Mid-Semester Test (30% CA)

Name : ______________________________ Adm. No. : _______________

Class : ________________________ Class S/N : ______________

Date : ________________________ Time allowed : 1 hour

Maximum mark : 100

Instructions

Answer all 4 questions. Take g = 9.80 m/s^2.

This question paper consists of 3 printed pages including 1 page of formulae.

You are reminded that cheating during this test is a serious offence.

All working in support of your answer must be shown. Answers must be to appropriate significant figures.

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1. a) The acceleration of an object has the formula $a = \dfrac{m^x v^y}{r^z}$ where *m*, *v* and *r* are mass, velocity and distance respectively. Using dimensional analysis, determine the values of *x*, *y*, and *z* and hence find the formula for the acceleration.

   b) A particle is under the influence of a force **F** = (3**i** + 4**j**) N. The displacement vector of the particle is **s** = (7**i** + 24**j**) m. Calculate the dot product of **F** and **s** and hence find the angle between the two vectors.

(20 marks)

a) The dimension of a is $\dfrac{L}{T^2}$

The dimension of $\dfrac{m^x v^y}{r^z}$ is $\dfrac{M^x L^y T^{-y}}{L^z}$

$$\frac{L}{T^2} = \frac{M^x L^y T^{-y}}{L^z} = \frac{M^x L^{y-z}}{T^y}$$

Equating the exponents we get

$x = 0, y = 2$

$y - z = 1,$

$z = 1$

hence $a = \dfrac{v^2}{r}$

b) $\vec{F}.\vec{s} = 21 + 96 = 117\,\text{N.m}$

$$\cos\theta = \frac{\vec{F}.\vec{s}}{|\vec{F}||\vec{s}|} = \frac{21+96}{5\times 25} = 0.936$$

$\theta = \cos^{-1}(0.936) = 20.6^{\circ}$

2. A sprinter is running with a constant speed 10.4 m/s on a straight track and passes a stationary sports car which immediately begins to race with constant acceleration 7.9 m/s^2.

a) How much time does the car require to catch up with the sprinter?

b) How far does the car travel before catching up with sprinter?

c) Sketch the motion of the car and the sprinter on the same *x-t* graph from the time the car starts moving until it has caught up with the sprinter.

(25 marks)

a) for the sprinter: $v_{s,x} = 10.4\,\text{m/s}$

$x_s = v_{s,x} t$

for the car: $v_{c,0,x} = 0$, $a_{c,x} = 7.9\,\text{m/s}^2$

$$x_c = \frac{1}{2} a_{c,x} t^2$$

car catches up with sprinter when $x_c = x_s$

(let t_1 be the time taken for catching up)

$$\Rightarrow \frac{1}{2} a_{c,x} t_1^{\ 2} = v_{s,x} t_1$$

$$\Rightarrow t_1 (\frac{1}{2} a_{c,x} t_1 - v_{s,x}) = 0$$

(discard $t_1 = 0$ solution, this is when the car starts to move)

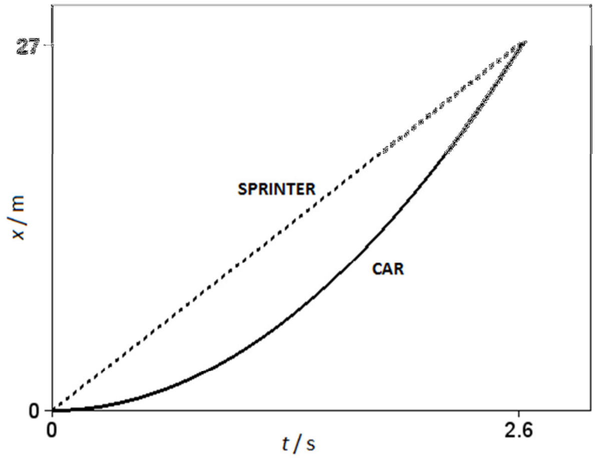
$$\Rightarrow t_1 = \frac{2v_{s,x}}{a_{c,x}} = \frac{2\times 10.4}{7.9} = 2.6329...\text{seconds}$$

$\Rightarrow t_1 = 2.6\,\text{s}$ (correct s.f)

b) $x_c = \frac{1}{2} a_{c,x} t_1^{\ 2} = \frac{1}{2} \times 7.9 \times (2.6329..)^2$ (insert from part a)

$\Rightarrow x_c = 27$ m

c)



3. An object is shot at a speed $v_0 = 35.0$ m/s at an angle 60^o to the horizontal. Ignore air resistance.

   a) Find the position of the object and its velocity in component form (or in terms of unit vectors **i** and **j**) at $t = 4.00$ s.
   b) Find the time when the object reaches the highest point.
   c) Find the maximum height attained by the object.
   d) Find the maximum horizontal distance travelled by the object.

