Mid-Semester test

Name : ________________________ Adm No : _______________

Class : ________________________ Class S/N : ______________

Date : ________________________ Time allowed : 1 hour

Instructions

Answer all 4 questions. Each question carries **25 marks**. Take g = 9.80 m/s^2

This question paper consists of 3 printed pages including 1 page of formulas.

You are reminded that cheating during test is a serious offence.

All working in support of your answer must be shown. Answers must be to appropriate significant figures.

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1. a) i) What is a derived physical quantity?
   ii) List any three derived physical quantities and state their derived units. You may use the special names of derived units.

   b) State the dimension and SI unit of each of the following:
   i) difference in lengths, i.e. $L_2 - L_1$.
   ii) weight, W.
   iii) efficiency, e.

   c) The maximum elastic potential energy stored in the spring of an oscillating spring-mass system is given by $U_{max} = \frac{1}{2}kA^2$, where k is the spring constant in N/m and A is the amplitude in m. Determine the dimension of U_{max}.

   d) The equation for the period of a simple pendulum is $T = 2\pi\sqrt{\dfrac{l}{g}}$.

   Explain why the equation remains homogeneous even if 2π is missing.

a) i) Derived quantities are products or division of base quantities.
ii) Any 3, including those not listed below
density = mass/volume (kg/m^3).
speed = distance/time (m/s).
force = mass × acceleration (newton).
power = work done/time (joules).
energy = work done = force × distance

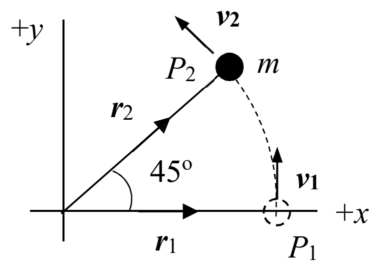
b) i) Dimension of $L_2 - L_1$ is [L] and SI unit is m.
ii) Dimension of W is [M][L][T]$^{-2}$ and SI unit is N or kg m/s^2.

iii) There is no dimension for efficiency and no SI unit.

c) Dimension of $U_{max} = \frac{1}{2}kA^2$ is $([M][L][T]^{-2}/[L]) \times [L]^2 = [M][L]^2[T]^{-2}$

d) Homogeneous means the two sides of the equation have the same dimension and 2π is dimensionless.

2. A 1.0 kg mass m moves at constant speed of 2.0 m/s from P_1 to P_2 along the circumference of a circle of radius 1.0 m on the x-y plane as shown.



a) Calculate the time taken from P_1 to P_2. [Hint: arc length = radius × angle in radians]

b) Express the following vectors in terms of **i** and **j**.

i) the vectors $\boldsymbol{r}_1$, $\boldsymbol{r}_2$ and $\Delta\boldsymbol{r}$.
ii) the velocity vectors $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$.
iii) the average velocity $\boldsymbol{v}_{av}$.
iv) the change in velocity $\Delta\boldsymbol{v}$.

c) Did the mass have an acceleration during its journey from P_1 to P_2? Give reason for your answer.

a) t = distance/speed = $(\frac{1}{4}\pi \times 1.0)/2.0 = \pi/8.0$ s = 0.383 s

b) i) $\boldsymbol{r}_1 = 1.0\mathbf{i}$ m
$\boldsymbol{r}_2 = 0.707\mathbf{i} + 0.707\mathbf{j}$ m
$\Delta\boldsymbol{r} = -\,0.293\mathbf{i} + 0.707\mathbf{j}$ m
ii) $\boldsymbol{v}_1 = 2.0\mathbf{j}$ m/s
$\boldsymbol{v}_2 = -\,1.414\mathbf{i} + 1.414\mathbf{j}$ m/s
iii) $\boldsymbol{v}_{av} = \Delta\boldsymbol{r} / t = (-\,0.293\mathbf{i} + 0.707\mathbf{j}) / 0.383$
$= -\,0.765\mathbf{i} + 1.845\mathbf{j}$ m/s
iv) $\Delta\boldsymbol{v} = \boldsymbol{v}_2 - \boldsymbol{v}_1 = (-\,1.414\mathbf{i} + 1.414\mathbf{j}) - 2.0\mathbf{j}$
$= -\,1.414\mathbf{i} - 0.586\mathbf{j}$ m/s

c) Since there is a change in velocity, there is an acceleration.

3. a) The diagram shows a ball dropping onto point A of a smooth spherical dish and projected out at point B with speed 10.0 m/s at an angle of 30° with respect to the horizontal.

