

Assignment 1

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1 PROBLEM 1

Question taken from

cbse/math/10/2006/Set 1 Q.no.16
Available at: <https://tinyurl.com/yfbrdbex>

Show that the points $\mathbf{A} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 \\ 8 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$ are the vertices of a square.

2 SOLUTION

There are two basic properties of square-

Lemma 2.1. 1) The angle made by two sides of a square is always 90°

2) The diagonals of a square are equal and bisect each other at the right angles

3) If two lines (Say \mathbf{AB} and \mathbf{BC}) are perpendicular to each other, The value of $(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C})=0$

Let us compute the values of the same—

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (2.0.1)$$

$$\mathbf{B} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \quad (2.0.2)$$

$$\mathbf{C} = \begin{pmatrix} 3 \\ 8 \end{pmatrix} \quad (2.0.3)$$

$$\mathbf{D} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} \quad (2.0.4)$$

1) We know if AB and BC are perpendicular then

$$(\mathbf{A} - \mathbf{B})^T(\mathbf{B} - \mathbf{C}) = 0 \quad (2.0.5)$$

$$(2.0.6)$$

Then for given vertices

$$(\mathbf{A} - \mathbf{B})^T(\mathbf{B} - \mathbf{C}) = \begin{pmatrix} -4 & -2 \end{pmatrix} \begin{pmatrix} 2 \\ -4 \end{pmatrix} = 0 \quad (2.0.7)$$

2) Also, if BC and CD are perpendicular then

$$(\mathbf{B} - \mathbf{C})^T(\mathbf{C} - \mathbf{D}) = 0 \quad (2.0.8)$$

$$(2.0.9)$$

Then for given vertices

$$(\mathbf{B} - \mathbf{C})^T(\mathbf{C} - \mathbf{D}) = \begin{pmatrix} 2 & -4 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} = 0 \quad (2.0.10)$$

3) Similarly, if CD and DA are perpendicular then

$$(\mathbf{C} - \mathbf{D})^T(\mathbf{D} - \mathbf{A}) = 0 \quad (2.0.11)$$

$$(2.0.12)$$

Then for given vertices

$$(\mathbf{C} - \mathbf{D})^T(\mathbf{D} - \mathbf{A}) = \begin{pmatrix} 4 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \end{pmatrix} = 0 \quad (2.0.13)$$

4) Similarly, if DA and AB are perpendicular then

$$(\mathbf{D} - \mathbf{A})^T(\mathbf{A} - \mathbf{B}) = 0 \quad (2.0.14)$$

$$(2.0.15)$$

Then for given vertices

$$(\mathbf{D} - \mathbf{A})^T(\mathbf{A} - \mathbf{B}) = \begin{pmatrix} 2 & 4 \end{pmatrix} \begin{pmatrix} 4 \\ -2 \end{pmatrix} = 0 \quad (2.0.16)$$

Therefore, The angle between AB and BC , BC and CD , CD and DA , DA and AB is 90°

Also, "The diagonals of a square are equal and bisect each other at right angles"

5)

if AC and BD are diagonals of the square, They must bisect each other at right angles

$$(\mathbf{A} - \mathbf{C})^T(\mathbf{B} - \mathbf{D}) = 0 \quad (2.0.17)$$

$$(2.0.18)$$

Putting the available values

$$(\mathbf{A} - \mathbf{C})^T(\mathbf{B} - \mathbf{D}) = \begin{pmatrix} -2 & 6 \end{pmatrix} \begin{pmatrix} 6 \\ -2 \end{pmatrix} = 0 \quad (2.0.19)$$

This fulfills the property that the "The diagonals of a square are equal and bisect each other at right angles"

Thus it can be claimed that $\mathbf{A} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 3 \\ 8 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$ are the vertices of a square.
Please see Figure 5 which verifies that A,B,C,D form a square

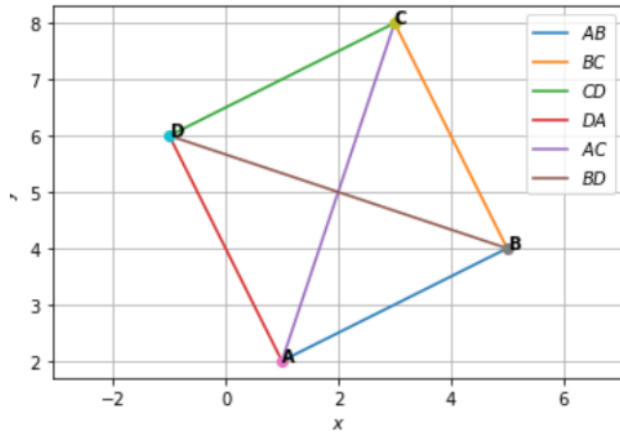


Fig. 5: Square ABCD