

## GATE SOLVED PAPER - EC

### COMMUNICATION SYSTEM

**2013****ONE MARK**

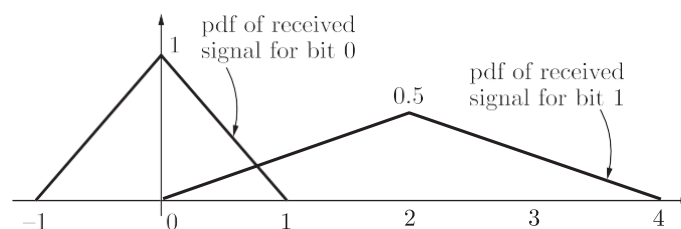
- Q. 1** The bit rate of a digital communication system is  $R$  kbits/s. The modulation used is 32-QAM. The minimum bandwidth required for ISI free transmission is
- (A)  $R/10$  Hz (B)  $R/10$  kHz  
(C)  $R/5$  Hz (D)  $R/5$  kHz

**2013****TWO MARKS**

- Q. 2** Let  $U$  and  $V$  be two independent zero mean Gaussain random variables of variances  $\frac{1}{4}$  and  $\frac{1}{9}$  respectively. The probability  $P\{3V \leq 2U\}$  is
- (A)  $4/9$  (B)  $1/2$   
(C)  $2/3$  (D)  $5/9$
- Q. 3** Consider two identically distributed zero-mean random variables  $U$  and  $V$ . Let the cumulative distribution functions of  $U$  and  $2V$  be  $F(x)$  and  $G(x)$  respectively. Then, for all values of  $x$
- (A)  $F(x) - G(x) \neq 0$  (B)  $F(x) - G(x) \leq 0$   
(C)  $F(x) - G(x) \geq 0$  (D)  $F(x) - G(x) \leq 0$
- Q. 4** Let  $U$  and  $V$  be two independent and identically distributed random variables such that  $P\{U = +1\} = P\{U = -1\} = \frac{1}{2}$ . The entropy  $H(U + V)$  in bits is
- (A)  $3/4$  (B)  $1$   
(C)  $3/2$  (D)  $\log_2 3$

#### Common Data for Questions 5 and 6:

Bits 1 and 0 are transmitted with equal probability. At the receiver, the pdf of the respective received signals for both bits are as shown below.



- Q. 5** If the detection threshold is 1, the BER will be
- (A)  $\frac{1}{2}$  (B)  $\frac{1}{4}$   
(C)  $\frac{1}{8}$  (D)  $\frac{1}{16}$

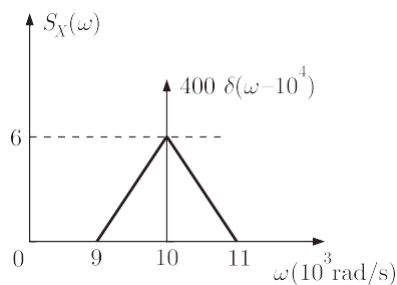
Q. 6 The optimum threshold to achieve minimum bit error rate (BER) is

- (A)  $\frac{1}{2}$  (B)  $\frac{4}{5}$   
(C) 1 (D)  $\frac{3}{2}$

2012

ONE MARK

Q. 7 The power spectral density of a real process  $X(t)$  for positive frequencies is shown below. The values of  $E[X^2(t)]$  and  $E[X(t)]$ , respectively, are



- (A)  $6000/p$  (B)  $6400/p$ , 0  
(C)  $6400/p$ ,  $20/(p^2)$  (D)  $6000/p$ ,  $20/(p^2)$

Q. 8 In a baseband communications link, frequencies upto 3500 Hz are used for signaling. Using a raised cosine pulse with 75% excess bandwidth and for no inter-symbol interference, the maximum possible signaling rate in symbols per second is

- (A) 1750 (B) 2625  
(C) 4000 (D) 5250

Q. 9 A source alphabet consists of  $N$  symbols with the probability of the first two symbols being the same. A source encoder increases the probability of the first symbol by a small amount  $e$  and decreases that of the second by  $e$ . After encoding, the entropy of the source

- (A) increases (B) remains the same  
(C) increases only if  $N = 2$  (D) decreases

Q. 10 Two independent random variables  $X$  and  $Y$  are uniformly distributed in the interval  $[-1, 1]$ . The probability that  $\max\{X, Y\}$  is less than  $1/2$  is

- (A)  $3/4$  (B)  $9/16$   
(C)  $1/4$  (D)  $2/3$

2012

TWO MARKS

Q. 11 A BPSK scheme operating over an AWGN channel with noise power spectral density of  $N_0/2$  uses equiprobable signals  $s_1(t) = \sqrt{\frac{2E}{T}} \sin(\omega_c t)$  and  $s_2(t) = -\sqrt{\frac{2E}{T}} \sin(\omega_c t)$  over the symbol interval  $(0, T)$ . If the local oscillator in a coherent receiver is ahead in phase by  $45^\circ$  with respect to the received signal, the probability of error in the resulting system is

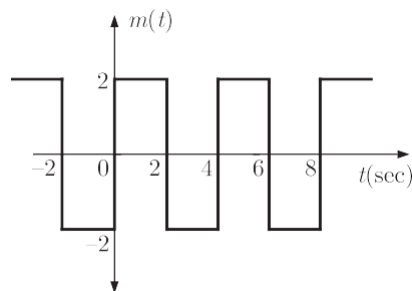
- (A)  $Q\left(\sqrt{\frac{2E}{N_0 T}}\right)$  (B)  $Q\left(\sqrt{\frac{E}{N_0 T}}\right)$

(C)  $Q_c \frac{E}{2N}$  m

(D)  $Q_c \frac{E}{4N_0}$  m

- Q. 12** A binary symmetric channel (BSC) has a transition probability of  $1/8$ . If the binary symbol  $X$  is such that  $P(X=0) = 9/10$ , then the probability of error for an optimum receiver will be
- (A)  $7/80$  (B)  $63/80$   
(C)  $9/10$  (D)  $1/10$

- Q. 13** The signal  $m(t)$  as shown is applied to both a phase modulator (with  $k_p$  as the phase constant) and a frequency modulator (with  $k_f$  as the frequency constant) having the same carrier frequency.



- The ratio  $k_p/k_f$  (in rad/Hz) for the same maximum phase deviation is
- (A)  $8p$  (B)  $4p$   
(C)  $2p$  (D)  $p$

**Statement for Linked Answer Question 14 and 15 :**

The transfer function of a compensator is given as

$$G_c(s) = \frac{s+a}{s+b}$$

- Q. 14**  $G_c(s)$  is a lead compensator if
- (A)  $a = 1, b = 2$  (B)  $a = 3, b = 2$   
(C)  $a = -3, b = -1$  (D)  $a = 3, b = 1$
- Q. 15** The phase of the above lead compensator is maximum at
- (A)  $\sqrt{2}$  rad/s (B)  $\sqrt{3}$  rad/s  
(C)  $\sqrt{6}$  rad/s (D)  $1/\sqrt{3}$  rad/s

**2011**

**ONE MARK**

- Q. 16** An analog signal is band-limited to 4 kHz, sampled at the Nyquist rate and the samples are quantized into 4 levels. The quantized levels are assumed to be independent and equally probable. If we transmit two quantized samples per second, the information rate is
- (A) 1 bit/sec (B) 2 bits/sec  
(C) 3 bits/sec (D) 4 bits/sec
- Q. 17** The **Column -1** lists the attributes and the **Column -2** lists the modulation systems. Match the attribute to the modulation system that best meets it.

## Column -1

- P. Power efficient transmission of signals  
 Q. Most bandwidth efficient transmission of voice signals  
 R. Simplest receiver structure  
 S. Bandwidth efficient transmission of signals with significant dc component

## Column -2

1. Conventional AM  
 2. FM  
 3. VSB  
 4. SSB-SC

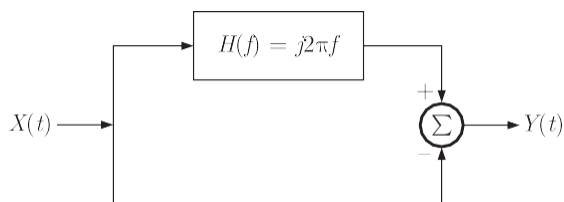
- (A) P-4, Q-2, R-1, S-3  
 (B) P-2, Q-4, R-1, S-3  
 (C) P-3, Q-2, R-1, S-4  
 (D) P-2, Q-4, R-3, S-1

2011

TWO MARKS

Q. 18

$X(t)$  is a stationary random process with auto-correlation function  $R_X(t) = \exp(-\rho t^2)$ . This process is passed through the system shown below. The power spectral density of the output process  $Y(t)$  is



- (A)  $(4\rho^2 f^2 + 1) \exp(-\rho f^2)$   
 (B)  $(4\rho^2 f^2 - 1) \exp(-\rho f^2)$   
 (C)  $(4\rho^2 f^2 + 1) \exp(-\rho f)$   
 (D)  $(4\rho^2 f^2 - 1) \exp(-\rho f)$

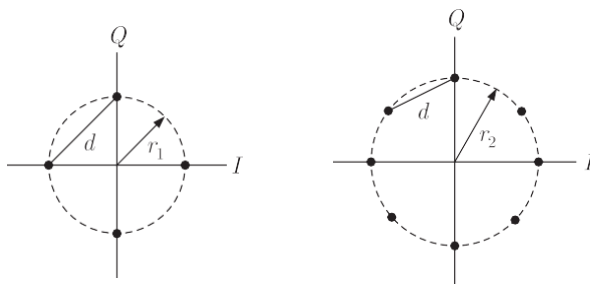
Q. 19

A message signal  $m(t) = \cos 2000\pi t + 4\cos 4000\pi t$  modulates the carrier  $c(t) = \cos 2\pi f_c t$  where  $f_c = 1$  MHz to produce an AM signal. For demodulating the generated AM signal using an envelope detector, the time constant  $RC$  of the detector circuit should satisfy

- (A)  $0.5 \text{ ms} < RC < 1 \text{ ms}$   
 (B)  $1 \mu\text{s} \ll RC < 0.5 \text{ ms}$   
 (C)  $RC \ll 1 \mu\text{s}$   
 (D)  $RC \gg 0.5 \text{ ms}$

## Statement for Linked Answer Questions: 20 and 21

A four-phase and an eight-phase signal constellation are shown in the figure below.



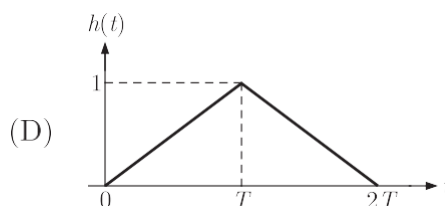
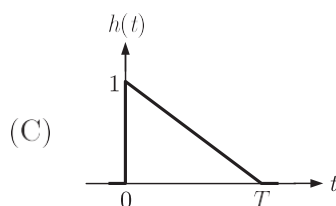
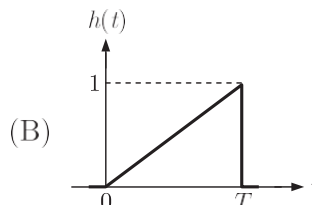
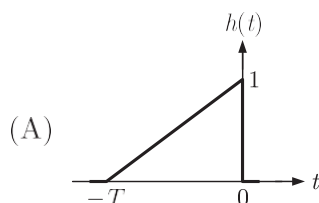
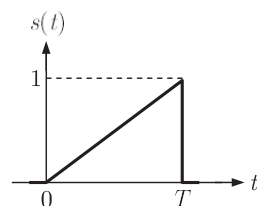
- Q. 20 For the constraint that the minimum distance between pairs of signal points be  $d$  for both constellations, the radii  $r_1$ , and  $r_2$  of the circles are  
 (A)  $r_1 = 0.707d, r_2 = 2.782d$  (B)  $r_1 = 0.707d, r_2 = 1.932d$   
 (C)  $r_1 = 0.707d, r_2 = 1.545d$  (D)  $r_1 = 0.707d, r_2 = 1.307d$
- Q. 21 Assuming high SNR and that all signals are equally probable, the additional average transmitted signal energy required by the 8-PSK signal to achieve the same error probability as the 4-PSK signal is  
 (A) 11.90 dB (B) 8.73 dB  
 (C) 6.79 dB (D) 5.33 dB

2010

ONE MARK

- Q. 22 Suppose that the modulating signal is  $m(t) = 2\cos(2\pi f_m t)$  and the carrier signal is  $x_c(t) = A_c \cos(2\pi f_c t)$ , which one of the following is a conventional AM signal without over-modulation  
 (A)  $x(t) = A_c m(t) \cos(2\pi f_c t)$   
 (B)  $x(t) = A_c [1 + m(t)] \cos(2\pi f_c t)$   
 (C)  $x(t) = A_c \cos(2\pi f_c t) + \frac{A_c}{4} m(t) \cos(2\pi f_c t)$   
 (D)  $x(t) = A_c \cos(2\pi f_m t) \cos(2\pi f_c t) + A_c \sin(2\pi f_m t) \sin(2\pi f_c t)$
- Q. 23 Consider an angle modulated signal  

$$x(t) = 6 \cos[2\pi \times 10^6 t + 2 \sin(800\pi t)] + 4 \cos(800\pi t)$$
  
 The average power of  $x(t)$  is  
 (A) 10 W (B) 18 W  
 (C) 20 W (D) 28 W
- Q. 24 Consider the pulse shape  $s(t)$  as shown below. The impulse response  $h(t)$  of the filter matched to this pulse is



2010

TWO MARKS

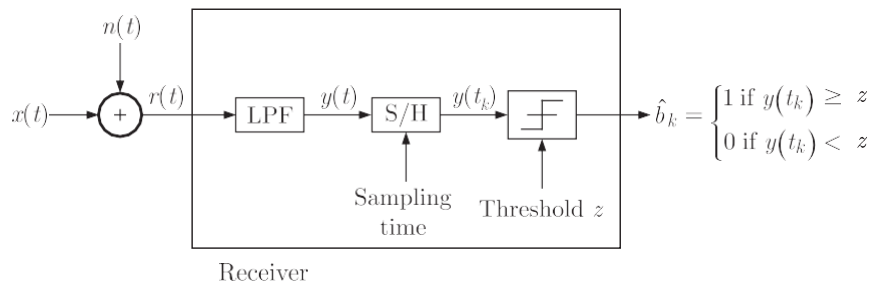
**Statement for linked Answer Question : 25 and 26 :**

Consider a baseband binary PAM receiver shown below. The additive channel noise  $n(t)$  is with power spectral density  $S_n(f) = N_0/2 = 10^{-20}$  W/Hz. The low-pass filter is ideal with unity gain and cut-off frequency 1 MHz. Let  $Y_k$  represent the random variable  $y(t_k)$ .

$$Y_k = N_k, \text{ if transmitted bit } b_k = 0$$

$$Y_k = a + N_k \text{ if transmitted bit } b_k = 1$$

Where  $N_k$  represents the noise sample value. The noise sample has a probability density function,  $P_{N_k}(n) = 0.5\alpha e^{-\alpha|n|}$  (This has mean zero and variance  $2/\alpha^2$ ). Assume transmitted bits to be equiprobable and threshold  $z$  is set to  $a/2 = 10^{-6}$  V.

**Q. 25**

The value of the parameter  $\alpha$  (in  $V^{-1}$ ) is

- (A)  $10^{10}$  (B)  $10^7$   
(C)  $1.414 \times 10^{-10}$  (D)  $2 \times 10^{-20}$

**Q. 26**

The probability of bit error is

- (A)  $0.5 \times e^{-3.5}$  (B)  $0.5 \times e^{-5}$   
(C)  $0.5 \times e^{-7}$  (D)  $0.5 \times e^{-10}$

**Q. 27**

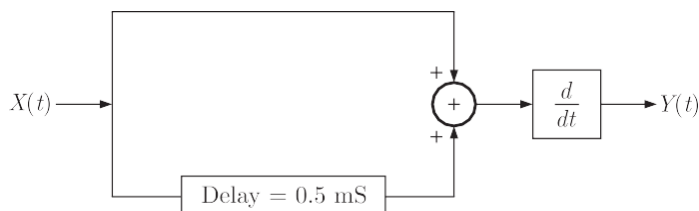
The Nyquist sampling rate for the signal

$$s(t) = \frac{\sin(500\pi t)}{\pi t} + \frac{\sin(700\pi t)}{\pi t}$$

- (A) 400 Hz (B) 600 Hz  
(C) 1200 Hz (D) 1400 Hz

**Q. 28**

$X(t)$  is a stationary process with the power spectral density  $S_x(f) > 0$ , for all  $f$ . The process is passed through a system shown below



Let  $S_y(f)$  be the power spectral density of  $Y(t)$ . Which one of the following statements is correct

- (A)  $S_y(f) > 0$  for all  $f$   
(B)  $S_y(f) = 0$  for  $|f| > 1$  kHz

(C)  $S_y(f) = 0$  for  $f = nf_0, f_0 = 2 \text{ kHz}$ ,  $n$  any integer

(D)  $S_y(f) = 0$  for  $f = (2n + 1)f_0 = 1 \text{ kHz}$ ,  $n$  any integer

**2009****ONE MARK**

**Q. 29** For a message signal  $m(t) = \cos(2\pi f_m t)$  and carrier of frequency  $f_c$ , which of the following represents a single side-band (SSB) signal?

(A)  $\cos(2\pi f_m t) \cos(2\pi f_c t)$

(B)  $\cos(2\pi f_c t)$

(C)  $\cos[2\pi(f_c + f_m)t]$

(D)  $[1 + \cos(2\pi f_m t)] \cos(2\pi f_c t)$

**2009****TWO MARKS**

**Q. 30** Consider two independent random variables  $X$  and  $Y$  with identical distributions. The variables  $X$  and  $Y$  take values 0, 1 and 2 with probabilities  $\frac{1}{2}, \frac{1}{4}$  and  $\frac{1}{4}$  respectively. What is the conditional probability  $P(X + Y = 2 | X - Y = 0)$ ?

(A) 0

(B)  $1/16$

(C)  $1/6$

(D) 1

**Q. 31** A discrete random variable  $X$  takes values from 1 to 5 with probabilities as shown in the table. A student calculates the mean  $X$  as 3.5 and her teacher calculates the variance of  $X$  as 1.5. Which of the following statements is true?

$k$	1	2	3	4	5
$P(X = k)$	0.1	0.2	0.3	0.4	0.5

(A) Both the student and the teacher are right

(B) Both the student and the teacher are wrong

(C) The student is wrong but the teacher is right

(D) The student is right but the teacher is wrong

**Q. 32** A message signal given by  $m(t) = \left(\frac{1}{2}\right)\cos w_1 t - \left(\frac{1}{2}\right)\sin w_2 t$  amplitude - modulated with a carrier of frequency  $w_c$  to generator  $s(t) = [1 + m(t)] \cos w_c t$ . What is the power efficiency achieved by this modulation scheme?

(A) 8.33%

(B) 11.11%

(C) 20%

(D) 25%

**Q. 33** A communication channel with AWGN operating at a signal to noise ratio  $SNR \gg 1$  and bandwidth  $B$  has capacity  $C_1$ . If the  $SNR$  is doubled keeping constant, the resulting capacity  $C_2$  is given by

(A)  $C_2 = 2C_1$

(B)  $C_2 = C_1 + B$

(C)  $C_2 = C_1 + 2B$

(D)  $C_2 = C_1 + 0.3B$

### Common Data For Q. 34 and 35 :

The amplitude of a random signal is uniformly distributed between -5 V and 5 V.

**Q. 34** If the signal to quantization noise ratio required in uniformly quantizing the signal is 43.5 dB, the step of the quantization is approximately

(A) 0.033 V

(B) 0.05 V

(C) 0.0667 V

(D) 0.10 V

- Q. 35** If the positive values of the signal are uniformly quantized with a step size of 0.05 V, and the negative values are uniformly quantized with a step size of 0.1 V, the resulting signal to quantization noise ratio is approximately  
 (A) 46 dB (B) 43.8 dB  
 (C) 42 dB (D) 40 dB

2008

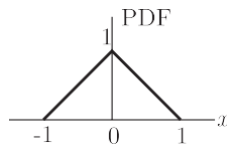
ONE MARK

- Q. 36** Consider the amplitude modulated (AM) signal  $A_c \cos w_c t + 2 \cos w_m t \cos w_c t$ . For demodulating the signal using envelope detector, the minimum value of  $A_c$  should be  
 (A) 2 (B) 1  
 (C) 0.5 (D) 0

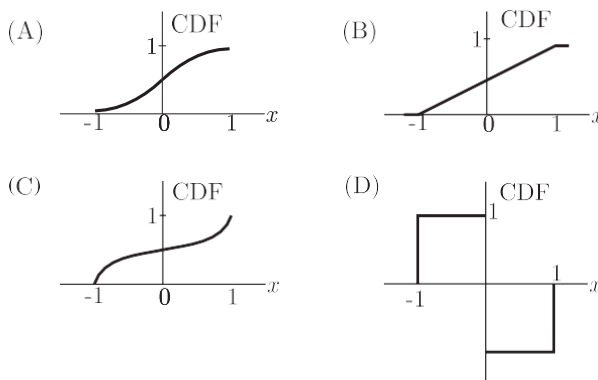
2008

TWO MARKS

- Q. 37** The probability density function (pdf) of random variable is as shown below



The corresponding cumulative distribution function CDF has the form



- Q. 38** A memoryless source emits  $n$  symbols each with a probability  $p$ . The entropy of the source as a function of  $n$   
 (A) increases as  $\log n$  (B) decreases as  $\log(\frac{1}{n})$   
 (C) increases as  $n$  (D) increases as  $n \log n$

- Q. 39** Noise with double-sided power spectral density on  $K$  over all frequencies is passed through a  $RC$  low pass filter with 3 dB cut-off frequency of  $f_c$ . The noise power at the filter output is  
 (A)  $K$  (B)  $Kf_c$   
 (C)  $kp f_c$  (D) 3

- Q. 40** Consider a Binary Symmetric Channel (BSC) with probability of error being  $p$ . To transmit a bit, say 1, we transmit a sequence of three 1s. The receiver will interpret the received sequence to represent 1 if at least two bits are 1. The probability that the transmitted bit will be received in error is



- (A)  $p^3 + 3p^2(1 - p)$  (B)  $p^3$   
 (C)  $(1 - p^3)$  (D)  $p^3 + p^2(1 - p)$
- Q. 41 Four messages band limited to  $W, W, 2W$  and  $3W$  respectively are to be multiplexed using Time Division Multiplexing (TDM). The minimum bandwidth required for transmission of this TDM signal is  
 (A)  $W$  (B)  $3W$   
 (C)  $6W$  (D)  $7W$
- Q. 42 Consider the frequency modulated signal  
 $10\cos[2\pi \times 10^5 t + 5\sin(2\pi \times 1500 t) + 7.5\sin(2\pi \times 1000 t)]$   
 with carrier frequency of  $10^5$  Hz. The modulation index is  
 (A) 12.5 (B) 10  
 (C) 7.5 (D) 5
- Q. 43 The signal  $\cos w_c t + 0.5\cos w_m t \sin w_c t$  is  
 (A) FM only (B) AM only  
 (C) both AM and FM (D) neither AM nor FM

**Common Data For Q. 40 to 46 :**

A speech signal, band limited to 4 kHz and peak voltage varying between +5 V and -5 V, is sampled at the Nyquist rate. Each sample is quantized and represented by 8 bits.

