Mid-Semester Test (30% CA)

Name : ______________________________ Adm No : _______________

Class : ________________________ Class S/N : ______________

Date : ________________________ Time allowed : 1.5 hour

Maximum Marks :100

Instructions

Answer all 7 questions. Take g = 9.80 m/s^2.

This question paper consists of 3 printed pages including 1 page of formulae.

You are reminded that cheating during test is a serious offence.

All working in support of your answer must be shown to appropriate significant figures.

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1. a) Calculate $56 + (32.00)/[1.2465 + 3.45]$ to the appropriate significant figures.

   b) The force F acting on an object of mass m moving in a circle with velocity v and at a distance r from the centre of the circle is given by $F = m^x v^y r^z$. Using dimensional analysis show that $x = 1$, $y = 2$ and $z = -1$ and hence the formula for F.

(15 marks)

Solution

1. a) $56 + (32.00)/[1.2465 + 3.45] = 56 + (32.00)/[1.2465 + 3.45]$

$= 56 + (32.00)/[4.70] = 56 + 6.81$

$= 63$

b) $F = m^x v^y r^z$

$$[MLT^{-2}] = [M^x]\left[\frac{L^y}{T^y}\right][L^z] = [M^x L^{y+z} T^{-y}]$$

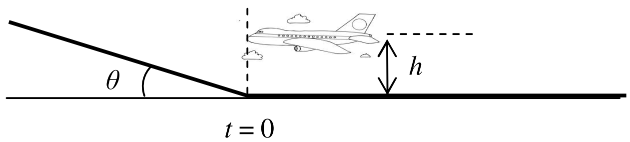
$x = 1$

$y + z = 1$

$y = 2$

Hence, $z = -1$. Therefore $F = mv^2 r^{-1} = mv^2 / r$

2. An airplane flies horizontally at 1500 km/h, at height h = 35 m above the ground. At time t = 0, the pilot approaches an inclined ground (see figure) that is sloping upward at angle $\theta = 4.3^{\circ}$. If the pilot does not change the airplane's direction, at what time will the plane crash?



(15 marks)

Solution

2. $\tan\theta = h/d$

$d = 35/\tan 4.3^{\circ} = 470$ m

1500 km/h = 417 m/s

Hence the plane will strike the ground in 470/417 = 1.1 s

3. Two particles A and B move along the x axis. The position of particle A (in m)is given by $x = 6.00t^2 + 3.00t + 2.00$. The acceleration of particle B (in m/s^2) is given by a = $-8.00t$. At t = 0, the velocity of particle B is 15 m/s. When will the velocity of the two particles be equal?

(15 marks)

Solution

3. $x_1 = 6.00t^2 + 3.00t + 2.00$

$$v_1 = \frac{d}{dt}x_1 = 12.0t + 3.00$$

$a_2 = -8.00t$

$$v_2 = \int a_2 dt = -4.00t^2 + c$$

When $t = 0$, $c = 15$

Hence $v_2 = -4.00t^2 + 15$

When the two velocities are equal,

$12.0t + 3.00 = -4.00t^2 + 15$

$4.00t^2 + 12t - 12 = 0$

$t^2 + 3t - 3 = 0$

$$t = \frac{-3 \pm \sqrt{9 + 4 \times 1 \times 3}}{2} = \frac{-3 \pm \sqrt{21}}{2}$$

$$t = \frac{-3 + 4.526}{2} = 0.80 \text{ s}$$

4. Three forces act on a particle that moves with unchanging velocity (in m/s) $\mathbf{v} = 2\mathbf{i} - 7\mathbf{j}$. Two of the forces (in N) are $\mathbf{F_1} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $\mathbf{F_2} = -5\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}$. What is the third force that acts on the particle?

