# Solutions MST AY1213S1

1. (a) Calculate the following to the appropriate significant figures :

   (i) $(7.899 \times 1.90 \times 9.7878)$

   (ii) $\left(\dfrac{78.35}{214.9} + 56.134 - 8.94550\right)$

   (b) The damping ratio (D) is given by $D = \dfrac{c}{2\sqrt{km}}$, where c is the damping coefficient (SI unit of ‘c’ is N s/m), k is the spring constant (SI unit of ‘k’ is N/m)and m is mass. Show that the damping ratio is dimensionless.

(15 marks)

**Solution**

1. (a) (i) 147

   (ii) 47.553

   (b) The SI unit of force is N and 1 N = 1 kg m/s$^2$. Therefore the dimension of force is $MLT^{-2}$.

   The spring constant k has a SI unit of is N/m. The dimension of k is therefore

$$\frac{MLT^{-2}}{L} = MT^{-2}$$

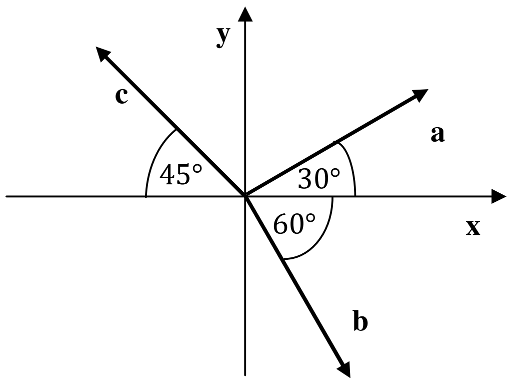
   The SI unit of c is N.s/m

   The dimension of c is $\dfrac{MLT^{-2}T}{L} = MT^{-1}$

   Therefore the damping ratio should have $\dfrac{\frac{MLT^{-2}}{L}}{\sqrt{MT^{-2}M}} = 1$

   This means damping ratio is dimensionless.

2. The magnitude of vectors **a**, **b**, and **c** in the below figure are all equal to 1. Find the direction of the vector **R** = **a** + **b** – **c**.




(10 marks)

**Solution**

We find the component of the three vectors via trigonometry: For a vector **A**, the components are $A_x = A\cos\theta$ and $A_y = A\sin\theta$ or also written as

$$\mathbf{A} = \begin{bmatrix} A\cos\theta \\ A\sin\theta \end{bmatrix}$$

$$\mathbf{a} = \begin{bmatrix} \sqrt{3}/2 \\ 1/2 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1/2 \\ -\sqrt{3}/2 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

Then, adding components:

$$\mathbf{R} = \mathbf{a} + \mathbf{b} - \mathbf{c} = \begin{bmatrix} \frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{1}{\sqrt{2}} \\ \frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{\sqrt{2}}{2} \\ \frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 + \sqrt{2} + \sqrt{3} \\ 1 - \sqrt{2} - \sqrt{3} \end{bmatrix}$$

The direction of the vector, $\theta$ will then be:

$$\tan\theta = \frac{1 - \sqrt{2} - \sqrt{3}}{1 + \sqrt{2} + \sqrt{3}}$$

Thus,

$\theta = \tan^{-1}\left(\frac{1-\sqrt{2}-\sqrt{3}}{1+\sqrt{2}+\sqrt{3}}\right) \approx -0.478\ (radians) \approx \mathbf{-27.368}°\ (degrees)$ with respect to positive x-axis.

3. A particle of mass 100 g moves in a circle of radius 20 cm. Its linear speed is given by v = 2t where t is measured in seconds and v in m/s. At time t = 3s, find the magnitude of
   (i) centripetal acceleration.
   (ii) centripetal force on the particle.
   (iii) tangential acceleration.

(15 marks)

**Solution**

(i) centripetal acceleration = $a_r$ = $v^2/r$ = $4t^2/r$. At t = 3s, $a_r$ = 180 m/s$^2$.

(ii) centripetal force = f = $mv^2/r$ = $m4t^2/r$. At t = 3s, f = 18 N.

