Solutions

1. Evaluate i) $\dfrac{25.2\times1374}{33.3}$

ii) $24.36 + 0.0623 + 256.2$

Solution

i) $\dfrac{25.2\times1374}{33.3} = 1039.7838....$

However the answer should have 3 significant digits.

Therefore the answer is 1040

ii) $24.36 + 0.0623 + 256.2 = 280.6223$

However the answer should contain only upto 1 decimal place.

Therefore the answer is rounded off and is 280.6

2. Newton's law of universal gravitation is represented by $\mathrm{F}=\dfrac{\mathrm{GMm}}{\mathrm{r}^2}$, where F is the force, M and m are masses of the two objects and r is the centre to centre distance between the two objects. Find the dimension of G and hence state its SI unit.

Solution

$$F=\frac{GMm}{r^2}$$

or $G=\dfrac{Fr^2}{Mm}$

The dimension of G is $\left[\dfrac{MLT^{-2}L^2}{M^2}\right] = L^3M^{-1}T^{-2}$

The SI unit of G is $\dfrac{N.m^2}{kg^2}$ which can be therefore further written as $\dfrac{m^3}{kg.s^2}$

3. When a spherical object moves through a liquid, the liquid opposes the motion with a force F. The magnitude of F depends on the coefficient of viscosity, η of the liquid (whose dimensions are $ML^{-1}T^{-1}$), the speed, v of the object and the radius, r of the object. Assuming that F is proportional to different powers of these quantities, guess a formula for F using the method of dimensions.

Solution

Let the formula be $F = k\,\eta^a r^b v^c$

Then $MLT^{-2} = [ML^{-1}T^{-1}]^a L^b \left[\dfrac{L}{T}\right]^c$

$= M^a L^{-a+b+c} T^{-a-c}$

Equating the exponents of M, L and T from both sides we get

$$a = 1$$
$$-a + b + c = 1$$
$$-a - c = -2$$

Solving $a = b = c = 1$

Therefore the formula is $F = k\eta r v$

4. The position of a particle (x) moving along the x-axis as function of time (t) is given by $x = at^3 + bt^2 + ct + d$. The numerical values of a, b, c and d are $1, 4, -2$ and 5 respectively and SI units are used for x and t. Find:
   i) the SI unit of a, b, c and d.
   ii) the velocity of the particle at t = 4s.
   iii) the acceleration of the particle at t = 4s.
   iv) the average velocity between t = 0 and t = 4s

Solution

i) Since all the terms are used to express position, whose units of measure is m
the units of the four components must therefore be:

a is measured in m/s^3
b is measured in m/s^2
c is measured in m/s^1
d is measured in m

ii) The velocity of the particle at $t = 4s$.
Since $x = t^3 + 4t^2 - 2t + 5$, $v = \frac{dx}{dt}$

$$v(t) = 3t^2 + 8t - 2$$
$$\therefore v(4) = 78\ m/s$$

iii) The acceleration of the particle at $t = 4s$.
Since $v(t) = 3t^2 + 8t - 2$, $a = \frac{dv}{dt}$

$$a(t) = 6t + 8$$
$$\therefore a(4) = 32\ m/s^2$$

iv) The average velocity between $t = 0$ and $t = 4$
The average velocity would be given by:

$$\bar{v} = \frac{\text{Total displacement}}{\text{Total time}}$$
$$\bar{v} = \frac{1}{4}[x(4) - x(0)]$$
$$\bar{v} = \frac{120}{4} = 30\ m/s$$

5. A particle moves in the X-Y plane with a constant acceleration of 1.5 m/s^2 in the direction making at an angle of 37^o with the X-axis. At t = 0 the particle is at the origin and its velocity is 8.0 m/s along the X-axis. Find
   i) the velocity of the particle at t = 4.0 s.
   ii) the position of the particle at t = 4.0 s.

Solution

(always ensure that your calculator is in correct mode such as degree mode in this question)

$a_x = (1.5 m/s^2)\cos 37^o$

$= 1.2\ m/s^2$

$a_y = (1.5 m/s^2)\sin 37^o$

$= 0.9\ m/s^2$

The initial velocity has components $v_{0x} = 8.0\ m/s$ and $v_{0y} = 0.0\ m/s$.