- Q. 44 If the bits 0 and 1 are transmitted using bipolar pulses, the minimum bandwidth required for distortion free transmission is  
 (A) 64 kHz (B) 32 kHz  
 (C) 8 kHz (D) 4 kHz
- Q. 45 Assuming the signal to be uniformly distributed between its peak to peak value, the signal to noise ratio at the quantizer output is  
 (A) 16 dB (B) 32 dB  
 (C) 48 dB (D) 4 kHz
- Q. 46 Assuming the signal to be uniformly distributed between its peak to peak value, the signal to noise ratio at the quantizer output is  
 (A) 1024 (B) 512  
 (C) 256 (D) 64

**2007****ONE MARK**

- Q. 47 If  $R(\tau)$  is the auto correlation function of a real, wide-sense stationary random process, then which of the following is NOT true  
 (A)  $R(\tau) = R(-\tau)$   
 (B)  $|R(\tau)| \leq R(0)$   
 (C)  $R(\tau) = -R(-\tau)$   
 (D) The mean square value of the process is  $R(0)$
- Q. 48 If  $S(f)$  is the power spectral density of a real, wide-sense stationary random

process, then which of the following is ALWAYS true?

- (A)  $S(0) \neq S(f)$  (B)  $S(f) \neq 0$   
 (C)  $S(-f) = -S(f)$  (D)  $\int_{-3}^3 S(f) df = 0$

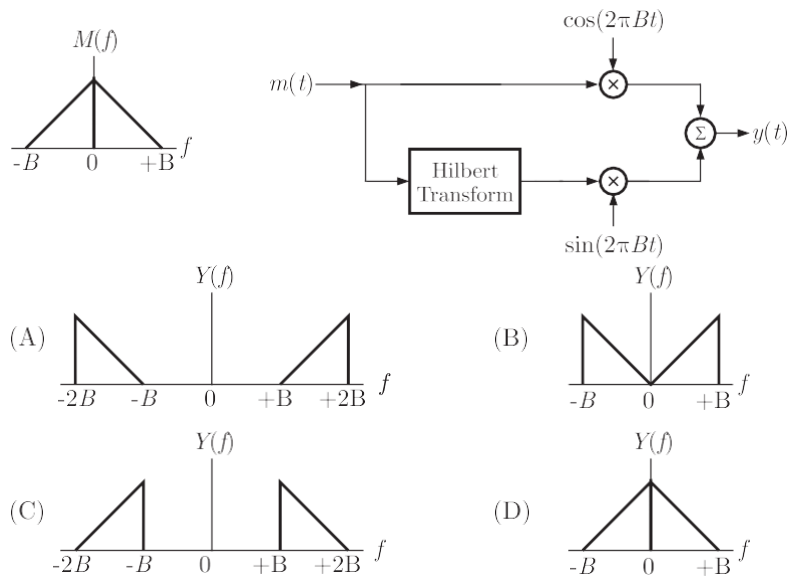
- Q. 49 If  $E$  denotes expectation, the variance of a random variable  $X$  is given by  
 (A)  $E[X^2] - E^2[X]$  (B)  $E[X^2] + E^2[X]$   
 (C)  $E[X^2]$  (D)  $E^2[X]$

2007

TWO MARKS

- Q. 50 A Hilbert transformer is a  
 (A) non-linear system (B) non-causal system  
 (C) time-varying system (D) low-pass system
- Q. 51 In delta modulation, the slope overload distortion can be reduced by  
 (A) decreasing the step size (B) decreasing the granular noise  
 (C) decreasing the sampling rate (D) increasing the step size
- Q. 52 The raised cosine pulse  $p(t)$  is used for zero ISI in digital communications. The expression for  $p(t)$  with unity roll-off factor is given by  

$$p(t) = \frac{\sin 4\pi W t}{4\pi W t (1 - 16W^2 t^2)}$$
  
 The value of  $p(t)$  at  $t = \frac{1}{4W}$  is  
 (A)  $-0.5$  (B)  $0$   
 (C)  $0.5$  (D)  $3$
- Q. 53 In the following scheme, if the spectrum  $M(f)$  of  $m(t)$  is as shown, then the spectrum  $Y(f)$  of  $y(t)$  will be



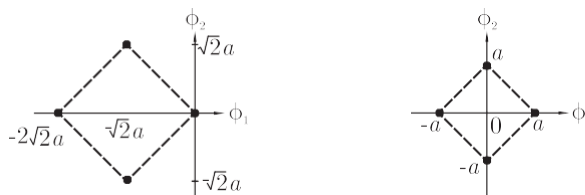
- Q. 54 During transmission over a certain binary communication channel, bit errors occur independently with probability  $p$ . The probability of AT MOST one bit in error in a block of  $n$  bits is given by

- (A)  $p^n$  (B)  $1 - p^n$   
 (C)  $np(1-p)^{n-1} + (1+p)^n$  (D)  $1 - (1-p)^n$

- Q. 55** In a GSM system, 8 channels can co-exist in 200 kHz bandwidth using TDMA. A GSM based cellular operator is allocated 5 MHz bandwidth. Assuming a frequency reuse factor of  $\frac{1}{5}$ , i.e. a five-cell repeat pattern, the maximum number of simultaneous channels that can exist in one cell is  
 (A) 200 (B) 40  
 (C) 25 (D) 5
- Q. 56** In a Direct Sequence CDMA system the chip rate is  $1.2288 \times 10^6$  chips per second. If the processing gain is desired to be AT LEAST 100, the data rate  
 (A) must be less than or equal to  $12.288 \times 10^3$  bits per sec  
 (B) must be greater than  $12.288 \times 10^3$  bits per sec  
 (C) must be exactly equal to  $12.288 \times 10^3$  bits per sec  
 (D) can take any value less than  $122.88 \times 10^3$  bits per sec

**Common Data For Q. 57 and 58 :**

Two 4-array signal constellations are shown. It is given that  $\phi_1$  and  $\phi_2$  constitute an orthonormal basis for the two constellation. Assume that the four symbols in both the constellations are equiprobable. Let  $\frac{N_0}{2}$  denote the power spectral density of white Gaussian noise.



Constellation 1

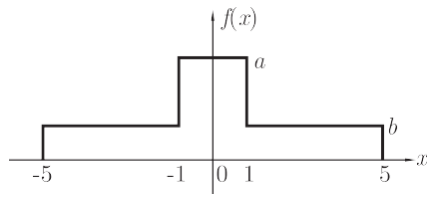
Constellation 2

- Q. 57** The ratio of the average energy of Constellation 1 to the average energy of Constellation 2 is  
 (A)  $4a^2$  (B) 4  
 (C) 2 (D) 8
- Q. 58** If these constellations are used for digital communications over an AWGN channel, then which of the following statements is true?  
 (A) Probability of symbol error for Constellation 1 is lower  
 (B) Probability of symbol error for Constellation 1 is higher  
 (C) Probability of symbol error is equal for both the constellations  
 (D) The value of  $N_0$  will determine which of the constellations has a lower probability of symbol error

**Statement for Linked Answer Question 59 and 60 :**

An input to a 6-level quantizer has the probability density function  $f(x)$  as shown in the figure. Decision boundaries of the quantizer are chosen so as to maximize the entropy of the quantizer output. It is given that 3 consecutive decision boundaries

are '1', '0' and '1'.



Q. 59

The values of  $a$  and  $b$  are

(A)  $a = \frac{1}{6}$  and  $b = \frac{1}{12}$

(B)  $a = \frac{1}{5}$  and  $b = \frac{3}{40}$

(C)  $a = \frac{1}{4}$  and  $b = \frac{1}{16}$

(D)  $a = \frac{1}{3}$  and  $b = \frac{1}{24}$

Q. 60

Assuming that the reconstruction levels of the quantizer are the mid-points of the decision boundaries, the ratio of signal power to quantization noise power is

(A)  $\frac{152}{9}$

(B)  $\frac{64}{3}$

(C)  $\frac{76}{3}$

(D) 28

2006

ONE MARK

Q. 61

A low-pass filter having a frequency response  $H(j\omega) = A(\omega)e^{j\mathcal{H}(\omega)}$  does not produce any phase distortions if

(A)  $A(\omega) = C\omega^3, \mathcal{H}(\omega) = k\omega^3$

(B)  $A(\omega) = C\omega^2, \mathcal{H}(\omega) = k\omega$

(C)  $A(\omega) = C\omega, \mathcal{H}(\omega) = k\omega^2$

(D)  $A(\omega) = C, \mathcal{H}(\omega) = k\omega^{-1}$

2006

TWO MARKS

Q. 62

A signal with bandwidth 500 Hz is first multiplied by a signal  $g(t)$  where

$$g(t) = \sum_{k=-3}^3 (-1)^k d(t - 0.5 \times 10^{-4}k)$$

The resulting signal is then passed through an ideal lowpass filter with bandwidth 1 kHz. The output of the lowpass filter would be

(A)  $d(t)$

(B)  $m(t)$

(C) 0

(D)  $m(t)d(t)$

Q. 63

The minimum sampling frequency (in samples/sec) required to reconstruct the following signal from its samples without distortion

$$x(t) = 5 \cdot \frac{\sin 2\pi 100t}{\pi t} j^3 + 7 \cdot \frac{\sin 2\pi 100t}{\pi t} j^2 \text{ would be}$$

(A)  $2 \times 10^3$

(B)  $4 \times 10^3$

(C)  $6 \times 10^3$

(D)  $8 \times 10^3$

Q. 64

The minimum step-size required for a Delta-Modulator operating at 32k samples/sec to track the signal (here  $u(t)$  is the unit-step function)

$$x(t) = 125[u(t) - u(t-1) + (250t)[u(t-1) - u(t-2)]]$$

so that slope-overload is avoided, would be

(A)  $2^{-10}$

(B)  $2^{-8}$

(C)  $2^{-6}$

(D)  $2^{-4}$

- Q. 65 A zero-mean white Gaussian noise is passes through an ideal lowpass filter of bandwidth 10 kHz. The output is then uniformly sampled with sampling period  $t_s = 0.03$  msec. The samples so obtained would be  
 (A) correlated (B) statistically independent  
 (C) uncorrelated (D) orthogonal

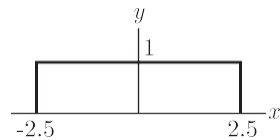
- Q. 66 A source generates three symbols with probabilities 0.25, 0.25, 0.50 at a rate of 3000 symbols per second. Assuming independent generation of symbols, the most efficient source encoder would have average bit rate is  
 (A) 6000 bits/sec (B) 4500 bits/sec  
 (C) 3000 bits/sec (D) 1500 bits/sec

- Q. 67 The diagonal clipping in Amplitude Demodulation (using envelop detector) can be avoided if RC time-constant of the envelope detector satisfies the following condition, (here  $W$  is message bandwidth and  $w$  is carrier frequency both in rad/sec)  
 (A)  $RC < \frac{1}{W}$  (B)  $RC > \frac{1}{W}$   
 (C)  $RC < \frac{1}{w}$  (D)  $RC > \frac{1}{w}$

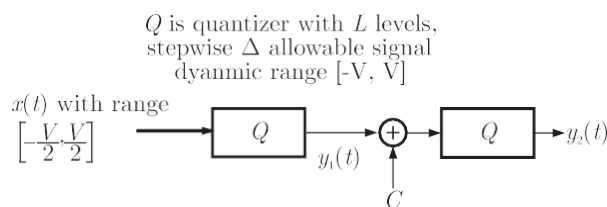
- Q. 68 A uniformly distributed random variable  $X$  with probability density function

$$f_x(x) = \frac{1}{10} p[u(x+5) - u(x-5)]$$

where  $u(\cdot)$  is the unit step function is passed through a transformation given in the figure below. The probability density function of the transformed random variable  $Y$  would be



- (A)  $f_y(y) = \frac{1}{5} [u(y+2.5) - u(y-2.5)]$   
 (B)  $f_y(y) = 0.5d(y) + 0.5d(y-1)$   
 (C)  $f_y(y) = 0.25d(y+2.5) + 0.25d(y-2.5) + 5d(y)$   
 (D)  $f_y(y) = 0.25d(y+2.5) + 0.25d(y-2.5) + \frac{1}{10} [u(y+2.5) - u(y-2.5)]$
- Q. 69 In the following figure the minimum value of the constant " $C$ ", which is to be added to  $y_1(t)$  such that  $y_1(t)$  and  $y_2(t)$  are different, is



- (A) 3 (B)  $\frac{3}{2}$   
 (C)  $\frac{3^2}{12}$  (D)  $\frac{3}{L}$

- Q. 70** A message signal with bandwidth 10 kHz is Lower-Side Band SSB modulated with carrier frequency  $f_{c1} = 10^6$  Hz. The resulting signal is then passed through a Narrow-Band Frequency Modulator with carrier frequency  $f_{c2} = 10^9$  Hz. The bandwidth of the output would be
- (A)  $4 \times 10^4$  Hz (B)  $2 \times 10^6$  Hz  
(C)  $2 \times 10^9$  Hz (D)  $2 \times 10^{10}$  Hz

**Common Data For Q. 71 and 72 :**

Let  $g(t) = p(t) * (pt)$ , where  $*$  denotes convolution &  $p(t) = u(t) - u(t-1)$  with  $u(t)$  being the unit step function

- Q. 71** The impulse response of filter matched to the signal  $s(t) = g(t) - d(1-2) * g(t)$  is given as :
- (A)  $s(1-t)$  (B)  $-s(1-t)$   
(C)  $-s(t)$  (D)  $s(t)$
- Q. 72** An Amplitude Modulated signal is given as
- $$x_{AM}(t) = 100 [p(t) + 0.5g(t)] \cos \omega_c t$$
- in the interval  $0 \leq t \leq 1$ . One set of possible values of modulating signal and modulation index would be
- (A)  $t, 0.5$  (B)  $t, 1.0$   
(C)  $t, 2.0$  (D)  $t^2, 0.5$

**Common Data For Q. 73 and 74 :**

The following two question refer to wide sense stationary stochastic process

- Q. 73** It is desired to generate a stochastic process (as voltage process) with power spectral density  $S(w) = 16 / (16 + w^2)$  by driving a Linear-Time-Invariant system by zero mean white noise (As voltage process) with power spectral density being constant equal to 1. The system which can perform the desired task could be
- (A) first order lowpass R-L filter  
(B) first order highpass R-C filter  
(C) tuned L-C filter  
(D) series R-L-C filter
- Q. 74** The parameters of the system obtained in previous Q would be
- (A) first order R-L lowpass filter would have  $R = 4\Omega$   $L = 1H$   
(B) first order R-C highpass filter would have  $R = 4\Omega$   $C = 0.25F$   
(C) tuned L-C filter would have  $L = 4H$   $C = 4F$   
(D) series R-L-C lowpass filter would have  $R = 1\Omega$ ,  $L = 4H$ ,  $C = 4F$

**Common Data For Q. 75 and 76 :**

Consider the following Amplitude Modulated (AM) signal, where  $f_m < B$

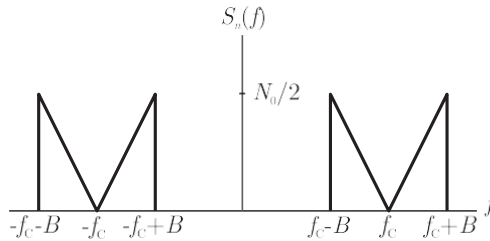
$$X_{AM}(t) = 10(1 + 0.5 \sin 2\pi f_m t) \cos 2\pi f_c t$$

- Q. 75** The average side-band power for the AM signal given above is

- (A) 25 (B) 12.5  
(C) 6.25 (D) 3.125

Q. 76

The AM signal gets added to a noise with Power Spectral Density  $S_n(f)$  given in the figure below. The ratio of average sideband power to mean noise power would be :



- (A)  $\frac{25}{8N_0B}$  (B)  $\frac{25}{4N_0B}$   
(C)  $\frac{25}{2N_0B}$  (D)  $\frac{25}{N_0B}$

2005

ONE MARK

Q. 77

Find the correct match between group 1 and group 2.

Group 1

Group 2

P.  $\{1 + km(t)A \sin(w_c t)\}$ 

W. Phase modulation

Q.  $km(t)A \sin(w_c t)$ 

X. Frequency modulation

R.  $A \sin\{w_c t + km(t)\}$ 

Y. Amplitude modulation

S.  $A \sin\{w_c t + k \int m(t) dt\}$ 

Z. DSB-SC modulation

- (A) P – Z, Q – Y, R – X, S – W  
(B) P – W, Q – X, R – Y, S – Z  
(C) P – X, Q – W, R – Z, S – Y  
(D) P – Y, Q – Z, R – W, S – X

Q. 78

Which of the following analog modulation scheme requires the minimum transmitted power and minimum channel bandwidth ?

- (A) VSB (B) DSB-SC  
(C) SSB (D) AM

2005

TWO MARKS

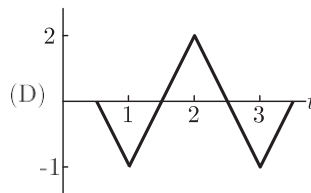
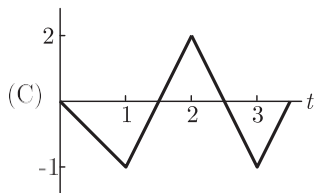
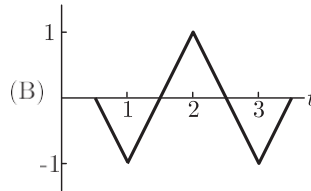
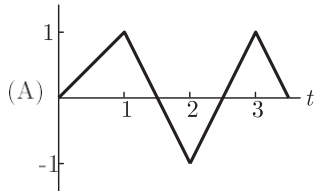
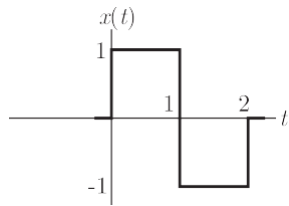
Q. 79

A device with input  $X(t)$  and output  $y(t)$  is characterized by:  $Y(t) = x^2(t)$ . An FM signal with frequency deviation of 90 kHz and modulating signal bandwidth of 5 kHz is applied to this device. The bandwidth of the output signal is

- (A) 370 kHz (B) 190 kHz  
(C) 380 kHz (D) 95 kHz

Q. 80

A signal as shown in the figure is applied to a matched filter. Which of the following does represent the output of this matched filter ?



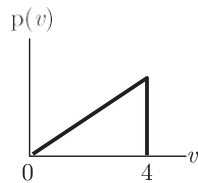
Q. 81

Noise with uniform power spectral density of  $N_0$  W/Hz is passed through a filter  $H(\omega) = 2 \exp(-j\omega t_d)$  followed by an ideal pass filter of bandwidth  $B$  Hz. The output noise power in Watts is

- (A)  $2N_0B$  (B)  $4N_0B$   
(C)  $8N_0B$  (D)  $16N_0B$

Q. 82

An output of a communication channel is a random variable  $v$  with the probability density function as shown in the figure. The mean square value of  $v$  is



- (A) 4 (B) 6  
(C) 8 (D) 9

Q. 83

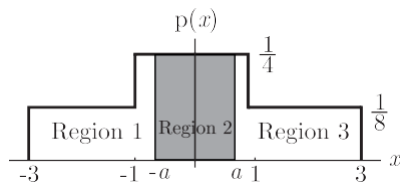
A carrier is phase modulated (PM) with frequency deviation of 10 kHz by a single tone frequency of 1 kHz. If the single tone frequency is increased to 2 kHz, assuming that phase deviation remains unchanged, the bandwidth of the PM signal is

- (A) 21 kHz (B) 22 kHz  
(C) 42 kHz (D) 44 kHz

#### Common Data For Q. 84 and 85 :

Asymmetric three-level midtread quantizer is to be designed assuming equiprobable occurrence of all quantization levels.



