Fuzzy vs. Probability

Fuzzy: When we say about certainty of thing

Example: A patient come to the doctor and he has to diagnose so that medicine can be prescribed.
Doctor prescribed a medicine with certainty 60% that the patient is suffering from flue. So, the disease will be cured with certainty of 60% and uncertainty of 40%. Here, instead of flue, other diseases with some other certainties may be.

Probability: When we say about the chance of an event to occur.

Example: India will win this T20 World Cup with a chance 60% means that out of 100 matches, India won 60 matches.

Prediction vs. Forecasting

The Fuzzy vs. Probability is analogical to Prediction vs. Forecasting

Prediction: When you start guessing about the things.
Forecasting: When you take the information from the past job and apply it to new job.

The main differences

Prediction is based on the **best guess from experiences.**
Forecasting is based on data you have actually recorded and packed from previous job.

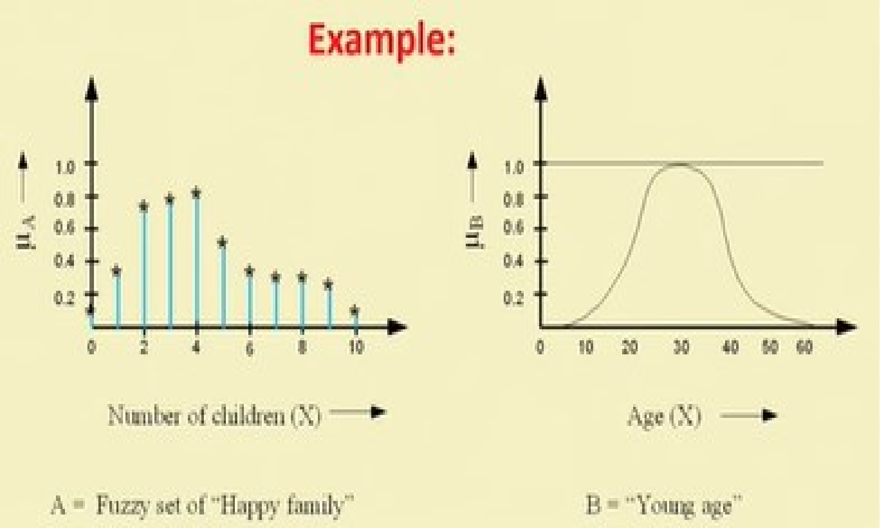
Fuzzy membership functions

- A fuzzy set is completely characterized by its membership function (sometimes abbreviated as MF and denoted as μ). So, it would be important to learn how a membership function can be expressed (mathematically or otherwise)

