THE LEGEND OF TOWERS OF HANOI

The legend says that at the beginning of time, the priests in the temple were given a stack of ***64 gold disks***, each one a little smaller than the one beneath it. Their assignment was to transfer the ***64*** **disks** from one of three poles to another, with one important rule: a large **disk** can never be placed on top of a smaller one. The priests worked very efficiently, day and night. When they finished their work, the myth said, the temple would crumble into dust, and the world would vanish.

An **old legend** tells of a ***Hindu temple*** (earlier) where the pyramid puzzle might have been used for the mental discipline of young priests. According to this **myth** (uttered long before the Vietnam War), there is a ***Buddhist monastery*** (current) at ***Hanoi***, which contains a large room with three timeworn posts in it surrounded by golden **disks**. Monks, acting out the command of an ancient prophecy, have been moving these **disks**, in accordance with the rules of the puzzle, once every day since the monastery was founded over a thousand years ago. They are said to believe that when the last move of the puzzle is completed, the world will end in a clap of thunder. Fortunately, they are nowhere close to being done. There is a similar "***myth***" about similar towers in the Hindu city of ***Benares*** in India about end of the world. According to ***Hindu Mythology***, ***God Brahma*** is the creator of the world and creator of all the creatures.

At the creation of the world, the priests were given a brass platform on which were ***3*** diamond needles. On the first needle were stacked ***64*** golden **disks**, each one slightly smaller than the one under it. (The less exotic version sold in ***Europe*** had ***8*** cardboard **disks** and wooden posts.) The priests were assigned the task of moving all the golden **disks** from the first needle to the third, subject to the condition that only one **disk** can be moved at a time, and that no disk is ever allowed to be placed on top of a smaller disk. The priests were told that when they had finished moving the ***64*** disks, it would signify the end of the world.

EARLY INVENTIONS

In ***1883***, ***Francoise Edouard Anatole Lucas*** (***1842 - 1891***), a ***French*** mathematician, invented a game called the **Tower of Hanoi** (sometimes referred to as the ***Tower of Brahma*** or the End of the World Puzzle). The puzzle appeared in ***1883*** under the name of ***M. Claus***. ***Claus*** is an anagram of Lucas! Lucas' four volumes on recreational mathematics is a classic. He was inspired by a legend that tells of a ***Hindu temple*** where the pyramid puzzle might have been used for the mental discipline of young priests. In th nineteenth century, a game called the ***Towers of Hanoi*** appeared in ***Europe*** together with promotional material explaining that the game represented a task is underway in the ***Temple of Brahma.***

Edouard Lucas

1842 - 1891

THE GAME OF MYTH

The game begins with a number of **disks**, arranged on one of three poles. Each **disk** is smaller than the **disk** below it. The object is to move all the **disks** from the starting ***tower*** to one of the remaining ***towers***. Only one **disk** can be moved at a time, and a larger **disk** can never be placed on top of a smaller one. Use the lowest number of possible moves.

The number of separate transfers of single disks the priests must make to transfer the tower is ***2*** to the ***64th*** power minus ***1 (2 ^64 - 1)***, or ***18,446,744,073,709,551,615*** moves! If the priests worked day and night, making one move every second it would take slightly more than ***580*** billion years to accomplish the job! Most probably the end of the world will come before the end of the ***580*** billion years!

The solution given here for ***Towers of Hanoi*** not only produces a complete solution to the task, but also produces the best possible solution. To show uniqueness to the irreducible solution, at every stage, the task to be done can be summarized as to move a certain number of **disks** from one ***tower*** to another. In finding how many times the recursion will proceed before starting to return and back out with ***n = 64 disks***, it is evident that each recursive call reduces the value of n by 1. It is easy to calculate how many instructions are needed to move ***64 disks***.

Number of moves $= 1 + 2 + 4 + \ldots + 2^{63}$

$$= \mathbf{2^{64} - 1}$$

$$= \mathbf{18{,}446{,}744{,}073{,}709{,}551{,}615}$$

THE TOWER OF HANOI SOLUTION

The idea that gives a solution is to concentrate our attention not on the first step, but rather on the hardest step, moving the bottom **disk**. There is no way to reach the bottom **disk** until the top ***63*** **disks** have moved, and, furthermore, they must all be on needle ***2*** so that the bottom **disk** from needle ***1*** can be moved to needle ***3***. This is because only one **disk** can be moved at a time and bottom one can never be on top of any other, when the bottom one is moved; there can be no other **disk** on needles ***1*** or ***3***.

Somehow if the ***63*** **disks** can be successfully moved from one needle to another needle, the final step is to move just one final bottom **disk**. If we know how to move ***62*** **disks**, the final step would be to move the **disk** ***63*** from one needle to another. This is exactly the idea of **recursion**. The obvious stopping rule is that, when there are no **disks** to be moved on the starting needle and all the **disks** are successfully moved to the destination.

