ( $S_1(t) - S_2(t)$ )

$$\text{Now, } S(t) = S_1(t) - S_2(t)$$

(33)

$$\text{So, } P_{\min} = Q \left[ \sqrt{\frac{1}{4} \cdot \frac{\text{Ed}}{N_0/2}} \right]$$

$$P_{\min} = Q \left[ \sqrt{\frac{\text{Ed}}{2N_0}} \right]$$

\* From M.F op<sup>n</sup>:

$\frac{a_1 + a_2}{\sigma_{n_0}^2}$  is maximised to  $(\text{Ed}/N_0/2)$  so that

$P_e$  will be minimised

Expe of ON-OFF SIGNALLING SCHEMES:

$$J \rightarrow S_1(t) = A_c$$

$$O \rightarrow S_2(t) = 0V$$

$$\text{So, } P_e = Q \left[ \sqrt{\frac{\text{Ed}}{2N_0}} \right]$$

$\text{Ed} = \text{Energy} [S_1(t) - S_2(t)]$

$$= \int_0^{T_b} \{S_1(t) - S_2(t)\}^2 dt$$

$$= \int_0^{T_b} (A_c - 0)^2 dt$$

$$\boxed{\text{Ed} = A_c^2 T_b}$$

$$\text{So, } P_e = Q \left[ \sqrt{\frac{A_c^2 T_b}{2 N_0}} \right] = P_e = Q[a_1]$$

\* Pe of NRZ operating scheme

$$1 \rightarrow S_1(t) = A_c$$

$$0 \rightarrow S_2(t) = -A_c$$

(332)

$$\begin{aligned} E_d &= \int_0^{T_b} \{S_1(t) - S_2(t)\}^2 dt \\ &= \int_0^{T_b} (2A_c)^2 dt \end{aligned}$$

$$E_d = 4A_c^2 T_b$$

$$\text{So, } P_e = Q \left[ \sqrt{\frac{4A_c^2 T_b}{2 N_0}} \right]$$

$$P_e = Q \left[ \sqrt{\frac{2A_c^2 T_b}{N_0}} \right] \Rightarrow P_e = Q[x_2]$$

\* As  $x_2 \uparrow \rightarrow Q[x] \downarrow$

Conclusion:

$$\text{As } x_2 > x_1$$

$$\text{So, } P_e(\text{NRZ}) < P_e(\text{ON-OFF})$$

\* Pe of ASK:

$$1 \rightarrow S_1(t) = A_c \cos 2\pi f_c t$$

$$0 \rightarrow S_2(t) = 0$$

$$\text{So, } E_d = \int_0^{T_b} \{S_1(t) - S_2(t)\}^2 dt$$

$$= \int_0^{T_b} \{A_c \cos 2\pi f_c t\}^2 dt$$

$$= \int_0^{T_b} \frac{A_c^2}{2} dt + \frac{A_c^2}{2} \int_0^{T_b} \cancel{\cos 2\pi f_c t} dt \quad \left\{ \because f_c \text{ is integer multiple of } f_s \right.$$

$P_e$  of complete cycles}

$$E_d = \frac{A_c^2}{2} T_b$$

$$\text{So, } P_e = \Phi \left[ \int \frac{E_d}{2N_0} \right]$$

(333)

$$\boxed{P_e = \Phi \left[ \int \frac{A_c^2 T_b}{4 N_0} \right]}$$

\*Pe OF PSK:

$$1 \rightarrow s_1(t) = A_c \cos 2\pi f_c t$$

$$0 \rightarrow s_2(t) = -A_c \cos 2\pi f_c t$$

$$\begin{aligned} \text{So, } E_d &= \int_0^{T_b} \{s_1(t) - s_2(t)\}^2 dt \\ &= \int_0^{T_b} \{2A_c \cos 2\pi f_c t\}^2 dt \\ &= \cancel{2A_c} \int_0^{T_b} 4A_c^2 \cos^2 2\pi f_c t dt \\ &= \frac{4A_c^2}{2} T_b + \frac{4A_c^2}{2} \int_0^{T_b} \cos 4\pi f_c t dt \end{aligned}$$

