Simulation studies of Black Holes (BH) originating in Globular clusters

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Abstract

Black holes (BHs) are celestial objects characterized by their extraordinary gravitational forces, which are so powerful that they prevent even light from escaping. Within globular clusters, densely packed groups of stars held together by gravity, the presence and evolution of BHs significantly influence the dynamical processes unfolding in these systems. This study aims to examine the behavior of BHs originating in globular clusters, with a particular focus on understanding their dynamics as they move rapidly through the cluster. By creating a realization of a Plummer sphere, we analyze the equilibrium state of the system and introduce a massive particle to represent a BH. Our investigation delves into the dynamics of this massive particle as it traverses the Plummer sphere, taking into account the effects of dynamical friction. The findings from our simulations offer valuable insights into the dynamics of BHs in globular clusters, enhancing our understanding of their behavior within these dense stellar environments.

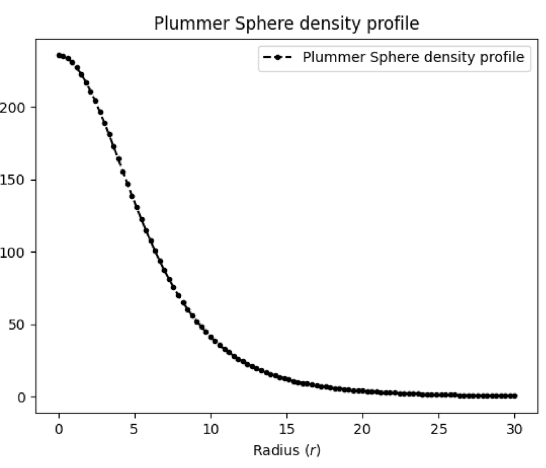
Problem statement

Probe how the dynamics of massive compact objects such as Black holes(BH) moving rapidly through globular cluster changes ?
Does it slow down enough to be captured by the cluster or is able to escape the system/cluster?

Modelling Globular clusters

We have modelled our cluster using the Plummer sphere. It is a dynamically stable model described by the following potential-density pair relation.

$$\begin{cases} \rho(r) &= \dfrac{3M}{4\pi b^3}\left(1+\dfrac{r^2}{a^2}\right)^{-5/2} \\ \Phi(r) &= \dfrac{GM}{\sqrt{r^2+b^2}} \end{cases} \tag{1}$$



Realisation of Plummer sphere

- Generate particles in the *spatial subspace* as a random realisation of the plummer sphere mass distribution.
- Assign each particle a velocity vector in the *velocity subspace* such that the velocity distribution follows the energy distribution function.

Populating *spatial subspace*

Cumulative mass distribution function for plummer sphere is given by

$$M(r) = \int_0^\infty 4\pi r^2 \rho(r) dr = \frac{M_0 r^3}{(r^2+b^2)^{3/2}} = \frac{M_0 (r/b)^3}{(1+r^2/b^2)^{3/2}} \tag{2}$$

