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Assignment 2

Ques 1 Recursive Procedure of insertion sort.

void insertionSort(arr[], n) {

if $n \leq 1$

return

insertionSort(arr, n-1)

last = arr[n-1]

$j = n-2$;

while ($j > 0$ and $arr[j] > last$)

$arr[j+1] = arr[j]$

$j--$

$arr[j+1] = last$

$$T(n) = T(n-1) + n$$

Ans

Ques 2:-

Algo:-

Search(arr[], length, sum) {

mergeSort(arr, 1, length)

for ($i = 1$ to length)

index = BinarySearch(arr, $n - arr[i]$)

if (index $\neq i$) and index $\neq i$

return true

return false

Ques 3:-

(a) Sorting sublists:- for input of size K
insertion sort run on $\Theta(K^2)$ worst
case time so worst case time to
sort n/k sublists each of length K
will be $n/k \cdot \Theta(K^2) = \Theta(nK)$

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(b) merging sublists:-

We have n elements divided into n/k sorted sublists each of length k . To merge these n/k sorted sublists of length n , we have to take two sublists at a time and continue to merge them.

This will result in $\lg(n/k)$ steps. And in every step we are eventually going to compare n elements. So the whole process will run a $\Theta(n \lg(n/k))$.

(c) Largest value of k :

For the modified algorithm to have the same asymptotic running time as standard merge sort $\Theta(nk + n \lg(n/k))$ must be same as $\Theta(n \lg n)$.

To satisfy the condition k can't grow faster than $\lg n$ asymptotically if it does then because of the nk from the algorithm will run at worst asymptotic time then $\Theta(n \lg n)$.

So, let's assume $k = \Theta(\lg n)$

$$\begin{aligned}\Theta(nk + n \lg(n/k)) &= \Theta(nk + n \lg n - n \lg k) \\ &= \Theta(n \lg n + n \lg n - n \lg \lg n)\end{aligned}$$

$$= O(n \log n)$$

neglect $\log \log n$ very small.

(d) Practical value of k :-

To determine Practical value of k . It has to be largest input size for which insertion sort runs faster than merge sort to get exact value we need to calculate the exact running time expression with constant factor and use the method described in.

Qust 10 Given Array

2	3	8	6	1
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Index 1 2 3 4 5

5 inversion $\rightarrow (1,5), (2,5), (3,4), (3,5)$ and $(4,5)$ Σ used index have

11 Given Array :- 1, 2, 3, ..., n

Array will be most no. of inversion max array sorted in descending order

$$\text{no. of inversion} = \frac{n(n+1)}{2} \left\{ \begin{array}{l} \because \text{1st} \rightarrow n-1 \\ \text{2nd} \rightarrow n-2 \\ \vdots \\ n-1 \rightarrow 1 \end{array} \right.$$

12 If no. of Inversion in an array is more than inner loop of Insertion sort run more time

run more time so the higher the no. of inversions in an array, the longer insertion sort will take to sort the array.

④ Algorithms to count inversion
count-Inversions (A, P, r)

{

if ($P \geq r$)

return 0

$q = (P + r) / 2$

left = count-Inversions (A, P, q)

right = count-Inversions ($A, q+1, r$)

total = left + right + merge (A, P, q, r)

return total;

}

merge (A, P, q, r) {

$n_1 = q - P + 1$

$n_2 = r - q$

int $L[1 \dots n_1]$, $R[1 \dots n_2]$

for ($i = 1$ to n_1)

$L[i] = A[P + i - 1]$

for $j = 1$ to n_2

$R[j] = A[q + j]$

$L[n_1 + 1] = \infty$

$R[n_2 + 1] = \infty$

$i = 1$

$j = 1$


```

count = 0
for (k = P to r) {
    if (L[i] ≤ R[j]) {
        A[k] = L[i]
        i = i + 1
    }
    else {
        count = count + (n1 - i + 2)
        A[k] = R[j]
        j = j + 1
    }
}
return count;

```

Ques 5

- ① Insertion sort, merge sort, are stable and Heapsort, and quicksort are not.
- ② To make my sorting algorithm stable we can preprocess, replacing each element of an array with an ordered pair. The first entry will be the value of the element and second value will be the index of the element.

eg. Array

2	1	1	3	4	4	4
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 would become

(2,1)	(1,2)	(1,3)	(3,4)	(4,5)	(4,6)	(4,7)
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We now interpret $(i, j) < (k, m)$
if $(i < k)$ or
 $i = k$ and
 $j < m$.

This doubles the space requirement
but the running time will be
asymptotically unchanged.

Ques 6

Step 1: Run through the list of
integers and convert each one to
base n .

Step 2: Radix Sort

∵ Each number will have at most
 $\log_n 8 \approx 3$ digit, so will be only
3 passes.

Step 3: For each pass, there
are n possible values which
can be taken on, so we
can use counting sort to
sort them in $O(n)$.