Mid-Semester Test (30% CA)

Name : ______________________________ Adm No : _______________

Class : ________________________ Class S/N : ______________

Date : ________________________ Time allowed : 1.5 hour

Maximum mark : 100

Instructions

Answer all 6 questions. Take g = 9.80 m/s^2.

This question paper consists of 4 printed pages including 1 page of formulae.

You are reminded that cheating during test is a serious offence.

All working in support of your answer must be shown. Answers must be to appropriate significant figures.

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1. a) Calculate $98.1+\dfrac{40.00}{2.24+3.95}$ to the appropriate significant figures.

   b) The rest energy E of an object with rest mass m is given by Einstein's famous equation $E = m\times c^2$, where $c = 2.99792458\times10^8$ m/s is the speed of light in vacuum. Find E for an electron of mass $m = 9.11\times10^{-31}$ kg.

   c) Determine the dimension and state the SI unit for :

      i) Force

      ii) Power

(10 marks)

a) $98.1+\dfrac{40.00}{2.24+3.95}=98.1+6.46=104.6$

b) $E=9.11\times10^{-31}\times(2.99792458\times10^{8})^2$

$=8.19\times10^{-14}$ J

c) i) MLT^{-2} (kg m/s^2 or newton)

ii) $\dfrac{MLT^{-2}L}{T}=ML^2T^{-3}$ (kg m^2/s or watt)

2. a) Define instantaneous velocity $\vec{v}$ and acceleration $\vec{a}$ using calculus notations.

   b) Two particles A and B move along the x axis. The position of A (in m) is given by $x_A = 4.00t + 3.00$ and that of B is $x_B = 1.00t^2$.

      i) What are the positions of A and B at time $t = 0.0$ s and $t = 2.0$ s?

      ii) What are the accelerations of A and B?

      iii) At what time does B overtake A?

(20 marks)

a) Instantaneous velocity, $\vec{v} = \frac{d}{dt}\vec{r}$

Instantaneous acceleration, $\vec{a} = \frac{d}{dt}\vec{v}$

b) i) $x_A = 4.00t + 3.00$

$x_B = 1.00t^2$

$x_A(0.0) = 3.00$ m $\quad x_A(2.0) = 11.0$ m

$x_B(0.0) = 0.0$ m $\quad x_B(2.0) = 4.0$ m

ii) $\vec{a}_A = \frac{d}{dt}\vec{v}_A = 0.0 \text{ m/s}^2$

$\vec{a}_B = \frac{d}{dt}\vec{v}_B = 2.0 \text{ m/s}^2$

iii) At time of overtaking, their position must be the same

$x_A(t) = x_B(t)$

$4.00t + 3.00 = 1.00t^2$

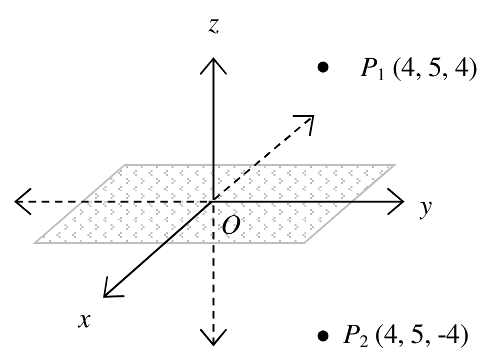
$1.00t^2 - 4.00t - 3.00 = 0$

$$t_{1,2} = \frac{4.00 \pm \sqrt{4.00^2 + 4(1.00)(3.00)}}{2}$$

$$= \frac{4.00 \pm \sqrt{28.0}}{2}$$

$t_1 = 4.65$ s or $t_2 = -0.65$ s (reject)

3. The diagram shows two points P_1 and P_2 in 3 dimensional Cartesian space with the origin at O.



a) Write the position vectors, $\vec{r}_1$ and $\vec{r}_2$ for P_1 and P_2 respectively in terms of $\hat{i}$, $\hat{j}$ and $\hat{k}$, which are unit vectors in the direction of positive x, y and z axes respectively.

b) If a particle takes 2.0 s to move from P_1 to P_2, what is its displacement vector $\vec{r}_2 - \vec{r}_1$ and its average velocity $\vec{v}_{ave}$.

c) Find the unit vector in the direction of OP_1.