$$(E_d = 2A_c^2 T_b)$$

$$\text{So, } P_e = \Phi \left[ \int \frac{E_d}{2N_0} \right]$$

$$\boxed{P_e = \Phi \left[ \int \frac{A_c^2 T_b}{N_0} \right]}$$

\*Pe OF FSK:

$$1 \rightarrow s_1(t) = A_c \cos 2\pi f_1 t \quad (f_1 > f_2)$$

$$0 \rightarrow s_2(t) = A_c \cos 2\pi f_2 t$$

$$E_d = \int_0^{T_b} \{s_1(t) - s_2(t)\}^2 dt$$

$$E_d = \int (A_c (\cos 2\pi f_1 t - A_c \cos 2\pi f_2 t)) dt$$

$$= \int_{0}^{T_b} A_c^2 \cos 2\pi f_1 t dt + \int_{0}^{T_b} A_c^2 \cos 2\pi f_2 t dt \\ - 2 \int_{0}^{T_b} A_c^2 \cos 2\pi f_1 t \cos 2\pi f_2 t dt$$

$$= \frac{A_c^2}{2} \int_{0}^{T_b} dt + \frac{A_c^2}{2} \int_{0}^{T_b} \cancel{\cos 2\pi f_1 t} dt + \frac{A_c^2}{2} \int_{0}^{T_b} dt + \frac{A_c^2}{2} \int_{0}^{T_b} \cancel{\cos 2\pi f_2 t} dt \\ - 2 \frac{A_c^2}{2} \int_{0}^{T_b} \left\{ \cos 2\pi(f_1 + f_2)t + \cos 2\pi(f_1 - f_2)t \right\} dt$$

$$E_d = \frac{A_c^2 T_b}{2}$$

$$\text{So, } P_e = Q \left[ \int \frac{E_d}{2 N_0} \right]$$

$$P_e = Q \left[ \int \frac{A_c^2 T_b}{4 N_0} \right]$$

$$P_e = Q \left[ \int \frac{A_c^2 T_b}{2 N_0} \right]$$

$$P_e = Q \left[ \int \frac{A_c^2 T_b}{4 N_0} \right]$$

Conclusion:

Signalling scheme

ASK

$P_e$

$$Q \left[ \int \frac{A_c^2 T_b}{4 N_0} \right]$$

FPSK

$$Q \left[ \int \frac{A_c^2 T_b}{2 N_0} \right]$$

PSK

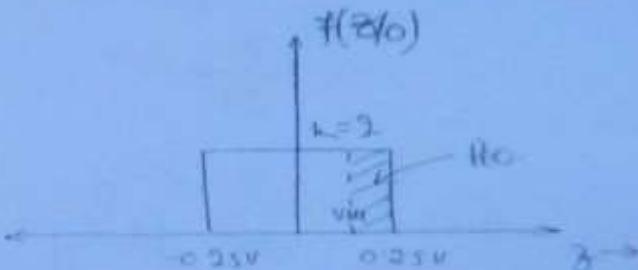
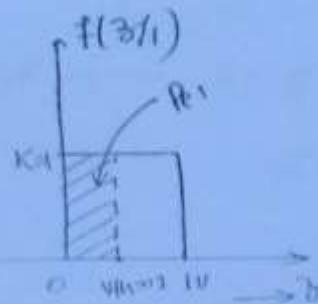
$$Q \left[ \int \frac{A_c^2 T_b}{N_0} \right]$$

Sol:

$$\boxed{P_e(\text{bit}1) < P_e(\text{bit}0) < P_e(\text{ACK})}$$

(335)

- Q1. A Binary Tx is transmitting 2 possible binary symbols specified by 0 & 1. When Binary 1 was transmitted, the received signal voltage at the S/P of Threshold Comparator will be in b/w 0V & 1V with equal probability. When Binary 0 was transmitted signal voltage lies b/w -0.25V & 0.25V with equal probability. Threshold voltage is given by 0.2V. Find avg. Pe?

