Assignment 1

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1 Problem 1

Show that the points$\mathbf{A} = \begin{pmatrix}1\\2\end{pmatrix}, \mathbf{B} = \begin{pmatrix}5\\4\end{pmatrix}, \mathbf{C} = \begin{pmatrix}3\\8\end{pmatrix}, \mathbf{C} = \begin{pmatrix}-1\\6\end{pmatrix}$ are the vertices of a sqaure.

2 Solution

Let us compute the values of the same—-

$$\mathbf{A} = \begin{pmatrix}1\\2\end{pmatrix} \quad (2.0.1)$$

$$\mathbf{B} = \begin{pmatrix}5\\4\end{pmatrix} \quad (2.0.2)$$

$$\mathbf{C} = \begin{pmatrix}3\\8\end{pmatrix} \quad (2.0.3)$$

$$\mathbf{D} = \begin{pmatrix}-1\\-6\end{pmatrix} \quad (2.0.4)$$

We know if AB and BC are perpendicular then

$$(\mathbf{A}-\mathbf{B})^{\mathbf{T}}(\mathbf{B}-\mathbf{C}) = 0 \quad (2.0.5)$$
$$(2.0.6)$$

Then for given vertices

$$(\mathbf{A}-\mathbf{B})^{\mathbf{T}}(\mathbf{B}-\mathbf{C}) = \begin{pmatrix}-4 & -2\end{pmatrix}\begin{pmatrix}2\\-4\end{pmatrix} \implies 0 \quad (2.0.7)$$

Also, if BC and CD are perpendicular then

$$(\mathbf{B}-\mathbf{C})^{\mathbf{T}}(\mathbf{C}-\mathbf{D}) = 0 \quad (2.0.8)$$
$$(2.0.9)$$

Then for given vertices

$$(\mathbf{B}-\mathbf{C})^{\mathbf{T}}(\mathbf{C}-\mathbf{D}) = \begin{pmatrix}2 & -4\end{pmatrix}\begin{pmatrix}4\\2\end{pmatrix} \implies 0 \quad (2.0.10)$$

Similarly, if CD and DA are perpendicular then

$$(\mathbf{C}-\mathbf{D})^{\mathbf{T}}(\mathbf{D}-\mathbf{A}) = 0 \quad (2.0.11)$$
$$(2.0.12)$$

Then for given vertices

$$(\mathbf{C}-\mathbf{D})^{\mathbf{T}}(\mathbf{D}-\mathbf{A}) = \begin{pmatrix}4 & 2\end{pmatrix}\begin{pmatrix}2\\4\end{pmatrix} \implies 0 \quad (2.0.13)$$

$(\mathbf{D}-\mathbf{A})^{\mathbf{T}}(\mathbf{A}-\mathbf{B}) = 0$

Then for given vertices

$$(\mathbf{D}-\mathbf{A})^{\mathbf{T}}(\mathbf{A}-\mathbf{B}) = \begin{pmatrix}2 & 4\end{pmatrix}\begin{pmatrix}4\\-2\end{pmatrix} \implies 0 \quad (2.0.14)$$

Therefore, The angle between AB and BC, BC and CD, CD and DA , DA and AB is 90°

Also, ”The diagonals of a square are equal and bisect each other at right angles”

if AC and BD are diagonals of the sqaure, They must bisect each other at right angles

$$(\mathbf{A}-\mathbf{C})^{\mathbf{T}}(\mathbf{B}-\mathbf{D}) = 0 \quad (2.0.15)$$
$$(2.0.16)$$

Putting the available values

$$(\mathbf{A}-\mathbf{C})^{\mathbf{T}}(\mathbf{B}-\mathbf{D}) = \begin{pmatrix}-2 & 6\end{pmatrix}\begin{pmatrix}6\\-2\end{pmatrix} \implies 0 \quad (2.0.17)$$

This fulfills the property that the ”The diagonals of a square are equal and bisect each other at right angles”

Thus it can be claimed that $\mathbf{A} = \begin{pmatrix}1\\2\end{pmatrix}, \mathbf{B} = \begin{pmatrix}5\\4\end{pmatrix}, \mathbf{C} = \begin{pmatrix}3\\8\end{pmatrix}, \mathbf{C} = \begin{pmatrix}-1\\6\end{pmatrix}$ are the vertices of a sqaure.

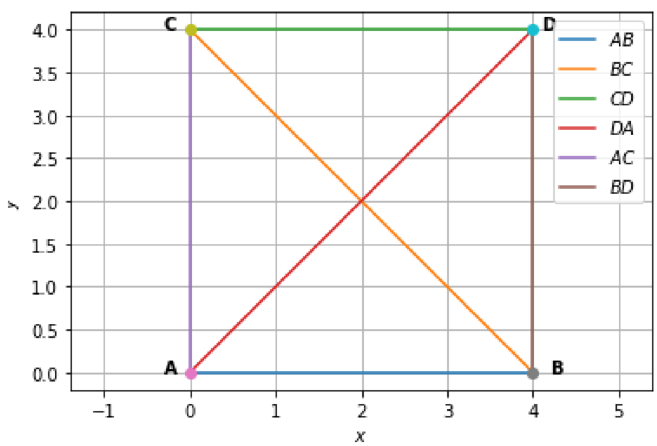

Fig. 0: Answer Image