6.S093 Visual Recognition through Machine Learning Competition

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Acknowledgment: Many slides from David Sontag and Machine Learning Group of UT Austin

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kirkh.deviantart.com

What is Machine Learning???

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According to the Wikipedia,
Machine learning concerns the construction and study of systems that can learn from data.

What is ML?

- Classification
  - From data to discrete classes

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- Regression
  - From data to a continuous value

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  - Comparing items

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- Clustering
  - Discovering structure in data

What is ML?

- Classification
  - From data to discrete classes
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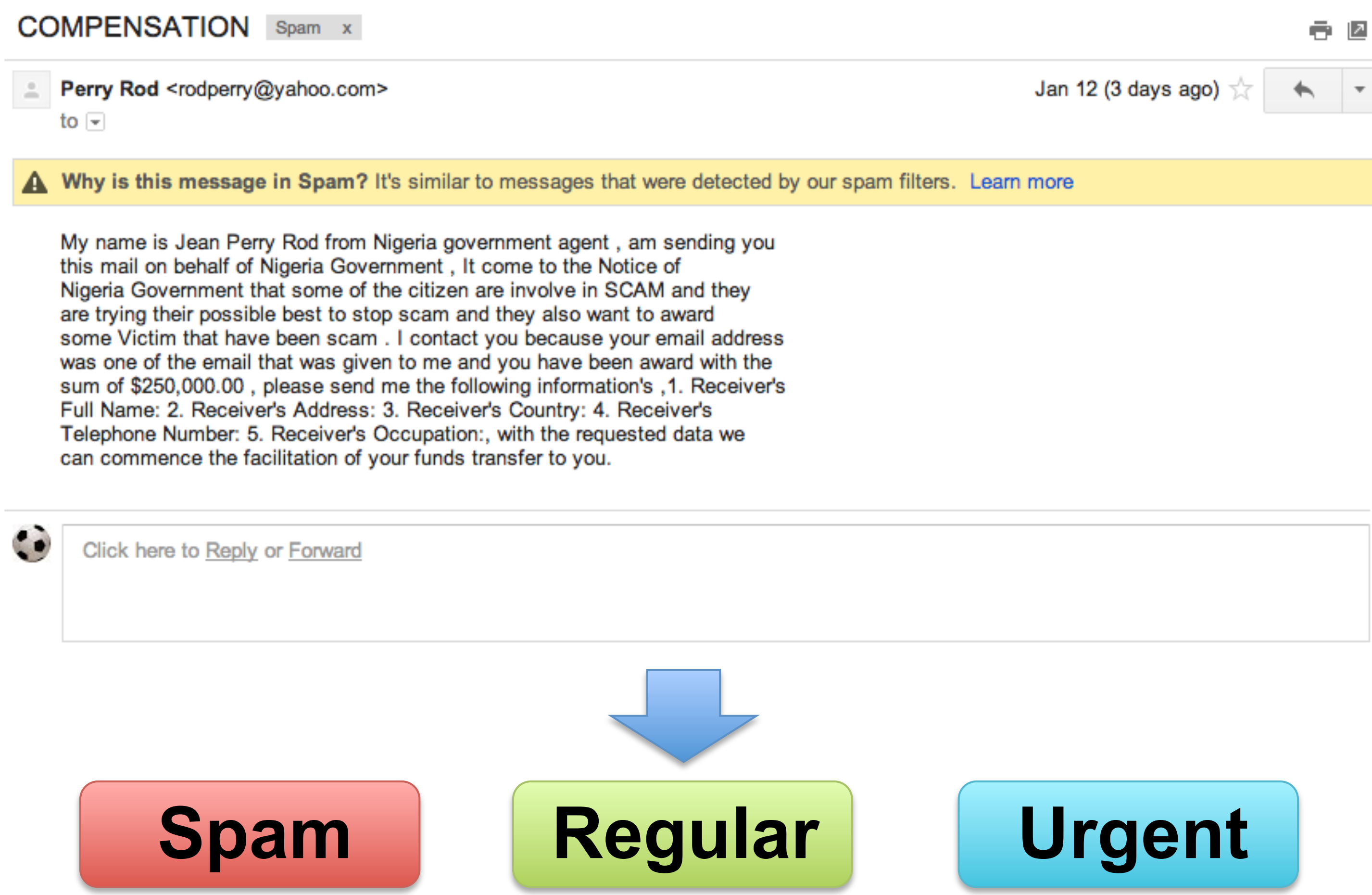
  - From data to a continuous value
- Ranking
  - Comparing items
- Clustering
  - Discovering structure in data
- Others