At $t=0, x=0$ and $y=0$.

$v_x = v_{0x} + a_x t = 12.8\ m/s$

$v_y = v_{0y} + a_y t = 3.6\ m/s$

The velocity of the particle at $t=4.0\,s$ is given by v $= \sqrt{v_x^{\ 2} + v_y^{\ 2}} = 13.3\ m/s$

The direction of the velocity is given by $\tan\theta = \dfrac{v_y}{v_x}\ \ or\,\theta = 15.8^o$

The x-coordinate at $t=4.0\,s$ is $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = 41.6\ m$

The y-coordinate at $t=4.0\,s$ is $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = 7.2\ m$

Therefore the position of the particle is $(41.6\,m, 7.2\,m)$ at $t=4.0\,s$

6. A stone is thrown vertically upward from a point on a bridge located 40 m above the water. Knowing that it strikes the water 4 s after release, determine
   i) the speed with which the stone was thrown upward,
   ii) the speed with which the stone strikes the water.

Solution

i) $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

$t = 4\,s$

$a_y = -g = -9.8\ m/s^2$

$-40\,m = 0 + v_{0y}(4s) - \frac{1}{2}(9.8)\ m/s^2(4s)^2$

or $v_{0y} = 9.6\ m/s$ (+ve sign indicating upward direction)

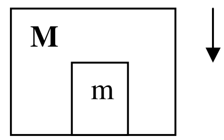
ii) $v = v_{0y} + a_y t$

$= 9.6\ m/s - (9.8 m/s^2)\,4s$

or $v = -29.6\ m/s$ (−ve sign indicating downward direction)

7. A box of mass **M** contains an object of mass **m** as shown in the below.

What should be the acceleration of the box during its descent if the object exerts a force of **mg**/4 on the floor of the box?



Solution

We know that the object exerts a force on the box equal to $\frac{mg}{4}$. From Newton's third law the box also exerts the same magnitude force of $\frac{mg}{4}$ on the object.

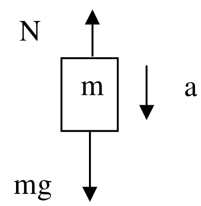
Therefore the normal reaction is $N = \frac{mg}{4}$

The free-body diagram of the object of mass **m** is

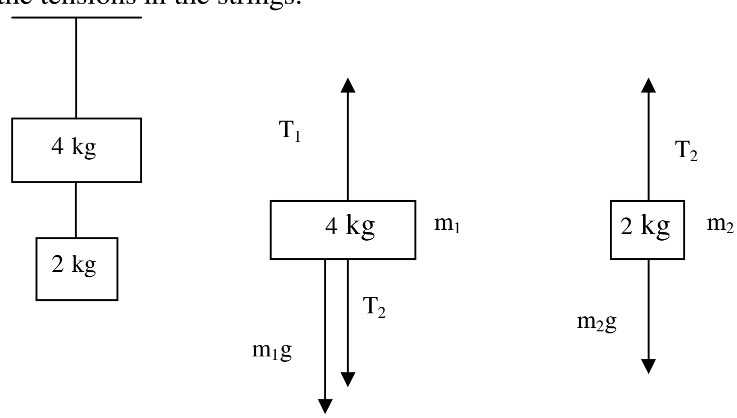
$$mg - N = ma$$

$$mg - \frac{mg}{4} = ma$$

$$\text{or } a = \frac{3g}{4}$$



8. A 4 kg block is suspended from the roof of an elevator. A 2 kg block is suspended from the 4 kg block (see figure below) . When the elevator accelerates upwards 2.2 m/s^2, find the magnitudes of the tensions in the strings.