(10 marks)

Solution

4. $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

$2\hat{i} + 3\hat{j} - 2\hat{k} - 5\hat{i} + 8\hat{j} - 2\hat{k} + F_x\hat{i} + F_y\hat{j} + F_z\hat{k} = 0$

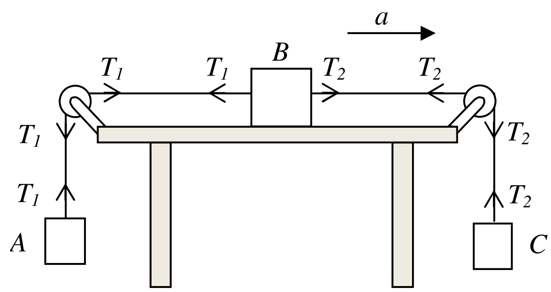
$2 - 5 + F_x = 0$

$3 + 8 + F_y = 0$

$-2 - 2 + F_z = 0$

Hence $\vec{F} = 3\hat{i} - 11\hat{j} + 4\hat{k}$

5. The diagram below shows three blocks A, B and C attached by chords that loop over frictionless pulleys. Block B lies on a frictionless table. The masses are $m_A = 6.00$ kg, $m_B = 8.00$ kg and $m_C = 10.00$ kg. The tension in the chord connecting A and B is T_1 while the tension in the chord connecting B and C is T_2. When the blocks are released, what is the tension T_2 in the chord on the right in terms of the acceleration due to gravity "g"?. [Hint : Assume the system of blocks to accelerate at "a" as shown].



(15 marks)

Solution

5. For m_C, $10g - T_2 = 10a$ (1)

For m_B, $T_2 - T_1 = 8.0a$ (2)

For m_A, $T_1 - 6.0g = 6.0a$ (3)

From eqn (2) and (3),

$T_2 - 6.0g = 14.0a$ (4)

From eqn (2) and (3),

$a = \frac{1}{6}g$

Sub into eqn (1),

$T_2 = \frac{25}{3}g$

6. A man playing basket ball stands 2.50 m in front of the ring. He jumps up so that his hands are level with the ring and launches the ball at 5.00 m/s at an angle of 75^o above the horizontal. Will he be able to put the ball into the ring? Explain with calculation.

(15 marks)

Solution

6. Let us calculate the time taken to reach the same level as horizontal. y = 0

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

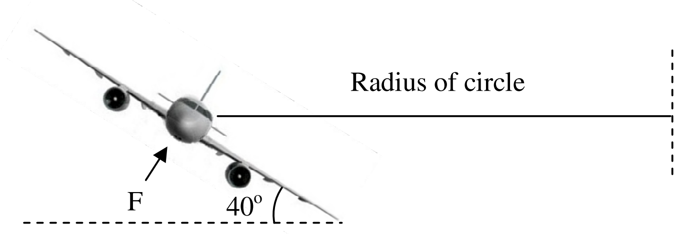
$$y = y_0 = 0,\ v_{0y} = 5\sin 75^o,\ a_y = -9.80\,m/s^2$$

solving t = 0.99 s

$$x = v_{0x}t = (5\cos 75^o)(0.99) = 1.3\,m$$

This implies the ball cannot be put into the ring and fall before reaching the ring which is at a horizontal distance of 2.50 m from the where the ball was thrown.

7. The figure shows an aeroplane flying in a horizontal circle at a speed of 600 km/h. If its wings are tilted at an angle θ = 40^o to the horizontal, what is the radius of the circle in which the plane is flying? Assume that the required force is provided entirely by an “aerodynamic lift” that is perpendicular to the wing surface.
[Hint : Resolve F in the vertical and horizontal direction].



(15 marks)

Solution

7.

