# Assignment No.5

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Download latex-tikz codes from

https://github.com/shishirNIPER/ASSIGNMENT05/blob/main/main.tex

Download python codes from

https://github.com/shishirNIPER/ASSIGNMENT05/blob/main/ellipse.py

question taken from

quadratic_forms, exercise 2.28

## 1 Question No 1

Find the equation of the ellipse whose vertices are $\begin{pmatrix} \pm 13 \\ 0 \end{pmatrix}$ and foci are $\begin{pmatrix} \pm 5 \\ 0 \end{pmatrix}$

## 2 Solution

We have been provided with values for vertices and foci
Let

$$\mathbf{p} = \begin{pmatrix} \pm 13 \\ 0 \end{pmatrix}, \mathbf{q} = \begin{pmatrix} \pm 5 \\ 0 \end{pmatrix}$$

Also, The giver coordinate of foci are $\begin{pmatrix} \pm 5 \\ 0 \end{pmatrix}$ In general, the equation of ellipse passing through p and q can be expressed as

$$(X - C)^T D(X - C) = 1$$

Where
C is the centre, D is the diagonal matrix p,q satisfies above equation

$$(p - C)^T D(p - C) = 1$$

$$(q - C)^T D(p - C) = 1$$

which can be simplified as

$$2(\mathbf{p} - \mathbf{q})^T DC = p^T D(\mathbf{p} - \mathbf{q})^T Dq$$

Using identity

$$2(\mathbf{p} - \mathbf{q})^T DC = (\mathbf{p} - \mathbf{q})^T D(\mathbf{p} + \mathbf{q})^T Dq$$

$$\implies (\mathbf{p} - \mathbf{q})^T D(2C - (\mathbf{p} + \mathbf{q}) = 0$$

We have values of (p+q), The value of m i.e (p-q) and C

$$\mathbf{p} - \mathbf{q} = \begin{pmatrix} \pm 8 \\ 0 \end{pmatrix}$$

$$\mathbf{p} + \mathbf{q} = \begin{pmatrix} \pm 18 \\ 0 \end{pmatrix}$$

We know that foci= p=$\begin{pmatrix} \pm \beta \\ 0 \end{pmatrix}$=$\begin{pmatrix} \pm c \\ 0 \end{pmatrix}$ Thus

$$C = 5$$

C can parametrically be expressed as

$$C = 1/2[\mathbf{p} + \mathbf{q} + KD^{-1}m] \tag{2.0.1}$$

Where

$$K = constant$$

And

$$(\mathbf{p} - \mathbf{q})^T m = 0$$

Substituting these values in equation (2.0.1)

$$\begin{pmatrix} \pm \beta \\ 0 \end{pmatrix} = \frac{1}{2}\left( \begin{array}{c} 18 \\ \hline 0 \end{array} \right) + K \begin{pmatrix} \frac{1}{\lambda 1} & 0 \\ 0 & \frac{1}{\lambda 2} \end{pmatrix} \begin{pmatrix} \pm 8 \\ 0 \end{pmatrix}$$

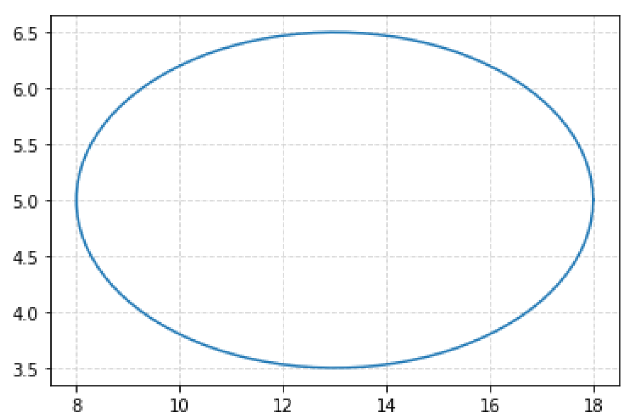


Fig. 0: Ellipse