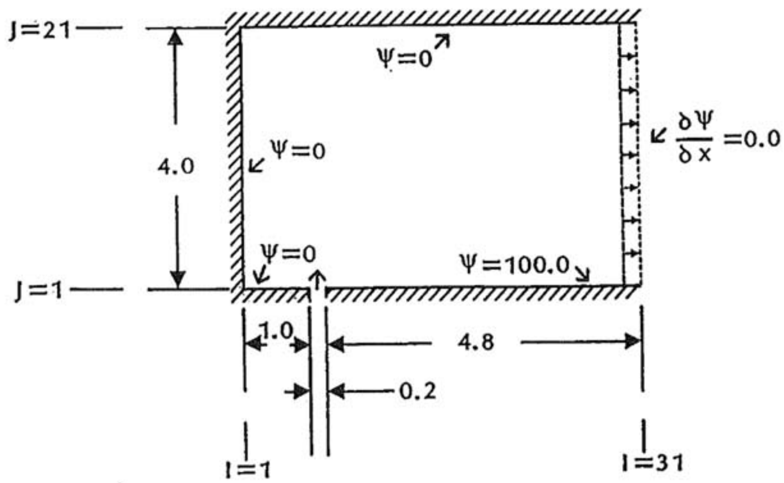
FLORIDA INSTITUTE OF TECHNOLOGY
AEROSPACE, PHYSICS, AND SPACE SCIENCES DEPARTMENT

**AEE 5150-E1: Computational Fluid Dynamics**
Fall 2021

# Coding Project 2

Due October 14, 2021

A two-dimensional inviscid, incompressible fluid is flowing steadily through a chamber between the inlet and the outlet, as shown in the figure. It is required to determine the streamline pattern within the chamber.




For a two-dimensional, incompressible flow, the continuity equation is expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

A stream function $\Psi$ may be defined such that

$$u = \frac{\partial \Psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \Psi}{\partial x}$$

Recall that a streamline is a line of constant stream function. Furthermore, vorticity is defined as

$$\vec{\Omega} = \nabla \times \vec{V}$$

for which

$$\Omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

For an irrotational flow, the vorticity is zero. Therefore,

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

Substituting the definitions of the stream function into the above equations yields

$$\frac{\partial}{\partial x}\left(-\frac{\partial \Psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial \Psi}{\partial y}\right) = 0$$

or

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = 0$$

The goal in this problem is to obtain the solution of this elliptic partial differential equation using the various numerical techniques discussed earlier. The solution will provide the streamline pattern within the chamber.

Since the chamber walls are streamlines, i.e. lines of constant $\Psi$, we will assign values for these streamlines, as shown in the figure. Solve this problem with codes using the following techniques:

a) Point Gauss-Seidel
b) Line Gauss-Seidel
c) Point SOR
d) Line SOR

For all methods, the step sizes are specified as

$$\Delta x = 0.2, \qquad \Delta y = 0.2 \qquad ERRORMAX = 0.01$$

with convergence criterion *ERROR* < *ERRORMAX*, where

$$ERROR = \sum_{\substack{i=2 \\ j=2}}^{\substack{j=JM-1 \\ i=IM-1}} \left| \Psi_{i,j}^{k+1} - \Psi_{i,j}^{k} \right|$$

Note the different kind of boundary condition on the outflow. Be sure to embed BC's accordingly.

Print the converged solution for each scheme for all $y$ locations at $x$ = 0.0, 2.0, 4.0, and 6.0. Numerical values should be formatted into decimal-aligned columns (with headers) and each non-integer number should have exactly 3 decimal places (including trailing zeros if necessary). Use an initial data distribution of $\Psi = 0.0$. Plot the streamline pattern (lines of constant $\Psi$). Rerun the SOR codes for several values of the relaxation parameter and plot the relaxation parameter versus the number of iterations for these two schemes. In each case, determine the optimal value of the relaxation parameter to three decimal places.

Submit an electronic copy of your code on Canvas. Submit hardcopies of your data and plots.

**Note: Except for the purpose of creating plots, MATLAB, Excel, or other commercially available software may not be used for this assignment. You may otherwise program in any language you wish except Python.**