# **Exposure Mapping Function Learning for Peer Effect Estimation**

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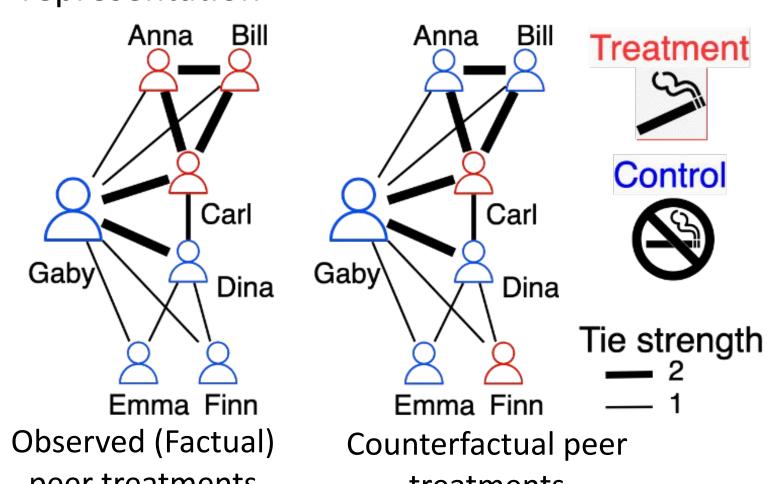
exposure mechanism

cannot be explained totally

with motifs structures only.

#### Introduction

- *Peer effect*: difference in counterfactual outcomes of an individual for different levels of *peer exposure*
- Peer exposure: the extent to which an individual is exposed to the treatments, actions, or behaviors of peers
- Exposure mapping function: maps peer treatments and other relevant contexts to peer exposure representation



peer treatments treatments
Gaby's ego network with observed and counterfactual treatments

Example peer exposures				
Exposure Type	Exposure Value			
	Factual	Counter- factual		
Binary (at least a peer treated)	1	1		
Fraction of treated peers	3/6	2/6		
Linear threshold (40%)	1	0		
Weighted fraction (tie strengths)	4/8	3/8		
Weighted fraction (attribute similarity: female)	1/3	0/3		
Local structure: Clustering coefficient of treated peers	1	0		
Local structure: Structural diversity of treated peers (connected components)	1	2		

Example peer exposures on Gaby for observed and counterfactual treatments.

#### **Research Goal**

To learn the exposure mapping function to capture underlying peer influence mechanisms for robust peer effect estimation.

#### **Causal Inference Problem Setup**

- Attributed network G=(V, ε) with node attributes X,
   edge attributes Z, and N=|V| nodes
- Treatment random variables  $\mathbf{T} = \langle T_1, ..., T_i, ..., T_N \rangle$ with assignments  $\mathbf{T} = \langle T_1, ..., T_i, ..., T_N \rangle$
- Y outcome variable for node v
- Individual Peer effect (IPE) for node  $v_i$  due to peer treatments  $T_{N(i)} = \pi_{N(i)}$  vs  $T_{N(i)} = \pi'_{N(i)}$  on outcome  $Y_i$  conditioned on *effect modifiers*  $C_i$

$$\begin{split} \delta_{i} &= E[Y_{i}(T_{i} = \pi_{i}, P_{N(i)} = \phi_{e}(\pi_{N(i)}, G, Z)) | \mathbf{C}_{i}] - \\ &= E[Y_{i}(T_{i} = \pi_{i}, P_{N(i)} = \phi_{e}(\pi'_{N(i)}, G, Z) | \mathbf{C}_{i}], \text{ where} \end{split}$$

- P<sub>N(i)</sub> is random variable for peer exposure
- $\circ \varphi$  is an exposure mapping function that maps
- $\circ$   $C_i = \phi_f(G, X, Z)$  captures confounders and effect modifiers and  $\phi_f$  is feature mapping function
- After standard causal inference assumptions, peer effects can be estimated as:

$$\begin{split} \delta_{i} &= E[Y_{i} | T_{i} = \pi_{i}, P_{N(i)} = \phi_{e}(\pi_{N(i)}, G, Z), C_{i}] - \\ &E[Y_{i} | T_{i} = \pi_{i}, P_{N(i)} = \phi_{e}(\pi'_{N(i)}, G, Z), C_{i}] \end{split}$$

## **Experimental Setup**

#### **Evaluation:**

Setting: Observed peer treatments versus flipped peer treatments

Metric: Precision in the estimation of heterogeneous effect (PEHE)

$$\mathbf{\epsilon}_{\mathsf{PEHE}} = \sqrt{rac{1}{N} \sum_i (\delta_i - \hat{\delta_i})^2}$$