Classification: data to discrete classes

- Spam filtering



Classification:
data to discrete classes

- Object classification

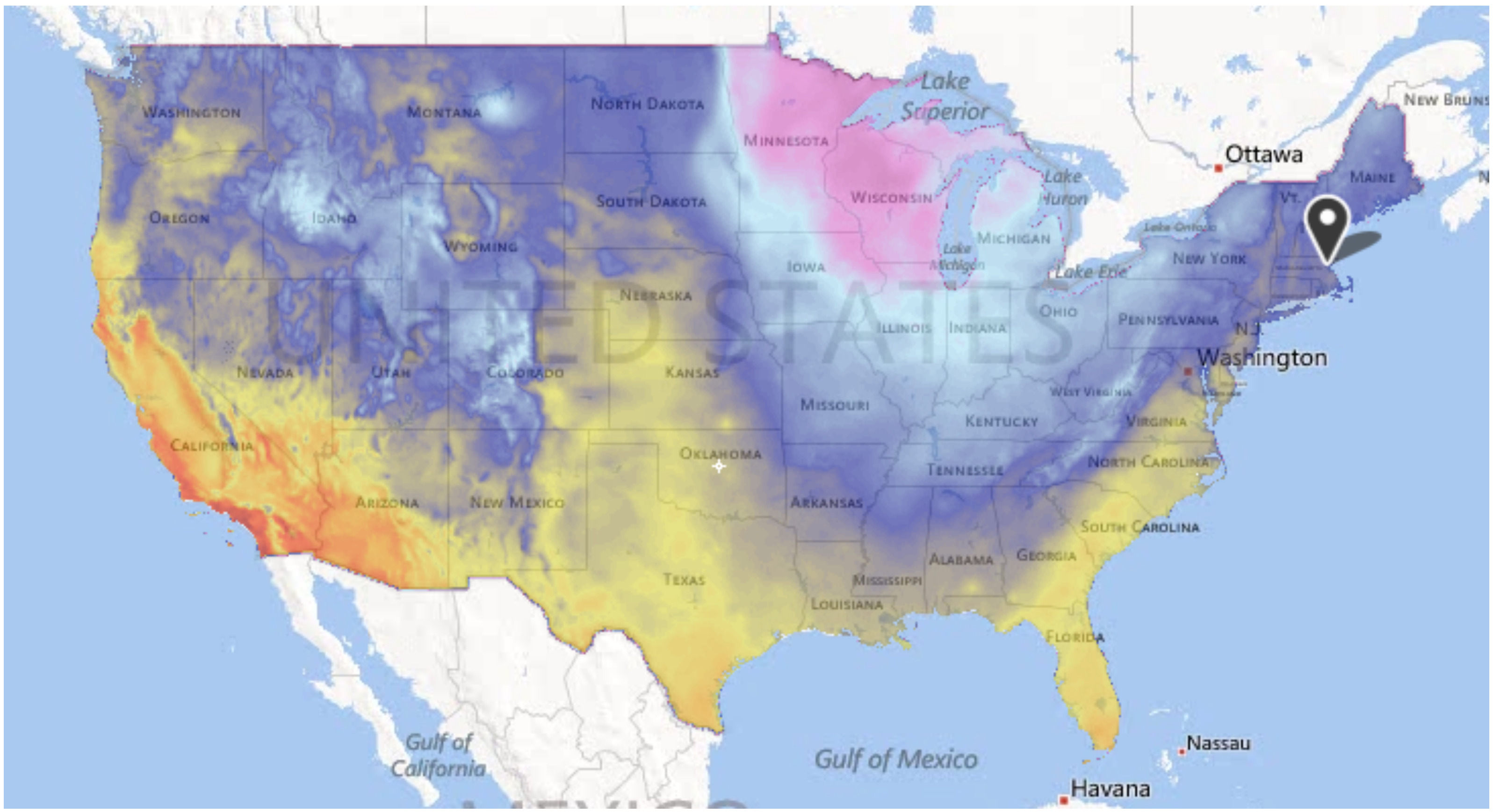
Fries | Hamburger | None

Regression: data to a value

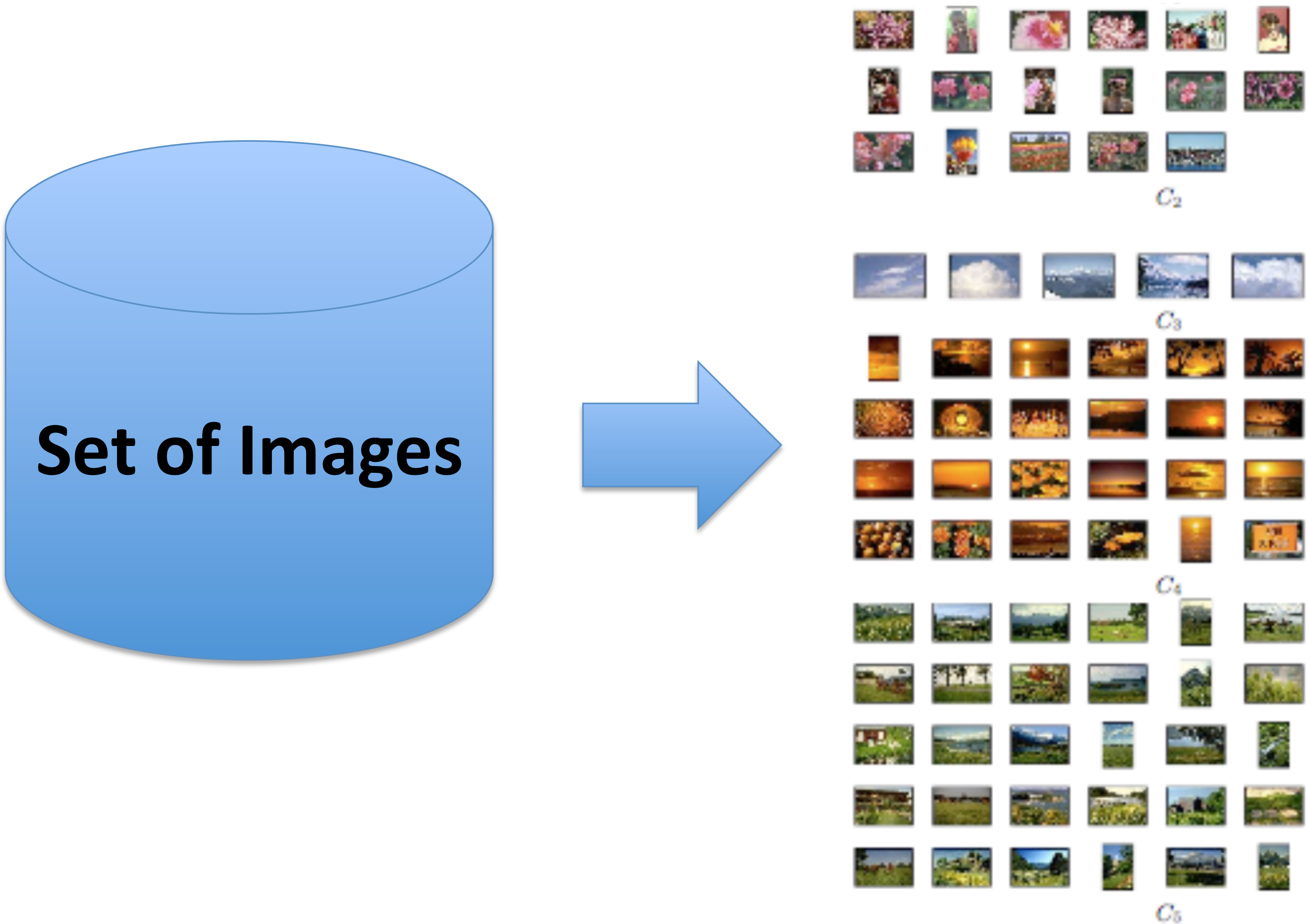
- Stock market

Regression: data to a value

- Weather prediction



Clustering: Discovering structure



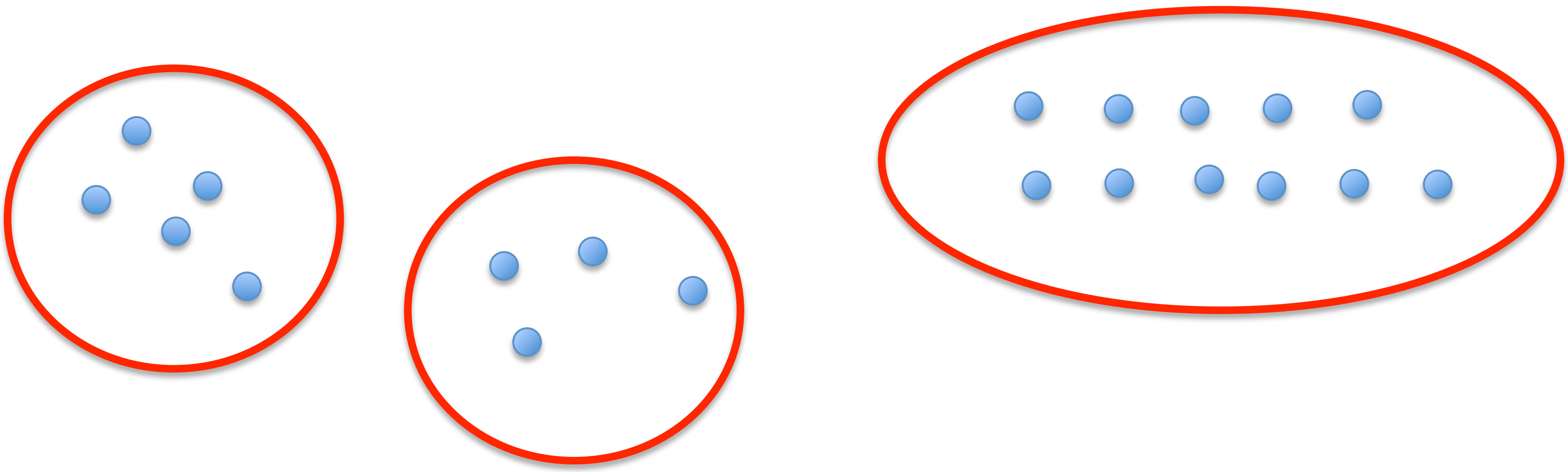
Clustering