Soln:

$$\text{Now, } P_{e\text{avg}} = \frac{P_{e1} + P_{e0}}{2}$$

channel is not BSC:

$$\begin{aligned} P_{e1} &= P(z < V_{th}) = \int_{-\infty}^{V_{th}} f(z|1) dz \\ &= \int_0^{0.2} 1 \cdot dz = 0.2 \end{aligned}$$

Now,

$$\begin{aligned} P_{e0} &= P(z > V_{th}) = \int_{V_{th}}^{\infty} f(z|0) dz \\ &= \int_{0.25}^{0.25} 2 dz = 0.1 \end{aligned}$$

$$\text{So, } P_{e\text{avg}} = \frac{P_{e0} + P_{e1}}{2} = \frac{0.1 + 0.2}{2}$$

$$\boxed{P_{e\text{avg}} = 0.15}$$

## \* Complementary Error Function; erfc(x)

Mathematically given as:

(36)

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-y^2} dy$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-z^2/2} dz ; z = \text{dummy variable}$$

$$\left. \begin{array}{l} \text{let } \frac{z}{\sqrt{2}} = y \\ z = \sqrt{2}y \\ dz = \sqrt{2}dy \end{array} \right| \quad \begin{array}{l} z=x \Rightarrow y = x/\sqrt{2} \\ z=\infty \Rightarrow y=\infty \end{array}$$

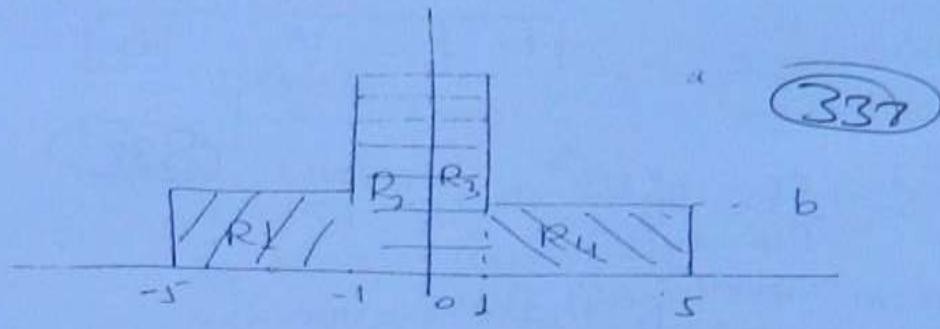
$$\text{So, } \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{x/\sqrt{2}}^{\infty} e^{-y^2} \cdot \sqrt{2} dy$$

$$\Phi(x) = \frac{1 \times 1 \times 2}{2 \sqrt{\pi}} \int_{x/\sqrt{2}}^{\infty} e^{-y^2} dy$$

$$\boxed{\Phi(x) = \frac{1}{2} \text{erfc}(x/\sqrt{2})}$$

And,

$$\boxed{\text{erfc}[x] = \frac{e^{-x^2}}{x\sqrt{\pi}}}$$



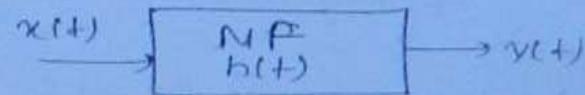
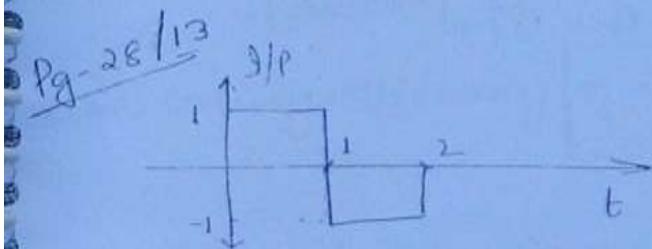
(337)

