SOLUTIONS

Mid-Semester Test

Name : ______________________________ Adm. No. : _______________

Class : ________________________ Class S/N : ______________

Date : ________________________ Time allowed : 1 hour

Maximum mark : 100

Instructions

Answer all 4 questions. Take g = 9.80 m/s^2.

This question paper consists of 3 printed pages including 1 page of formulae.

You are reminded that cheating during this test is a serious offence.

All working in support of your answer must be shown. Answers must be to appropriate significant figures.

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1. a) In the below equation, the SI units of x and x_0 are in metres, t and t_0 are in seconds, v_0 is m/s and a is m/s^2. Using the dimensional analysis show whether the equation is homogenous or not.

$$x = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$$

b) A particle is under a constant force **F** = (-2**i** + 4**j** + **k**) N as it moves from a point (5, 1, 2) m to another point (-2, 3, 4) m. Find the work done on the particle.

c) What is the angle between the displacement and force vectors in 1(b)?

(25 marks)

SOLUTIONS

(a) LHS $= x$

Therefore the dimension is $[L]$

$$\text{RHS} = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$$

The dimension in RHS is $[L] + [LT^{-1}][T] + [LT^{-2}][T^2]$

$$= [L] + [L] + [L] = [L] = \text{LHS}$$

Since it is a dimensionally consistent equation, it is homogenous.

(b) Displacement vector, $\vec{s} = (-2\hat{i} + 3\hat{j} + 4\hat{k}) - (5\hat{i} + \hat{j} + 2\hat{k})$

$$= (-7\hat{i} + 2\hat{j} + 2\hat{k})$$

Work done = $\vec{F} \odot \vec{s} = (-2\hat{i} + 4\hat{j} + \hat{k}).(-7\hat{i} + 2\hat{j} + 2\hat{k})$

$$= 14 + 8 + 2 = 24\,\text{J}$$

(c) $\vec{F} \odot \vec{s} = |\vec{F}||\vec{s}|\cos(\theta)$

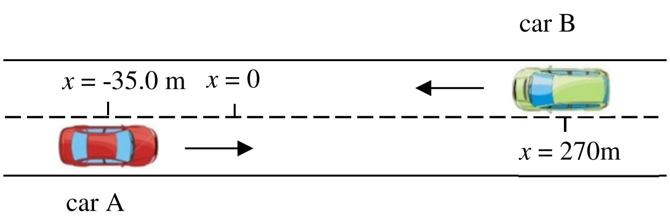
$$\cos(\theta) = \frac{\vec{F}.\vec{s}}{|\vec{F}||\vec{s}|} = \frac{24}{\sqrt{21}\sqrt{57}}$$

$$\theta = \cos^{-1}\left(\frac{24}{\sqrt{21}\sqrt{57}}\right) = 46°$$

2. In the figure below, car A and car B, moves towards each other in adjacent lanes and parallel to an x axis. At time $t = 0$, car A is at $x = -35.0$ m and accelerates uniformly from rest at 2.00 m/s^2 while car B is at $x = 270.0$ m moving at a constant speed of 20.0 m/s.

a) When do the cars meet?
b) Where do the cars meet?
c) What is the speed of car A when it crosses car B?
d) Sketch the displacement-time plots of the two cars on the same graph.

(25 marks)



SOLUTIONS

(a) Let "t" be the time when the cars meet at a point "x"

The equations for the cars would be

$x_A = 270 - 20.0 \times t$

$x_B = -35 + \frac{1}{2}(2)t^2$

Solving the above equations (with $x_A = x_B$), we get $t = 10.1\text{s}$ when the cars meet.

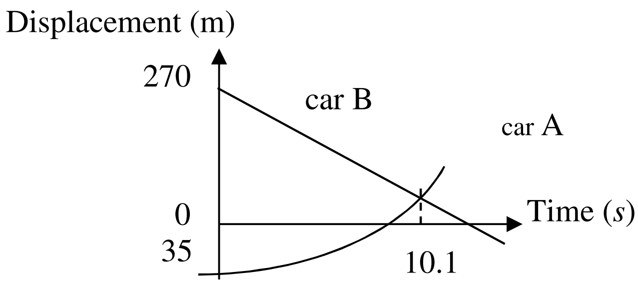
(b) Substituting t in one of the above two equations we get $x = 68\,\text{m}$ where the cars meet.

