SOLUTIONS

Mid-Semester Test (30% CA)

Name : ______________________________ Adm. No. : _______________

Class : ________________________ Class S/N : ______________

Date : ________________________ Time allowed : 1 hour

Maximum mark : 100

Instructions

Answer all 4 questions. Take g = 9.80 m/s^2.

This question paper consists of 3 printed pages including 1 page of formulae.

You are reminded that cheating during this test is a serious offence.

All working in support of your answer must be shown. Answers must be to appropriate significant figures.

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1. a) From an experiment, a formula $t = k\sqrt{\dfrac{ml^3}{a}}$ is established where *t*, *m*, *l* and *a* are time, mass, distance and acceleration respectively. Find the dimension of the quantity *k*. What is the SI unit of *k*?

   b) Find the cross product of **A** = (2**i** + 3**j** + **4k**) and **B** = (4**i** + 6**j**).

   c) Find the angle between the two vectors **A** and **B** in 1 (b).

(25 marks)

a) $k = t\sqrt{\dfrac{a}{ml^3}}$

The dimension of a is $\dfrac{L}{T^2}$, while the dimensions of t is T, m is M and l is L.

Therefore the dimension of k is

$$T\sqrt{\frac{L}{T^2ML^3}} = M^{-1/2}L^{-1}$$

The SI unit of k is (kg)$^{-1/2}$ m^{-1}.

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2. A rocket is accelerated from rest at 5 m/s^2 vertically for 5 seconds before it accelerates at 2 m/s^2 for another 5 seconds.
   a) What is the total vertical distance travelled in 10 seconds?
   b) Sketch the velocity-time graph of the rocket for the first 10 seconds.
   c) If at the end of the second acceleration, the engine is turned off, how long will it take for the rocket to return to the launch pad, i.e. count from the time the engine is turned off?

(25 marks)

a) $y = y_0 + v_{0y}t + \frac{1}{2}ayt^2$

$y_1 = 0 + \frac{1}{2}5\times5^2 = 62.5$ m

$v_y = v_{0y} + a_y t$

$v_{1y} = 0 + 5\times5 = 25.0$ m/s

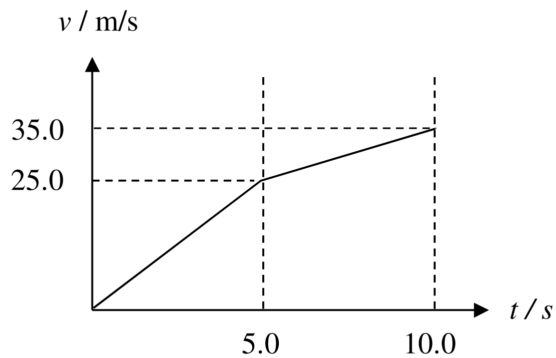
$y_2 = 25\times5 + \frac{1}{2}2\times5^2 = 150.0$ m (Take end of y_1 as origin)

$v_{2,y} = 25.0 + 2.0\times5 = 35.0$ m/s

Total height at which engine is turned off from

the ground = 212.5 m

b)



c) To return to the ground, $y = -212.5$ m and $a_y = -9.80$ m/s^2

$-212.5 = 35.0t - \frac{1}{2}\times9.8\times t^2$

$t = 11.1$ s

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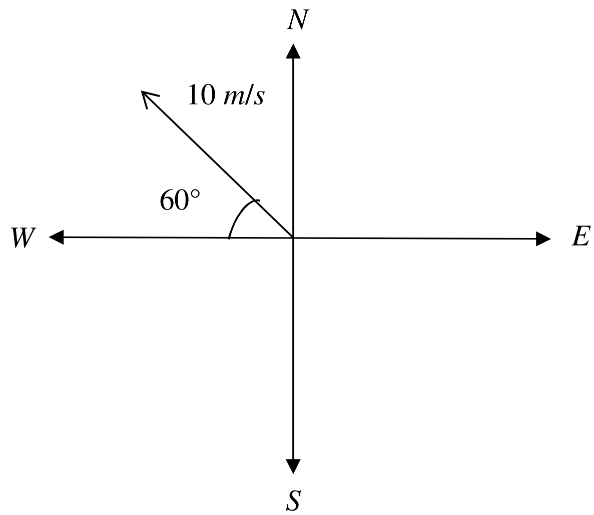
3. An object moves at 10 m/s in a direction 60^{o} north of west.

a) Draw the velocity vector, clearly labelling the directions N, S, E and W.
b) Calculate the velocities in the direction of north and west respectively?
c) How long does it take to go 1000 m west?
d) By the time it is 1000 m west, what is the total distance it has it travelled?

