CS 105: Department Introductory Course on Discrete Structures

Instructor : S. Akshay

Aug 20, 2024

Lecture 10 – Basic structures: Countable and Uncountable sets

Logistics

Quiz 1

- VENUE: LH 101, 102, 301, 302
- Date and time: Aug 28th, 8.25am

Countable and countably infinite sets

Definition

- For a given set C, if there is a bijection from C to $\mathbb{N}$, then C is called countably infinite.
- A set is countable if it is finite or countably infinite.

Examples: even numbers, number of horses,...

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What are the other properties of countable sets?

Some questions...

Are the following sets countable?

That is, is there a bijection from these sets to $\mathbb{N}$?

- the set of all integers $\mathbb{Z}$
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- $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$
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To show these it suffices to show that

- there is an injection from these sets to $\mathbb{N}$
- or there is a surjection from $\mathbb{N}$ (or any countable set) to these sets.

Unions of countable sets is countable

Let $A = \{a_0, \ldots,\}$ be a countably infinite set and B be a set. Then, is $A \cup B$ countable, under the following conditions?

1. $B = \{b_0\}$ is a singleton
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- Is this correct?

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Hint: Show that $f(a, b) = \begin{cases} a/b \text{ if } b \neq 0 \\ 0 \text{ if } b = 0 \end{cases}$, is a surjection. How does the result follow?

Pop Quiz?

Exercises

1. Show that set of primes P is countable.

2. Show that $((\mathbb{Z} \times \mathbb{N}) \cup (\mathbb{N} \times \mathbb{Q}) \cup \{\pi, \sqrt{2}\})$ is countable.

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   - Proof 2 Show $f : P \to \mathbb{N}$ by f maps i^{th} prime to i is a bijection
2. Show that $((\mathbb{Z} \times \mathbb{N}) \cup (\mathbb{N} \times \mathbb{Q}) \cup \{\pi, \sqrt{2}\})$ is countable.

Countable sets and functions

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- S and $f(j)$ differ at position j, for any j.
- Thus, $S \neq f(j)$ for all $j \in \mathbb{N}$, which is a contradiction! $\square$

Cantor's diagonalization

Does this proof look familiar??

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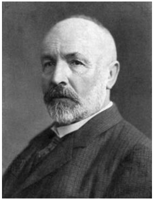

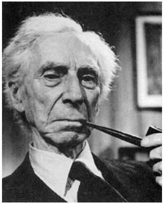

Figure: Cantor and Russell

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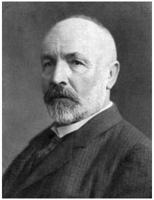

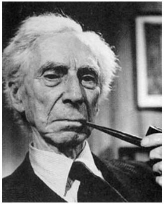

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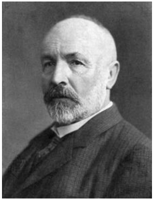 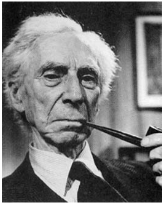

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- If $\exists j \in \mathbb{N}$ such that $f(j) = S$, then we have a contradiction.
  - If $j \in S$, then $j \notin f(j) = S$.
  - If $j \notin S$, then $j \notin f(j)$, which implies $j \in S$.

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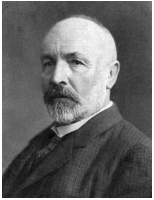
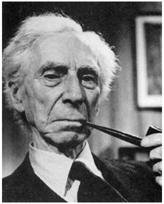

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- There cannot be a bijection between any set and its power set (i.e., its set of subsets).(H.W)
- So there is an infinite hierarchy of "larger" infinities...

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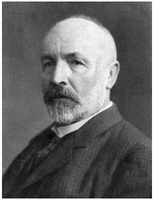
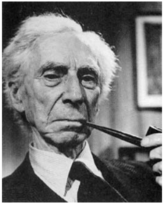

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- There cannot be a bijection between any set and its power set (i.e., its set of subsets).(H.W)
- So there is an infinite hierarchy of "larger" infinities...
- There is no bijection from $\mathbb{R}$ to $\mathbb{N}$ (H.W). Moreover, there is a bijection from $\mathbb{R}$ to set of subsets of $\mathbb{N}$.

One infinity is “strictly” bigger than another!

Theorem (Cantor, 1891)

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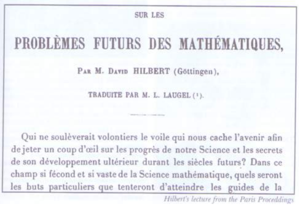
Cantor’s Continuum hypothesis

There is no set whose “cardinality” is strictly between $\mathbb{N}$ and $\mathcal{P}(\mathbb{N})$ (i.e., between naturals and reals).



SUR LES

PROBLÈMES FUTURS DES MATHÉMATIQUES,

PAR M. DAVID HILBERT (Göttingen),

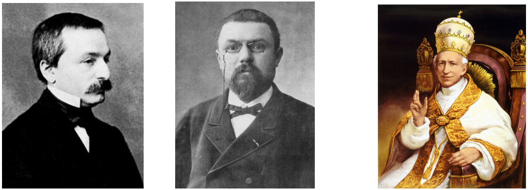
TRADUITE PAR M. L. LAUGEL (1).

Qui ne soulèverait volontiers le voile qui nous cache l'avenir afin de jeter un coup d'œil sur les progrès de notre Science et les secrets de son développement ultérieur durant les siècles futurs? Dans ce champ si fécond et si vaste de la Science mathématique, quels seront les buts particuliers que tenteront d'atteindre les guides de la

Hilbert's lecture from the Paris Proceddings

Figure: 1st of Hilbert’s 23 problems for the 20th century in 1900.

What did the world think about these proofs (in 1890s?)



(a) Kronecker (b) Poincare (c) Theologians

- Kronecker: Only constructive proofs are proofs! "Scientific Charlatan", "Corruptor of youth"!
- Poincare: Set theory is a "disease" from which mathematics will be cured.
- Christian Theologians: God=Uniqueness of an absolute infinity. So, what is all this different infinities...?!

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- Hilbert: No one can expel us from the paradise that Cantor has created for us.

Summary and moving on...

- Finite and infinite sets.
- Using functions to compare sets: focus on bijections.
- Countable, countably infinite and uncountable sets.
- Cantor's diagonalization argument (A new powerful proof technique!).

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Next: Basic Mathematical Structures – Relations