(c) $v = v_0 + at$

$= 0 + 2 \times 10.1$

$v = 20.2$ m/s

(d)



3. The position vector of a particle of mass 2.0 kg moving on the x-y plane is $\vec{r} = 2t\,\hat{i} + 2\sin(\pi t/4)\,\hat{j}$, with $\vec{r}$ in metres and t in seconds. Calculate in component form the

a) particle's average velocity during the first second of its journey.
b) particle's velocity at $t = 1.0$ s.
c) force acting on the particle at $t = 1.0$ s.

(25 marks)

SOLUTIONS

(a) $\vec{r}(t) = 2t\,\hat{i} + 2\sin(\pi t/4)\,\hat{j}$

$\vec{r}(0.0) = 0\hat{i} + 0\,\hat{j}$

$\vec{r}(1.0) = 2\hat{i} + 2\sin(\pi/4)\,\hat{j}$

$$\vec{v}_{\text{average}}(t) = \frac{\Delta\vec{r}}{\Delta t} = \frac{\{\vec{r}(1.0) - \vec{r}(0.0)\}}{1.0 - 0.0}$$

$$= \{2\hat{i} + 2\sin(\pi/4)\,\hat{j}\}\ \text{m/s}$$

(b) $\vec{v}(t) = 2\hat{i} + 2\times\pi/4\cos(\pi t/4)\,\hat{j}$

$= 2\hat{i} + \pi/2\cos(\pi t/4)\,\hat{j}$

$\vec{v}(1.0) = 2\hat{i} + \pi/2\cos(\pi/4)\,\hat{j}$

$$= 2\hat{i} + \pi/2\times\frac{1}{\sqrt{2}}\,\hat{j}$$

$$= 2\hat{i} + \frac{\pi}{2\sqrt{2}}\,\hat{j}\ \text{m/s.}$$

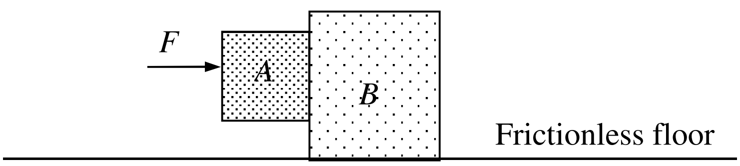
(c) $\vec{a}(t) = -2\times(\pi/4)^2\sin(\pi t/4)\,\hat{j}$

$$\vec{a}(1.0) = -2\times(\pi/4)^2\times\frac{1}{\sqrt{2}}\,\hat{j}\ \text{m/s}^2 = -0.87\hat{j}\ \text{m/s}^2$$

Hence $F = ma = -1.74\hat{j}$ N

4. The two blocks (A = 16 kg and B = 88 kg) in the figure below are not attached to each other. The coefficient of static friction between the two blocks is $\mu_s = 0.33$, but the surface beneath the larger block is frictionless.

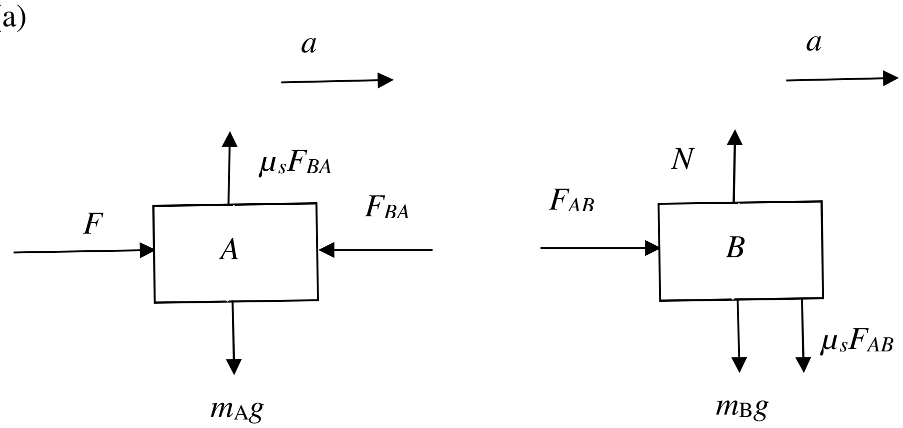
a) Draw the free-body diagram for blocks A and B if a force F acts on A as shown in the figure.
b) What is the magnitude of the minimum force F required to keep block A from slipping down block B?
c) If F acts on block B instead of block A, what is the magnitude of F so that block A does not fall off block B?