(25 marks)

a)



Take *x* as west (left positive) and *y* as north (upward negative).

b) $\mathbf{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} 10\ \cos 60^{o} \\ 10\ \sin 60^{o} \end{pmatrix} = \begin{pmatrix} 5\ m/s \\ 8.67\, m/s \end{pmatrix}$

c) $x = v_x \cos\theta t$

$t = \dfrac{1000}{5} = 200\ s$

d) $y = v_x \sin\theta t$

$= 8.67 \times 200 = 1734\ m.$

$\text{Total distance travelled} = \sqrt{1000^2 + 1734^2} = 2002\ m$

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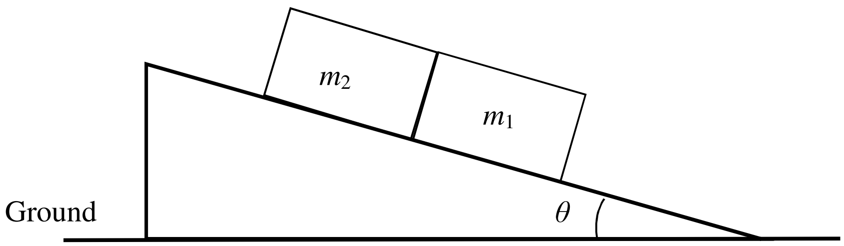
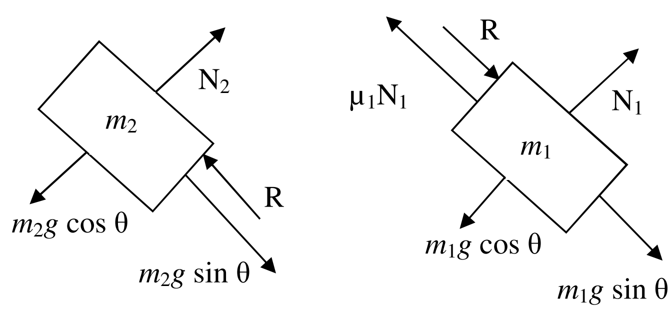
4. There are two objects of masses m_1 and m_2 that are touching each other on an inclined plane as shown in the figure. The coefficient of friction between m_1 and the inclined plane is μ_1 while there is no friction between m_2 and the inclined plane.

   a) Draw the free body diagram of the objects.
   b) Find the interaction force between the two objects.
   c) Find the minimum angle θ at which the objects start sliding.
   d) If there is a friction between m_2 and the inclined plane, how would it affect the interaction force and the acceleration of the objects? Explain without giving numerical answer to this question.



(25 marks)

a)



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b) $N_1 = m_1 g\cos\theta$

$F_{r1} = \mu_1 N_1 = \mu_1 m_1 g\cos\theta$

Let 'a' be the acceleration of the system and 'R' be the interaction force.

$m_1 g\sin\theta + R - \mu_1 m_1 g\cos\theta = m_1 a$ (1)

$N_2 = m_2 g\cos\theta$

$m_2 g\sin\theta - R = m_2 a$ (2)

Multiply (1) with m_2 and (2) with m_1 we get

$m_2 m_1 g\sin\theta + Rm_2 - \mu_1 m_1 m_2 g\cos\theta = m_1 m_2 a$ (3)

$m_1 m_2 g\sin\theta - Rm_1 = m_1 m_2 a$ (4)

$(3)-(4)$ yields

$$R = \frac{\mu_1 m_1 m_2 g\cos\theta}{(m_1 + m_2)}$$

c) Let 'α' be the minimum angle when the objects slide. This will happen when a is zero. Therefore in (1) we substitute $a = 0$

$m_1 g\sin\alpha + R - \mu_1 m_1 g\cos\alpha = 0$

Substituting $R = \dfrac{\mu_1 m_1 m_2 g\cos\alpha}{(m_1 + m_2)}$ we get

$$\tan\alpha = \frac{m_1\mu_1}{(m_1 + m_2)}$$

$$\alpha = \tan^{-1}\left(\frac{m_1\mu_1}{(m_1 + m_2)}\right)$$

d) The interaction force will reduce because of the presence of friction which will decrease the acceleration of the system.

