

Practical - 1

Aim:- Write examples based on probability.

Description:-

1) Probability Definition:-

If the sample space S of an experiment is such that each elementary event is the equally likely then to each other elementary event we can assign the probability as $\frac{1}{n}$ where n is the sample size.

2) Sample Space definition:-

A sample space of an experiment is the set of all possible outcomes of the experiment.

3) Event Definition:-

An Event is a set of outcomes i.e. zero subset of the sample space to which a probability is assign.

• Types of Events:-

i) Elementary Events:-

If an Event E_i only one sample points such an event E is called an elementary event.

ii) Complementary Events :-

A complement of an Event E denoted by E' is the set of all basic outcomes in the sample space that do not belong to E .

iii) Compound Events :-

A compound event is any subset of the sample space.

iv) Independent Events :-

Two events A and B are independent if the probability of the succeeding events is not affected by the outcome of the proceeding event.

4) Union Definition :-

$A \cup B$ is the event that consists of all sample points that are either in A or in B or in both A and B . The event $A \cup B$ is called the union of events A and B .

• Formula of Union :-

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Simply, the number of elements in the union of set A and B is equal to the sum of cardinal numbers of the sets A and B , minus that of their intersection.

5) Addition and Multiplication Formula with description:-

i) Addition theorem on probability: If A and B are any two events then the probability of happening of at least one of the events is defined as $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

ii) According to the multiplication rule of probability, the probability of occurrence of both the events A and B is equal to the product of the probability of B occurring and the conditional probability that event A occurring given that event B occurs.

$$P(A \cap B) = P(A) \cdot P(B|A)$$

If A and B are independent

$$P(A \cap B) = P(A) \cdot P(B)$$

- > library(prob)
- > tosscoin(1)
- > tosscoin(3)
- > rolldie(1)
- > rolldie(3)
- > head(cards())
- > urnsamples(1:3, size = 2, replace = TRUE,
ordered = TRUE)

Ordered without replacement

- > urnsamples(1:3, size = 2, replace = FALSE,
ordered = TRUE)

Unordered Without Replacement

- > urnsamples(1:3, size = 2, replace = FALSE,
ordered = FALSE)

Events

- > S <- tosscoin(2, makespace = TRUE)
- > S[1:3]
- > S <- cards()
- > subset(S, rank%in% 7:9)

Union

- > `s = cards()`
- > `A = subset(s, suit = "Heart")`
- > `B = subset(s, rank % in % 1:9)`
- > `Union(A, B)`
- > `intersect(A, B)`
- > `setdiff(A, B)`

Practical - 2

Aim:- Condition Probability and Independence

```

> outcomes <- rolldie(1)
> rolldie(1)
> p <- rep(1/6, times = 6)
> probspace
> probspace(outcomes, probs = p)
> probspace(1:6, probs = p)
> probspace(1:6)
> rolldie(1, makespace = TRUE)
> probspace(tosscoin(1), probs = c(0.1, 0.3))
> S <- cards(makespace = TRUE)
> A <- subset(S, suit == "Heart")
> A
> B <- subset(S, rank %in% 1:9)
> B
> prob(A)
> prob(S, suit == "Heart")
> nsamp(n=3, k=2, replace=TRUE, ordered=TRUE)
> nsamp(n=3, k=2, replace=FALSE, ordered=TRUE)
> nsamp(n=3, k=2, replace=FALSE, ordered=FALSE)
> nsamp(n=3, k=2, replace=TRUE, ordered=FALSE)
> n <- c(11, 7, 31)
> n
> k <- c(3, 4, 3)
> k
> r <- c(FALSE, FALSE, TRUE)
> r

```



```

> x <- nsamp(n, k, rep = r, ord = TRUE)
> x
> prod(x)
> (11 * 10 * 9) * (1 * 6 * 5 * 4) * 313
> prod(9:11) * prod(4:7) * 313
> prod(factorial(c(11, 7)) / factorial(c(8, 3))) * 313
> library(Rcmdr)
> library(RcmdrPlugin.IPSUR)
> g <- Vectorize(pbirthday.ipsur)
> plot(1:50, g(1:50),
+ xlab = "Number of people in room",
+ ylab = "Prob(at least one match)",
+ main = "The Birthday Problem")
> abline(h = 0.5)
> abline(v = 23, lty = 2)
> library(prob)
> S <- rolldie(2, makespace = TRUE)
> head(S)
> A <- subset(S, x1 == x2)
> B A <- subset
> B <- subset(S, x1 + x2 >= 8)
> B
> prob(A, given = B)
> prob(B, given = A)
> prob(S, x1 == x2, given = (x1 + x2 >= 8))
> prob(S, x1 + x2 >= 8, given = (x1 == x2))
> library(prob)
> L <- cards()