#### **Baselines:**

Handling influence mechanisms due to local neighborhood structures

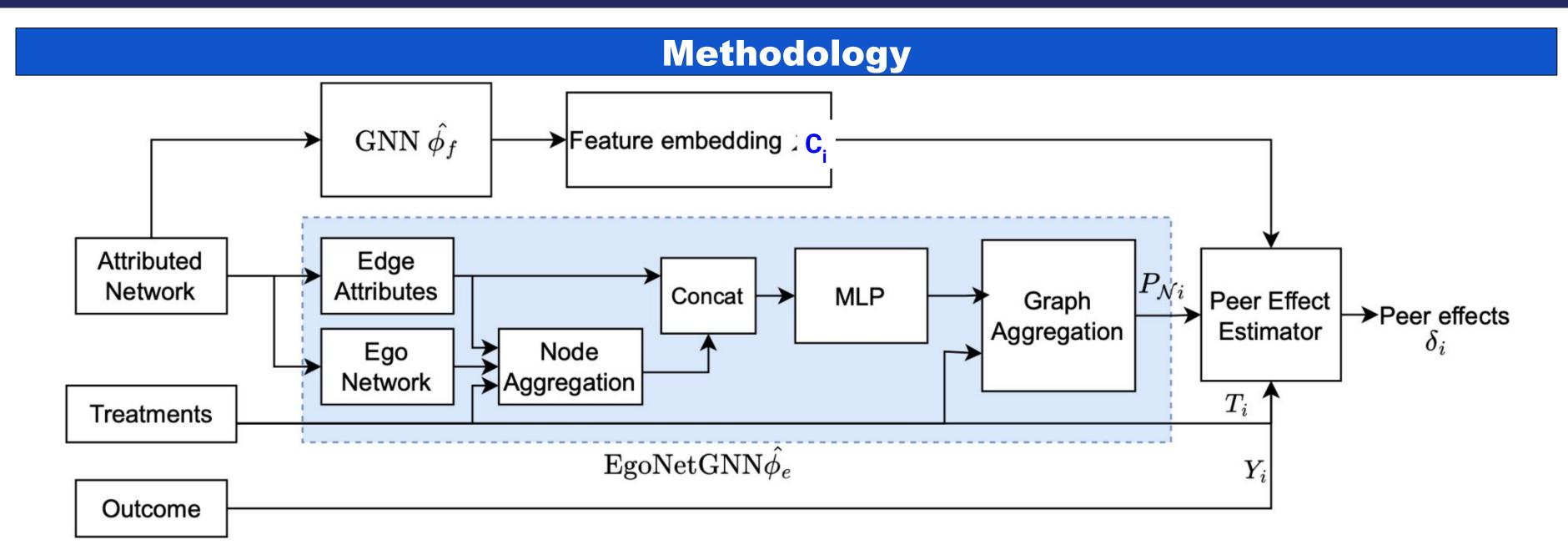
- GNN\_TARNet\_MOTIFS (Yuan et al., WWW'21)
- INE\_TARNet (Adhikari and Zheleva, Machine Learning Journal 2025)

Potentially misspecified peer exposure mapping

- *1GNN\_HSIC* (Ma et al., AISTATS'21)
- **DWR** (Zhao et al., TKDD'24)