Clustering

- Unsupervised learning
- Requires data, but **NO LABELS**
- Detect patterns
  - Group emails or search results
  - Regions of images
- Useful when don’t know what you’re looking for

Clustering

- Basic idea: group together similar instances
- Example: 2D point patterns



Clustering

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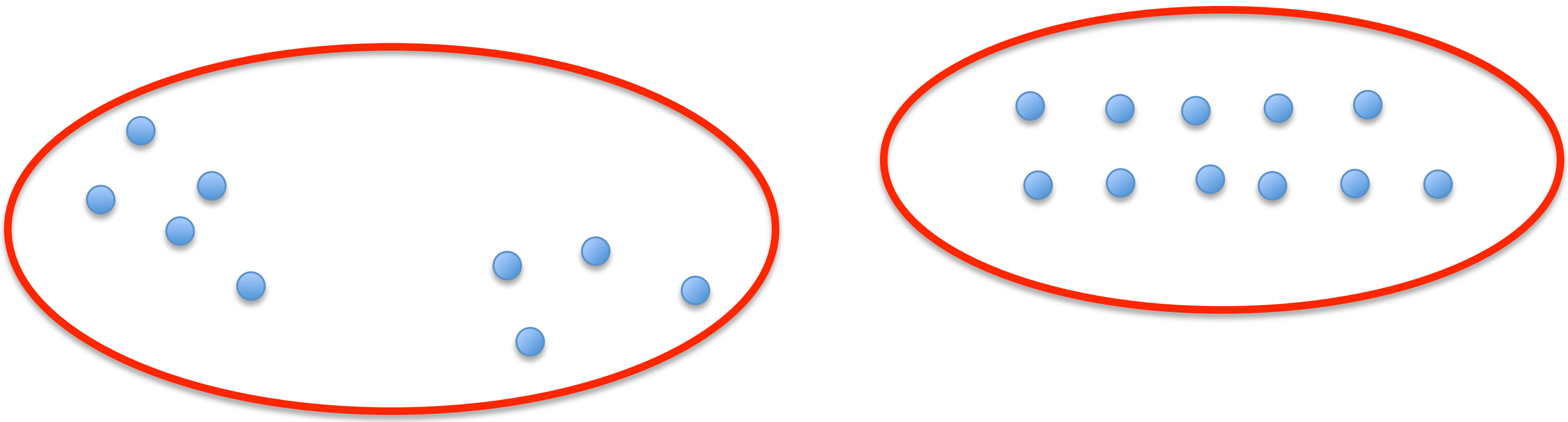
- Example: 2D point patterns



