

問題 7.5. (省略)

問題 7.6.

(i) 方程式 $3x + 2y + 2z = -1$ が $\tilde{z} = 0$ となるように座標変換する^{*1}. たとえば,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = P \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} + \vec{v} = \begin{pmatrix} \frac{4}{\sqrt{34}} & 0 & \frac{3}{\sqrt{17}} \\ -\frac{3}{\sqrt{34}} & \frac{1}{\sqrt{2}} & \frac{2}{\sqrt{17}} \\ -\frac{3}{\sqrt{34}} & -\frac{1}{\sqrt{2}} & \frac{2}{\sqrt{17}} \end{pmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

(ii) 視点 S を同次座標で表し, さらに $\tilde{x}\tilde{y}\tilde{z}$ -座標に変換する;

$$\begin{aligned} S = \begin{pmatrix} 2 \\ 3 \\ 7 \\ 1 \end{pmatrix} &= \begin{bmatrix} 2 \\ 3 \\ 7 \\ 1 \end{bmatrix} \xrightarrow{\text{座標変換}} \tilde{S} = \left(\begin{array}{ccc|c} P & \vec{v} \\ \hline 0 & 0 & 0 & 1 \end{array} \right)^{-1} \begin{bmatrix} 2 \\ 3 \\ 7 \\ 1 \end{bmatrix} \\ &= \left(\begin{array}{ccc|c} {}^tP & -{}^tP\vec{v} \\ \hline 0 & 0 & 0 & 1 \end{array} \right) \begin{bmatrix} 2 \\ 3 \\ 7 \\ 1 \end{bmatrix} \\ &= \left(\begin{array}{ccc|c} \frac{4}{\sqrt{34}} & -\frac{3}{\sqrt{34}} & -\frac{3}{\sqrt{34}} & \frac{7}{\sqrt{34}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{3}{\sqrt{17}} & \frac{2}{\sqrt{17}} & \frac{2}{\sqrt{17}} & \frac{1}{\sqrt{17}} \\ \hline 0 & 0 & 0 & 1 \end{array} \right) \begin{bmatrix} 2 \\ 3 \\ 7 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} -\frac{15}{\sqrt{34}} \\ -\frac{5}{\sqrt{2}} \\ \frac{27}{\sqrt{17}} \\ 1 \end{bmatrix} = \begin{bmatrix} -15 \\ -5\sqrt{17} \\ 27\sqrt{2} \\ \sqrt{34} \end{bmatrix} \end{aligned}$$

(iii) $\tilde{z} = 0$ への透視投影 (視点 \tilde{S}) を表す行列をつくる;

$$\varphi_{\tilde{S}} = \begin{pmatrix} -27\sqrt{2} & 0 & -15 & 0 \\ 0 & -27\sqrt{2} & -5\sqrt{17} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{34} & -27\sqrt{2} \end{pmatrix}$$

^{*1} 12 月 8 日の授業ノートおよび第 6 回小テスト [2] を参照.

(iv) xyz -座標における π への透視投影を表す行列を計算する；

$$\begin{aligned}
 \Phi_S &= \left(\begin{array}{ccc|c} P & & & \vec{v} \\ \hline 0 & 0 & 0 & 1 \end{array} \right) \times \varphi_{\tilde{S}} \times \left(\begin{array}{ccc|c} {}^tP & & & -{}^tP\vec{v} \\ \hline 0 & 0 & 0 & 1 \end{array} \right) \\
 &= \left(\begin{array}{ccc|c} \frac{4}{\sqrt{34}} & 0 & \frac{3}{\sqrt{17}} & -1 \\ \frac{-3}{\sqrt{34}} & \frac{1}{\sqrt{2}} & \frac{2}{\sqrt{17}} & 1 \\ \frac{-3}{\sqrt{34}} & -\frac{1}{\sqrt{2}} & \frac{2}{\sqrt{17}} & 0 \\ \hline 0 & 0 & 0 & 1 \end{array} \right) \left(\begin{array}{cccc} -27\sqrt{2} & 0 & -15 & 0 \\ 0 & -27\sqrt{2} & -5\sqrt{17} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{34} & -27\sqrt{2} \end{array} \right) \\
 &\quad \times \left(\begin{array}{ccc|c} \frac{4}{\sqrt{34}} & -\frac{3}{\sqrt{34}} & -\frac{3}{\sqrt{34}} & \frac{7}{\sqrt{34}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{3}{\sqrt{17}} & \frac{2}{\sqrt{17}} & \frac{2}{\sqrt{17}} & \frac{1}{\sqrt{17}} \\ \hline 0 & 0 & 0 & 1 \end{array} \right) \\
 &= \sqrt{2} \begin{pmatrix} -21 & 4 & 4 & 2 \\ 9 & -21 & 6 & 3 \\ 21 & 14 & -13 & 7 \\ 3 & 2 & 2 & -26 \end{pmatrix}
 \end{aligned}$$

(v) 各点 A, B, C, D, E, F を同次座標で表し，行列 Φ_S をかける；

$$\begin{aligned}
 \Phi_S(A) &= \begin{bmatrix} -3\sqrt{2} \\ 9\sqrt{2} \\ 3\sqrt{2} \\ -15\sqrt{2} \end{bmatrix}, \quad \Phi_S(B) = \begin{bmatrix} 39\sqrt{2} \\ -9\sqrt{2} \\ -39\sqrt{2} \\ -21\sqrt{2} \end{bmatrix}, \quad \Phi_S(C) = \begin{bmatrix} 31\sqrt{2} \\ 33\sqrt{2} \\ -67\sqrt{2} \\ -25\sqrt{2} \end{bmatrix}, \\
 \Phi_S(D) &= \begin{bmatrix} -11\sqrt{2} \\ 51\sqrt{2} \\ -25\sqrt{2} \\ -19\sqrt{2} \end{bmatrix}, \quad \Phi_S(E) = \begin{bmatrix} 8\sqrt{2} \\ 12\sqrt{2} \\ -\frac{25\sqrt{2}}{2} \\ -23\sqrt{2} \end{bmatrix}, \quad \Phi_S(F) = \begin{bmatrix} 20\sqrt{2} \\ 30\sqrt{2} \\ -\frac{103\sqrt{2}}{2} \\ -17\sqrt{2} \end{bmatrix}.
 \end{aligned}$$

(vi) 各点の Φ_S による像を直交座標で表す；

$$\begin{aligned}
 \Phi_S(A) &= \begin{pmatrix} \frac{1}{5} \\ -\frac{3}{5} \\ -\frac{1}{5} \end{pmatrix}, \quad \Phi_S(B) = \begin{pmatrix} -\frac{13}{7} \\ \frac{3}{7} \\ \frac{13}{7} \end{pmatrix}, \quad \Phi_S(C) = \begin{pmatrix} -\frac{31}{25} \\ -\frac{33}{25} \\ \frac{67}{25} \end{pmatrix}, \\
 \Phi_S(D) &= \begin{pmatrix} \frac{11}{19} \\ -\frac{51}{19} \\ \frac{25}{19} \end{pmatrix}, \quad \Phi_S(E) = \begin{pmatrix} -\frac{8}{23} \\ -\frac{12}{23} \\ \frac{25}{46} \end{pmatrix}, \quad \Phi_S(F) = \begin{pmatrix} -\frac{20}{17} \\ -\frac{30}{17} \\ \frac{103}{34} \end{pmatrix}.
 \end{aligned}$$