Project on

"An analytical model to evaluate perforation Impedance at higher temperatures "

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Mechanical Engineering with Honours

By

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CERTIFICATE

This is to certify that **Mr. Shivam Mishra** bearing admission number **14JE000117** has successfully carried out his B. Tech. project entitled **"An analytical model to evaluate perforation impedance at higher temperatures"** under the guidance of **Dr. Rabindra Nath Hota** (Associate Professor, IIT ISM Dhanbad). This thesis is an authentic record, carried out as requirement for the award of Bachelor of Technology in Mechanical Engineering at Indian Institute of Technology (ISM) Dhanbad. The results in this thesis have not been submitted elsewhere for the award of any other degree.

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Contents

Acknowledgement	4
List of Symbols	5
Abstract	6
Introduction	7 - 9
Literature Review	10-12
Theory	13-15
Results & Discussion	16-18
Conclusion and Future Scope	19
References	20-21
Appendix 1	22-23
Appendix 2	24-25

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Bachelor of Technology

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List of Symbols

ρ	Density of the fluid
ϕ	Pressure gradient
ω	Excitation Angular frequency
j	Square root of minus 1
t	Liner thickness
ΔP	Pressure differential across the perforated hole
ΔT	Temperature differential
J_1	Bessel function of order one
μ	Coefficient of dynamic viscosity
T_{amb}	Ambient temperature
T	Temperature inside the duct
d	Diameter of the hole
γ	Isentropic compressibility of air
C	Sutherland’s constant
V	Mean velocity
Z	Impedance

Abstract

Acoustic liners are widely used to damp sound waves propagating inside a duct or an enclosure. They are generally found in a high-temperature environment; e.g. car exhaust mufflers, exhaust ducts of aircraft jet engines, and inside the combustion chamber of gas turbine engines.

Bias-flow acoustic liners are used to suppress thermo-acoustic instabilities developed in gas turbines by increasing the acoustic losses of the system. Instabilities rise from the interaction between acoustic waves and unsteady combustion. The acoustic performance of liners is mathematically expressed in terms of its impedance. An apt impedance model is required to effectively design the acoustic liner. There are many impedance models available to estimate the liner impedance but most of them are semi-empirical and are based on the experiments which are carried out at room temperatures. Hence, none takes the effect of high temperature and temperature gradients across the liner into account which is the case for most of the practical scenarios especially for gas-turbine applications where the liner is subjected to temperatures as high as 1500 K.

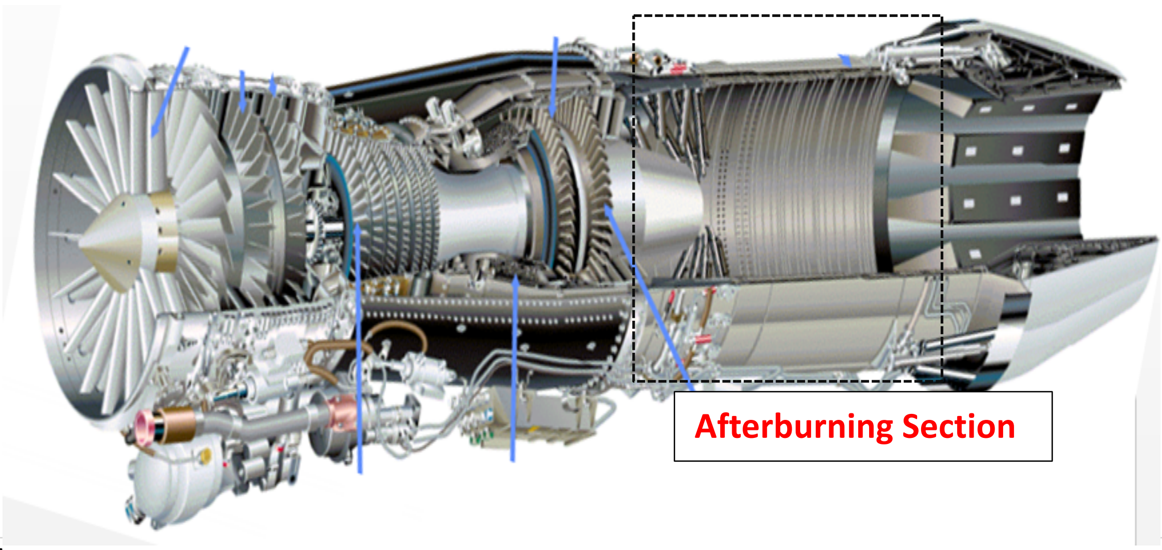
The aim of the current work is to develop a formula to estimate the liner impedance at a higher temperature by making use of existing impedance models.

Introduction

Gas turbine reheat thrust augmenters known as afterburners are used to provide additional thrust during emergencies, like combat, or in supersonic flight. Afterburners provide a lightweight, low-capital cost method to greatly increase engine thrust. Afterburning is achieved by injecting additional fuel into the jet pipe downstream of the turbine. The reheat flame which thus produced is a powerful source of sound, and leads to the formation of a variety of acoustically driven combustion instabilities. The two most common instabilities are referred to by the terms 'buzz' and 'screech'. Buzz is a low frequency, self-excited oscillation that can occur above a certain fuel-air ratio.