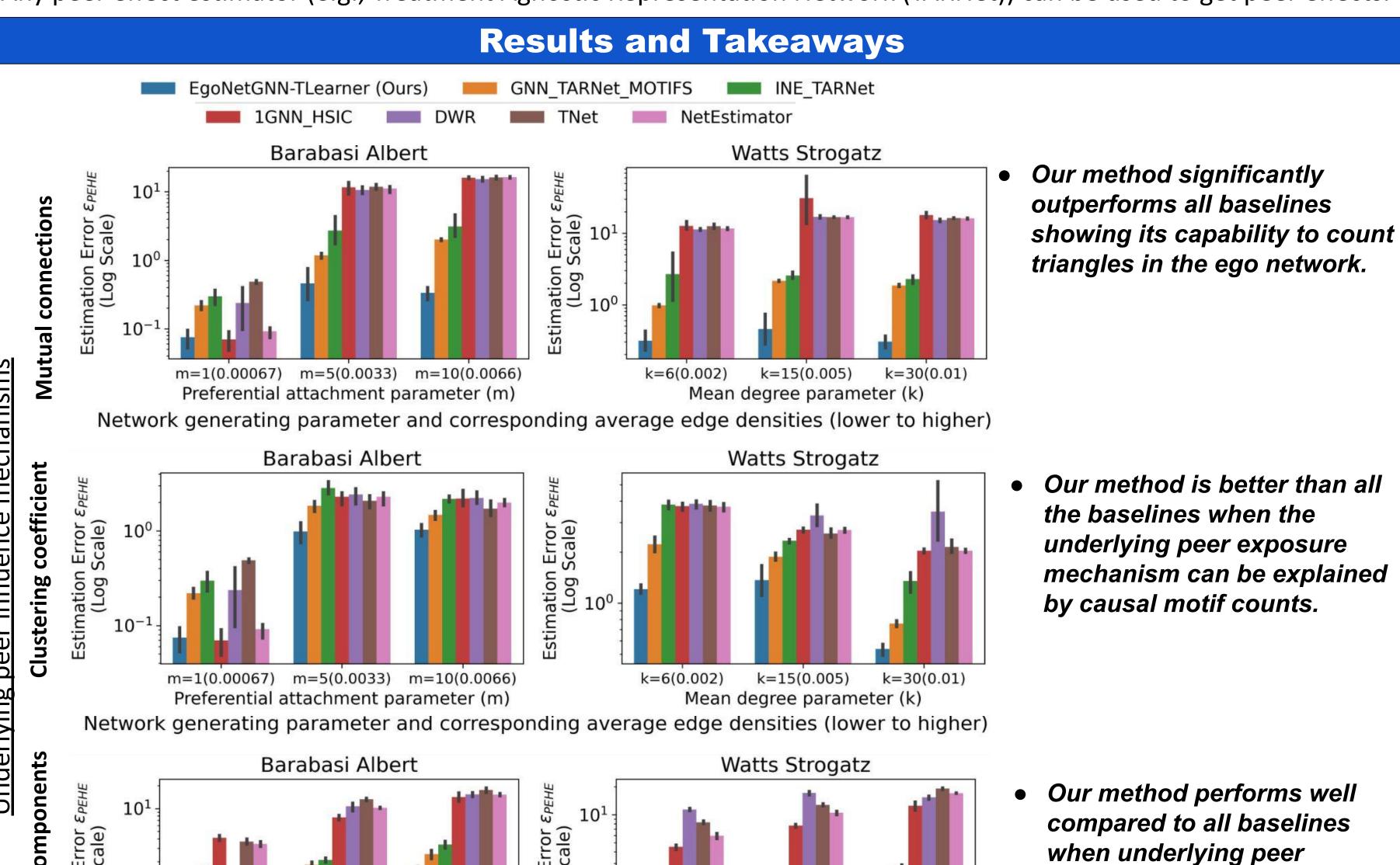
Homogeneous exposure mapping based on fraction of treated peers

- TNet (Chen et al., ICML'24)
- *NetEstimator* (Jiang and Sun, CIKM'22)



An overview of the proposed EgoNetGNNmodel to learn exposure mapping function for peer effect estimation

- EgoNetGNN extracts ego networks, for each node, with peer treatments as node attributes and existing edge attributes.
- Node-level aggregation, encoder MLP, and graph-level aggregation capture relevant local neighborhood contexts.
- Any peer effect estimator (e.g., Treatment Agnostic Representation Network (TARNet)) can be used to get peer effects.



Network generating parameter and corresponding average edge densities (lower to higher)

m=1(0.00067) m=5(0.0033) m=10(0.0066)

Preferential attachment parameter (m)

#### Conclusion

k=6(0.002) k=15(0.005)

Mean degree parameter (k)

EgoNetGNN, improves the estimation of peer effects compared to state-of-the-art baselines by learning an exposure mapping function that captures unknown underlying peer influence mechanisms accounting for peer treatments, edge weights and neighborhood structure.

# **Exposure Mapping Function Learning for Peer Effect Estimation**

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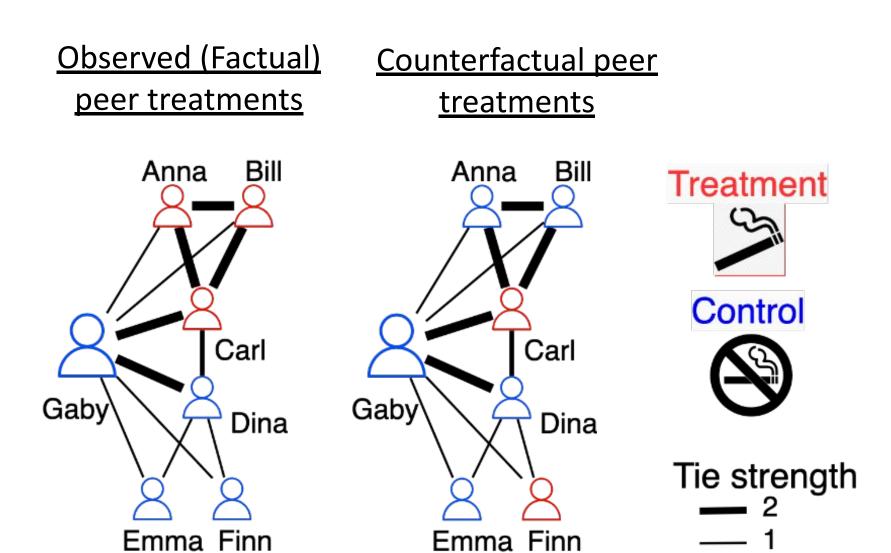
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#### Introduction

- *Peer effect*: difference in counterfactual outcomes of an individual for different levels of *peer exposure*
- **Peer exposure:** aggregated peer treatment, the extent to which an individual is exposed to the treatments or actions of peers
- Exposure mapping function: maps peer treatments and relevant contexts to peer exposure representation

Existing research assumes that the exposure mapping function is known a priori. In reality, it is often unknown and can be misspecified.