Clustering

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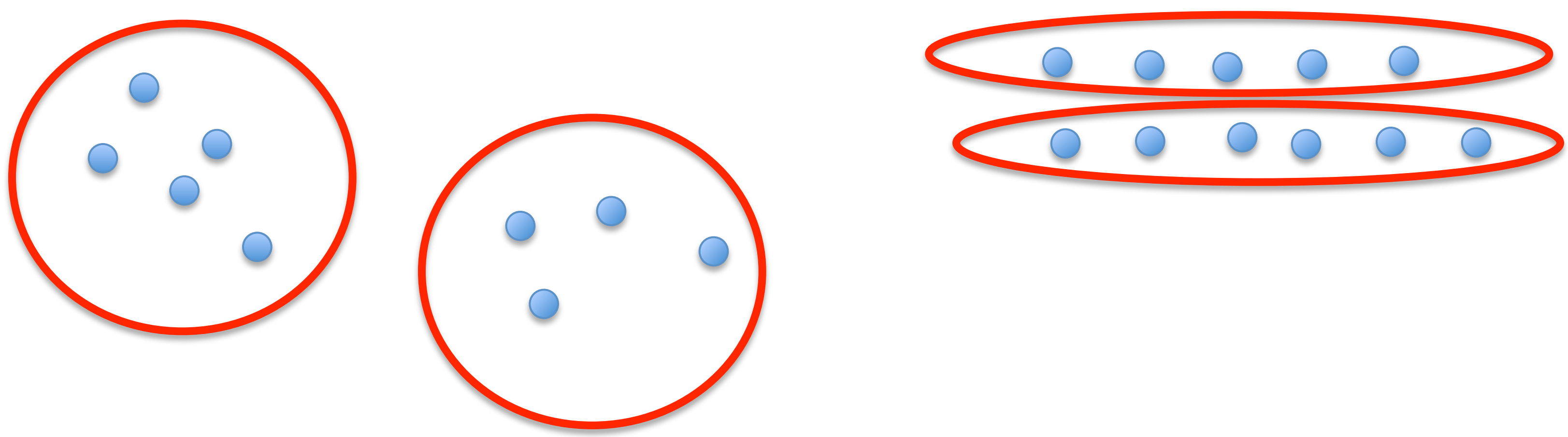
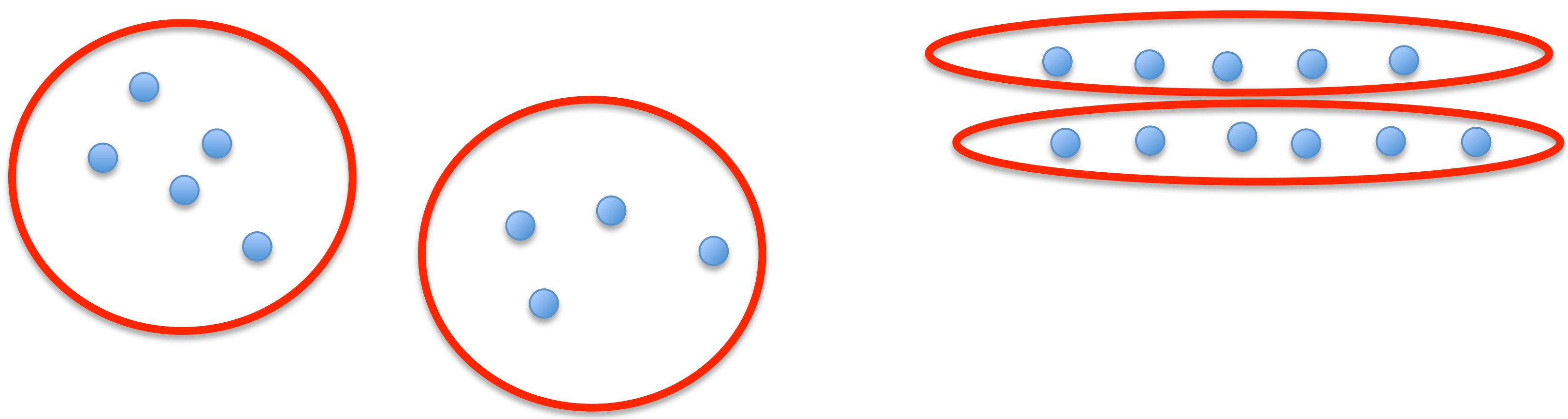
- Example: 2D point patterns



Clustering

- Basic idea: group together similar instances
- Example: 2D point patterns



- What could "similar" mean?
  - One option: small Euclidean distance
  - Clustering is crucially dependent on the measure of similarity (or distance) between "points"

K-Means

- An iterative clustering algorithm
  - Initialize: Pick K random points as cluster centers
  - Alternate:
    - Assign data points to closest cluster center
    - Change the cluster center to the average of its assigned points
  - Stop when no points' assignments change

Animation is from Andrey A. Shabalin's website

K-Means

- An iterative clustering algorithm
  - Initialize: Pick K random points as cluster centers
  - Alternate:
    - Assign data points to closest cluster center

$$S_i^{(t)} = \{x_p : \|x_p - m_i^{(t)}\|^2 \le \|x_p - m_j^{(t)}\|^2 \ \forall j, 1 \le j \le k\},$$