Screech occurs at higher frequencies, when the flame excites a transverse mode of oscillation. Screech is accompanied by high frequency pressure oscillations that may be of such magnitude as to cause rapid deterioration of the burner. It's onset is invariably followed by rapid mechanical failure. This failure evinces itself in the tearing of the sheet metal, or if the screech is mild, persistent breakage of bolts or slackening the nuts [1].

Combustion-driven flow oscillations that arise in combustors and the afterburners are difficult to predict [2]. Because of destructive nature of screeching combustion considerable effort is required to find methods of mitigating or preventing the occurrence of screech. Screech is associated with transverse oscillations. Perforated liners with a bias flow through the apertures are effective in mitigating transverse oscillations over the full operable range of fuel-air ratio for burner [3].



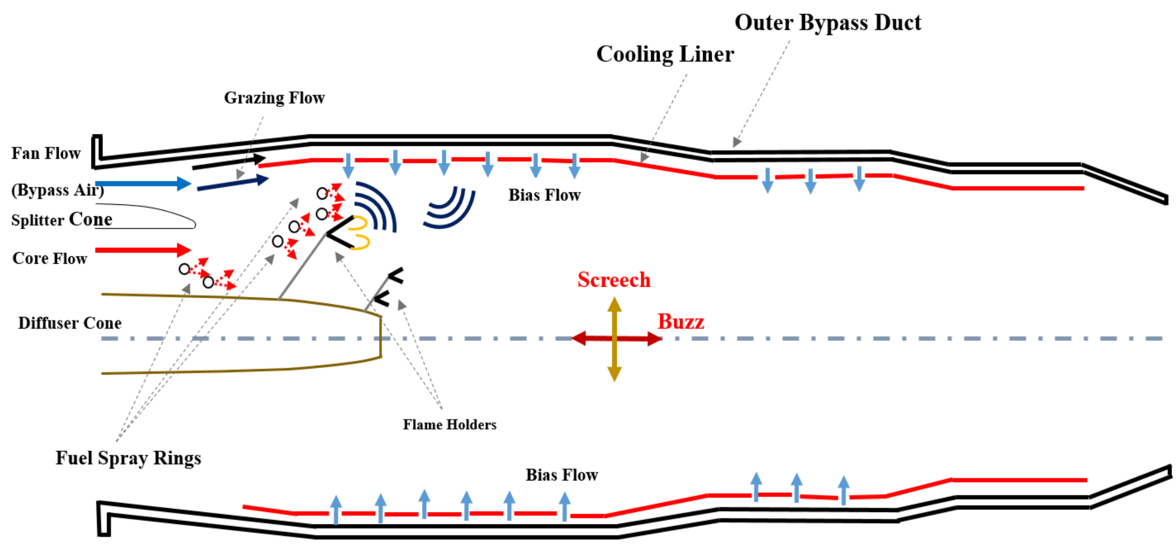


Figure 1 : Cross-sectional schematic of an Afterburning Turbofan Engine [4]

The acoustic performance of liners is mathematically expressed in terms of its impedance. The acoustic impedance is a measure of the amount by which the motion induced by a pressure applied to a surface is impeded; or in other words, a measure of the lumpiness of the surface. Since frictional forces are proportional to velocity, a natural choice for this measure would be the ratio between the pressure and velocity. The acoustic impedance is defined as the ratio of the acoustic pressure in a medium to the associated normal particle velocity. The normalized normal-incident impedance is a useful dimensionless quantity of the impedance with respect to the characteristic impedance of the medium (density of the fluid, ρ , multiplied by the speed of sound, c, in that fluid).

The impedance consists of a real part, the resistance, and an imaginary part, the reactance. The most important factor that controls attenuation of noise by liners is the real (resistive) part of the impedance, whereas the imaginary (reactive) part controls the frequency of maximum attenuation.

Afterburner liners are designed based on the impedance models developed from experimental studies. There are many different models in the literature which describe the hole impedance under different surrounding conditions such as the acoustic pressure field and the flow grazing to the surface of the perforate and through the holes. Most of these semi-empirical models are based on the experiments conducted at room temperatures and hence, do not take into account the effect of high temperature or the thermal differential set up between main and by pass ducts due to the hot exhaust gases. There is not much published work on the effect of high temperatures and temperature differential inside the cavity. A reason for that could be the difficulty of performing pressure measurements under hot conditions.

Perforates present in liner dissipate sound energy by viscous dissipation mechanisms inside the holes. As the temperature rises viscosity of air also increases [5], making the viscous impedance across the holes more and more relevant for acoustic losses in the system. Dependency of impedances other than viscous impedance on temperature is not considered in this work. The objective of the present work is to estimate the liner viscous impedance at higher temperatures using the impedance models present at room temperature. It is reported that temperature effect on the impedance can be predicted quite well by changing the fluid properties[6]. Impedance-model in this work is attempted by varying the relevant fluid properties with respect to temperature.