#### Variety of possible peer exposures for Gaby's ego network

Exposure mapping	Peer exposure		
function	Factual	Counterfactual	
Binary (at least one peer treated)	1	1	
Fraction of treated peers	3/6	2/6	
Linear threshold (40%)	1	0	
Weighted fraction (tie strengths)	4/8	3/8	
Weighted fraction (attribute similarity: female)	1/3	0/3	
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#### **Research Goal**

To learn the exposure mapping function to capture underlying peer influence mechanisms for robust peer effect estimation.

#### **Causal Inference Problem Setup**

- Attributed network G=(V, ε) with node attributes X,
   edge attributes Z, and N=|V| nodes
- Treatment random variables  $T = < T_1, ..., T_i, ..., T_N >$  with assignments  $\mathbf{\pi} = < \pi_1, ..., \pi_i, ..., \pi_N >$
- Outcome **Y**=<Y<sub>1</sub>, ..., Y<sub>i</sub>, ..., Y<sub>N</sub>>
- Individual Peer effect (IPE) for node  $v_i$  due to peer treatments  $T_{N(i)} = \pi_{N(i)}$  vs  $T_{N(i)} = \pi'_{N(i)}$  on outcome  $Y_i$  conditioned on *effect modifiers*  $C_i$

$$\delta_{i} = E[Y_{i}(T_{i}=\pi_{i}, P_{N(i)}=\phi_{e}(\pi_{N(i)}, G, \mathbf{Z})) | \mathbf{C}_{i}] - E[Y_{i}(T_{i}=\pi_{i}, P_{N(i)}=\phi_{e}(\pi'_{N(i)}, G, \mathbf{Z}) | \mathbf{C}_{i}], \text{ where}$$

- P<sub>N(i)</sub> is a random variable for peer exposure
- φ is an exposure mapping function
- $\circ$   $C_i = \phi_f(G, X, Z)$  is a feature mapping function that captures confounders and effect modifiers
- Assuming unconfoundedness, consistency, and positivity, peer effects can be estimated as:

$$\delta_{i} = E[Y_{i} | T_{i} = \pi_{i}, P_{N(i)} = \phi_{e}(\pi_{N(i)}, G, Z), C_{i}] - E[Y_{i} | T_{i} = \pi_{i}, P_{N(i)} = \phi_{e}(\pi'_{N(i)}, G, Z), C_{i}]$$

#### **Experimental Setup**

#### **Evaluation:**

Setting: Observed peer treatments versus flipped peer treatments

Metric: Precision in the estimation of heterogeneous effect (PEHE)

$$oldsymbol{arepsilon}_{\mathsf{PEHE}} = \sqrt{rac{1}{N} \sum_i (\delta_i - \hat{\delta_i})^2}$$