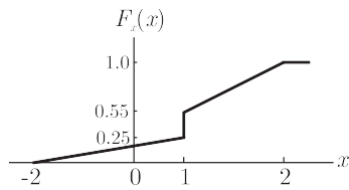


- Q. 84** If the probability density function is divided into three regions as shown in the figure, the value of  $a$  in the figure is
- (A)  $\frac{1}{3}$  (B)  $\frac{2}{3}$   
 (C)  $\frac{1}{2}$  (D)  $\frac{1}{4}$
- Q. 85** The quantization noise power for the quantization region between  $-a$  and  $+a$  in the figure is
- (A)  $\frac{4}{81}$  (B)  $\frac{1}{9}$   
 (C)  $\frac{5}{81}$  (D)  $\frac{2}{81}$

2004

ONE MARK

- Q. 86** In a PCM system, if the code word length is increased from 6 to 8 bits, the signal to quantization noise ratio improves by the factor
- (A)  $\frac{8}{6}$  (B) 12  
 (C) 16 (D) 8
- Q. 87** An AM signal is detected using an envelop detector. The carrier frequency and modulating signal frequency are 1 MHz and 2 kHz respectively. An appropriate value for the time constant of the envelop detector is
- (A) 500 msec (B) 20 msec  
 (C) 0.2 msec (D) 1 msec
- Q. 88** An AM signal and a narrow-band FM signal with identical carriers, modulating signals and modulation indices of 0.1 are added together. The resultant signal can be closely approximated by
- (A) broadband FM (B) SSB with carrier  
 (C) DSB-SC (D) SSB without carrier
- Q. 89** In the output of a DM speech encoder, the consecutive pulses are of opposite polarity during time interval  $t_1$  to  $t_2$ . This indicates that during this interval
- (A) the input to the modulator is essentially constant  
 (B) the modulator is going through slope overload  
 (C) the accumulator is in saturation  
 (D) the speech signal is being sampled at the Nyquist rate
- Q. 90** The distribution function  $F_x(x)$  of a random variable  $x$  is shown in the figure. The probability that  $X = 1$  is



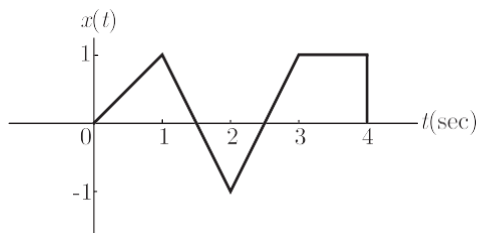
- (A) zero (B) 0.25  
(C) 0.55 (D) 0.30

2004

TWO MARKS

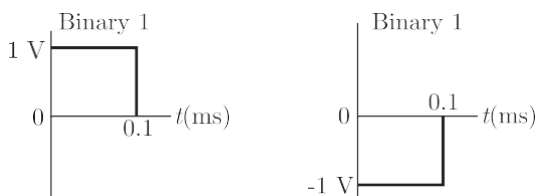
- Q. 91** A 1 mW video signal having a bandwidth of 100 MHz is transmitted to a receiver through cable that has 40 dB loss. If the effective one-side noise spectral density at the receiver is  $10^{-20}$  Watt/Hz, then the signal-to-noise ratio at the receiver is  
(A) 50 dB (B) 30 dB  
(C) 40 dB (D) 60 dB

- Q. 92** Consider the signal  $x(t)$  shown in Fig. Let  $h(t)$  denote the impulse response of the filter matched to  $x(t)$ , with  $h(t)$  being non-zero only in the interval 0 to 4 sec. The slope of  $h(t)$  in the interval  $3 < t < 4$  sec is



- (A)  $\frac{1}{2} \text{ sec}^{-1}$  (B)  $-1 \text{ sec}^{-1}$   
(C)  $-\frac{1}{2} \text{ sec}^{-1}$  (D)  $1 \text{ sec}^{-1}$

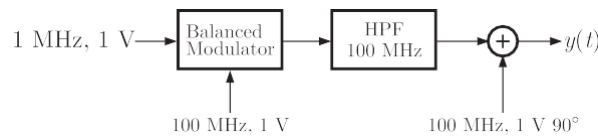
- Q. 93** A source produces binary data at the rate of 10 kbps. The binary symbols are represented as shown in the figure. The source output is transmitted using two modulation schemes, namely Binary PSK (BPSK) and Quadrature PSK (QPSK). Let  $B_1$  and  $B_2$  be the bandwidth requirements of the above rectangular pulses is 10 kHz,  $B_1$  and  $B_2$  are



- (A)  $B_1 = 20 \text{ kHz}$ ,  $B_2 = 20 \text{ kHz}$  (B)  $B_1 = 10 \text{ kHz}$ ,  $B_2 = 20 \text{ kHz}$   
(C)  $B_1 = 20 \text{ kHz}$ ,  $B_2 = 10 \text{ kHz}$  (D)  $B_1 = 10 \text{ kHz}$ ,  $B_2 = 10 \text{ kHz}$

- Q. 94** A 100 MHz carrier of 1 V amplitude and a 1 MHz modulating signal of 1 V amplitude are fed to a balanced modulator. The output of the modulator is passed through an ideal high-pass filter with cut-off frequency of 100 MHz. The output of the filter is added with 100 MHz signal of 1 V amplitude and 90° phase

shift as shown in the figure. The envelope of the resultant signal is



- (A) constant (B)  $\sqrt{1 + \sin(2\pi \cdot 10^6 t)}$   
 (C)  $\frac{5}{4} - \sin(2\pi \cdot 10^6 t)$  (D)  $\frac{5}{4} + \cos(2\pi \cdot 10^6 t)$

**Q. 95** Two sinusoidal signals of same amplitude and frequencies 10 kHz and 10.1 kHz are added together. The combined signal is given to an ideal frequency detector. The output of the detector is

- (A) 0.1 kHz sinusoid (B) 20.1 kHz sinusoid  
 (C) a linear function of time (D) a constant

**Q. 96** Consider a binary digital communication system with equally likely 0's and 1's. When binary 0 is transmitted the detector input can lie between the levels -0.25 V and +0.25 V with equal probability : when binary 1 is transmitted, the voltage at the detector can have any value between 0 and 1 V with equal probability. If the detector has a threshold of 0.2 V (i.e., if the received signal is greater than 0.2 V, the bit is taken as 1), the average bit error probability is

- (A) 0.15 (B) 0.2  
 (C) 0.05 (D) 0.5

**Q. 97** A random variable  $X$  with uniform density in the interval 0 to 1 is quantized as follows :

$$\begin{aligned} \text{If } 0 \leq X < 0.3, & \quad x_q = 0 \\ \text{If } 0.3 \leq X < 1, & \quad x_q = 0.7 \end{aligned}$$

where  $x_q$  is the quantized value of  $X$ .

The root-mean square value of the quantization noise is

- (A) 0.573 (B) 0.198  
 (C) 2.205 (D) 0.266

**Q. 98** Choose the current one from among the alternative A, B, C, D after matching an item from Group 1 with the most appropriate item in Group 2.

Group 1

1. FM
2. DM
3. PSK
4. PCM

Group 2

- P. Slope overload
- Q.  $m$ -law
- R. Envelope detector
- S. Hilbert transform
- T. Hilbert transform
- U. Matched filter

- (A) 1 - T, 2 - P, 3 - U, 4 - S (B) 1 - S, 2 - U, 3 - P, 4 - T  
 (C) 1 - S, 2 - P, 3 - U, 4 - Q (D) 1 - U, 2 - R, 3 - S, 4 - Q

**Q. 99** Three analog signals, having bandwidths 1200 Hz, 600 Hz and 600 Hz, are sampled at their respective Nyquist rates, encoded with 12 bit words, and time division multiplexed. The bit rate for the multiplexed signal is

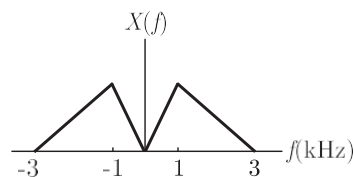
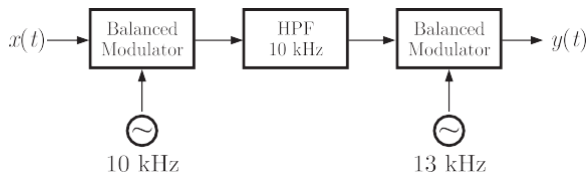
- (A) 115.2 kbps (B) 28.8 kbps

(C) 57.6 kbps

(D) 38.4 kbps

Q. 100

Consider a system shown in the figure. Let  $X(f)$  and  $Y(f)$  denote the Fourier transforms of  $x(t)$  and  $y(t)$  respectively. The ideal HPF has the cutoff frequency 10 kHz.



The positive frequencies where  $Y(f)$  has spectral peaks are

(A) 1 kHz and 24 kHz

(B) 2 kHz and 24 kHz

(C) 1 kHz and 14 kHz

(D) 2 kHz and 14 kHz

2003

ONE MARK

Q. 101

The input to a coherent detector is DSB-SC signal plus noise. The noise at the detector output is

(A) the in-phase component

(B) the quadrature - component

(C) zero

(D) the envelope

Q. 102

The noise at the input to an ideal frequency detector is white. The detector is operating above threshold. The power spectral density of the noise at the output is

(A) raised - cosine

(B) flat

(C) parabolic

(D) Gaussian

Q. 103

At a given probability of error, binary coherent FSK is inferior to binary coherent PSK by.

(A) 6 dB

(B) 3 dB

(C) 2 dB

(D) 0 dB

2003

TWO MARKS

Q. 104

Let  $X$  and  $Y$  be two statistically independent random variables uniformly distributed in the ranges  $(-1, 1)$  and  $(-2, 1)$  respectively. Let  $Z = X + Y$ . Then the probability that  $(z \neq -1)$  is

(A) zero

(B)  $\frac{1}{6}$ (C)  $\frac{1}{3}$ (D)  $\frac{1}{12}$

**Common Data For Q. 105 and 106 :**

$X(t)$  is a random process with a constant mean value of 2 and the auto correlation function  $R_{xx}(T) = 4(e^{-0.2|T|} + 1)$ .

- Q. 105** Let  $X$  be the Gaussian random variable obtained by sampling the process at  $t = t_i$  and let

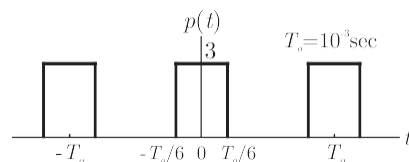
$$Q(a) = \frac{1}{\sqrt{2\pi}} \int_a^{\infty} e^{-\frac{y^2}{2}} dy$$

The probability that  $X \geq 1$  is

- (A)  $1 - Q(0.5)$  (B)  $Q(0.5)$   
 (C)  $Q\left(\frac{1}{2}\right)$  (D)  $1 - Q\left(\frac{1}{2}\right)$
- Q. 106** Let  $Y$  and  $Z$  be the random variable obtained by sampling  $X(t)$  at  $t = 2$  and  $t = 4$  respectively. Let  $W = Y - Z$ . The variance of  $W$  is
- (A) 13.36 (B) 9.36  
 (C) 2.64 (D) 8.00

- Q. 107** A sinusoidal signal with peak-to-peak amplitude of 1.536 V is quantized into 128 levels using a mid-rise uniform quantizer. The quantization-noise power is
- (A) 0.768 V (B)  $48 \times 10^{-6} V^2$   
 (C)  $12 \times 10^{-6} V^2$  (D) 3.072 V

- Q. 108** Let  $x(t) = 2\cos(800\pi t) + \cos(1400\pi t)$ .  $x(t)$  is sampled with the rectangular pulse train shown in the figure. The only spectral components (in kHz) present in the sampled signal in the frequency range 2.5 kHz to 3.5 kHz are



- (A) 2.7, 3.4 (B) 3.3, 3.6  
 (C) 2.6, 2.7, 3.3, 3.4, 3.6 (D) 2.7, 3.3
- Q. 109** A DSB-SC signal is to be generated with a carrier frequency  $f_c = 1$  MHz using a non-linear device with the input-output characteristic  $V_0 = a_0 v_i + a_1 v_i^3$  where  $a_0$  and  $a_1$  are constants. The output of the non-linear device can be filtered by an appropriate band-pass filter. Let  $V_i = A_i \cos(2\pi f_i t) + m(t)$  is the message signal. Then the value of  $f_i$  (in MHz) is
- (A) 1.0 (B) 0.333  
 (C) 0.5 (D) 3.0

**Common Data For Q. 110 and 111 :**

Let  $m(t) = \cos[(4\pi \times 10^3)t]$  be the message signal &  
 $c(t) = 5 \cos[(2\pi \times 10^6)t]$  be the carrier.

- Q. 110**  $c(t)$  and  $m(t)$  are used to generate an AM signal. The modulation index of the generated AM signal is 0.5. Then the quantity  $\frac{\text{Total sideband power}}{\text{Carrier power}}$  is

- (A)  $\frac{1}{2}$  (B)  $\frac{1}{4}$   
 (C)  $\frac{1}{3}$  (D)  $\frac{1}{8}$

**Q. 111**  $c(t)$  and  $m(t)$  are used to generate an FM signal. If the peak frequency deviation of the generated FM signal is three times the transmission bandwidth of the AM signal, then the coefficient of the term  $\cos[2\pi(1008 + 10^3t)]$  in the FM signal (in terms of the Bessel coefficients) is

- (A)  $5J_4(3)$  (B)  $\frac{5}{2}J_8(3)$   
 (C)  $\frac{5}{2}J_8(4)$  (D)  $5J_4(6)$

**Q. 112** Choose the correct one from among the alternative A, B, C, D after matching an item in Group 1 with most appropriate item in Group 2.

- | Group 1                       | Group 2                   |
|-------------------------------|---------------------------|
| P. Ring modulator             | 1. Clock recovery         |
| Q. VCO                        | 2. Demodulation of FM     |
| R. Foster-Seely discriminator | 3. Frequency conversion   |
| S. Mixer                      | 4. Summing the two inputs |
|                               | 5. Generation of FM       |
|                               | 6. Generation of DSB-Sc   |
- (A)  $P-1; Q-3; R-2; S-4$  (B)  $P-6; Q=5; R-2; S-3$   
 (C)  $P-6; Q-1; R-3; S-2$  (D)  $P-5; Q-6; R-1; S-3$

**Q. 113** A superheterodyne receiver is to operate in the frequency range 550 kHz - 1650 kHz, with the intermediate frequency of 450 kHz. Let  $R = C_{\max}/C_{\min}$  denote the required capacitance ratio of the local oscillator and  $I$  denote the image frequency (in kHz) of the incoming signal. If the receiver is tuned to 700 kHz, then

- (A)  $R = 4.41, I = 1600$  (B)  $R = 2.10, I = 1150$   
 (C)  $R = 3.0, I = 600$  (D)  $R = 9.0, I = 1150$

**Q. 114** If  $E_b$ , the energy per bit of a binary digital signal, is  $10^{-5}$  watt-sec and the one-sided power spectral density of the white noise,  $N_0 = 10^{-6}$  W/Hz, then the output SNR of the matched filter is

- (A) 26 dB (B) 10 dB  
 (C) 20 dB (D) 13 dB

**Q. 115** The input to a linear delta modulator having a step-size  $\Delta = 0.628$  is a sine wave with frequency  $f_m$  and peak amplitude  $E_m$ . If the sampling frequency  $f_s = 40$  kHz, the combination of the sine-wave frequency and the peak amplitude, where slope overload will take place is

- | $E_m$     | $f_m$ |
|-----------|-------|
| (A) 0.3 V | 8 kHz |
| (B) 1.5 V | 4 kHz |
| (C) 1.5 V | 2 kHz |
| (D) 3.0 V | 1 kHz |

**Q. 116** If  $S$  represents the carrier synchronization at the receiver and  $r$  represents the bandwidth efficiency, then the correct statement for the coherent binary PSK is

- (A)  $r = 0.5$ ,  $S$  is required (B)  $r = 1.0$ ,  $S$  is required  
 (C)  $r = 0.5$ ,  $S$  is not required (D)  $r = 1.0$ ,  $S$  is not required

- Q. 117 A signal is sampled at 8 kHz and is quantized using 8 - bit uniform quantizer. Assuming  $SNR_q$  for a sinusoidal signal, the correct statement for PCM signal with a bit rate of  $R$  is
- (A)  $R = 32$  kbps,  $SNR_q = 25.8$  dB  
 (B)  $R = 64$  kbps,  $SNR_q = 49.8$  dB  
 (C)  $R = 64$  kbps,  $SNR_q = 55.8$  dB  
 (D)  $R = 32$  kbps,  $SNR_q = 49.8$  dB

2002

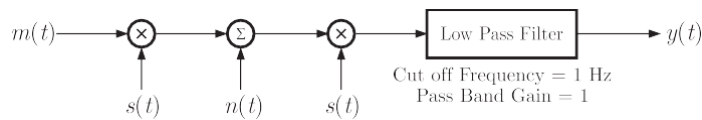
ONE MARK

- Q. 118 A 2 MHz sinusoidal carrier amplitude modulated by symmetrical square wave of period 100 msec. Which of the following frequencies will NOT be present in the modulated signal ?
- (A) 990 kHz (B) 1010 kHz  
 (C) 1020 kHz (D) 1030 kHz
- Q. 119 Consider a sample signal  $y_3(t) = 5 \times 10^{-6} \sum_{n=-3}^{+3} d(t - nT_s)$  where  $x(t) = 10 \cos(8\pi \times 10^3 t)$  and  $T = 100 \mu$  sec.
- When  $y(t)$  is passed through an ideal lowpass filter with a cutoff frequency of 5 KHz, the output of the filter is
- (A)  $5 \times 10^{-6} \cos(8\pi \times 10^3 t)$  (b)  $5 \times 10^{-5} \cos(8\pi \times 10^3 t)$   
 (C)  $5 \times 10^{-1} \cos(8\pi \times 10^3 t)$  (D)  $10 \cos(8\pi \times 10^3 t)$
- Q. 120 For a bit-rate of 8 Kbps, the best possible values of the transmitted frequencies in a coherent binary FSK system are
- (A) 16 kHz and 20 kHz (C) 20 kHz and 32 kHz  
 (D) 20 kHz and 40 kHz (D) 32 kHz and 40 kHz
- Q. 121 The line-of-sight communication requires the transmit and receive antennas to face each other. If the transmit antenna is vertically polarized, for best reception the receiver antenna should be
- (A) horizontally polarized  
 (B) vertically polarized  
 (C) at 45° with respect to horizontal polarization  
 (D) at 45° with respect to vertical polarization

2002

TWO MARKS

- Q. 122 An angle-modulated signal is given by
- $$s(t) = \cos 2\pi (2 \times 10^6 t + 30 \sin 150t + 40 \cos 150t).$$
- The maximum frequency and phase deviations of  $s(t)$  are
- (A) 10.5 kHz,  $140\pi$  rad (B) 6 kHz,  $80\pi$  rad  
 (C) 10.5 kHz,  $100\pi$  rad (D) 7.5 kHz,  $100\pi$  rad
- Q. 123 In the figure  $m(t) = \frac{2 \sin 2\pi t}{t}$ ,  $s(t) = \cos 200\pi t$  and  $n(t) = \frac{\sin 199\pi t}{t}$ . The output  $y(t)$  will be



(A)  $\frac{\sin 2pt}{t}$

(B)  $\frac{\sin 2pt}{t} + \frac{\sin pt}{t} \cos 3pt$

(C)  $\frac{\sin 2pt}{t} + \frac{\sin 0.5pt}{t} \cos 1.5pt$

(D)  $\frac{\sin 2pt}{t} + \frac{\sin pt}{t} \cos 0.75pt$

Q. 124

A signal  $x(t) = 100 \cos(24\pi \times 10^3 t)$  is ideally sampled with a sampling period of 50  $\mu$ sec and then passed through an ideal lowpass filter with cutoff frequency of 15 kHz. Which of the following frequencies is/are present at the filter output?