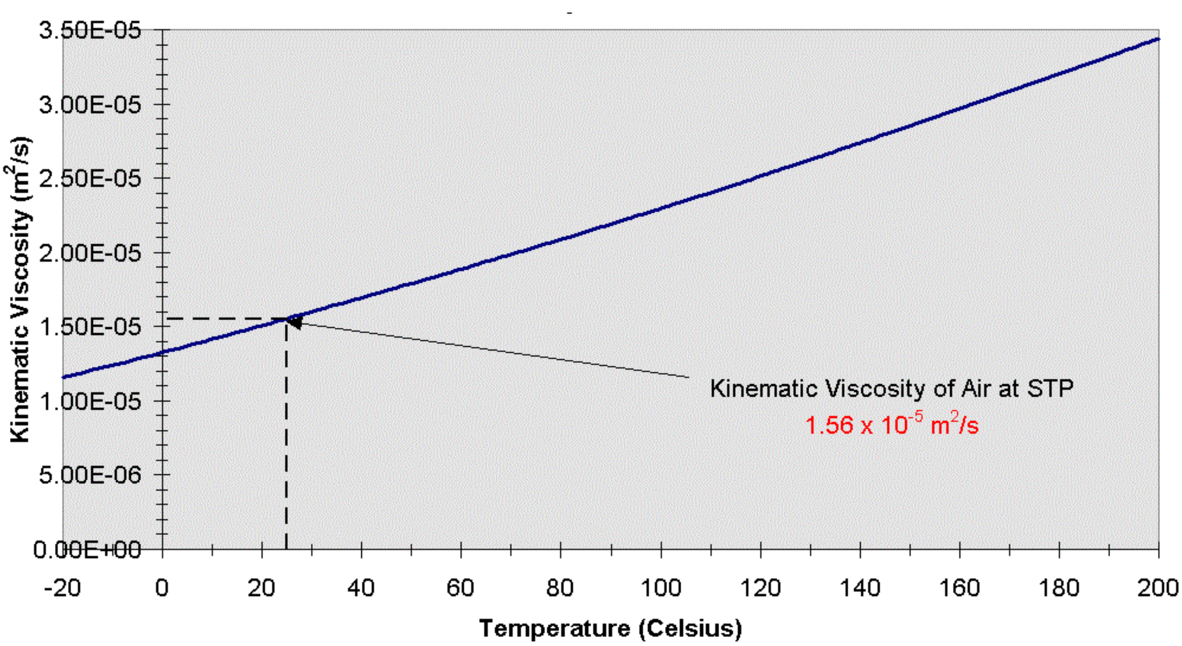


Figure 2:Kinematic viscosity of air at 1 atm as a function of temperature

Literature Review

In the early seventies, researchers began to investigate the no flow impedance of single-degree of-freedom (SDOF) liners. It consists of a sandwich structure: a solid back-plate and a perforated face sheet enclose a honeycomb cellular separator. In this paper, the theoretical approach is based on Crandall's theory of acoustic propagation in perforates [7]. Melling [8] and Guess [9] used this approach, together with some measurement results from Ingård [10] and Sivian [11]. In order to derive the expression of the viscous impedance due to lumped face-sheet resistance and mass reactance of an array of orifices, Crandall first assumed the perforate hole to be an infinite duct, then end corrections are added to account for the finite length of the perforate holes.

Later on, other approaches have been used to develop other models. Motsinger & Kraft [12] assumed an acoustic resistance equivalent to a DC flow resistance inside the hole. Therefore, there was no frequency dependence for the resistive part in their impedance model. This works well for low frequencies. Hersh, Walker & Celano [13] derived a simple, one dimensional, fluid mechanical-lumped element model applying the conservation of mass and momentum across a control volume just outside the orifice. They assumed the flow inside the orifice to be consisting of a uniform part within an inviscid core, and a laminar boundary layer part.

Tam and Kurbatskii [14] investigated the flow around and inside a typical liner resonator under the excitation of an incident acoustic field by Direct Numerical Simulation. Numerical experiments revealed that at low sound intensity, acoustic dissipation comes mainly from viscous losses in the jet-like unsteady laminar boundary layers adjacent to the walls of the resonator opening. At high sound intensity, dissipation is due to the shedding of micro vortices at the mouth of the resonator. The energy dissipation rate associated with the shedding of micro-vortices is found to be very high compared to viscous losses. These observations support the assumption made by Crandall for the linear regime.

Inside aircraft engines, there is high mean flow, which is considered grazing to the liner surface. This aspect has received special attention due to the strong effect of grazing flow on the acoustic behaviour of perforated structures. Generally speaking, the dependence of the hole impedance on the grazing flow can be explained as follows: The flow blows off some of the end effects which result in a decrease of the so-called attached mass, which means that some of the kinetic energy stored in the oscillating medium across the orifice is lost; and this results in an increase in the acoustical resistance of the hole and a shift in the resonator resonance frequency. In short, it has been found that the orifice resistance increases with mean flow speed while the reactance decreases.