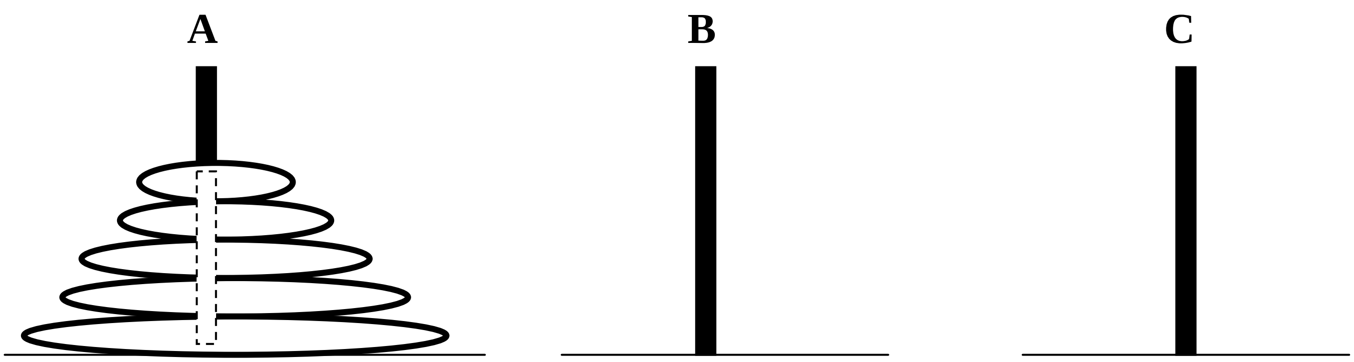
Temple Pura Ulu Danau, Bali

A RECURSIVE APPROACH TO TOWERS OF HANOI

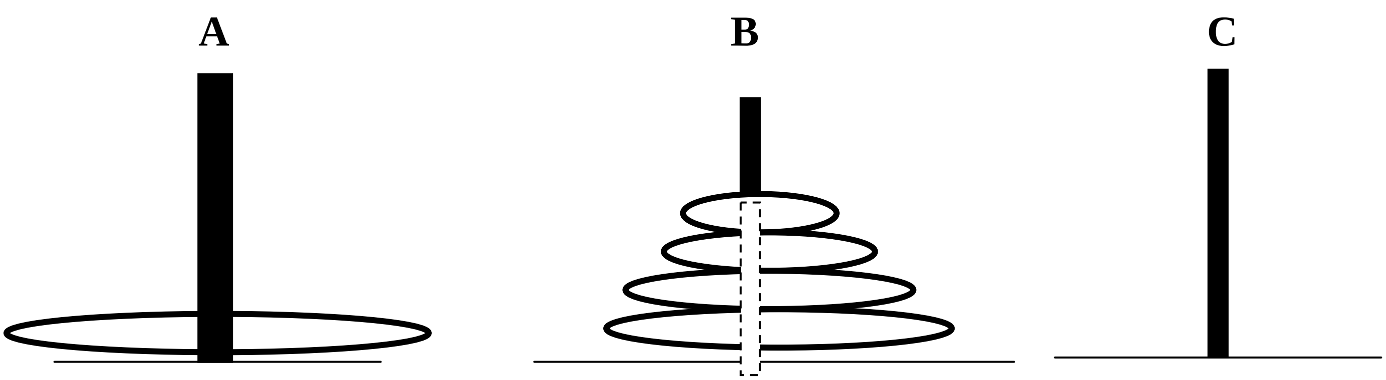
Simplified problem seeking a recursive solution:

The problem of ***towers of Hanoi*** has five **disks** of differing graded diameters are placed on peg ***A*** so that a larger **disk** is always below a smaller **disk**. The objective is to move the five disks to peg ***C***, using peg ***B*** as auxiliary. Only the top **disk** on any peg may be moved to any other peg, and a larger **disk** may never rest on a smaller one.

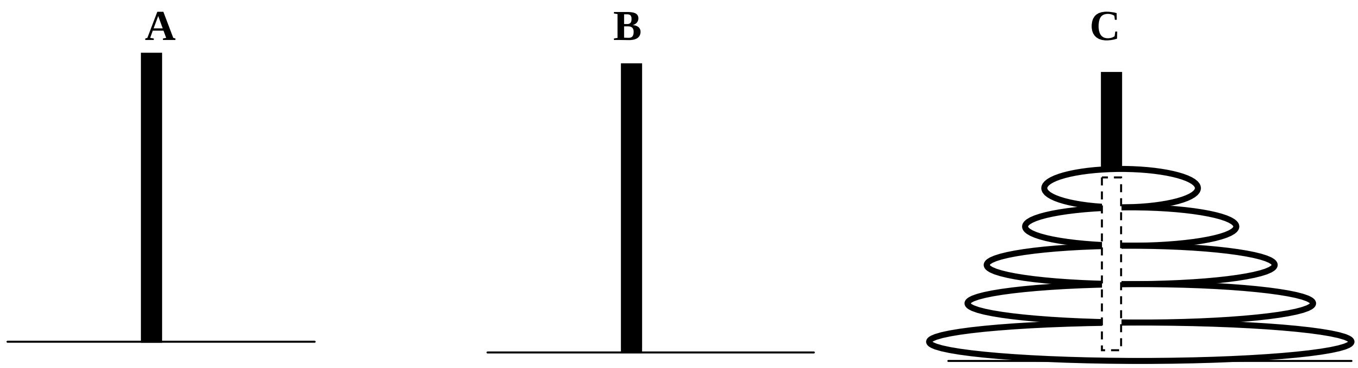
If there is a solution for ***(n –1)*** **disks**, then the solution could be stated for ***n*** **disks** in terms of the solution for ***n – 1*** **disks**. In the trivial case of one **disk**, the solution is simple: merely moving the single **disk** from peg ***A*** to peg ***C*****.**



Initial set up of the Towers of Hanoi



Intermediate set up of the Towers of Hanoi



Final set up of the Towers of Hanoi

RECURSIVE APPROACH

Algorithm to move n disks from A to C, using B as auxiliary:

if n == 1, move the single disk from A to C and stop.
move the top n – 1 disks ***recursively*** from A to B, using C as auxiliary.
move the remaining disk from A to C.
move the n – 1 disks ***recursively*** from B to C, using A as auxiliary.