Solution

Consider m_1,

$T_1 - T_2 - m_1g = 4 \times 2.2$

$T_1 - T_2 = 8.8 + 4 \times 10 = 48.8$ N ....(1)

Consider m_2,

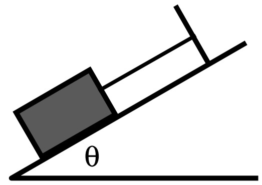
$T_2 - m_2g = 2 \times 2.2$

$T_2 = 4.4 + 2 \times 10 = 24.4$ N

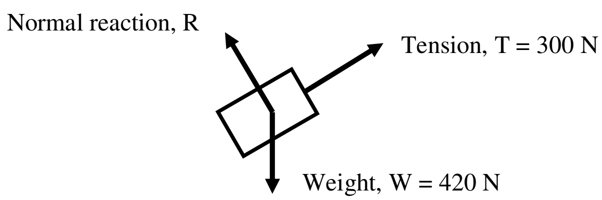
From (1),

$T_1 = 48.8 + 24.4 = 73.2$ N

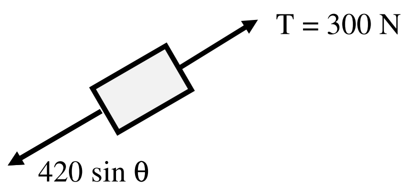
9. A block of mass 42 kg rests on a slope with a smooth surface. It is tied by a string to a post. The string can withstand a maximum tension of 300 N. Determine the angle at which the string is about to snap.



Solution

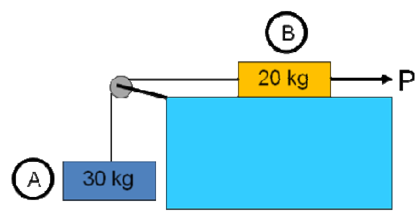


Considering forces along the ramp only, we have



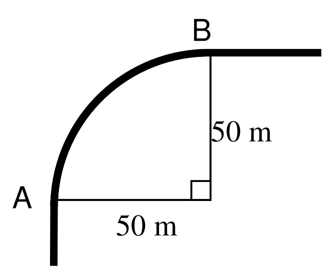
Thus, $300 = 420 \sin \theta$, which gives $\theta = 45.6^o$.

10. A force P is acting on block B as shown. If P = 200 N, find the tension of the string and the acceleration of the system.



Solution
Drawing the free-body diagrams and defining the direction of acceleration:
We can form two equations with Newton's Second Law:
For Block A: $300 - T = 30a$
For Block B: $T - 200 = 20a$
Solving we get $T = 240$ N and $a = 2$ m s^{-2} to the left

11. A car of mass 1000 kg is travelling along a road at a constant speed of 72 km h^{-1}. It enters a bend at A and exits at B. The bend has a radius of 50 m.
(a) What is the magnitude of the centripetal force that is experienced by the car?
(b) If your answer in (a) is the largest force that the car



can withstand, what is the maximum speed of the car if the radius is 32 m instead?

Solution

(a) Speed of the car = 72 km h^{-1} = 20 m s^{-1} or Centripetal acceleration = v^2/R = 8 m/s^2.
Therefore centripetal force is mv^2/R = 8000 N

(b) If radius is 32 m and centripetal force is 8000 N,
mv^2/R = 8000 N
then the maximum speed of the car is 16 m/s or 57.6 km/hr

12. A clock has its minute hand 4.0cm long. Find the average velocity of the tip of the minute hand
(a) Between 6:00am to 6:30am and
(b) Between 6:00am to 6:30pm.

Solution

(a) $\frac{8cm}{0.5hr} = 16cmhr^{-1}$

(b) $\frac{8cm}{12.5hr} = 0.64cmhr^{-1}$

(you can express the answer in other units such as cm/s and m/s).

13. A particle travelled between two points. It travels half the distance with a velocity v_0 while the remaining part of the distance was covered with velocity v_1 for half the time and with velocity v_2 for the other half of the time. Find the average velocity during the journey. (Assume the particle travels in a straight line.)

Solution

Let d be the distance between the two points.

