

## Appendix I. COUNTER-EXAMPLES OF STOCHASTIC PROCESSES

If the carbon price stochastic process doesn't meet the sufficient conditions proposed in Section III, the corresponding properties of NSD strategy cannot be assured. Several simple counter-examples are provided as follows (Although more complicated examples can be constructed).

### (1) Without Condition 1, Property 1 cannot be assured

Suppose there is a stochastic process that meets all sufficient conditions from day  $d^*$  to  $D$ . Then we have  $A_{\text{buy},d}(\lambda_{C,d})$  and  $V_{\text{buy},d}(\lambda_{C,d})$  are both nondecreasing in  $\lambda_{C,d}$  for all  $d \geq d^*$ .

Choose  $x_1, x_2, y_1, y_2$  that satisfy

$$A_{\text{buy},d^*}(y_1) < x_1 < A_{\text{buy},d^*}(y_2) < x_2, y_1 < y_2$$

Violate Condition 1 by setting

$$P(\lambda_{C,d^*}=y_2 | \lambda_{C,d^*-1}=x_1)=1, P(\lambda_{C,d^*}=y_1 | \lambda_{C,d^*-1}=x_2)=1$$

Then we have

$$\begin{aligned} V_{\text{buy},d^*-1}(x_1) &= A_{\text{buy},d^*}(y_2) \\ V_{\text{buy},d^*-1}(x_2) &= A_{\text{buy},d^*}(y_1) \\ A_{\text{buy},d^*-1}(x_1) &= \min\{x_1, V_{\text{buy},d^*-1}(x_1)\} \\ &= \min\{x_1, A_{\text{buy},d^*}(y_2)\} \\ &= x_1 \\ A_{\text{buy},d^*-1}(x_2) &= \min\{x_2, V_{\text{buy},d^*-1}(x_2)\} \\ &= \min\{x_2, A_{\text{buy},d^*}(y_1)\} \\ &= A_{\text{buy},d^*}(y_1) \end{aligned}$$

So, Property 1 doesn't hold as

$$\begin{aligned} V_{\text{buy},d^*-1}(x_2) &< V_{\text{buy},d^*-1}(x_1) \\ A_{\text{buy},d^*-1}(x_2) &< A_{\text{buy},d^*-1}(x_1) \end{aligned}$$

### (2) Without Condition 1b, Property 1 cannot be assured

Suppose there is a stochastic process that meets all sufficient conditions from day  $d^*$  to  $D$ . Then we have  $A_{\text{buy},d}(\lambda_{C,d})$  and  $V_{\text{buy},d}(\lambda_{C,d})$  are both nondecreasing in  $\lambda_{C,d}$  for all  $d \geq d^*$ .

Choose  $x_1, x_2, y_1, y_2$  that satisfy

$$\begin{aligned} y_2 - y_1 &> x_2 - x_1 > 0, x_2 > y_2, x_1 > y_1 \\ A_{\text{buy},d^*}(y_2) - A_{\text{buy},d^*}(y_1) &> x_2 - x_1 \end{aligned}$$

Violate Condition 1b by setting

$$P(\lambda_{C,d^*}=y_1 | \lambda_{C,d^*-1}=x_1)=1, P(\lambda_{C,d^*}=y_2 | \lambda_{C,d^*-1}=x_2)=1$$

Then we have

$$\begin{aligned} A_{\text{buy},d^*-1}(x_1) &= \min\{x_1, V_{\text{buy},d^*-1}(x_1)\} \\ &= \min\{x_1, A_{\text{buy},d^*}(y_1)\} \\ &= A_{\text{buy},d^*}(y_1) \end{aligned}$$

$$\begin{aligned} A_{\text{buy},d^*-1}(x_2) &= \min\{x_2, V_{\text{buy},d^*-1}(x_2)\} \\ &= \min\{x_2, A_{\text{buy},d^*}(y_2)\} \\ &= A_{\text{buy},d^*}(y_2) \end{aligned}$$

So, Property 2 doesn't hold as

$$\begin{aligned} \Delta A_{\text{buy},d^*-1}(x_2) - \Delta A_{\text{buy},d^*-1}(x_1) \\ &= x_2 - A_{\text{buy},d^*-1}(x_2) - [x_1 - A_{\text{buy},d^*-1}(x_1)] \\ &= x_2 - x_1 - [A_{\text{buy},d^*}(y_2) - A_{\text{buy},d^*}(y_1)] < 0 \end{aligned}$$

### (3) Without Condition 2, Property 3 cannot be assured

Suppose there is a stochastic process that meets all sufficient conditions from day  $d^*$  to  $D$ . Then we have  $A_{\text{buy},d}(\lambda_{C,d})$  and  $V_{\text{buy},d}(\lambda_{C,d})$  are both nondecreasing in  $\lambda_{C,d}$  for all  $d \geq d^*$ .

