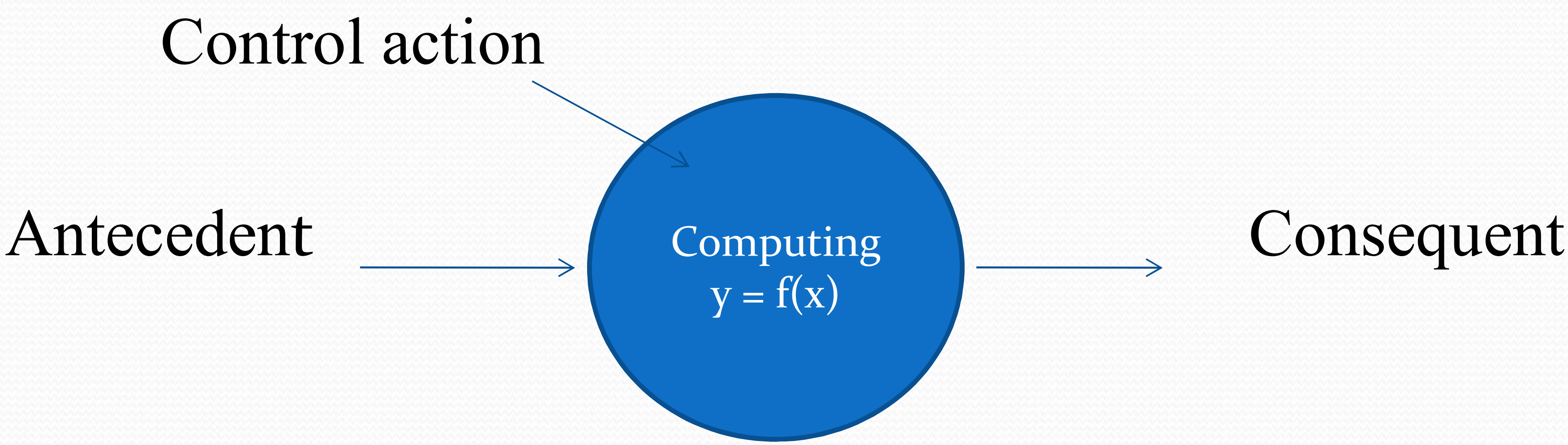
Introduction to Soft Computing

Introduction to Soft Computing

- Concept of computation
- Hard computing
- Soft computing
- How soft computing?
- Hard computing vs. Soft computing
- Hybrid computing

Concept of Computation



y = f(x), f is a mapping function

f is also called a formal method or an algorithm to solve a problem.

Important characteristics of computing

- Should provide precise solution.
- Control action should be unambiguous and accurate.
- Suitable for problem, which is easy to model mathematically.

Hard Computing

- In 1996, L. A. Zade (LAZ) introduced the term hard computing.
- According to LAZ: We term a computing as Hard computing, if-
  - Precise result is guaranteed.
  - Control action is unambiguous.
  - Control action is formally defined (i.e., with mathematical model or algorithm).

Example of Hard Computing

- Solving numerical problems (e.g., roots of polynomials, integration, etc.).
- Searching and sorting techniques.
- Solving computational geometry problems (e.g., shortest tour in a graph, finding closest pair of points given a set of points, etc.).
- Many more……

Soft Computing

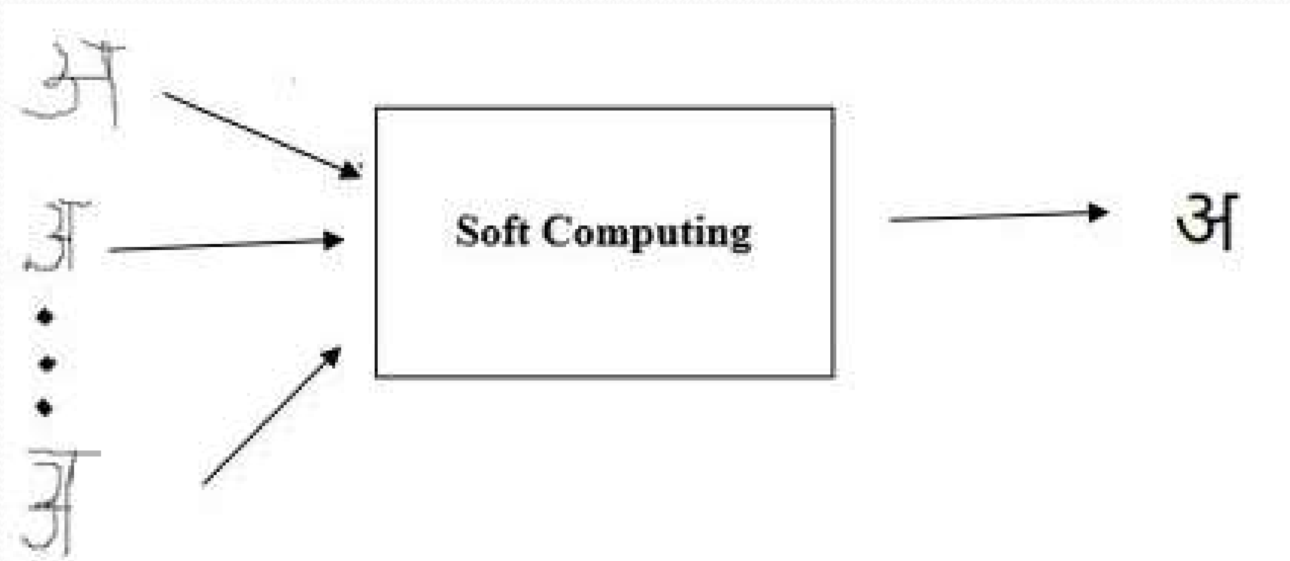
- The term Soft computing was proposed by the inventor of fuzzy logic, Lotfi A. Zadeh. He describes it as follows-

Definition 1: Soft Computing
Soft computing is a collection of methodologies that aim to exploit the tolerance for imprecision and uncertainty to achieve tractability, robustness, and low solution cost. Its principal constituents are fuzzy logic, neuro-computing, and probabilistic reasoning. The role model for soft computing is the human mind.

Characteristics of soft Computing

- It does not require any mathematical modeling of problem solving.
- It may not yield the precise solution.
- Algorithms are adaptive (i.e., it can adjust to change of dynamic environment).
- Use some biological inspired methodologies such as genetics, evolution, Ant's behaviors, particles swarming, human nervous system, etc.).

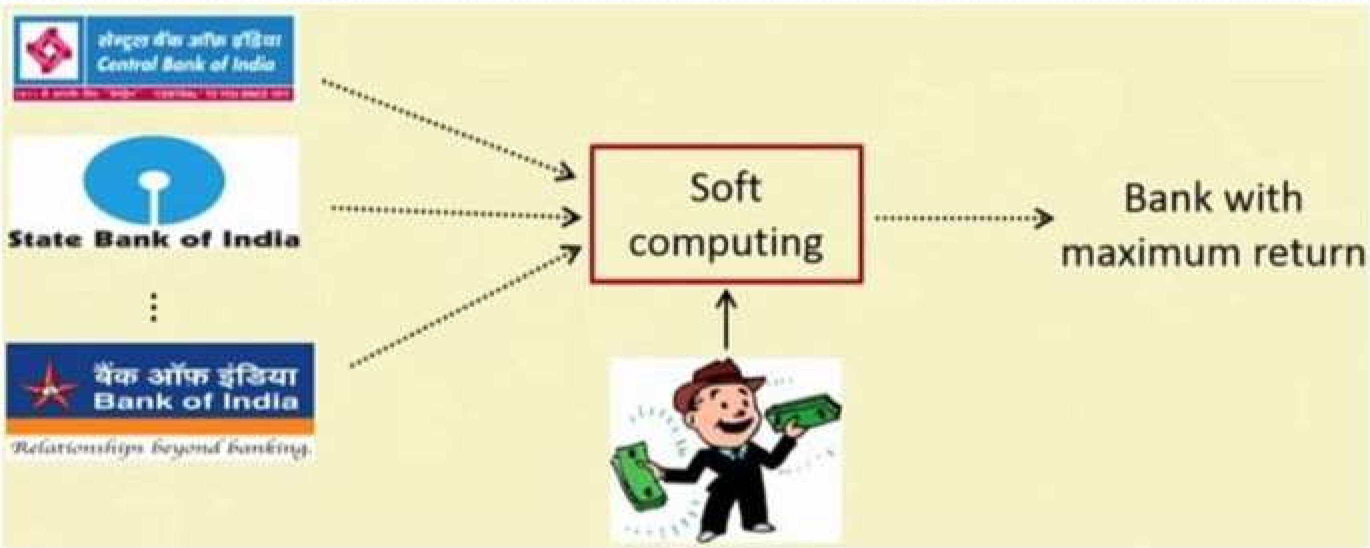
Examples of Soft Computing



Example: Hand written character recognition

(Artificial Neural Network)