$$\text{Prob}(R_1) = \frac{1}{4} = \text{Area}(R_1)$$

$$\Rightarrow 4b = \frac{1}{4} \Rightarrow b = \frac{1}{16}$$

$$\text{Area}(R_2) = \text{Prob}(R_2) = \frac{1}{4}$$

$$a = \frac{1}{4}$$



$$y(t) = a(t) * h(t)$$

$$h(t) = s(t-2)$$

when  $a$  is not given, then  
Take  $T = \text{Total duration of given Signal} = 2$

$$T=2$$

$$\text{So, } h(t) = s(2-t)$$

$$\text{So, } y(t) = x(t) * s(2-t)$$

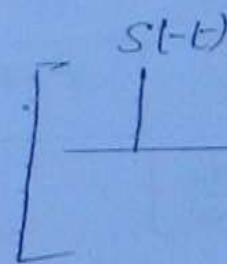
Firme  
Reversal

$$s(t) \rightarrow s(2-t)$$

$$s(t)$$

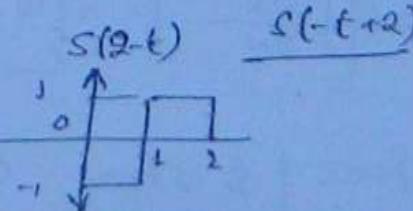
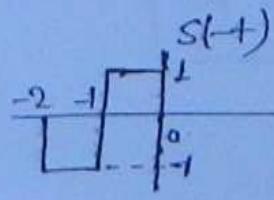
↓ delay by a unit

$$s(-(t-2)) = s(-t+2)$$



$s(-t)$  delay by 2 units

$$\text{out } t = (t-2) \text{ we get}$$



$$s(-t+2)$$

$$\boxed{\text{So, } y(t) = x(t) * s(2-t)}$$

N = 10<sup>-4</sup> watt

Given. Pt = 1mW

PL = 6dB.

Pe

$$S_i = \frac{P_t}{P_L} = \frac{1 \times 10^{-3}}{10^6} = 0.1 \mu\text{W}$$

N = 10<sup>-4</sup> watt.

$$P_e = Q \left[ \sqrt{\frac{A_c^2 T_b}{2 N_0}} \right]$$

of ASK

∴ No data is given regarding above.

Ans. Output NF is matched to output energy P<sub>ED</sub> of noise

$$S/N = E/(N_0/2)$$

Now,

$$\text{Energy/bit, } E_b = \frac{A_c^2 T_b}{2} \quad \text{+ } P_e = Q \left[ \sqrt{\frac{E_b}{2 N_0}} \right].$$

So,  $E_b \rightarrow S$

$$\frac{N_0}{2} \rightarrow N$$

$$\text{So, } P_e = Q \left[ \sqrt{\frac{S}{2 \cdot 2 N}} \right] \quad ; \text{ Before matched filter operation}$$

②

For PSK

$$P_e = Q \left[ \sqrt{\frac{A_c^2 T_b}{N_0}} \right]$$

$$E_b = \frac{A_c^2 T_b}{2}$$

$$P_e = Q \left[ \sqrt{\frac{2 E_b}{N_0}} \right]$$

$$= Q \left[ \sqrt{\frac{25}{2 N}} \right] = \left[ Q \left[ \sqrt{\frac{S}{N}} \right] : P_e \right]$$

Pe  
2nd model

$$P_b = 2.5 \times 10^{-6} \text{ bits/sec}$$

$$\frac{N_0}{2} = 10^{-20} \text{ watt/Hz}$$

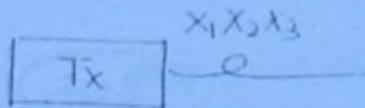
$$A_c = 1 \text{ mV/lt}$$

$$P_e \text{ of FSK} = Q \left[ \sqrt{\frac{A_c^2 T_b}{2 N_0}} \right]$$

$$= Q \left[ \sqrt{\frac{1 \times 10^{-12} \times 2.5 \times 10^{-6} \times 2 \times 2 \times 10^{-20}}{2 \times 10^6 \times 2 \times 10^{-20}}} \right]$$

$$A_c = Q[2] = \frac{1}{2} \operatorname{erfc} \left[ \frac{x}{\sqrt{2}} \right]$$

## \* SOURCE CODING THEOREM



(339)

$$x_1 = 001 \rightarrow \text{code length} = 3$$

$$x_2 = 0010 \rightarrow \text{code length} = 4$$

$$x_3 = 0011 \rightarrow \text{code length} = 4$$

\* how efficiently is the code working is calculated by the coding efficiency.

