Assignment 1

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1 Problem 1

Show that the points $\mathbf{A} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 \\ 8 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$ are the vertices of a square.

2 Solution

There are two basic properties of square-

Lemma 2.1. *1) The angle made by two sides of a square is always 90°*
2) The diagonals of a square are equal and bisect each other at the right angles
3) If two lines (Say $\mathbf{A}$ *and* $\mathbf{B}$*) are perpendicular to each other, The value of* $(A-B)^T(B-C)$*=0*

Let us compute the values of the same—-

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{B} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{C} = \begin{pmatrix} 3 \\ 8 \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{D} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} \tag{2.0.4}$$

1) We know if AB and BC are perpendicular then

$$(\mathbf{A}-\mathbf{B})^{\mathbf{T}}(\mathbf{B}-\mathbf{C}) = 0 \tag{2.0.5}$$
$$\tag{2.0.6}$$

Then for given vertices

$$(\mathbf{A}-\mathbf{B})^{\mathbf{T}}(\mathbf{B}-\mathbf{C}) = \begin{pmatrix} -4 & -2 \end{pmatrix} \begin{pmatrix} 2 \\ -4 \end{pmatrix} = 0 \tag{2.0.7}$$

2) Also, if BC and CD are perpendicular then

$$(\mathbf{B}-\mathbf{C})^{\mathbf{T}}(\mathbf{C}-\mathbf{D}) = 0 \tag{2.0.8}$$
$$\tag{2.0.9}$$

Then for given vertices

$$(\mathbf{B}-\mathbf{C})^{\mathbf{T}}(\mathbf{C}-\mathbf{D}) = \begin{pmatrix} 2 & -4 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} = 0 \tag{2.0.10}$$

3) Similarly, if CD and DA are perpendicular then

$$(\mathbf{C}-\mathbf{D})^{\mathbf{T}}(\mathbf{D}-\mathbf{A}) = 0 \tag{2.0.11}$$
$$\tag{2.0.12}$$

Then for given vertices

$$(\mathbf{C}-\mathbf{D})^{\mathbf{T}}(\mathbf{D}-\mathbf{A}) = \begin{pmatrix} 4 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \end{pmatrix} = 0 \tag{2.0.13}$$

4) Similarly, if DA and AB are perpendicular then

$$(\mathbf{D}-\mathbf{A})^{\mathbf{T}}(\mathbf{A}-\mathbf{B}) = 0 \tag{2.0.14}$$
$$\tag{2.0.15}$$

Then for given vertices

$$(\mathbf{D}-\mathbf{A})^{\mathbf{T}}(\mathbf{A}-\mathbf{B}) = \begin{pmatrix} 2 & 4 \end{pmatrix} \begin{pmatrix} 4 \\ -2 \end{pmatrix} = 0 \tag{2.0.16}$$

Therefore, The angle between AB and BC, BC and CD, CD and DA , DA and AB is 90°
Also, "The diagonals of a square are equal and bisect each other at right angles"

5)

if AC and BD are diagonals of the square, They must bisect each other at right angles

$$(\mathbf{A}-\mathbf{C})^{\mathbf{T}}(\mathbf{B}-\mathbf{D}) = 0 \tag{2.0.17}$$
$$\tag{2.0.18}$$

Putting the available values

$$(\mathbf{A}-\mathbf{C})^{\mathbf{T}}(\mathbf{B}-\mathbf{D}) = \begin{pmatrix} -2 & 6 \end{pmatrix} \begin{pmatrix} 6 \\ -2 \end{pmatrix} = 0 \tag{2.0.19}$$

This fulfills the property that the "The diagonals of a square are equal and bisect each other at right angles"

Thus it can be claimed that $\mathbf{A} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 \\ 8 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$ are the vertices of a square.

Please see Figure 5 which verifies that A,B,C,D form a square

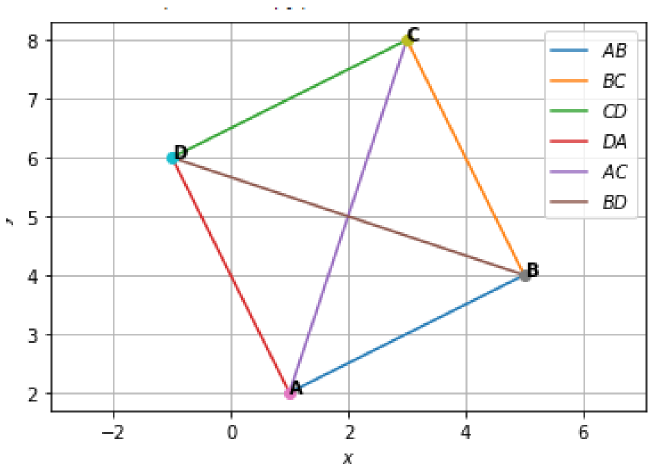

Fig. 5: Square ABCD