(30 marks)

a) $$\mathbf{r} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} v_0 \cos\theta t \\ v_0 \sin\theta t - 0.5gt^2 \end{pmatrix}$$

$$\mathbf{r}(4.0) = \begin{pmatrix} 70.0\ m \\ 42.8m \end{pmatrix} = [70.0\hat{\mathbf{i}} + (42.8)\hat{\mathbf{j}}]\ m$$

$$\mathbf{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} v_0 \cos\theta \\ v_0 \sin\theta - gt \end{pmatrix}$$

$$\mathbf{v}(4.0) = \begin{pmatrix} 17.5\ m/s \\ -8.89\ m/s \end{pmatrix} = [17.5\hat{\mathbf{i}} + (-8.89)\hat{\mathbf{j}}]\ m/s$$

b) At the maximum height, $v_y = 0$

$$\Rightarrow\ v_0 \sin\theta - gt = 0 \Rightarrow t = \frac{v_0 \sin\theta}{g} = 3.09\,s$$

c) $$v_y^{\ 2} = (v_0 \sin\theta)^2 - 2gH$$

maximum height $= H = \dfrac{(v_0 \sin\theta)^2}{2g} = 46.9\,m$

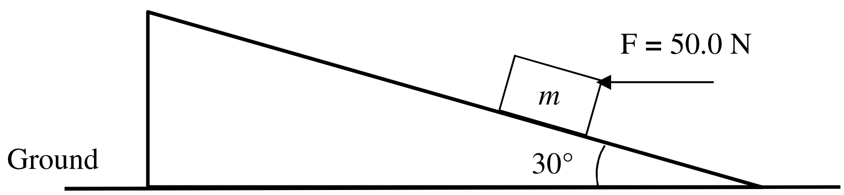
d) $$y = y_0 + v_0 \sin\theta t - 0.5gt^2$$

When the object reaches the ground, $y = y_0 = 0$. Therefore the total time is

$$0 = 0 + v_0 \sin\theta t - 0.5gt^2\ \Rightarrow\ t = \frac{2v_0 \sin\theta}{g}$$

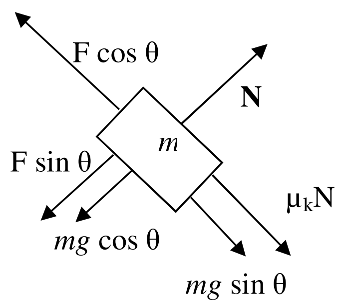
Maximum horizontal distance = $v_0 \cos\theta\, t = (v_0 \cos\theta)\dfrac{2v_0 \sin\theta}{g} = 108\,m$

4. A block with a mass $m = 2.50$ kg is pushed up an incline (that is fixed to the ground) by a horizontal force F = 50.0 N (see the below figure). The coefficient of kinetic friction between the block and the incline is 0.300.
   a) Draw the free body diagram of the block.
   b) Find the magnitude of the normal force due to the incline on the block.
   c) Find the magnitude of the friction force.
   d) Find the acceleration of the block.



(25 marks)

a)



b) $N = mg\cos\theta + F\sin\theta = 2.5 \times 9.80 \times \cos 30^{o} + 50 \times \sin 30^{o}$

$= 21.2 + 25 = 46.2$ N

c) $F_r = \mu_k N = 0.30 \times 46.2 = 13.9$ N

d) $F\cos\theta - mg\sin\theta - \mu_k N = ma$

$50 \times \cos 30^{o} - 2.5 \times 9.80 \times \sin 30^{o} - 13.9 = ma$

$a = 6.9 \text{ m/s}^2$

***** End of Paper *****

Formula sheet

Name: ________________________ Admin. No.: _______________ Seat No.: ________

Kinematics

$$v_x = v_{0x} + a_x t$$
$$v_x^{\,2} = v_{0x}^{\,2} + 2a_x(x - x_0)$$
$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$
$$\vec{v} = \frac{d\vec{r}}{dt}, \quad \vec{a} = \frac{d\vec{v}}{dt}$$
$$y = (tan\,\theta\,)x - (\frac{g}{2v^2 cos^2\theta})x^2$$
$$R = \frac{v^2 \sin 2\theta}{g}$$