(15 marks)

a) $\vec{r}_1 = 4.0\hat{i} + 5.0\hat{j} + 4.0\hat{k}$

$\vec{r}_2 = 4.0\hat{i} + 5.0\hat{j} - 4.0\hat{k}$

b) $\vec{r}_2 - \vec{r}_1 = 4.0\hat{i} + 5.0\hat{j} - 4.0\hat{k} - (4.0\hat{i} + 5.0\hat{j} + 4.0\hat{k})$

$= -8.0\hat{k}$

$$\vec{v}_{\text{ave}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{-8.0\hat{k}}{2.0} = -4.0\hat{k}$$

c) Unit vector $= \frac{1}{\sqrt{57}}(4\hat{i} + 5\hat{j} + 4\hat{k})$

4. The figure below shows the acceleration of a 2.00 kg particle as an applied force F_x moves it from rest along an x axis from x = 0.0 m to x = 9.0 m.

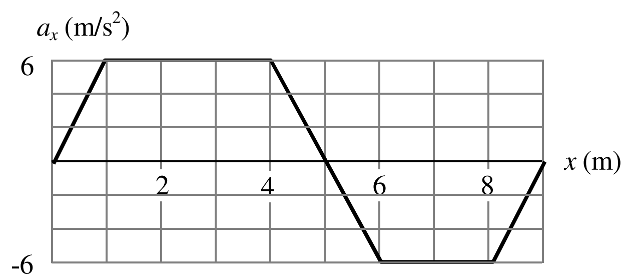
a) Write the acceleration a_x as a function of x for
   i) x = 0.0 m to x = 1.0 m
   ii) x = 1.0 m to x = 4.0 m

b) What is the particle's speed when it reaches x = 1.0 m?

c) How much work has the force done on the particle when it reaches
   i) x = 4.0 m
   ii) x = 9.0 m

Hint : For 4 (b), work done from x = 0.0 m to 1.0 m is $W = \int_0^1 F(x)dx$.

For c (ii) use symmetry of the graph to avoid some integrations.



(15 marks)

a) i) From $x = 0.0$ to $x = 1.0$, $a_x = 6x$ m/s^2

ii) From $x = 1.0$ to $x = 4.0$, $a_x = 6$ m/s^2

b) $W = \int_0^1 F(x)dx = 2.0\int_0^1 6x dx = 6.0$ J

$\frac{1}{2}mv^2 = W = 6.0$ J

$v = \sqrt{6.0} = 2.5$ m/s

c) i) $W = \int_0^1 F(x)dx + \int_1^4 F(x)dx$

$W = 2.0(\int_0^1 6x dx + \int_1^4 6 dx) = 42 \text{ J}$

ii) From $x = 6.0$ to $x = 8.0$, $a_x = -6 \text{ m/s}^2$

From $x = 8.0$ to $x = 9.0$, $a_x = 6x - 54 \text{ m/s}^2$

$W = 2.0(-\int_6^8 6dx + \int_8^9 (6x - 54)dx) = -30 \text{ J}$

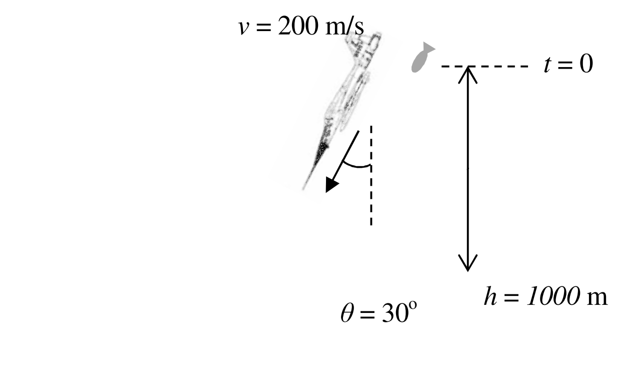
Total work done = 12 J.