\* Avg. code length =  $L$  = bits/symbol.  
Mathematically given as:-

$$L = \sum_i n_i P(x_i)$$

\* So, coding efficiency,  $\eta = \frac{L_{\min}}{L}$

\* According to the Source coding theorem  $\Rightarrow L \geq H$

$$\therefore \eta \leq \frac{H}{L}$$

\* If avg. code length is small, then it is called to be as that the coding efficiency is high.

$$H = - \sum_i P(x_i) \log P(x_i)$$

g. variable length - now coding will be done using all possible symbols with probabilities  $0.3, 0.25, 0.2, 0.12, 0.08, 0.05$   
 Find coding efficiency

(340)

Step 1:

Step 1 → Arrange all prob. in decreasing order

0.3  
0.25  
0.2  
0.12  
0.08  
0.05

Step 2 → divide the whole prob. in such that the 2 sets have equiprobable probabilities (i.e almost close prob.).

	0.3	0
0.25	0.25	0
	0.2	1
0.45	0.12	1
	0.08	1
	0.05	1

Note: Assign 0 to above set and 1 to the lower set or vice-versa, but only one has to be followed.

Step 3: Again divide the 2 set with the same process as in 2nd step

0.3	0	0	$\rightarrow m_1 = 2$
0.25	0	1	$\rightarrow m_2 = 2$
0.2	1	0	$\rightarrow m_3 = 2$
0.12	1	1	$\rightarrow m_4 = 3$
0.08	1	1	$\rightarrow m_5 = 4$
0.05	1	1	$\rightarrow m_6 = 4$

Step 4: Repeat 3.

Step 5:

$$L = \sum_{i=1}^6 n_i P(x_i)$$

$$L = (2 \times 0.3 + 2 \times 0.25 + 2 \times 0.2 + 3 \times 0.12 + 4 \times 0.08 + 1 \times 0.05)$$

$$\boxed{L = 2.38 \text{ bits/symbol}}$$

(34)

Step 6:

$$H = -\sum_{i=1}^6 P(x_i) \log \{P(x_i)\}$$

$$\boxed{H = 2.36 \text{ bits/symbol}}$$

So,

$$\therefore \eta = H/L$$

$$\therefore \eta = \frac{2.36}{2.38} \times 100$$

$$\boxed{\therefore \eta = 99.1\% \text{ Ans}}$$

Q2. Construct HOFFMAN coding for the above problem.

Q3:

Step 1: Arrange in decreasing order

0.3  
0.25  
0.2  
0.12  
0.08  
0.05

Step 2: Sum the last 2 prob.

$$\text{ie } 0.08 + 0.05 = 0.13$$

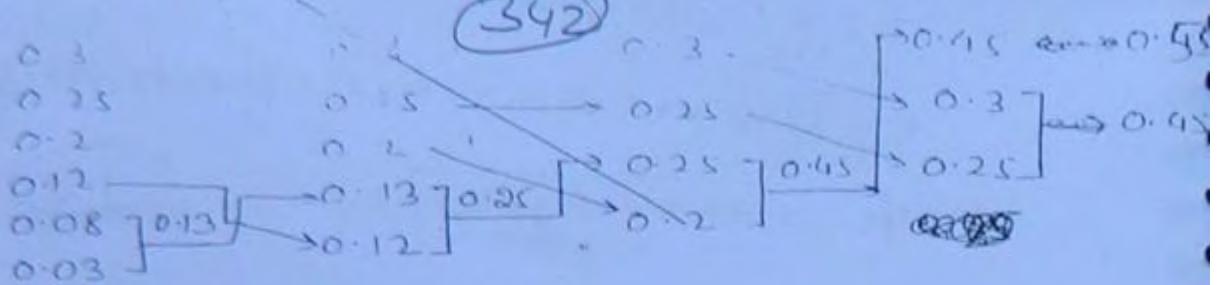
0.3  
0.25  
0.2  
0.12  
0.08  
0.05 ] - 0.13