(A) 12 kHz only

(B) 8 kHz only

(C) 12 kHz and 9 kHz

(D) 12 kHz and 8 kHz

Q. 125

If the variance  $\sigma^2$  of  $d(n) = x(n) - x(n-1)$  is one-tenth the variance  $\sigma^2$  of stationary zero-mean discrete-time signal  $x(n)$ , then the normalized autocorrelation function  $\frac{R_{xx}(k)}{\sigma_x^2}$  at  $k = 1$  is

(A) 0.95

(B) 0.90

(C) 0.10

(D) 0.05

2001

ONE MARK

Q. 126

A bandlimited signal is sampled at the Nyquist rate. The signal can be recovered by passing the samples through

(A) an RC filter

(B) an envelope detector

(C) a PLL

(D) an ideal low-pass filter with the appropriate bandwidth

Q. 127

The PDF of a Gaussian random variable  $X$  is given by

$$p_x(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-4)^2}{18}}$$

(A)  $\frac{1}{2}$

(B)  $\frac{1}{3\sqrt{2\pi}}$

(C) 0

(D)  $\frac{1}{4}$

2001

TWO MARKS

Q. 128

A video transmission system transmits 625 picture frames per second. Each frame consists of a 400  $\times$  400 pixel grid with 64 intensity levels per pixel. The data rate of the system is

(A) 16 Mbps

(B) 100 Mbps

(C) 600 Mbps

(D) 6.4 Gbps

Q. 129

The Nyquist sampling interval, for the signal  $\sin c(700t) + \sin c(500t)$  is

(A)  $\frac{1}{350}$  sec

(B)  $\frac{1}{350}$  sec

(C)  $\frac{1}{700}$  sec

(D)  $\frac{1}{175}$  sec



- Q. 130** During transmission over a communication channel, bit errors occur independently with probability  $p$ . If a block of  $n$  bits is transmitted, the probability of at most one bit error is equal to  
 (A)  $1 - (1 - p)^n$  (B)  $p + (n - 1)(1 - p)$   
 (C)  $np(1 - p)^{n-1}$  (D)  $(1 - p)^n + np(1 - p)^{n-1}$
- Q. 131** The PSD and the power of a signal  $g(t)$  are, respectively,  $S_g(w)$  and  $P_g$ . The PSD and the power of the signal  $ag(t)$  are, respectively,  
 (A)  $a^2 S_g(w)$  and  $a^2 P_g$  (B)  $a^2 S_g(w)$  and  $a P_g$   
 (C)  $a S_g(w)$  and  $a^2 P_g$  (D)  $a S_g(w)$  and  $a P_g$

2000

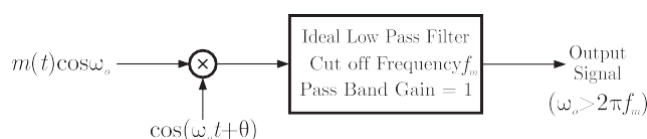
ONE MARK

- Q. 132** The amplitude modulated waveform  $s(t) = A_c [1 + K_a m(t)] \cos w_c t$  is fed to an ideal envelope detector. The maximum magnitude of  $K_a m(t)$  is greater than 1. Which of the following could be the detector output?  
 (A)  $A_c m(t)$  (B)  $A_c^2 [1 + K_a m(t)]^2$   
 (C)  $[A_c (1 + K_a m(t))]$  (D)  $A_c [1 + K_a m(t)]^2$
- Q. 133** The frequency range for satellite communication is  
 (A) 1 KHz to 100 KHz (B) 100 KHz to 10 KHz  
 (C) 10 MHz to 30 MHz (D) 1 GHz to 30 GHz

2000

TWO MARKS

- Q. 134** In a digital communication system employing Frequency Shift Keying (FSK), the 0 and 1 bit are represented by sine waves of 10 KHz and 25 KHz respectively. These waveforms will be orthogonal for a bit interval of  
 (A) 45 msec (B) 200 msec  
 (C) 50 msec (D) 250 msec
- Q. 135** A message  $m(t)$  bandlimited to the frequency  $f_m$  has a power of  $P_m$ . The power of the output signal in the figure is



- (A)  $\frac{P_m \cos q}{2}$  (B)  $\frac{P_m}{4}$   
 (C)  $\frac{P_m \sin^2 q}{4}$  (D)  $\frac{P_m \cos^2 q}{4}$
- Q. 136** The Hilbert transform of  $\cos w_1 t + \sin w_2 t$  is  
 (A)  $\sin w_1 t - \cos w_2 t$  (B)  $\sin w_1 t + \cos w_2 t$   
 (C)  $\cos w_1 t - \sin w_2 t$  (D)  $\sin w_1 t + \sin w_2 t$
- Q. 137** In a FM system, a carrier of 100 MHz modulated by a sinusoidal signal of 5 KHz. The bandwidth by Carson's approximation is 1 MHz. If  $y(t)$  = (modulated waveform)<sup>3</sup>, then by using Carson's approximation, the bandwidth of  $y(t)$  around 300 MHz and the spacing of spectral components are, respectively.

- (A) 3 MHz, 5 KHz  
(C) 3 MHz, 15 KHz

- (B) 1 MHz, 15 KHz  
(D) 1 MHz, 5 KHz

1999

ONE MARK

- Q. 138** The input to a channel is a bandpass signal. It is obtained by linearly modulating a sinusoidal carrier with a single-tone signal. The output of the channel due to this input is given by

$$y(t) = (1/100) \cos(100t - 10^{-6}) \cos(10^6 t - 1.56)$$

The group delay ( $t_g$ ) and the phase delay ( $t_p$ ) in seconds, of the channel are

- (A)  $t_g = 10^{-6}, t_p = 1.56$  (B)  $t_g = 1.56, t_p = 10^{-6}$   
(C)  $t_g = 10^8, t_p = 1.56 \times 10^{-6}$  (D)  $t_g = 10^8, t_p = 1.56$

- Q. 139** A modulated signal is given by  $s(t) = m_1(t) \cos(2\pi f_c t) + m_2(t) \sin(2\pi f_c t)$  where the baseband signal  $m_1(t)$  and  $m_2(t)$  have bandwidths of 10 kHz, and 15 kHz, respectively. The bandwidth of the modulated signal, in kHz, is

- (A) 10 (B) 15  
(C) 25 (D) 30

- Q. 140** A modulated signal is given by  $s(t) = e^{-at} \cos[(\omega_c + D\omega)t] u(t)$ , where  $a, \omega_c$  and  $D\omega$  are positive constants, and  $\omega_c \gg D\omega$ . The complex envelope of  $s(t)$  is given by

- (A)  $\exp(-at) \exp[j(\omega_c + D\omega)t] u(t)$   
(B)  $\exp(-at) \exp(jD\omega t) u(t)$   
(C)  $\exp(jD\omega t) u(t)$   
(D)  $\exp[j\omega_c + D\omega)t]$

1999

TWO MARKS

- Q. 141** The Nyquist sampling frequency (in Hz) of a signal given by  $6 \times 10^4 \sin^2(400t) * 10^6 \sin^3(100t)$  is

- (A) 200 (B) 300  
(C) 500 (D) 1000

- Q. 142** The peak-to-peak input to an 8-bit PCM coder is 2 volts. The signal power-to-quantization noise power ratio (in dB) for an input of  $0.5 \cos(\omega_m t)$  is

- (A) 47.8 (B) 49.8  
(C) 95.6 (D) 99.6

- Q. 143** The input to a matched filter is given by
- $$s(t) = \begin{cases} 10 \sin(2\pi \times 10^6 t) & 0 < t < 10^{-4} \text{ sec} \\ 0 & \text{otherwise} \end{cases}$$

The peak amplitude of the filter output is

- (A) 10 volts (B) 5 volts  
(C) 10 millivolts (D) 5 millivolts

- Q. 144** Four independent messages have bandwidths of 100 Hz, 200 Hz and 400 Hz, respectively. Each is sampled at the Nyquist rate, and the samples are time division multiplexed (TDM) and transmitted. The transmitted sample rate (in

- Hz) is  
 (A) 1600 (B) 800  
 (C) 400 (D) 200

1998

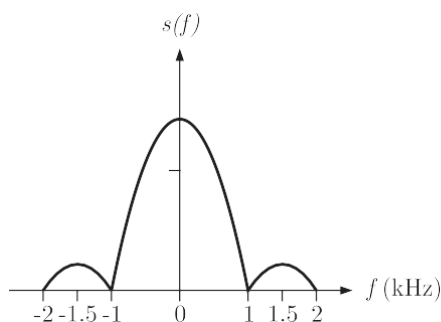
ONE MARK

- Q. 145** The amplitude spectrum of a Gaussian pulse is  
 (A) uniform (B) a sine function  
 (C) Gaussian (D) an impulse function
- Q. 146** The ACF of a rectangular pulse of duration  $T$  is  
 (A) a rectangular pulse of duration  $T$   
 (B) a rectangular pulse of duration  $2T$   
 (C) a triangular pulse of duration  $T$   
 (D) a triangular pulse of duration  $2T$
- Q. 147** The image channel selectivity of superheterodyne receiver depends upon  
 (A) IF amplifiers only  
 (B) RF and IF amplifiers only  
 (C) Preselector, RF and IF amplifiers  
 (D) Preselector, and RF amplifiers only
- Q. 148** In a PCM system with uniform quantisation, increasing the number of bits from 8 to 9 will reduce the quantisation noise power by a factor of  
 (A) 9 (B) 8  
 (C) 4 (D) 2
- Q. 149** Flat top sampling of low pass signals  
 (A) gives rise to aperture effect (B) implies oversampling  
 (C) leads to aliasing (D) introduces delay distortion
- Q. 150** A DSB-SC signal is generated using the carrier  $\cos(\omega_c t + \phi)$  and modulating signal  $x(t)$ . The envelope of the DSB-SC signal is  
 (A)  $x(t)$  (B)  $|x(t)|$   
 (C) only positive portion of  $x(t)$  (D)  $x(t)\cos\phi$
- Q. 151** Quadrature multiplexing is  
 (A) the same as FDM  
 (B) the same as TDM  
 (C) a combination of FDM and TDM  
 (D) quite different from FDM and TDM
- Q. 152** The Fourier transform of a voltage signal  $x(t)$  is  $X(f)$ . The unit of  $|X(f)|$  is  
 (A) volt (B) volt-sec  
 (C) volt/sec (D)  $\text{volt}^2$
- Q. 153** Compression in PCM refers to relative compression of  
 (A) higher signal amplitudes (B) lower signal amplitudes  
 (C) lower signal frequencies (D) higher signal frequencies

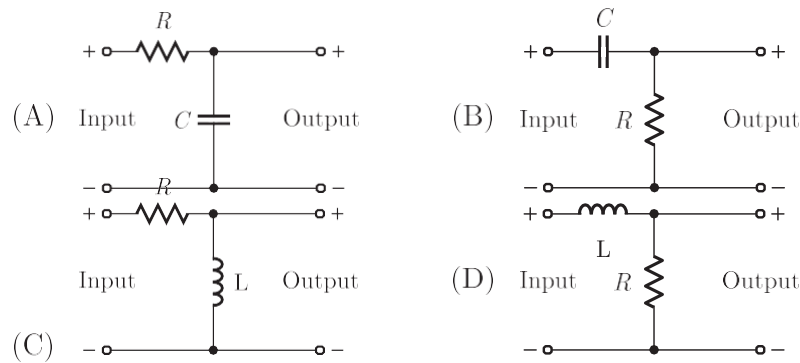
- Q. 154** For a give data rate, the bandwidth  $B_p$  of a BPSK signal and the bandwidth  $B_0$  of the OOK signal are related as  
 (A)  $B_p = \frac{B_0}{4}$  (B)  $B_p = \frac{B_0}{2}$   
 (C)  $B_p = B_0$  (D)  $B_p = 2B_0$
- Q. 155** The spectral density of a real valued random process has  
 (A) an even symmetry (B) an odd symmetry  
 (C) aconjugate symmetry (D) no symmetry
- Q. 156** The probability density function of the envelope of narrow band Gaussian noise is  
 (A) Poisson (B) Gaussian  
 (C) Rayleigh (D) Rician

**1997****ONE MARK**

- Q. 157** The line code that has zero dc component for pulse transmission of random binary data is  
 (A) Non-return to zero (NRZ)  
 (B) Return to zero (RZ)  
 (C) Alternate Mark Inversion (AM)  
 (D) None of the above
- Q. 158** A probability density function is given by  $p(x) = Ke^{-x^2/2} - 3 < x < 3$ . The value of K should be  
 (A)  $\frac{1}{2\sqrt{p}}$  (B)  $\frac{2}{p}$   
 (C)  $\frac{1}{2\sqrt{p}}$  (D)  $\frac{1}{\sqrt{2}}$
- Q. 159** A deterministic signal has the power spectrum given in the figure is, The minimum sampling rate needed to completely represent this signal is



- (A) 1 kHz (B) 2 kHz  
 (C) 3 kHz (D) None of these
- Q. 160** A communication channel has first order low pass transfer function. The channel is used to transmit pulses at a symbol rate greater than the half-power frequency of the low pass function. Which of the network shown in the figure is can be used to equalise the received pulses?



- Q. 161** The power spectral density of a deterministic signal is given by  $[\sin(f)/f^2]$  where  $f$  is frequency. The auto correlation function of this signal in the time domain is  
 (A) a rectangular pulse (B) a delta function  
 (C) a sine pulse (D) a triangular pulse

1996

ONE MARK

- Q. 162** A rectangular pulse of duration  $T$  is applied to a filter matched to this input. The output of the filter is a  
 (A) rectangular pulse of duration  $T$   
 (B) rectangular pulse of duration  $2T$   
 (C) triangular pulse  
 (D) sine function
- Q. 163** The image channel rejection in a superheterodyne receiver comes from  
 (A)  $IF$  stages only (B)  $RF$  stages only  
 (C) detector and  $RF$  stages only (D) detector  $RF$  and  $IF$  stages

1996

TWO MARKS

- Q. 164** The number of bits in a binary PCM system is increased from  $n$  to  $n + 1$ . As a result, the signal to quantization noise ratio will improve by a factor  
 (A)  $\frac{n+1}{n}$  (B)  $2^{(n+1)/n}$   
 (C)  $2^{2(n+1)/n}$  (D) which is independent of  $n$
- Q. 165** The auto correlation function of an energy signal has  
 (A) no symmetry (B) conjugate symmetry  
 (C) odd symmetry (D) even symmetry

\*\*\*\*\*

# SOLUTIONS

Sol. 1

Option (B) is correct.

In ideal Nyquist Channel, bandwidth required for ISI (Inter Symbol reference) free transmission is

$$W = \frac{R_b}{2}$$

Here, the used modulation is 32 - QAM (Quantum Amplitude modulation) i.e.,

$$q = 32$$

or  $2^v = 32$

$$v = 5 \text{ bits}$$

So, the signaling rate (sampling rate) is

$$R_b = \frac{R}{5} \quad (R \text{ is given bit rate})$$

Hence, for ISI free transmission, minimum bandwidth is

$$W = \frac{R_b}{2} = \frac{R}{10} \text{ kHz}$$

Sol. 2

Option (B) is correct.

Given, random variables  $U$  and  $V$  with mean zero and variances  $\frac{1}{4}$  and  $\frac{1}{9}$

i.e.,  $U = V = 0$

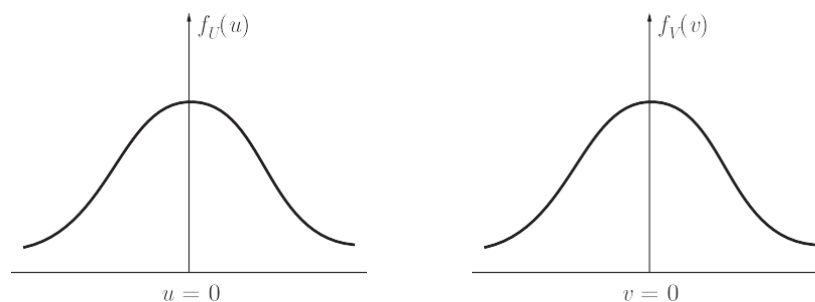
$$s_u^2 = \frac{1}{4}$$

and  $s_v^2 = \frac{1}{9}$

so,  $P^{U \leq 0} = \frac{1}{2}$

and  $P^{V \leq 0} = \frac{1}{2}$

The distribution is shown in the figure below



$$f_u(u) = \frac{1}{\sqrt{2\pi s_u^2}} e^{-u^2/2s_u^2}$$

$$f_v(v) = \frac{1}{\sqrt{2\pi s_v^2}} e^{-v^2/2s_v^2}$$

We can express the distribution in standard form by assuming

$$X = \frac{u - 0}{s_u} = \frac{u}{2} = 2U$$

and  $Y = \frac{v - 0}{s_v} = \frac{v}{3} = 3V$

for which we have

$$X = 2U = 0$$

$$Y = 2V = 0$$

and  $\frac{X^2}{4} = U^2 = 1$

also,  $\frac{Y^2}{4} = V^2 = 1$

Therefore,  $X - Y$  is also a normal random variable with

$$\overline{X - Y} = 0$$

Hence,

$$P(X - Y = 0) = P(X = Y = 0) = \frac{1}{2}$$

or, we can say

$$P(2U - 2V = 0) = \frac{1}{2}$$

Thus,  $P(3V = 2U) = \frac{1}{2}$

**Sol. 3**

Option (C) is correct.

The mean of random variables  $U$  and  $V$  are both zero

i.e.,  $\overline{U} = \overline{V} = 0$

Also, the random variables are identical

i.e.,  $f_U(u) = f_V(v)$

or,  $F_U(u) = F_V(v)$

i.e., their cdf are also same. So,

$$F_U(u) = F_V(v)$$

i.e., the cdf of random variable  $2V$  will be also same but for any instant

$$2V = U$$

Therefore,

but,  $G_X(x) = F_X(x)$

or,  $6F_X(x) - G_X(x) \neq 0$

**Sol. 4**

Option (C) is correct.

Given,  $P(U = +1) = P(U = -1) = \frac{1}{2}$

where  $U$  is a random variable which is identical to  $V$  i.e.,

$$P(V = +1) = P(V = -1) = \frac{1}{2}$$

So, random variable  $U$  and  $V$  can have following values

$$U = +1, -1; V = +1, -1$$

Therefore the random variable  $U + V$  can have the following values,

$$-2 \text{ When } U = V = -1$$

$$U + V = 0 \text{ When } U = 1, V = 1 \text{ or } U = -1, V = -1$$

$$2 \text{ When } U = V = 1$$

Hence, we obtain the probabilities for  $U + V$  as follows

$U + V$	$P(U + V)$
-2	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
0	$\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$
2	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

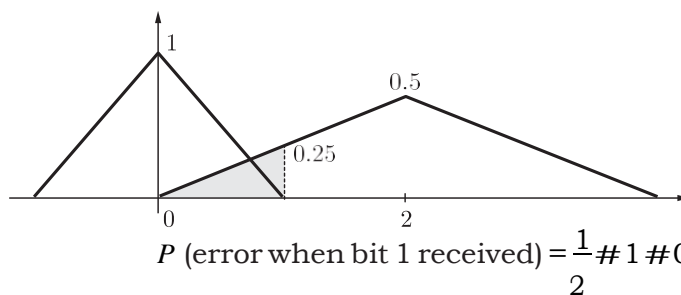
Therefore, the entropy of the  $U + V$  is obtained as

$$\begin{aligned}
 H^U + Vh &= - \frac{1}{P^U + Vh} \log_2 \frac{1}{P^U + Vh} \\
 &= -\frac{1}{4} \log_2 4 + \frac{1}{2} \log_2 2 + \frac{1}{4} \log_2 4 \\
 &= \frac{2}{4} + \frac{1}{2} + \frac{2}{4} \\
 &= \frac{3}{2}
 \end{aligned}$$

**Sol. 5**

Option (D) is correct.

For the shown received signal, we conclude that if 0 is the transmitted signal then the received signal will be also zero as the threshold is 1 and the pdf of bit 0 is not crossing 1. Again, we can observe that there is an error when bit 1 is received as it crosses the threshold. The probability of error is given by the area enclosed by the 1 bit pdf (shown by shaded region)



$$P(\text{error when bit 1 received}) = \frac{1}{2} \times 1 \times 0.25 = \frac{1}{8}$$

or

$$P_{b \frac{\text{received 1}}{\text{transmitted 0}}} = \frac{1}{8}$$

Since, the 1 and 0 transmission is equiprobable:

$$\text{i.e., } P^{0h} = P^{1h} = \frac{1}{2}$$

Hence bit error rate (BER) is

$$\begin{aligned}
 \text{BER} &= P_{b \frac{\text{received 0}}{\text{transmitted 1}}} P^{0h} + P_{b \frac{\text{received 1}}{\text{transmitted 0}}} P^{1h} \\
 &= 0 + \frac{1}{8} \times \frac{1}{2} \\
 &= \frac{1}{16}
 \end{aligned}$$

**Sol. 6**

Option (B) is correct.

The optimum threshold is the threshold value for transmission as obtained at the intersection of two pdf. From the shown pdf. We obtain at the intersection

$$(\text{transmitted, received}) = \left( \frac{4}{5}, \frac{1}{5} \right)$$

we can obtain the intersection by solving the two linear eqs

$$x + y = 1 \quad \text{pdf of received bit 0}$$

$$y = \frac{0.5}{2}x \quad \text{pdf of received bit 1}$$

Hence for threshold  $\frac{4}{5}$ , we have

$$\begin{aligned}
 \text{BER} &= P_{b \frac{\text{received 1}}{\text{transmitted 0}}} P^{0h} + P_{b \frac{\text{received 0}}{\text{transmitted 1}}} P^{1h} \\
 &= \left( \frac{1}{2} \times \frac{1}{5} \times \frac{1}{2} \right) + \left( \frac{1}{2} \times \frac{4}{5} \times \frac{1}{2} \right) \\
 &= \frac{1}{20} < (\text{BER for threshold} = 1)
 \end{aligned}$$



Hence, optimum threshold is  $\frac{4}{5}$

Sol. 7

Option (A) is correct.

The mean square value of a stationary process equals the total area under the graph of power spectral density, that is

$$E[X^2(t)] = \int_{-\infty}^{\infty} S_X(f) df$$

$$\text{or, } E[X^2(t)] = \int_{-\infty}^{\infty} S_X(w) dw$$

$$\text{or, } E[X^2(t)] = \frac{2p}{2p-1} \int_0^p S_X(w) dw \quad (\text{Since the PSD is even})$$

$$= \frac{1}{p} [\text{area under the triangle} + \text{integration of delta function}]$$

$$= \frac{1}{p} \left[ 2 \times \frac{1}{2} \times 1 \times 10^3 + 400 \right]$$

$$= \frac{1}{p} [6000 + 400] = \frac{6400}{p}$$

$|E[X(t)]|$  is the absolute value of mean of signal  $X(t)$  which is also equal to value of  $X(w)$  at  $(w=0)$ .

From given PSD

$$S_X(w) \Big|_{w=0} = 0$$

$$S_X(w) = |X(w)|^2 = 0$$

$$|X(w)|^2 \Big|_{w=0} = 0$$

$$|X(w)| \Big|_{w=0} = 0$$

Sol. 8

Option (C) is correct.

For raised cosine spectrum transmission bandwidth is given as

$$B_T = W(1+a) \quad a \text{ " Roll of factor}$$

$$B_T = \frac{R_b}{2}(1+a) \quad R_b \text{ " Maximum signaling rate}$$

$$3500 = \frac{R_b}{2}(1+0.75)$$

$$R_b = \frac{3500 \times 2}{1.75} = 4000$$

Sol. 9

Option (D) is correct.

Entropy function of a discrete memory less system is given as

$$H = - \sum_{k=0}^N P_k \log_b \frac{1}{P_k}$$

where  $P_k$  is probability of symbol  $S_k$ .