```



```

> M <- urnsamples(L, size = 2)
> N <- probspace(M)
> prob(N, all(rank == "A"))
> library(prob)
> L <- rep(c("red", "green"), times = c(7, 3))
> M <- urnsamples(L, size = 3, replace = FALSE,
ordered = TRUE)
> N <- probspace(M)
> prob(N, isrep(N, "red", 3))
> prob(N, isrep(N, "red", 2))
> prob(N, isin(N, c("red", "green", "red"),
ordered = TRUE))
> prob(N, isin(N, c("red", "green", "red")))
> S <- tosscoin(10, makespace = TRUE)
> A <- subset(S, isrep(S, vals = "T", nrep = 10))
> 1 - prob(A)
> iidspace(c("H", "T"), ntrials = 3, probs = c(0.7, 0.3))
> prior <- c(0.6, 0.3, 0.1)
> like <- c(0.003, 0.007, 0.01)
> post <- prior * like
> post / sum(post)
> newprior <- post
> post <- newprior * like^7
> post / sum(post)
> fastpost <- prior * like^8
> fastpost / sum(fastpost)

```


Practical - 3

Aim:- Discrete Random Variable

Description:-

- a) Probability distribution of discrete random variable.
→ The probability of a discrete random variable X is a list of each possible value of X together with the probability that X takes that value in one trial of the experiment.
- b) Probability mass function.
→ A probability mass function (pmf) is a function over the sample space of a discrete random variable X which gives the probability that X is equal to a certain value.

Creating discrete random variables

- ```
> (X <- RV(outcomes = 1:6, probs = 1/6))
> library(rv) (discrete RV)
> (X <- RV(outcomes = 1:6, probs = 1/6))
> (X <- RV(1:6))
> pois.func <- function(y, lambda) { return (lambda^y *
 exp(-lambda) / factorial(y)) }
> (Y <- RV(outcomes = c(0, Inf), probs = pois.func,
 lambda = 2))
> (Y <- RV("poison", lambda = 2))
> (X.loaded <- RV(outcomes = 1:6, odds = c(4, 1, 1, 1, 1, 1)))
⇒ Probability Calculations
> P(X == 2)
```



- >  $P(X < 3)$
- >  $P(X < 3 \mid X < 4)$
- >  $\text{delta} <- 3$
- >  $\text{lambda} <- 2$
- >  $P((Y \geq \text{lambda} - \text{delta}) \% \text{AND}\% (Y \leq \text{lambda} + \text{delta}))$
- >  $P((Y - \text{lambda})^2 \leq \text{delta}^2)$
- >  $E(X)$
- >  $V(X)$
- >  $E((X - E(X))^2)$

### Joint Distributions

- >  $(A \text{ and } B <- \text{jointRV}(\text{outcomes} = \text{list}(1:3, 0:2), \text{probs} = 1:9 / \text{sum}(1:9)))$
- >  $A <- \text{marginal}(A \text{ and } B, 1)$
- >  $B <- \text{marginal}(A \text{ and } B, 2)$
- >  $P(A < B)$
- >  $P(A == 2 \mid B < 0)$
- >  $P(A == 2 \mid B > 0)$
- >  $P(A == 2 \mid (B == 1) \% \text{OR}\% (B == 2))$
- >  $\text{independent}(A, B)$
- >  $A \mid (A > 1)$
- >  $A \mid (B == 2)$
- >  $E(A \mid (B == 2))$
- >  $(X2 <- \text{iid}(X, n = 2))$
- >  $(X3 <- \text{iid}(X, n = 3))$
- >  $(X2 <- \text{SoftIID}(X, n = 2))$
- >  $(X20 <- \text{SoftIID}(X, n = 20, \text{progress} = \text{FALSE}))$
- >  $\text{RV}(1:6) + \text{RV}(1:6)$