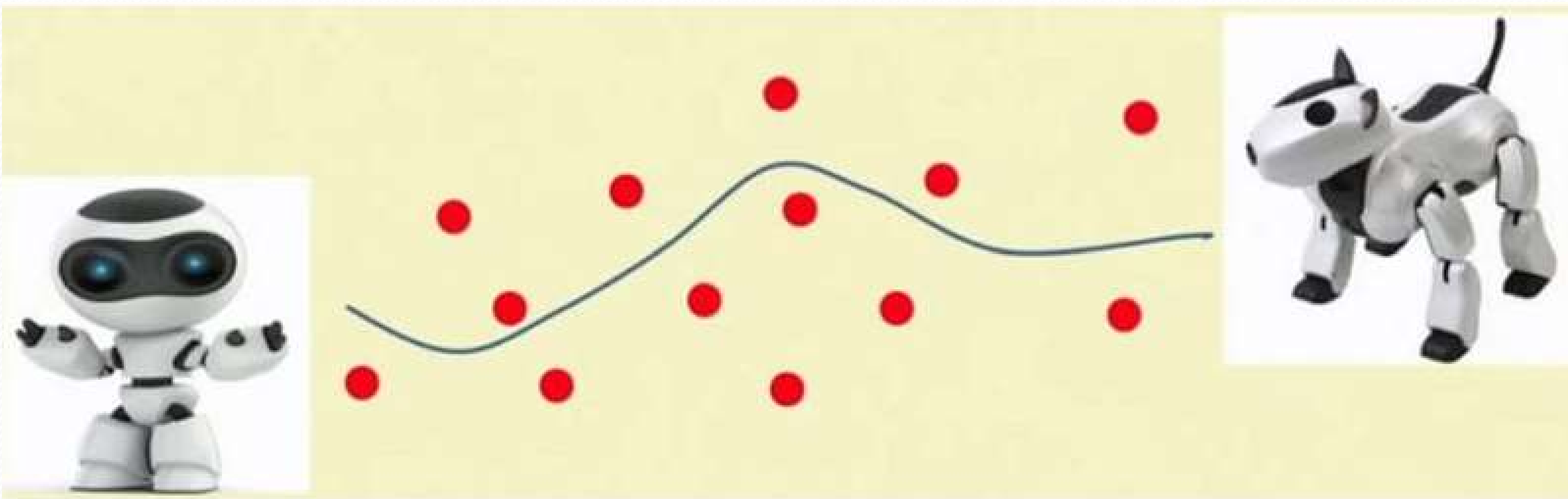
Examples of Soft Computing



Example: Money allocation problem

(Evolutionary Computing)

Examples of Soft Computing



Example: Robot movement

(Fuzzy Logic)

How Soft Computing?

- How a student learns from his teacher?
  - Teacher asks questions and tell the answers then.
  - Teacher puts questions and hints answers and asks weather the answers are correct or not.
  - Student thus learn a topic and store in his memory.
  - Based on the knowledge he solves new problems.
- This is the way how human brain works.
- Based on this concept Artificial Neural Network is used to solve problems.

How Soft Computing?

- How world selects the best?
  - It starts with a population (random).
  - Reproduces another population (next generation).
  - Rank the population and selects the superior individuals.
- Genetic algorithm is based on this natural phenomena.
  - Population is synonymous to solutions.
  - Selection of superior solution is synonymous to exploring optimal solution.

How Soft Computing?

- How a doctor treats his patient?
  - Doctor asks the patient about suffering.
  - Doctor find the symptoms of diseases.
  - Doctor prescribed tests and medicines.
- This is exactly the way Fuzzy Logic works.
  - Symptoms are correlated with diseases with uncertainty.
  - Doctor prescribes tests/medicines fuzzily.

Hard Computing vs. Soft Computing

Hard Computing	Soft Computing
It requires a precisely stated analytical model and often a lot of computation time.	It is tolerant of imprecision, uncertainty, partial truth, and approximation.
It is based on binary logic, crisp system, numerical analysis and crisp software.	It is based on fuzzy logic, neural nets and probabilistic reasoning.
It has the characteristics of precision and categoricity.	It has the characteristics of approximation and dispositionality.

Hybrid Computing

- It is a combination of the conventional hard computing and emerging soft computing.

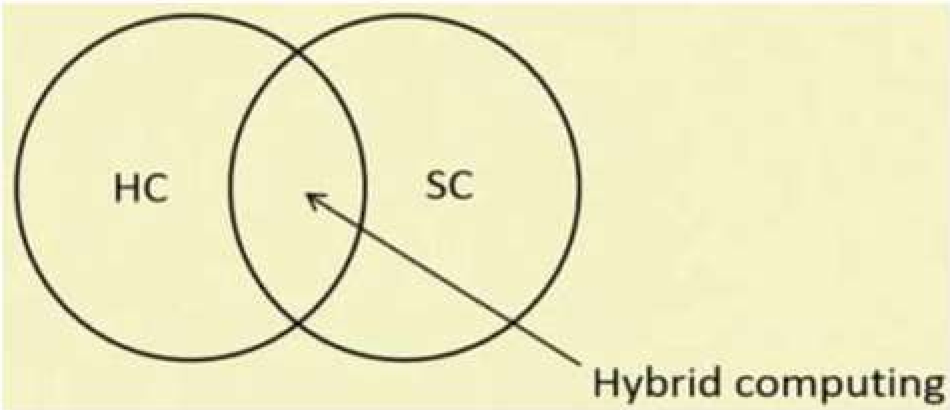


Figure: Concept of Hybrid Computing

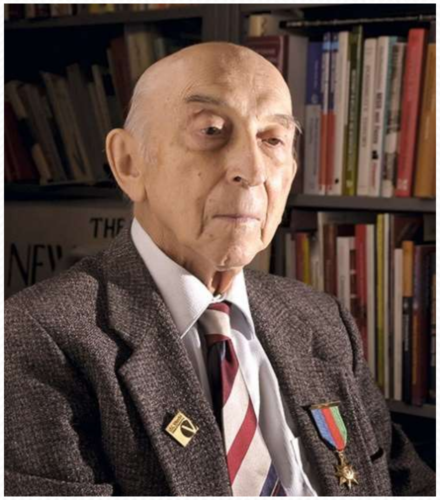
Introduction to Fuzzy Logic

What is Fuzzy Logic?

- Fuzzy logic is a mathematical language to express something.
  - This means it has grammar, syntax, semantic like a language for communication.
- There are some other mathematical languages also known
  - **Relational algebra** (Operation on set).
  - **Boolean algebra** (Operation on Boolean variables).
  - **Predicate algebra** (Operation on well formed formulea (wff), also called predicate propositions).
- **Fuzzy logic deals with** Fuzzy set or Fuzzy algebra .