The difficulties associated with the theoretical modelling of the acoustic behaviour of perforates subjected to grazing and bias flow have resulted in a reliance on empirical models based on experimental data. However, these experiments do not consider the high temperature reached during the practical scenarios.

The temperature in a combustion chamber is very high and little is known on the effect of high temperature on the acoustic performance of a bias flow liner. It is not only that the temperature is very high, but there is also a significant difference in the temperatures of the bias flow and the grazing flow. The temperature of the grazing flow is increased, while the bias flow remains constant at approximately ambient. Very few publications are available, that treat the influence of temperature on the acoustic properties of a liner. Their findings are summarized here.

Christie [15] studies the acoustic absorption of porous materials at high temperatures. The quantities that determine the temperature dependence are the flow resistance, the density of the fluid, and the speed of sound. He finds that the Delany Bazley model is capable of providing useful predictions, when the empirical constants are adjusted to the high temperature condition.

Rademaker et al. [16] present a series of impedance measurements of perforated, linear, and porous liners at varying temperatures up to 773 K and including a grazing flow with Mach numbers up to 0.4. It was found that the resistance of the perforated liner was nearly independent of the temperature, while the resistance of the linear and the porous liner is significantly increasing with the temperature. All configurations exhibit a reduction of the mass reactance with higher temperatures.

Sun et al. [17] confirm these observations regarding the impedance in a comprehensive experimental and theoretical study with metallic porous materials. Furthermore, their results show consistently, that the sound absorption is reduced when the temperature is increased.

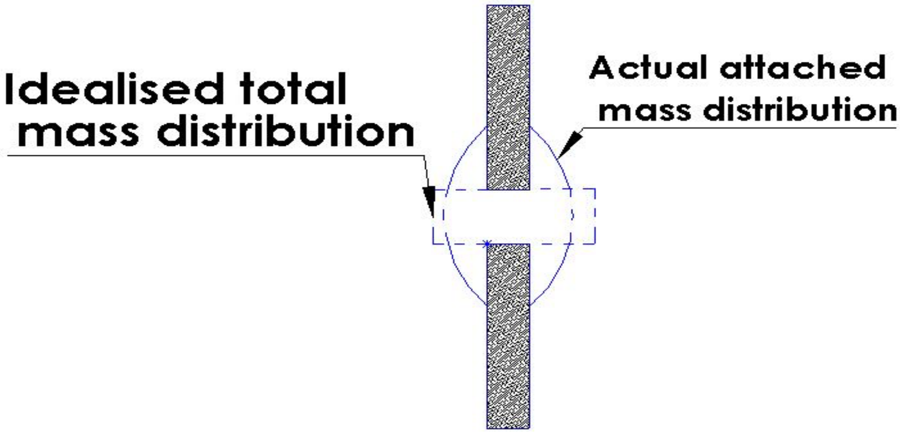
Elnady et al. [6] measure the acoustic properties of a single orifice and cavity, which they placed in an oven. The temperature is increased up to 573 K and measurements are carried out at 120 dB (linear regime) as well as at 140 dB (nonlinear regime). In the linear regime the resistance is increasing with the temperature and its maximum is shifted to higher frequencies. At 140 dB only the resonance frequency changes, while the maximum level remains constant with temperature. The reactance is reduced with higher temperatures at both sound pressure levels.

The results are compared to a standard impedance model. While the reactance agrees well with the measured data, the resistance is over predicted at higher frequencies. This mismatch occurs already for ambient temperature, so that it is not related to the temperature influence. However, the general trend when increasing the temperature is captured by the impedance model. They conclude that the temperature effect is sufficiently described by adjusting the properties of air (i.e. density, viscosity, and speed of sound) accordingly.

The results can be interpreted as follows. The absorption mechanism of linear or porous liners is mainly based on viscous dissipation. The viscosity increases with the temperature, so that the resistance grows accordingly.

Theory

The perforated face-sheet can be modelled as an array of Helmholtz resonators. The mass of the air inside the perforate hole is the resonator mass and the air inside the cavity performs as the resonator spring. When thickness and the hole diameter are small compared to the wavelength, then under the influence of the incident wave the air in the hole move as lumped mass, that is it behaves as a small piston of air.



Let's consider a tube of fluid of length **t** (i.e. thickness of liner) in which a shear layer is set up as a result of viscous retardation of the fluid near the wall. The total driving force on an annular ring of fluid, at radius r, is balanced by the inertial and resistive forces. The equation of motion at any temperature T can be written as

$$\rho(T)[j\omega(2\pi r)dr * t]v(r,T) + \frac{\partial}{\partial r}\left[-\rho(T)2\pi rt\mu(T)\frac{\partial v(r,T)}{\partial r}\right]dr = (\Delta P)2\pi rdr \qquad [1]$$

$\Delta P = \emptyset t$, $\emptyset$ is the pressure gradient across the holes at temperature T. v(r, T) is the velocity of fluid particles at a distance r in the annular ring of fluid at any temperature T. The terms varying with the temperature are ρ, μ and ΔP.

