FWER

	True H_is	False H_is	Total
Accepted H_is	U	T	$M - R$
Rejected H_is	V	S	R
Total	m_0	$\mathrm{m} - \mathrm{m}_0$	m

Familywise (type I) **error rate**:

$$\text{FWER} = P(V > 0)$$

Controlling FWER at level α:

$$\text{FWER} = P(V > 0) \leq \alpha$$

How?

FWER

Familywise (type I) **error rate**:

$$\text{FWER} = P(V > 0)$$

Controlling FWER at level α:

$$\text{FWER} = P(V > 0) \leq \alpha$$

How?

$\alpha_1, \ldots, \alpha_m$ – significance levels for $H_1, \ldots, H_m$

We have to select them to ensure $\text{FWER} \leq \alpha$.

Bonferroni correction

Bonferroni method:

$$\alpha_1 = \cdots = \alpha_m = \frac{\alpha}{m}$$

Comparing α_i and p_i is the same as comparing original α and **adjusted p-value**

$$\tilde{p}_i = \min(1, mp_i)$$

H_i is rejected when $\tilde{p}_i \leq \alpha$.

Bonferroni correction

Theorem. If H_i is rejected when $p_i \leq \alpha/m$, then FWER $\leq \alpha$.

Proof.

$$\text{FWER} = P(V > 0) = P\left(\bigcup_{i \in \boldsymbol{M}_0} \left\{p_i \leq \frac{\alpha}{m}\right\}\right) \leq$$

$$\leq \sum_{i \in \boldsymbol{M}_0} P\left(p_i \leq \frac{\alpha}{m}\right) \leq$$

$$\leq \sum_{i \in \boldsymbol{M}_0} \frac{\alpha}{m} = \frac{m_0}{m}\alpha \leq \alpha$$

Model experiment

50 samples from $N(1,1)$, 150 samples from $N(0,1)$, $n = 20$
$H_i\colon \mathbb{E}X_i = 0$, $H_i'\colon \mathbb{E}X_i \neq 0$, one sample t-test

No corrections:

	True H_is	False H_is	Total
Accepted H_is	143	0	143
Rejected H_is	7	50	57
Total	150	50	200

Bonferroni correction:

	True H_is	False H_is	Total
Accepted H_is	150	19	169
Rejected H_is	0	31	31
Total	150	50	200

Can we do better?

Bonferroni method:

$$\alpha_1 = \cdots = \alpha_m = \frac{\alpha}{m}$$

A more powerful method is possible if we allow α_is to vary.

Step-down methods

Sorted p-values:

$$p_{(1)} \leq \cdots \leq p_{(m)}$$

$H_{(1)}, \dots, H_{(m)}$ – corresponding hypotheses

Step-down procedure:

1. If $p_{(1)} > \alpha_1$, accept $H_{(1)}, \dots, H_{(m)}$ and stop; otherwise reject $H_{(1)}$ and continue
2. If $p_{(2)} > \alpha_2$, accept $H_{(2)}, \dots, H_{(m)}$ and stop; otherwise reject $H_{(2)}$ and continue
3. …

Holm's method

Holm's method – a step-down procedure with

$$\alpha_1 = \frac{\alpha}{m}, \alpha_2 = \frac{\alpha}{m-1}, \ldots, \alpha_i = \frac{\alpha}{m-i+1}, \ldots, \alpha_m = \alpha$$