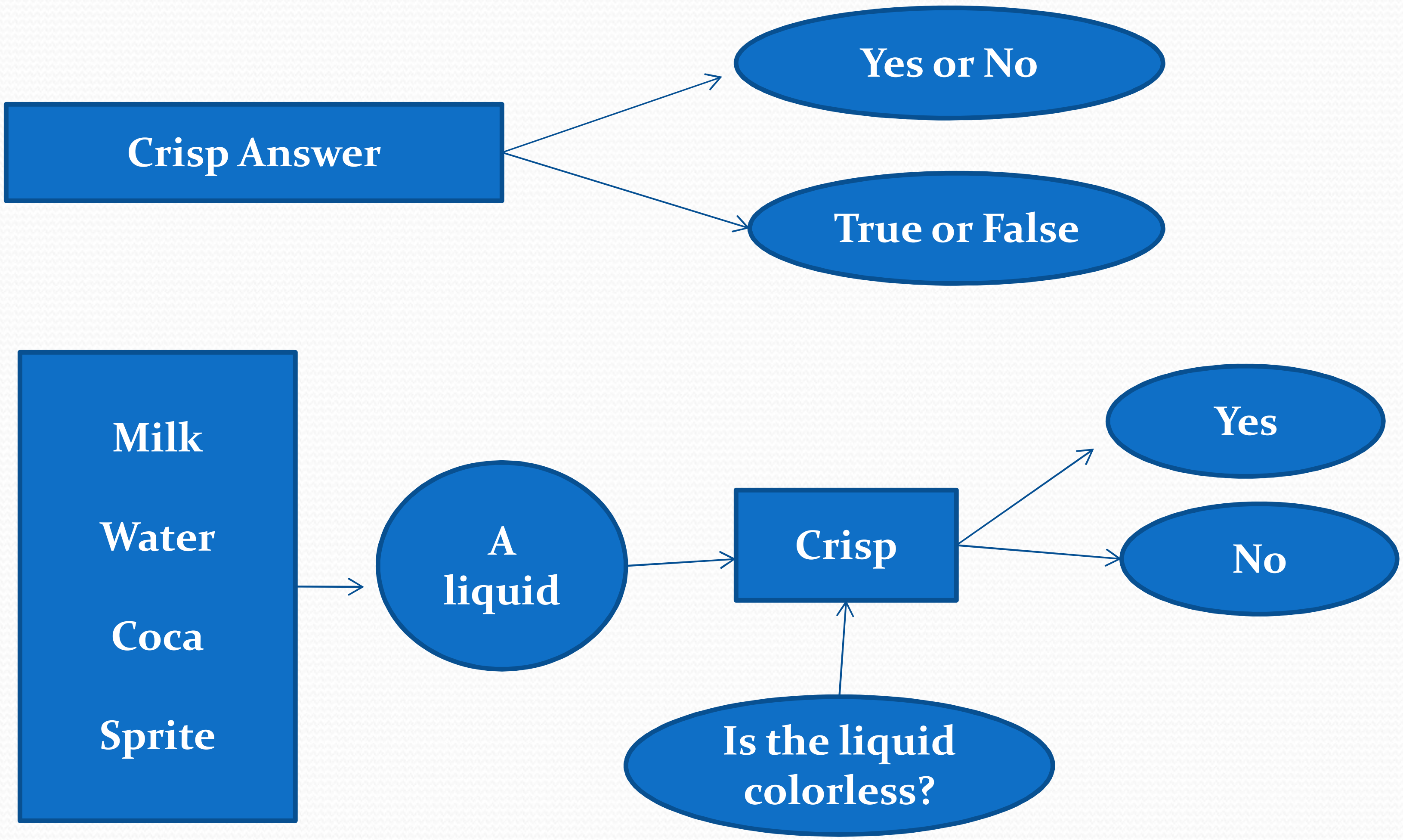
What is fuzzy?

- Dictionary meaning of fuzzy is **“not clear”, “noisy”** etc.
  - Example: Is the picture on this slide is fuzzy?

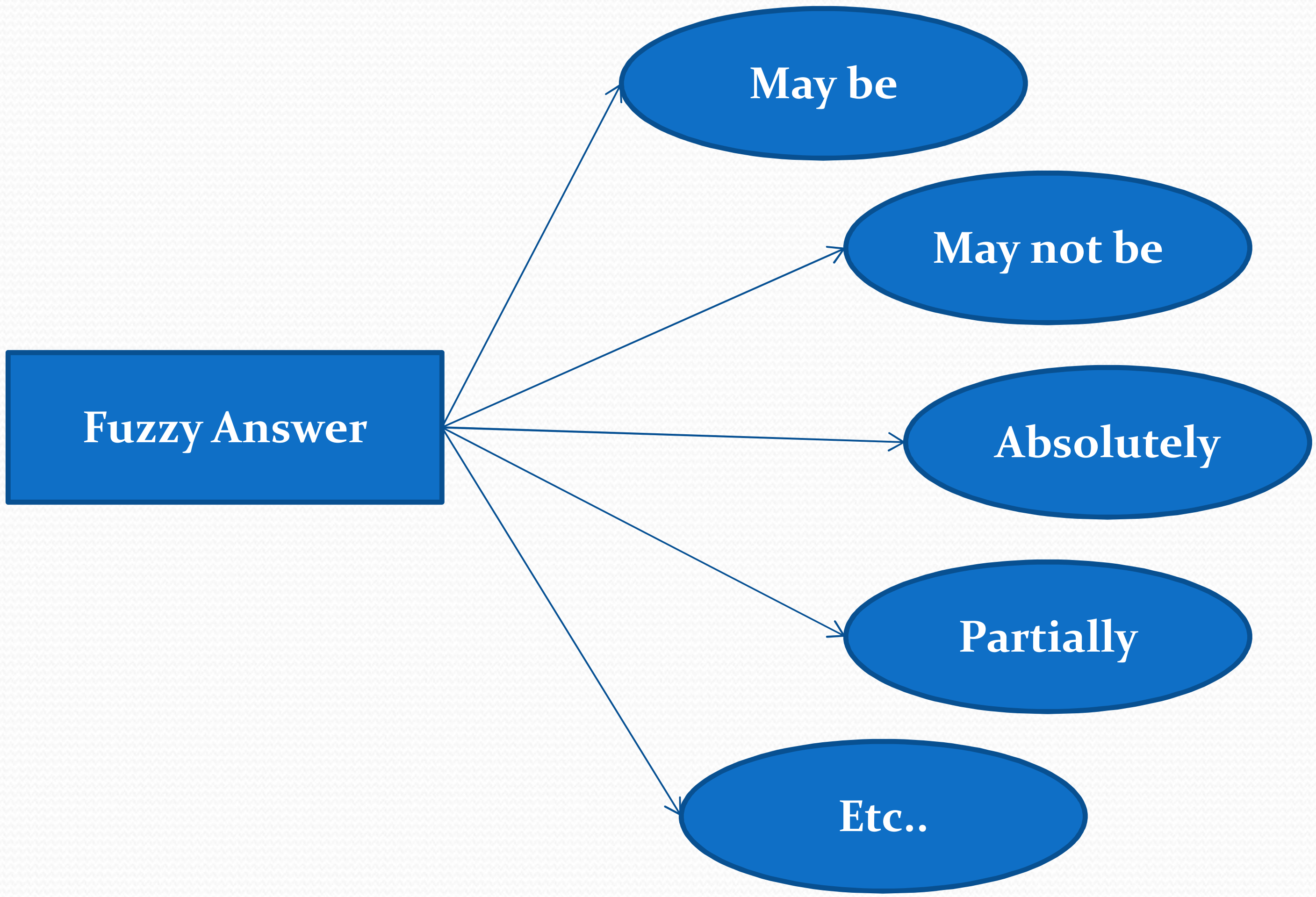


- Antonym of fuzzy is **“crisp”**
  - Example: Are the chips crisp?

