Mid-Semester Test (25% CA) + Solutions

Name : ______________________________ Adm No : ______________

Class : ______________________________ Class S/N : ______________

Date : ______________________________ Time allowed : 1.0 hour

Maximum mark : 100

Instructions

Answer all 4 questions. Take $g = 9.80$ m/s^2.

This paper consists of 6 printed pages including 1 page of formulae.

You are reminded that cheating during test is a serious offence.

All working in support of your answer must be shown. Answers must be to appropriate significant figures.

---

1. (a) The rate of heat transfer Q/t (measured in watts) through a slab of area A and thickness L is given by

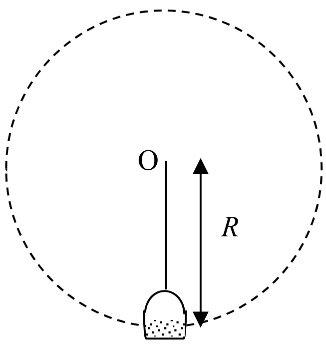
$$\frac{Q}{t} = \frac{kA}{L}(T_2 - T_1)$$

where T_1 and T_2 are the surface temperature on opposite sides of the slab and k is the thermal conductivity of the slab material. The dimension of temperature is $[\theta]$ and the SI unit of temperature is Kelvin (K).

i) Show that the SI unit of k is W/m K.

ii) What is the dimensions of k and its units expressed in terms of SI base units?

(b) A pail of water is swung around in a vertical circle as shown. At what minimum speed will water pour out of the pail when it is at the top of the swing? The mass of the pail and water is m, the radius is R and the acceleration due to gravity is g.



(25 marks)

1. (a) i) $k = \dfrac{QL}{tA(T_2 - T_1)}$

SI unit of k is $\dfrac{\text{J m}}{\text{s m}^2\ \text{K}} = \dfrac{\text{J}}{\text{s m K}}$ or W/m K

ii) Dimension of k is $\dfrac{[\text{M}][\text{L}][\text{T}]^{-2}[\text{L}]}{[\text{T}][\text{L}][\theta]} = [\text{M}][\text{T}]^{-3}[\text{L}][\theta]^{-1}$

Base SI unit of k is kg m/(s^{3}K)

(b) At the top, both the tension and weight will act downward,

$$T + W = \frac{mv^2}{r}$$

To get the minimum speed we set $T = 0$, $W = mg = \dfrac{mv^2}{r}$

$$v = \sqrt{gr}$$

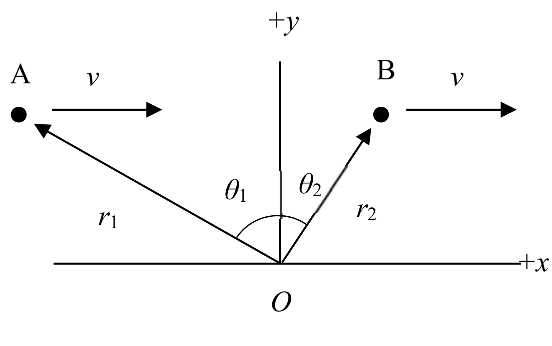
2. Angular momentum $\vec{L}$ as defined as $\vec{L} = \vec{r} \times \vec{p}$ where $\vec{r}$ is the position vector and $\vec{p} = m\vec{v}$ is the linear momentum of a particle of mass m. A particle of mass m = 1.0 kg moves with velocity v = 10.0 m/s from A to B as shown in the below figure. Given r_1 = 10.0 m, θ_1 = 60°, r_2 = 5.0 m and θ_2 = 30°. Take right as $+x$ (unit vector $\hat{i}$), up as $+y$ (unit vector $\hat{j}$) and out of the paper as $+z$ (unit vector $\hat{k}$).

i) Determine the angular momentum $\vec{L}_1$ of the particle at A.

ii) Determine the angular momentum $\vec{L}_2$ of the particle at B.

iii) Determine the change in angular momentum $\Delta\vec{L}$.

(25 marks)



2

i) $\vec{L}_1 = \vec{r}_1 \times \vec{p}_1 = (-10.0\cos 30°\hat{i} + 10.0\sin 30°\hat{j}) \times 10.0\hat{i}$

$= (-8.66\hat{i} + 5.00\hat{j}) \times 10.0\hat{i}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -8.66 & 5.00 & 0 \\ 10.0 & 0 & 0 \end{vmatrix}$$

$$= \hat{k}\begin{vmatrix} -8.66 & 5.00 \\ 10.0 & 0 \end{vmatrix} = -50\hat{k} \text{ kg m}^2\text{/s}$$

ii) $\vec{L}_2 = \vec{r}_2 \times \vec{p}_2 = (5.00\sin 30\hat{i} + 5.00\cos 30\hat{j}) \times 10.0\hat{i}$