Note: A membership function can be on-

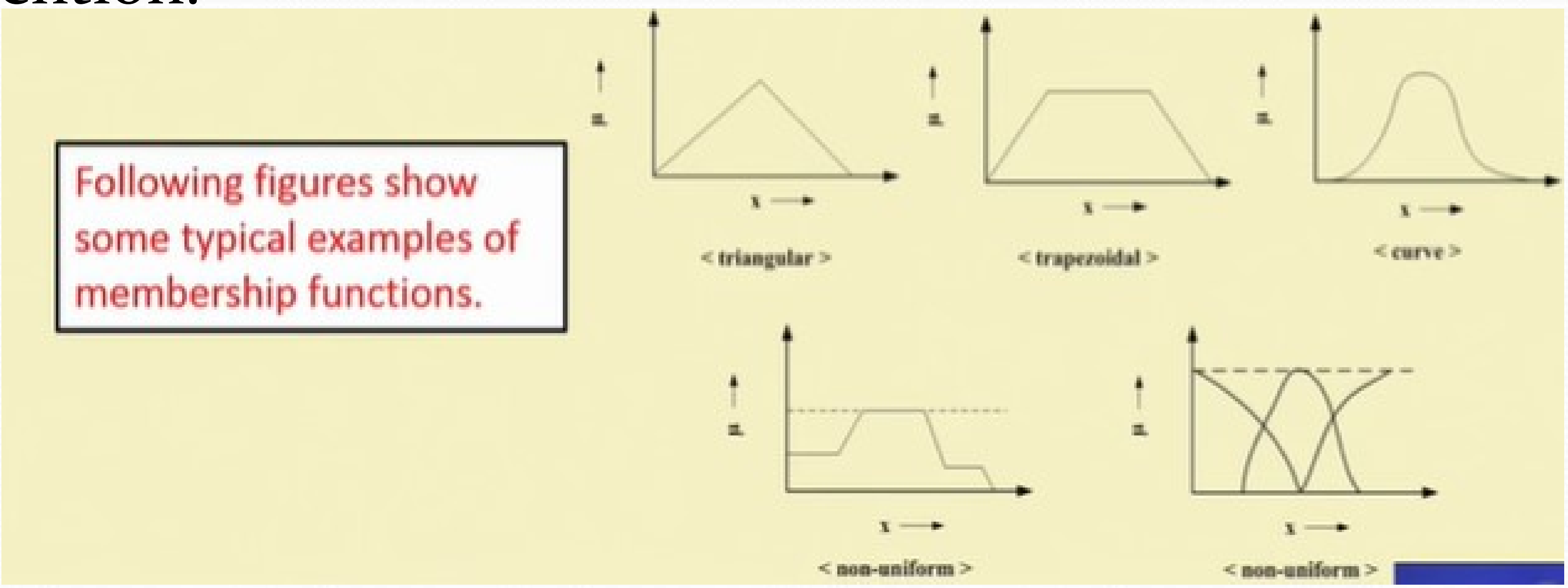
a) a discrete universe of discourse and

b) a continuous universe of discourse



Fuzzy membership functions

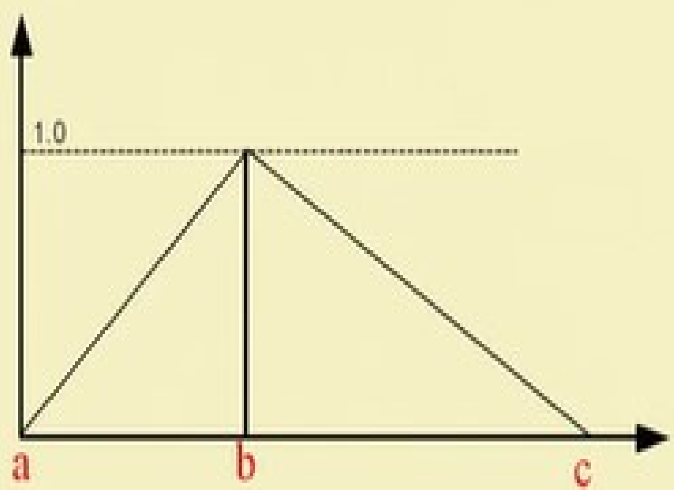
So membership function on a discrete universe of course is trivial. However a membership function on a continuous universe of discourse needs a special attention.



Fuzzy MFs: Formulation & Parameterization

In the following, we try to parameterize the different MFs on a continuous universe of discourse.

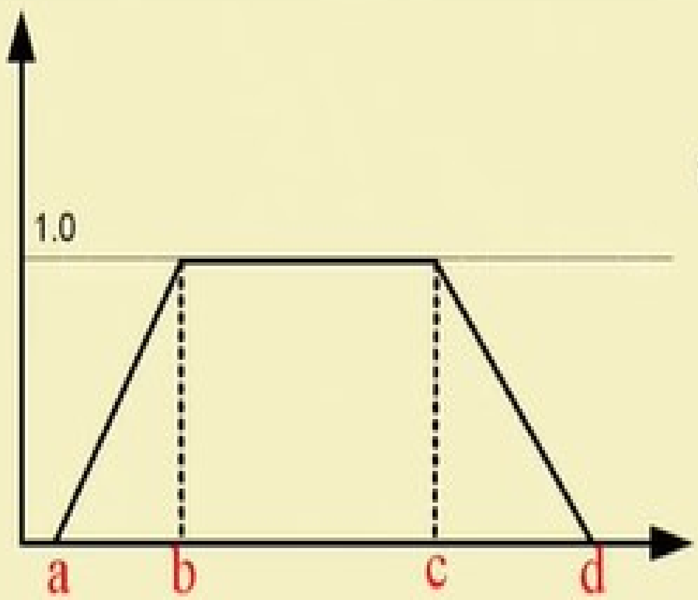
Triangular MFs: A triangular MF is specified by three parameter {a, b, c} and can be formulated as follows-



$$\text{triangle}(x; a, b, c) = \begin{cases} 0 & if\ x \le a \\ \dfrac{x-a}{b-a} & if\ a \le x \le b \\ \dfrac{c-x}{c-b} & if\ b \le x \le c \\ 0 & if\ c \le x \end{cases}$$

