Mid-Semester Test – Solutions

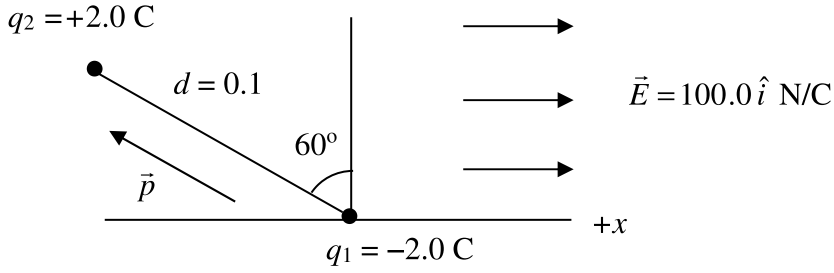
---

1. An electric dipole is made up of two charges q_1 and q_2 of equal magnitude q (measured in coulombs) but with opposite signs and separated by distance d.

When placed in an electric field $\vec{E}$, an electric dipole experiences a torque given by $\vec{\tau} = \vec{p} \times \vec{E}$.

where $\vec{p}$ is the electric dipole moment vector with magnitude $|\vec{p}| = |qd|$ and its direction is from the negative charge to the positive charge.

The diagram shows an electric dipole with $q_1 = -2.0$ C, $q_2 = +2.0$ C and $d = 0.1$ m in an electric field $\vec{E}$.



a) Write the vector $\vec{p}$ and $\vec{E}$ in component form.
b) Calculate the torque $\vec{\tau} = \vec{p} \times \vec{E}$ acting on the dipole in this position.
c) Show that the torque has a unit m N.
d) Show that $\vec{p} \times \vec{E}$ has the dimension of $[M][L]^2[T]^{-2}$.

(25 marks)

a) $\vec{p} = 2.0(-0.1\sin 60°\,\hat{i} + 0.1\cos 60°\,\hat{j})$

$= -0.17\hat{i} + 0.10\hat{j}$ C m

$\vec{E} = 100.0\hat{i}$ N/C

b) $$\vec{\tau} = \vec{p} \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -0.17 & 0.10 & 0 \\ 100.0 & 0 & 0 \end{vmatrix}$$

$$= \hat{k}\begin{vmatrix} -0.17 & 0.10 \\ 100.0 & 0 \end{vmatrix} = -10\hat{k} \text{ N m}$$

c) Unit of $\vec{p} \times \vec{E}$ is $\dfrac{\text{C m N}}{\text{C}} = \text{m N}$

d) Dimension of $\vec{p} \times \vec{E}$ is $[M][L][T]^{-2}[L]$

$= [M][L]^2[T]^{-2}$

2. The position of a particle of mass 0.5 kg moving along the x-axis is given by $x = \alpha t^4 - \beta t^2 - k$, where α, β and k are constants, t is measured in seconds while x is measured in metres.

   a) Find the SI units of α, β and k.
   b) Find the average velocity of the particle between $t = 0$ and $t = 3$ s.
   c) Find the instantaneous acceleration of the particle at $t = 3$ s.
   d) Find the net force acting on the particle at $t = 3$ s.

(25 marks)

a) The SI units of α, β and k are $\mathrm{m/s^4}$, $\mathrm{m/s^2}$ and m respectively.

b) $x = \alpha t^4 - \beta t^2 - k$

$x(0) = -k$

$x(3) = 81\alpha - 9\beta - k$

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x(3) - x(0)}{3 - 0} = \frac{(81\alpha - 9\beta - k) - (-k)}{3} = 27\alpha - 3\beta$$

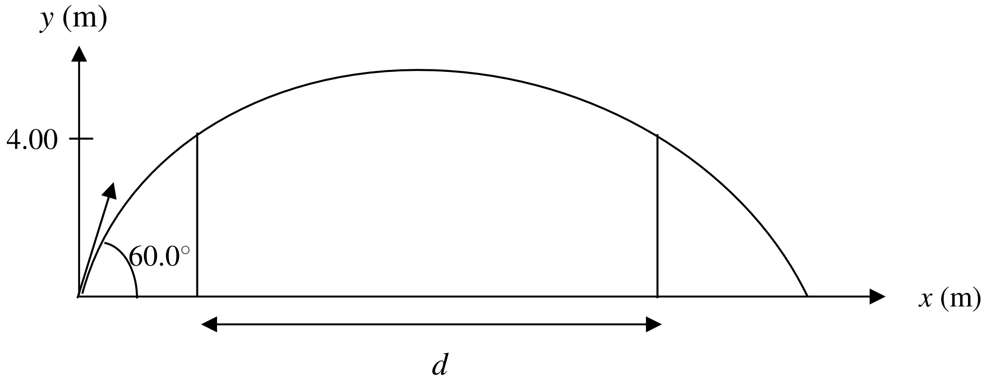
c) $v = \dfrac{dx}{dt} = 4\alpha t^3 - 2\beta t$

