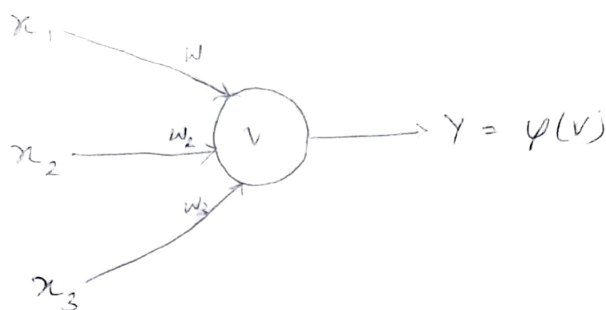


Ques: (1) Below is a diagram of a single artificial neuron (unit):



Single unit with inputs

The node has three inputs $x = (x_1, x_2, x_3)$ that receive only binary signals (either 0 or 1) given number of inputs?

Sol for three inputs the number of combinations of 0 and 1 is 8:

x_1	0	1	0	1	0	1	0	1
x_2	0	0	1	1	0	0	1	1
x_3	0	0	0	0	1	1	1	1

and for four inputs the number of combinations is 16:

x_1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
x_2	0	0	1	1	0	0	1	1	0	0	1	1	0	0
x_3	0	0	0	0	1	1	1	1	0	0	0	0	1	1
x_4	0	0	0	0	0	0	0	1	1	1	1	1	1	1

You may check that for five inputs the number of combinations will be 32. Note that $8 = 2^3$, $16 = 2^4$ and $32 = 2^5$ (for three, four and five inputs). Thus, the formula for number of binary input pattern is 2^n , where n is the number of inputs.

Ques ②. Consider the unit shown on figure 1.

$$\begin{array}{l} w_1 = 2 \\ w_2 = -4 \\ w_3 = 1 \end{array}$$

$$\varphi(v) = \begin{cases} 1 & \text{if } v \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Pattern	P_1	P_2	P_3	P_4
x_1	1	0	1	1
x_2	0	1	0	1
x_3	0	1	1	1

Sol To find the output value y of the unit for each of the following input patterns:

To find the output value y for each pattern we have to:

a) Calculate the weighted sum: $v = \sum_i w_i x_i = w_1 \cdot x_1 + w_2 \cdot x_2 + w_3 \cdot x_3$

b) Apply the activation function to v

The calculations for each input pattern are:

$$P_1: v = 2 \cdot 1 - 4 \cdot 0 + 1 \cdot 0 = 2 \quad (2 \geq 0), \quad y = \varphi(2) = 1$$

$$P_2: v = 2 \cdot 0 - 4 \cdot 1 + 1 \cdot 1 = -3 \quad (-3 < 0), \quad y = \varphi(-3) = 0$$

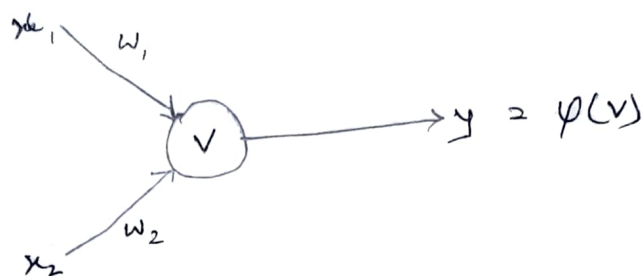
$$P_3: v = 2 \cdot 1 - 4 \cdot 0 + 1 \cdot 1 = 3, \quad (3 \geq 0), \quad y = \varphi(3) = 1$$

$$P_4: v = 2 \cdot 1 - 4 \cdot 1 + 1 \cdot 1 = -1, \quad (-1 < 0), \quad y = \varphi(-1) = 0$$

Ques 3 Logical operator (i.e. NOT, AND, OR, XOR, etc) are

the following table

x_1	0	1	0	1
x_2	0	0	1	1
$x_1 \text{ AND } x_2$	0	0	0	1



a) Test how the neural AND function works.

Sol:

$$P_1: v = 1 \cdot 0 + 1 \cdot 0 = 0, (0 < 2), y = \phi(0) = 0$$

$$P_2: v = 1 \cdot 1 + 1 \cdot 0 = 1, (1 < 2), y = \phi(1) = 0$$

$$P_3: v = 1 \cdot 0 + 1 \cdot 1 = 1, (1 < 2), y = \phi(1) = 0$$

$$P_4: v = 1 \cdot 1 + 1 \cdot 1 = 2, (2 = 2), y = \phi(2) = 1$$

b) Suggest how to change either the weights or the threshold OR function.

x_1	0	1	0	1
x_2	0	0	1	1
$x_1 \text{ OR } x_2$	0	1	1	1

Sol One solution is to increase the weights of the unit: $w_1 = 2$ and $w_2 = 2$

$$P_1: v = 2 \cdot 0 + 2 \cdot 0 = 0, (0 < 2), y = \phi(0) = 0$$

$$P_2: v = 2 \cdot 1 + 2 \cdot 0 = 2, (2 = 2), y = \phi(2) = 1$$

$$P_3: v = 2 \cdot 0 + 2 \cdot 1 = 2, (2 = 2), y = \phi(2) = 1$$

$$P_4: v = 2 \cdot 1 + 2 \cdot 1 = 4, (4 > 2), y = \phi(4) = 1$$

Alternatively, we could reduce the threshold to 1:

$$\psi(v) = \begin{cases} 1 & \text{if } v \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