Step 3: by taking step 2 in considera-

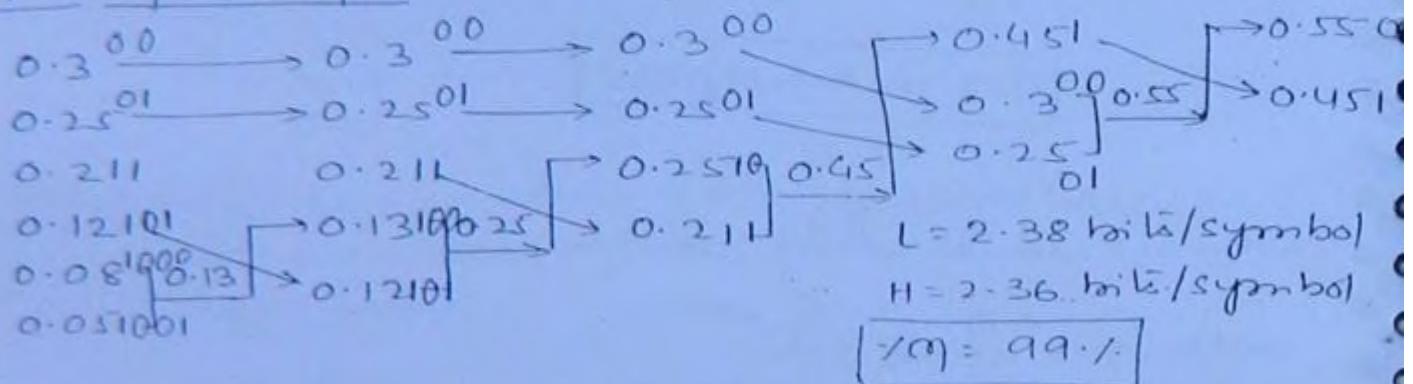
tion again arrange prob in decreasing order

0.3	0.3
0.25	0.25
0.2	0.2
0.12	0.12
0.08	0.08
0.05	0.05

Step 3 Repeat step 2



Step 4 Repeat Step 3



Step 5 Associate '0' to all prob corresponding to 0.55 & 1 to prob corresponding to 0.45

Step 6 Move Backward and repeat above.

p34/1

$$\text{i. i)} H = - \sum_{i=1}^8 p(x_i) \log \{p(x_i)\}.$$

$$= 1.96 \text{ bits/symbol.}$$

$$\text{ii)} \text{Prob. of occur of '0'} = (3 \times \frac{1}{2} + 2 \times \frac{1}{4} + 2 \times \frac{1}{8} + \dots) \times \frac{1}{3}$$

Prob. of occurring of '1' = 0.2

$$\text{iii)} \eta = \frac{H}{H_{\max}} = \frac{1.96}{\log 2.8} \quad | \because \text{no coding technique was given, so } L = H_{\max}$$

$$\eta = \frac{1.96}{3}$$

$$\eta = 65.3\%.$$

iv) by Shannon formula

343

$$\text{v) } \eta = 100\%$$

$\eta_1$	$x_1$	$\downarrow$	$0.5$	$0$	$\eta_1 = 1$
$\eta_2$	$x_2$	$\downarrow$	$0.4$	$1$	$\underline{0}$
$\eta_3$	$x_3$	$\downarrow$	$0.1$	$1$	$1$

$$\eta_2 = 2$$

$$\eta_3 = 2$$

$$\begin{aligned} \text{So, } I &= 1 \times 0.5 + 2 \times 0.4 + 2 \times 0.1 \\ &= 0.5 + 0.8 + 0.1 \\ &= 1.4 \text{ bili/symbol} \end{aligned}$$

$$\eta = 90.7\%$$

$\times$  2nd order convolution code:

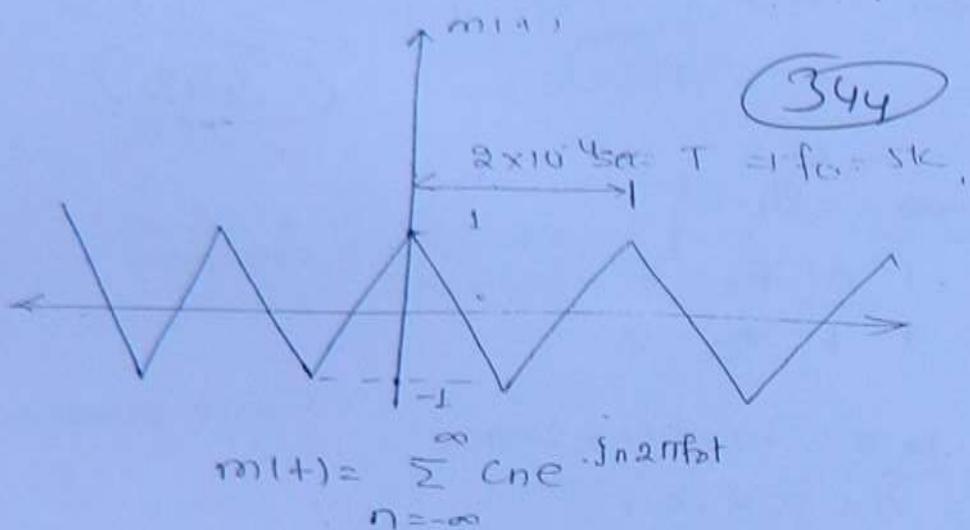
	$P_e$
$x_1 x_4$	$\rightarrow 0.25$
$x_1 x_2$	$\rightarrow 0.2$
$x_1 x_3$	$\rightarrow 0.05$
$x_2 x_4$	$\rightarrow 0.2$
$x_2 x_3$	$\rightarrow 0.16$
$x_2 x_3$	$\rightarrow 0.04$
$x_3 x_4$	$\rightarrow 0.05$
$x_3 x_2$	$\rightarrow 0.04$
$x_3 x_3$	$\rightarrow 0.01$

S.F Coding

Q. Given  $z = x+y$ , where  $x$  and  $y$  are random variables having density function in the form of rectangular pulse. Density function of  $z$  will be:

- a) Rectangular pulse
- b) Triangular pulse
- c) Gaussian pulse
- d) None

$$\left\{ \begin{array}{l} \because f(z) = f(x) * f(y) \\ \text{矩形} * \text{矩形} \\ f(z) = \text{梯形} \end{array} \right.$$



upto 3rd Harmonic freqn.

$$m(t) = 3f_0, 2f_0, f_0 \dots \quad \left. \begin{array}{l} \text{multitone} \\ f_{\max} = 1.5 < 1.75 \{ 3f_0 \} \end{array} \right\} \text{modulation}$$

Now,  $K_f = 2\pi \times 10^5$ ;  $K_p = 5\pi$

The units are not mentioned. & 11 terms are involved.

$$\omega_i = \omega_c + K_f m(t)$$

but all the analysis was done for  
 $f_i = f_c + K_p m(t)$

$$\text{So, } \frac{\omega_i}{2\pi} = \frac{\omega_c}{2\pi} + \frac{K_f m(t)}{2\pi}$$

$$\therefore K_f = \frac{2\pi \times 10^5}{2\pi}; K_p = 5\pi / 2\pi \\ = 1.0 \times 10^5 \quad K_p = 5/2$$

$$\begin{aligned} \text{B.F.M.} &= 2(\Delta f + f_{\max}) \\ &= 2 \left\{ \frac{K_f A_m - f_{\max}}{2\pi} \right\} \\ &= 2 \left\{ 2 \times 10^5 \times 1.15 \times 10^3 \right\} \\ &= 230 K \end{aligned}$$