    - Change the cluster center to the average of its assigned points

$$m_i^{(t+1)} = \frac{1}{|S_i^{(t)}|} \sum_{x_j \in S_i^{(t)}} x_j$$

  - Stop when no points' assignments change

Properties of K-means algorithm

- Guaranteed to converge in a finite number of iterations

- Running time per iteration:
  - Assign data points to closest cluster center

    O(KN)
  - Change the cluster center to the average of its assigned points

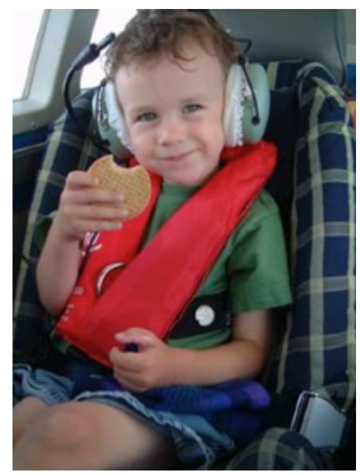
    O(N)

Example: K-Means for Segmentation

K=2

Goal of Segmentation is to partition an image into regions each of which has reasonably homogenous visual appearance.

Original

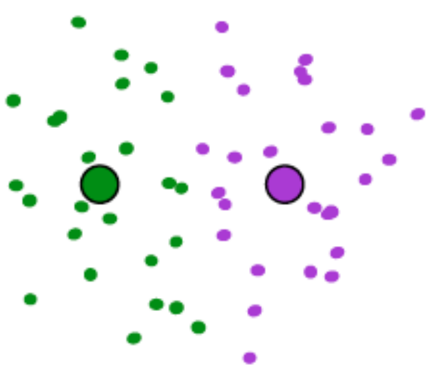


Example: K-Means for Segmentation

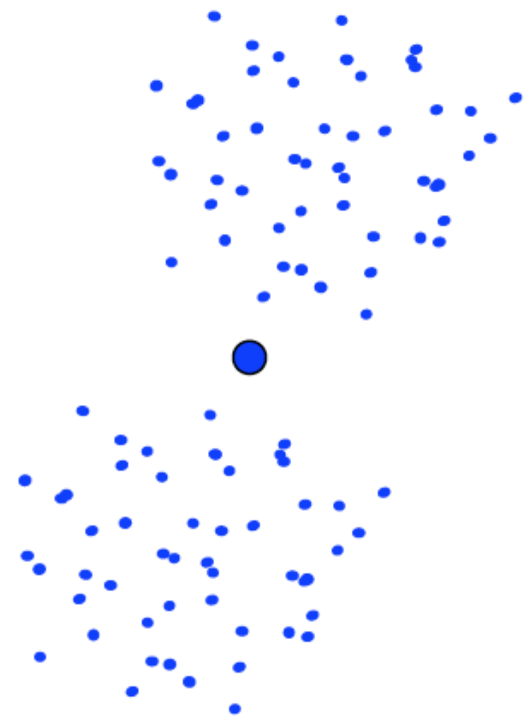
Pitfalls of K-Means

- K-means algorithm is heuristic
  - Requires initial means
  - It does **matter** what you pick!