Using the above relation, one can solve for $r(m)$ as

$$\boxed{r(m) = \frac{b}{\sqrt{(M_0/m)^{-2/3}-1}}} \tag{3}$$

Particles can be arranged as a random realisation of eqn.3

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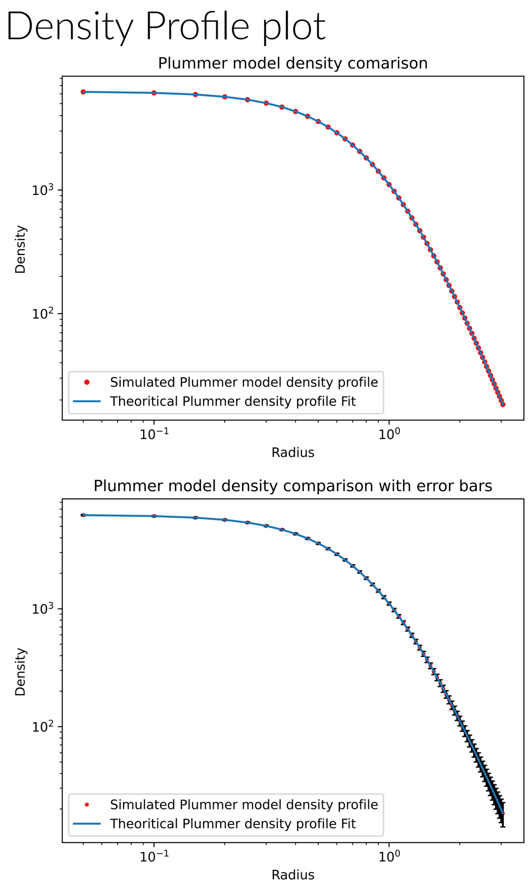
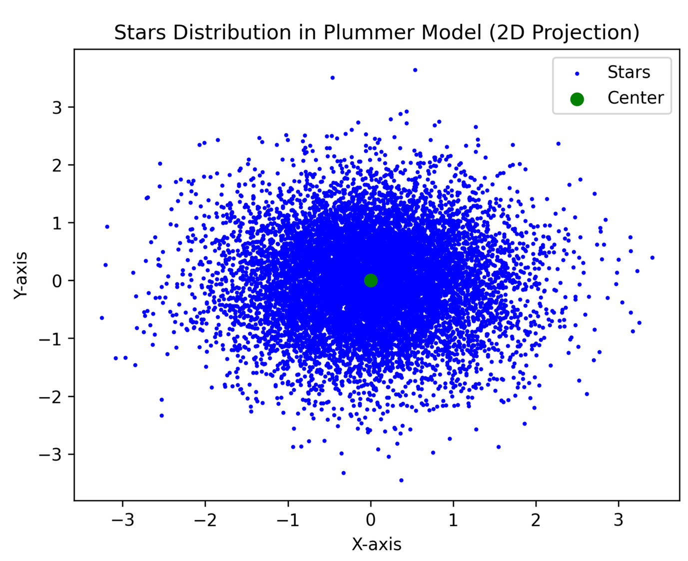
Spatial subspace plot of plummer sphere

Density Profile plot



Plummer Sphere 2D Projection



Populating *velocity subspace*

Energy distribution function for particles in plummer sphere is

$$f(\vec{r},\vec{v})d\vec{r}d\vec{v} = f(E(r,v))4\pi r^2 dr 4\pi v^2 dv = \frac{384\sqrt{2}}{7\pi m}(-E)^{7/2} r^2 v^2 dr dv \tag{4}$$

Transforming the above equation for the magnitude of velocity , we get

$$g(q) = (1-q^2)^{7/2} q^2 \text{ where } q = \frac{v}{v_e} \tag{5}$$

Looks similar to random realisation problem, but analytical inverse of eqn.5 doesn't exists. Using *Jon Von Neumann rejection technique* one can solve for q and hence for $v(r)$ as

$$\boxed{v = q \times v_e \implies v(r) = \sqrt{2}q(r^2+1)^{-1/4}} \tag{6}$$

NBody simulations

We conducted N-Body simulations on our numerically generated Plummer sphere, as well as upon introducing a black hole within the Plummer sphere, in order to comprehensively examine the underlying dynamics of these systems. We used *Barnes-Hut algorithm* for NBody simulations.

Barnes-Hut algorithm

The Barnes-Hut algorithm is a hierarchical, tree-based method for approximating N-body simulations, which significantly reduces computational complexity. By dividing the simulation space into a hierarchical structure called an octree (in 3D) or quadtree (in 2D), it enables the calculation of gravitational interactions among particles with a time complexity of $\mathcal{O}N\log N$ instead of the standard $\mathcal{O}N^2$ required for direct methods. A very brief summary of how this algorithm works is listed below:

1. Initialisation of populating the octree structure
   - Vertical cubical division of empty space into sub(cells)
   - Construction of the tree structure by
     - Discarding empty subcells
     - Accepting subcells with only one occupant
     - Recursively dividing subcells with shared occupancy
   - Performing this tree construction *"ab-initio"* every time.
2. Force calculation
   The interaction between the particle and the cluster of particles is determined by the hyper-parameter called θ defined as ratio of length of cell to the distance between particle and cluster. The force terms are calculated by taylor expanding cluster potential around its center of mass and is given by the following eqn.
   $$\boxed{\vec{a} = -GM\frac{\hat{r}}{r^2} + \frac{G}{r^4}\vec{Q}\cdot\hat{r} - \frac{5G}{2}(\hat{r}\cdot\hat{r})\frac{\hat{r}}{r^4}} \tag{7}$$
3. Update the position and velocity of each particle
   One can employ various integrators, such as the Leapfrog integrator, to efficiently update the position and velocity of each particle following every time step, allowing for accurate and flexible simulation dynamics.