$a = \dfrac{dv}{dt} = 12\alpha t^2 - 2\beta$

$a(3) = 108\alpha - 2\beta$

d) $F = ma = (0.5)(108\alpha - 2\beta) = 54\alpha - \beta$

3. A boy threw a stone with an initial speed of 15.0 m/s at 60.0° with respect to the ground. The stone just missed the tops of two 4.00 m tall wall that are separated by distance d. The below figure (not drawn to scale) depicts the scenario



a) What is the maximum height reached by the stone?
b) How long did the stone take to reach the maximum height?
c) Determine the separation d of the two walls?

(25 marks)

a)
$$v_y^{\,2} = v_{0y}^{\,2} - 2g(y - y_0)$$
$$0 = (15\sin 60°)^2 - 2\times 9.8\times y$$
$$y = 8.61 \text{ m}$$

b)
$$v_y = v_{0y} - gt$$
$$0 = 15\sin 60° - 9.8\times t$$
$$t = 1.33 \text{ s}$$

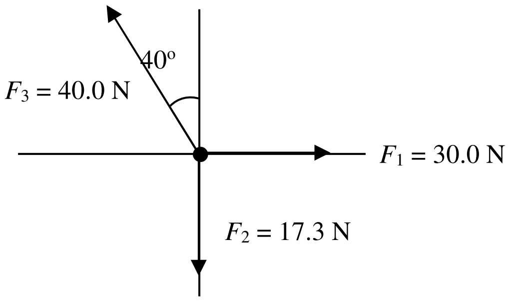
c)
$$y = (\tan\theta)x - \left(\frac{g}{2v^2\cos^2\theta}\right)x^2$$

$$4.0 = 1.73x - \frac{9.8x^2}{2\times 15.0^2\cos^2 60°}$$
$$4.0 = 1.73x - 0.087x^2$$
$$0.087x^2 - 1.73x + 4 = 0$$
$$x = \frac{1.73 \pm \sqrt{1.73^2 - 4\times 0.087\times 4}}{2\times 0.087}$$
$$= 2.67 \text{ or } 17.2 \text{ m}$$

Hence separation between walls is $17.2 - 2.67 = 14.5$ m

4. a) An elevator and its load have a combined mass of 1000 kg. The elevator initially moving downward at 8.0 m/s decelerates constantly to a stop after traveling a distance of 20.0 m. What is the tension T in the supporting cable while the elevator is being brought to rest?

b) A point is subjected to three forces as shown. Find the force (magnitude, direction as well as its components) needed to keep the point in equilibrium.



(25 marks)

a) $T - mg = ma$

$T = m(g + a)$

$v_y^{\,2} = v_{0y}^{\,2} + 2a_y(y - y_0)$

$0 = (-8)^2 + 2a_y(-20) \Rightarrow a_y = \dfrac{64}{40} = 1.6\,\text{m/s}^2$

$T = 1000(9.8 + 1.6) = 11400\,\text{N}$

b) $\vec{F}_1 = \begin{pmatrix} 30.0 \\ 0 \end{pmatrix},\ \vec{F}_2 = \begin{pmatrix} 0 \\ -17.3 \end{pmatrix},\ \vec{F}_3 = \begin{pmatrix} -40\sin 40° \\ 40\cos 40° \end{pmatrix}$

$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \begin{pmatrix} 30 - 40\sin 40° \\ -17.3 + 40\cos 40° \end{pmatrix} = \begin{pmatrix} 4.29 \\ 13.3 \end{pmatrix}$

To balance $\vec{F}_{net}$, a force of $-\vec{F}_{net}$ will be required

$\Rightarrow \vec{F} = -\vec{F}_{net} = \begin{pmatrix} -4.29 \\ -13.3 \end{pmatrix}$

The magnitude of $\vec{F} = \sqrt{(-4.29)^2 + (-13.3)^2} = 14\,\text{N}$

The direction of $\vec{F}$ is $\theta = \tan^{-1}\left(\dfrac{-13.3}{-4.29}\right)$

$= 252°$ with respect to $+$x-axis

************* End ************