- K-means can get stuck



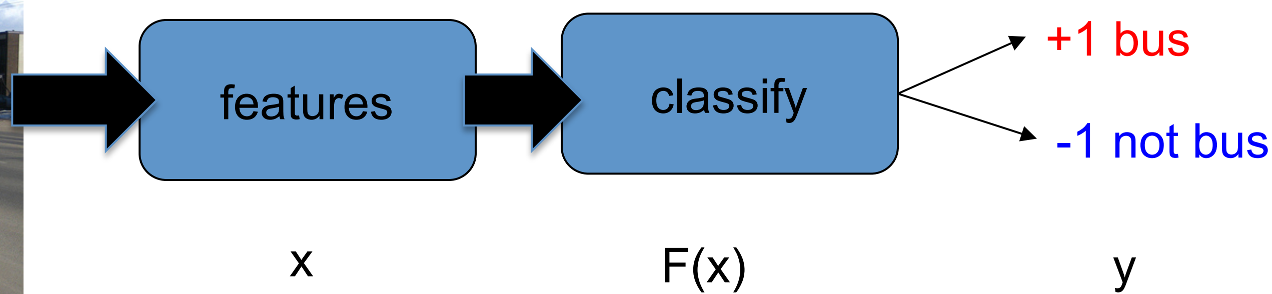
K=1 should be better



K=2 should be better

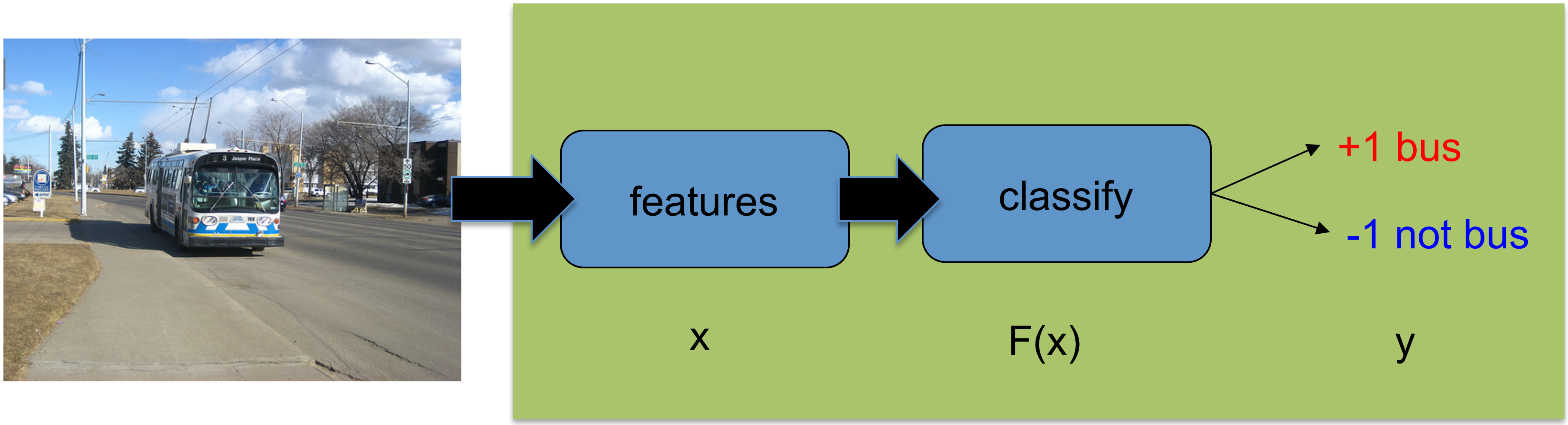
Classification

Typical Recognition System



- Extract features from an image
- A classifier will make a decision based on extracted features

Typical Recognition System



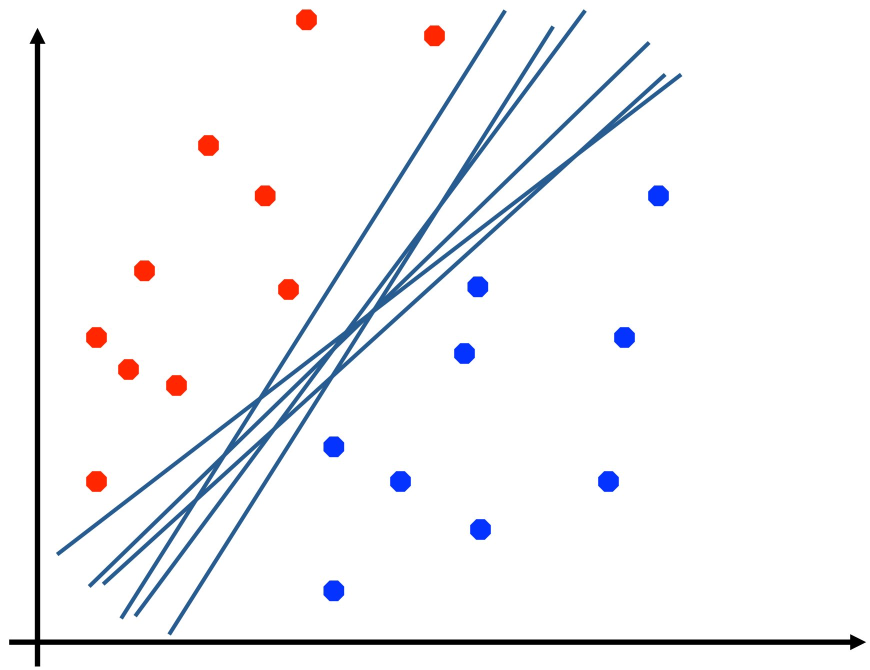
- Extract features from an image
- A classifier will make a decision based on extracted features

Classification

- Supervised learning
- Requires data, AND **LABELS**
- Useful when you know what you're looking for

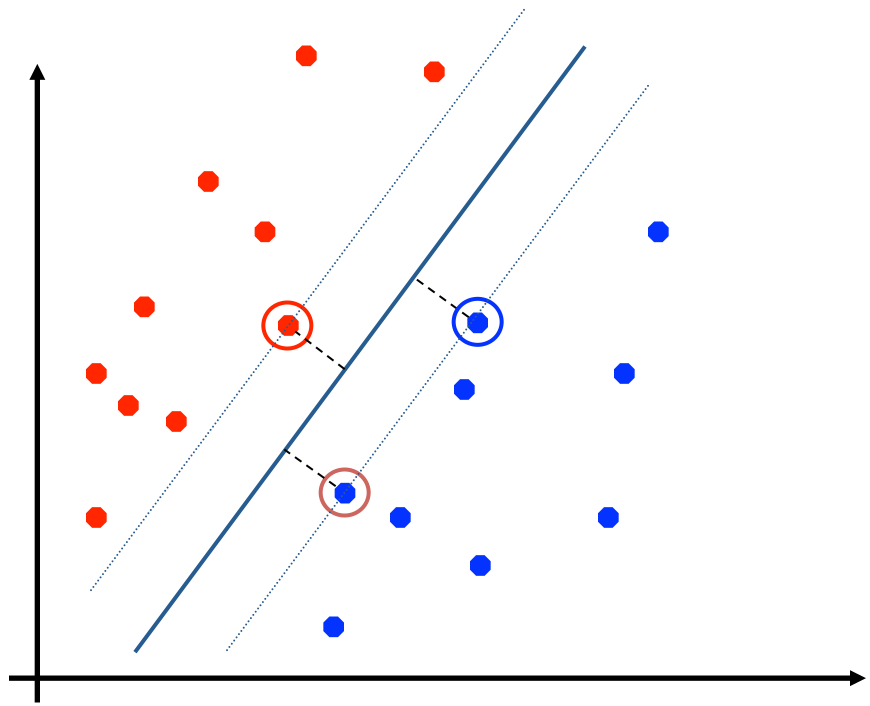
Linear classifier

- Which of these linear separators is optimal?



