Assignment No.5

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Download latex-tikz codes from

https://github.com/shishirNIPER/ASSIGNMENT05 /blob/main/main.tex

Download python codes from

https://github.com/shishirNIPER/ASSIGNMENT05 /blob/main/ellipse.py

question taken from

quadratic_forms, exercise 2.28

1 Question No 1

Find the equation of the ellipse whose vertices are

$$\begin{bmatrix} \pm 13 \\ 0 \end{bmatrix}$$

and foci are

$$\begin{bmatrix} \pm 5 \\ 0 \end{bmatrix}$$

2 Solution

We have been provided with values for vertices and foci
Let

$$\mathbf{p} = \begin{bmatrix} \pm 13 \\ 0 \end{bmatrix}, \mathbf{q} = \begin{bmatrix} \pm 5 \\ 0 \end{bmatrix}$$

Also, The given coordinate of foci are

$$\begin{bmatrix} \pm 5 \\ 0 \end{bmatrix}$$

In general, the equation of ellipse passing through p and q can be expressed as

$$(\mathbf{X} - \mathbf{C})^T \mathbf{D} (\mathbf{X} - \mathbf{C}) = 1$$

Where
C is the centre, D is the diagonal matrix p,q satisfies above equation

$$(\mathbf{p} - \mathbf{C})^T \mathbf{D} (\mathbf{p} - \mathbf{C}) = 1$$

$$(\mathbf{q} - \mathbf{C})^T \mathbf{D} (\mathbf{p} - \mathbf{C}) = 1$$

which can be simplified as

$$2(\mathbf{p} - \mathbf{q})^T \mathbf{DC} = \mathbf{p}^T \mathbf{D} (\mathbf{p} - \mathbf{q})^T \mathbf{Dq}$$

Using identity

$$2(\mathbf{p} - \mathbf{q})^T \mathbf{DC} = (\mathbf{p} - \mathbf{q})^T \mathbf{D} (\mathbf{p} + \mathbf{q})^T \mathbf{Dq}$$

$$\Rightarrow (\mathbf{p} - \mathbf{q})^T \mathbf{D} (2\mathbf{C} - (\mathbf{p} + \mathbf{q}) = 0$$

We have values of (p+q), The value of m i.e (p-q) and C

$$\mathbf{p} - \mathbf{q} = \begin{bmatrix} \pm 8 \\ 0 \end{bmatrix}$$

$$\mathbf{p} + \mathbf{q} = \begin{bmatrix} \pm 18 \\ 0 \end{bmatrix}$$

We know that

$$foci = \mathbf{p} = \begin{bmatrix} \pm \beta \\ 0 \end{bmatrix} = \begin{bmatrix} \pm \mathbf{C} \\ 0 \end{bmatrix}$$

Thus

$$\mathbf{C} = 5$$

C can parametrically be expressed as

$$\mathbf{C} = 1/2[\mathbf{p} + \mathbf{q} + K\mathbf{D}^{-1} m] \tag{2.0.1}$$

Where

$$K = constant$$

And

$$(\mathbf{p} - \mathbf{q})^T m = 0$$

Substituting these values in equation (2.0.1)

$$\begin{bmatrix} \pm \beta \\ 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 18 \\ 0 \end{bmatrix} + K \begin{bmatrix} \frac{1}{\lambda 1} & 0 \\ 0 & \frac{1}{\lambda 2} \end{bmatrix} \begin{bmatrix} \pm 8 \\ 0 \end{bmatrix}$$

The possible ellipse satisfying the above conditions are plotted below

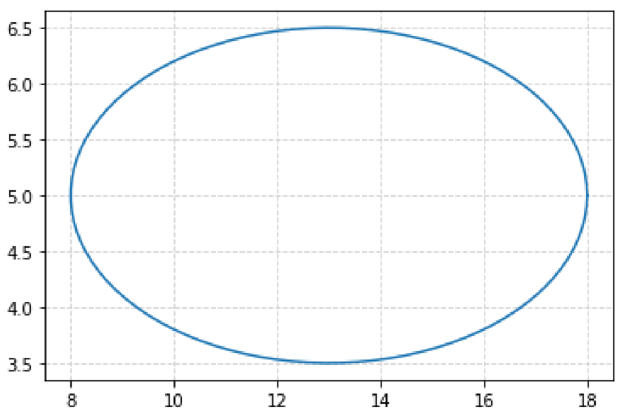

Fig. 0: Ellipse