Results

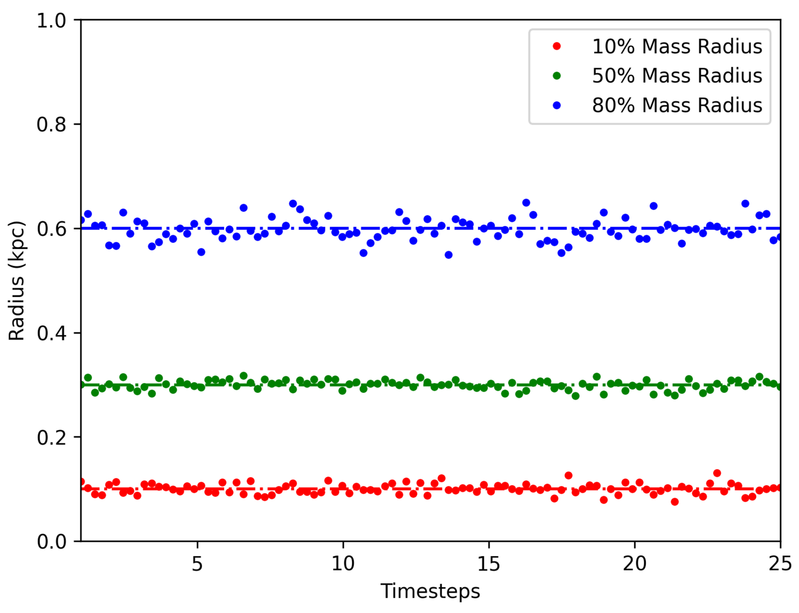
NBody simulation of plummer sphere

We simulated the plummer sphere using *Barnes-Hut algorithm* to establish the time-stationary nature of plummer sphere.

Evolution and equilibrium of the system

Snapshots

Plots



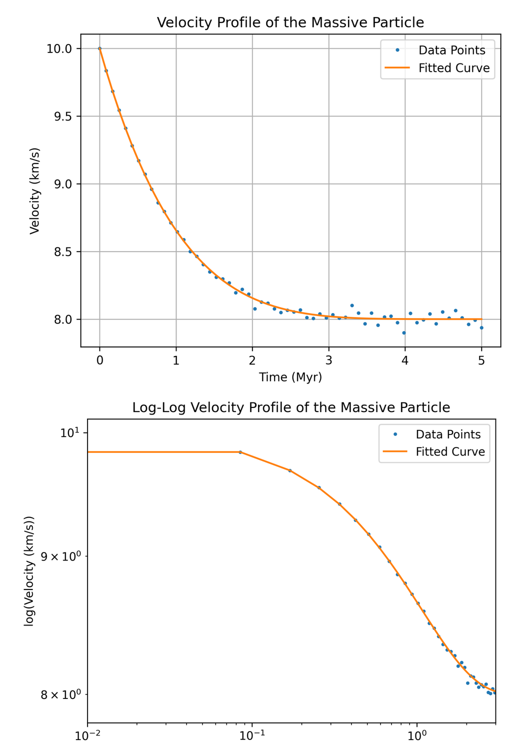
NBody simulation of BH moving in plummer sphere

We try to understand the dynamics of BH moving through the plummer by modeling its velocity profile under the influence of dynamical friction and compare it to the analytically calculated profile. Analytical expression for a massive particle moving through a homogenous background of stars is given by

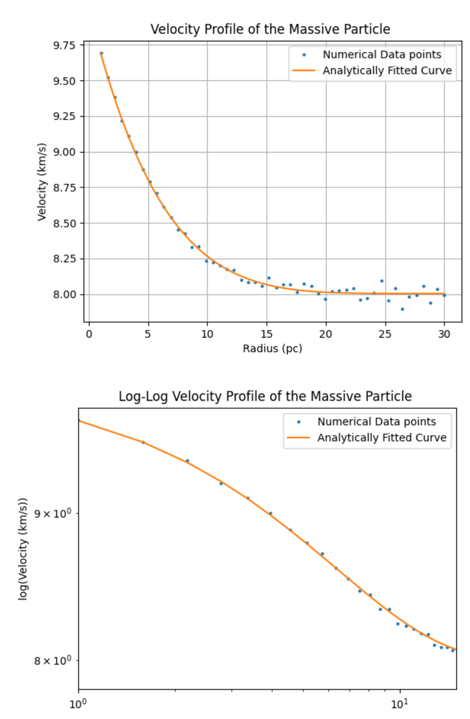
$$\boxed{\frac{d\vec{v}_m}{dt} = -16\pi^2 G^2 m_*(m+m_*)\log(\Lambda)\left[\int_0^{v_m} f(v_*)v_*^2 dv\right]\frac{\vec{v_m}}{v_m^3}} \tag{8}$$

Velocity profile of BH moving the cluster

Velocity Profile



Velocity Profile



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