Choose  $x_1$  and  $y_1$  that satisfy

$$V_{\text{buy},d^*}(y_1) < x_1 < y_1$$

Violate Condition 2 by setting

$$P(\lambda_{C,d^*}=y_1 | \lambda_{C,d^*-1}=x)=1, \forall x$$

So, Property 3 doesn't hold as

$$V_{\text{buy},d^*-1}(x_1) = A_{\text{buy},d^*}(y_1) = V_{\text{buy},d^*}(y_1) > V_{\text{buy},d^*}(x_1)$$

$$A_{\text{buy},d^*-1}(x_1) = \min\{x_1, A_{\text{buy},d^*}(y_1)\} = A_{\text{buy},d^*}(y_1) > A_{\text{buy},d^*}(x_1)$$

### (4) Without Condition 2, Property 4 cannot be assured

Suppose there is a stochastic process that meets all sufficient conditions from day  $d^*$  to  $D$ . Then we have  $A_{\text{buy},d}(\lambda_{C,d})$  and  $V_{\text{buy},d}(\lambda_{C,d})$  are both nondecreasing in  $\lambda_{C,d}$  for all  $d \geq d^*$ . And there is a unique  $\theta_d^*$  that satisfy  $V_{\text{buy},d^*}(\theta_d^*) = \theta_d^*$ .

Choose  $x_1, x_2, x_3$  that satisfy

$$\begin{aligned} \theta_{d^*} &< x_1 < x_2 < x_3 \\ A_{\text{buy},d^*}(x_2) &= x_1, A_{\text{buy},d^*}(x_3) > \theta_3 \end{aligned}$$

Violate Condition 2 by setting

$$\begin{aligned} P(\lambda_{C,d^*}=x_2 | \lambda_{C,d^*-1}=x_1) &= 1 \\ P(\lambda_{C,d^*}=\theta_{d^*} | \lambda_{C,d^*-1}=x_1) &= 1 \\ P(\lambda_{C,d^*}=x_3 | \lambda_{C,d^*-1}=x_1) &= 1 \end{aligned}$$

Then we have

$$\begin{aligned} V_{\text{buy},d^*-1}(x_1) &= A_{\text{buy},d^*}(x_2) = x_1 \\ V_{\text{buy},d^*-1}(x > x_1) &= A_{\text{buy},d^*}(x_3) > x_1 \\ V_{\text{buy},d^*-1}(x < x_1) &= A_{\text{buy},d^*}(\theta_{d^*}) < x_1 \end{aligned}$$

So, Property 4 doesn't hold as

$$\theta_{d^*-1} = x_1 > \theta_{d^*}$$

## Appendix II. PARAMETERS USED IN THE SIMULATIONS

### A. Generator Parameters

The weight and price of gas are calculated in the state of LNG.