## Practical - 4

Aim:- Discrete Distributions

Description:-

- i) Discrete random variables -
- i) Probability mass functions:- A probability mass function (pmf) is a function over the sample space of a discrete random variable  $X$  which gives the probability that  $X$  is equal to a certain value.  $f(x) = P[X = x]$ .
- ii) Mean, Variance and Standard Deviation:- Variance is a measure of how data points vary from the mean, whereas standard deviation is the measure of the distribution of statistical data.
- 2) The Binomial Distribution:- The binomial distribution is based on a Bernoulli trial, which is a random experiment in which there are only two possible outcomes: success (S) and failure (F).
- 3) An empirical distribution:- It is one for which each possible event is assigned a probability derived from experimental observation.



```

> x <- c(0, 1, 2, 3)
> f <- c(1/8, 3/8, 3/8, 1/8)
> mu <- sum(x * f)
> mu
> sigma2 <- sum((x - mu)^2 * f)
> sigma2
> sigma <- sqrt(sigma2)
> sigma
> F = cumsum(f)
> F
> library(distrEx)
> X <- DiscreteDistribution(supp = 0:3, prob = c(1, 3, 3, 1)/8)
> E(X); var(X); sd(X)
> Pr
> pbinom(9, size = 12, prob = 1/6) - pbinom(6, size =
12, prob = 1/6)
> diff(pbinom(c(6, 9), size = 12, prob = 1/6))
> library(distr)
> X <- Binom(size = 3, prob = 1/2)
> X
> d(X)(1)
> p(X)(2)
> X <- Binom(size = 3, prob = 0.45)
> library(distrEx)
> E(X)
> E(3 * X + 4)
> var(X)
> sd(X)

```



```
> x <- c(4, 7, 9, 11, 12)
> ecdf(x)
> plot(ecdf(x))
> epdf <- function(x) function(t) { sum(x %in% t) /
length(x) }
> x <- c(0, 0, 1)
> epdf(x)(0)
> x <- c(0, 0, 1)
> sample(x, size = 7, replace = TRUE)
```



# Practical - 5

Aim:- Continuous Distributions

Description:-

1) Continuous Random Variables:-

A random variable is called continuous if its range is uncountably infinite.

2) The Continuous Uniform Distribution:-

This is one of the simplest probability distributions in statistics. It is a continuous distribution, this means that it takes values within a specified range, eg. between 0 and 1.

3) The Normal Distribution:-

A normal distribution is a type of continuous probability distribution in which most data points cluster toward the middle of the range, while the rest taper off symmetrically toward either extreme.

4) The Chi-Square Distribution:-

A chi-square ( $\chi^2$ ) distribution is a continuous probability distribution that is used in many hypothesis tests. This distribution has a single parameter called the degrees of freedom.



```

> f <- function(x) 3 * x^2
> integrate(f, lower = 0.14, upper = 0.71)
> g <- function(x) 3 / x^3
> integrate(g, lower = 1, upper = Inf)
> library(distr)
> ff <- function(x) 3 * x^2
> X <- AbscountDistribution(d = f, low1 = 0, up1 = 1)
> p(x)(0.71) - p(x)(0.14)
> library(distrEx)
> E(x)
> var(x)
> 3 / 80
> pnorm(1:3) - pnorm(-(1:3))
> g <- function(x) pnorm(x, mean = 100, sd = 15) - 0.99

```



```

> uniroot(g, interval = c(130, 145))
> qnorm(0.99, mean = 100, sd = 15)
> qnorm(c(0.025, 0.01, 0.005), lower.tail = FALSE)
> library(distr)
> X <- Norm(mean = 0, sd = 1)
> Y <- 4 - 3 * X
> Y
> Y <- exp(X)
> Y
> W <- sin(exp(X) + 27)
> W
> p(W)(0.5)
> W <- sin(exp(X) + 27)
> p(W)(0.5)
> curve(dchisq(x, df = 3), from = 0, to = 20, ylab = "Y")
> ind <- c(4, 5, 10, 15)

```



- > for(i in ind) curve(dchisq(x, df=i), 0, 20, add = TRUE)
- > library(actuar)
- > mgamma(1:4, shape = 13, rate = 1)
- > plot(function(x) {  
+ mgfgamma(x, shape = 13, rate = 1)  
+ }, from = -0.1, to = 0.1, ylab = "gamma mgf")
- > pnorm(2.64, lower.tail = FALSE)
- > pnorm(0.87) - 1/2
- > 2 \* pnorm(-1.39)