3. **IDENTIFY:** For SHM the motion is sinusoidal.

SET UP: $x(t) = A\cos(\omega t)$.

EXECUTE: $x(t) = A\cos(\omega t)$, where $A = 0.320\text{ m}$ and $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.900\text{ s}} = 6.981\text{ rad/s}$.

(a) $x = 0.320\text{ m}$ at $t_1 = 0$. Let t_2 be the instant when $x = 0.160\text{ m}$. Then we have $0.160\text{ m} = (0.320\text{ m})\cos(\omega t_2)$. $\cos(\omega t_2) = 0.500$. $\omega t_2 = 1.047\text{ rad}$. $t_2 = \dfrac{1.047\text{ rad}}{6.981\text{ rad/s}} = 0.150\text{ s}$. It takes $t_2 - t_1 = 0.150\text{ s}$.

(b) Let t_3 be when $x = 0$. Then we have $\cos(\omega t_3) = 0$ and $\omega t_3 = 1.571\text{ rad}$. $t_3 = \dfrac{1.571\text{ rad}}{6.981\text{ rad/s}} = 0.225\text{ s}$. It takes $t_3 - t_2 = 0.225\text{ s} - 0.150\text{ s} = 0.0750\text{ s}$.

EVALUATE: Note that it takes twice as long to go from $x = 0.320\text{ m}$ to $x = 0.160\text{ m}$ than to go from $x = 0.160\text{ m}$ to $x = 0$, even though the two distances are the same, because the speeds are different over the two distances.

4. **IDENTIFY:** The general expression for $v_x(t)$ is $v_x(t) = -\omega A\sin(\omega t + \phi)$. We can determine ω and A by comparing the equation in the problem to the general form.

SET UP: $\omega = 4.71\text{ rad/s}$. $\omega A = 3.60\text{ cm/s} = 0.0360\text{ m/s}$.

EXECUTE: **(a)** $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi\text{ rad}}{4.71\text{ rad/s}} = 1.33\text{ s}$

(b) $A = \dfrac{0.0360\text{ m/s}}{\omega} = \dfrac{0.0360\text{ m/s}}{4.71\text{ rad/s}} = 7.64\times10^{-3}\text{ m} = 7.64\text{ mm}$

(c) $a_{\max} = \omega^2 A = (4.71\text{ rad/s})^2(7.64\times10^{-3}\text{ m}) = 0.169\text{ m/s}^2$

(d) $\omega = \sqrt{\dfrac{k}{m}}$ so $k = m\omega^2 = (0.500\text{ kg})(4.71\text{ rad/s})^2 = 11.1\text{ N/m}$.

EVALUATE: The overall positive sign in the expression for $v_x(t)$ and the factor of $-\pi/2$ both are related to the phase factor ϕ in the general expression.

5. The spring constant is the same regardless of what mass is attached to the spring.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \;\rightarrow\; \frac{k}{4\pi^2} = mf^2 = \text{constant} \;\rightarrow\; m_1 f_1^2 = m_2 f_1^2 \;\rightarrow$$

$$(m\text{ kg})(0.83\text{ Hz})^2 = (m\text{ kg} + 0.68\text{ kg})(0.60\text{ Hz})^2 \;\rightarrow\; m = \frac{(0.68\text{ kg})(0.60\text{ Hz})^2}{(0.83\text{ Hz})^2 - (0.60\text{ Hz})^2} = 0.74\text{ kg}$$

6. **IDENTIFY:** Initially part of the energy is kinetic energy and part is potential energy in the stretched spring. When $x = \pm A$ all the energy is potential energy and when the glider has its maximum speed all the energy is kinetic energy. The total energy of the system remains constant during the motion.

SET UP: Initially $v_x = \pm 0.815\text{ m/s}$ and $x = \pm 0.0300\text{ m}$.

EXECUTE: (a) Initially the energy of the system is

$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}(0.175\text{ kg})(0.815\text{ m/s})^2 + \frac{1}{2}(155\text{ N/m})(0.0300\text{ m})^2 = 0.128\text{ J}$. $\frac{1}{2}kA^2 = E$ and

$A = \sqrt{\dfrac{2E}{k}} = \sqrt{\dfrac{2(0.128\text{ J})}{155\text{ N/m}}} = 0.0406\text{ m} = 4.06\text{ cm}$.

(b) $\frac{1}{2}mv_{\max}^2 = E$ and $v_{\max} = \sqrt{\dfrac{2E}{m}} = \sqrt{\dfrac{2(0.128\text{ J})}{0.175\text{ kg}}} = 1.21\text{ m/s}$.

(c) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{155\text{ N/m}}{0.175\text{ kg}}} = 29.8\text{ rad/s}$

EVALUATE: The amplitude and the maximum speed depend on the total energy of the system but the angular frequency is independent of the amount of energy in the system and just depends on the force constant of the spring and the mass of the object.