For first two symbols probability is same, so

$$H = - P_1 \log_b \frac{1}{P_1} - P_2 \log_b \frac{1}{P_2} - \sum_{k=3}^N P_k \log_b \frac{1}{P_k}$$

$$= - P_1 \log P_1 - P_2 \log P_2 - \sum_{k=3}^N P_k \log P_k$$

$$= - 2P \log P - \sum_{k=3}^{N-1} P_k \log P_k \quad (P_1 = P_2 = P)$$

Now,  $P_1 = P + e, P_2 = P - e$

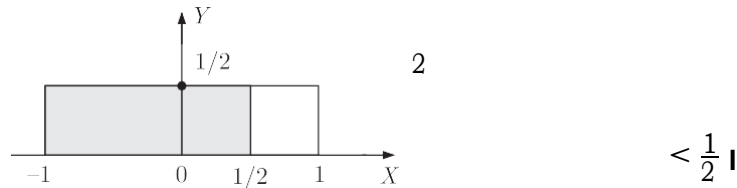
$$\text{So, } H' = -(P+e)\log(P+e) + (P-e)\log(P-e) + \sum_{k=3}^{N-1} P_k \log P_k$$

By comparing,  $H' < H$ , Entropy of source decreases.

Sol. 10

Option (B) is correct.

Probability density function of uniformly distributed variables  $X$  and  $Y$  is shown as



$$P[\max(x, y)] < \frac{1}{2}$$

Since  $X$  and  $Y$  are independent.

$$P[\max(x, y)] < \frac{1}{2} = P[X < \frac{1}{2}] P[Y < \frac{1}{2}]$$

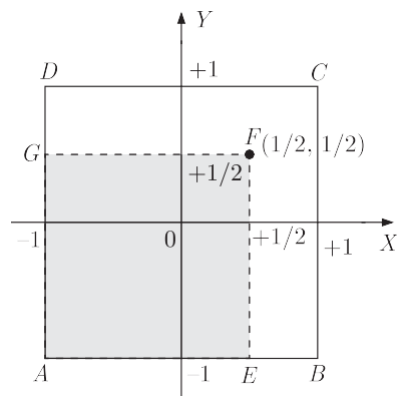
$$P[X < \frac{1}{2}] = \text{shaded area} = \frac{3}{4}$$

Similarly for  $Y$ :  $P[Y < \frac{1}{2}] = \frac{3}{4}$

$$\text{So } P[\max(x, y)] < \frac{1}{2} = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

#### Alternate Method:

From the given data since random variables  $X$  and  $Y$  lies in the interval  $[-1, 1]$  as from the figure  $X, Y$  lies in the region of the square  $ABCD$ .



Probability for  $\max(X, Y) < 1/2$ : The points for  $\max(X, Y) < 1/2$  will be inside the region of square  $AEFG$ .

$$\text{So, } P[\max(X, Y) < \frac{1}{2}] = \frac{\text{Area of } AEFG}{\text{Area of square } ABCD}$$

$$= \frac{\frac{3}{2} \times \frac{3}{2}}{2 \times 2} = \frac{9}{16}$$

Sol. 11

Option (B) is correct.

In a coherent binary PSK system, the pair of signals  $s_1(t)$  and  $s_2(t)$  used to represent binary system 1 and 0 respectively.

$$s_1(t) = \sqrt{\frac{2E}{T}} \sin w_c t$$

$$s_2(t) = -\sqrt{\frac{2E}{T}} \sin w_c t$$

where  $0 \leq t \leq T$ ,  $E$  is the transmitted energy per bit.

General function of local oscillator

$$f_1(t) = \sqrt{\frac{2}{T}} \sin(w_c t), 0 \leq t < T$$

But here local oscillator is ahead with  $45^\circ$ . so,

$$f_1(t) = \sqrt{\frac{2}{T}} \sin(w_c t + 45^\circ)$$

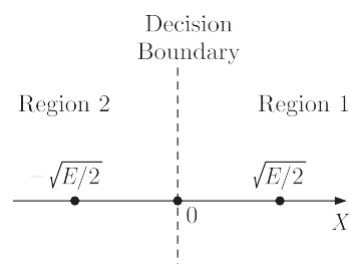
The coordinates of message points are

$$\begin{aligned} s_{11} &= \int_0^T s_1(t) f_1(t) dt \\ &= \int_0^T \sqrt{\frac{2E}{T}} \sin w_c t \sqrt{\frac{2}{T}} \sin(w_c t + 45^\circ) dt \\ &= \frac{2E}{T} \int_0^T \sin(w_c t) \sin(w_c t + 45^\circ) dt \\ &= \frac{2E}{T} \int_0^T \frac{1}{2} [\sin 45^\circ + \sin(2w_c t + 45^\circ)] dt \\ &= \frac{1}{T} \sqrt{E} \int_0^T \frac{1}{\sqrt{2}} dt + \frac{1}{T} \sqrt{E} \int_0^T \sin(2w_c t + 45^\circ) dt \\ &= \frac{E}{2} \end{aligned}$$

Similarly,

$$s_{21} = -\sqrt{\frac{E}{2}}$$

Signal space diagram



Now here the two message points are  $s_{11}$  and  $s_{21}$ .

The error at the receiver will be considered.

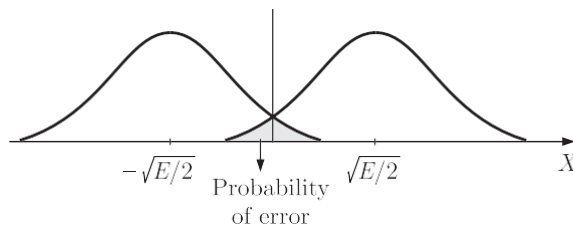
When : (i)  $s_{11}$  is transmitted and  $s_{21}$  received

(ii)  $s_{21}$  is transmitted and  $s_{11}$  received

So, probability for the 1<sup>st</sup> case will be as :

$$\begin{aligned} P_{b_s}^{s_{21} \text{ received}} &= P(X < 0) \text{ (as shown in diagram)} \\ &= P\left[-\sqrt{E/2} + N < 0\right] \\ &= P\left[N < -\sqrt{E/2}\right] \end{aligned}$$

Taking the Gaussian distribution as shown below :



Mean of the Gaussian distribution =  $E/2$

$$\text{Variance} = \frac{N_0}{2}$$

Putting it in the probability function :

$$P_{bN} < -\frac{E}{2} = \int_{-\infty}^0 \frac{1}{\sqrt{2\pi \frac{N_0}{2}}} e^{-\frac{(x + E/2)^2}{2 \cdot \frac{N_0}{2}}} dx$$

$$= \int_{-\infty}^0 \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(x + E/2)^2}{N_0}} dx$$

Taking,  $\frac{x + E/2}{\sqrt{N_0/2}} = t$

$$dx = \frac{N_0}{2} dt$$

$$\text{So, } P_{bN} < -\frac{E}{2} = \int_{-\infty}^0 \frac{1}{\sqrt{\pi N_0}} e^{-\frac{t^2}{2}} dt = Q_c \left( \frac{E}{N_0} \right)$$

where  $Q$  is error function.

Since symbols are equiprobable in the 2<sup>nd</sup> case

So,

$$P \frac{s_{11} \text{ received}}{s_{21} \text{ transmitted}} = Q_c \sqrt{\frac{E}{N_0}}$$

So the average probability of error

$$= \frac{1}{2} \left[ P_{b \frac{s_{21} \text{ received}}{s_{11} \text{ transmitted}}} + P_{b \frac{s_{11} \text{ received}}{s_{21} \text{ transmitted}}} \right]$$

$$= \frac{1}{2} = Q_c \sqrt{\frac{E}{N_0}} + Q_c \sqrt{\frac{E}{N_0}} = Q_c \sqrt{\frac{E}{N_0}}$$

**Sol. 12**

Option (A) is correct.

**Sol. 13**

Option (B) is correct.

General equation of FM and PM waves are given by

$$f_{FM}(t) = A_c \cos \left[ \omega_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right]$$

$$f_{PM}(t) = A_c \cos [\omega_c t + k_p m(t)]$$

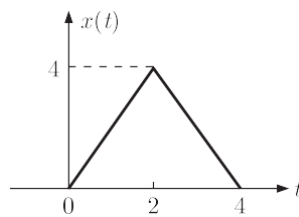
For same maximum phase deviation.

$$k_p [m(t)]_{\max} = 2\pi k_f \int_0^t m(\tau) d\tau_{\max}$$

$$k_p \cdot 2 = 2\pi k_f [x(t)]_{\max}$$

where,

$$x(t) = \int_0^t m(\tau) d\tau$$



$$[x(t)]_{\max} = 4$$

So,

$$k_p \neq 2 = 2pk_f \neq 4$$

$$\frac{k_p}{k_f} = 4p$$

**Sol. 14**

Option (A) is correct.

$$G_C(s) = \frac{s+a}{s+b} = \frac{jw+a}{jw+b}$$

Phase lead angle

$$\phi = \tan^{-1} \frac{w}{a} - \tan^{-1} \frac{w}{b}$$

$$\phi = \tan^{-1} \frac{J \frac{w}{a} - \frac{w}{b}}{1 + \frac{w^2}{ab}} = \tan^{-1} \frac{w(b-a)}{ab + w^2}$$

For phase-lead compensation  $\phi > 0$

$$b - a > 0$$

$$b > a$$

**Note:** For phase lead compensator zero is nearer to the origin as compared to pole, so option (C) can not be true.

**Sol. 15**

Option (A) is correct.

$$\phi = \tan^{-1} \frac{w}{a} - \tan^{-1} \frac{w}{b}$$

$$\frac{d\phi}{dw} = \frac{1/a}{1 + \frac{w^2}{a^2}} - \frac{1/b}{1 + \frac{w^2}{b^2}} = 0$$

$$\frac{1}{a} + \frac{w^2}{ab^2} = \frac{1}{b} + \frac{1}{b} \frac{w^2}{a^2}$$

$$\frac{1}{a} - \frac{1}{b} = \frac{w^2}{ab} \left( \frac{1}{a} - \frac{1}{b} \right)$$

$$w = \sqrt{ab} = \sqrt{1 \times 2} = \sqrt{2} \text{ rad/sec}$$

**Sol. 16**

Option (D) is correct.

Quantized 4 level require 2 bit representation i.e. for one sample 2 bit are required. Since 2 sample per second are transmitted we require 4 bit to be transmitted per second.

**Sol. 17**

Option (B) is correct.

In FM the amplitude is constant and power is efficiently transmitted. No variation in power.

There is most bandwidth efficient transmission in SSB-SC, because we transmit only one side band.

Simple Diode in Non linear region (Square law) is used in conventional AM that is simplest receiver structure.

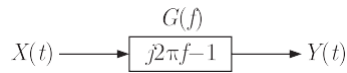
In VSB dc. component exists.

Sol. 18

Option (A) is correct.

We have 
$$S_x(f) = F\{R_x(t)\} = F\{\exp(-pt^2)\}$$
$$= e^{-pf^2}$$

The given circuit can be simplified as



Power spectral density of output is

$$\begin{aligned} S_y(f) &= |G(f)|^2 S_x(f) \\ &= |j2\pi f - 1|^2 e^{-pf^2} \\ &= (4\pi^2 f^2 + 1) e^{-pf^2} \end{aligned}$$

or

$$S_y(f) = (4\pi^2 f^2 + 1) e^{-pf^2}$$

Sol. 19

Option (B) is correct.

Highest frequency component in  $m(t)$  is  $f_m = 4000p/2p = 2000$  Hz

Carrier frequency  $f_c = 1$  MHz

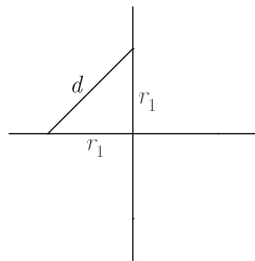
For Envelope detector condition

$$\begin{aligned} 1/f_c &\ll RC \ll 1/f_m \\ 1 \mu s &\ll RC \ll 0.5 \text{ ms} \end{aligned}$$

Sol. 20

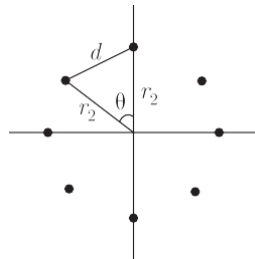
Option (D) is correct.

Four phase signal constellation is shown below



Now

$$\begin{aligned} d^2 &= r_1^2 + r_1^2 \\ d^2 &= 2r_1^2 \\ r_1 &= d/\sqrt{2} = 0.707d \end{aligned}$$



$$q = \frac{2p}{M} = \frac{2p}{8} = \frac{p}{4}$$

Applying Coorine law we have

$$d^2 = r_1^2 + r_2^2 - 2r_1 r_2 \cos \frac{\pi}{4}$$

$$= 2r_1^2 - 2r_1^2 \frac{1}{\sqrt{2}} = (2 - \frac{1}{\sqrt{2}}) r_1^2$$

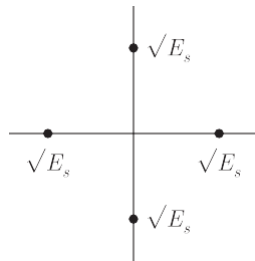
or

$$r_2 = \frac{d}{\sqrt{2} - \frac{1}{\sqrt{2}}} = 1.3065d$$

**Sol. 21**

Option (D) is correct.

Here  $P_e$  for 4 PSK and 8 PSK is same because  $P_e$  depends on  $d$ . Since  $P_e$  is same,  $d$  is same for 4 PSK and 8 PSK.



Additional Power SNR

$$= (SNR)_2 - (SNR)_1$$

$$= 10 \log_b \frac{E_{s2}}{N_0} - 10 \log_b \frac{E_{s1}}{N_0}$$

$$= 10 \log_b \frac{E_{s2}}{E_{s1}}$$

$$= 10 \log_a \frac{r_2^2}{r_1^2} \text{ k} \& 20 \log_a \frac{r_2}{r_1} \text{ k} = 20 \log \frac{1.3065d}{0.707d}$$

Additional SNR = 5.33 dB

**Sol. 22**

Option (C) is correct.

Conventional AM signal is given by

$$x(t) = A_c [1 + m \cos(2\pi f_m t)] \cos(2\pi f_c t)$$

Where  $m < 1$ , for no over modulation.

In option (C)

$$x(t) = A_c [1 + \frac{1}{4} m \cos(2\pi f_m t)] \cos(2\pi f_c t)$$

Thus  $m = \frac{1}{4} < 1$  and this is a conventional AM-signal without over-modulation

**Sol. 23**

Option (B) is correct.

Power

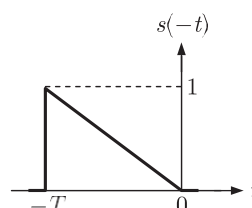
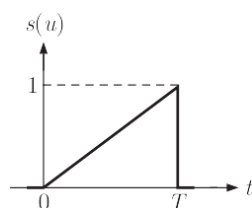
$$P = \frac{(6)^2}{2} = 18 \text{ W}$$

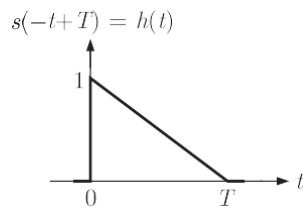
**Sol. 24**

Option (C) is correct.

Impulse response of the matched filter is given by

$$h(t) = S(T - t)$$





Sol. 25

Option (B) is correct.

Let response of LPF filters

$$H(f) = \begin{cases} 1, & |f| < 1 \text{ MHz} \\ 0, & \text{elsewhere} \end{cases}$$

Noise variance (power) is given as  $\bar{P} = \int_0^{f_o} H^2(f) N df = \frac{2}{a^2}$  (given)

$$\int_0^{1 \times 10^6} 2 \times 10^{-20} df = \frac{2}{a^2}$$

$$2 \times 10^{-20} \times 10^6 = \frac{2}{a^2}$$

$$a^2 = 10^{14}$$

or

$$a = 10^7$$

Sol. 26

Option (D) is correct.

Probability of error is given by

$$P_e = \frac{1}{2} [P(0/1) + P(1/0)]$$

$$P(0/1) = \int_{-3}^{a/2} 0.5e^{-a|n-a|} dn = 0.5e^{-10}$$

where  $a = 2 \times 10^{-6} \text{ V}$  and  $a = 10^7 \text{ V}^{-1}$ 

$$P(1/0) = \int_{a/2}^3 0.5e^{-a|n|} dn = 0.5e^{-10}$$

$$P_e = 0.5e^{-10}$$

Sol. 27

Option (C) is correct.

$$S(t) = \sin c(500t) \sin c(700t)$$

$S(f)$  is convolution of two signals whose spectrum covers  $f_1 = 250 \text{ Hz}$  and  $f_2 = 350 \text{ Hz}$ . So convolution extends

$$f = 25 + 350 = 600 \text{ Hz}$$

Nyquist sampling rate

$$N = 2f = 2 \times 600 = 1200 \text{ Hz}$$

Sol. 28

Option (D) is correct.

For the given system, output is written as

$$y(t) = \frac{d}{dt} [x(t) + x(t - 0.5)]$$

$$y(t) = \frac{dx(t)}{dt} + \frac{dx(t - 0.5)}{dt}$$

Taking Laplace on both sides of above equation

$$Y(s) = sX(s) + se^{-0.5s}X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = s(1 + e^{-0.5s})$$



$$H(f) = jf(1 + e^{-0.5 \times 2\pi f}) = jf(1 + e^{-\pi f})$$

Power spectral density of output

$$S_Y(f) = |H(f)|^2 S_X(f) = f^2 (1 + e^{-\pi f})^2 S_X(f)$$

$$\text{For } S_Y(f) = 0, \quad 1 + e^{-\pi f} = 0$$

$$f = (2n + 1)f_0$$

or

$$f_0 = 1 \text{ KHz}$$

**Sol. 29**

Option (C) is correct.

$$\cos(2\pi f_m t) \cos(2\pi f_c t) \text{ \$ DSB suppressed carrier}$$

$$\cos(2\pi f_c t) \text{ \$ Carrier Only}$$

$$\cos[2\pi (f_c + f_m) t] \text{ \$ USB Only}$$

$$[1 + \cos(2\pi f_m t) \cos(2\pi f_c t)] \text{ \$ USB with carrier}$$

**Sol. 30**

Option (C) is correct.

We have

$$\begin{aligned} p(X=0) &= p(Y=0) = \frac{1}{2} \\ p(X=1) &= p(Y=1) = \frac{1}{4} \\ p(X=2) &= p(Y=2) = \frac{1}{4} \end{aligned}$$

Let

$$X + Y = 2 \text{ \$ } A$$

and

$$X - Y = 0 \text{ \$ } B$$

Now

$$P(X + Y = 2 | X - Y = 0) = \frac{P(A+B)}{P(B)}$$

Event  $P(A+B)$  happen when  $X+Y=2$  and  $X-Y=0$ . It is only the case when  $X=1$  and  $Y=1$ .

$$\text{Thus } P(A+B) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

Now event  $P(B)$  happen when

$$X - Y = 0 \text{ It occurs when } X = Y, \text{ i.e.}$$

$$X = 0 \text{ and } Y = 0 \text{ or}$$

$$X = 1 \text{ and } Y = 1 \text{ or}$$

$$X = 2 \text{ and } Y = 2$$

$$\text{Thus } P(B) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{4} = \frac{6}{16}$$

$$\text{Now } \frac{P(A+B)}{P(B)} = \frac{1/16}{6/16} = \frac{1}{6}$$

**Sol. 31**

Option (B) is correct.

The mean is

$$\bar{X} = \sum x_i p_i(x)$$

$$= 1 \times 0.1 + 2 \times 0.2 + 3 \times 0.4 + 4 \times 0.2 + 5 \times 0.1$$

$$= 0.1 + 0.4 + 1.2 + 0.8 + 0.5 = 3.0$$

$$\bar{X}^2 = \sum x_i^2 p_i(x)$$

$$= 1 \times 0.1 + 4 \times 0.2 + 9 \times 0.4 + 16 \times 0.2 + 25 \times 0.1$$

$$= 0.1 + 0.8 + 3.6 + 3.2 + 2.5 = 10.2$$

Variance  $s_x^2 = \overline{X^2} - \bar{X}^2$

$$= 10.2 - (3)^2 = 1.2$$

Sol. 32

Option (C) is correct.

$$m(t) = \frac{1}{2} \cos w_1 t - \frac{1}{2} \sin w_2 t$$

$$s_{AM}(t) = [1 + m(t)] \cos w_c t$$

Modulation index  $= \frac{|m(t)|_{\max}}{V_c}$

$$m = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} = \frac{1}{\sqrt{2}}$$

$$h = \frac{m^2}{m^2 + 2} \times 100\% = \frac{\frac{1}{2}}{\frac{1}{2} + 2} \times 100\% = 20\%$$

Sol. 33

Option (B) is correct.

We have  $C_1 = B \log_2 \left(1 + \frac{S}{N}\right)$

$$= B \log_2 \frac{S}{N} \quad \text{As } \frac{S}{N} \gg 1$$

If we double the  $\frac{S}{N}$  ratio then

$$C_2 = B \log_2 \left(\frac{2S}{N}\right)$$

$$= B \log_2 2 + B \log_2 \frac{S}{N}$$

$$= B + C_1$$

Sol. 34

Option (C) is correct.

We have  $SNR = 1.76 + 6n$

or  $43.5 = 1.76 + 6n$

$$6n = 43.5 - 1.76$$

$$6n = 41.74 \Rightarrow n = 7$$

No. of quantization levels is

$$2^7 = 128$$

Step size required is

$$= \frac{V_H - V_L}{128} = \frac{5 - (-5)}{128} = \frac{10}{128}$$

$$= 0.078125$$

$$= 0.0667$$

Sol. 35

Option (B) is correct.