#### **Baselines:**

Handling influence mechanisms due to local neighborhood structures

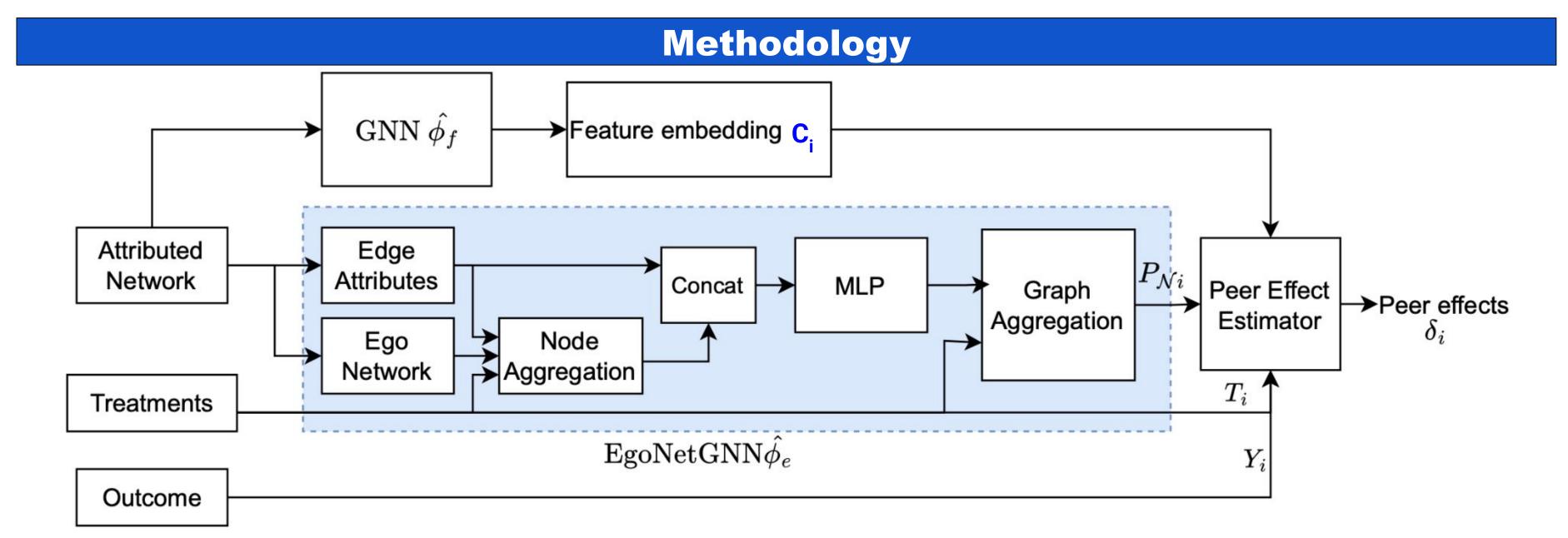
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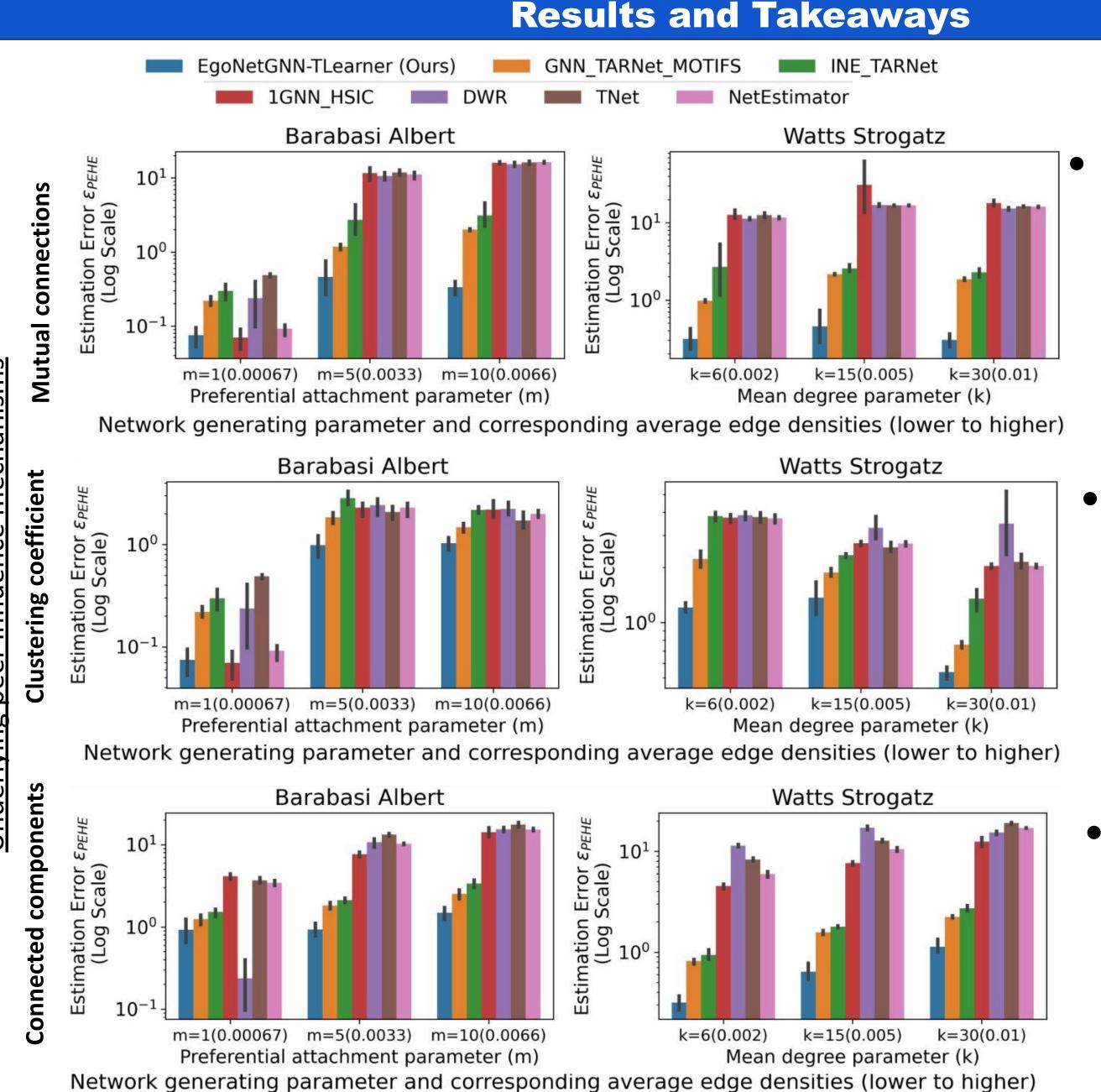
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An overview of the proposed EgoNetGNNmodel to learn exposure mapping function for peer effect estimation

- EgoNetGNN extracts ego networks, for each node, with peer treatments as node attributes and existing edge attributes.
- Node-level aggregation, encoder MLP, and graph-level aggregation capture relevant local neighborhood contexts.
- Any peer effect estimator (e.g., Treatment Agnostic Representation Network (TARNet)) can be used to get peer effects.