Fuzzy MFs: Trapezoidal

A Trapezoidal MF is specified by four parameters {a, b, c, d} and can be defined as follows:

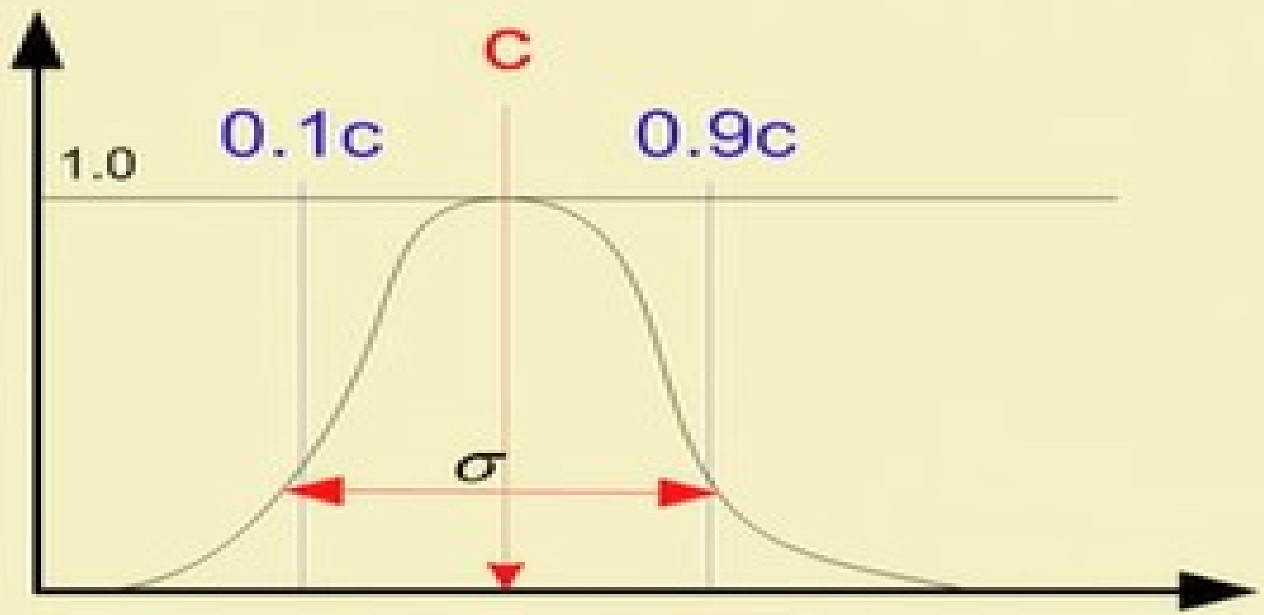


$$\text{trapozoid}\,(x; a, b, c, d) = \begin{cases} 0 & if\ x \le a \\ \dfrac{x-a}{b-a} & if\ a \le x \le b \\ 1 & if\ b \le x \le c \\ \dfrac{d-x}{d-c} & if\ c \le x \le d \\ 0 & if\ d \le x \end{cases}$$

Fuzzy MFs: Gaussian

A Gaussian MF is specified by two parameters {c, σ} and can be defined as below:

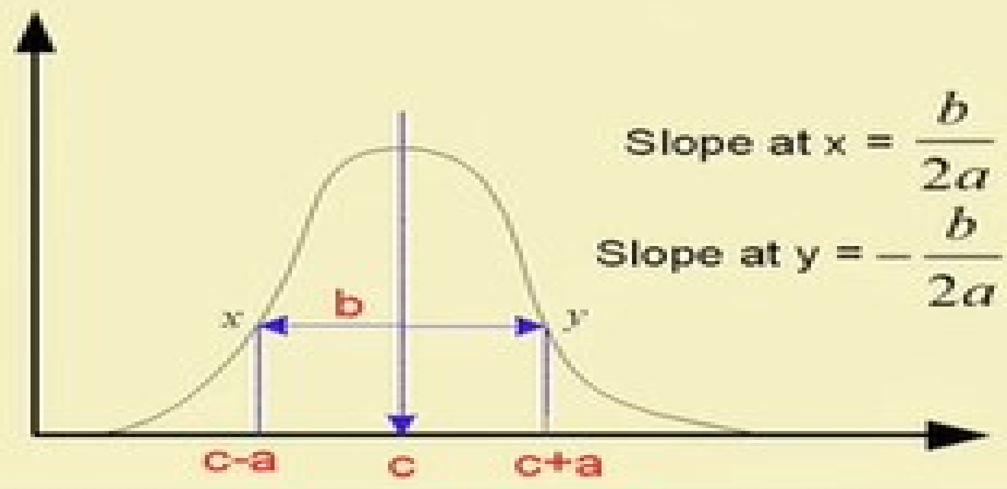
$$gaussian\,(x; c, \sigma) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}$$