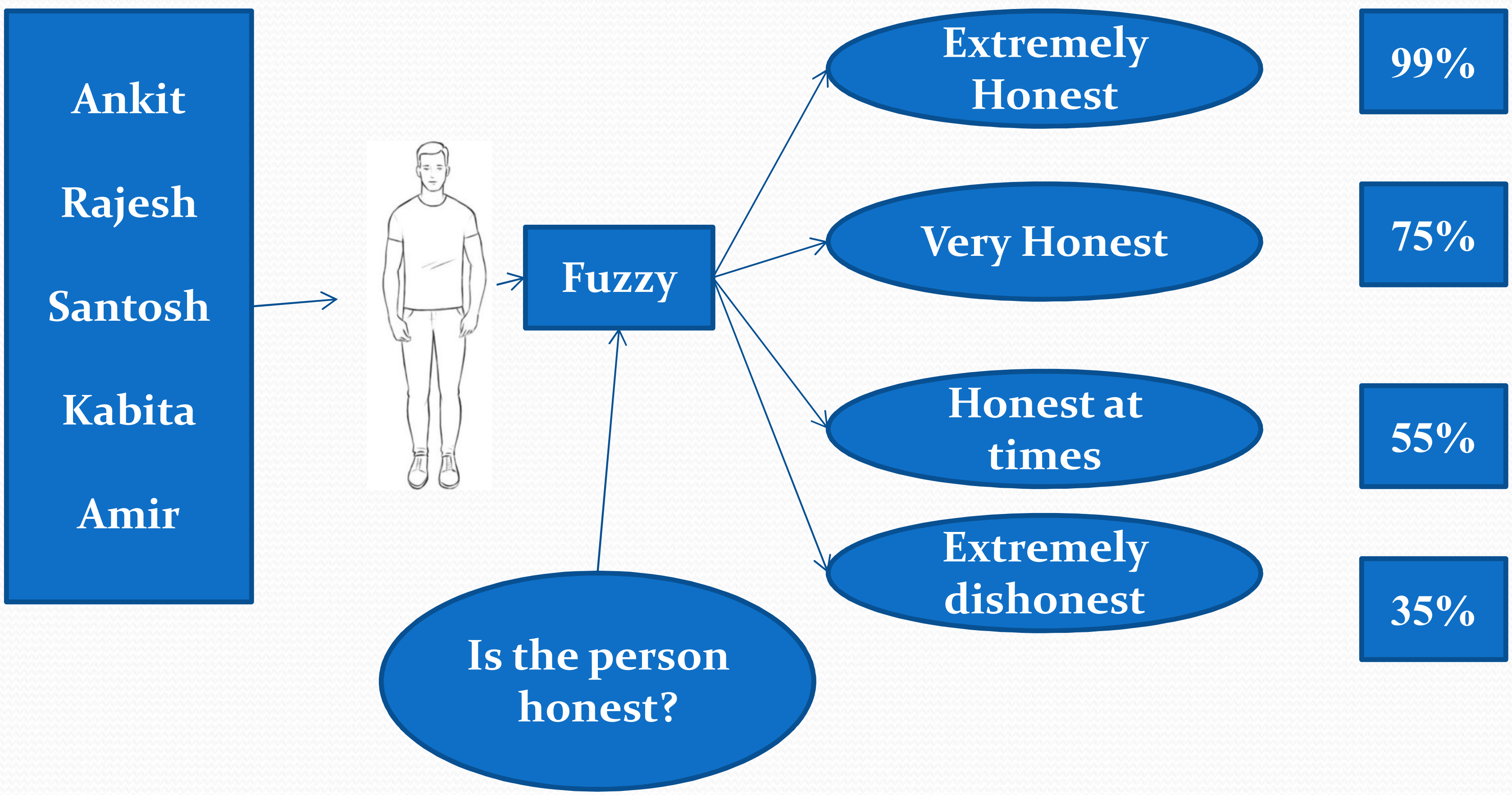
Example: Fuzzy logic vs. Crisp logic



Example: Fuzzy logic vs. Crisp logic

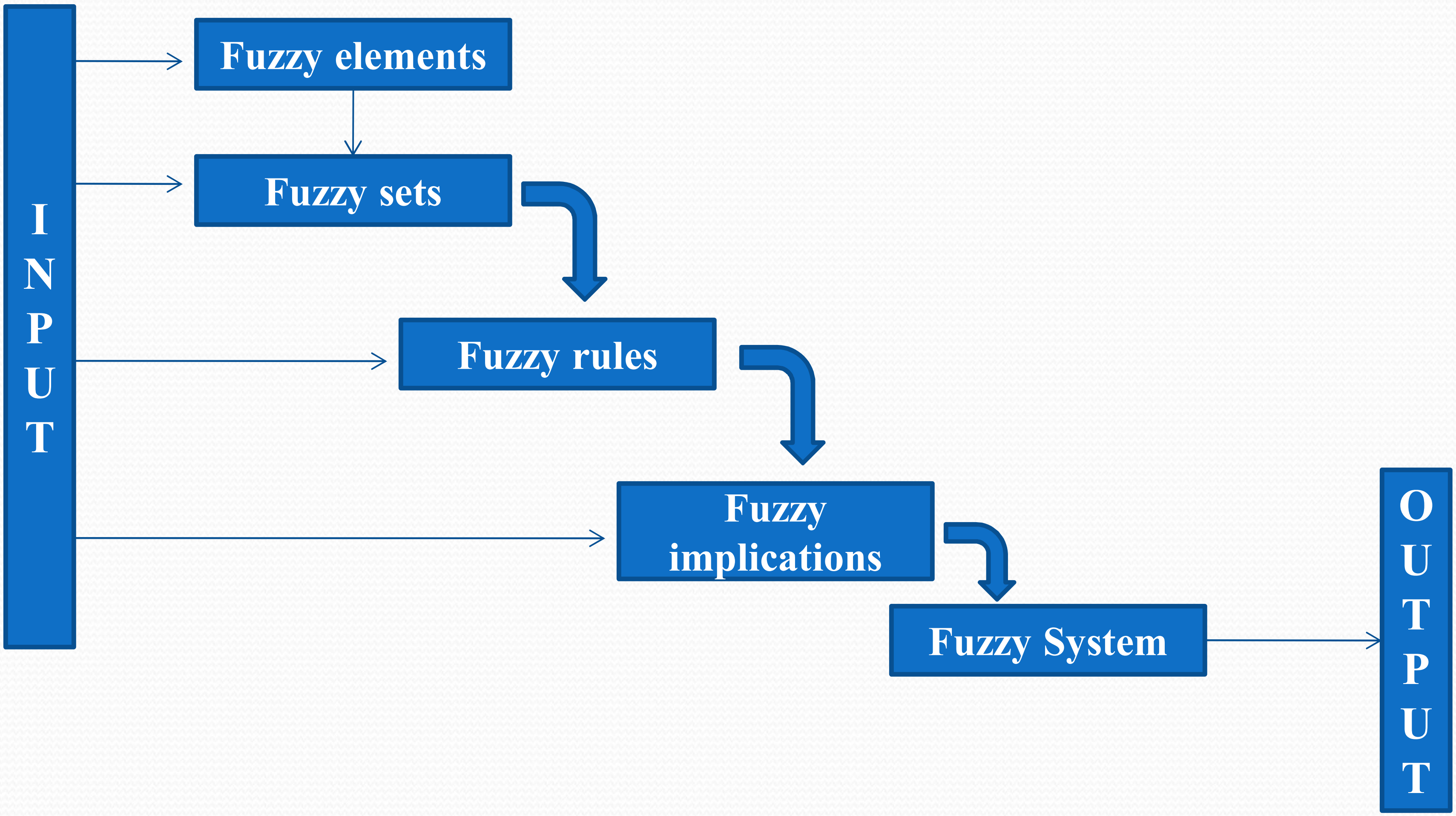


Example: Fuzzy logic vs. Crisp logic



World is Fuzzy !!!