(25 marks)

SOLUTIONS

(a)



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(b) For A not to fall off, $\mu_s F_{BA} = m_A g$

$\therefore F_{BA} = 16g / 0.33$

Taking A and B accelerating as one object

$F = (16+88)a = 104a$

$a = F / 104$

Taking A accelerating alone

$F - F_{BA} = 16a$

$F = F_{BA} + 16a$

$= \dfrac{16g}{0.33} + 16\dfrac{F}{104}$

$F = 5.62\times10^2$ N

(c) If F acts on B, then

$F - F_{AB} = 88a$

$F = F_{AB} + 88a$

Since $F_{AB} = F_{BA}$

$F = \dfrac{16g}{0.33} + 88\dfrac{F}{104}$

$F = 3.09\times10^3$ N

***** End of Paper *****

SOLUTIONS

Formula sheet

Kinematics

$v_x = v_{0x} + a_x t$

$v_x{}^2 = v_{0x}{}^2 + 2a_x(x - x_0)$

$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

$\vec{v} = \frac{d\vec{r}}{dt}, \quad \vec{a} = \frac{d\vec{v}}{dt}$

$y = (\tan\theta)x - \left(\frac{g}{2v^2\cos^2\theta}\right)x^2$

$R = \frac{v^2 \sin 2\theta}{g}$

Dynamics

$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}, \; F = \mu N$

$a = \frac{dv}{dt}, \; a = \frac{v^2}{r}, F = m\frac{v^2}{r}$

$\vec{J} = \int \vec{F}dt = \Delta\vec{p}$

$W = \int \vec{F}.\vec{dr}, \; W_{net} = K_f - K_i$

$KE = \frac{1}{2}mv^2, \; PE = mgh$

$P = \frac{W}{t}, \; P = \frac{dW}{dt}$

Linear momentum

$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$

Static electricity

$F = k\frac{q_1q_2}{r^2}, \; k = \frac{1}{4\pi\varepsilon_o}$

$F = qE$

$V = k\frac{q}{r}, \; U = qV$

$\Phi_E = \oint \vec{E}.d\vec{A} = \frac{q}{\varepsilon_o}$

$V = Ed, \; W = qV, \; E = \frac{kq}{r^2}$

Current electricity

$Q = It \quad V = IR$

$P = VI = I^2R = \frac{V^2}{R}$

Magnetism & electromagnetism

$\vec{F} = q\vec{v} \times \vec{B} \quad \vec{F} = i\vec{L} \times \vec{B}$

$e.m.f. = -N\frac{d\Phi_B}{dt} \quad \Phi_B = BA$

Thermodynamics

$\Delta U = Q - W$

$W = \int p\,dV$

$Q_V = nC_V\Delta T$ const vol

$Q_p = nC_p\Delta T$ const pressure

$Q = mC\Delta T$

$Q = mL$

Ideal Gas

$pV = nRT$

$pV^\gamma = c$ (adiabatic)

$\gamma = \frac{C_p}{C_V}, \; C_p - C_v = R$

$W = pV \ln\frac{V_2}{V_1} = nRT \ln\frac{V_2}{V_1}$

$W = \frac{1}{\gamma - 1}(p_1V_1 - p_2V_2)$

Rotational Motion

$\omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt}$

$\omega = \omega_0 + \alpha t$

$\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$I = \sum_i^n m_i r_i^2, \; I = \int r^2 dm, \; K = \frac{1}{2}I\omega^2$

SHM & waves

$T = \frac{1}{f} \quad v = f\lambda \quad \omega = 2\pi f$

$\omega = \frac{2\pi}{T} \quad k = \frac{2\pi}{\lambda}$

$\omega = \sqrt{k/m} \quad \omega = \sqrt{g/L}$

$x = A\cos(\omega t + \phi)$

$x = A\sin(\omega t + \phi)$

$y(x,t) = A\cos(\omega t \pm kx)$

$y(x,t) = A\sin(\omega t \pm kx)$

$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

Circuits

$R = R_1 + R_2 + R_3 + \ldots$ series

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$ parallel

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$ series

$C = C_1 + C_2 + C_3 + \ldots$ parallel

$Q = CV \quad U = \frac{1}{2}CV^2$

Constants

Charge on electron
$e = -1.60 \times 10^{-19}$ C

Coulomb's constant
$k = 9 \times 10^9$ N m^2 C^{-2}

Ideal gas constant
$R = 8.314$ J mol^{-1} K^{-1}

Mass of proton
$m_p = 1.67 \times 10^{-27}$ kg

Mass of electron
$m_e = 9.11 \times 10^{-31}$ kg

Permeability of free space
$\mu_o = 4\pi \times 10^{-7}$ N A^{-2}

Permittivity of free space
$\varepsilon_o = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}

Speed of light in vacuum
$c = 3.00 \times 10^8$ m s^{-1}