Fuzzy MFs: Generalized Bell

It is also called Cauchy MFs. A Generalized bell MF is specified by three parameters {a, b, c}and is defined as follows-

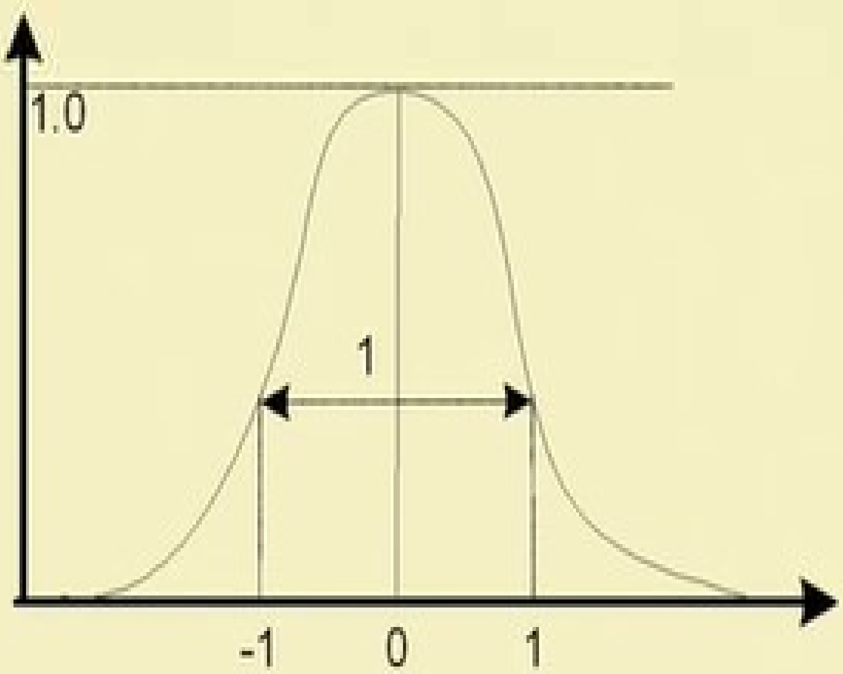
$$bell(x;\, a, b, c) = \frac{1}{1 + \left|\frac{x - c}{a}\right|^{2b}}$$



Example: Generalized Bell MF

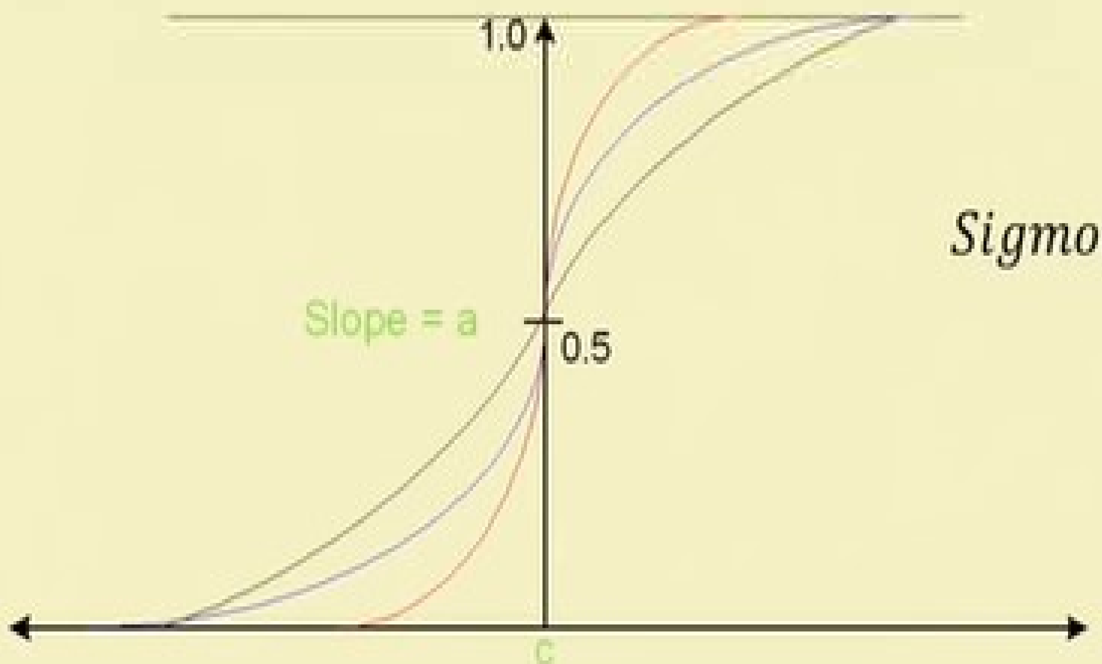
Example: $\mu(x) = \frac{1}{1+|x|^{2}}$;