Concept of fuzzy system



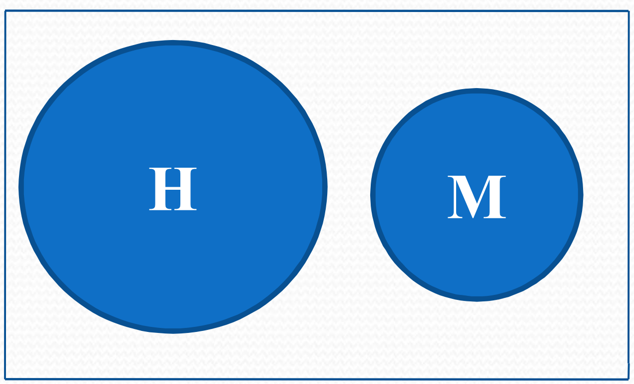
Concept of fuzzy set

To understand the concept of fuzzy set it is better, if we first clear our idea of crisp set.

X = The entire population of India.

H = All Hindu population = $\{h_1, h_2, h_3, \ldots\ldots, h_L\}$

M = All Muslim population = $\{m_1, m_2, m_3, \ldots\ldots, m_N\}$



Here, all are the sets of finite numbers of individuals.

This is a crisp set.

Example of fuzzy set

Let us discuss about fuzzy set.

X = All students in MNNIT

S = All Good Students.

$S = \{(s, g(s)) \mid s \in X\}$ and $g(s)$ is measurement of goodness of the student s.

Example: S = { (Rajat, 0.8), (Kabita, 0.7), (Ankit, 0.9)} etc.

Fuzzy set vs. Crisp set

Crisp Set	Fuzzy Set
•$S = \{s \mid s \in X\}$	•$F = (s, \mu(s)) \mid s \in X$ and $\mu(s)$ is the degree of s.
•It is a collection of elements.	•It is a collection of ordered pairs.
•Inclusion of an element $s \in x$ into S is crisp, that is, has strict boundary yes or no.	•Inclusion of an element $s \in x$ into F is fuzzy, that is, if present, then with a degree of membership.

Fuzzy set vs. Crisp set

Note: A crisp set is fuzzy set, but a fuzzy set is not necessarily a crisp set.

Example:

$H = \{(h_1, 1), (h_2, 1), \ldots\ldots (h_L, 1)\}$

$Person = \{(p_1, 0), (p_2, 0), \ldots.., (p_N, 0)\}$

In case of a crisp set, the elements are with extreme values of degree of membership namely either 1 or 0.

Degree of membership

How to decide the degree of memberships elements in a fuzzy set?

City	Bangalore	Bombay	Kolkata	Delhi	Madras	Allahabad
μ	0.95	0.90	0.80	0.75	0.65	0.70

How the cities of comfort can be judged?