• EgoNetGNN significantly outperforms all baselines showing its capability to count triangles in the ego network.

• EgoNetGNN is better than all the baselines when the underlying peer exposure mechanism can be explained by causal motif counts.

• EgoNetGNN performs well compared to all baselines when underlying peer exposure mechanism cannot be explained totally with motif structures only.

#### Conclusion

EgoNetGNN improves the estimation of peer effects compared to state-of-the-art baselines by learning an exposure mapping function that captures unknown underlying peer influence mechanisms accounting for peer treatments, unknown edge weights and neighborhood structure.

Exposure mapping	Peer exposure		
function	Factual	Counterfactual	
Binary (at least one peer treated)	1	1	
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#### Introduction

- *Interference* refers to the phenomenon in which the actions of peers in a network can influence an individual's outcome.
- **Peer effect** refers to the difference in counterfactual outcomes of an individual for different levels of peer exposure.
- *Peer exposure* captures the extent to which an individual is exposed to the treatments, actions, or behaviors of peers.
- Peer effect *estimation* necessitates determining how to represent peer exposure.
- Exposure mapping is a function that maps peer treatments and other contexts to peer exposure, a representation that summarizes exposure to peer treatments, reduces high dimensionality, and is invariant to irrelevant contexts.

#### **Research Goal**

The underlying peer influence mechanism, and hence, the best representation of peer exposure, is unknown. Our work aims to learn the exposure mapping function to capture underlying peer influence mechanisms for robust peer effect estimation.

#### **Causal Inference Problem Setup**

- Network  $G=(V, \mathcal{E})$  with N=|V| nodes with node attributes X and edge attributes Z
- Treatment random variables  $T = < T_1, ..., T_i, ..., T_N >$  with assignments  $\mathbf{T} = \langle \Pi_1, ..., \Pi_i, ..., \Pi_N \rangle$
- Y, outcome variable for node v,
- Individual Peer effect (IPE) for node v, due to peer treatments  $T_{N(i)} = \pi_{N(i)}$  vs  $T_{N(i)} = \pi'_{N(i)}$  on outcome Y for effect modifiers C.

$$\begin{split} \boldsymbol{\delta}_{i} &= E[Y_{i}(T_{i} = \boldsymbol{\pi}_{i}, P_{N(i)} = \boldsymbol{\phi}_{e}(\boldsymbol{\pi}_{N(i)}, G, \boldsymbol{Z})) \mid \boldsymbol{C}_{i}] - \\ &E[Y_{i}(T_{i} = \boldsymbol{\pi}_{i}, P_{N(i)} = \boldsymbol{\phi}_{e}(\boldsymbol{\pi'}_{N(i)}, G, \boldsymbol{Z}) \mid \boldsymbol{C}_{i}], \text{ where} \end{split}$$

- Y<sub>i</sub>(...) denotes counterfactual outcome
- $\circ$  P<sub>N(i)</sub> is random variable for peer exposure and  $\varphi_{\alpha}$  is an exposure mapping function that maps peer treatments and other contexts to peer exposure value
- $\circ$   $C_i = \phi_f(G, X, Z)$  captures confounders and effect modifiers
- After standard causal inference assumptions, peer effects can be estimated as follows:

$$\begin{split} \boldsymbol{\delta}_{i} &= E[Y_{i} | T_{i} = \boldsymbol{\pi}_{i}, P_{N(i)} = \boldsymbol{\phi}_{e}(\boldsymbol{\pi}_{N(i)}, G, \boldsymbol{Z}), \boldsymbol{C}_{i}] - \\ &E[Y_{i} | T_{i} = \boldsymbol{\pi}_{i}, P_{N(i)} = \boldsymbol{\phi}_{e}(\boldsymbol{\pi'}_{N(i)}, G, \boldsymbol{Z}), \boldsymbol{C}_{i}] \end{split}$$

