CS105 (DIC on Discrete Structures)
Problem set 7

- Attempt *all* questions.
- Apart from things proved in lecture, you cannot assume anything as "obvious". Either quote previously proved results or provide clear justification for each statement.

Basic

1. The drama club has m members and the dance club has n members. For the upcoming paf, a committee of k people needs to be formed with at least one member from each club. If the clubs have exactly r members in common, what is the number of ways the committee may be chosen? Justify your answer.

2. Let $B(n)$ be the no. of bit strings of length n that contain the string 01.
   (a) Write a recurrence relation for $B(n)$.
   (b) Determine the initial conditions.
   (c) How many strings are there of length 7 are there that contain 01?
   (d) Solve the recurrence to obtain an expression for $B(n)$ in terms of n.

3. Consider the standard deck of 52 playing cards. A balanced hand is a subset of 13 cards containing four cards of one suit and three cards of each of the remaining three suits. Find N , the number of balanced hands. Find the number of ways of dealing the cards to four (distinguishable) players so that each player gets a balanced hand. Is this number equal to $N(N-1)(N-2)(N-3)$?

4. Find the number of integral solutions for $x_1 + x_2 + x_3 + x_4 + x_5 = 21$ with each $x_i \geq 2$. Next, generalize this to find the number of solutions for $\sum_{i=1}^{k} x_i = n$ with each $x_i \geq t$ and express it in a closed form involving n, t, and k.

5. Find the coefficients of x^{10} in

(a) $(1+x)^{12}$

(b) the power series of $x^4/(1-3x)^3$

Advanced

6. Write a recurrence for the number of derrangements. That is, no. of ways to arrange n letters into n addressed envelopes such that no letter goes to the correct envelope.

7. Consider a grid from $(0,0)$ to (n,n). Starting from the point $(0,0)$, we wish to take units steps ONLY in the direction of the positive X and Y axes (i.e., right and up), and reach the point (n,n). Find a recurrence relation for the number of ways of doing so, if we are not allowed to go above the line joining $(0,0)$ and (n,n), i.e, the diagonal.

Programming assignment (Out of Syllabus)

These questions are not for the exam, but just for your own interest. Solve them at your own risk! *(Also see Piazza for student-posted solutions!)*

1. Write a program to draw the Pascal's triangle of any size. That is, the program should take as input an integer n and write down the numbers of the first n rows of the triangle (as a triangle). For example, for input $n = 10$, the output should look like:

```
                         1
                       1   1
                     1   2   1
                   1   3   3   1
                 1   4   6   4   1
               1   5  10  10   5   1
             1   6  15  20  15   6   1
           1   7  21  35  35  21   7   1
         1   8  28  56  70  56  28   8   1
       1   9  36  84 126 126  84  36   9   1
     1  10  45 120 210 252 210 120  45  10   1
```

2. Now, modify the program to replace every odd number in the triangle by the symbol $*$ and every even number by an empty blank space. Print the output of this new program for $n = 17$ and $n = 50$. Can you see something special about this picture? Can you explain this phenomenon mathematically?