Mid-Semester Test (25% CA)

Name : ________________________ Adm No : ______________

Class : ________________________ Class S/N : ______________

Date : ________________________ Time allowed : 1.0 hour

Maximum mark : 100

Instructions

Answer all 4 questions. Take g = 9.80 m/s^2.

This question paper consists of 4 printed pages including 1 page of formulae.

You are reminded that cheating during test is a serious offence.

All working in support of your answer must be shown. Answers must be to appropriate significant figures.

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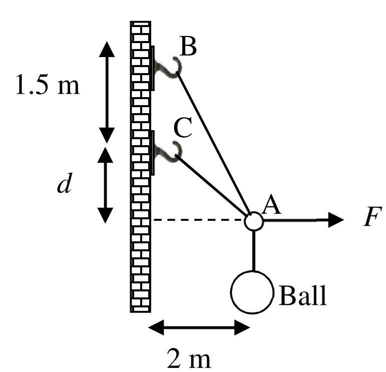
1. (a) Determine the dimensions of the following physical quantities and state their SI units.
   i) Density
   ii) Rate of change of velocity
   iii) Work

   (b) In the below figure, a ball of mass 20 kg is connected to a massless ring. A force, F = 100 N is applied horizontally to the ring at A. There are two cables that also connect the ring to the walls via the hooks at B and C.
   i) Draw the free body diagram for the ring. Label the tension in cable AB as T_1 and that in cable AC as T_2.
   ii) Determine d so that the tension in cable AC is zero.
   iii) Determine the tension in AB when the tension in cable AC is zero.



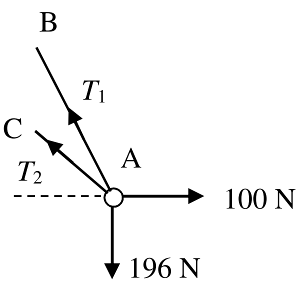
(25 marks)

1. (a) i) Density – $[M]\,[L]^{-3}$ kg/m^3

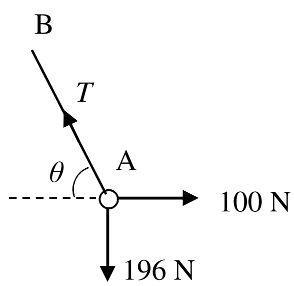
ii) Rate of change of velocity – $[L]\,[T]^{-2}$ m/s^2

iii) Work – $[M]\,[L]^2\,[T]^{-2}$ $kg\ m^2/s^2$

(b) i)



(b) If there is no force in cable AC, then the free body diagram reduces to



Resolving forces

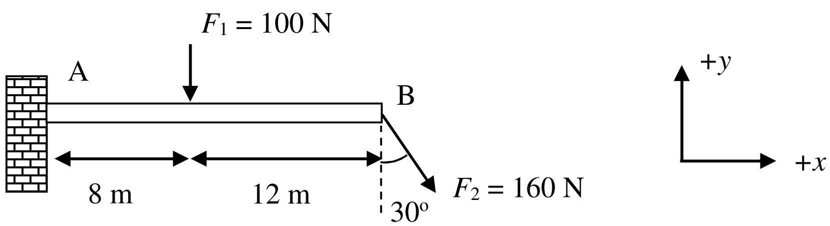
$T\cos\theta = 100$ … (1)
$T\sin\theta = 196$ … (2)

$\tan\theta = 1.96 = (1.5 + d)/2$
$d = 2.42$ m

$T = 220$ N

2. (a) Express each of the two forces $\vec{F}_1$ and $\vec{F}_2$ in the below diagram in terms of unit vectors .

(b) Determine the torque $\vec{\tau}$ about the point A of each of the two forces acting on the beam, where $\vec{\tau} = \vec{r} \times \vec{F}$ and $\vec{r}$ is the position vector. Take right as $+x$ (unit vector $\hat{i}$), up as $+y$ (unit vector $\hat{j}$) and out of the paper as $+z$ (unit vector $\hat{k}$).