For positive values step size

$$s_+ = 0.05 \text{ V}$$

For negative value step size

$$s_- = 0.1 \text{ V}$$

No. of quantization in +ive is

$$= \frac{5}{s_+} = \frac{5}{0.05} = 100$$

Thus  $2^{n^+} = 100$   $\therefore n^+ = 7$

No. of quantization in  $-ve$

$$Q_1 = \frac{5}{s} = \frac{5}{0.1} = 50$$

Thus  $2^{n^-} = 50$   $\therefore n^- = 6$

$$\left(\frac{S}{N}\right)_{j_+} = 1.76 + 6n^+ = 1.76 + 42 = 43.76 \text{ dB}$$

$$\left(\frac{S}{N}\right)_{j_-} = 1.76 + 6n^- = 1.76 + 36 = 37.76 \text{ dB}$$

Best  $\left(\frac{S}{N}\right)_{j_0} = 43.76 \text{ dB}$

**Sol. 36**

Option (A) is correct.

$$\begin{aligned} \text{We have } x_{AM}(t) &= A_c \cos w_c + 2 \cos w_m t \cos w_c t \\ &= A_c \cos 1 + \frac{2}{A_c} \cos w_m t \cos w_c t \end{aligned}$$

For demodulation by envelope demodulator modulation index must be less than or equal to 1.

$$\begin{aligned} \text{Thus } \frac{2}{A_c} &\leq 1 \\ A_c &\geq 2 \end{aligned}$$

Hence minimum value of  $A_c = 2$

**Sol. 37**

Option (A) is correct.

CDF is the integration of PDF. Plot in option (A) is the integration of plot given in question.

**Sol. 38**

Option (A) is correct.

The entropy is

$$H = \sum_{i=1}^m p_i \log_2 \frac{1}{p_i} \text{ bits}$$

$$\text{Since } p_1 = p_2 = \dots = p_n = \frac{1}{n}$$

$$H = \sum_{i=1}^n \frac{1}{n} \log n = \log n$$

**Sol. 39**

Option (C) is correct.

$$\text{PSD of noise is } \frac{N_0}{2} = K \quad \dots(1)$$

The 3-dB cut off frequency is

$$f_c = \frac{1}{2\pi RC} \quad \dots(2)$$

Output noise power is

$$= \frac{N_0}{4RC} = \frac{N_0}{2} \frac{1}{2\pi RC} = Kpf$$

**Sol. 40**

Option (D) is correct.

At receiving end if we get two zero or three zero then its error.

Let  $p$  be the probability of 1 bit error, the probability that transmitted bit error is

$$= \text{Three zero} + \text{two zero and single one}$$

$$= 3C_3p^3 + 3C_2p^2(1-p)$$

$$= p^3 + p^2(1-p)$$

**Sol. 41** Option (D) is correct.  
Bandwidth of TDM is

$$= \frac{1}{2} (\text{sum of Nyquist Rate})$$

$$= \frac{1}{2} [2W + 2W + 4W + 6W] = 7W$$

**Sol. 42** Option (B) is correct.

We have  $q_i = 2p10^5 t + 5 \sin(2p1500t) + 7.5 \sin(2p1000t)$

$$w_i = \frac{dq_i}{dt} = 2p10^5 + 10p1500 \cos(2p1500t) + 15p1000 \cos(2p1000t)$$

Maximum frequency deviation is

$$3w_{\max} = 2p(5 \# 1500 + 7.5 \# 1000)$$

$$\text{Modulation index is } = \frac{3f_{\max}}{f_m} = \frac{15000}{1500} = 10$$

**Sol. 43** Option (C) is correct.

**Sol. 44** Option (B) is correct.

$$f_m = 4 \text{ KHz}$$

$$f_s = 2f_m = 8 \text{ kHz}$$

Bit Rate  $R_b = \eta f_s = 8 \# 8 = 64 \text{ kbps}$

The minimum transmission bandwidth is

$$BW = \frac{R_b}{2} = 32 \text{ kHz}$$

**Sol. 45** Option (C) is correct.

$$\frac{S_0}{N_0} = 1.76 + 6n \text{ dB}$$

$$= 1.76 + 6 \# 8 = 49.76 \text{ dB}$$

We have  $n = 8$

**Sol. 46** Option (B) is correct.

As Noise  $\propto \frac{1}{L^2}$

To reduce quantization noise by factor 4, quantization level must be two times i.e.  $2L$ .

Now  $L = 2^n = 2^8 = 256$

Thus  $2L = 512$

**Sol. 47** Option (C) is correct.

Autocorrelation is even function.

**Sol. 48** Option (B) is correct.

Power spectral density is non negative. Thus it is always zero or greater than zero.

**Sol. 49** Option (A) is correct.

The variance of a random variable  $x$  is given by

$$E[X^2] - E^2[X]$$

Sol. 50

Option (A) is correct.

A Hilbert transformer is a non-linear system.

Sol. 51

Option (D) is correct.

Slope overload distortion can be reduced by increasing the step size

$$\frac{3}{T_s} \text{ slope of } x(t)$$

Sol. 52

Option (C) is correct.

We have  $p(t) = \frac{\sin(4pWt)}{4pWt(1-16W^2t^2)}$

at  $t = \frac{1}{4W}$  it is  $\frac{0}{0}$  form. Thus applying L'Hospital rule

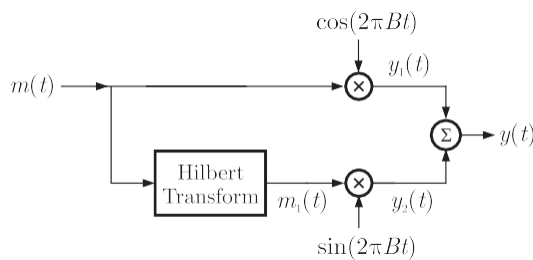
$$p\left(\frac{1}{4W}\right) = \frac{4pW \cos(4pWt)}{4pW[1-48W^2t^2]}$$

$$= \frac{\cos(4pWt)}{1-48W^2t^2} = \frac{\cos p}{1-3} = 0.5$$

Sol. 53

Option (B) is correct.

The block diagram is as shown below



Here

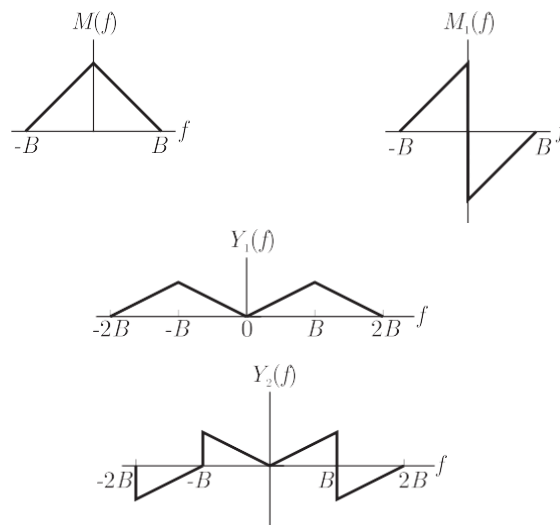
$$M_1(f) = \hat{M}(f)$$

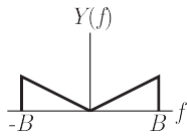
$$Y_1(f) = M(f) \frac{e^{j2pB} + e^{-j2pB}}{2}$$

$$Y_2(f) = M_1(f) \frac{e^{j2pB} - e^{-j2pB}}{2}$$

$$Y(f) = Y_1(f) + Y_2(f)$$

All waveform is shown below





Sol. 54

Option (C) is correct.

By Binomial distribution the probability of error is

$$p_e = {}^n C_r p^r (1-p)^{n-r}$$

Probability of at most one error

$$= \text{Probability of no error} + \text{Probability of one error}$$

$$= {}^n C_0 p^0 (1-p)^{n-0} + {}^n C_1 p^1 (1-p)^{n-1}$$

$$= n(1-p)^n + np(1-p)^{n-1}$$

Sol. 55

Option (B) is correct.

Bandwidth allocated for 1 Channel = 5 MHz

Average bandwidth for 1 Channel  $\frac{5}{5} = 1$  MHzTotal Number of Simultaneously Channel =  $\frac{1\text{M} \times 8}{200k} = 40$  Channel

Sol. 56

Option (A) is correct.

Chip Rate  $R_C = 1.2288 \times 10^6$  chips/secData Rate  $R_b = \frac{R_C}{G}$ Since the processing gain  $G$  must be at least 100, thus for  $G_{\min}$  we get

$$R_{b \max} = \frac{R_C}{G_{\min}} = \frac{1.2288 \times 10^6}{100} = 12.288 \times 10^3 \text{ bps}$$

Sol. 57

Option (B) is correct.

Energy of constellation 1 is

$$\begin{aligned} E_{g1} &= (0)^2 + (-\sqrt{2}a)^2 + (-\sqrt{2}a)^2 + (\sqrt{2}a)^2 + (-2\sqrt{2}a)^2 \\ &= 2a^2 + 2a^2 + 2a^2 + 8a^2 = 16a^2 \end{aligned}$$

Energy of constellation 2 is

$$\begin{aligned} E_{g2} &= a^2 + a^2 + a^2 + a^2 = 4a^2 \\ \text{Ratio} &= \frac{E_{g1}}{E_{g2}} = \frac{16a^2}{4a^2} = 4 \end{aligned}$$

Sol. 58

Option (A) is correct.

Noise Power is same for both which is  $\frac{N_0}{2}$ .

Thus probability of error will be lower for the constellation 1 as it has higher signal energy.

Sol. 59

Option (A) is correct.

Area under the pdf curve must be unity

$$\text{Thus } 2a + 4a + 4b = 1$$

$$2a + 8b = 1 \quad \dots(1)$$

For maximum entropy three region must be equiprobable thus

$$2a = 4b = 4b \quad \dots(2)$$

From (1) and (2) we get

$$b = \frac{1}{12} \text{ and } a = \frac{1}{6}$$

**Sol. 60** Option (\*) is correct.

**Sol. 61** Option (B) is correct.

A LPF will not produce phase distortion if phase varies linearly with frequency.

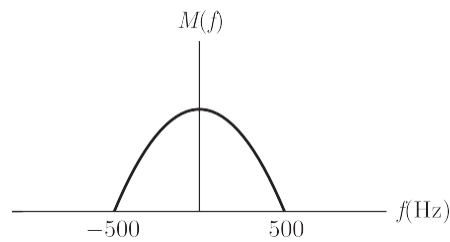
$$\angle f(w) \propto w$$

i.e.

$$\angle f(w) = kw$$

**Sol. 62** Option (B) is correct.

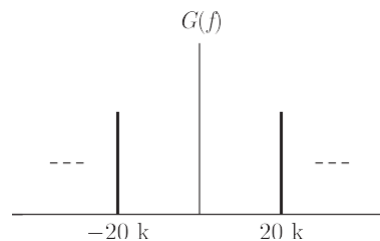
Let  $m(t)$  is a low pass signal, whose frequency spectra is shown below



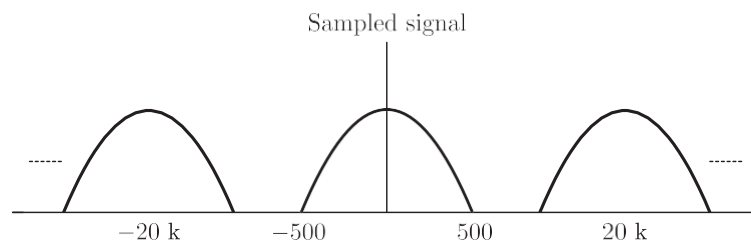
Fourier transform of  $g(t)$

$$G(f) = \frac{1}{0.5 \times 10^{-4}} \int_{-3}^3 \delta(f - 20 \times 10^3 k) dk$$

Spectrum of  $G(f)$  is shown below



Now when  $m(t)$  is sampled with above signal the spectrum of sampled signal will look like.



When sampled signal is passed through a LP filter of BW 1 kHz, only  $m(t)$  will remain.

**Sol. 63** Option (C) is correct.

The highest frequency signal in  $x(t)$  is  $1000 \times 3 = 3 \text{ kHz}$  if expression is expanded. Thus minimum frequency requirement is

$$f = 2 \times 3 \times 10^3 = 6 \times 10^3 \text{ Hz}$$

**Sol. 64** Option (B) is correct.

We have

$$x(t) = 125t[u(t) - u(t-1)] + (250 - 125t)[u(t-1) - u(t-2)]$$

The slope of expression  $x(t)$  is 125 and sampling frequency  $f_s$  is 32#1000 samples/sec.

Let  $\Delta$  be the step size, then to avoid slope overload

$$\begin{aligned} \frac{\Delta}{T_s} &\leq \text{slope } x(t) \\ 3f_s \Delta &\leq \text{slope } x(t) \\ 3 \times 32000 \Delta &\leq 125 \\ \Delta &\leq \frac{125}{32000} \\ \Delta &= 2^{-8} \end{aligned}$$

**Sol. 65** Option (A) is correct.

The sampling frequency is

$$f_s = \frac{1}{0.03\text{m}} = 33 \text{ kHz}$$

Since  $f_s \geq 2f_m$ , the signal can be recovered and are correlated.

**Sol. 66** Option (B) is correct.

We have  $p_1 = 0.25$ ,  $p_2 = 0.25$  and  $p_3 = 0.5$

$$\begin{aligned} H &= - \sum_{i=1}^3 p_i \log_2 \frac{1}{p_i} \text{ bits/symbol} \\ &= p_1 \log_2 \frac{1}{p_1} + p_2 \log_2 \frac{1}{p_2} + p_3 \log_2 \frac{1}{p_3} \\ &= 0.25 \log_2 \frac{1}{0.25} + 0.25 \log_2 \frac{1}{0.25} + 0.5 \log_2 \frac{1}{0.5} \\ &= 0.25 \log_2 4 + 0.25 \log_2 4 + 0.5 \log_2 2 \\ &= 0.5 + 0.5 + \frac{1}{2} = \frac{3}{2} \text{ bits/symbol} \end{aligned}$$

$$R_b = 3000 \text{ symbol/sec}$$

$$\begin{aligned} \text{Average bit rate} &= R_b H \\ &= \frac{3}{2} \times 3000 = 4500 \text{ bits/sec} \end{aligned}$$

**Sol. 67** Option (A) is correct.

The diagonal clipping in AM using envelop detector can be avoided if

$$\frac{1}{w_c} \ll RC \ll \frac{1}{W}$$

$$\text{But from } \frac{1}{RC} \leq \frac{Wm \sin Wt}{1 + m \cos Wt}$$

We can say that  $RC$  depends on  $W$ , thus

$$RC < \frac{1}{W}$$

**Sol. 68** Option (B) is correct.

**Sol. 69** Option (B) is correct.

When  $\Delta/2$  is added to  $y(t)$  then signal will move to next quantization level.

Otherwise if they have step size less than  $\frac{\Delta}{2}$  then they will be on the same quantization level.

**Sol. 70** Option (C) is correct.

After the SSB modulation the frequency of signal will be  $f_c - f_m$  i.e.



$$1000 - 10 \text{ kHz} \sim 1000 \text{ kHz}$$

The bandwidth of FM is

$$BW = 2(b + 1) \beta f$$

For  $NBFM \beta \ll 1$ , thus

$$BW_{NBFM} \sim 2 \beta f = 2(10^9 - 10^6) \sim 2 \times 10^9$$

Sol. 71

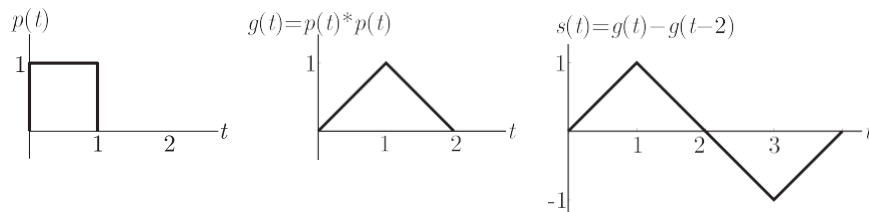
Option (A) is correct.

We have  $p(t) = u(t) - u(t - 1)$

$$g(t) = p(t) * p(t)$$

$$s(t) = g(t) - g(t - 2) \quad g(t) = g(t) - g(t - 2)$$

All signal are shown in figure below :



The impulse response of matched filter is

$$h(t) = s(T - t) = s(1 - t)$$

Here  $T$  is the time where output SNR is maximum.

Sol. 72

Option (A) is correct.

We have  $x_{AM}(t) = 10[P(t) + 0.5g(t)] \cos w_c t$

where  $p(t) = u(t) - u(t - 1)$

and  $g(t) = r(t) - 2r(t - 1) + r(t - 2)$

For desired interval  $0 \leq t \leq 1$ ,  $p(t) = 1$  and  $g(t) = t$ , Thus we have,

$$x_{AM}(t) = 100(1 - 0.5t) \cos w_c t$$

Hence modulation index is 0.5

Sol. 73

Option (A) is correct.

We know that  $S_{YY}(w) = |H(w)|^2 \cdot S_{XX}(w)$

Now  $S_{YY}(w) = \frac{16}{16 + w^2}$  and  $S_{XX}(w) = 1$  white noise

Thus  $\frac{16}{16 + w^2} = |H(w)|^2$

or  $|H(w)| = \frac{4}{\sqrt{16 + w^2}}$

or  $H(s) = \frac{4}{4 + s}$

which is a first order low pass RL filter.

Sol. 74

Option (A) is correct.

We have  $\frac{R}{R + sL} = \frac{4}{4 + s}$

or  $\frac{\frac{R}{L}}{\frac{R}{L} + s} = \frac{4}{4 + s}$

Comparing we get  $L = 1 \text{ H}$  and  $R = 4 \text{ W}$

Sol. 75

Option (C) is correct.

We have  $x_{AM}(t) = 10(1 + 0.5 \sin 2\pi f_m t) \cos 2\pi f_c t$

The modulation index is 0.5

$$\text{Carrier power } P_c = \frac{(10)^2}{2} = 50$$

$$\text{Side band power } P_s = \frac{(10)^2}{2} = 50$$

$$\text{Side band power } P_s = \frac{m^2 P_c}{2} = \frac{(0.5)^2(50)}{2} = 6.25$$

**Sol. 76**

Option (B) is correct.

Mean noise power = Area under the PSD curve

$$= 4 \cdot \frac{1}{2} \# B \# \frac{N_o}{2} E = BN_o$$

The ratio of average sideband power to mean noise power is

$$\frac{\text{Side Band Power}}{\text{Noise Power}} = \frac{6.25}{N_o B} = \frac{25}{4N_o B}$$

**Sol. 77**

Option (D) is correct.

$\{1 + km(t)\}A \sin(w_c t)$  \$ Amplitude modulation

$dm(t)A \sin(w_c t)$  \$ DSB-SC modulation

$A \sin\{\cos t + km(t)\}$  \$ Phase Modulation

$A \sin[w_c t + k \int_{-3}^t m(t) dt]$  \$ Frequency Modulation

**Sol. 78**

Option (C) is correct.

VSB \$  $f_m + f_c$

DSB - SC \$  $2f_m$

SSB \$  $f_m$

AM \$  $2f_m$

Thus SSB has minimum bandwidth and it require minimum power.

**Sol. 79**

Option (A) is correct.

Let  $x(t)$  be the input signal where

$$x(t) = \cos(\cos t + b_1 \cos w_m t)$$

$$y(t) = x^2(t) = \frac{1}{2} + \frac{\cos(2w_c t + 2b_1 \cos w_m t)}{2}$$

$$\text{Here } b = 2b_1 \text{ and } b_1 = \frac{3f}{f_m} = \frac{90}{5} = 18$$

$$BW = 2(b + 1)f_m = 2(2 \# 18 + 1) \# 5 = 370 \text{ kHz}$$

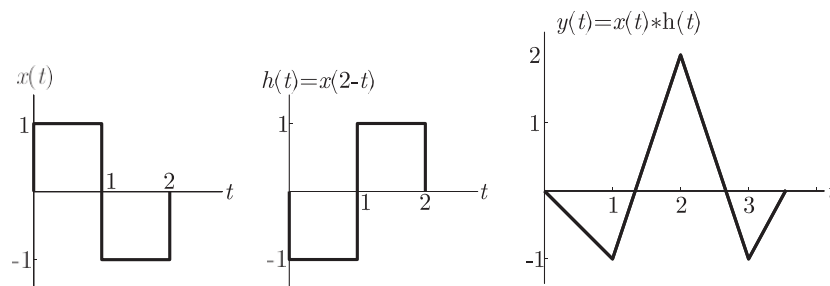
**Sol. 80**

Option (C) is correct.

The transfer function of matched filter is

$$h(t) = x(t - t) = x(2 - t)$$

The output of matched filter is the convolution of  $x(t)$  and  $h(t)$  as shown below



Sol. 81

Option (B) is correct.

We have

$$H(f) = 2e^{-j\omega t_d}$$

$$|H(f)| = 2$$

$$G_o(f) = |H(f)|^2 G_i(f)$$

$$= 4N_o \text{ W/Hz}$$

The noise power is

$$= 4N_o B$$

Sol. 82

Option (C) is correct.

As the area under pdf curve must be unity

$$\frac{1}{2} (4 - k) = 1 \Rightarrow k = \frac{1}{2}$$

Now mean square value is

$$s_v^2 = \int_{-3}^{+3} v^2 p(v) dv$$

$$= \int_0^4 v^2 \frac{v}{8} dv$$

$$= \int_0^4 \frac{v^3}{8} dv = 8$$

$$\text{as } p(v) = \frac{1}{8} v$$

Sol. 83

Option (D) is correct.

The phase deviation is

$$b = \frac{3f}{f_m} = \frac{10}{1} = 10$$

If phase deviation remain same and modulating frequency is changed

$$BW = 2(b+1)f'_m = 2(10+1)2 = 44 \text{ kHz}$$

Sol. 84

Option (B) is correct.

As the area under pdf curve must be unity and all three region are equiprobable.