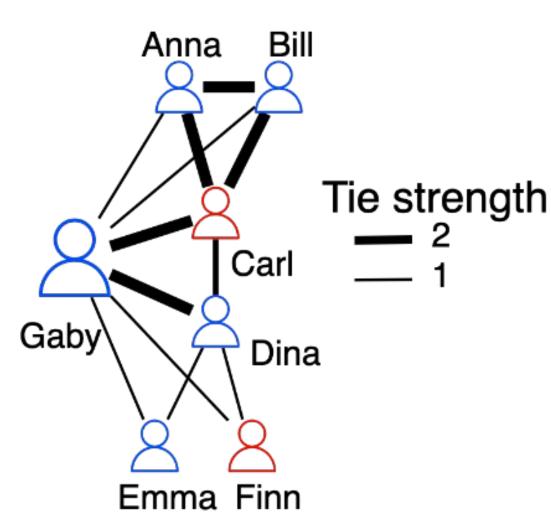
#### Methodology GNN $\hat{\phi}_f$ ➤ Feature embedding C Attributed Edge **Attributes** Network Concat Aggregation Estimator Node Ego Aggregation **Treatments** EgoNetGNN $\hat{\phi_e}$ Outcome

<u>An overview of the proposed EgoNetGNNmodel to learn exposure mapping function for peer effect estimation</u>

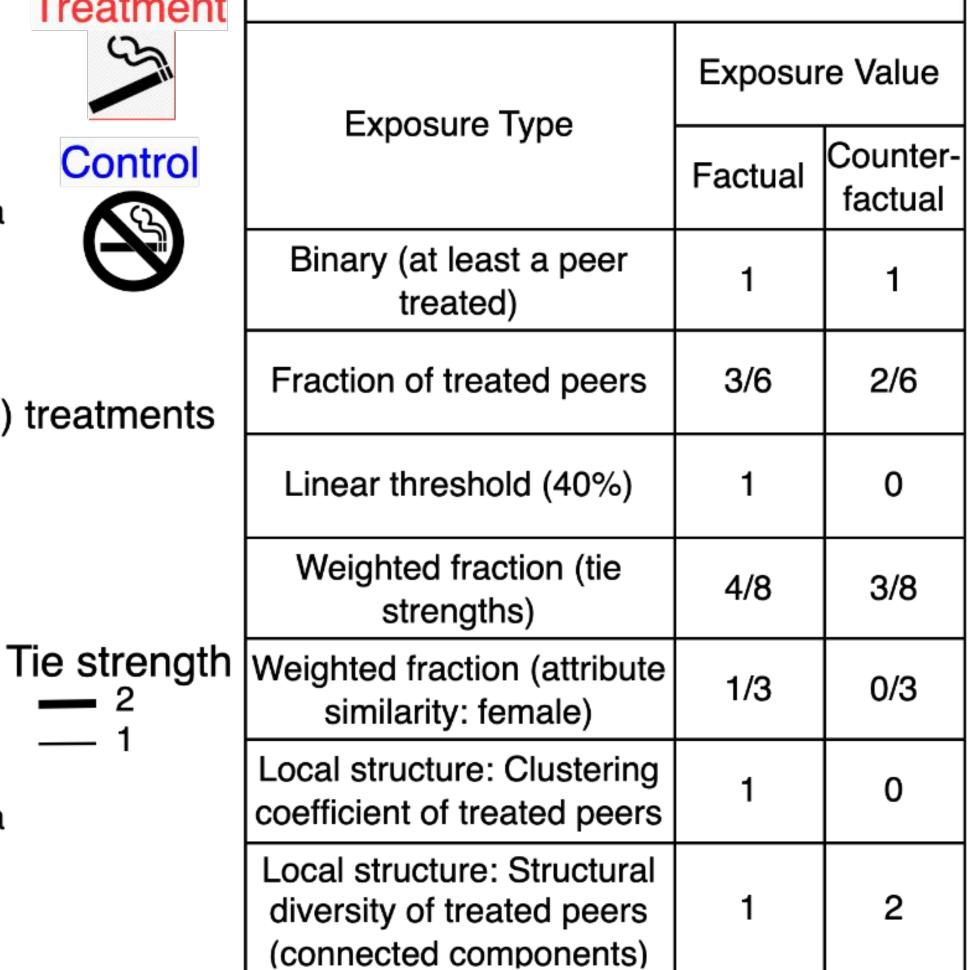
- EgoNetGNN extracts ego networks, for each node, with peer treatments as node attributes and existing edge attributes.
- Node-level aggregation, encoder MLP, and graph-level aggregation capture relevant local neighborhood contexts.
- Any peer effect estimator (e.g., Treatment Agnostic Representation Network (TARNet)) can be used to get peer effects.

# Control Emma Finn

### Factual (observed) treatments



Counterfactual treatments Gaby's ego network with observed and counterfactual treatments.



Example peer exposures

Example peer exposures on Gaby for the observed and counterfactual treatments.

