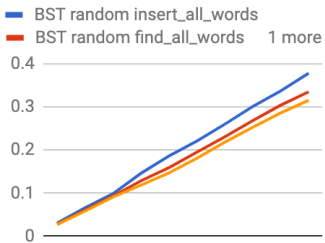


Binary Search Tree (BST) Unsorted:



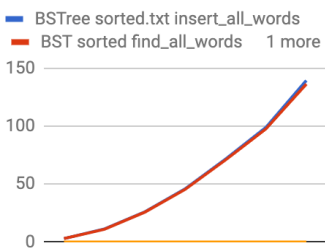
Observed time complexity:

Insert: $O(\log n)$. Inserting n words takes $O(\log n) * n = O(n \log n)$ time, as observed in the graph.

Find: $O(\log n)$. Finding n words takes $O(\log n) * n = O(n \log n)$ time, as observed in the graph.

Remove: $O(\log n)$. Removing n words takes $O(\log n) * n = O(n \log n)$ time.

Binary Search Tree (BST) Sorted:



Observed time complexity:

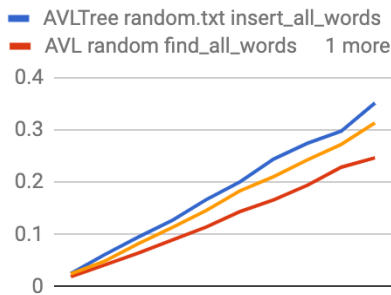
Insert: $O(n)$. Inserting n words takes $O(n) * n = O(n^2)$ time, as observed in the graph.

Find: $O(n)$. Finding n words takes $O(1) * n = O(n^2)$ time, as observed in the graph.

Remove: $O(1)$. Removing n words takes $O(1) * n = O(n)$ time, as observed in the graph.

Analysis: Insert and find take $O(n)$ time because an unbalanced tree can degrade to $O(n)$ performance when inserting in a sorted order. Remove likely takes $O(1)$ time because removing is in the same order as inserting, and so it is effectively removing the first element each time.

AVL Tree Unsorted:



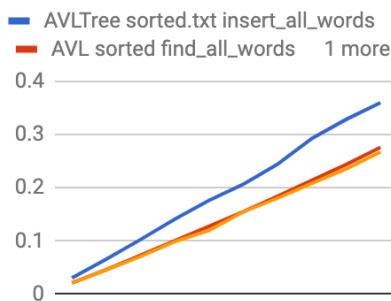
Observed time complexity:

Insert: $O(\log n)$. Inserting n words takes $O(\log n) * n = O(n \log n)$ time, as observed in the graph.

Find: $O(\log n)$. Finding n words takes $O(\log n) * n = O(n \log n)$ time, as observed in the graph.

Remove: $O(\log n)$. Removing n words takes $O(\log n) * n = O(n \log n)$ time.

AVL Tree Sorted:



Observed time complexity:

Insert: $O(\log n)$. Inserting n words takes $O(\log n) * n = O(n \log n)$ time, as observed in the graph.

Find: $O(\log n)$. Finding n words takes $O(\log n) * n = O(n \log n)$ time, as observed in the graph.

Remove: $O(\log n)$. Removing n words takes $O(\log n) * n = O(n \log n)$ time.

Analysis: As expected, keeping the tree balanced makes the most significant impact when inserting into the tree in a sorted order. The AVL tree was able to perform much better.