(25 marks)

2

$$\vec{F}_1 = -100\hat{j} \text{ N}$$

$$\vec{F}_2 = 160\sin 30^{\circ}\hat{i} - 160\cos 30^{\circ}\hat{j} \text{ N}$$

$$\vec{r}_1 = 8\hat{i} \text{ m}$$

$$\vec{r}_2 = 20\hat{i} \text{ m}$$

$$\vec{\tau}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 0 & 0 \\ 0 & -100 & 0 \end{vmatrix} = -800\hat{k} \text{ N m}$$

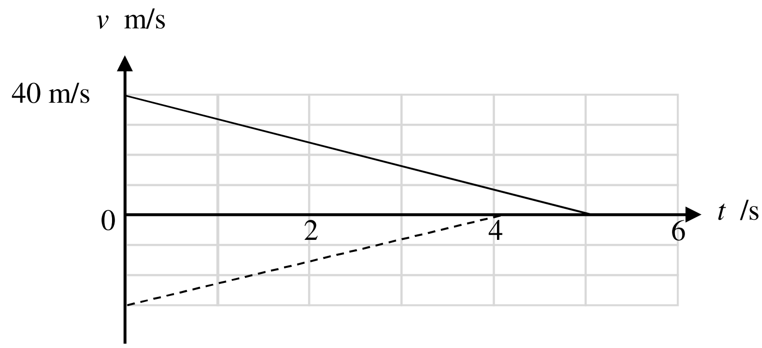
$$\vec{\tau}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 20 & 0 & 0 \\ 80 & -138.6 & 0 \end{vmatrix} = -2772\hat{k} \text{ N m}$$

3. (a) A particle moves so that its position as a function of time is $\vec{r} = \hat{i} + 4t^2\hat{j} + t\hat{k}$.

i) Write expressions for its instantaneous velocity and acceleration as functions of time.

ii) What is the average velocity and average acceleration between $t = 0$ s and $t = 2$ s?

(b) As two trains move along a straight track, their conductors suddenly notice that they are headed towards each other. The figure below gives their velocities vs time graphs as the conductors slow the trains. The slowing processes begins when the trains are 200 m apart. What is their separation when both trains have stopped?



(25 marks)

3 (a) (i) $\vec{v} = \dfrac{\mathrm{d}}{\mathrm{d}t}\vec{r} = 8t\hat{j} + \hat{k}$

$\vec{a} = \dfrac{\mathrm{d}}{\mathrm{d}t}\vec{v} = 8\hat{j}$

(ii) $\vec{v}_{ave} = \dfrac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \dfrac{\hat{i} + 4(2)^2\hat{j} + (2)\hat{k} - \hat{i}}{2}$

$= 8\hat{j} + \hat{k}$ m/s

$\vec{a}_{ave} = \dfrac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \dfrac{8(2)\hat{j} + \hat{k} - \hat{k}}{2}$

$= 8\hat{j}$ m/s^2

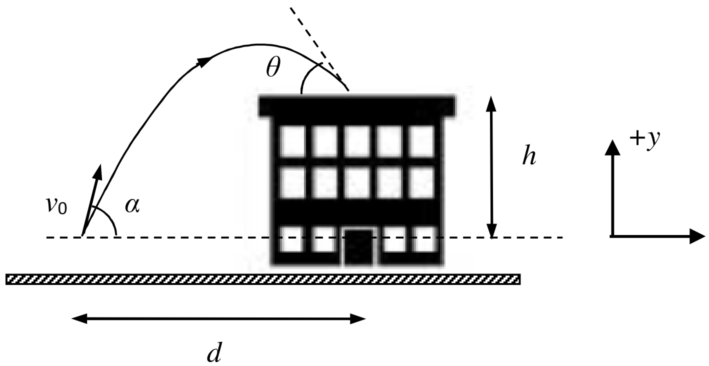
(b) From the velocity time-graphs, the distance travelled by the two trains are 100 m and 60 m. Since the trains were 200 m apart, they will be 40 m apart when both come to a stop.

4. In the below figure a ball is thrown up onto a roof, landing 4.50 s later at height h = 20.0 m above the release level. The ball's path just before landing is angled at θ = 60.0° with the roof.

i) Write the horizontal and vertical components of v_0 in terms of unit vectors $\hat{i}$ and $\hat{j}$

ii) Find the horizontal distance d it travels.

iii) What is the magnitude of the ball's initial velocity v_0 and angle α (relative to the horizontal)?