$$F \times \sin 40^o = \frac{mv^2}{r} \qquad (1)$$

$$F \times \cos 40^o = mg \qquad (2)$$

$$\text{eqn (1)} \div \text{eqn (2)}$$

$$\tan 40^o = \frac{v^2}{gr}$$

$$r = \frac{v^2}{g \times \tan 40^o} = \frac{166.678^2}{10 \times \tan 40^o}$$

$$= 3{,}310.43 \text{ m}$$

*************** END of Paper ***************

Formula sheet for MS811M

Name: ________________________ Admin. No.: _______________ Seat No.: ________

Kinematics

$v_x = v_{0x} + a_x t$

$v_x^{\,2} = v_{0x}^{\,2} + 2a_x(x - x_0)$

$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

$\vec{v} = \frac{d\vec{r}}{dt}, \quad \vec{a} = \frac{d\vec{v}}{dt}$

$y = (tan\,\theta)x - (\frac{g}{2v^2 cos^2\theta})x^2$

$R = \frac{v^2 sin\,2\theta}{g}$

Dynamics

$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a},\ F = \mu N$

$a = \frac{dv}{dt},\ a = \frac{v^2}{r}, F = m\frac{v^2}{r}$

$\vec{J} = \int \vec{F}dt = \Delta\vec{p}$

$W = \int \vec{F}.\,d\vec{r},\ W_{net} = K_f - K_i$

$KE = \frac{1}{2}mv^2,\ PE = mgh$

$P = \frac{W}{t},\quad P = \frac{dW}{dt}$

Linear momentum

$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$

Static electricity

$F = k\frac{q_1 q_2}{r^2},\ k = \frac{1}{4\pi\varepsilon_o}$

$F = qE$

$V = k\frac{q}{r}, U = qV$

$\Phi_E = \oint \vec{E}.d\vec{A} = \frac{q}{\varepsilon_o}$

$V = Ed, W = qV, E = \frac{kq}{r^2}$

Current electricity

$Q = It \qquad V = IR$

$P = VI = I^2R = \frac{V^2}{R}$

Magnetism & electromagnetism

$\vec{F} = q\vec{v}\times\vec{B} \qquad \vec{F} = i\vec{L}\times\vec{B}$

$e.m.f. = -N\frac{d\Phi_B}{dt}$

$\Phi_B = BA$

Thermodynamics

$\Delta U = Q - W$

$W = \int pdV$

$Q_V = nC_V\Delta T$ const vol

$Q_p = nC_p\Delta T$ const pressure

$Q = mC\Delta T$

$Q = mL$

Ideal Gas

$pV = nRT$

$pV^\gamma = c$ (adiabatic)

$\gamma = \frac{C_p}{C_V},\ C_p - C_v = R$

$W = pV\,ln\frac{V_2}{V_1} = nRT\,ln\frac{V_2}{V_1}$

$W = \frac{1}{\gamma - 1}(p_1V_1 - p_2V_2)$

Rotational Motion

$\omega = \frac{d\theta}{dt},\quad \alpha = \frac{d\omega}{dt}$

$\omega = \omega_0 + \alpha t$

$\omega^2 = \omega_0^{\,2} + 2\alpha(\theta - \theta_0)$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\mathrm{I} = \sum_i^n m_i r_i^2,\ \mathrm{I} = \int r^2 dm,\ K = \frac{1}{2}I\omega^2$

SHM & waves

$T = \frac{1}{f} \quad v = f\lambda \quad \omega = 2\pi f$

$\omega = \frac{2\pi}{T} \quad k = \frac{2\pi}{\lambda}$

$\omega = \sqrt{k/m} \quad \omega = \sqrt{g/L}$

$x = A\cos(\omega t + \phi)$

$x = A\sin(\omega t + \phi)$

$y(x,t) = A\cos(\omega t \pm kx)$

$y(x,t) = A\sin(\omega t \pm kx)$

$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

Circuits

$R = R_1 + R_2 + R_3 + \ldots$ series

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$ parallel

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$ series

$C = C_1 + C_2 + C_3 + \ldots$ parallel

$Q = CV \qquad U = \frac{1}{2}CV^2$

Constants

Charge on electron
$e = -1.60\times10^{-19}$ C

Coulomb's constant
$k = 9\times10^9$ N m^2 C^{-2}

Ideal gas constant
$R = 8.314$ J mol^{-1} K^{-1}

Mass of proton
$m_p = 1.67\times10^{-27}$ kg

Mass of electron
$m_e = 9.11\times10^{-31}$ kg

Permeability of free space
$\mu_o = 4\pi\times10^{-7}$ N A^{-2}

Permittivity of free space
$\varepsilon_o = 8.85\times10^{-12}$ C^2 N^{-1} m^{-2}

Speed of light in vacuum
$c = 3.00\times10^8$ m s^{-1}