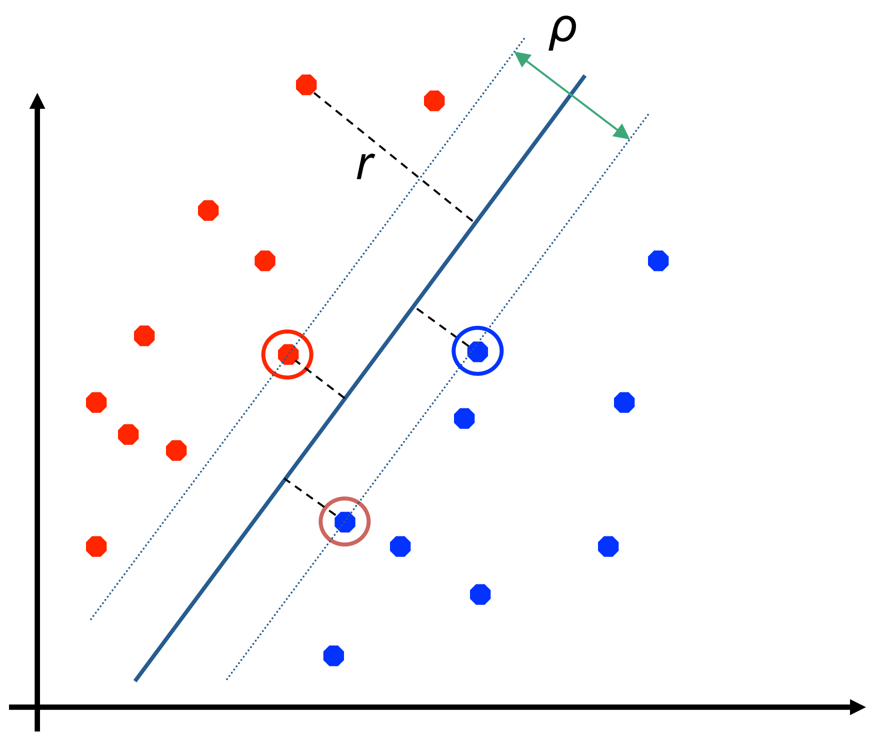
Maximum Margin Classification

- Maximizing the margin is good according to intuition and PAC theory.
- Implies that only support vectors matter; other training examples are ignorable.



Classification Margin

- Distance from example $\mathbf{x}_i$ to the separator is $r = \frac{\mathbf{w}^T\mathbf{x}_i + b}{\|\mathbf{w}\|}$
- Examples closest to the hyperplane are ***support vectors***.
- ***Margin*** ρ of the separator is the distance between support vectors.



Linear SVM Mathematically

- Let training set $\{(\mathbf{x}_i, y_i)\}_{i=1..n}$, $\mathbf{x}_i \in \mathbf{R}^d$, $y_i \in \{-1, 1\}$ be separated by a hyperplane with margin ρ. Then for each training example $(\mathbf{x}_i, y_i)$:

$$\begin{array}{ll} \mathbf{w}^\mathbf{T}\mathbf{x}_i + b \leq -\rho/2 & \text{if } y_i = -1 \\ \mathbf{w}^\mathbf{T}\mathbf{x}_i + b \geq \rho/2 & \text{if } y_i = 1 \end{array} \quad \Leftrightarrow \quad y_i(\mathbf{w}^\mathbf{T}\mathbf{x}_i + b) \geq \rho/2$$

- For every support vector $\mathbf{x}_s$ the above inequality is an equality. After rescaling $\mathbf{w}$ and b by $\rho/2$ in the equality, we obtain that distance between each $\mathbf{x}_s$ and the hyperplane is $r = \frac{y_s(\mathbf{w}^T\mathbf{x}_s + b)}{\|\mathbf{w}\|} = \frac{1}{\|\mathbf{w}\|}$

- Then the margin can be expressed through (rescaled) $\mathbf{w}$ and b as:

$$\rho = 2r = \frac{2}{\|\mathbf{w}\|}$$

Linear SVMs Mathematically (cont.)

- Then we can formulate the *quadratic optimization problem:*

Find **w** and b such that

$$\rho = \frac{2}{\|\mathbf{w}\|} \text{ is maximized}$$

and for all $(\mathbf{x}_i, y_i)$, $i=1..n$: $y_i(\mathbf{w}^T\mathbf{x}_i + b) \geq 1$

