Assignment 1

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1 Problem 1

Show that the points $\mathbf{A} = \begin{pmatrix}1\\2\end{pmatrix}, \mathbf{B} = \begin{pmatrix}5\\4\end{pmatrix}, \mathbf{C} = \begin{pmatrix}3\\8\end{pmatrix}, \mathbf{D} = \begin{pmatrix}-1\\6\end{pmatrix}$ are the vertices of a square.

2 Solution

There are two basic properties of square-

Lemma 2.1. *1) The angle made by two sides of a square is always 90°*
2) The diagonals of a square are equal and bisect each other at the right angles
3) If two lines (Say **AB** *and* **BC***) are perpendicular to each other, The value of* $(A-B)^T(B-C)=0$

Let us compute the values of the same—-

$$\mathbf{A} = \begin{pmatrix}1\\2\end{pmatrix} \tag{2.0.1}$$

$$\mathbf{B} = \begin{pmatrix}5\\4\end{pmatrix} \tag{2.0.2}$$

$$\mathbf{C} = \begin{pmatrix}3\\8\end{pmatrix} \tag{2.0.3}$$

$$\mathbf{D} = \begin{pmatrix}-1\\6\end{pmatrix} \tag{2.0.4}$$

1) We know if AB and BC are perpendicular then

$$(\mathbf{A}-\mathbf{B})^{\mathbf{T}}(\mathbf{B}-\mathbf{C}) = 0 \tag{2.0.5}$$
$$\tag{2.0.6}$$

Then for given vertices

$$(\mathbf{A}-\mathbf{B})^{\mathbf{T}}(\mathbf{B}-\mathbf{C}) = \begin{pmatrix}-4 & -2\end{pmatrix}\begin{pmatrix}2\\-4\end{pmatrix} = 0 \tag{2.0.7}$$

2) Also, if BC and CD are perpendicular then

$$(\mathbf{B}-\mathbf{C})^{\mathbf{T}}(\mathbf{C}-\mathbf{D}) = 0 \tag{2.0.8}$$
$$\tag{2.0.9}$$

Then for given vertices

$$(\mathbf{B}-\mathbf{C})^{\mathbf{T}}(\mathbf{C}-\mathbf{D}) = \begin{pmatrix}2 & -4\end{pmatrix}\begin{pmatrix}4\\2\end{pmatrix} = 0 \tag{2.0.10}$$

3) Similarly, if CD and DA are perpendicular then

$$(\mathbf{C}-\mathbf{D})^{\mathbf{T}}(\mathbf{D}-\mathbf{A}) = 0 \tag{2.0.11}$$
$$\tag{2.0.12}$$

Then for given vertices

$$(\mathbf{C}-\mathbf{D})^{\mathbf{T}}(\mathbf{D}-\mathbf{A}) = \begin{pmatrix}4 & 2\end{pmatrix}\begin{pmatrix}2\\4\end{pmatrix} = 0 \tag{2.0.13}$$

4) Similarly, if DA and AB are perpendicular then

$$(\mathbf{D}-\mathbf{A})^{\mathbf{T}}(\mathbf{A}-\mathbf{B}) = 0 \tag{2.0.14}$$
$$\tag{2.0.15}$$

Then for given vertices

$$(\mathbf{D}-\mathbf{A})^{\mathbf{T}}(\mathbf{A}-\mathbf{B}) = \begin{pmatrix}2 & 4\end{pmatrix}\begin{pmatrix}4\\-2\end{pmatrix} = 0 \tag{2.0.16}$$

Therefore, The angle between AB and BC, BC and CD, CD and DA , DA and AB is 90°
Also, "The diagonals of a square are equal and bisect each other at right angles"

5)

if AC and BD are diagonals of the square, They must bisect each other at right angles

$$(\mathbf{A}-\mathbf{C})^{\mathbf{T}}(\mathbf{B}-\mathbf{D}) = 0 \tag{2.0.17}$$
$$\tag{2.0.18}$$

Putting the available values

$$(\mathbf{A}-\mathbf{C})^{\mathbf{T}}(\mathbf{B}-\mathbf{D}) = \begin{pmatrix}-2 & 6\end{pmatrix}\begin{pmatrix}6\\-2\end{pmatrix} = 0 \tag{2.0.19}$$

This fulfills the property that the "The diagonals of a square are equal and bisect each other at right angles"

Thus it can be claimed that $\mathbf{A} = \begin{pmatrix}1\\2\end{pmatrix}, \mathbf{B} = \begin{pmatrix}5\\4\end{pmatrix}, \mathbf{C} = \begin{pmatrix}3\\8\end{pmatrix}, \mathbf{C} = \begin{pmatrix}-1\\6\end{pmatrix}$ are the vertices of a square.

Please see Figure 5 which verifies that A,B,C,D form a square

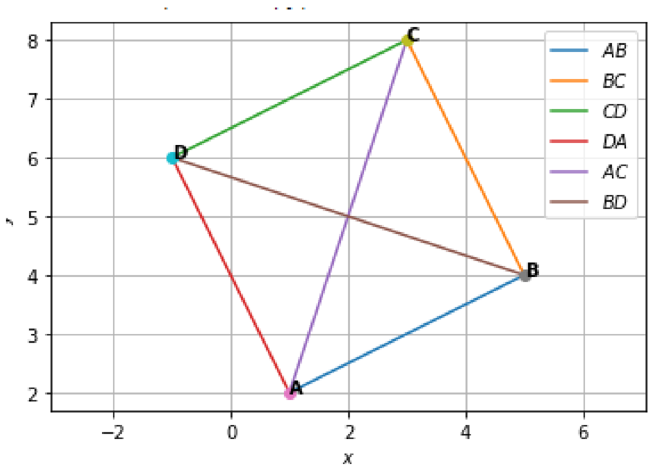

Fig. 5: Square ABCD