If t_0 is the time taken to reach the half-way point, then

$$v_0 = \frac{d}{2t_0} \rightarrow t_0 = \frac{d}{2v_0}$$

Then, if t_1 and t_2 are the other respective times, then,

$$\frac{d}{2} = v_1 t_1 + v_2 t_2$$

However, $t_1 = t_2$, so

$$\frac{d}{2} = t_1(v_1 + v_2)$$

But

$$\frac{d}{2} = v_0 t_0$$

Setting these equal to each other yields

$$v_0 t_0 = t_1(v_1 + v_2)$$

So

$$t_1 = \frac{v_0 t_0}{v_1 + v_2}$$

Since average velocity is,

$$\bar{v} = \frac{d}{t_0 + 2t_1}$$

We substitute the above value of t_1 into the equation to get

$$\bar{v} = \frac{d}{t_0 + \frac{2v_0 t_0}{v_1 + v_2}} = \frac{d}{\frac{t_0(v_1 + v_2) + 2v_0 t_0}{v_1 + v_2}} = \frac{d(v_1 + v_2)}{t_0(v_1 + v_2) + 2v_0 t_0}$$

Finally, since $t_0 = \frac{d}{2v_0}$, average velocity becomes

$$\bar{v} = \frac{d(v_1 + v_2)}{\frac{d}{2v_0}(v_1 + v_2) + d} = \frac{v_1 + v_2}{\frac{(v_1 + v_2) + 2v_0}{2v_0}} = \frac{2v_0(v_1 + v_2)}{(v_1 + v_2) + 2v_0}$$

14. The sum of two vectors, **p** and **q** is perpendicular to **p**. Find the angle between **p** and **q**.

Solution

We know that $\vec{\boldsymbol{p}} \cdot (\vec{\boldsymbol{p}} + \vec{\boldsymbol{q}}) = 0$ (as the sum of **p** and **q** is 90° to **p)** and want to express the angle, θ in terms of $\vec{\boldsymbol{p}}$ and $\vec{\boldsymbol{q}}$.

Since $\vec{\boldsymbol{p}} \cdot \vec{\boldsymbol{q}} = \big||\vec{\boldsymbol{p}}|\big|\big||\vec{\boldsymbol{q}}|\big|\cos\theta$, we can write

$$\cos\theta = \frac{\vec{\boldsymbol{p}} \cdot \vec{\boldsymbol{q}}}{\big||\vec{\boldsymbol{p}}|\big|\big||\vec{\boldsymbol{q}}|\big|}$$

Because $\vec{\boldsymbol{p}} \cdot (\vec{\boldsymbol{p}} + \vec{\boldsymbol{q}}) = 0$, distributing the dot product gives,

$$\vec{\boldsymbol{p}} \cdot \vec{\boldsymbol{p}} + \vec{\boldsymbol{p}} \cdot \vec{\boldsymbol{q}} = 0$$
$$\vec{\boldsymbol{p}} \cdot \vec{\boldsymbol{q}} = -||\vec{\boldsymbol{p}}||^2$$

Combining the two equations give,

$$\cos\theta = \frac{-||\vec{\boldsymbol{p}}||^2}{\big||\vec{\boldsymbol{p}}|\big|\big||\vec{\boldsymbol{q}}|\big|} = -\frac{\big||\vec{\boldsymbol{p}}|\big|}{\big||\vec{\boldsymbol{q}}|\big|}$$

$$\theta = \cos^{-1}\left(-\frac{\big||\vec{\boldsymbol{p}}|\big|}{\big||\vec{\boldsymbol{q}}|\big|}\right)$$

15. An aircraft is flying horizontally with speed v at altitude h. If it drops a packet to be collected by a man standing on the ground, at what distance from the man should the packet be dropped? (The man stands in the vertical plane of the aircraft's motion.)

Solution

The packet will accelerate vertically with g, thus its vertical height can be expressed As (note that the vertical component of the velocity is zero:

$$h(t) = 1/2gt^2$$

Thus, the package will hit the ground (preferably not the man) at,

$$t = \sqrt{\frac{2h}{g}}$$

Since horizontal and vertical motion are independent of each other, the packet should be released at a distance d = tv from the man,

$$d = v\sqrt{\frac{2h}{g}}$$

Therefore the distance from the man to the packet

when it was first dropped $= \sqrt{d^2 + h^2} = \sqrt{\frac{2u^2 h}{g} + h^2}$

16. A 50-kg box, starting from rest, is pulled for 2 m across a floor with a constant horizontal force of 200 N. The coefficient of kinetic friction is 0.03.
    (a) What is the net force acting on the object?
    (b) Find the final velocity of the box.

