HW 3 Question 1

Given $\vec{x}^0 = [2,2]^T$, $f(x_1,x_2) = \frac{x_1^4}{4} + \frac{x_2^4}{9}$

$$\nabla f = \begin{bmatrix} x_1^3 \\ 4x_2^3/9 \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix}$$

$$H = \begin{pmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{pmatrix} = \begin{pmatrix} 3x_1^2 & 0 \\ 0 & 4x_2^2/3 \end{pmatrix}$$

$$H|_{x_0} = \begin{pmatrix} 12 & 0 \\ 0 & 48/9 \end{pmatrix}, \quad g|_{x_0} = \begin{bmatrix} 8 \\ 32/9 \end{bmatrix}$$

$$H_n \cdot P_n = g_n \Rightarrow P_n = H_n^{-1} \cdot g_n$$

$$\Rightarrow P_n = \begin{bmatrix} 2/3 \\ 2/3 \end{bmatrix} = P_0$$

$$X_1 = X_0 - 1 \cdot P_n \Rightarrow X_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 2/3 \\ 2/3 \end{bmatrix} = \begin{bmatrix} 4/3 \\ 4/3 \end{bmatrix}$$

→

$\text{abs}(x_1 - x_0) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$4 \cdot \left(\frac{16}{9}\right) = \frac{64}{9} \cdot \frac{1}{9} = \frac{64}{81}$

Now calc x_2!

$x_1 = \begin{bmatrix} 4/3 \\ 4/3 \end{bmatrix}$ $H|_{x_1} = \begin{pmatrix} 16/9 & 0 \\ 0 & 64/81 \end{pmatrix}$ $\nabla f|_{x_1} = \begin{bmatrix} 64/27 \\ 256/243 \end{bmatrix} = g_1$

$p_1 = H_1^{-1} g_1 = \begin{bmatrix} 4/9 \\ 4/3 \end{bmatrix}$

$x_2 = x_1 - \begin{bmatrix} 4/9 \\ 4/3 \end{bmatrix} = \begin{bmatrix} 8/9 \\ 8/9 \end{bmatrix}$

Two iterations complete ✓

$x_0 = (2, 2)$ Now for $f(x_1, x_2) = \frac{x_1^2}{4} + \frac{x_2^2}{9}$

$\nabla f = g = \begin{bmatrix} x_1/2 \\ 2x_2/9 \end{bmatrix}$ $H = \begin{bmatrix} 1/2 & 0 \\ 0 & 2/9 \end{bmatrix}$

$g|_{x_0} = \begin{bmatrix} 1 \\ 4/9 \end{bmatrix}$ $H|_{x_0} = \begin{bmatrix} 1/2 & 0 \\ 0 & 2/9 \end{bmatrix}$

$p_n = H^{-1} \cdot g = \underset{2\times2}{\begin{bmatrix} 2 & 0 \\ 0 & 9/2 \end{bmatrix}} \underset{2\times1}{\begin{bmatrix} 1 \\ 4/9 \end{bmatrix}} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$x_1 = x_0 - p_n = \begin{bmatrix} 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ complete.