## **Experimental Setup**

#### **Datasets (Synthetic networks):**

- Barabasi Albert (BA)
- Watts Strogatz (WS)
- Stochastic Block (SB)

#### **Evaluation:**

Setting: Observed peer treatments versus flipped peer treatments Metric: Precision in the estimation of heterogeneous effect (PEHE)

ε<sub>PEHE</sub>=

#### **Baselines:**

Handling influence mechanisms due to locgl he ghoorhood structures

- **GNN\_TARNet\_MOTIFS** (Yuan et al., WWW'21)
- INE\_TARNet (Adhikari and Zheleva, Machine Learning Journal 2025) Potentially misspecified peer exposure mapping
- **1GNN\_HSIC** (Ma et al., AISTATS'21)
- **DWR** (Zhao et al., TKDD'24) Homogeneous exposure mapping

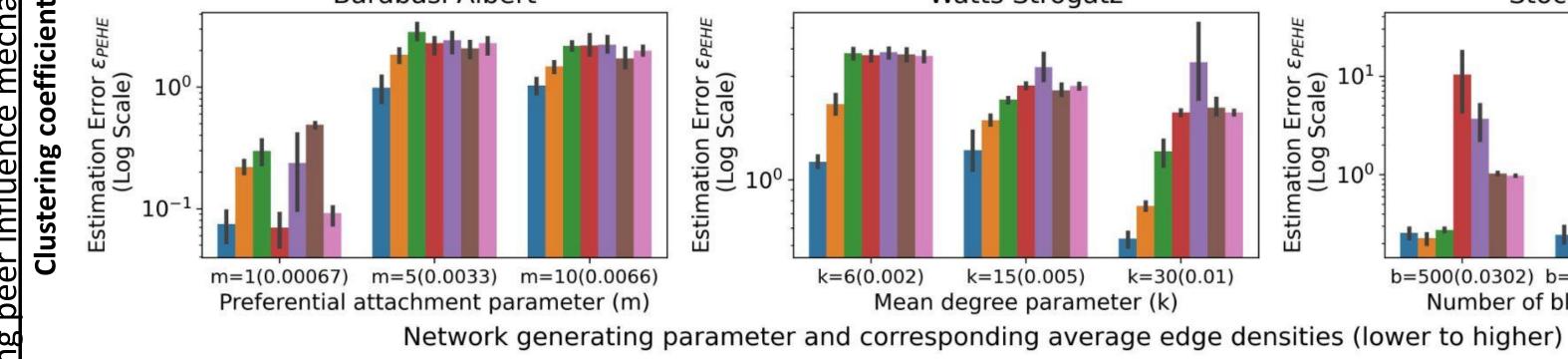
based on fraction of treated peers

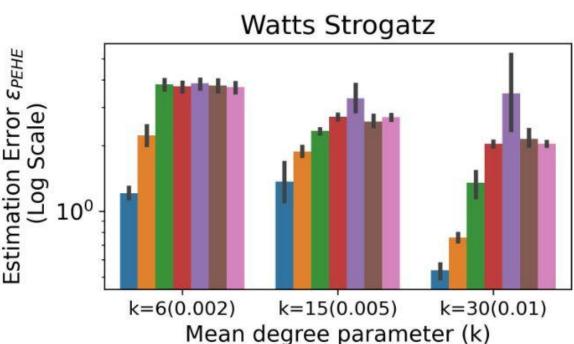
<u>Unde</u> ponents

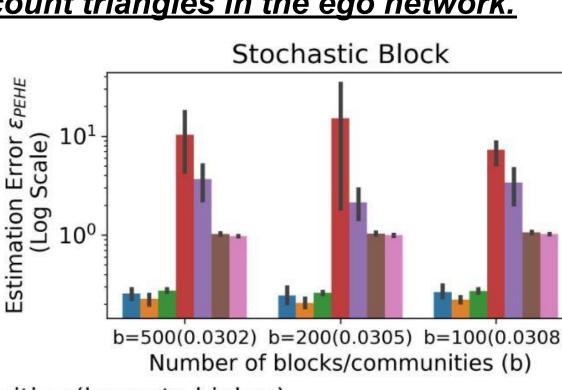
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## **Results and Takeaways** Barabasi Albert Stochastic Block Watts Strogatz k=30(0.01)Number of blocks/communities (b) Preferential attachment parameter (m) Mean degree parameter (k) Network generating parameter and corresponding average edge densities (lower to higher)

Our method significantly outperforms all baselines showing its capability to count triangles in the ego network. Barabasi Albert Stochastic Block Watts Strogatz







Our method is better than or competitive to motif-count based baseline when the underlying peer exposure

mechanism can be explained by causal motif counts. Stochastic Block Barabasi Albert Watts Strogatz m=5(0.0033) m=10(0.0066)k=15(0.005)k=30(0.01)b=500(0.0302) b=200(0.0305) b=100(0.0308)Number of blocks/communities (b) Preferential attachment parameter (m) Mean degree parameter (k)

Network generating parameter and corresponding average edge densities (lower to higher)

Our method performs well compared to all baselines when underlying peer exposure mechanism cannot be explained totally with motifs structures only.