(iii) tangential acceleration = $a_t$ = dv/dt = 2 m/s$^2$.

4. A ball is dropped from the top of a 50.0 m high cliff. At the same time a carefully aimed stone is launched vertically upwards from the bottom of the cliff with a speed of 24.0 m/s. The ball and stone collide.
   (i) When does the collision take place?
   (ii) How far above the base of the cliff does the collision take place?

(15 marks)

**Solution**

Choose up to be the positive direction. Let the bottom of the cliff be $y_0 = 0$.
The equation of motion for the dropped ball is

$y_{\text{ball}} = y_0 + v_0 t + \frac{1}{2}at^2 = 50.0\,\text{m} + \frac{1}{2}\left(-9.80\,\text{m/s}^2\right)t^2.$

The equation of motion for the thrown stone is

$y_{\text{stone}} = y_0 + v_0 t + \frac{1}{2}at^2 = \left(24.0\,\text{m/s}\right)t + \frac{1}{2}\left(-9.80\,\text{m/s}^2\right)t^2.$

Set the two equations equal and solve for the time of the collision. Then use that time to find the location of either object.

(i) $y_{\text{ball}} = y_{\text{stone}} \quad \rightarrow \quad 50.0\,\text{m} + \frac{1}{2}\left(-9.80\,\text{m/s}^2\right)t^2 = \left(24.0\,\text{m/s}\right)t + \frac{1}{2}\left(-9.80\,\text{m/s}^2\right)t^2 \quad \rightarrow$

$50.0\,\text{m} = \left(24.0\,\text{m/s}\right)t \quad \rightarrow \quad t = \dfrac{50.0\,\text{m}}{24.0\,\text{m/s}} = 2.083\,\text{s}$

(ii) $y_{\text{ball}} = y_0 + v_0 t + \frac{1}{2}at^2 = 50.0\,\text{m} + \frac{1}{2}\left(-9.80\,\text{m/s}^2\right)\left(2.083\,\text{s}\right)^2 = 28.7\,\text{m}$

5. A ball is thrown at an angle of $30^o$ above the horizon with a speed of 20 m/s at a vertical wall which is 30 m away. Find the
   (i) time taken for the ball to hit the wall.
   (ii) position of the point of contact when the ball hit the wall.
   (iii) magnitude and direction of the velocity vector when the ball hits the wall.

(15 marks)

**Solution**

The initial velocity is 20 m/s. Let the initial position coordinates be $(x_0,y_0) = (0,0)$
Therefore the x- component of the initial velocity (which is always a constant) is
$v_{0x} = v_x = 20\cos(30^o) = 17.3$ m/s

(i) The time taken for the ball to hit the wall = $x/v_x = 30/17.3 = 1.7$ s

(ii)The x-coordinate of the point of contact is 30. The y coordinate of the point of contact can be calculated as $y = y_0 + v_{0y}\,t - ½\,gt^2 = y = y_0 + v_0 \sin(30^o)\,t - ½\,gt^2$

$y = 20\sin(30^o)(1.7) - ½ \times 9.8 \times (1.7)^2 = 2.8$ m

(iii) $v_x = 17.3$ m/s, $v_y = 20\sin(30^o) - 9.80 \times 1.7 = -6.7$m/s
Magnitude of the velocity $= \sqrt{(17.3)^2 + (-6.7)^2} = 18.6$ m/s
Direction of the velocity $= \theta = \tan^{-1}(-6.7/17.3) = -21.1^o$ (i.e the ball is in its downward motion).

6. A 3.0-kg object is under the influence of two forces which are $\mathbf{F_1} = (16\,\mathbf{i} + 12\,\mathbf{j})$N and $\mathbf{F_2} = (-10\,\mathbf{i} + 22\,\mathbf{j})$ N. If the object is initially at rest, what is the
   (i) net force vector acting on the object?
   (ii) acceleration vector at $t = 3.0$ s?
   (iii) velocity vector at $t = 3.0$ s?

(15 marks)

**Solution**

Find the net force by adding the force vectors. Divide that net force by the mass to find the acceleration, and then find the velocity at the given time.