© The XOR function (exclusive or) - This can be represented by the following table:

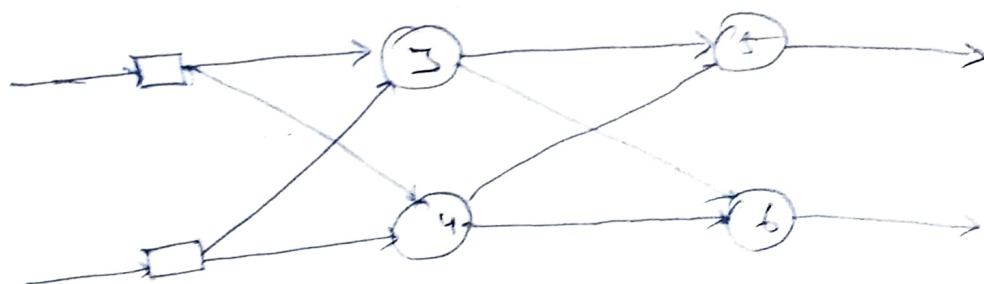
x_1	0	1	0	1
x_2	0	0	1	1
$x_1 \text{ XOR } x_2$	0	1	1	0

Do you think it is possible to implement this function using a single unit? A network of several units?

Sol This is a difficult question, and it puzzled scientists for some time because it is actually impossible to implement the XOR function neither by a single unit nor by a single-layer feed-forward network (single-layer perceptron). This was known as the XOR problem.

The solution was found using a feed-forward network with a hidden layer. The XOR network uses two hidden nodes and one output node.

Ques 4 The following diagram represents a feed-forward neural network with one hidden layer:



Pattern	P_1	P_2	P_3	P_4
Node 1	0	1	0	1
Node 2	0	0	1	1

Solution In order to find the output of the network it is necessary to calculate weighted sum of hidden nodes 3 and 4

$$V_3 = w_{13}x_1 + w_{23}x_2 ; V_4 = w_{14}x_1 + w_{24}x_2$$

Then find the outputs from hidden nodes using activation function ϕ :

$$y_3 = \phi(V_3), y_4 = \phi(V_4)$$

Use the outputs of the hidden nodes y_3 and y_4 as the input values to the output layer (nodes 5 and 6), and find weighted sums of output nodes 5 and 6:

$$V_5 = w_{35}y_3 + w_{45}y_4, V_6 = w_{36}y_3 + w_{46}y_4$$

finally, find the outputs from node 5 and 6 (also using ϕ):

$$y_5 = \phi(V_5), y_6 = \phi(V_6)$$

The output pattern will be (y_5, y_6) . Perform these calculation for each input pattern:

P_1 : Input pattern (0, 0)

$$\begin{aligned} V_3 &= -2 \cdot 0 + 3 \cdot 0 = 0, & y_3 &= \phi(0) = 1 \\ V_4 &= 4 \cdot 0 - 1 \cdot 0 = 0, & y_4 &= \phi(0) = 1 \\ V_5 &= 1 \cdot 1 - 1 \cdot 1 = 0, & y_5 &= \phi(0) = 1 \\ V_6 &= -1 \cdot 1 + 1 \cdot 1 = 0, & y_6 &= \phi(0) = 1 \end{aligned}$$

The output of the network is (1, 1)

P_2 : Input pattern (1, 0)

$$V_3 = -2 \cdot 1 + 3 \cdot 0 = -2$$

$$V_4 = 4 \cdot 1 - 1 \cdot 0 = 4$$

$$V_5 = 1 \cdot 0 - 1 \cdot 1 = -1$$

$$V_6 = -1 \cdot 0 + 1 \cdot 1 = 1$$

$$Y_3 = \varphi(-2) = 0$$

$$Y_4 = \varphi(4) = 1$$

$$Y_5 = \varphi(-1) = 0$$

$$Y_6 = \varphi(1) = 1$$

The output of the network is (0, 1).

P_3 : Input pattern (0, 1)

$$V_3 = -2 \cdot 0 + 3 \cdot 1 = 3$$

$$V_4 = 4 \cdot 0 - 1 \cdot 1 = -1$$

$$V_5 = 1 \cdot 1 - 1 \cdot 0 = 1$$

$$V_6 = -1 \cdot 1 + 1 \cdot 0 = -1$$

$$Y_3 = \varphi(3) = 1$$

$$Y_4 = \varphi(-1) = 0$$

$$Y_5 = \varphi(1) = 1$$

$$Y_6 = \varphi(-1) = 0$$

The output of the network is (1, 0)

Qus ⑤ X What is training set and how is it used to train neural network?

sol Training set is set pairs of input patterns with corresponding desired output patterns. The network is trained to respond correctly to each input pattern from the training set. Training algorithms that use training set are called supervised learning algorithms.

The error is used is used to adjust the weights in the network so that next time the error was smaller.