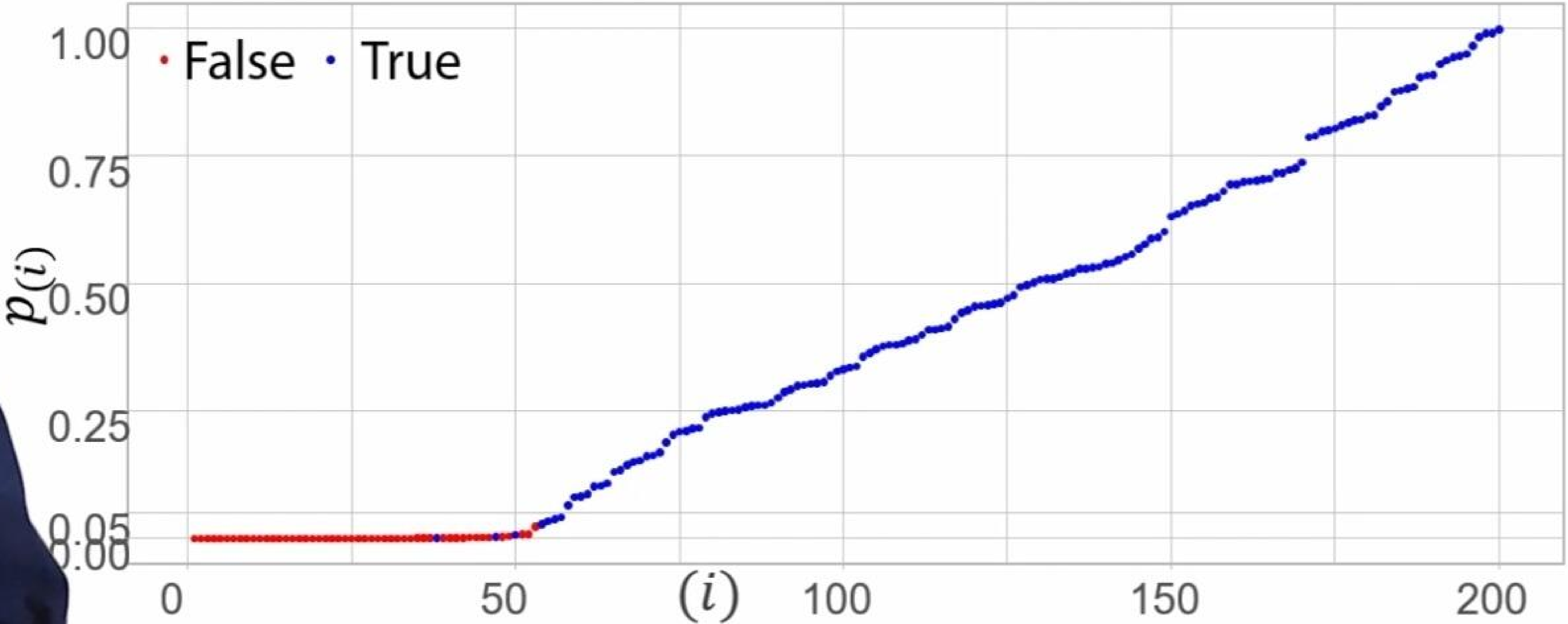
Adjusted p-values:

$$\tilde{p}_{(i)} = \min\left(1, \max\left((m-i+1)p_{(i)}, \tilde{p}_{(i-1)}\right)\right)$$

- FWER $\leq \alpha$ is guaranteed

Model experiment

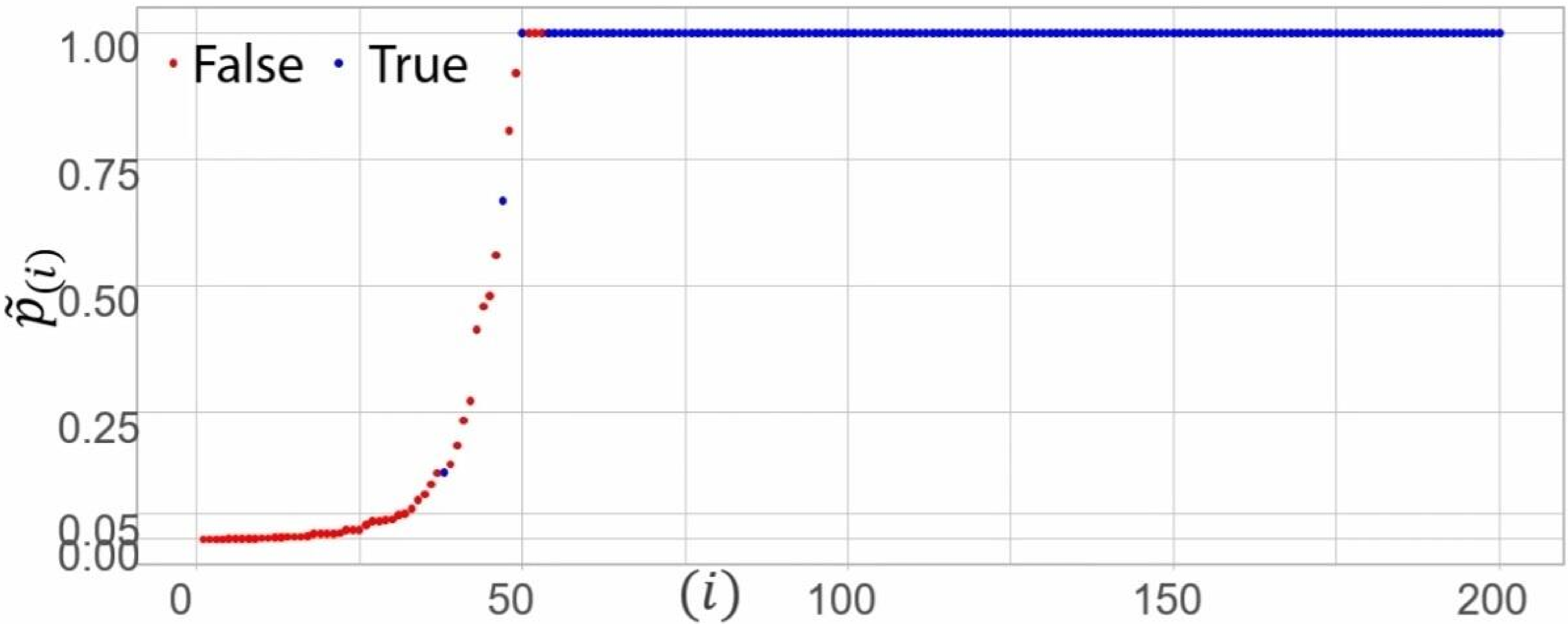
No corrections:



	True H_is	False H_is	Total
Accepted H_is	143	0	143
Rejected H_is	7	50	57
Total	150	50	200

Model experiment

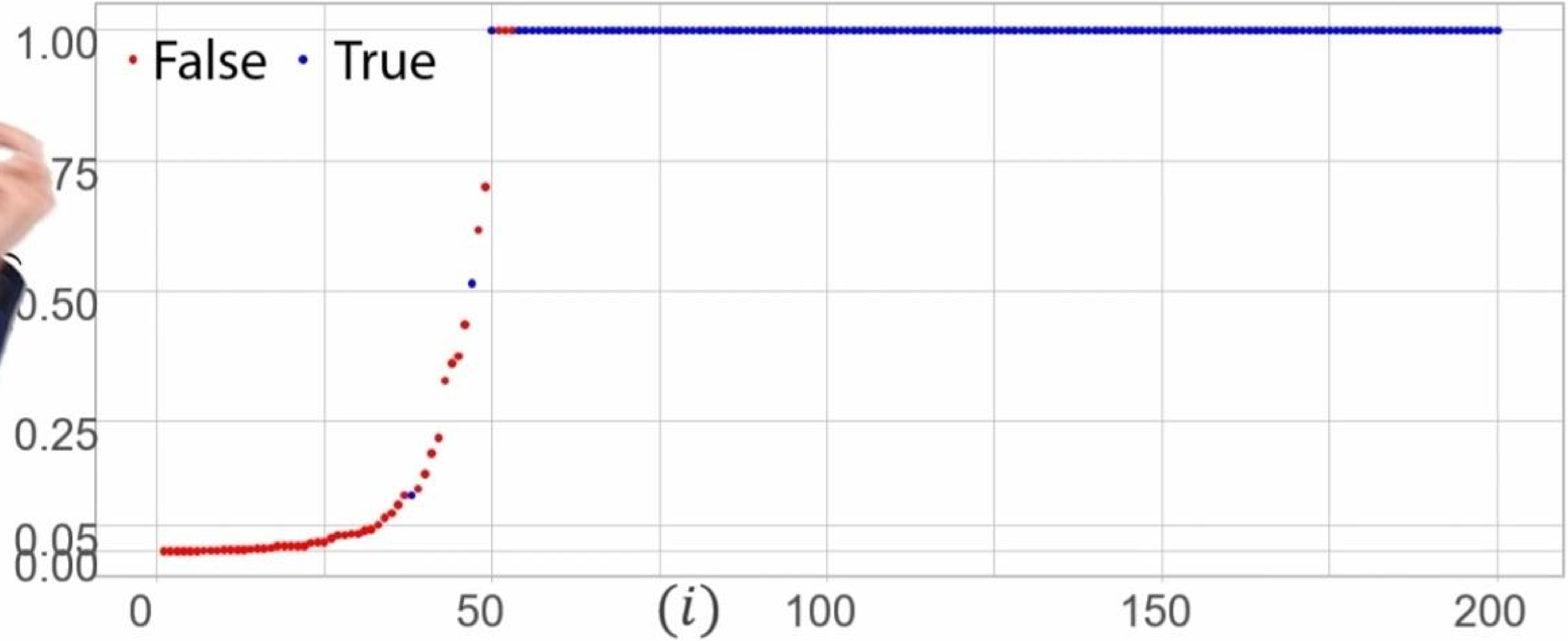
Bonferroni correction:



	True H_is	False H_is	Total
Accepted H_is	150	19	169
Rejected H_is	0	31	31
Total	150	50	200

Model experiment

Holm's method:



	True H_is	False H_is	Total
Accepted H_is	150	18	168
Rejected H_is	0	32	32
Total	150	50	200

Model experiment

Bonferroni correction:

	True H_is	False H_is	Total
Accepted H_is	150	19	169
Rejected H_is	0	31	31
Total	150	50	200

Holm's method:

	True H_is	False H_is	Total
Accepted H_is	150	18	168
Rejected H_is	0	32	32
Total	150	50	200

Takeaways about FWER

- Control FWER if it's very important not to make ANY type I error
- Use Holm's method instead of Bonferroni to reject more hypotheses FOR FREE