a = b = 1 and c = 0;



Fuzzy MFs: Sigmoidal MFs

Parameters {a, c} ; where c = crossover point and a = slope at c



$$Sigmoid(x; a, c) = \frac{1}{1 + e^{-\left[\frac{x-c}{a}\right]}}$$

Fuzzy MFs: Example

Example: Consider the following grading system for a course

Excellent = Marks $\geq$ 90

Very Good = 75 $\leq$ Marks $\leq$ 90

Good = 60 $\leq$ Marks $\leq$ 70

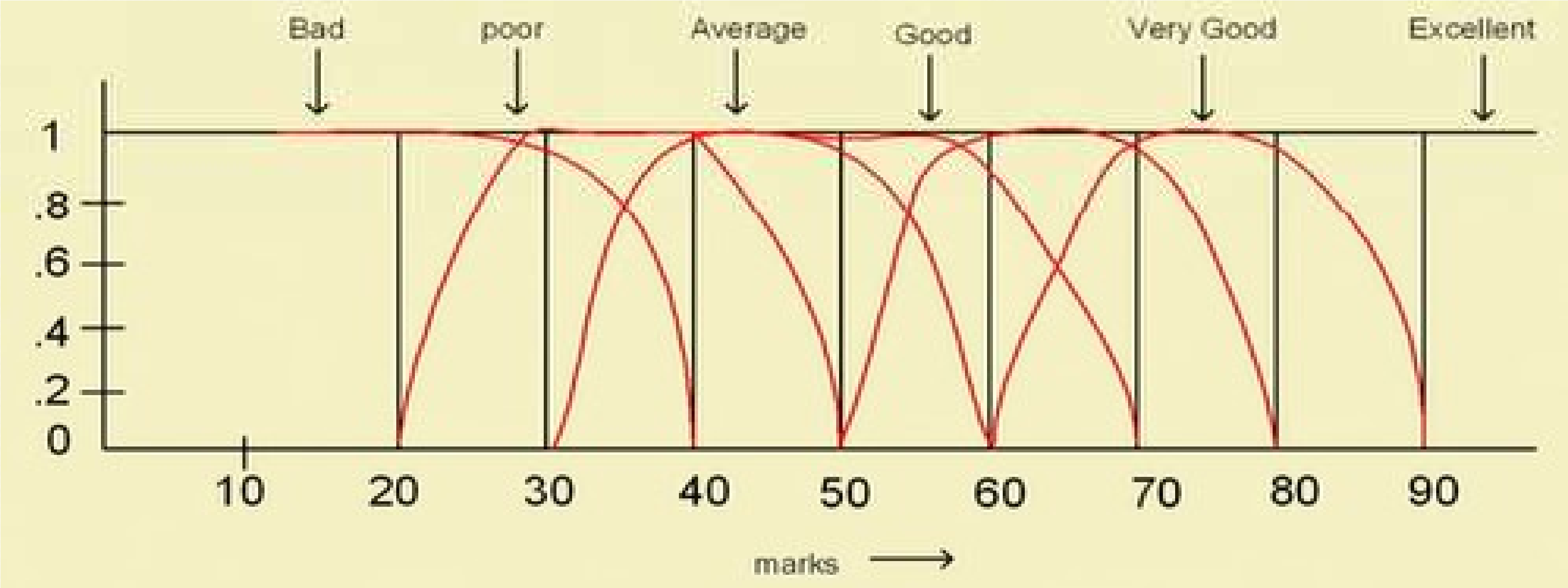
Average = 50 $\leq$ Marks $\leq$ 60

Poor = 35 $\leq$ Marks $\leq$ 50

Bad = Marks $\leq$ 35

Grading System

A Fuzzy implementation look like the following



You can decide a standard fuzzy MF for each fuzzy grade

Generation of MFs

Given a membership function of a fuzzy set representing a linguistic hedge , we can derive many more MFs representing several other linguistic hedges using the concept of **Concentration** and **Dilation**.

