Assignment No.5

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question taken from

quadratic_forms, exercise 2.28

1 QUESTION No 1

Find the equation of the ellipse whose vertices are

$$\begin{bmatrix} \pm 13 \\ 0 \end{bmatrix}$$

and foci are

$$\begin{bmatrix} \pm 5 \\ 0 \end{bmatrix}$$

2 SOLUTION

We have been provided with values for vertices and foci

Let

$$\mathbf{p} = \begin{bmatrix} \pm 13 \\ 0 \end{bmatrix}, \mathbf{q} = \begin{bmatrix} \pm 5 \\ 0 \end{bmatrix}$$

Also, The given coordinate of foci are

$$\begin{bmatrix} \pm 5 \\ 0 \end{bmatrix}$$

In general, the equation of ellipse passing through p and q can be expressed as

$$(X - C)^T D(X - C) = 1$$

Where
C is the centre, D is the diagonal matrix p,q satisfies above equation

$$(p - C)^T D(p - C) = 1$$

$$(q - C)^T D(p - C) = 1$$

which can be simplified as

$$2(\mathbf{p} - \mathbf{q})^T DC = p^T D(\mathbf{p} - \mathbf{q})^T Dq$$

Using identity

$$2(\mathbf{p} - \mathbf{q})^T DC = (\mathbf{p} - \mathbf{q})^T D(\mathbf{p} + \mathbf{q})^T Dq$$

$$\Rightarrow (\mathbf{p} - \mathbf{q})^T D(2C - (\mathbf{p} + \mathbf{q}) = 0$$

We have values of (p+q), The value of m i.e (p-q) and C

$$\mathbf{p} - \mathbf{q} = \begin{bmatrix} \pm 8 \\ 0 \end{bmatrix}$$

$$\mathbf{p} + \mathbf{q} = \begin{bmatrix} \pm 18 \\ 0 \end{bmatrix}$$

We know that

$$foci = p = \begin{bmatrix} \pm\beta \\ 0 \end{bmatrix} = \begin{bmatrix} \pm C \\ 0 \end{bmatrix}$$

Thus

$$C = 5$$

C can parametrically be expressed as

$$C = 1/2[\mathbf{p} + \mathbf{q} + KD^{-1}m] \qquad (2.0.1)$$

Where

$$K = constant$$

And

$$(\mathbf{p} - \mathbf{q})^T m = 0$$

Substituting these values in equation (2.0.1)

$$\begin{bmatrix} \pm\beta \\ 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 18 \\ 0 \end{bmatrix} + K\begin{bmatrix} \dfrac{1}{\lambda 1} & 0 \\ 0 & \dfrac{1}{\lambda 2} \end{bmatrix}\begin{bmatrix} \pm 8 \\ 0 \end{bmatrix}$$

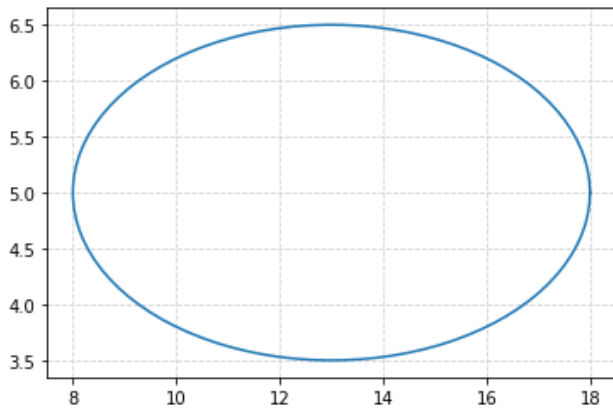The possible ellipse satisfying the above conditions are plotted below

Fig. 0: Ellipse