Thus are under each region must be  $\frac{1}{3}$ .

$$2a \times \frac{1}{4} = \frac{1}{3} \Rightarrow a = \frac{2}{3}$$

Sol. 85

Option (A) is correct.

$$N_q = \int_{-a}^{+a} x^2 p(x) dx = 2 \int_0^a x^2 \frac{1}{4} dx = \frac{1}{2} \left[ \frac{x^3}{3} \right]_0^a = \frac{a^3}{6}$$

Substituting  $a = \frac{2}{3}$  we have

$$N_q = \frac{4}{81}$$

Sol. 86

Option (C) is correct.

When word length is 6

$$\frac{S}{N} \Big|_{N=6} = 2^{2 \times 6} = 2^{12}$$

When word length is 8

$$\frac{S}{N} \Big|_{N=8} = 2^{2 \times 8} = 2^{16}$$

Now  $\frac{S}{N} \Big|_{N=6} = \frac{2^{16}}{2^{12}} = 2^4 = 16$

Thus it improves by a factor of 16.

**Sol. 87**

Option (B) is correct.

Carrier frequency  $f_c = 1 \times 10^6$  Hz

Modulating frequency

$$f_m = 2 \times 10^3 \text{ Hz}$$

For an envelope detector

$$2\pi f_c > \frac{1}{RC} > 2\pi f_m$$

$$\frac{1}{2\pi f_c} < RC < \frac{1}{2\pi f_m}$$

$$\frac{1}{2\pi f_c} < RC < \frac{1}{2\pi f_m}$$

$$\frac{1}{2\pi \times 10^6} < RC < \frac{1}{2\pi \times 10^3}$$

$$1.59 \times 10^{-7} < RC < 7.96 \times 10^{-5}$$

so, 20 msec sec best lies in this interval.

**Sol. 88**

Option (B) is correct.

$$S_{AM}(t) = A_c [1 + 0.1 \cos w_m t] \cos w_c t$$

$$S_{NBFM}(t) = A_c \cos [w_c t + 0.1 \sin w_m t]$$

$$s(t) = S_{AM}(t) + S_{NBFM}(t)$$

$$= A_c [1 + 0.1 \cos w_m t] \cos w_c t + A_c \cos (w_c t + 0.1 \sin w_m t)$$

$$= A_c \cos w_c t + A_c 0.1 \cos w_m t \cos w_c t$$

$$+ A_c \cos w_c t \cos (0.1 \sin w_m t) - A_c \sin w_c t \cdot \sin (0.1 \sin w_m t)$$

As  $0.1 \sin w_m t \rightarrow +0.1$  to  $-0.1$

so,  $\cos(0.1 \sin w_m t) \approx 1$

As when  $q$  is small  $\cos q \approx 1$  and  $\sin q \approx q$ , thus

$$\sin (0.1 \sin w_m t) = 0.1 \sin w_m t \cos w_c t + A_c \cos w_c t$$

$$- A_c 0.1 \sin w_m t \sin w_c t$$

$$= 2A_c \cos w_c t + 0.1A_c \cos (w_c + w_m)t$$

$$\underbrace{1}_{\text{cosec}} \underbrace{442443}_{\text{cosec}} \quad \underbrace{144444}_{\text{USB}} \underbrace{244444}_{\text{USB}} \underbrace{43}_{\text{cosec}}$$

Thus it is SSB with carrier.

**Sol. 89**

Option (A) is correct.

Consecutive pulses are of same polarity when modulator is in slope overload.

Consecutive pulses are of opposite polarity when the input is constant.

**Sol. 90**

Option (D) is correct.

$$P(x_1 \leq X < x_2) = P(X = x_2) - P(X = x_1)$$

or  $P(X = 1) = P(X = 1^+) - P(X = 1^-)$

$$= 0.55 - 0.25 = 0.30$$

Sol. 91

Option (A) is correct.

The SNR at transmitter is

$$SNR_{tr} = \frac{P_{tr}}{NB}$$

$$\frac{10^{-3}}{10^{-20} \# 100 \# 10^6} = 10^9$$

$$\text{In dB} \quad SNR_{tr} = 10 \log 10^9 = 90 \text{ dB}$$

$$\text{Cable Loss} = 40 \text{ dB}$$

At receiver after cable loss we have

$$SNR_{Rc} = 90 - 40 = 50 \text{ dB}$$

Sol. 92

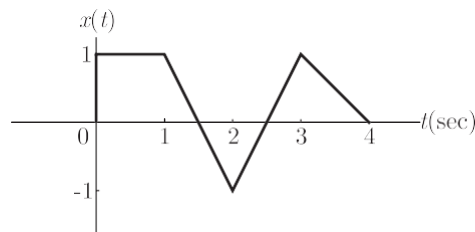
Option (B) is correct.

The impulse response of matched filter is

$$h(t) = x(T - t)$$

Since here  $T = 4$ , thus

$$h(t) = x(4 - t)$$

The graph of  $h(t)$  is as shown below.From graph it may be easily seen that slope between  $3 < t < 4$  is  $-1$ .

Sol. 93

Option (C) is correct.

The required bandwidth of  $M$  array PSK is

$$BW = \frac{2R_b}{n}$$

where  $2^n = M$  and  $R_b$  is bit rate

$$\text{For BPSK,} \quad M = 2 = 2^n \quad \& \quad n = 1$$

$$\text{Thus} \quad B_1 = \frac{2R_b}{1} = 2 \# 10 = 20 \text{ kHz}$$

$$\text{For QPSK,} \quad M = 4 = 2^n \quad \& \quad n = 2$$

$$\text{Thus} \quad B_2 = \frac{2R_b}{2} = 10 \text{ kHz}$$

Sol. 94

Option (C) is correct.

We have

$$f_c = 100 \text{ MHz} = 100 \# 10^6 \text{ and } f_m = 1 \text{ MHz} \\ = 1 \# 10^6$$

The output of balanced modulator is

$$V_{BM}(t) = [\cos w_c t][\cos w_m t] \\ = \frac{1}{2} [\cos(w_c + w_m)t + \cos(w_c - w_m)t]$$

If  $V_{BM}(t)$  is passed through HPF of cut off frequency  $f_H = 100 \# 10^6$ , then only  $(w_c + w_m)$  passes and output of HPF is

$$V_{HP}(t) = \frac{1}{2} \cos(w_c + w_m) t$$

$$\begin{aligned} \text{Now } V_0(t) &= V_{HP}(t) + \sin(2\pi \times 100 \times 10^6 t) \\ &= \frac{1}{2} \cos[2\pi \times 100 \times 10^6 + 2\pi \times 1 \times 10^6 t] + \sin(2\pi \times 100 \times 10^6 t) \\ &= \frac{1}{2} \cos[2\pi \times 10^8 + 2\pi \times 10^6 t] + \sin(2\pi \times 10^8 t) \\ &= \frac{1}{2} [\cos(2\pi \times 10^8 t) \cos(2\pi \times 10^6 t) - \sin(2\pi \times 10^8 t) \sin(2\pi \times 10^6 t) + \sin(2\pi \times 10^8 t)] \\ &= \frac{1}{2} \cos(2\pi \times 10^6 t) \cos(2\pi \times 10^8 t) + \frac{1}{2} - \frac{1}{2} \sin(2\pi \times 10^6 t) \sin(2\pi \times 10^8 t) \end{aligned}$$

This signal is in form

$$= A \cos 2\pi \times 10^8 t + B \sin 2\pi \times 10^8 t$$

The envelope of this signal is

$$\begin{aligned} &= \sqrt{A^2 + B^2} \\ &= \sqrt{\left(\frac{1}{2} \cos(2\pi \times 10^6 t)\right)^2 + \left(1 - \frac{1}{2} \sin(2\pi \times 10^6 t)\right)^2} \\ &= \sqrt{\frac{1}{4} \cos^2(2\pi \times 10^6 t) + 1 + \frac{1}{4} \sin^2(2\pi \times 10^6 t) - \sin(2\pi \times 10^6 t)} \\ &= \frac{1}{4} + 1 - \sin(2\pi \times 10^6 t) = \frac{5}{4} - \sin(2\pi \times 10^6 t) \end{aligned}$$

**Sol. 95**

Option (A) is correct.

$$s(t) = A \cos[2\pi \times 10 \times 10^3 t] + A \cos[2\pi \times 10.1 \times 10^3 t]$$

Here

$$T_1 = \frac{1}{10^3} = 100 \text{ msec}$$

and

$$T_2 = \frac{1}{10.1 \times 10^3} = 99 \text{ msec}$$

Period of added signal will be LCM  $[T_1, T_2]$

Thus

$$T = \text{LCM}[100, 99] = 9900 \text{ msec}$$

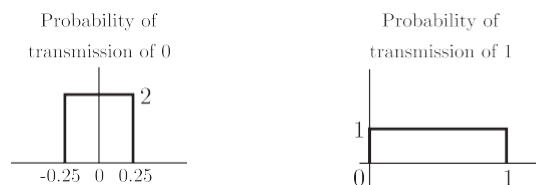
Thus frequency

$$f = \frac{1}{9900 \text{ msec}} = 0.1 \text{ kHz}$$

**Sol. 96**

Option (A) is correct.

The pdf of transmission of 0 and 1 will be as shown below :



Probability of error of 1

$$P(0 \text{ # } X \text{ # } 0.2) = 0.2$$

Probability of error of 0 :

$$P(0.2 \text{ # } X \text{ # } 0.25) = 0.05 \times 2 = 0.1$$

$$\begin{aligned} \text{Average error} &= \frac{P(0 \text{ # } X \text{ # } 0.2) + P(0.2 \text{ # } X \text{ # } 0.25)}{2} \\ &= \frac{0.2 + 0.1}{2} = 0.15 \end{aligned}$$

Sol. 97

Option (B) is correct.

The square mean value is

$$\begin{aligned}
 s^2 &= \int_{-3}^3 (x - x_q)^2 f(x) dx \\
 &= \int_0^1 (x - x_q)^2 f(x) dx \\
 &= \int_{0.3}^{0.3} (x - 0)^2 f(x) dx + \int_{0.3}^{0.1} (x - 0.7)^2 f(x) dx \\
 &= \int_{0.3}^{0.3} x^3 + \int_{0.3}^{0.1} x^2 + 0.49x - 14 \int_{0.3}^{0.1} x^2
 \end{aligned}$$

or

$$s^2 = 0.039$$

$$\text{RMS} = \sqrt{s^2} = \sqrt{0.039} = 0.198$$

Sol. 98

Option (C) is correct.

FM \$ Capture effect

DM \$ Slope over load

PSK \$ Matched filter

PCM \$  $m$  - law

Sol. 99

Option (C) is correct.

Since  $f_s = 2f_m$ , the signal frequency and sampling frequency are as follows

$$f_{m1} = 1200 \text{ Hz } \$ 2400 \text{ samples per sec}$$

$$f_{m2} = 600 \text{ Hz } \$ 1200 \text{ samples per sec}$$

$$f_{m3} = 600 \text{ Hz } \$ 1200 \text{ samples per sec}$$

Thus by time division multiplexing total 4800 samples per second will be sent.

Since each sample require 12 bit, total 4800 # 12 bits per second will be sent

$$\text{Thus bit rate } R_b = 4800 \times 12 = 57.6 \text{ kbps}$$

Sol. 100

Option (B) is correct.

The input signal  $X(f)$  has the peak at 1 kHz and  $-1$  kHz. After balanced modulator the output will have peak at  $f_c \pm 1$  kHz i.e.:

$$10 \pm 1 \$ 11 \text{ and } 9 \text{ kHz}$$

$$10 \pm (-1) \$ 9 \text{ and } 11 \text{ kHz}$$

9 kHz will be filtered out by HPF of 10 kHz. Thus 11 kHz will remain. After passing through 13 kHz balanced modulator signal will have 13  $\pm$  11 kHz signal i.e. 2 and 24 kHz.

Thus peak of  $Y(f)$  are at 2 kHz and 24 kHz.

Sol. 101

Option (A) is correct.

The input is a coherent detector is DSB - SC signal plus noise. The noise at the detector output is the in-phase component as the quadrature component  $n_q(t)$  of the noise  $n(t)$  is completely rejected by the detector.

Sol. 102

Option (C) is correct.

The noise at the input to an ideal frequency detector is white. The PSD of noise at the output is parabolic

Sol. 103

Option (B) is correct.

$$\text{We have } P_e = \frac{1}{2} \text{erfc}_c \sqrt{\frac{E_d}{2h}}$$

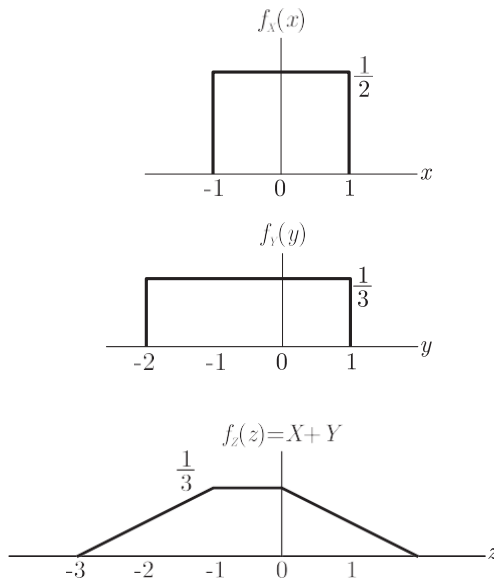
Since  $P_e$  of Binary FSK is 3 dB inferior to binary PSK

Sol. 104

Option (D) is correct.

The pdf of  $Z$  will be convolution of pdf of  $X$  and pdf of  $Y$  as shown below.

$$\begin{aligned} \text{Now } p[Z \# z] &= \int_{-\infty}^{\infty} f_X(z) f_Y(z-x) dx \\ p[Z \# -2] &= \int_{-\infty}^{\infty} f_Z(z) dz \\ &= \text{Area } [z \# -2] \\ &= \frac{1}{2} \times \frac{1}{6} \times 1 = \frac{1}{12} \end{aligned}$$



Sol. 105

Option (D) is correct.

We have

$$R_{XX}(T) = 4(e^{-0.2|T|} + 1)$$

$$R_{XX}(0) = 4(e^{-0.2|0|} + 1) = 8 = s^2$$

or

$$s = 2\sqrt{2}$$

Given

mean

$$m = 0$$

Now

$$P(x \# 1) = F_x(1)$$

$$= 1 - Q_c \frac{X-m}{s}$$

at  $x = 1$

$$= 1 - Q_c \frac{1-0}{2\sqrt{2}} = 1 - Q_c \frac{1}{2\sqrt{2}}$$

Sol. 106

Option (C) is correct.

$$W = Y - Z$$

$$E[W^2] = E[Y - Z]^2$$

$$= E[Y^2] + E[Z^2] - 2E[YZ]$$

$$= s_w^2$$

We have

$$E[X^2(t)] = R_x(10)$$

$$= 4[e^{-0.2|0|} + 1] = 4[1 + 1] = 8$$

$$E[Y^2] = E[X^2(2)] = 8$$



$$E[Z^2] = E[X^2(4)] = 8$$

$$E[YZ] = R_{XX}(2) = 4[e^{-0.2(4-2)} + 1] = 6.68$$

$$E[W^2] = s_w^2 = 8 + 8 - 2 \times 6.68 = 2.64$$

Sol. 107

Option (C) is correct.

$$\text{Step size } d = \frac{2m_p}{L} = \frac{1.536}{128} = 0.012 \text{ V}$$

$$\begin{aligned} \text{Quantization Noise power} &= \frac{d^2}{12} = \frac{(0.012)^2}{12} \\ &= 12 \times 10^{-6} \text{ V}^2 \end{aligned}$$

Sol. 108

Option (D) is correct.

The frequency of pulse train is

$$f \frac{1}{10^{-3}} = 1 \text{ kHz}$$

The Fourier Series coefficient of given pulse train is

$$\begin{aligned} C_n &= \frac{1}{T_o} \int_{-T_o/2}^{T_o/2} A e^{-j n \omega_o t} dt \\ &= \frac{1}{T_o} \int_{-T_o/6}^{T_o/6} A e^{-j n \omega_o t} dt \\ &= \frac{A}{T_o (-j n \omega_o)} [e^{-j n \omega_o t}]_{-T_o/6}^{T_o/6} \\ &= \frac{A}{(-j 2 \pi n)} (e^{-j n \omega_o T_o/6} - e^{j n \omega_o T_o/6}) \\ &= \frac{A}{j 2 \pi n} (e^{j n \pi/3} - e^{-j n \pi/3}) \end{aligned}$$

or

$$C_n = \frac{A}{\pi n} \sin \frac{n \pi}{3}$$

From  $C_n$  it may be easily seen that 1, 2, 4, 5, 7, harmonics are present and 0, 3, 6, 9, ... are absent. Thus  $p(t)$  has 1 kHz, 2 kHz, 4 kHz, 5 kHz, 7 kHz, ... frequency component and 3 kHz, 6 kHz, ... are absent.

The signal  $x(t)$  has the frequency components 0.4 kHz and 0.7 kHz. The sampled signal of  $x(t)$  i.e.  $x(t) * p(t)$  will have

$$1 \times 0.4 \text{ and } 1 \times 0.7 \text{ kHz}$$

$$2 \times 0.4 \text{ and } 2 \times 0.7 \text{ kHz}$$

$$4 \times 0.4 \text{ and } 4 \times 0.7 \text{ kHz}$$

Thus in range of 2.5 kHz to 3.5 kHz the frequency present is

$$2 + 0.7 = 2.7 \text{ kHz}$$

$$4 - 0.7 = 3.3 \text{ kHz}$$

Sol. 109

Option (C) is correct.

$$v_i = A_c \cos(2\pi f_c t) + m(t)$$

$$v_0 = a_0 v_i + a v_i^3$$

$$\begin{aligned} v_0 &= a_0 [A_c \cos(2\pi f_c t) + m(t)] + a_1 [A_c \cos(2\pi f_c t) + m(t)]^3 \\ &= a_0 A_c \cos(2\pi f_c t) + a_0 m(t) + a_1 [A_c^3 \cos^3(2\pi f_c t) \\ &\quad + (A_c^2 \cos(2\pi f_c t) t)^2 m(t) + 3 A_c \cos(2\pi f_c t) m^2(t) + m^3(t)] \\ &= a_0 A_c \cos(2\pi f_c t) + a_0 m(t) + a_1 [A_c^3 \cos^3(2\pi f_c t) \\ &\quad + 3 A_c \cos(2\pi f_c t) m^2(t) + m^3(t)] \\ &\quad + 3 a_1 A_c^2 \cos(2\pi f_c t) m(t) \end{aligned}$$

The term  $3a_1 A_c (\cos 4\pi f_c t) m^2(t) + m^3(t)$  is a DSB-SC signal having carrier frequency 1. MHz.  
Thus  $2f_c = 1 \text{ MHz}$  or  $f_c = 0.5 \text{ MHz}$

**Sol. 110** Option (D) is correct.

$$P_T = P_c \left( 1 + \frac{a^2}{2} \right)$$

$$P_{sb} = \frac{P_c a^2}{2} = \frac{P_c (0.5)^2}{2}$$

or 
$$\frac{P_{sb}}{P_c} = \frac{1}{8}$$

**Sol. 111** Option (D) is correct.

$$\text{AM Band width} = 2f_m$$

$$\text{Peak frequency deviation} = 3(2f_m) = 6f_m$$

$$\text{Modulation index } b = \frac{6f_m}{f_m} = 6$$

The FM signal is represented in terms of Bessel function as

$$x_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(b) \cos(w_c - nw_n)t$$

$$w_c + nw_m = 2\pi (1008 \times 10^3)$$

$$2\pi 10^6 + n4\pi \times 10^3 = 2\pi (1008 \times 10^3), n = 4$$

Thus coefficient =  $5J_4(6)$

**Sol. 112** Option (B) is correct.

Ring modulation \$ Generation of DSB - SC

VCO \$ Generation of FM

Foster seely discriminator \$ Demodulation of fm

mixer \$ frequency conversion

**Sol. 113** Option (A) is correct.

$$f_{\max} = 1650 + 450 = 2100 \text{ kHz}$$

$$f_{\min} = 550 + 450 = 1000 \text{ kHz}$$

or 
$$f = \frac{1}{2RC}$$

frequency is minimum, capacitance will be maximum

$$R = \frac{C_{\max}}{C_{\min}} = \frac{f_{\max}^2}{f_{\min}^2} = (2.1)^2$$

or 
$$R = 4.41$$

$$f_i = f_c + 2f_{IF} = 700 + 2(455) = 1600 \text{ kHz}$$

**Sol. 114** Option (D) is correct.

$$E_b = 10^{-6} \text{ watt-sec}$$

$$N_o = 10^{-5} \text{ W/Hz}$$

$$(\text{SNR})_{\text{matched filter}} = \frac{E_o}{\frac{N_o}{2}} = \frac{10^6}{2 \times 10^{-5}} = .05$$

$$(\text{SNR})_{dB} = 10 \log_{10} (0.05) = 13 \text{ dB}$$

**Sol. 115** Option (B) is correct.

For slopeoverload to take place  $E_m \leq \frac{3f_s}{2pf_m}$

This is satisfied with  $E_m = 1.5$  V and  $f_m = 4$  kHz

**Sol. 116**

Option (A) is correct.

If  $s$  " carrier synchronization at receiver  
 $r$  " represents bandwidth efficiency  
 then for coherent binary PSK  $r = 0.5$  and  $s$  is required.

**Sol. 117**

Option (B) is correct.

$$\text{Bit Rate} = 8 \times 8 = 64 \text{ kbps}$$

$$(\text{SNR})_q = 1.76 + 6.02n \text{ dB}$$

$$= 1.76 + 6.02 \times 8 = 49.8 \text{ dB}$$

**Sol. 118**

Option (C) is correct.