1. Concentration: $A^K = [\mu_A (x)]^K$; $K > 1$

2. Dilation: $A^K = [\mu_A (x)]^K$; $K < 1$

Example: Age = { Young, Middle-aged, old}

Thus, corresponding to Young, we have : Not Young, Very Young, Not Very Young and so on.

Similarly, with old, we can have : Not Old, Very Old, Very very Old etc.

Thus, $\mu_{\text{Extremely old}} (x) = (((\mu_{old} (x))^2)^2)^2$

Or, $\mu_{\text{more or less old}} (x) = (\mu_{old} (x))^{0.5}$

Linguistic variables & values

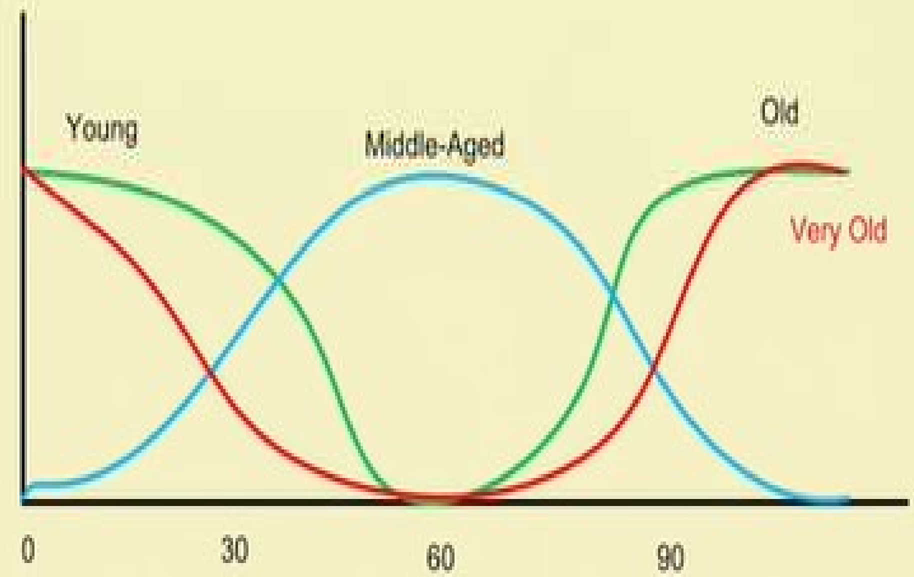
$\mu_{young}(x)$ =bell(x,20,2,0) = $\frac{1}{1+(\frac{x}{20})^4}$

$\mu_{old}(x)$ =bell(x,30,3,100) = $\frac{1}{1+(\frac{x-100}{30})^6}$

$\mu_{middle-aged}(x)$ =bell(x,30,60,50)