5. The figure below shows a plane flying at an angle of $\theta = 30^\text{o}$ and speed = 200 m/s releasing a bomb at height = 1000 m.

a) Using kinematic equation for free fall under gravity, show that it takes about 5.05 s for the bomb to hit the ground.

b) What is the horizontal distance of the bomb from the plane's original position when the bomb reaches the ground?

c) If the plane makes a 300 m radius turn at the same speed after dropping the bomb, what is its centripetal acceleration?



(20 marks)

a) Vertical velocity $= 200\cos 30^\text{o} = 173.2$ m/s

$$s = ut + \frac{1}{2}gt^2$$

$$1000 = 173.2t + \frac{1}{2}9.80t^2$$

$$4.9t^2 + 173.2t - 1000 = 0$$

$$t = 5.05 \text{ s}$$

b) Horizontal velocity $= 200\sin 30^\text{o} = 100.0$ m/s

Horizontal distance = 505 m

c) $a = \frac{v^2}{r} = \frac{200^2}{300} = 133 \text{ m/s}^2$

6. The diagram below shows a block of mass m = 2.0 kg being kept stationary by a rope on a rough inclined plane. The coefficient of static friction between the block and plane is $\mu_s = 0.3$.

a) Draw the free body diagram of the stationary block.

b) Find the magnitude of

   i) the normal force on the block.

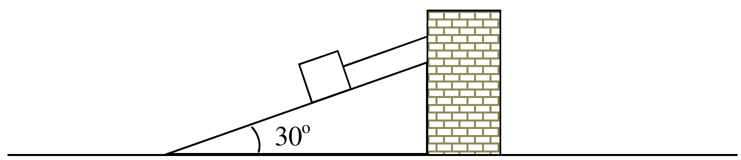
   ii) the force acting downward along the plane on the block.

   iii) the frictional force between the block and the plane.

   iv) the tension in the rope.

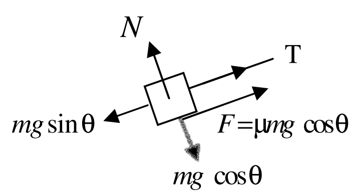
c) If the rope is cut, what is the initial acceleration of the block along the plane?

d) Name the conservative and non-conservative forces acting on the block as it slides down.



(20 marks)

a)



b) i) $N = mg\cos\theta = 2.0\times 9.80\times\cos 30^\circ$

$= 16.97$ N

ii) $F_r = \mu N = 0.3\times 16.97 = 5.09$ N

iii) $F = mg\sin\theta = 2.0\times 9.80\times\sin 30^\circ = 9.80$ N

iv) $T = mg\sin\theta - F_r = 9.80 - 5.09$

$= 4.71$ N

c) $F = ma$

$$a = \frac{F}{m} = \frac{4.709}{2.0} = 2.35 \text{ m/s}^2$$

d) Gravity – conservative

Friction – Non-conservative

***** End of Paper *****

Formula sheet for MS811M

Name: ________________________ Admin. No.: _______________ Seat No.: ________

Kinematics

$$v_x = v_{0x} + a_x t$$
$$v_x^{\ 2} = v_{0x}^{\ 2} + 2a_x(x - x_0)$$
$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$
$$\vec{v} = \frac{d\vec{r}}{dt}, \quad \vec{a} = \frac{d\vec{v}}{dt}$$
$$y = (\tan\theta)x - \left(\frac{g}{2v^2\cos^2\theta}\right)x^2$$
$$R = \frac{v^2\sin 2\theta}{g}$$