Example: Course evaluation in a crisp way

EX : Marks $\geq$ 90

A : 80 $\leq$ Marks < 90

B: 70 $\leq$ Marks < 80

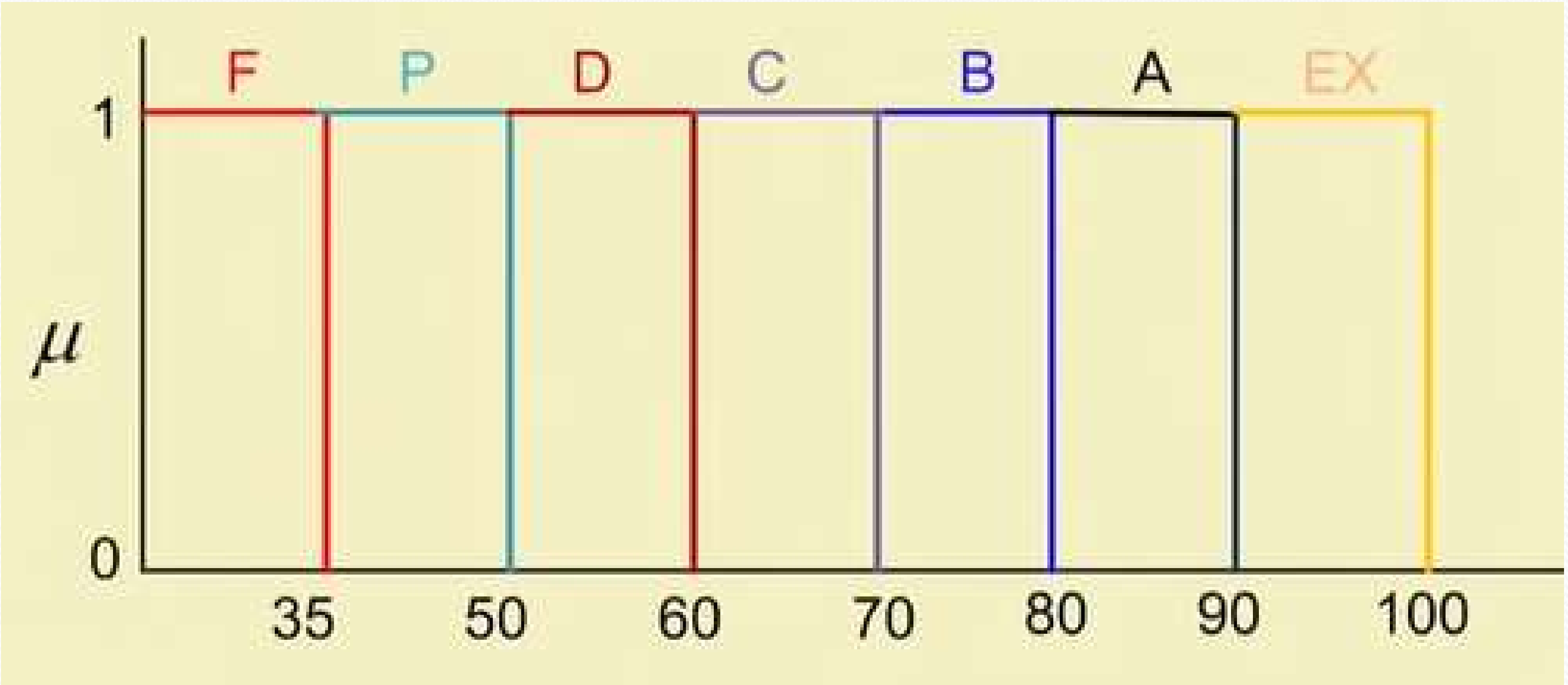
C: 60 $\leq$ Marks < 70

D: 50 $\leq$ Marks < 60

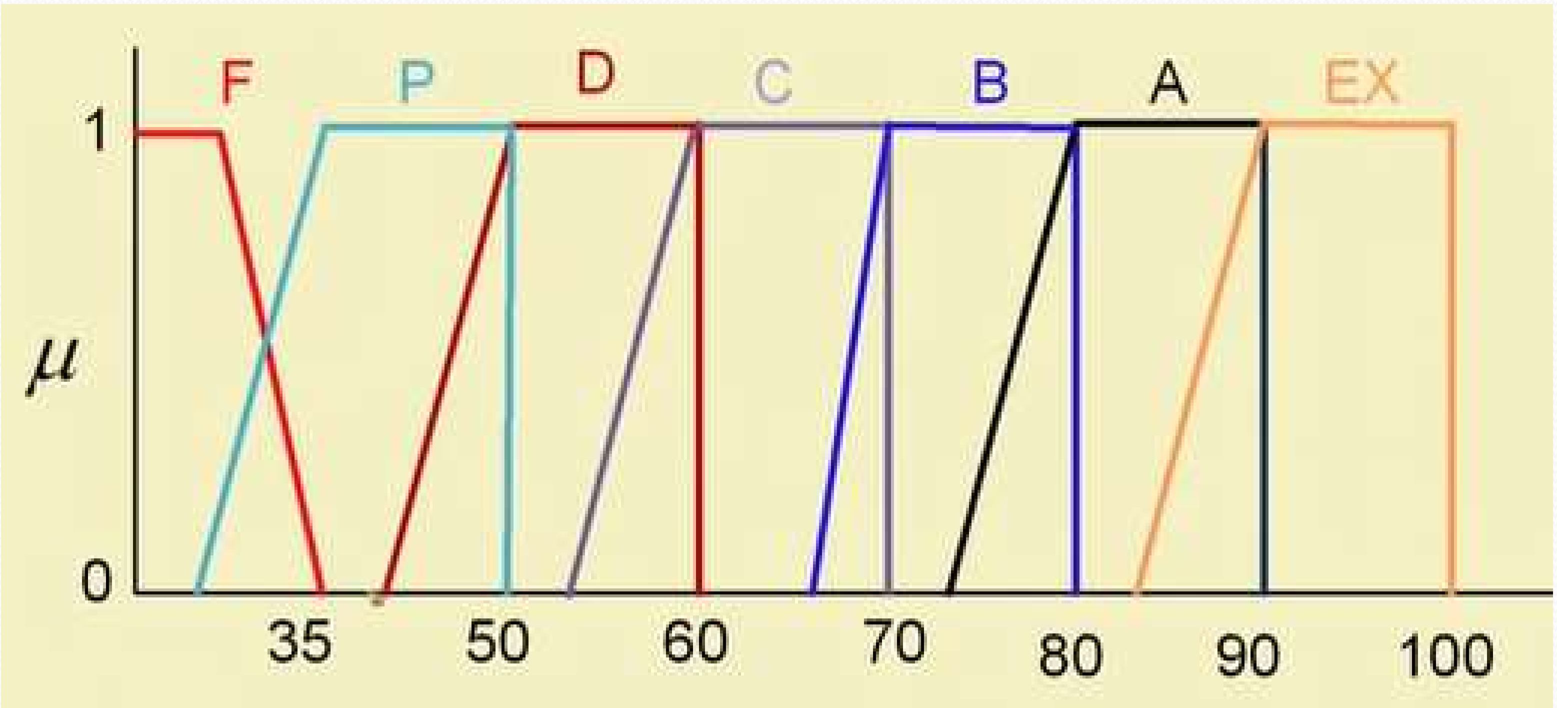
P: 35 $\leq$ Marks < 50

F: Marks $\leq$ 35

Example: Course evaluation in a crisp way



Example: Course evaluation in a fuzzy way



Few examples of fuzzy set

- High Temperature
- Low Pressure
- Color of Apple
- Sweetness of Orange
- Weight of Mango

Note: Degree of membership values lie in the range [0..1]

Some basic terminologies and notations

Definition 1: Membership function (and Fuzzy set)

If X is a universe of discourse and $x \in X$, then a fuzzy set A in X is defined as a set of ordered pairs, that is-
$A = \{(x, \mu_A (x)) \; x \in X \}$ where $\mu_A (x)$ is called the membership function for the fuzzy set A.

Note: $\mu_A (x)$ map each element of x onto a membership grade (or membership value) between 0 and 1 (both inclusive).

Question: How (and who) decides $\mu_A (x)$ for a fuzzy set A in X?

Some basic terminologies and notations

Example:

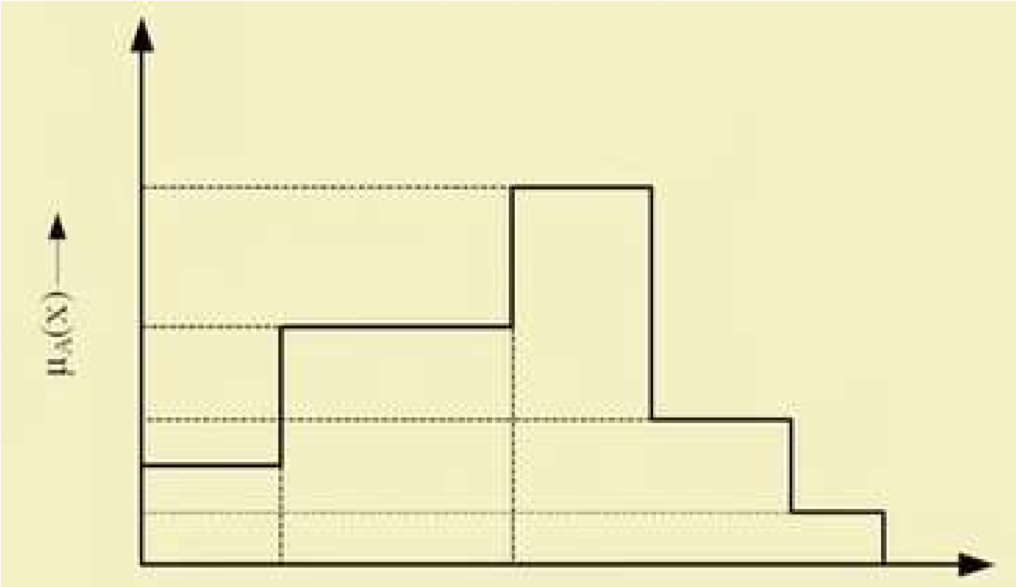
X = All cities in India.

A = City of comfort.

A = {(New Delhi, 0.7), (Bangalore, 0.9), (Chennai, 0.8), (Kolkata, 0.3), (Allahabad, 0.6)}

Membership function with discrete membership values

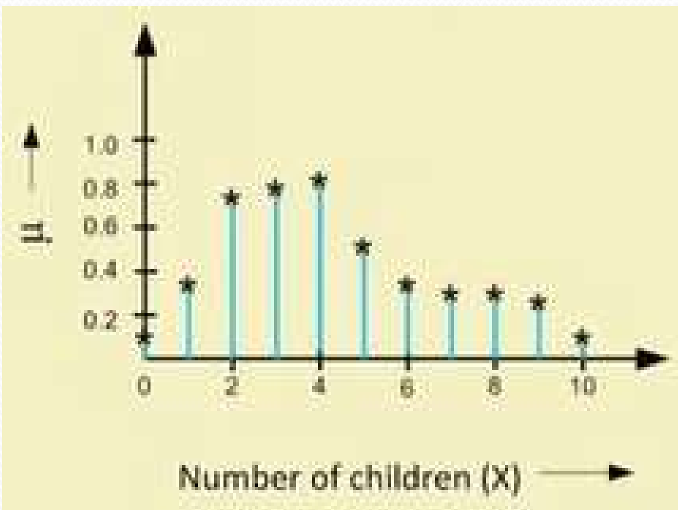
The membership values may be of discrete values



A fuzzy set with discrete values μ

Membership function with discrete membership values

Either elements or their membership values (or both) also may be discrete values



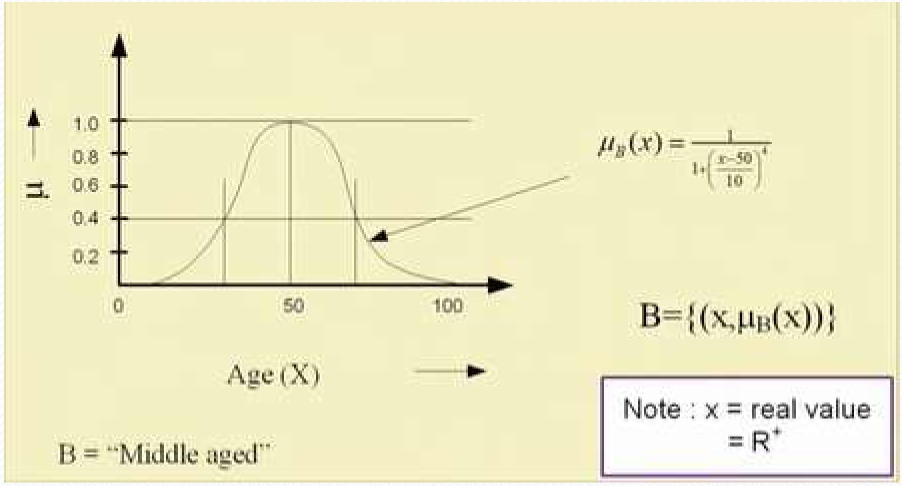
A = {(0, 0.1), (1, 0.30), (2, 0.78)….(10, 0.1)}

Note: X = discrete value.