(i) Net Force $= \sum \vec{\mathbf{F}} = \left(16\hat{\mathbf{i}} + 12\hat{\mathbf{j}}\right)\mathrm{N} + \left(-10\hat{\mathbf{i}} + 22\hat{\mathbf{j}}\right)\mathrm{N} = \left(6\hat{\mathbf{i}} + 34\hat{\mathbf{j}}\right)\mathrm{N}$

(ii) $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} = (3.0\,\mathrm{kg})\vec{\mathbf{a}} \rightarrow \vec{\mathbf{a}} = \dfrac{\left(6\hat{\mathbf{i}} + 34\hat{\mathbf{j}}\right)\mathrm{N}}{3.0\,\mathrm{kg}} = \left(2\hat{\mathbf{i}} + 11.3\hat{\mathbf{j}}\right)\mathrm{m/s^2}$

(iii) Since force and acceleration are constant of time or position we can use

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t = 0 + \frac{\left(6\hat{\mathbf{i}} + 34\hat{\mathbf{j}}\right)\mathrm{N}}{3.0\,\mathrm{kg}}(3.0\,\mathrm{s}) = \left(6\hat{\mathbf{i}} + 34\hat{\mathbf{j}}\right)\mathrm{m/s}$$

7. As shown in the below figures the masses attached to a pulley (you can assume the mass of the pulley and the string are negligible and the string is inextensible). When $M_1 = M$, the tension in the string is $T_o$ (see Figure 1). However, when $M_1 = 0.8$ M, the masses accelerate and the tension decreases by 0.300 N (see Figure 2). Find the value of the acceleration a and M. (15 marks)

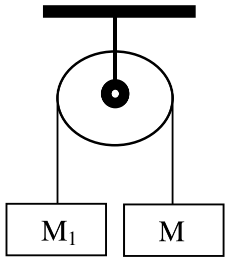



Figure 1

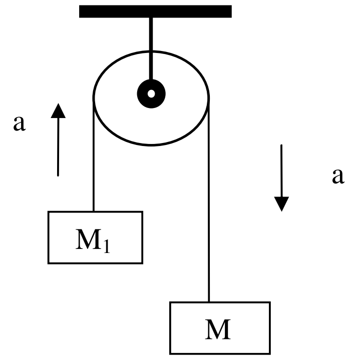
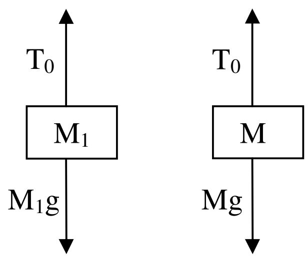



Figure 2

**Solution**

For Figure 1, $M_1 = M$, the free-body diagram for both masses will be similar

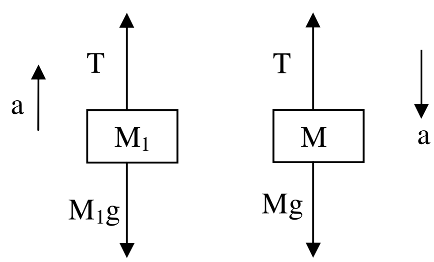



Applying Newton's second law for M we get $T_0 + (-Mg) = Ma$ or $T_0 = Mg$

For Figure 2, $M_1 = 0.8M$, let the tension be T. We know from the problem that

$$T_0 - T = 0.3 \text{ or } Mg - T = 0.3 \text{ or } T = Mg - 0.3$$

The free-body diagram for both masses are




From Newton's second law we get (we are taking –ve downward and +ve upward)

$$T + (-M_1g) = M_1a \quad (1)$$
$$T + (-Mg) = -Ma \quad (2)$$

Subtracting eqn (2) from (1) will yield $(M - M_1)g = (M + M_1)a$
or $a = 0.2g/1.8 = 1.09$ m/s$^2$