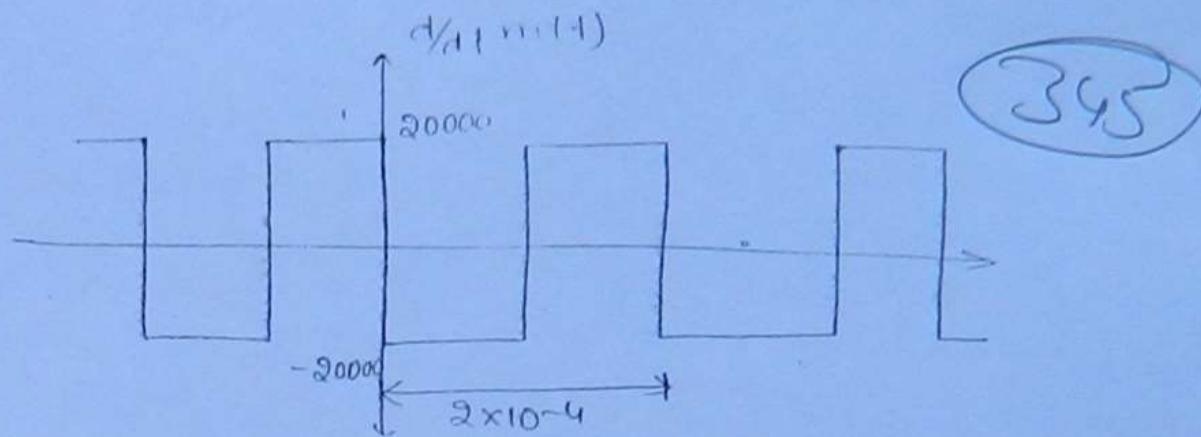
For PM, if the msg. is not sinusoidal, the formula of B.W & Power changes.

So,

$$\text{PM of } m(t) = \text{FM of } \frac{d}{dt} m(t).$$

$$\text{B.W of PM of } m(t) = \text{B.W of FM of } \frac{d}{dt} m(t)$$

Now,  $\frac{d}{dt} m(t)$  = Slope of  $m(t)$ :  $\frac{+2}{2 \times 10^{-4}} = 150,000$



$$B.W = 2 \{ \Delta f + f_{max} \}$$

$$= 2 \{ K_f A_m + f_{max} \}_{FM}$$

If  $B.W$  of  $PM$  is to be calculated by  $\frac{d}{dt} m(t)$ , then  $K_f$  replaced by  $K_P$

$$B.W = 2 \{ K_P A_m + f_{max} \}$$

$$= 2 \{ 5 \times \frac{10000}{20000} + 15000 \}$$

$$= 2 \times 55K$$

$$\boxed{B.W = 130K}$$

$$B.W = 2 (\Delta f + f_{max})$$

$$B.W = 2 (\Delta \phi^2 + f_{max})$$

346

PAM:

Generation  $\rightarrow$  AND gate  
demod.  $\rightarrow$  LPF (integrator)

PNN:

Generation  $\rightarrow$  Monostable M.V  
demod.  $\rightarrow$  LPF (integrator).

PPM:

Generation  $\rightarrow$  PWM  $\rightarrow$   $\frac{d}{dt}$   $\rightarrow$  clipper  $\rightarrow$  PPM  
demod.  $\rightarrow$  PPM  $\rightarrow$  Monostable M.V  $\rightarrow$  PWM.

\* Granular Noise power =  $\frac{\Delta^2}{3}$ ;  $\Delta$  = step size

\* % of modulation for FM =  $\frac{\Delta f}{f_{\text{max}}}$  ;  $\Delta f_{\text{max}} = 75K$   
standard.

\* For PAM  $\rightarrow$  Roll off factor ( $\alpha$ )

$$B.W = \frac{R_b(1+\alpha)}{2}$$