Let the temperature inside the duct be increased adiabatically from T to T+dT. Due to this temperature increment, temperature outside the duct will also increase but the increment will not be the same, let it be dT_1 and let dT and dT_1 be related by a constant k_1 as

$$dT_1 = dT/k_1 \qquad [2]$$

Pressure difference across the perforates is modified to

$$\Delta P_{new} = \rho(T)R\left[\frac{1}{k_1} - 1\right]dT + \emptyset t \qquad [3]$$

<u>Relation between density and temperature</u>

$$\rho = k_2 T^f \qquad [4]$$

$k_2 = \frac{\rho_0}{T_{amb}^f}$

$f = 1/(\gamma - 1)$
<u>Variation of viscosity μ with temperature [ref]</u>

$$\mu = k_3 \frac{T^{1.5}}{T+C} \qquad [5]$$

$k_3 = \mu_0 \frac{(T_0+C)}{(T_0)^{3/2}}$, where T_0, C and μ_0 are 291.15K, 120K and $1.827 * 10^{-5}$ Pa-s respectively.

Equation [1] after temperature increment dT can be written as

$$\rho(T)[j\omega(2\pi r)dr * t]v(r,T) + \frac{\partial}{\partial r}\left[-\rho(T)2\pi r t\mu \frac{\partial v(r,T)}{\partial r}\right]dr = (\Delta P)_{new} 2\pi r dr \qquad [6]$$

From [3], [4], [5] and [6]

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v(r,T)}{\partial r} - \frac{j\omega k_2}{k_3} T^{(f-1.5)}(T+C)v(r,T) = -\frac{Rk_2}{k_3 t}\left[\frac{1}{k_1} - 1\right]T^{(f-1.5)}(T+C)dT - \frac{\emptyset t}{k_3 t}T^{(-1.5)}(T+C)$$

$$k_4 = -\frac{j\omega k_2}{k_3}$$

Solving the above equation [Appendix 1], we have

$$v(r,T) = \left[a_0[J_0(e)] + \frac{\lambda^2}{k_4}[J_0(e) - 1]\right] * \frac{1}{\lambda^2 k_3 t}\left[Rk_2\left\{\frac{1}{k_1} - 1\right\}\Delta T + \emptyset t\, T^{(-f)}\right]$$

e = $k_4\, m^2$
m = $T^{(f-1.5)}(T+C)r^2$

a_0 and λ are any arbitrary constants whose values are found using the boundary conditions. Using the boundary condition that v(d/2, T) = 0 we obtain the value of a_0.

$$a_0 = \frac{-\lambda^2}{k_4} * \left[1 - \frac{1}{J_0\left(e_{\frac{d}{2}}\right)}\right]$$

Particle velocity v(r,T) is integrated over the cross-sectional area to give mean velocity V which is a function of perforation diameter and temperature.

$$\mathbf{V} = \left[(a_0 e_{\frac{d}{2}}\ J_1\left(e_{\frac{d}{2}}\right)) + \left(\frac{\lambda^2}{k_4} e_{\frac{d}{2}}\ J_1\left(e_{\frac{d}{2}}\right)\right) - \left(\frac{\lambda^2}{k_4} * \frac{{e_{\frac{d}{2}}}^2}{2}\right) \right] * A(T)\ \ * \frac{1}{\pi * \frac{d^2}{4}} * \text{constant}$$

$A(T)\ = \frac{1}{\lambda^2 k_3 t}\left[R k_2 \left[\frac{1}{k_1} - 1\right] \Delta T + \emptyset t T^{(-f)}\right]$ and constant = $\left[8/k_4 (T + C) T^{(f-1.5)} d^2\right]$

$$\Delta \mathbf{P}_{\mathbf{new}} = k_2 R \left[\frac{1}{k_1} - 1\right] (T^{f+1} - T_{amb}^{f+1}) \frac{1}{f+1} + \emptyset t$$

Impedance **Z** = $\Delta \mathbf{P}_{\mathbf{new}} / \mathbf{V}$, Z is the viscous impedance due to a single hole at any temperature T and aperture diameter d. This model can be used to develop the total acoustic impedance of the liner at higher temperatures.

$\emptyset t = \mathbf{z_0}\ \mathbf{v_0}$, where z_0 and v_0 are impedance and mean velocity for the perforate at temperature T, which is determined by existing impedance models, such as Bauer impedance model [18] , Howe Model (HM) [19,20] , Betts impedance model [21,22] , Bellucci impedance model [23,24] which do not consider temperature as varying parameter.

Results and Discussions

A formula has been derived to determine the viscous impedance values of a perforated liner at higher temperatures by making use of the impedance models or experimental studies carried out at cold conditions when there is no onset of temperature gradient across the perforation hole. Existence of a temperature gradient is very likely to exist inside the cavity in both the real applications and experiments.

