Part V.I : CNN

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Task 1 : Given, (RGB Image)

Input image : 32×28
Filter : 5×5

Padding : 0
Stride : 1

Number of filters : 10

W.K.T,

output dim = (In dim + 2P − Filter)/stride + 1

for height ⇒

$$\text{Out } H = (32 + 2 \times 0 - 5)/1 + 1 = 27 + 1 = \underline{28}$$

$$\text{Out } W = (28 + 2 \times 0 - 5)/1 + 1 = 23 + 1 = \underline{24}$$

∴ output size ⇒ $\boxed{28 \times 24 \times 10}$ (∵ 10 filters)

Task 2 : Given, to find total learnable parameters

Filter : 5×5
Input channels : 3 (for RGB)

∴ for each filter,

we have, weights $= 5 \times 5 \times 3 = 75$
(also with bias)

⇒ Total per filter $= 75 + 1 = \underline{76}$

∴ Total Parameters $= 10 \times 76 = \underline{\underline{760}}$ for 10 filters //

Task 3 : Similar to task 1,

Padding = 1

$$\Rightarrow \text{out } H = (32 + 2 \times 1 - 5)/1 + 1 = 29 + 1 = \underline{\underline{30}}$$

$$\text{out } W = (28 + 2 \times 1 - 5)/1 + 1 = 25 + 1 = \underline{\underline{26}}$$

∴ with 10 filters, output size ⇒ $\boxed{30 \times 26 \times 10}$

Task 4 : Given,

to find No of parameters with Greyscale image input,

Input channel : 1

∴ for each filter,

Parameters : $5 \times 5 \times 1 = \underline{25}$
(with bias)

⇒ Total parameters per filter : $25 + 1 = \underline{26}$

∴ for 10 filters ⇒

Total Parameters $= 26 \times 10 = \underline{260}$

Task 5 : Given the task, the most suitable activation function for the output layer is,

$\boxed{\text{Softmax}}$

We choose Softmax for its ability to convert raw outputs (logits) into probability that sum to 1, which is essential for "multi-class classification tasks".

In the given Scenario, other activation functions such as,

- Sigmoid ⇒ outputs values b/w 0 & 1 but doesn't ensure it sums to 1, which makes it better fit to binary classification.

- Relu ⇒ outputs non-negative values but does not bound the values or provide a problistic interpretation.

- Tanh ⇒ outputs values btw −1 & 1 which is not valid.

Softmax $\sigma(z_i) = \dfrac{e^{z_i}}{\sum_{j=1}^{5} e^{z_j}}$ for $i = 1, 2 \ldots 5$ Also, $\sum_{i=1}^{5} \sigma(z_i) = 1$

Task 6 : Proof of Shift Invariance of Softmax

to prove : Adding a constant c to every input does not change the output probability.

W.K.T, $\sigma(z_i) = \frac{e^{z_i}}{\sum_{j=1}^{k} e^{z_j}}$

Now lets consider shifted inputs $\Rightarrow z_i' = z_i + c$

lets compute softmax for shifted inputs,

$$\sigma(z_i') = \frac{e^{z_i+c}}{\sum_{j=1}^{k} e^{z_j+c}} = \frac{e^{z_i} \cdot e^{c}}{\sum_{j=1}^{k} e^{z_j} \cdot e^{c}}$$

$$= \frac{\cancel{e^c}}{\cancel{e^c}} \cdot \frac{e^{z_i}}{\sum_{j=1}^{k} e^{z_j}} = \frac{e^{z_i}}{\sum_{j=1}^{k} e^{z_j}} = \sigma(z_i)$$

, hence proof

Adding a constant c to every element of the input vector does not change the output of softmax. This shift invariance is significant because it ensures that the softmax probabilities remain consistent regardless of any offsets, which arises from bais terms or numerical adjustments.