How you measure happiness ?

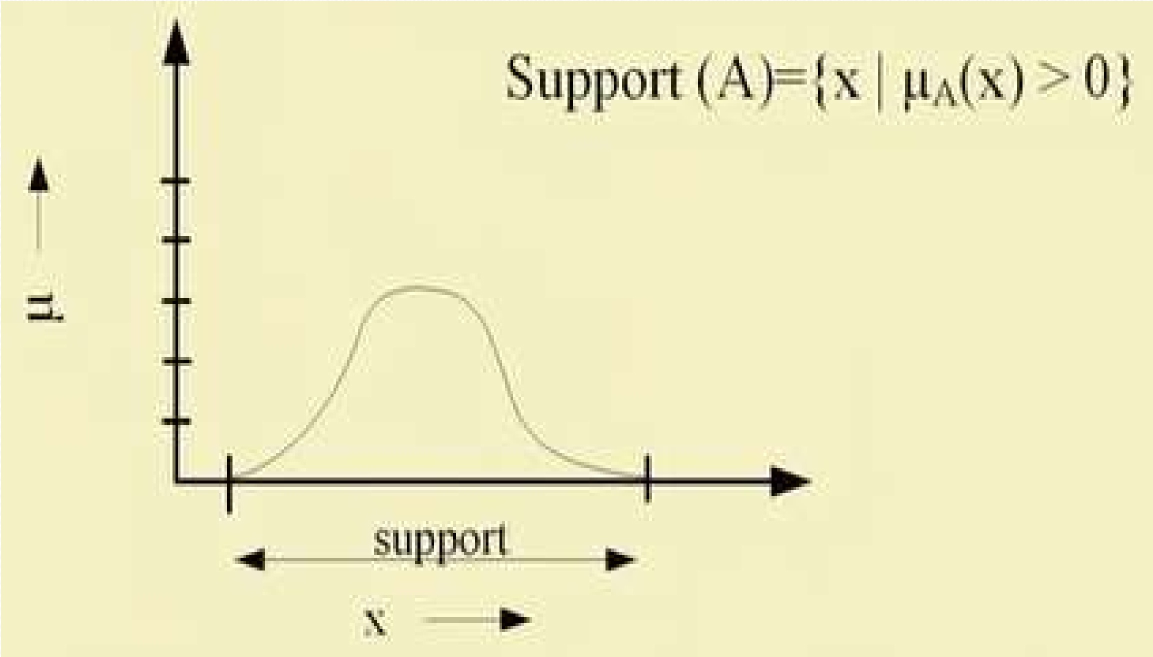
A = “Happy Family”

Membership function with continuous membership values



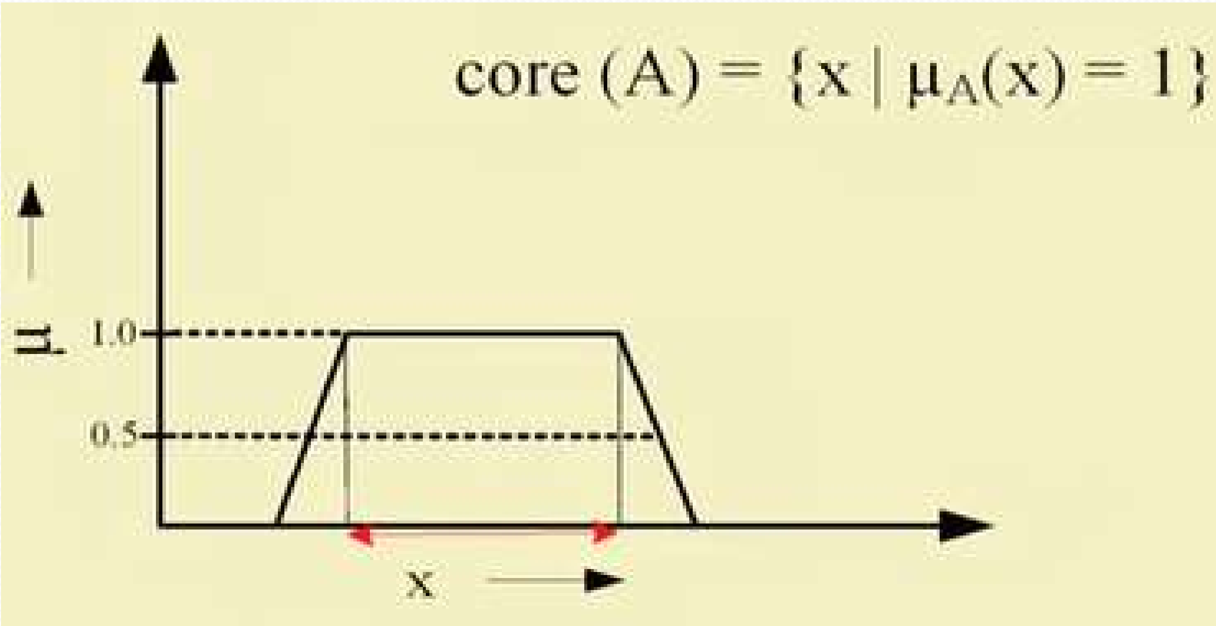
Fuzzy terminologies: Support

Support: The support of a fuzzy set A is the set of all points $x \in X$ such that $\mu_A(x) > 0$



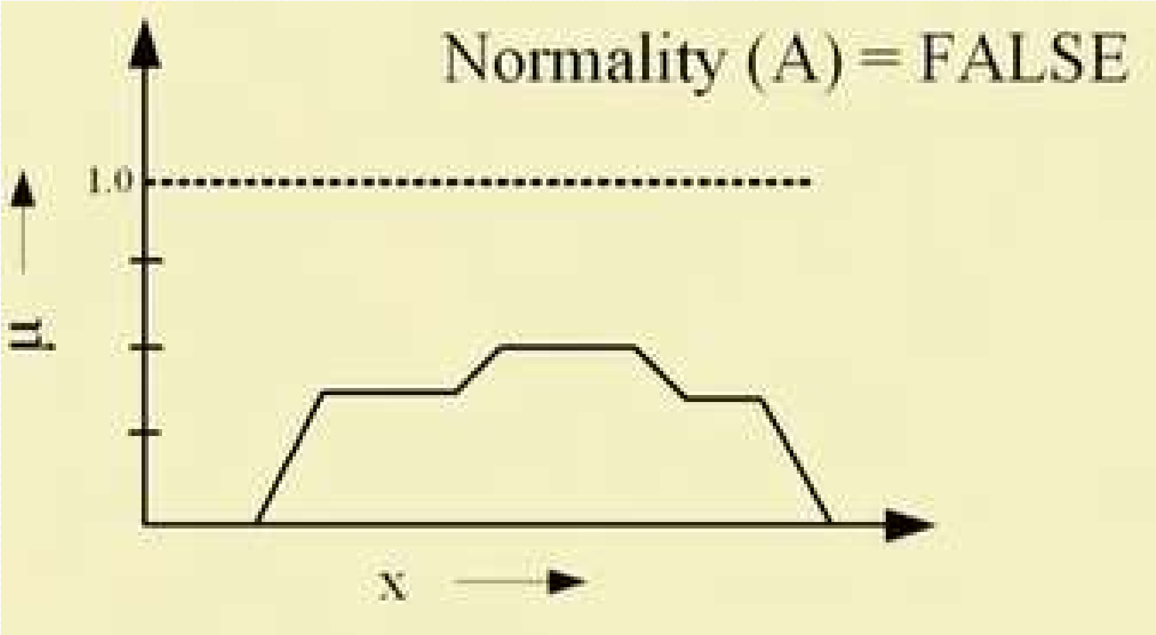
Fuzzy terminologies: Core

Core: The support of a fuzzy set A is the set of all points $x \in X$ such that $\mu_A(x) = 1$