Inside the liners installed in the exhaust duct of gas turbines, the temperature of the flowing gases can reach up to several hundreds of degrees, whereas the temperature on the outside of the engine can be below 0 degree Celsius during flight, or at least ambient when the aircraft is close to the ground. So, the temperature might drop a lot within a very short distance of a few centimetres. In this work the temperature gradient across the holes is dealt by taking the mean temperatures across them.

The derived formula for the impedance **Z** of a single-hole perforation is

$$\mathbf{Z} = \frac{k_2 R\left[\frac{1}{k_1} - 1\right](T^{f+1} - T_{amb}^{f+1})\frac{1}{f+1} + z_0 v_0}{\left[(a_0 e_{\frac{d}{2}}\ J_1\left(e_{\frac{d}{2}}\right)) + \left(\frac{1}{k_4} e_{\frac{d}{2}}\ J_1\left(e_{\frac{d}{2}}\right)\right) - \left(\frac{1}{k_4} * \frac{{e_{\frac{d}{2}}}^2}{2}\right)\right] * A(T) \ * \frac{1}{\pi * \frac{d^2}{4}} * \text{constant}}$$

The term $z_0 v_0$ is calculated based on existing impedance models or by conducting experimental studies at room temperatures in the absence of any temperature differential or gradient. At higher temperatures, the variation is only in the fluid properties and hence in viscous dissipation terms, other impedances like the non-linear impedance, cavity reactance do not change with temperature and hence are incorporated into the above formulation by the $z_0 v_0$, as these terms will remain same for experiments at room temperature or practical scenarios of high temperature.

Graphs are plotted for the real and imaginary part of the impedance against the angular frequency at three different temperatures that is 1000 K, 800 K and 600 K. T_{amb} is assumed to be around 300 K which is the case for aircraft when it is close to the ground. v_0 is assumed to be around 0.2 Mach and z_0 is found using the fluid properties at 300 K by using the model developed by Elandy and Boden [26]. Matlab programme for the plot is written down in Appendix 2.