$= (2.50\hat{i} + 4.33\hat{j}) \times 10.0\hat{i}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2.50 & 4.33 & 0 \\ 10.0 & 0 & 0 \end{vmatrix}$$

$$= \hat{k}\begin{vmatrix} 2.50 & 4.33 \\ 10.0 & 0 \end{vmatrix} = -43.3\hat{k} \text{ kg m}^2\text{/s}$$

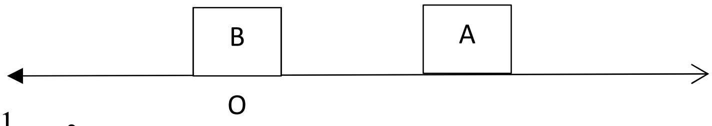
iii) $\Delta\vec{L} = \vec{L}_2 - \vec{L}_1 = (-43.3\hat{k} - (-50\hat{k}) = 6.70\hat{k} \text{ kg m}^2\text{/s}$

3. Two cars, A and B are moving in the same direction along the x-axis. When $t = 0$, their respective velocities are 1.0 m/s and 3.0 m/s while their respective uniform accelerations are 2.0 m/s^2 and 1.0 m/s^2. Car A is 1.5 m ahead of car B at $t = 0$.

i) Draw a diagram showing the positions of the two cars at $t = 0$ along the x-axis clearly labelling the origin.
ii) When will A and B be side by side?
iii) What are the speeds of A and B when they are side by side?
iv) How far would each car have travelled when they are side by side?

(25 marks)

3 i) Diagram



5

ii) $s_A = u_A t + \frac{1}{2} a_A t^2 + s_{A0}$

$s_B = u_B t + \frac{1}{2} a_B t^2$

They will be side by side when $s_A = s_B$

$1.0t + \frac{1}{2} 2t^2 + 1.5 = 3.0t + \frac{1}{2} t^2$

$t = 1.0 \text{ s}$ or $t = 3.0 \text{ s}$

iii) When $t = 1.0$ s,

$v_A = u_A + a_A t = 1.0 + 2.0 \times 1.0$

$= 3.0 \text{ m/s}$

$v_B = u_B + a_B t = 3.0 + 1.0 \times 1.0$

$= 4.0 \text{ m/s}$

When $t = 3.0$ s,

$v_A = u_A + a_A t = 1.0 + 2.0 \times 3.0$

$= 7.0 \text{ m/s}$

$v_B = u_B + a_B t = 3.0 + 1.0 \times 3.0$

$= 6.0 \text{ m/s}$

iv) When $t = 1.0$ s,

$s_A = 1.0 \times 1.0 + \frac{1}{2} 2 \times 1.0^2 = 2.0 \text{ m}$

$s_B = 3.0 \times 1.0 + \frac{1}{2} \times 1.0^2 = 3.5 \text{ m}$

When $t = 3.0$ s,

$s_A = 1.0 \times 3.0 + \frac{1}{2} 2 \times 3.0^2 = 12.0 \text{ m}$

$s_B = 3.0 \times 3.0 + \frac{1}{2} \times 3.0^2 = 13.5 \text{ m}$

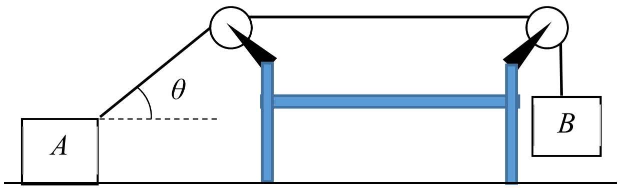
4. In the below figure, block A of mass m is dragged along the floor by a rope (inclined at an angle θ) that passes over a smooth pulley. The coefficient of kinetic friction between block A and the floor is μ_k. Block B has a mass M and is accelerating downwards. The tension in the rope is T and you can assume the rope and pulleys to be massless.

(25 marks)

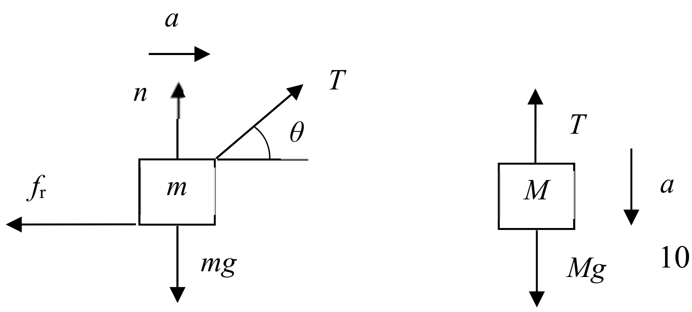
i) Draw the free body diagrams for block A and block B.

ii) Find the frictional force exerted by the floor on block A in terms of T, θ, m, μ_k and g.

iii) Find T.