***** End of Paper *****

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Formula sheet

Name: ________________________ Admin. No.: _______________ Seat No.: ________

Kinematics

$v_x = v_{0x} + a_x t$

$v_x{}^2 = v_{0x}{}^2 + 2a_x(x - x_0)$

$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

$\vec{v} = \frac{d\vec{r}}{dt}, \quad \vec{a} = \frac{d\vec{v}}{dt}$

$y = (\tan\theta)x - (\frac{g}{2v^2\cos^2\theta})x^2$

$R = \frac{v^2 \sin 2\theta}{g}$

Dynamics

$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a},\ F = \mu N$

$a = \frac{dv}{dt},\ a = \frac{v^2}{r}, F = m\frac{v^2}{r}$

$\vec{J} = \int \vec{F}dt = \Delta\vec{p}$

$W = \int \vec{F}.\vec{dr},\ W_{net} = K_f - K_i$

$KE = \frac{1}{2}mv^2,\ PE = mgh$

$P = \frac{W}{t},\ \ P = \frac{dW}{dt}$

Linear momentum

$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$

Static electricity

$F = k\frac{q_1 q_2}{r^2},\ k = \frac{1}{4\pi\varepsilon_o}$

$F = qE$

$V = k\frac{q}{r}, U = qV$

$\Phi_E = \oint \vec{E}.d\vec{A} = \frac{q}{\varepsilon_o}$

$V = Ed, W = qV, E = \frac{kq}{r^2}$

Current electricity

$Q = It \quad V = IR$

$P = VI = I^2R = \frac{V^2}{R}$

Magnetism & electromagnetism

$\vec{F} = q\vec{v}\times\vec{B} \quad \vec{F} = i\vec{L}\times\vec{B}$

$e.m.f. = -N\frac{d\Phi_B}{dt} \quad \Phi_B = BA$

Thermodynamics

$\Delta U = Q - W$

$W = \int p\,dV$

$Q_V = nC_V\Delta T$ const vol

$Q_p = nC_p\Delta T$ const pressure

$Q = mC\Delta T$

$Q = mL$

Ideal Gas

$pV = nRT$

$pV^\gamma = c$ (adiabatic)

$\gamma = \frac{C_p}{C_V}, C_p - C_v = R$

$W = pV\,ln\frac{V_2}{V_1} = nRT\,ln\frac{V_2}{V_1}$

$W = \frac{1}{\gamma - 1}(p_1V_1 - p_2V_2)$

Rotational Motion

$\omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt}$

$\omega = \omega_0 + \alpha t$

$\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$I = \sum_i^n m_i r_i^2,\ I = \int r^2 dm,\ K = \frac{1}{2}I\omega^2$

SHM & waves

$T = \frac{1}{f} \quad v = f\lambda \quad \omega = 2\pi f$

$\omega = \frac{2\pi}{T} \quad k = \frac{2\pi}{\lambda}$

$\omega = \sqrt{k/m} \quad \omega = \sqrt{g/L}$

$x = A\cos(\omega t + \phi)$

$x = A\sin(\omega t + \phi)$

$y(x,t) = A\cos(\omega t \pm kx)$

$y(x,t) = A\sin(\omega t \pm kx)$

$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

Circuits

$R = R_1 + R_2 + R_3 + \ldots$ series

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$ parallel

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$ series

$C = C_1 + C_2 + C_3 + \ldots$ parallel

$Q = CV \qquad U = \frac{1}{2}CV^2$

Constants

Charge on electron
$e = -1.60\times10^{-19}$ C

Coulomb's constant
$k = 9\times10^9$ N m^2 C^{-2}

Ideal gas constant
$R = 8.314$ J mol^{-1} K^{-1}

Mass of proton
$m_p = 1.67\times10^{-27}$ kg

Mass of electron
$m_e = 9.11\times10^{-31}$ kg

Permeability of free space
$\mu_o = 4\pi\times10^{-7}$ N A^{-2}

Permittivity of free space
$\varepsilon_o = 8.85\times10^{-12}$ C^2 N^{-1} m^{-2}

Speed of light in vacuum
$c = 3.00\times10^8$ m s^{-1}

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