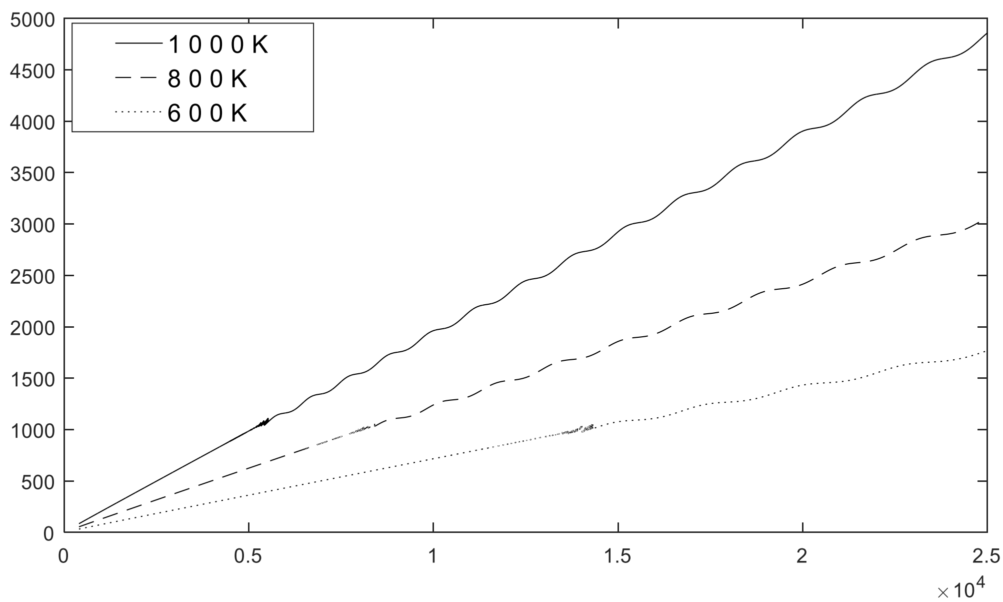


Figure 3: Imaginary part of impedance vs Angular frequency

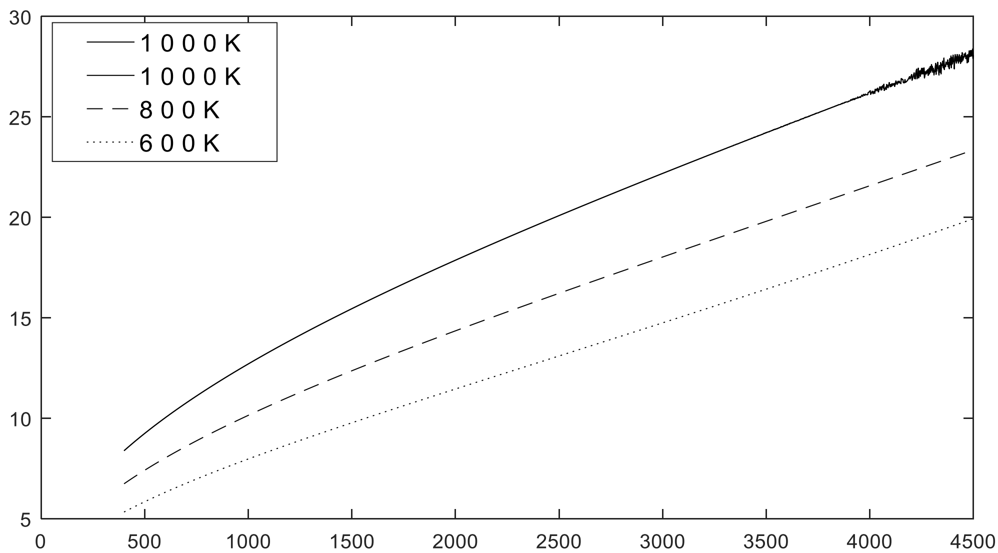


Figure 4: Real part of impedance vs Angular frequency

As can be seen from the two plots, the real and imaginary parts of impedance increases with increase in temperature. This follows that at higher temperatures, both the attenuation capability as well as the maximum attenuation frequency is raised.

A key difference between the references and the experiment here is of course the bias flow. Additionally, the bias flow is not heated, but constantly at around 288 K, while the given temperature corresponds to the mean grazing flow temperature. This approach with different bias and grazing flow temperatures reflects the conditions at a combustor liner in a gas turbine, where, however, both temperatures are at a much higher level. Presence of constant k_1 deals with that in discrepancy but it is always a difficult task to determine the value of this constant, which may vary with temperature as well. To solve this problem impedance can be calculated for different logical values of k_1, and then liner can be designed.

The increase in liner resistance and reactance with temperature is in accordance with the work of Elandy [27] and Boden[25].

Conclusions and Future work

Perforated liners are widely used to eliminate sound waves and, most of the times finds its application in high temperature environment. Although there is a great deal of literature available for the modelling liner impedance, very few considers the effect of temperature on the impedance. In this work an impedance model is developed which can predict impedance at higher temperatures based upon the experimental studies and models available at cold conditions. This is done by developing a differential equation to determine viscous impedance at any temperature T.

Although the formulation is not validated with any experimental or numerical results, the trend for acoustic reactance and resistance appears to be same as for the previously conducted experiments. Both the real and imaginary parts of the impedance increases with the temperature.

The planned future work is to validate the above formulation and to extend it to a complete liner using the analytical approach developed by Sullivan and Crocker [28] to predict the transmission loss of concentric-tube resonators. If derived, the impedance value across a single perforate would be enough to predict the total acoustic impedance of the liner at any temperature.

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Appendix 1

Solution of Differential Equation

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v(r,T)}{\partial r} - \frac{j\omega k_2}{k_3} T^{(f-1.5)}(T+C)v(r,T) = -\frac{Rk_2}{k_3 t}\left[\frac{1}{k_1} - 1\right] T^{(f-1.5)}(T+C)dT - \frac{\emptyset t}{k_3 t} T^{(-1.5)}(T+C)$$

Let m = $T^{(f-1.5)}(T+C)r^2$

$\frac{\partial v}{\partial r} = \frac{\partial v}{\partial m}\frac{\partial m}{\partial r} + \frac{\partial v}{\partial T}\frac{\partial T}{\partial r}$

$\frac{\partial T}{\partial r} = 0$, Temperature and radius r are assumed to be independent from each other.

$\frac{\partial v}{\partial r} = 2\ r\ T^{(f-1.5)}(T+C)\frac{\partial v}{\partial m}$

$\frac{\partial[\frac{\partial v}{\partial r}]}{\partial r} = \frac{\partial[\frac{\partial v}{\partial r}]}{\partial m}\frac{\partial m}{\partial r} + \frac{\partial[\frac{\partial v}{\partial r}]}{\partial T}\frac{\partial T}{\partial r}$ $\qquad \frac{\partial T}{\partial r} = 0$

$\frac{\partial[\frac{\partial v}{\partial r}]}{\partial r} = \frac{\partial^2 v}{\partial m^2} 4r^2 T^{2(f-1.5)}(T+C)^2 + \frac{\partial v}{\partial m} 2T^{(f-1.5)}(T+C)$

Substituting the value of $\frac{\partial v}{\partial r}$, $\frac{\partial[\frac{\partial v}{\partial r}]}{\partial r}$ and r^2 = m/$T^{(f-1.5)}(T+C)$, equation is reduced to,

$4m\frac{\partial^2 v}{\partial m^2} + 4\frac{\partial v}{\partial m} + k_4 v = \frac{1}{k_3 t}\left[Rk_2\left[\frac{1}{k_1} - 1\right]dT - \emptyset t T^{(-f)}\right]$

Above equation is solved by assuming v= A(T) B(m). Substituting v = A(T) B(m) in above equation we have,

$4m\frac{\partial^2 B}{\partial m^2} + 4\frac{\partial B}{\partial m} + k_4 B = \frac{1}{A\, k_3 t}\left[Rk_2\left[\frac{1}{k_1} - 1\right]dT - \emptyset t T^{(-f)}\right]$

Term on the LHS is the function of m only while the term on the RHS is a function of T only, hence $4m\frac{\partial^2 B}{\partial m^2} + 4\frac{\partial B}{\partial m} + k_4 B = -\lambda^2 = \frac{1}{A\, k_3 t}\left[Rk_2\left[\frac{1}{k_1} - 1\right]dT - \emptyset t T^{(-f)}\right]$ where λ is any arbitrary constant. A and B can be found from the two set of differential equation and hence, v can be determined.