Not young=$\overline{\mu_{young}(x)} = 1 - \mu_{young}(x)$

Young but not too young = $\mu_{young}(x) \cap \overline{\mu_{young}(x)}$



Fuzzy Operations

Basic Fuzzy set operation: Union

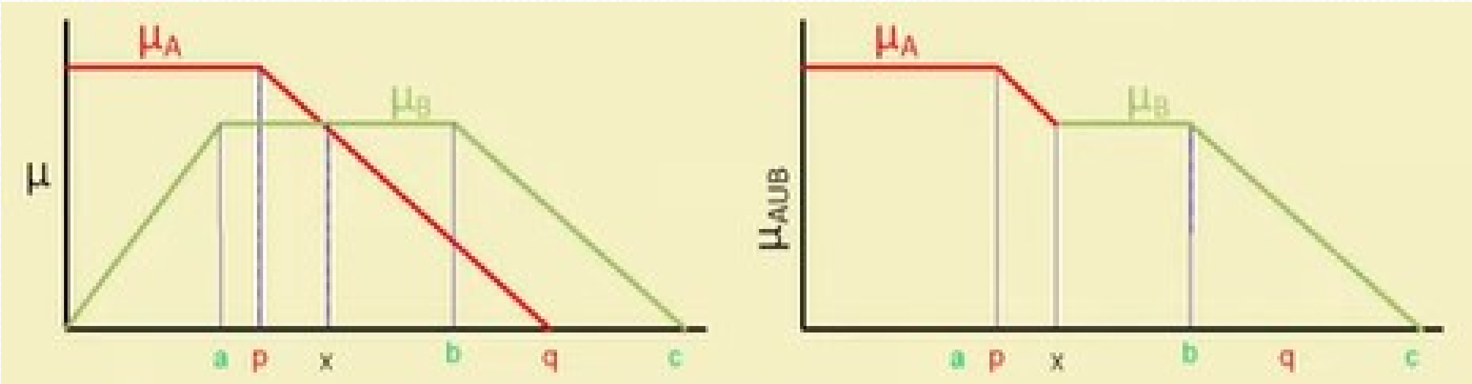
Union ($A \cup B$): $\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$

Example:

$A = \{(x_1, 0.5), (x_2, 0.1), (x_3, 0.4)\}$ and

$B = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.5)\}$;

$C = A \cup B = \{(x_1, 0.5), (x_2, 0.3), (x_3, 0.5)\}$



Basic Fuzzy set operation: Intersection

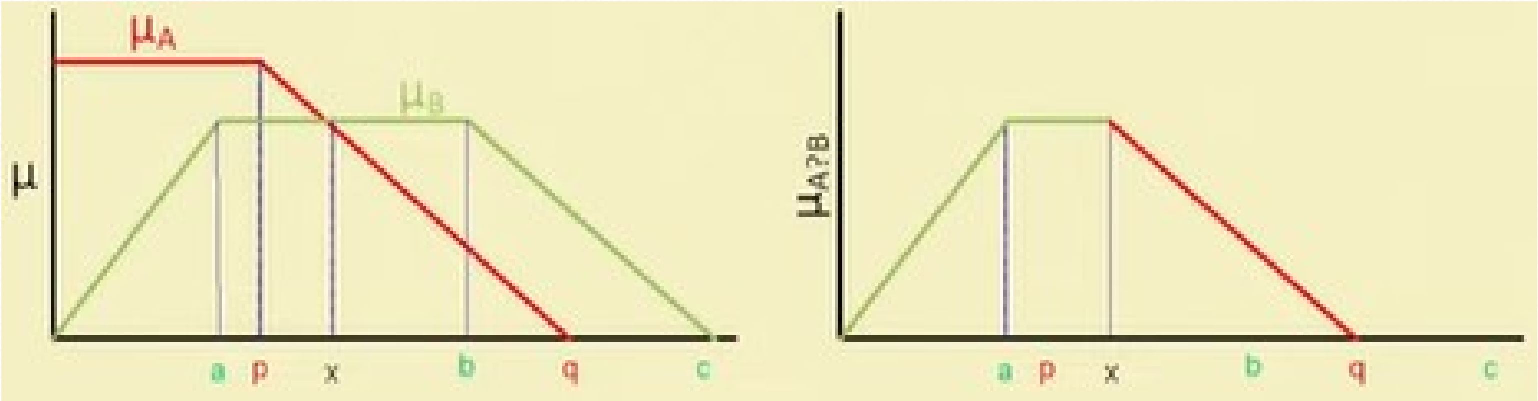
Intersection $(A \cap B)$: $\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$

Example:

$A = \{(x_1, 0.5), (x_2, 0.1), (x_3, 0.4)\}$ and

$B = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.5)\}$;