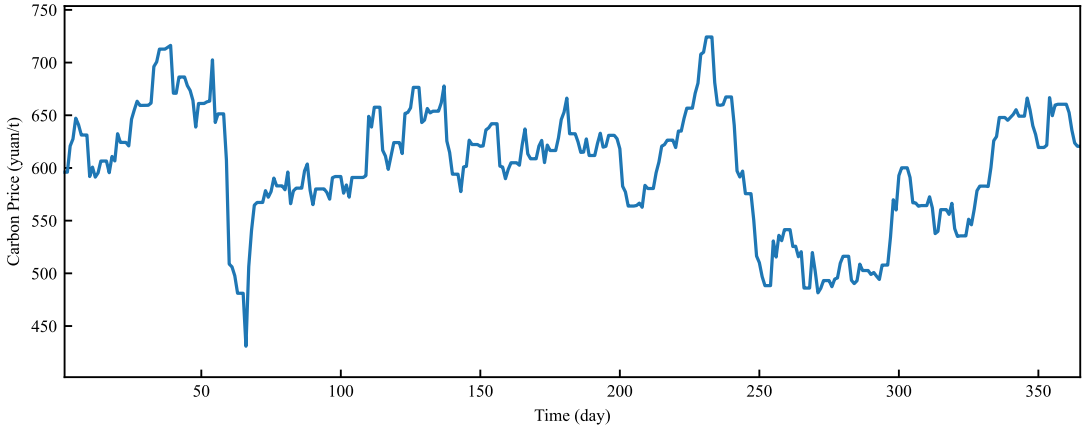
g	Fuel	$\lambda_{W,g}$ (yuan/t)	$P_{\text{max},g}$ (MW)	$P_{\text{min},g}$ (MW)	$P_{U,g}$ (MW)	$P_{D,g}$ (MW)	$h_{U,g}$ (hour)	$h_{D,g}$ (hour)	$\lambda_{U,g}$ (yuan)	$\lambda_{D,g}$ (yuan)	$\alpha_{g,2}$ (t/MW <sup>2</sup> h)	$\alpha_{g,1}$ (t/MWh)	$\alpha_{g,0}$ (t/h)	$k_g$ (t/t)	$k_{\text{free},g}$ (t/MWh)
1	Coal	600	320	120	120	120	4	4	800000	180000	3.00E-05	0.300	11.200	2.262	0
2	Gas	3000	300	100	600	600	1	1	100000	100000	3.93E-05	0.093	6.115	3.082	0

B. Miscellaneous Parameters

$\bar{\lambda}$	$\eta$	$\sigma_M$	$\sigma_B$	$\sigma_{TVN,d}$	$\lambda_{pen}$	$D$	$I$	$N$	$D_F$
602.78	0.041	16.24	16.24	81.18	$2\bar{\lambda}$	365	10000	5	0

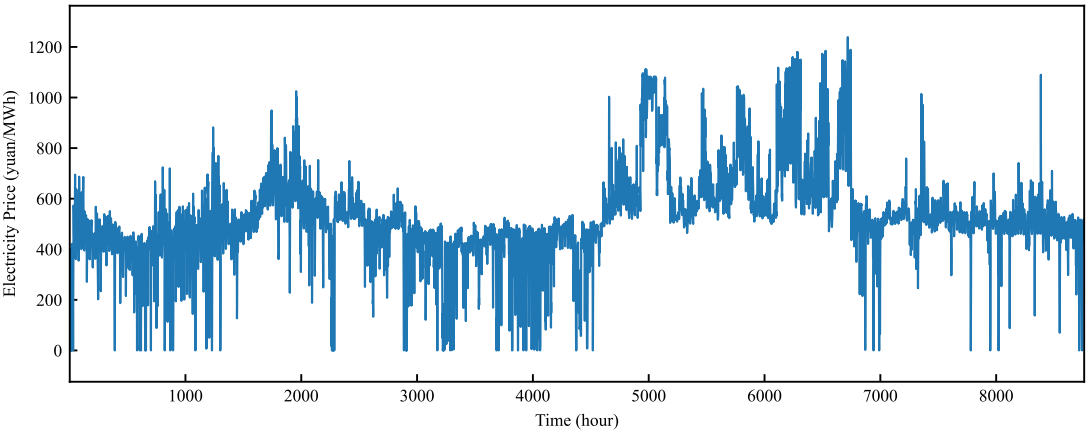
C. Daily Carbon Prices of EU ETS, 2022

Converted from EUR to Chinese Yuan at a ratio of 1:7.39.

