Algorithms and complexity HW 1

1. Consider the polynomial $P(x) = x^3 - 5x^2 + 7x - 2$

a. Compute the derivative $P'(x)$ usual way and plug in $x_0 = 2$

$P'(x) = 3x^2 - 10x + 7 \Rightarrow P'(2) = 12 - 20 + 7 = \boxed{-1}$

b. insert the dual number $x = 2 + \epsilon$ into $P(x)$ and use the algebra obeyed by dual numbers to compute the value of the derivative $P'(x_0 = 2)$

$$P(2+\epsilon) = (2+\epsilon)^3 - 5(2+\epsilon)^2 + 7x - 2$$

$$(x+y)^3 = (x+y)(x^2+2xy+y^2) = (x^3+2x^2y+xy^2+x^2y+2xy^2+y^3)$$
$$= (x^3+3x^2y+3y^2x+y^3)$$

$$\therefore (2+\epsilon)^3 = 8 + 12\epsilon + 6\epsilon^2 + \epsilon^3$$

$$\text{Also: } (2+\epsilon)(2+\epsilon)^2 = (2+\epsilon)(4+2\epsilon+2\epsilon+\epsilon^2) = (2+\epsilon)(4+4\epsilon+\epsilon^2)$$
$$= 8 + 8\epsilon + 2\epsilon^2 + 4\epsilon + 4\epsilon^2 + \epsilon^3 = 8 + 12\epsilon + 6\epsilon^2 + \epsilon^3 \checkmark$$

$$P(2+\epsilon) = 8 + 12\epsilon + 6\epsilon^2 + \epsilon^3 - 20 - 20\epsilon - 5\epsilon^2 + 14 + 7\epsilon - 2$$
$$= \epsilon^3 + \epsilon^2 - \epsilon \Rightarrow \epsilon^2, \epsilon^3 = 0 \Rightarrow \boxed{P(2+\epsilon) = -\epsilon}$$
$$\therefore P'(2) = -1$$