Solution

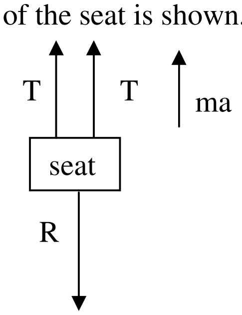
(a) Find the frictional force f = μmg = 15 N
Net force = 200 – 15 = 185 N

(b) The acceleration = a = Net force/mass = 185/50 = 3.7 m/s^2
$v^2 = 2\,a\,s$ or v = $(2 \times 3.7 \times 2)^{0.5}$ = 3.8 m/s

17. A 35 kg child swings to and fro on a swing supported by two chains each 3 m long. The tension in each chain at the lowest point of the motion is 280 N. At the lowest point, find the
    (a) force exerted by the seat on the child (Ignore the mass of the seat).
    (b) child's speed assuming that it does not hold the chain at the lowest point.

Solution

(a) Let "R" be the force that the child exerts on the seat. The Free body diagram of the seat is shown.

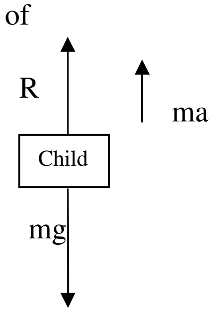


$2T - R - m_{seat}g = m_{seat}a$

But mass of the seat is negligible.
Therefore $m_{seat} = 0$ or
2T = R = 560 N
From Newton's third law, R should be the force that the seat exerts on the child.

Note that ma is not a separate force but the resultant of mg and 2T.

(b) The Free body diagram of the child is shown.
(note that the child is not holding to the chain).

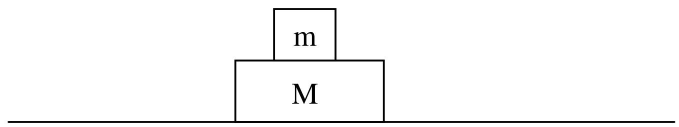


R – mg = ma

= mv^2/r

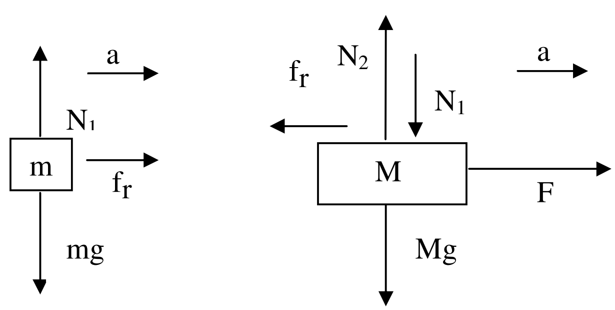
or v = 4.3 m/s

18. In the figure below a block of mass M lies on a surface that is smooth. A smaller block of mass m lies on the bigger block. The coefficient of static friction between the two blocks is μ. What is the maximum horizontal force that can be applied on the bigger block so that the blocks move together?



Solution

Draw the free-body diagram for both blocks. Note that the force of friction provides the horizontal force on small block. N_1 is the normal force between small and big blocks and N_2 is the normal force between the smooth surface and big block M. 'F' is the applied force on the big block.



Write the Newton's second law in the x and y directions (see below)

For the small block

$$F_y: \quad N_1 = mg \qquad \text{.............................(1)}$$

$$F_x: \quad f_r = ma = \mu mg \qquad \text{.............................(2)}$$

$$a = \mu g$$

For the big block

$$F_y: \quad N_2 = N_1 + mg \qquad \text{...........................(3)}$$

$$F_x: \quad F - f_r = Ma \qquad \text{.............................(4)}$$

$$F - \mu mg = M\mu g$$

$$\text{or} \qquad F = \mu g\left(m + M\right)$$

**** End ****