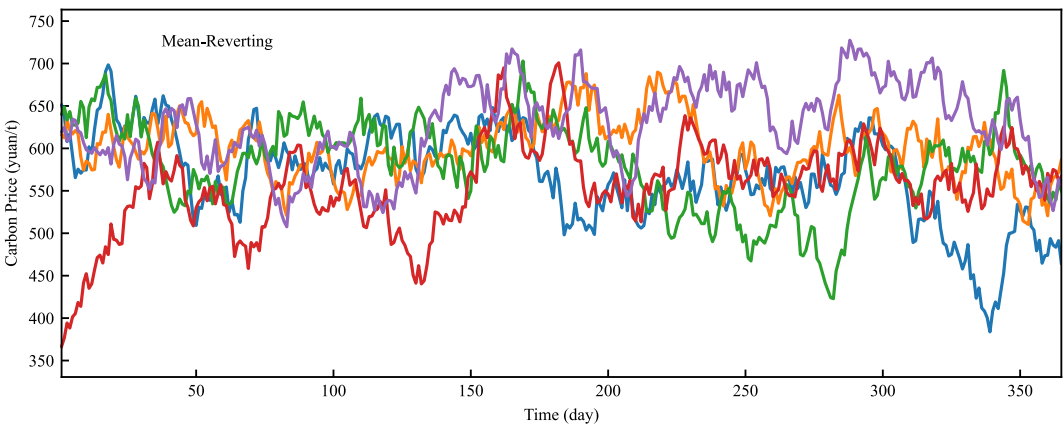


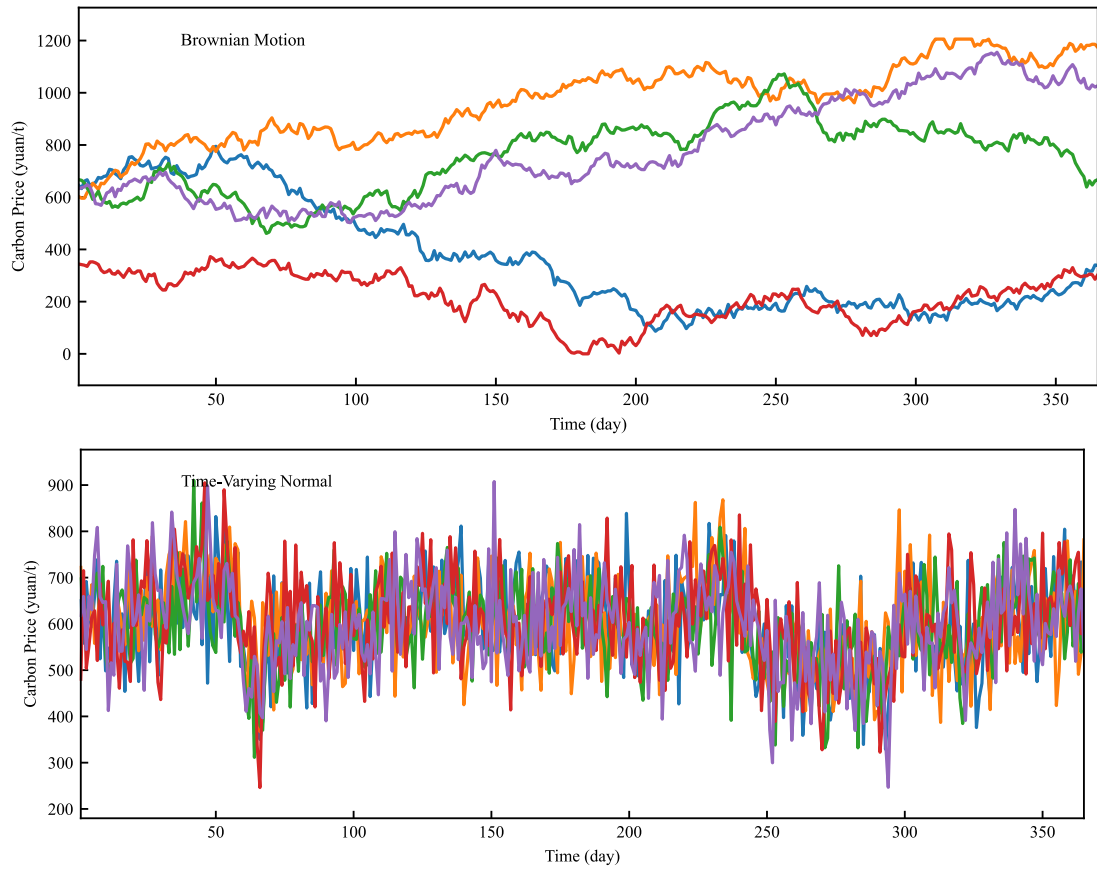
D. Hourly Electricity Prices of Guangdong, China, 2022

These prices are used as the means of a time-varying normal distribution to generate stochastic scenarios of electricity prices in the simulations.



Appendix III. EXAMPLES OF CARBON PRICE CURVES GENERATED BY STOCHASTIC PROCESSES





#### Appendix IV. THE FIRST 12 SCENARIOS OUT OF THE TOTAL 1000 SCENARIOS OF SYNERGISTIC DECISION COMPARISON

All 1000 scenarios are provided online at <https://github.com/shishouyuan/PaperData> **NSD Carbon Trading Strategy**