Dynamics

$$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a},\ F = \mu N$$
$$a = \frac{dv}{dt},\ a = \frac{v^2}{r}, F = m\frac{v^2}{r}$$
$$\vec{J} = \int \vec{F}dt = \Delta\vec{p}$$
$$W = \int \vec{F}.\vec{dr},\ W_{net} = K_f - K_i$$
$$KE = \frac{1}{2}mv^2,\ PE = mgh$$
$$P = \frac{W}{t}, \quad P = \frac{dW}{dt}$$

Linear momentum

$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

Static electricity

$$F = k\frac{q_1q_2}{r^2},\ k = \frac{1}{4\pi\varepsilon_o}$$
$$F = qE$$
$$V = k\frac{q}{r}, U = qV$$
$$\Phi_E = \oint \vec{E}.d\vec{A} = \frac{q}{\varepsilon_o}$$
$$V = Ed, W = qV,\ E = \frac{kq}{r^2}$$

Current electricity

$$Q = It \qquad V = IR$$
$$P = VI = I^2R = \frac{V^2}{R}$$

Magnetism & electromagnetism

$$\vec{F} = q\vec{v}\times\vec{B} \qquad \vec{F} = i\vec{L}\times\vec{B}$$
$$e.m.f. = -N\frac{d\Phi_B}{dt}$$
$$\Phi_B = BA$$

Thermodynamics

$$\Delta U = Q - W$$
$$W = \int p\,dV$$
$$Q_V = nC_V\Delta T \quad \text{const vol}$$
$$Q_p = nC_p\Delta T \quad \text{const pressure}$$
$$Q = mC\Delta T$$
$$Q = mL$$

Ideal Gas

$$pV = nRT$$
$$pV^\gamma = c \text{ (adiabatic)}$$
$$\gamma = \frac{C_p}{C_V},\ C_p - C_v = R$$
$$W = pV\ln\frac{V_2}{V_1} = nRT\ln\frac{V_2}{V_1}$$
$$W = \frac{1}{\gamma-1}(p_1V_1 - p_2V_2)$$

Rotational Motion

$$\omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt}$$
$$\omega = \omega_0 + \alpha t$$
$$\omega^2 = \omega_0^{\ 2} + 2\alpha(\theta - \theta_0)$$
$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$
$$I = \sum_i^n m_i r_i^2,\ I = \int r^2 dm,\ K = \frac{1}{2}I\omega^2$$

SHM & waves

$$T = \frac{1}{f} \quad v = f\lambda \quad \omega = 2\pi f$$
$$\omega = \frac{2\pi}{T} \quad k = \frac{2\pi}{\lambda}$$
$$\omega = \sqrt{k/m} \quad \omega = \sqrt{g/L}$$
$$x = A\cos(\omega t + \phi)$$
$$x = A\sin(\omega t + \phi)$$
$$y(x,t) = A\cos(\omega t \pm kx)$$
$$y(x,t) = A\sin(\omega t \pm kx)$$
$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

Circuits

$$R = R_1 + R_2 + R_3 + \ldots \quad \text{series}$$
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \quad \text{parallel}$$
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \quad \text{series}$$
$$C = C_1 + C_2 + C_3 + \ldots \text{ parallel}$$
$$Q = CV \qquad U = \frac{1}{2}CV^2$$

Constants

Charge on electron
$e = -1.60\times10^{-19}$ C

Coulomb's constant
$k = 9\times10^9$ N m^2 C^{-2}

Ideal gas constant
$R = 8.314$ J mol^{-1} K^{-1}

Mass of proton
$m_p = 1.67\times10^{-27}$ kg

Mass of electron
$m_e = 9.11\times10^{-31}$ kg

Permeability of free space
$\mu_o = 4\pi\times10^{-7}$ N A^{-2}

Permittivity of free space
$\varepsilon_o = 8.85\times10^{-12}$ C^2 N^{-1} m^{-2}

Speed of light in vacuum
$c = 3.00\times10^8$ m s^{-1}