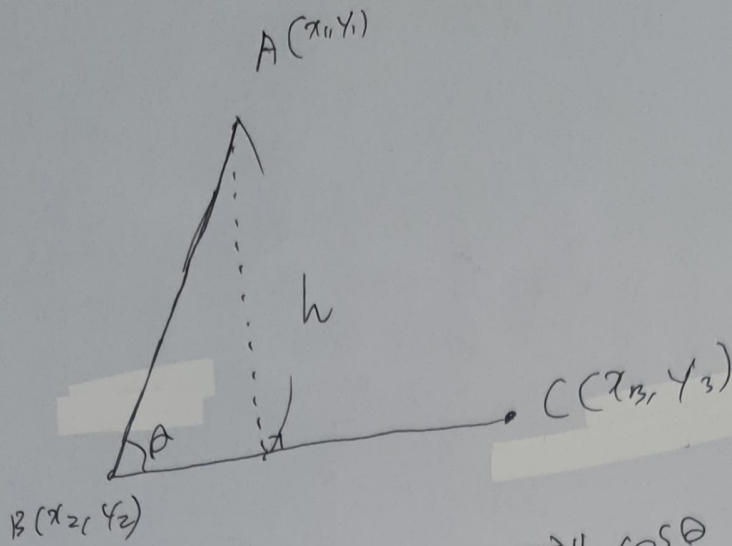


1)



$$\vec{BA} \cdot \vec{BC} = \|\vec{BA}\| \|\vec{BC}\| \cos \theta$$

$$\|\vec{BA}\| \cdot \sin \theta = h$$

$$\therefore h = \|\vec{BA}\| \sqrt{1 - \left(\frac{\vec{BA} \cdot \vec{BC}}{\|\vec{BA}\| \|\vec{BC}\|} \right)^2} \quad \left(\because \sin \theta > 0, \cos^2 \theta = 1 - \sin^2 \theta \right)$$

$$= \sqrt{\|\vec{BA}\|^2 \|\vec{BC}\|^2 - (\vec{BA} \cdot \vec{BC})^2}$$

$$= \frac{\|\vec{BC}\|}{\sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}} \cdot \left| x_2 y_1 - y_2 x_1 - x_3 y_1 + x_1 y_3 + y_2 x_3 - x_2 y_3 \right|$$

2) $\angle A = \varphi$ at 2h.

$$\Rightarrow \vec{AB} \cdot \vec{AC} = \|\vec{AB}\| \|\vec{AC}\| \cos \varphi$$

$$\therefore \cos \varphi = \frac{x_2 x_3 - x_1 x_3 + x_1^2 - x_2 x_1 + y_2 y_3 + y_1^2 - y_2 y_1 - y_3 y_1}{\sqrt{((x_1 - x_2)^2 + (y_1 - y_2)^2) \cdot ((x_1 - x_3)^2 + (y_1 - y_3)^2)}}$$

$$\therefore \angle A = \varphi = \cos^{-1} \left(\frac{x_2 x_3 - x_1 x_3 + x_1^2 - x_2 x_1 + y_2 y_3 + y_1^2 - y_2 y_1 - y_3 y_1}{\sqrt{((x_1 - x_2)^2 + (y_1 - y_2)^2) \cdot ((x_1 - x_3)^2 + (y_1 - y_3)^2)}} \right)$$