4. i)



10

ii) Apply Newton's second law for m in y-direction

$n = (mg - T\sin\theta)$

$f_r = \mu_k n = \mu_k(mg - T\sin\theta)$

iii) Apply Newton's second law for m in x-direction

$T\cos\theta - f_r = ma$

$T\cos\theta - \mu_k(mg - T\sin\theta) = ma \quad ...(1)$

Apply Newton's second law for M in y-direction

$Mg - T = Ma$

$$a = \frac{Mg - T}{M} \qquad ...(2)$$

Substituting (2) in (1) we get

$$T\cos\theta - \mu_k(mg - T\sin\theta) = m\frac{Mg - T}{M}$$

$$T\left(\cos\theta + \mu_k\sin\theta + \frac{m}{M}\right) = mg(\mu_k + 1)$$

$$T = \frac{mg(\mu_k + 1)}{\left(\cos\theta + \mu_k\sin\theta + \frac{m}{M}\right)}$$

************************End of paper************************

Formula sheet

Name: ________________________ Admin. No.: _______________ Seat No.: ________

Kinematics

$v_x = v_{0x} + a_x t$

$v_x{}^2 = v_{0x}{}^2 + 2a_x(x - x_0)$

$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

$\vec{v} = \frac{d\vec{r}}{dt}, \quad \vec{a} = \frac{d\vec{v}}{dt}$

$y = (tan\,\theta)x - (\frac{g}{2v^2 cos^2\theta})x^2$

$R = \frac{v^2 sin\,2\theta}{g}$

Dynamics

$\vec{F} = \frac{d(m\vec{v})}{dt}, F = \mu N$

$a = \frac{v^2}{r}, F = m\frac{v^2}{r}$

$W = \int \vec{F}\cdot d\vec{r}, W_{net} = K_f - K_i$

$KE = \frac{1}{2}mv^2, GPE = mgh$

$P = \frac{dW}{dt}$

Linear momentum

$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$

$\vec{J} = \int \vec{F}dt = \Delta\vec{p}$

Static electricity

$F = k\frac{q_1 q_2}{r^2}, \quad k = \frac{1}{4\pi\varepsilon_o}$

$F = qE$

$V = k\frac{q}{r}, U = qV$

$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_o}$

$V = Ed \;, W = qV$

Current electricity

$Q = It\;, \quad V = IR$

$P = VI = I^2 R = \frac{V^2}{R}$

Magnetism & electromagnetism

$\vec{F} = q\vec{v}\times\vec{B} \qquad \vec{F} = i\vec{L}\times\vec{B}$

$\varepsilon = -N\frac{d\Phi_B}{dt} \qquad \Phi_B = \vec{B}\cdot\vec{A}$

Thermodynamics

$\Delta U = Q - W$

$W = \int p dV$

$Q = mc\Delta T \quad , Q = ml$

$Q_V = nC_V\Delta T$ const vol

$Q_p = nC_p\Delta T$ const pressure

Ideal Gas

$pV = nRT$

$pV^\gamma = c$ (adiabatic)

$\gamma = \frac{c_p}{c_V}, \quad c_p - c_v = R$

$W = pV\ln\frac{V_2}{V_1} = nRT\ln\frac{V_2}{V_1}$

$W = \frac{1}{\gamma - 1}(p_1V_1 - p_2V_2)$

Rotational Motion

$\omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt}$

$\omega = \omega_0 + \alpha t$

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$I = \sum_i^n m_i r_i^2 \;, \; K = \frac{1}{2}I\omega^2$

SHM & waves

$T = \frac{1}{f} \quad v = f\lambda \quad \omega = 2\pi f$

$\omega = \frac{2\pi}{T} \quad k = \frac{2\pi}{\lambda}$

$\omega = \sqrt{k/m} \quad \omega = \sqrt{g/L}$

$x = A\cos(\omega t + \phi)$

$y(x,t) = A\cos(\omega t \pm kx)$

$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

Circuits

$R = R_1 + R_2 + R_3 + \ldots$ series

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$ parallel

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$ series

$C = C_1 + C_2 + C_3 + \ldots$ parallel

$Q = CV \;, \; U = \frac{1}{2}CV^2$

Constants

Charge on electron

$e = -1.60\times10^{-19}$ C

Coulomb's constant

$k = 9\times10^9$ N m^2 C^{-2}

Ideal gas constant

$R = 8.314$ J mol^{-1} K^{-1}

Mass of proton

$m_p = 1.67\times10^{-27}$ kg

Mass of electron

$m_e = 9.11\times10^{-31}$ kg

Permeability of free space

$\mu_o = 4\pi\times10^{-7}$ N A^{-2}

Permittivity of free space

$\varepsilon_o = 8.85\times10^{-12}$ C^2 N^{-1} m^{-2}

Speed of light in vacuum

$c = 3.00\times10^8$ m s^{-1}