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CORRECTION TO "ALGEBRAIC STRUCTURE OF SCHUR RINGS"

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Abstract. More general statements of some results stated in [1] concerning the Schur subrings are shown.

The transitivity of the relation $\mathcal{R}$ in [1. p. 58] is proved as a corollary of Theorem 1, this fills out a gap in [1, p. 59]. Furthermore, we show that Proposition 2.3 through Corollary 2.6 in [1] still hold whether subgroups H are normal or not.

Theorem 1. *Suppose $\mathcal{G} = (G; \{D_0, \cdots, D_d\})$ is an S–ring over G. Let $H \leq G$ and the natural mapping $\nu : G \to G/H$ (here G/H is the set of left cosets of H in G). Then $(i,j) \in \mathcal{R}$ if and only if $\nu(D_j) \subseteq \nu(D_i)$.*

Proof.

$(i,j) \in \mathcal{R}$

$\Leftrightarrow D_i^{(-1)} D_j \cap H \neq \emptyset \Leftrightarrow D_i H \cap D_j \neq \emptyset$

$\Leftrightarrow \exists d_j \in D_j$ such that $d_j = d_i h$ for some $d_i \in D_i, h \in H$

$\Leftrightarrow \overline{D_j}$ appears in the expression of $\overline{D_i}\,\overline{H}$ as a linear combination of $\overline{D}_k$'s

$\Leftrightarrow \forall d_j \in D_j \ \exists d_i \in D_i$ such that $d_j \in d_i H$.

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Corollary 2. *$(i,j) \in \mathcal{R}$ if and only if $\nu(D_i) = \nu(D_j)$. Hence $\mathcal{R}$ is an equivalence relation.*

Corollary 3. *Let $[i]$ be an equivalence class of $\mathcal{R}$ containing i. For $g \in G$, set $S(g) = \{j | gH \cap D_j \neq \emptyset\}$. Then $[i] = S(g)$ for any $g \in D_i$.*

Proof. For any fixed $g \in D_i$, suppose $j \in S(g)$ $\exists h \in H$ such that $gh \in D_j$. So we have $h \in g^{-1}D_j$ and $D_i^{(-1)}D_j \cap H \neq \emptyset$. Hence $(i,j) \in \mathcal{R}$ and $S(g) \subseteq [i]$. On the other hand, suppose $(i,j) \in \mathcal{R}$, then $j \in S(g)$ by Corollary 2. Hence $[i] = S(g)$.

Corollary 4. *If $H \leq G$, then $\nu(D_i)$ and $\nu(D_j)$ are either disjoint or identical.*

Proof. Suppose $\nu(D_i) \cap \nu(D_j) \neq \emptyset$, then $D_i^{(-1)}D_j \cap H \neq \emptyset$. Thus $(i,j) \in \mathcal{R}$. The corollary follows easily.

References

1. W. C. Shiu, Algebraic structure of Schur ring, *Chinese J. Math.* **21** (1993), 55–71.

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