The frequency of message signal is

$$f_c = 1000 \text{ kHz}$$

1 The frequency of message signal is

$$f_m = \frac{1}{10^{-6}} = 10^6 \text{ Hz} = 1 \text{ MHz}$$

Here message signal is symmetrical square wave whose FS has only odd harmonics i.e. 10 kHz, 30 kHz, 50 kHz. Modulated signal contains  $f_c \pm f_m$  frequency component. Thus modulated signal has

$$f_c \pm f_m = (1000 \pm 10) \text{ kHz} = 1010 \text{ kHz}, 990 \text{ kHz}$$

$$f_c \pm 3f_m = (1000 \pm 30) \text{ kHz} = 1030 \text{ kHz}, 970 \text{ kHz}$$

Thus, there is no 1020 kHz component in modulated signal.

**Sol. 119**

Option (C) is correct.

$$\text{We have } y(t) = 5 \times 10^{-6} x(t) \int_{n=-3}^{+3} d(t - nT_s)$$

$$x(t) = 10 \cos(8\pi \times 10^3 t)$$

$$T_s = 100 \mu \text{ sec}$$

The cut off  $f_c$  of LPF is 5 kHz

We know that for the output of filter

$$= \frac{x(t)y(t)}{T_s}$$

$$= \frac{10 \cos(8\pi \times 10^3 t) \times 5 \times 10^{-6}}{100 \times 10^{-6}}$$

$$= 5 \times 10^{-1} \cos(8\pi \times 10^3 t)$$

**Sol. 120**

Option (C) is correct.

Transmitted frequencies in coherent BFSK should be integral of bit rate 8 kHz.

**Sol. 121**

Option (B) is correct.

For best reception, if transmitting waves are vertically polarized, then receiver should also be vertically polarized i.e. transmitter and receiver must be in same polarization.

**Sol. 122**

Option (D) is correct.

$$s(t) = \cos 2\pi (2 \times 10^6 t + 30 \sin 150t + 40 \cos 150t)$$

$$= \cos \{4p10^6 t + 100p \sin(150t + q)\}$$

Angle modulated signal is

$$s(t) = A \cos \{w_c t + b \sin(w_m t + q)\}$$

Comparing with angle modulated signal we get

Phase deviations  $b = 100p$

Frequency deviations

$$3f = bf_m = 100p \# \frac{150}{2p} = 7.5 \text{ kHz}$$

**Sol. 123**

Option (\*) is correct.

We have

$$\begin{aligned} m(t)s(t) &= y_1(t) \\ &= \frac{2 \sin(2pt) \cos(200pt)}{t} \\ &= \frac{\sin(202pt) - \sin(198pt)}{t} \end{aligned}$$

$$y_1(t) + n(t) = y_2(t) = \frac{\sin 202pt - \sin 198pt}{t} + \frac{\sin 199pt}{t}$$

$$\begin{aligned} y_2(t)s(t) &= u(t) \\ &= \frac{[\sin 202pt - \sin 198pt + \sin 199pt] \cos 200pt}{t} \end{aligned}$$

$$= \frac{1}{2} [\sin(402pt) + \sin(2pt) - \{\sin(398pt) - \sin(2pt)\} + \sin(399pt) - \sin(pt)]$$

After filtering

$$\begin{aligned} y(t) &= \frac{\sin(2pt) + \sin(2pt) - \sin(pt)}{2t} \\ &= \frac{\sin(2pt) + 2 \sin(0.5t) \cos(1.5pt)}{2t} \\ &= \frac{\sin 2pt}{2t} + \frac{\sin 0.5pt}{t} \cos 1.5pt \end{aligned}$$

**Sol. 124**

Option (B) is correct.

The signal frequency is

$$f_m = \frac{24p10^3}{2p} = 12 \text{ kHz}$$

$$T_s = 50 \text{ m sec} \quad f_s = \frac{1}{T_s} = \frac{1}{50} \# 10^6 = 20 \text{ kHz}$$

After sampling signal will have  $f_s \pm f_m$  frequency component i.e. 32 and 12 kHz

At filter output only 8 kHz will be present as cutoff frequency is 15 kHz.

**Sol. 125**

Option (A) is correct.

$$d(n) = x(n) - x(n-1)$$

$$E[d(n)]^2 = E[x(n) - x(n-1)]^2$$

or  $E[d(n)]^2 = E[x(n)]^2 + E[x(n-1)]^2 - 2E[x(n)x(n-1)]$  as  $k=1$

or  $s_d^2 = s_x^2 + s_x^2 - 2R_{xx}(1)$

As we have been given  $s_d = \frac{s_x^2}{10}$ , therefore

$$\frac{s_x^2}{10} = s_x^2 + s_x^2 - 2R_{xx}(1)$$

or  $2R_{xx}(1) = \frac{19}{10} s_x^2$

or 
$$\frac{R_{xx}}{S_x^2} = \frac{19}{20} = 0.95$$

**Sol. 126**

Option (A) is correct.

An ideal low - pass filter with appropriate bandwidth  $f_m$  is used to recover the signal which is sampled at nyquist rate  $2f_m$ .

**Sol. 127**

Option (A) is correct.

For any PDF the probability at mean is  $\frac{1}{2}$ . Here given PDF is Gaussian random variable and  $X = 4$  is mean.

**Sol. 128**

Option (C) is correct.

We require 6 bit for 64 intensity levels because  $64 = 2^6$

$$\begin{aligned} \text{Data Rate} &= \text{Frames per second} \times \text{pixels per frame} \times \text{bits per pixel} \\ &= 625 \times 400 \times 6 = 600 \text{ Mbps sec} \end{aligned}$$

**Sol. 129**

Option (C) is correct.

We have

$$\sin c(700t) + \sin c(500t) = \frac{\sin(700\pi t)}{700\pi t} + \frac{\sin(500\pi t)}{500\pi t}$$

Here the maximum frequency component is  $2\pi f_m = 700\pi$  i.e.  $f_m = 350$  Hz

$$\begin{aligned} \text{Thus Nyquist rate} \quad f_s &= 2f_m \\ &= 2(350) = 700 \text{ Hz} \end{aligned}$$

$$\text{Thus sampling interval} = \frac{1}{700} \text{ sec}$$

**Sol. 130**

Option (D) is correct.

$$\text{Probability of error} = p$$

$$\text{Probability of no error} = q = (1 - p)$$

Probability for at most one bit error

$$\begin{aligned} &= \text{Probability of no bit error} \\ &\quad + \text{probability of 1 bit error} \\ &= (1 - p)^n + np(1 - p)^{n-1} \end{aligned}$$

**Sol. 131**

Option (A) is correct.

$$\text{If } g(t) \xrightarrow{FT} G(w)$$

then PSD of  $g(t)$  is

$$S_g(w) = |G(w)|^2$$

and power is

$$P = \int_{-\infty}^{\infty} S_g(w) dw$$

Now

$$ag(t) \xrightarrow{FT} aG(w)$$

PSD of  $ag(t)$  is

$$\begin{aligned} S_{ag}(w) &= |aG(w)|^2 \\ &= a^2 |G(w)|^2 \end{aligned}$$

or

$$S_{ag}(w) = a^2 S_g(w)$$

Similarly

$$P_{ag} = a^2 P_g$$

**Sol. 132**

Option (C) is correct.

The envelope of the input signal is  $[1 + k_a m(t)]$  that will be output of envelope

detector.

**Sol. 133** Option (D) is correct.

Frequency Range for satellite communication is 1 GHz to 30 GHz,

**Sol. 134** Option (B) is correct.

Waveform will be orthogonal when each bit contains integer number of cycles of carrier.

Bit rate

$$\begin{aligned} R_b &= HCF(f_1, f_2) \\ &= HCF(10k, 25k) \\ &= 5 \text{ kHz} \end{aligned}$$

Thus bit interval is

$$T_b = \frac{1}{R_b} = \frac{1}{5k} = 0.2 \text{ msec} = 200 \text{ msec}$$

**Sol. 135** Option (D) is correct.

We have

$$P_m = m^2(t)$$

The input to LPF is

$$\begin{aligned} x(t) &= m(t) \cos w_o t \cos (w_o t + q) \\ &= \frac{m(t)}{2} [\cos (2w_o t + q) + \cos q] \\ &= \frac{m(t) \cos (2w_o t + q)}{2} + \frac{m(t) \cos q}{2} \end{aligned}$$

The output of filter will be

$$y(t) = \frac{m(t) \cos q}{2}$$

Power of output signal is

$$P_y = y^2(t) = \frac{1}{4} m^2(t) \cos^2 q = \frac{P_m \cos^2 q}{4}$$

**Sol. 136** Option (A) is correct.

Hilbert transformer always adds  $-90^\circ$  to the positive frequency component and  $90^\circ$  to the negative frequency component.

Hilbert Transform

$$\begin{aligned} \cos w_1 t &\rightarrow \sin w_1 t \\ \sin w_1 t &\rightarrow -\cos w_1 t \end{aligned}$$

Thus

$$\cos w_1 t + \sin w_2 t \rightarrow \sin w_1 t - \cos w_2 t$$

**Sol. 137** Option (A) is correct.

We have

$$\begin{aligned} x(t) &= A_c \cos \{w_c t + b \sin w_m t\} \\ y(t) &= \{x(t)\}^3 \end{aligned}$$

$$= A_c^3 \cos^3 (w_c t + b \sin w_m t) = 3 A_c^2 \cos (w_c t + b \sin w_m t) \cos^2 (w_c t + b \sin w_m t)$$

Thus the fundamental frequency doesn't change but BW is three times.

$$BW = 2(3f) = 2(3f \pm 3) = 3 \text{ MHz}$$

**Sol. 138** Option (C) is correct.

**Sol. 139** Option (C) is correct.

This is Quadrature modulated signal. In QAM, two signals having bandwidth  $B_1$  &  $B_2$  can be transmitted simultaneously over a bandwidth of  $(B_1 + B_2)$  Hz

so

$$B.W. = (15 + 10) = 25 \text{ kHz}$$

**Sol. 140**

Option (B) is correct.

A modulated signal can be expressed in terms of its in-phase and quadrature component as

$$S(t) = S_1(t) \cos(2\pi f_c t) - S_Q(t) \sin(2\pi f_c t)$$

Here

$$\begin{aligned} S(t) &= [e^{-at} \cos D\omega t \cos \omega_c t - e^{-at} \sin D\omega t \sin \omega_c t] m(t) \\ &= [e^{-at} \cos D\omega t] \cos 2\pi f_c t - [e^{-at} \sin D\omega t] \sin 2\pi f_c t \\ &= S_1(t) \cos 2\pi f_c t - S_Q(t) \sin 2\pi f_c t \end{aligned}$$

Complex envelope of  $s(t)$  is

$$\begin{aligned} S(t) &= S_1(t) + jS_Q(t) \\ &= e^{-at} \cos D\omega t + j e^{-at} \sin D\omega t \\ &= e^{-at} [\cos D\omega t + j \sin D\omega t] \\ &= \exp(-at) \exp(jD\omega t) m(t) \end{aligned}$$

**Sol. 141**

Option (B) is correct.

Given function

$$g(t) = 6 \times 10^4 \sin^2(400t) \times 10^6 \sin^3(100t)$$

Let

$$g_1(t) = 6 \times 10^4 \sin^2(400t)$$

$$g_2(t) = (10^6) \sin^3(100t)$$

We know that  $g_1(t) \times g_2(t) \rightarrow G_1(\omega) G_2(\omega)$  occupies minimum of Bandwidth of  $G_1(\omega)$  or  $G_2(\omega)$

$$\text{Band width of } G_1(\omega) = 2 \times 400 = 800 \text{ rad/sec or } = 400 \text{ Hz}$$

$$\text{Band width of } G_2(\omega) = 3 \times 100 = 300 \text{ rad/sec or } 150 \text{ Hz}$$

Sampling frequency

$$= 2 \times 150 = 300 \text{ Hz}$$

**Sol. 142**

Option (B) is correct.

For a sinusoidal input SNR (dB) is PCM is obtained by following formulae.

$$SNR \text{ (dB)} = 1.8 + 6n \quad n \text{ is no. of bits}$$

Here

$$n = 8$$

So,

$$SNR \text{ (dB)} = 1.8 + 6 \times 8 = 49.8$$

**Sol. 143**

Option (D) is correct.

We know that matched filter output is given by

$$\begin{aligned} g_0(t) &= \int_{-3}^3 g(t) g(T_0 - t + \tau) d\tau \text{ at } t = T_0 \\ g_0(t)_{\max} &= \int_{-3}^3 g(\tau) g(\tau) d\tau = \int_{-3}^3 g^2(\tau) d\tau \\ &= \int_0^{1 \times 10^{-4}} [10 \sin(2\pi \times 10^6 \tau)]^2 d\tau \\ [g_0(t)]_{\max} &= \frac{1}{2} \times 100 \times 10^{-4} = 5 \text{ mV} \end{aligned}$$

**Sol. 144**

Option (B) is correct.

Sampling rate must be equal to twice of maximum frequency.

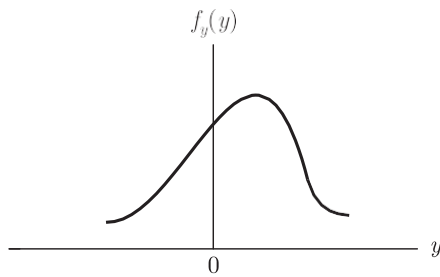
$$f_s = 2 \times 400 = 800 \text{ Hz}$$

**Sol. 145**

Option (C) is correct.

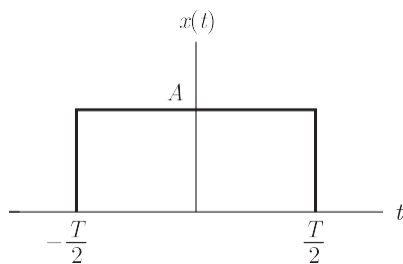
The amplitude spectrum of a gaussian pulse is also gaussian as shown in the fig.

$$f(y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right)$$



Sol. 146

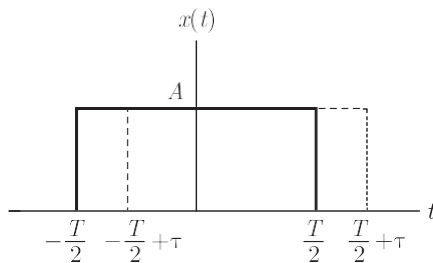
Option (C) is correct.  
Let the rectangular pulse is given as



Auto correlation function is given by  

$$R_{xx}(\tau) = \frac{1}{T} \int_{-T/2}^{T/2} x(t) x(t-\tau) dt$$

When  $x(t)$  is shifted to right ( $\tau > 0$ ),  $x(t-\tau)$  will be shown as dotted line.



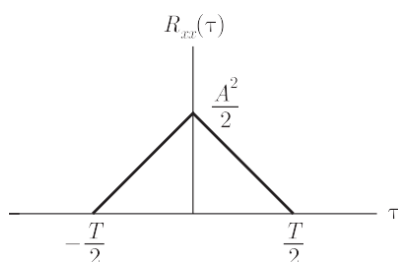
$$R_{xx}(\tau) = \frac{1}{T} \int_{-T/2}^{T/2+\tau} A^2 dt$$

$$= \frac{A^2}{T} \left( \frac{T}{2} + \frac{T}{2} - \tau \right) = \frac{A^2}{T} (T - \tau)$$

( $\tau$ ) can be negative or positive, so generalizing above equations

$$R_{xx}(\tau) = \frac{A^2}{T} (T - |\tau|)$$

$R_{xx}(\tau)$  is a regular pulse of duration  $T$ .





**Sol. 147** Option (B) is correct.

Selectivity refers to select a desired frequency while rejecting all others. In super heterodyne receiver selective is obtained partially by RF amplifier and mainly by IF amplifier.

**Sol. 148** Option (C) is correct.

In PCM,  $SNR \propto 2^{2n}$

so if bit increased from 8 to 9

$$\frac{(SNR)_1}{(SNR)_2} = \frac{2^{2 \times 8}}{2^{2 \times 9}} = 2^2 = \frac{1}{4}$$

so SNR will increased by a factor of 4

**Sol. 149** Option (A) is correct.

In flat top sampling an amplitude distortion is produced while reconstructing original signal  $x(t)$  from sampled signal  $s(t)$ . High frequency of  $x(t)$  are mostly attenuated. This effect is known as aperture effect.

**Sol. 150** Option (A) is correct.

$$\text{Carrier } C(t) = \cos(\omega_c t + \phi)$$

$$\text{Modulating signal} = x(t)$$

$$\text{DSB - SC modulated signal} = x(t) c(t) = x(t) \cos(\omega_c t + \phi)$$

$$\text{envelope} = |x(t)|$$

**Sol. 151** Option (D) is correct.

In Quadrature multiplexing two baseband signals can transmitted or modulated using  $I$  &  $Q$  phase & Quadrature carriers and its quite different from FDM & TDM.

**Sol. 152** Option (A) is correct.

Fourier transform perform a conversion from time domain to frequency domain for analysis purposes. Units remain same.

**Sol. 153** Option (A) is correct.

In PCM, SNR depends on step size (i.e. signal amplitude) SNR can be improved by using smaller steps for smaller amplitude. This is obtained by compressing the signal.

**Sol. 154** Option (C) is correct.

Band width is same for BPSK and APSK(OOK) which is equal to twice of signal Bandwidth.

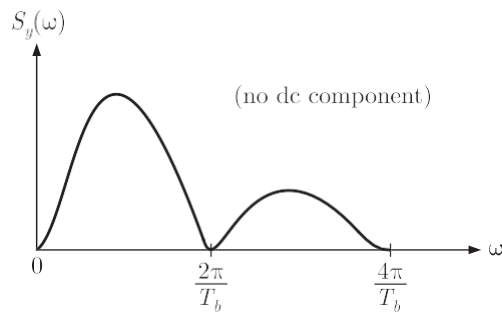
**Sol. 155** Option (A) is correct.

The spectral density of a real value random process symmetric about vertical axis so it has an even symmetry.

**Sol. 156** Option (A) is correct.

**Sol. 157** Option (C) is correct.

It is one of the advantage of bipolar signalling (AMI) that its spectrum has a dc null for binary data transmission PSD of bipolar signalling is

**Sol. 158**

Option (A) is correct.

Probability Density function (PDF) of a random variable  $x$  defined as

$$P(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

so here

$$K = \frac{1}{\sqrt{2\pi}}$$

**Sol. 159**

Option (C) is correct.

Here the highest frequency component in the spectrum is 1.5 kHz  
[at 2 kHz is not included in the spectrum]

$$\text{Minimum sampling freq.} = 1.5 \times 2 = 3 \text{ kHz}$$

**Sol. 160**

Option (B) is correct.

We need a high pass filter for receiving the pulses.

**Sol. 161**

Option (D) is correct.

Power spectral density function of a signal  $g(t)$  is fourier transform of its auto correlation function

$$R_g(t) \xrightarrow{F} S_g(\omega)$$

$$\text{here } S_g(\omega) = \sin^2(f)$$

so  $R_g(t)$  is a triangular pulse.

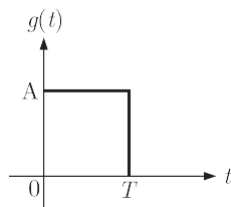
$$f[\text{triang.}] = \sin^2(f)$$

**Sol. 162**

Option (C) is correct.

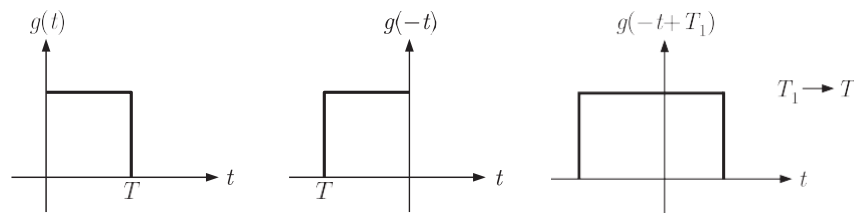
For a signal  $g(t)$ , its matched filter response given as

$$h(t) = g(T-t)$$

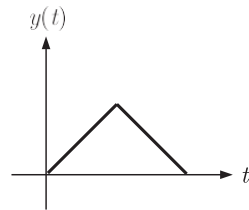
so here  $g(t)$  is a rectangular pulse of duration  $T$ .

output of matched filter

$$y(t) = g(t) \triangleright h(t)$$



if we shift  $g(-t)$  for convolution  $y(t)$  increases first linearly then decreases to zero.



**Sol. 163**

Option (C) is correct.

The difference between incoming signal frequency ( $f_c$ ) and its image frequency ( $f_i$ ) is  $2f_f$  (which is large enough). The RF filter may provide poor selectivity against adjacent channels separated by a small frequency differences but it can provide reasonable selectivity against a station separated by  $2f_f$ . So it provides adequate suppression of image channel.

**Sol. 164**

Option (C) is correct.

In PCM SNR is given by

$$SNR = \frac{3}{2} 2^{2n}$$

if no. of bits is increased from  $n$  to  $(n + 1)$  SNR will increase by a factor of  $2^{2(n+1)/n}$

**Sol. 165**

Option (D) is correct.

The auto correlation of energy signal is an even function.

auto correlation function is gives as

$$R(\tau) = \int_{-\tau}^{\tau} x(t)x(t+\tau)dt$$

$$R(-\tau) = \int_{-\tau}^{\tau} x(t)x(t-\tau)dt$$

put

Let

$$t - \tau = \alpha$$

$$dt = d\alpha$$

$$R(-\tau) = \int_{-\tau}^{\tau} x(\alpha + \tau)x(\alpha)d\alpha$$

Changing variable  $\alpha \rightarrow t$

$$R(-\tau) = \int_{-\tau}^{\tau} x(t)x(t+\tau)dt = R(\tau)$$

$$R(-\tau) = R(\tau) \text{ even function}$$

\*\*\*\*\*