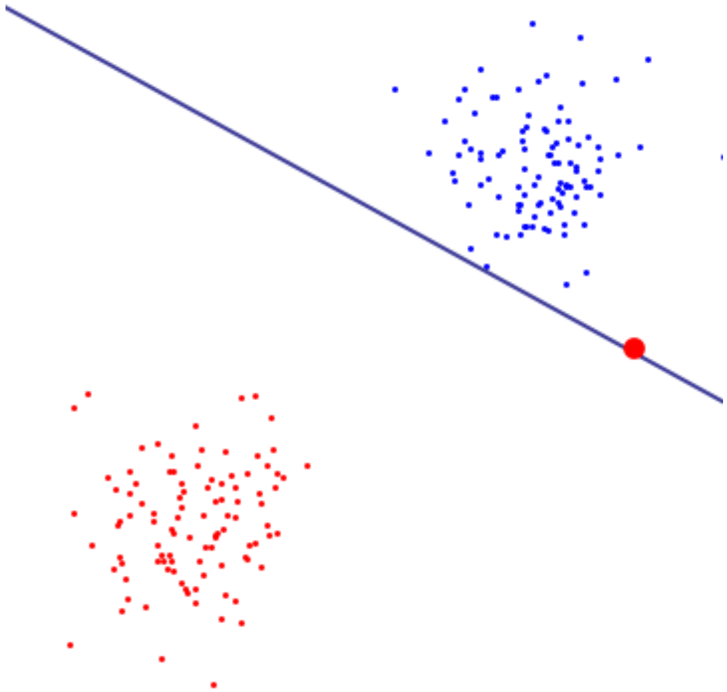
Find **w** and b such that

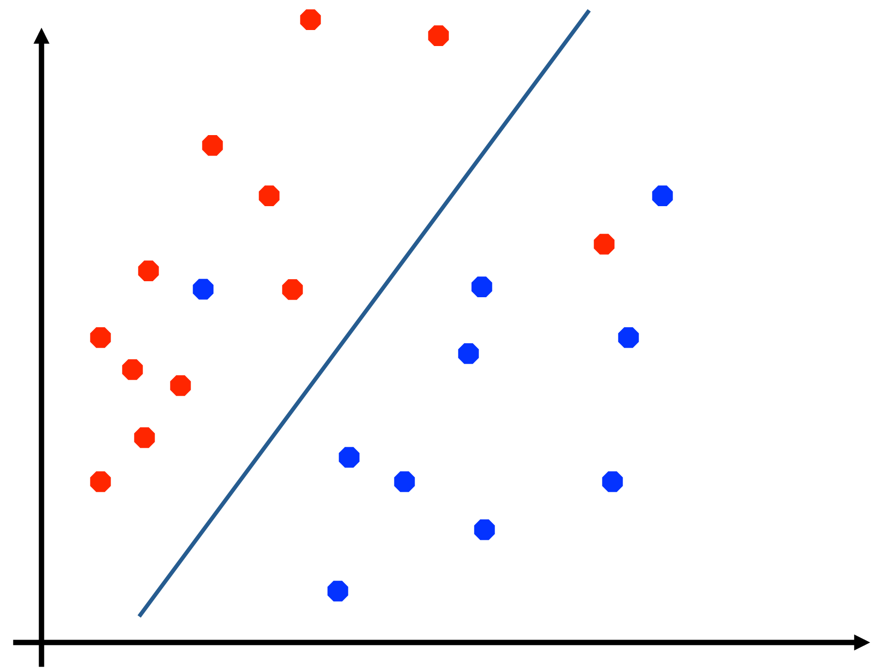
$\mathbf{\Phi}(\mathbf{w}) = ||\mathbf{w}||^2 = \mathbf{w}^T\mathbf{w}$ is minimized

and for all $(\mathbf{x}_i, y_i)$, $i=1..n$: $y_i(\mathbf{w}^T\mathbf{x}_i + b) \geq 1$

Is this perfect?

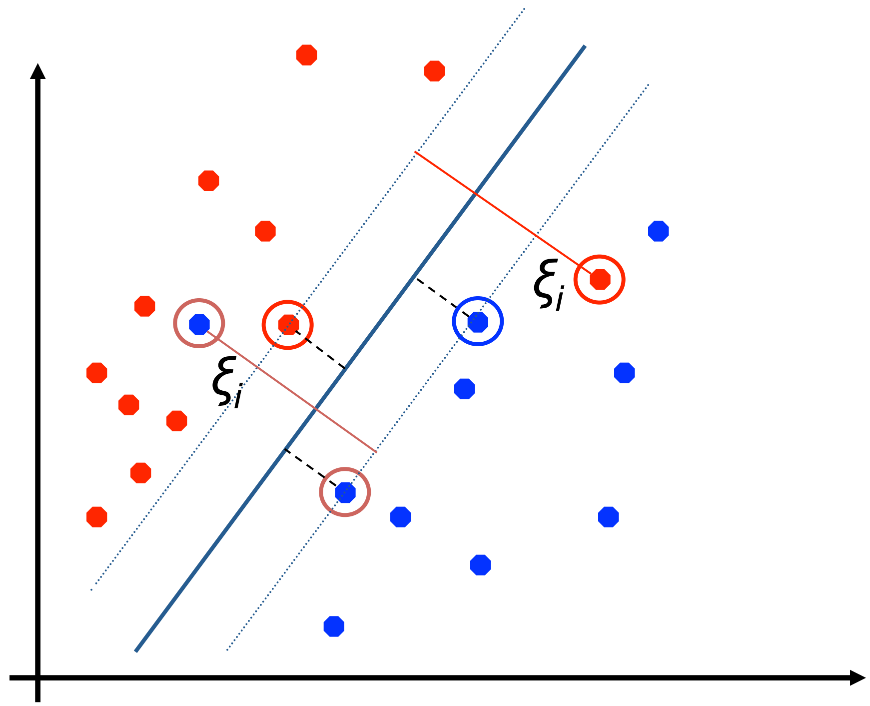
Is this perfect?





Soft Margin Classification

- What if the training set is not linearly separable?
- *Slack variables* ξ_i can be added to allow misclassification of difficult or noisy examples, resulting margin called *soft*.



Soft Margin Classification Mathematically

- The old formulation:

> Find **w** and b such that
> $\mathbf{\Phi}(\mathbf{w}) = \mathbf{w}^T\mathbf{w}$ is minimized
> and for all $(\mathbf{x}_i, y_i)$, $i=1..n$: $y_i(\mathbf{w}^T\mathbf{x}_i + b) \geq 1$

- Modified formulation incorporates slack variables:

> Find **w** and b such that
> $\mathbf{\Phi}(\mathbf{w}) = \mathbf{w}^T\mathbf{w} + C\Sigma\xi_i$ is minimized
> and for all $(\mathbf{x}_i, y_i)$, $i=1..n$: $y_i(\mathbf{w}^T\mathbf{x}_i + b) \geq 1 - \xi_i$, $\xi_i \geq 0$

- Parameter *C* can be viewed as a way to control overfitting: it "trades off" the relative importance of maximizing the margin and fitting the training data.

Exercise

- Exercise 1. Implement K-means
- Exercise 2. Play with SVM’s C-parameter