Assignment No.5

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question taken from

quadratic_forms, exercise 2.28

1 QUESTION No 1

Find the equation of the ellipse whose vertices are $\begin{pmatrix} \pm13 \\ 0 \end{pmatrix}$ and foci are $\begin{pmatrix} \pm5 \\ 0 \end{pmatrix}$

2 SOLUTION

We have been provided with values for vertices and foci
Let

$$p = \begin{pmatrix} \pm13 \\ 0 \end{pmatrix}, q = \begin{pmatrix} \pm5 \\ 0 \end{pmatrix}$$

Also, The giver coordinate of foci are $\begin{pmatrix} \pm5 \\ 0 \end{pmatrix}$ In general, the equation of ellipse passing through p and q can be expressed as

$$(X - C)^T D(X - C) = 1$$

Where
C is the centre, D is the diagonal matrix p,q satisfies above equation

$$(p - C)^T D(p - C) = 1$$

$$(q - C)^T D(p - C) = 1$$

which can be simplified as

$$2(p - q)^T DC = p^T Dp - q^T Dq$$

Using identity

$$2(p - q)^T DC = (p - q)^T D(p + q)^T Dq$$

$$\Rightarrow (p - q)^T D(2C - (p + q) = 0$$

We have values of (p+q), The value of m i.e (p-q) and C

$$(p - q) = \begin{pmatrix} \pm8 \\ 0 \end{pmatrix}$$

$$(p + q) = \begin{pmatrix} \pm18 \\ 0 \end{pmatrix}$$

We know that foci= p=$\begin{pmatrix} \pm\beta \\ 0 \end{pmatrix}$=$\begin{pmatrix} \pm c \\ 0 \end{pmatrix}$ Thus

$$C = 5$$

C can parametrically be expressed as

$$C = 1/2[p + q + KD^{-1}m] \qquad (2.0.1)$$

Where

$$K = constant$$

And

$$(p - q)^T m = 0$$

Substituting these values in equation (2.0.1)

$$\begin{pmatrix} \pm\beta \\ 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 18 \\ 0 \end{pmatrix} + K \begin{pmatrix} \frac{1}{\lambda 1} & 0 \\ 0 & \frac{1}{\lambda 2} \end{pmatrix} \begin{pmatrix} \pm8 \\ 0 \end{pmatrix}$$
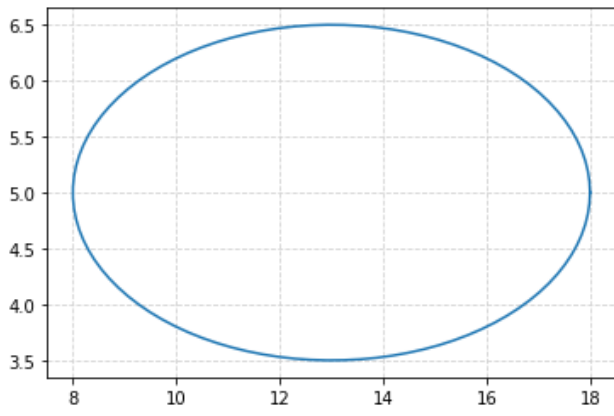
Fig. 0: Ellipse