CS105 Discrete Structures: Propositions, Theorems and Proofs
Exercise Problem Set 1

Part 1

1. Give the converse and contrapositive for the following propositions:
   (a) If it rains today, then my hostel room will leak.
   (b) If $|x| = x$, then $x \geq 0$.
   (c) If n is greater than 3, then n^2 is greater than 9.

   Which of the above statements/propositions are true? Are their converses also true?

2. True or False
   (a) A proposition is equivalent to its contrapositive.
   (b) A proposition is equivalent to its converse.
   (c) A proposition is equivalent to the converse of its converse.

3. For each of the following propositions, write their negation, such that the negated proposition begins with the quantifier: "there exists a natural number n" or "for all natural numbers n". Also convince yourself the negation is false if the original proposition is true, and vice-versa.
   (a) For all natural numbers n, n is a multiple of 2 or n is a multiple of 3.
   (b) For all natural numbers n, if n is prime, then it is odd.
   (c) There exists a natural number n which is greater than 100.
   (d) There exists a natural number n such that $n^2 = 7$.

4. For each of the following propositions, write its negation. Is the negation true?
   (a) If it rains today, then my hostel room will leak.
   (b) There exists $n \in \mathbb{N}$ such that $n \geq 5$ and $n^2 < 25$.
   (c) For all $n \in \mathbb{N}$, n is a prime or n^2 is a prime, but n^3 is not a prime.
   (d) All computer science students like coffee.

5. Prove or disprove the following:
   (a) For any real number x, if x^3 is irrational, then so is x.
   (b) For any real number x, if x is irrational, then so is x^3.
   (c) There exists a nonnegative integer $n^2 > 10^{1000}$. Is your proof constructive or non-constructive?

Part 2

6. Prove, by contradiction, that for all $a, b \in \mathbb{Z}$, we have $a^2 - 4b \neq 2$.

7. Prove or disprove: there is no rational solution to the equation $x^5 + x^4 + x^3 + x^2 + 1 = 0$.

8. Assume that the symbols $>$, $<$ and $=$ are given their natural interpretations as the "greater than", "less than" and "equal to" binary relations, and that the domain of discourse is over reals $\mathbb{R}$.

   (a) Convert the following to English.

      i. $(\forall x(x^2 > x)) \wedge (\exists x(x^2 = 2))$

      ii. $\forall x \forall y \exists z((x < y) \rightarrow ((x < z) \wedge (z < y)))$

      iii. $\forall x \forall y \forall z(((x = y) \wedge (y = z)) \rightarrow (x = z))$

   (b) Also, for each of the above statements, write their negation as propositions with quantifiers.