7. We assume that the collision of the bullet and block is so quick that there is no significant motion of the large mass or spring during the collision. Linear momentum is conserved in this collision. The speed that the combination has right after the collision is the maximum speed of the oscillating system. Then, the kinetic energy that the combination has right after the collision is stored in the spring when it is fully compressed, at the amplitude of its motion.

$$p_{\text{before}} = p_{\text{after}} \quad\to\quad mv_0 = (m+M)\,v_{\max} \quad\to\quad v_{\max} = \frac{m}{m+M}v_0$$

$$\tfrac{1}{2}(m+M)\,v_{\max}^2 = \tfrac{1}{2}kA^2 \quad\to\quad \tfrac{1}{2}(m+M)\left(\frac{m}{m+M}v_0\right)^2 = \tfrac{1}{2}kA^2 \quad\to$$

$$v_0 = \frac{A}{m}\sqrt{k(m+M)} = \frac{(9.460\times10^{-2}\,\text{m})}{(7.870\times10^{-3}\,\text{kg})}\sqrt{(142.7\,\text{N/m})(7.870\times10^{-3}\,\text{kg}+4.648\,\text{kg})}$$

$$= 309.8\,\text{m/s}$$

8. (a) If the surface is smooth

$$\text{W} = \frac{1}{2}\text{kx}^2$$

$$= \frac{1}{2}\times1.0\times10^3\times0.02^2$$

$$= 0.20\text{ J}$$

$$\frac{1}{2}\text{mv}^2 = \frac{1}{2}\text{kx}^2$$

$$\text{v}^2 = \frac{\text{kx}^2}{\text{m}} = \frac{1.0\times10^3\times0.02^2}{1.6}$$

$$= 0.25$$

$$\text{v} = 0.50\text{ m/s}$$

(b) If the surface is rough

$$\frac{1}{2}\text{mv}^2 = \frac{1}{2}\text{kx}^2 - \text{Fd}$$

$$= 0.20 - 4\times0.02 = 0.12$$

$$\text{v}^2 = \frac{2\times0.12}{1.6}$$

$$= 0.15$$

$$\text{v} = 0.39\text{ m/s}$$

9. **IDENTIFY:** $T = 2\pi\sqrt{L/g}$ is the time for one complete swing.

SET UP: The motion from the maximum displacement on either side of the vertical to the vertical position is one-fourth of a complete swing.

EXECUTE: (a) To the given precision, the small-angle approximation is valid. The highest speed is at the bottom of the arc, which occurs after a quarter period, $\frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{L}{g}} = 0.25$ s.

(b) The same as calculated in (a), 0.25 s. The period is independent of amplitude.

EVALUATE: For small amplitudes of swing, the period depends on L and g.

10. If we consider the pendulum as starting from its maximum displacement, then the equation of motion can be written as $\theta = \theta_0 \cos \omega t = \theta_0 \cos \frac{2\pi t}{T}$. Solve for the time for the position to decrease to half the amplitude.

$$\theta_{1/2} = \tfrac{1}{2}\theta_0 = \theta_0 \cos \frac{2\pi t_{1/2}}{T} \quad \rightarrow \quad \frac{2\pi t_{1/2}}{T} = \cos^{-1}\tfrac{1}{2} = \frac{\pi}{3} \quad \rightarrow \quad t_{1/2} = \tfrac{1}{6}T$$

It takes $\frac{1}{6}T$ for the position to change from $+10°$ to $+5°$. It takes $\frac{1}{4}T$ for the position to change from $+10°$ to 0. Thus it takes $\frac{1}{4}T - \frac{1}{6}T = \frac{1}{12}T$ for the position to change from $+5°$ to 0. Due to the symmetric nature of the cosine function, it will also take $\frac{1}{12}T$ for the position to change from 0 to $-5°$, and so from $+5°$ to $-5°$ takes $\frac{1}{6}T$. The second half of the cycle will be identical to the first, and so the total time spent between $+5°$ and $-5°$ is $\frac{1}{3}T$. So the pendulum spends one-third of its time between $+5°$ and $-5°$.

Answers

1 a) A = 4.00 m, f = 0.500 Hz, T = 2.00 s b) $v = -4.00\pi \sin(\pi t + \pi/4)$, $a = -4.00\pi^2 \cos(\pi t + \pi/4)$
c) x = –2.83 m, v = 8.89 m/s, a = 27.9 m/s^2 d) v_{max} =12.6 m/s, a_{max} = 39.5 m/s^2

2. a) T = 1.26 s b) v_{max} = 0.250 m/s
c) 1.25 m/s^2 d) x = 0.05 cos 5.00t, v = –0.250 sin 5.00t, a = –1.25 cos 5.00t

11. a) 5.31×10^3 N/m b) T = 0.695 s c) v_{max} = 0.452 m/s