$4m\frac{\partial^2 B}{\partial m^2} + 4\frac{\partial B}{\partial m} + k_4 B = -\lambda^2$ [1]

Complimentary function Equation [1] is $a_0\ J_0$ (e), where a_0 is any arbitrary constant, J_0 is bessel value of order 0.

$e^2 = k_4 m$

Particular integral for the equation [1] is determined by Power series method. Lengthy calculations for Power series methods is not listed here.

Particular integral obtained for equation [1] from Power Series method is $\frac{\lambda^2}{k_4}[J_0(e) - 1]$.

B(m)= $a_0[J_0(e)] + \frac{\lambda^2}{k_4}[J_0(e) - 1]$

Equation [2] obtained is

$$\frac{1}{A\, k_3 t}\left[Rk_2\left[\frac{1}{k_1} - 1\right] dT - \emptyset t T^{(-f)}\right] = -\lambda^2$$

Solving for A(T) we have, A(T) = $\frac{1}{\lambda^2 k_3 t}\left[Rk_2\left\{\frac{1}{k_1} - 1\right\}\Delta T + \emptyset t\ T^{(-f)}\right]$

v (r,T) = A(T) B(m)

so,

$$\mathbf{v\ (r,T) =}\ \left[\mathbf{a_0[J_0(e)]} + \frac{\boldsymbol{\lambda^2}}{\mathbf{k_4}}\mathbf{[J_0(e) - 1]}\right] * \frac{\mathbf{1}}{\boldsymbol{\lambda^2}\mathbf{k_3 t}}\left[\mathbf{Rk_2}\left\{\frac{\mathbf{1}}{\mathbf{k_1}} - \mathbf{1}\right\}\boldsymbol{\Delta}\mathbf{T} + \boldsymbol{\emptyset}\mathbf{t\ T^{(-f)}}\right]$$

Appendix 2

Matlab programme

```
%calculation of f
gamma=1.4;                %isoentropic index of air
f=(1/(gamma-1));          %value of f

%thickness of liner in metres
t=0.001;

%porosity ratio
sig=0.05;

%gas constant
R=287;

%calculation of k1
k1=input('temp relation k1');

%calculation of k2
rho0 =1.177;               %initial condition air density
T0 =300;                   %initial condition temperature
k2=(rho0)/(T0^f);          % k2 value

%calculation of k3
k3=1.512*(10^-6);          %k3 value

%calculation of k4
%k4 would be calculated inside the loop

%calculation for m
C=120;                     %sutherlands constant value is in kelvin
T=600;                     %desired elevated temperature
d=0.002;                   %aperture diameter in meters
m=(T^(f-1.5))*(T+C)*(d*d*0.25);
                           %m value

%input of rho0 ,mu0 and v0 required to calculate initial impedance
%rho0 already entered
mu0=1.846*(10^-5);         %initial viscosity
v0=74;                     %initial condition velocity


b=1;
for w=400:2:2000;

%calculation for initial impedance z0
k=((-1i*w*rho0)/(mu0))^0.5;
k00=k*d*0.5;
t01=(4*(besselj(0,k00)))/((k*d)*(besselj(0,k00)));
z0=(1i*w*rho0*t)/((sig)*(1-t01));

zinitial (b)=real(z0);
```

```
%calculation for k4
k4=(-1i*w*k2)/k3;

%calculation for e
e=(k4*m)^0.5;

%calculation for constant
co1=k4*(T+C)*(T^(f-1.5))*pi*d*d*0.25;
constant=2*pi/co1;

%calculation for P(T)
p1=R*k2*((1/k1)-1)*(T-T0);
p2=z0*v0*(T^(-f));
fun1=(p1+p2)/(k3*t);

%calculation for a0
a00=(1-(1/(besselj(0,e))));
a0=-1*a00/k4;

%calculation NUMERATOR
n1=k2*R*((1/k1)-1);
n2=((T^(f+1))-(T0^(f+1)))/(f+1);
n=(n1*n2)+(z0*v0);

%calculation DENOMINATOR
d1=e*(besselj(1,e))*(a0+(1/k4));
d2=((0.5)*e*e)/k4;
de=(sig)*(fun1)*(constant)*(d1-d2);

z=n/de;
zr(b)=real(z);
zi(b)=imag(z);



b=b+1;
end

po=400:2:2000;
plot(po,zr,po,zi);
```