$C = A \cap B = \{(x_1, 0.2), (x_2, 0.1), (x_3, 0.4)\}$



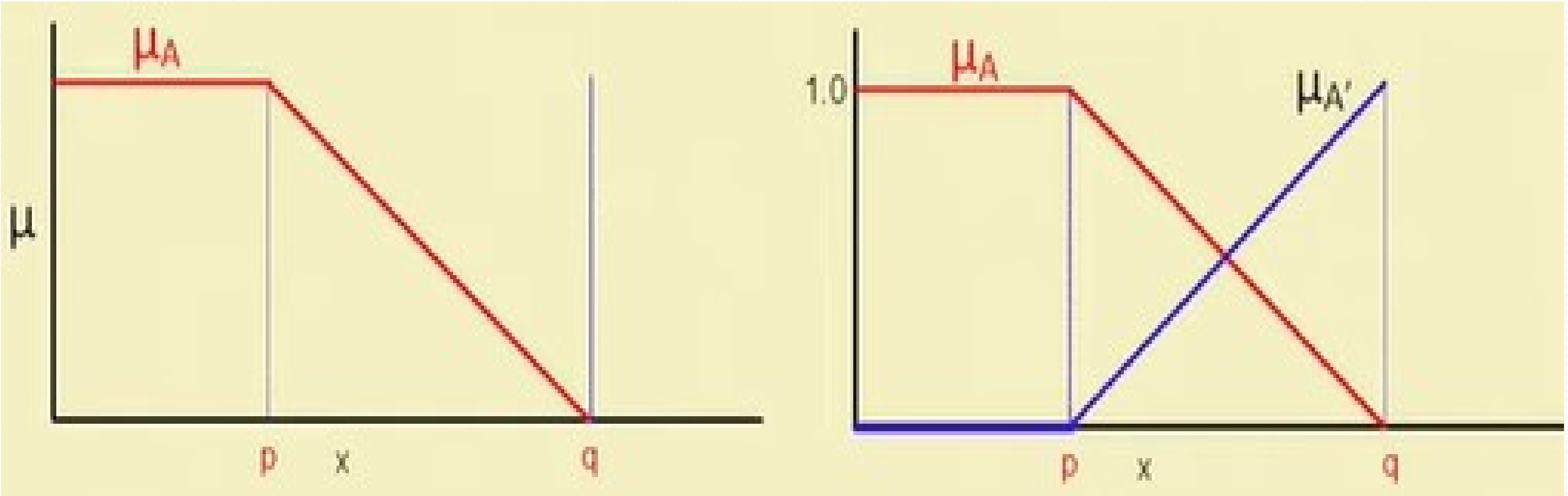
Basic Fuzzy set operation: Complement

Complement (A^C): $\mu_A{}^C(x) = 1 - \mu_A(x)$

Example:

A= {(x_1, 0.5), (x_2, 0.1), (x_3, 0.4)} and

C = A^C = {(x_1, 0.5), (x_2, 0.9), (x_3, 0.6)}



Basic Fuzzy set operation: Product

Algebric product or Vector product $(A \cdot B)$

$$\mu_{A \cdot B}(x) = \mu_A(x) \cdot \mu_A(x)$$

Scalar product $(\alpha \times A)$

$$\mu_{\alpha A}(x) = \alpha \times \mu_A(x)$$

Basic Fuzzy set operation: Sum & Difference

Sum (A + B):

$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \,.\, \mu_B(x)$

Difference ($A\text{-}B = A \cap B^C$):

$\mu_{A-B}(x) = \mu_{A \cap B^C}(x)$

Disjunctive sum:

$A \oplus B = (AC \cap B) \cup (A \cap BC)$

Bounded sum:

$| A(x) \oplus B(x)| = \mu_{| A(x) \oplus B(x) |} = \min\{ 1, \mu_A(x) + \mu_B(x)\}$

Bounded Difference:

$| A(x) \ominus B(x)| = \mu_{| A(x) \ominus B(x) |} = \max\{ 0, \mu_A(x) + \mu_B(x) - 1\}$

Basic Fuzzy set operation: Equality & Power

Equality (A = B):

$$\mu_A(x) = \mu_B(x)$$

Power of a Fuzzy set A^α

$$\mu_{A^\alpha}(x) = (\mu_A(x))^\alpha$$

- ✓ **If** $\alpha < 1$, then it is called dilation.
- ✓ **If** $\alpha > 1$, then it is called concentration.

Basic Fuzzy set operation: Cartesian product

Cartesian product (A × B):

$$\mu_{A\times B} = \min(\mu_A(x), \mu_B(x))$$

Example:

$A(x) = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.5), (x_4, 0.6)\}$

$B(y) = \{(y_1, 0.8), (y_2, 0.6), (y_3, 0.3)\}$

$$A \times B = \min(\mu_A(x), \mu_B(y)) = \begin{array}{c c} & \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} & \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.3 & 0.3 & 0.3 \\ 0.5 & 0.5 & 0.3 \\ 0.6 & 0.6 & 0.3 \end{bmatrix} \end{array}$$