

1. The force is the derivative of the momentum with respect to time.

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(4.8t^2\hat{i} - 8.0t\hat{j} - 8.9t\hat{k})}{dt} = (9.6t\hat{i} - 8.9\hat{k}) \text{ N}$$

2. (a) The impulse is the change in momentum. The direction of travel of the struck ball is the positive direction.

$$\Delta p = m\Delta v = (4.5 \times 10^{-2} \text{ kg})(45 \text{ m/s} - 0) = 2.0 \text{ kg}\cdot\text{m/s}, \text{ in the forward direction.}$$

- (b) The average force is the impulse divided by the interaction time.

$$\bar{F} = \frac{\Delta p}{\Delta t} = \frac{2.0 \text{ kg}\cdot\text{m/s}}{3.5 \times 10^{-3} \text{ s}} = 580 \text{ N}, \text{ in the forward direction.}$$

5. **IDENTIFY:** The  $x$  and  $y$  components of the momentum of the system of the two asteroids are separately conserved.

**SET UP:** The before and after diagrams are given in Figure and the choice of coordinates is indicated. Each asteroid has mass  $m$ .

**EXECUTE:** (a)  $P_{1x} = P_{2x}$  gives  $mv_{A1} = mv_{A2} \cos 30.0^\circ + mv_{B2} \cos 45.0^\circ$ .  $40.0 \text{ m/s} = 0.866v_{A2} + 0.707v_{B2}$  and  $0.707v_{B2} = 40.0 \text{ m/s} - 0.866v_{A2}$ .

$$P_{2y} = P_{1y} \text{ gives } 0 = mv_{A2} \sin 30.0^\circ - mv_{B2} \sin 45.0^\circ \text{ and } 0.500v_{A2} = 0.707v_{B2}.$$

Combining these two equations gives  $0.500v_{A2} = 40.0 \text{ m/s} - 0.866v_{A2}$  and  $v_{A2} = 29.3 \text{ m/s}$ . Then

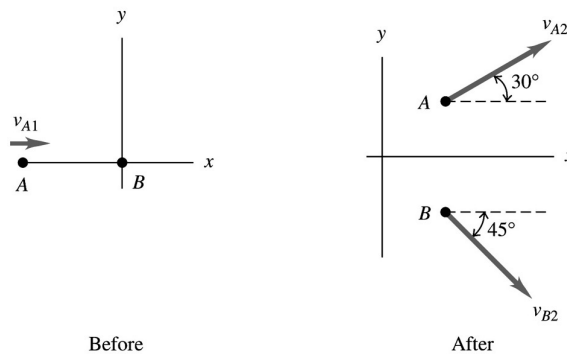
$$v_{B2} = \left( \frac{0.500}{0.707} \right) (29.3 \text{ m/s}) = 20.7 \text{ m/s}.$$

$$(b) K_1 = \frac{1}{2}mv_{A1}^2, K_2 = \frac{1}{2}mv_{A2}^2 + \frac{1}{2}mv_{B2}^2, \frac{K_2}{K_1} = \frac{v_{A2}^2 + v_{B2}^2}{v_{A1}^2} = \frac{(29.3 \text{ m/s})^2 + (20.7 \text{ m/s})^2}{(40.0 \text{ m/s})^2} = 0.804.$$

$$\frac{\Delta K}{K_1} = \frac{K_2 - K_1}{K_1} = \frac{K_2}{K_1} - 1 = -0.196.$$

19.6% of the original kinetic energy is dissipated during the collision.

**EVALUATE:** We could use any directions we wish for the  $x$  and  $y$  coordinate directions, but the particular choice we have made is especially convenient.



6. **IDENTIFY:** Apply conservation of energy to the motion before and after the collision and apply conservation of momentum to the collision.

**SET UP:** Let  $v$  be the speed of the mass released at the rim just before it strikes the second mass. Let each object have mass  $m$ .

**EXECUTE:** Conservation of energy says  $\frac{1}{2}mv^2 = mgR$ ;  $v = \sqrt{2gR}$ .

**SET UP:** This is speed  $v_1$  for the collision. Let  $v_2$  be the speed of the combined object just after the collision.

**EXECUTE:** Conservation of momentum applied to the collision gives  $mv_1 = 2mv_2$  so  $v_2 = v_1/2 = \sqrt{gR/2}$ .

**SET UP:** Apply conservation of energy to the motion of the combined object after the collision. Let  $y_3$  be the final height above the bottom of the bowl.

**EXECUTE:**  $\frac{1}{2}(2m)v_2^2 = (2m)gy_3$ .

$$y_3 = \frac{v_2^2}{2g} = \frac{1}{2g} \left( \frac{gR}{2} \right) = R/4.$$

**EVALUATE:** Mechanical energy is lost in the collision, so the final gravitational potential energy is less than the initial gravitational potential energy.

### Answers

3. a)  $7.5 \text{ m s}^{-1}$  west b) *car*:  $45 \text{ m s}^{-1}$  west, *truck*:  $5 \text{ m s}^{-1}$  east  
 4.  $1.33 \text{ m/s}$ ,  $1.67 \text{ m/s}$