Mid-Semester test

Name : _____ Adm No : _____

Class : _____ Class S/N : _____

Date : _____ Time allowed : 1 hour

Instructions

Answer all 4 questions. Take g = 9.80 m/s^2

This question paper consists of 3 printed pages including 1 page of formulas.

You are reminded that cheating during test is a serious offence.

All working in support of your answer must be shown. Answers must be to appropriate significant figures.

---

1. a) Write the dimensions of the following expressions.

   i) $p(V_2 - V_1)$, where p is the gas pressure (defined as force per unit area), V_2 and V_1 are the final and initial volume of the gas respectively.
   ii) ½ kx^2, where k is the spring constant in N/m and x is the extension in m.

   b) What do you notice about the dimensions obtained in i) and ii)?

   c) Write down the SI units of the expressions in i) and ii).

   d) Explain why an equation may be homogeneous but physically incorrect?

(25 marks)

a) i) Dimension of p is $\left[\frac{MLT^{-2}}{L^2}\right] = [ML^{-1}T^{-2}]$

Dimension of $V_2 - V_1$ is $[L^3]$

Dimension of $p(V_2 - V_1)$ is $[ML^2T^{-2}]$

ii) Dimension of k is $\left[\frac{MLT^{-2}}{L}\right]$

Dimension of x^2 is $[L^2]$

Dimension of $\frac{1}{2}kx^2$ is $[ML^2T^{-2}]$

b) Both have the same dimension.

c) SI units are both joule (J) or N m.

d) There could be dimensionless constants in the expression.

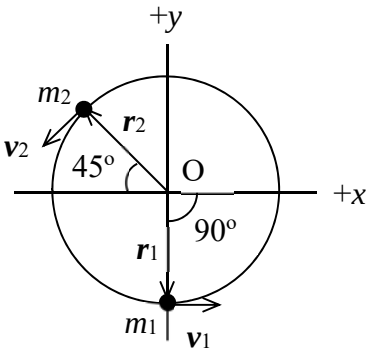
2. a) The diagram below shows two masses m_1 and m_2 performing uniform circular motion on the x-y plane about a common centre O. If m_1 and m_2 are 1.0 kg, r_1 and r_2 are 1.0 m and v_1 and v_2 are 1.0 m/s,

i) write $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ in terms of unit vectors **i** and **j**.
ii) write $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ in terms of unit vectors **i** and **j**.

b) Determine $\boldsymbol{L}_1$ and $\boldsymbol{L}_2$, the respective angular momentum of m_1 and m_2, where $\boldsymbol{L} = \boldsymbol{r} \times m\boldsymbol{v}$, i.e. angular Momentum is the vector cross product of $\boldsymbol{r}$ and $m\boldsymbol{v}$.

c) What do you notice about $\boldsymbol{L}_1$ and $\boldsymbol{L}_2$?

(25 marks)



a) i) $\boldsymbol{r}_1 = -1.0\ \mathbf{j}$ m

$$\boldsymbol{r}_2 = -1.0\cos 45^\circ \mathbf{i} + 1.0\sin 45^\circ \mathbf{j}$$
$$= -\frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}\ \text{m}$$

ii) $\boldsymbol{v}_1 = 1.0\ \mathbf{i}$ m/s

$$\boldsymbol{v}_2 = -1.0\cos 45^\circ \mathbf{i} - 1.0\sin 45^\circ \mathbf{j}$$
$$= -\frac{1}{\sqrt{2}}\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{j}\ \text{m/s}$$

b) $$\boldsymbol{L}_1 = 1.0 \times \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = 1.0\ \mathbf{k}\ \text{kg m}^2/\text{s}$$

$$\boldsymbol{L}_2 = 1.0 \times \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & -1/\sqrt{2} & 0 \end{vmatrix} = 1.0\ \mathbf{k}\ \text{kg m}^2/\text{s}$$

c) Both angular momenta are the same.

3. a) A cart moves on a straight rail at a constant speed of 2.0 m/s. At t = 0 s, it launches a stone at 45° to the horizontal with a velocity of 10 m/s.

i) Calculate the initial horizontal and vertical components of the velocity of the stone.
ii) How long does the stone take to hit a wall 10.0 m away from the launch point?
iii) At what height will the stone hit the wall?

b) A 5.0 kg mass is moving in +x direction with constant speed of 3.0 m/s. At t = 0 s, a constant force of 15 N is applied on the mass in the +y direction. After 2.0 s,

i) what is the acceleration of the mass in the +y direction?
ii) what is the velocity of the mass in the +y direction?
Express your answers in terms of **i** and **j**.

(25 marks)

3 a) i) $v_{\mathrm{H}} = v\cos 45^{\circ} + 2.0$

$= 10\cos 45^{\circ} + 2.0 = 9.07$ m/s

$v_{\mathrm{V}} = v\sin 45^{\circ}$

$= 10\sin 45^{\circ} = 7.07$ m/s

ii) $t = \dfrac{d}{v_{\mathrm{H}}} = \dfrac{10.0}{9.07}$

$= 1.10$ s

iii) $y = v_{0y}t + \dfrac{1}{2}a_y t^2 = v_{0y}t - \dfrac{1}{2}gt^2$

$= 7.07 \times 1.10 - 0.5 \times 9.80 \times 1.10^2$

$= 1.85$ m

b) i) $\boldsymbol{a}_y = 15 / 5.0 = 3.0\ \mathbf{j}$ m/s^2

ii) $\boldsymbol{v}_y = 0 + \boldsymbol{a}_y t$

$= 3.0 \times 2.0 = 6.0\ \mathbf{j}$ m/s

4. a) A man stands at the edge of a high cliff. He tossed a stone A vertically upward with an initial velocity of 10 m/s. Assume no air resistance.

   i) What is the maximum height (w.r.t. to the top of the cliff) reached by A?
   ii) How long does A take to return to the top of the cliff?

   b) Three seconds after the man tossed stone A, he dropped another stone B vertically downward. Determine the distance between A and B one second after he dropped B.

(25 marks)

a) i) $v_y^{\,2} = v_{0y}^{\,2} + 2a_y(y - y_0)$

$0 = 10^2 - 2 \times 9.80 \times (y - 0)$

$y = 5.10$ m

ii) $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

$0 = t(10 - 0.5 \times 9.80t)$

$t = 0$ (reject)

$t = 2.04$ s

b) $y_{\mathrm{A}} = v_{0\mathrm{A}}t_{\mathrm{A}} - \frac{1}{2}gt_{\mathrm{A}}^2$

$= 10 \times 4.0 - 0.5 \times 9.80 \times 4.0^2 = -38.4$ m

$y_{\mathrm{B}} = v_{0\mathrm{B}}t_{\mathrm{B}} - \frac{1}{2}gt_{\mathrm{B}}^2$

$= 0 \times 1.0 - 0.5 \times 9.80 \times 1.0^2 = -4.9\,\mathrm{m}$

$|y_{\mathrm{A}} - y_{\mathrm{B}}| = 33.5$ m

**********End**********