Mid-Semester Test – Solutions

1. a) Determine the dimensions and the SI (base) units of each of the following physical quantities :

    i)   displacement of a moving object
    ii) density of a cube
    iii) torque

    b) There are two points, C and D with position vectors $\overrightarrow{OC} = 2\hat{i} + 5\hat{j}$ and $\overrightarrow{OD} = -2\hat{i} + \hat{j}$ respectively. O is the origin.

    i)   Calculate the magnitude of $\overrightarrow{OC}$ and the magnitude of $\overrightarrow{OD}$,
    ii) determine the vector $\overrightarrow{CD}$, and
    iii) determine the angle between $\overrightarrow{CD}$ and $\overrightarrow{CO}$.

                                                                                          (25 marks)

1. a) i)   [L] and SI unit is m
         ii) $[M]/[L]^3$ and SI unit is kg/m^3
         iii) $[M][L]/[T]^2$ [L] = $[M][L]^2/[T]^2$ and SI unit is N m

                                                                                          (3 marks each)

    b) i) $|\overrightarrow{OC}| = \sqrt{2^2 + 5^2} = \sqrt{29}$

             $|\overrightarrow{OD}| = \sqrt{(-2)^2 + 1^2} = \sqrt{5}$

        ii) $\overrightarrow{OC} + \overrightarrow{CD} = \overrightarrow{OD}$

             $\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$

             $= (-2\hat{i} + \hat{j}) - (2\hat{i} + 5\hat{j})$

             $= -4\hat{i} - 4\hat{j}$

        iii) $\overrightarrow{OC} = 2\hat{i} + 5\hat{j}$

             $\overrightarrow{CO} = -2\hat{i} - 5\hat{j}$

             $\overrightarrow{CD} \cdot \overrightarrow{CO} = |\overrightarrow{CD}| |\overrightarrow{CO}| \cos\theta$

             $\theta = \cos^{-1} \dfrac{(-4\hat{i} - 4\hat{j}) \cdot (-2\hat{i} - 5\hat{j})}{\sqrt{32}\sqrt{29}}$

             $= 23.2^\circ$

2.   A car and a bus move off from rest at the same time on a straight track. The bus is initially 50.0 m behind the car. The car accelerates at a uniform rate of 2.00 m/s^2, and the bus accelerates at a uniform rate of 4.00 m/s^2.

    a)   How much time after moving off does the bus take to overtake the car?

    b)   How far will each vehicle have travelled during that time?

    c)   15.0 seconds after the bus has overtaken the car, how far ahead is the bus from the car? (assume that both vehicles are still accelerating).

                                                                                (25 marks)

    a) $x = x_o + ut + \dfrac{1}{2}at^2$

          For car, $x_o = 0$ and for bus, $x_o = -50.0$

          $s_c = 0 + 0t + \dfrac{1}{2}2t^2$

          $s_b = -50 + 0t + \dfrac{1}{2}4t^2$

          $t^2 = -50 + 0t + 2t^2$

          $t^2 = 50$

          $t = 7.07$ s

    b)   For car, $s_c = \dfrac{1}{2}2t^2 = 50.0$ m

          For bus, $s_b = 50 + 50 = 100.0$ m

    c)   For car the velocity at $t = 7.07$ s is $u_c = 2.00 \times 7.07 = 14.1$ m/s

          For bus the velocity at $t = 7.07$ s is $u_b = 4.00 \times 7.07 = 28.3$ m/s

          $s_c = 14.1 \times 15.0 + \dfrac{1}{2}2(15.0)^2 = 436.5$ m

          $s_b = 28.3 \times 15.0 + \dfrac{1}{2}4(15.0)^2 = 874.5$ m

          Bus is $874.5 - 436.5 = 438$ m ahead of car.

3. An airplane travelling at 180.0 km/h wants to drop food packets to flood victims isolated on a patch of land 196.0 m below.

   a) How many seconds before the plane is directly overhead the patch of land should the packets be dropped?

   b) From the time when the packets are released till they reach the ground, what horizontal distance did the airplane travel?

   c) From the location of the airplane when the packets are released, to the location where the packets hit the ground, what is the magnitude of the displacement vector between these two points?

   (25 marks)

   a) $y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$

   $196 = 0 + 0 + \frac{1}{2}(9.80)t^2$

 $t = \sqrt{\dfrac{2(196)}{9.80}} = 6.32$ s

   b) $v_{x0} = 180$ km/h $= 50$ m/s

   $x = v_{x0}t = (50)(6.32) = 316$ m

   c) The total distance $= \sqrt{196^2 + 316^2} = 371.8$ m

4. a) Two objects A and B with masses 3.0 kg and 6.0 kg, respectively, are connected by a string of negligible mass that passes over a frictionless pulley at the edge of a vertical drop (see the figure below). A is on a frictionless incline and B hangs freely.

   i) Draw the free body diagrams for A and B.
   ii) Find the acceleration of each object.
   iii) Find the tension in the string.

   b) If A is on a rough incline with coefficient of static friction $\mu_S = 0.8$, what is the maximum mass B can have without causing the blocks to move?

   (25 marks)
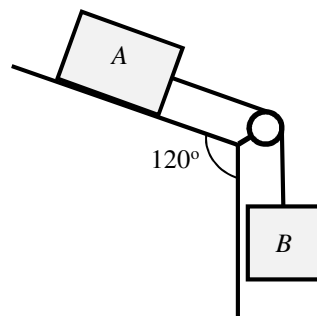
a)   i)
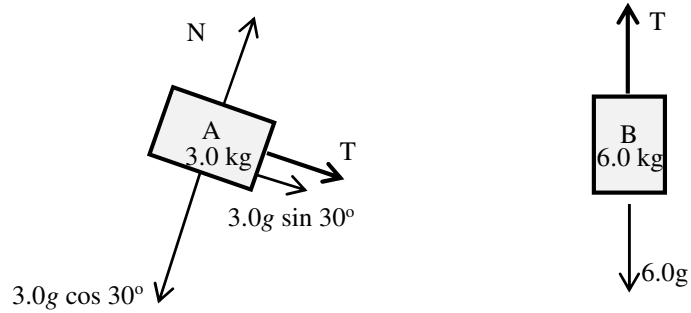
ii)   The magnitude of acceleration of each block is the same.

$$3.0a = T + 3.0g\sin(30°)$$
$$6.0a = 6.0g - T$$
$$\Rightarrow 9.0a = g(6.0 + 3.0\sin(30°))$$
$$\Rightarrow a = 8.17 \text{ m/s}^2$$

iii) $T = 6.0(g - a)$
        $\Rightarrow T = 9.78 \text{ N}$

b)   At the point of slipping:

A: $T + 3.0g\sin(30°) - \mu_S 3.0g\cos(30°) = 0$

B: $m_B g - T = 0$

$\Rightarrow m_B = 3.00(\mu_S \cos(30°) - \sin(30°))$

$\Rightarrow m_B = 0.578 \text{ kg}$

************* End *************