Substituting a in (2) and using $T = Mg - 0.3$, we get M = 0.28 kg

# Formula sheet for MS811M

## Kinematics

$v_x = v_{0x} + a_x t$

$v_x{}^2 = v_{0x}{}^2 + 2a_x(x - x_0)$

$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

$\vec{v} = \frac{d\vec{r}}{dt}, \quad \vec{a} = \frac{d\vec{v}}{dt}$

$y = (\tan\theta)x - \left(\frac{g}{2v^2\cos^2\theta}\right)x^2$

$R = \frac{v^2 \sin 2\theta}{g}$

## Dynamics

$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a},\ F = \mu N$

$a = \frac{dv}{dt},\ a = \frac{v^2}{r}, F = m\frac{v^2}{r}$

$\vec{J} = \int \vec{F}dt = \Delta\vec{p}$

$W = \int \vec{F}.\vec{dr},\ W_{net} = K_f - K_i$

$KE = \frac{1}{2}mv^2,\ PE = mgh$

$P = \frac{W}{t},\ \ P = \frac{dW}{dt}$

## Linear momentum

$\vec{p} = m\vec{v}$

$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$

## Static electricity

$F = k\frac{q_1 q_2}{r^2},\ k = \frac{1}{4\pi\varepsilon_o}$

$F = qE$

$V = k\frac{q}{r}, U = qV$

$\Phi_E = \oint \vec{E}.d\vec{A} = \frac{q}{\varepsilon_o}$

$V = Ed, W = qV, E = \frac{kq}{r^2}$

## Current electricity

$Q = It \quad V = IR$

$P = VI = I^2R = \frac{V^2}{R}$

## Magnetism & electromagnetism

$\vec{F} = q\vec{v}\times\vec{B} \qquad \vec{F} = i\vec{L}\times\vec{B}$

$e.m.f. = -N\frac{d\Phi_B}{dt}$

$\Phi_B = BA$

## Thermodynamics

$\Delta U = Q - W$

$W = \int pdV$

$Q_V = nC_V\Delta T$ const vol

$Q_p = nC_p\Delta T$ const pressure

$Q = mC\Delta T$

$Q = mL$

## Ideal Gas

$pV = nRT$

$pV^\gamma = c$ (adiabatic)

$\gamma = \frac{C_p}{C_V},\ C_p - C_v = R$

$W = pV\ln\frac{V_2}{V_1} = nRT\ln\frac{V_2}{V_1}$

$W = \frac{1}{\gamma - 1}(p_1V_1 - p_2V_2)$

## Rotational Motion

$\omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt}$

$\omega = \omega_0 + \alpha t$

$\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$I = \sum_i^n m_i r_i^2,\ I = \int r^2 dm,\ K = \frac{1}{2}I\omega^2$

## SHM & waves

$T = \frac{1}{f} \quad v = f\lambda \quad \omega = 2\pi f$

$\omega = \frac{2\pi}{T} \quad k = \frac{2\pi}{\lambda}$

$\omega = \sqrt{k/m} \quad \omega = \sqrt{g/L}$

$x = A\cos(\omega t + \phi)$

$x = A\sin(\omega t + \phi)$

$y(x,t) = A\cos(\omega t \pm kx)$

$y(x,t) = A\sin(\omega t \pm kx)$

$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

## Circuits

$R = R_1 + R_2 + R_3 + \ldots$ series

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$ parallel

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$ series

$C = C_1 + C_2 + C_3 + \ldots$ parallel

$Q = CV \qquad U = \frac{1}{2}CV^2$

## Constants

Charge on electron
$e = -1.60\times10^{-19}$ C

Coulomb's constant
$k = 9\times10^9$ N m$^2$ C$^{-2}$

Ideal gas constant
$R = 8.314$ J mol$^{-1}$ K$^{-1}$

Mass of proton
$m_p = 1.67\times10^{-27}$ kg

Mass of electron
$m_e = 9.11\times10^{-31}$ kg

Permeability of free space
$\mu_o = 4\pi\times10^{-7}$ N A$^{-2}$

Permittivity of free space
$\varepsilon_o = 8.85\times10^{-12}$ C$^2$ N$^{-1}$ m$^{-2}$

Speed of light in vacuum
$c = 3.00\times10^8$ m s$^{-1}$