Dynamics

$$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a},\ F = \mu N$$
$$a = \frac{dv}{dt},\ a = \frac{v^2}{r}, F = m\frac{v^2}{r}$$
$$\vec{J} = \int \vec{F}dt = \Delta\vec{p}$$
$$W = \int \vec{F}.\,\vec{dr},\ W_{net} = K_f - K_i$$
$$KE = \frac{1}{2}mv^2,\ \ PE = mgh$$
$$P = \frac{W}{t}, \quad P = \frac{dW}{dt}$$

Linear momentum

$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

Static electricity

$$F = k\frac{q_1 q_2}{r^2},\ k = \frac{1}{4\pi\varepsilon_o}$$
$$F = qE$$
$$V = k\frac{q}{r}, U = qV$$
$$\Phi_E = \oint \vec{E}.d\vec{A} = \frac{q}{\varepsilon_o}$$
$$V = Ed, W = qV, E = \frac{kq}{r^2}$$

Current electricity

$$Q = It \qquad V = IR$$
$$P = VI = I^2R = \frac{V^2}{R}$$

Magnetism & electromagnetism

$$\vec{F} = q\vec{v} \times \vec{B} \qquad \vec{F} = i\vec{L} \times \vec{B}$$
$$e.m.f. = -N\frac{d\Phi_B}{dt} \qquad \Phi_B = BA$$

Thermodynamics

$$\Delta U = Q - W$$
$$W = \int p\,dV$$
$Q_V = nC_V\Delta T$ const vol

$Q_p = nC_p\Delta T$ const pressure

$$Q = mC\Delta T$$
$$Q = mL$$

Ideal Gas

$$pV = nRT$$
$pV^\gamma = c$ (adiabatic)

$$\gamma = \frac{C_p}{C_V},\ C_p - C_v = R$$
$$W = pV\,ln\frac{V_2}{V_1} = nRT\,ln\frac{V_2}{V_1}$$
$$W = \frac{1}{\gamma - 1}(\ p_1V_1 - p_2V_2\ )$$

Rotational Motion

$$\omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt}$$
$$\omega = \omega_0 + \alpha t$$
$$\omega^2 = \omega_0^{\,2} + 2\alpha(\theta - \theta_0)$$
$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\mathrm{I} = \sum_i^n m_i r_i^2,\ \mathrm{I} = \int r^2 dm,\ K = \frac{1}{2}I\omega^2$$

SHM & waves

$$T = \frac{1}{f} \quad v = f\lambda \quad \omega = 2\pi f$$
$$\omega = \frac{2\pi}{T} \quad k = \frac{2\pi}{\lambda}$$
$$\omega = \sqrt{k/m} \quad \omega = \sqrt{g/L}$$
$$x = A\cos(\omega t + \phi)$$
$$x = A\sin(\omega t + \phi)$$
$$y(x,t) = A\cos(\omega t \pm kx)$$
$$y(x,t) = A\sin(\omega t \pm kx)$$
$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

Circuits

$R = R_1 + R_2 + R_3 + \ldots$ series

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$ parallel

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$ series

$C = C_1 + C_2 + C_3 + \ldots$ parallel

$$Q = CV \qquad U = \frac{1}{2}CV^2$$

Constants

Charge on electron
$e = -1.60 \times 10^{-19}$ C

Coulomb's constant
$k = 9 \times 10^9$ N m^2 C^{-2}

Ideal gas constant
$R = 8.314$ J mol^{-1} K^{-1}

Mass of proton
$m_p = 1.67 \times 10^{-27}$ kg

Mass of electron
$m_e = 9.11 \times 10^{-31}$ kg

Permeability of free space
$\mu_o = 4\pi \times 10^{-7}$ N A^{-2}

Permittivity of free space
$\varepsilon_o = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}

Speed of light in vacuum
$c = 3.00 \times 10^8$ m s^{-1}