

CS105 (DIC on Discrete Structures)

Problem set 9

- Attempt *all* questions.
 - Apart from things proved in lecture, you cannot assume anything as “obvious”. Either quote previously proved results or provide clear justification for each statement.
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Basic

1. Prove or disprove:
 - (a) Any finite simple graph with at least two vertices contains at least two vertices of the same degree.
 - (b) Every graph that is not connected, must have an isolated vertex.
 - (c) A graph is connected iff some vertex has an edge to all other vertices.
 - (d) A graph is connected iff some vertex is connected (i.e., has a path) to all other vertices.
 - (e) If a maximal trail in a graph is not closed, then its endpoints have odd degree.
 - (f) Every Eulerian bipartite graph has even number of edges.
 - (g) Every Eulerian simple graph with an even number of vertices has an even number of edges.
2. Anushka and her husband Virat gave a party at which there are 4 other married couples. Some pairs of people shake hands when they meet, but naturally no couple shake hands with each other. At the end of the party Virat asks everyone else how many people they have shaken hands with, and he receives nine different answers. How many people shook hands with Anushka? Model the problem as a graph and solve it.

Advanced

3. Consider any five points p_1, \dots, p_5 in the interior of a square S of side length 1. Prove that there are two of these five points which are separated by a distance of at most $\sqrt{2}/2$.
4. In any group of people, we are told that any two either don't know each other or, if they know each other, they like or dislike each other. In a group of 17 people, show that the following property P is true: there must exist 3 who know and mutually like each other or 3 who know but mutually dislike each other or 3 who don't know each other.
5. Let W be a closed walk of length greater than 1 that does not contain a cycle. Then show that some edge of W repeats immediately (once in each direction). Give two proofs: one, by induction and another without!