(25 marks)

i) Horizontal component of $v_o = v_o \cos\alpha\ \hat{i}$

Vertical component of $v_o = v_o \sin\alpha\ \hat{j}$

ii) $v_o \cos\alpha = \dfrac{d}{4.50}$ ...(1)

$s = (v_o \sin\alpha)t - \dfrac{1}{2}gt^2$

$20.0 = v_o \sin\alpha \times 4.50 - \dfrac{1}{2}9.8\times(4.50)^2$

$v_o \sin\alpha = 26.5$ ...(2)

On impact with the roof,

$v_{\text{vert}} = v_o \sin\alpha - gt$

$v_{\text{hor}} = v_o \cos\alpha$

$\tan(60^\circ) = \dfrac{-v_{\text{vert}}}{v_{\text{hor}}} = -\left(\dfrac{v_o \sin\alpha - gt}{v_o \cos\alpha}\right)$

$= -\left(\dfrac{26.5 - 9.8\times 4.50}{d/4.50}\right)$

$d = 45.7$ m

ii) From (1), $v_o \cos\alpha = \dfrac{d}{4.50} = 10.2$

From (2), $v_o \sin\alpha = 26.5$

Hence, $\alpha = 69.0^\circ$ and $v_o = 28.4$ m/s

Formula sheet

Name: ________________________ Admin. No.: _______________ Seat No.: ________

Kinematics

$v_x = v_{0x} + a_x t$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

$\vec{v} = \frac{d\vec{r}}{dt}, \quad \vec{a} = \frac{d\vec{v}}{dt}$

$y = (\tan\theta)x - (\frac{g}{2v^2\cos^2\theta})x^2$

$R = \frac{v^2 \sin 2\theta}{g}$

Dynamics

$\vec{F} = \frac{d(m\vec{v})}{dt}, F = \mu N$

$a = \frac{v^2}{r}, F = m\frac{v^2}{r}$

$W = \int \vec{F}\cdot d\vec{r}, W_{net} = K_f - K_i$

$KE = \frac{1}{2}mv^2, GPE = mgh$

$P = \frac{dW}{dt}$

Linear momentum

$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$

$\vec{J} = \int \vec{F}dt = \Delta\vec{p}$

Static electricity

$F = k\frac{q_1 q_2}{r^2}, \quad k = \frac{1}{4\pi\varepsilon_o}$

$F = qE$

$V = k\frac{q}{r}, U = qV$

$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_o}$

$V = Ed \ , W = qV$

Current electricity

$Q = It \ , \quad V = IR$

$P = VI = I^2 R = \frac{V^2}{R}$

Magnetism & electromagnetism

$\vec{F} = q\vec{v}\times\vec{B} \qquad \vec{F} = i\vec{L}\times\vec{B}$

$\varepsilon = -N\frac{d\Phi_B}{dt} \qquad \Phi_B = \vec{B}\cdot\vec{A}$

Thermodynamics

$\Delta U = Q - W$

$W = \int p dV$

$Q = mc\Delta T \quad ,Q = ml$

$Q_V = nC_V\Delta T$ const vol

$Q_p = nC_p\Delta T$ const pressure

Ideal Gas

$pV = nRT$

$pV^\gamma = c$ (adiabatic)

$\gamma = \frac{c_p}{c_V}, \quad c_p - c_v = R$

$W = pV\ln\frac{V_2}{V_1} = nRT\ln\frac{V_2}{V_1}$

$W = \frac{1}{\gamma - 1}(p_1V_1 - p_2V_2)$

Rotational Motion

$\omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt}$

$\omega = \omega_0 + \alpha t$

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$I = \sum_i^n m_i r_i^2 \ , \ K = \frac{1}{2}I\omega^2$

SHM & waves

$T = \frac{1}{f} \quad v = f\lambda \quad \omega = 2\pi f$

$\omega = \frac{2\pi}{T} \quad k = \frac{2\pi}{\lambda}$

$\omega = \sqrt{k/m} \quad \omega = \sqrt{g/L}$

$x = A\cos(\omega t + \phi)$

$y(x,t) = A\cos(\omega t \pm kx)$

$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

Circuits

$R = R_1 + R_2 + R_3 + ...$ series

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$ parallel

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$ series

$C = C_1 + C_2 + C_3 + ...$ parallel

$Q = CV \ , \ U = \frac{1}{2}CV^2$

Constants

Charge on electron

$e = -1.60\times10^{-19}$ C

Coulomb's constant

$k = 9\times10^9$ N m^2 C^{-2}

Ideal gas constant

$R = 8.314$ J mol^{-1} K^{-1}

Mass of proton

$m_p = 1.67\times10^{-27}$ kg

Mass of electron

$m_e = 9.11\times10^{-31}$ kg

Permeability of free space

$\mu_o = 4\pi\times10^{-7}$ N A^{-2}

Permittivity of free space

$\varepsilon_o = 8.85\times10^{-12}$ C^2 N^{-1} m^{-2}

Speed of light in vacuum

$c = 3.00\times10^8$ m s^{-1}