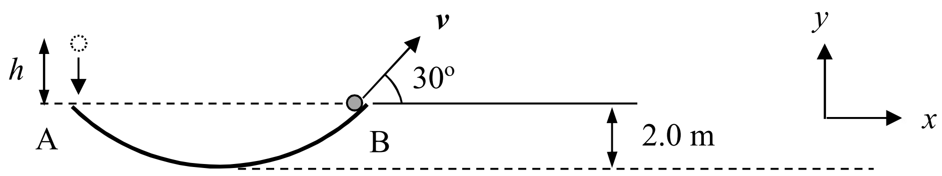
i) If the dish is not smooth, will it have any effect on the speed and exit angle of the ball at point B? Explain your answer. Assume the ball is projected out at point B.

ii) Write the vertical and horizontal components of $\boldsymbol{v}$ in terms of unit vectors **i** and **j**.

iii) How long will it take the ball to travel a horizontal distance of 2.0 m after exiting the dish at point B?

iv) What is the horizontal range of the ball at the level of point B?

b) How long (after exiting point B) will it take the ball to strike the surface 2.0 m below point B?



a) i) If the dish is rough, it will affect the magnitude of $\boldsymbol{v}$ but not its direction.

ii) $\boldsymbol{v}_x = 10\cos30^\circ\,\mathbf{i}$

$\boldsymbol{v}_y = 10\sin30^\circ\,\mathbf{j}$

iii) $v_x = 10\cos30^\circ = 2.0/t$

$t = 1/(5.0\cos30^\circ) = 0.23$ s

iv) $y = (\tan\alpha_0)\,x - \dfrac{1}{2v_0^2\cos^2\alpha_0}gx^2 = 0$

$$\sin\alpha_0 - \frac{1}{2v_0^2\cos\alpha_0}gx = 0$$

$$x = \frac{2v_0^2\cos\alpha_0\sin\alpha_0}{g} = \frac{v_0^2\sin2\alpha_0}{g} = 8.84\text{ m}$$

b) For vertical motion after exiting B,

$$y = v_0\sin(\alpha_0)t - \frac{1}{2}gt^2$$

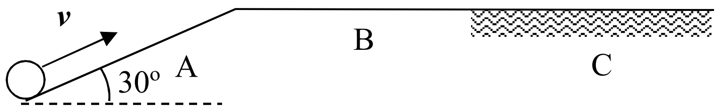
$$10\sin(30^\circ)t - \frac{1}{2}9.8t^2 = -2.0$$

$$4.9t^2 - 5t - 2.0 = 0$$

$$t = 1.328\text{ s}$$

4. a) State Newton's First Law of motion.

b) The diagram below shows three segments of a surface. All segments are 2.0 m long. Segment A is smooth and inclined at 30o. Segments B and C are both horizontal but B is smooth while C has coefficient of kinetic friction 0.25.



An object of mass 1.0 kg at the bottom of A is given an initial velocity directed along the inclined surface such that it can just reach the end of C.

i) Describe the motion of the object in the three segments.
ii) What is the initial velocity of the object?

c) What is the initial velocity of the object if it were to just stop at the top of A?

a) Every object remains stationary or moves with constant velocity unless a net force acts on it.

b) i) Along A, the object moves with constant deceleration of $g \sin \theta$. Along B, it moves with constant velocity and along C, it decelerates again uniformly.

ii) Let the minimum speed at the bottom of A be v_o.
Along A, $a = -g\sin(30^o) = -4.9$ m/s^2
If s is the displacement, then the speed at the top of A (and throughout B) is
$v^2 = v_o{}^2 + 2as = v_o{}^2 + 2\times -g\sin(30^o) \times 2$
$v = (\ v_o{}^2 + 2\times -g\sin 30^o \times 2)^{1/2} = (v_o{}^2 - 19.6)^{1/2}$
At C, there is friction with force of friction, $F_r = \mu_k N = \mu_k mg$
There is deceleration at C. $F_r = ma = \mu_k mg$ or $a = \mu_k g$
If s is the displacement, then at the end of C, the speed is zero.
$0^2 = v^2 + 2as = (v_o{}^2 - 19.6) + 2 \times (-\mu_k g) \times 2$
$v_o = 5.42$ m/s

c) Along A, $a = -g\sin(30^o) = -4.9$ m/s^2
$v^2 = v_o{}^2 + 2as$
$0 = v_o{}^2 - 2 \times 4.9 \times 2$
$v_o = 4.43$ m/s

**************End**************