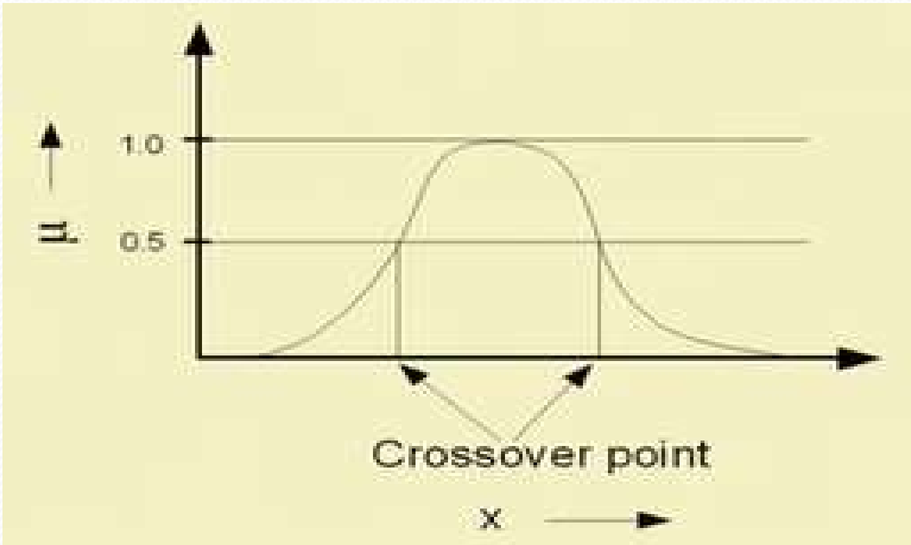
Fuzzy terminologies: Normality

Normality: A fuzzy set A is a normal if its core is nonempty. In other words, we can always find a point x ϵ X such that $\mu_A(x) = 1$



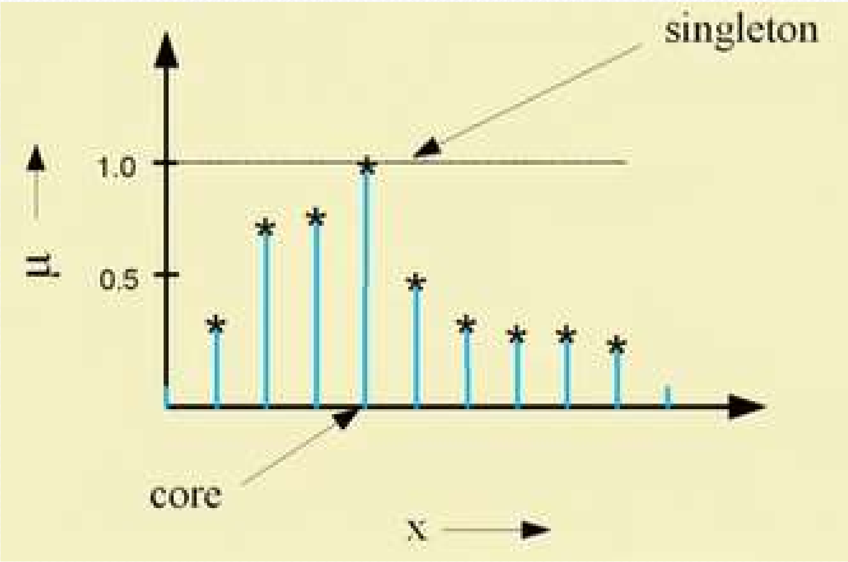
Fuzzy terminologies: Crossover points

Crossover point: A crossover point of a fuzzy set A is a point $x \in X$ at which $\mu_A(x) = 0.5$. That is crossover (A) = $\{x \mid \mu_A(x) = 0.5\}$



Fuzzy terminologies: Fuzzy Singleton

Fuzzy Singleton: A fuzzy set whose support is a single point in X with $\mu_A(x) = 1$ is called fuzzy singleton. That is $|A| = \{x \mid \mu_A(x) = 1\}$



Fuzzy terminologies: α-cut and strong α-cut

α-cut and strong α-cut:

- The α-cut of a fuzzy set A is a crisp set defined by

$$A_\alpha = \{ x | \mu_A(x) \geq \alpha \}$$

- Strong α-cut is defined similarly:

$$A'_\alpha = \{ x | \mu_A(x) > \alpha \}$$

Note: Support (A) = A_0' and Core (A) = A_1

Fuzzy terminologies: Bandwidth

Bandwidth:

For a fuzzy set, the bandwidth (or width) is defined as the distance between the two unique crossover points:

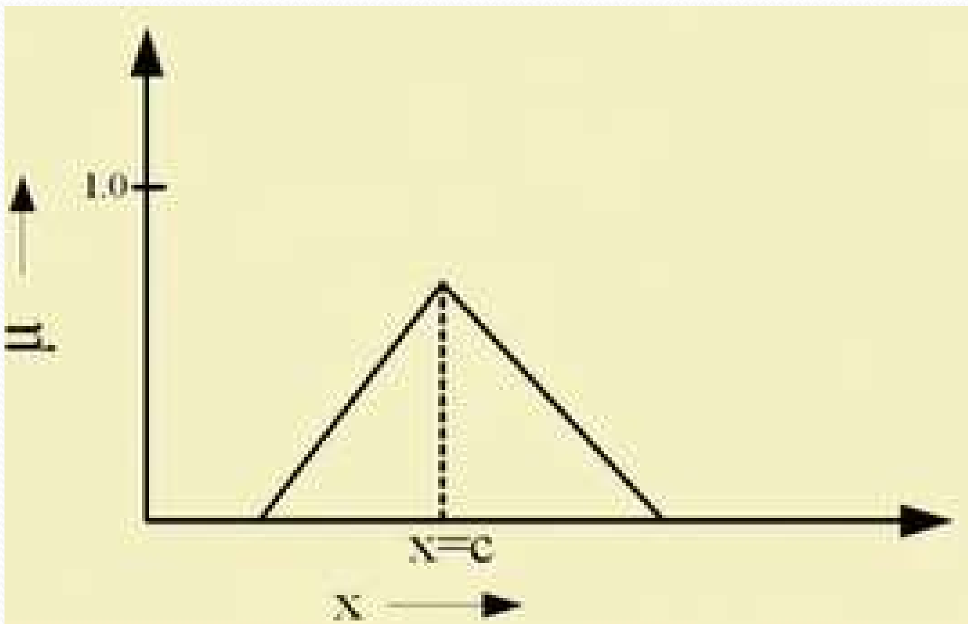
$$\text{Bandwidth: } | x_1 - x_2 |$$

Where $\mu_A(x_1) = \mu_A(x_2) = 0.5$

Fuzzy terminologies: Symmetry

Symmetry:

A fuzzy set A is symmetric if its membership function around a certain point x = c, namely $\mu_A(x + c) = \mu_A(x - c)$ for all $x \in X$



Fuzzy terminologies: Open and Closed

A fuzzy set A is

Open left : If $\lim x \to -\infty\ \mu_A(x) = 1$ and $\lim x \to +\infty\ \mu_A(x) = 0$

Open right: If $\lim x \to -\infty\ \mu_A(x) = 0$ and $\lim x \to +\infty\ \mu_A(x) = 1$

Closed: If $\lim x \to -\infty\ \mu_A(x) = \lim x \to +\infty\ \mu_A(x) = 0$