3. (a)

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$$y = A\sin(kx - \omega t)$$

$$\frac{\partial y}{\partial x} = Ak\cos(kx - \omega t)$$

$$\frac{\partial^2 y}{\partial x^2} = -Ak^2\sin(kx - \omega t) = \text{LHS}$$

$$\frac{\partial y}{\partial t} = -A\omega\cos(kx - \omega t)$$

$$\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = +A\omega^2\frac{1}{v^2}\sin(kx - \omega t) = \text{RHS}$$

But

$$k = \frac{\omega}{v}$$

$$\text{LHS} = \text{RHS}$$

(b) Amplitude = 2.00 cm

$$k = \frac{2\pi}{\lambda}$$

$$2.11 = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{2\pi}{2.11} = 2.99 \text{ m}$$

$$\omega = \frac{2\pi}{f}$$

$$3.62 = 2\pi f$$

$$f = \frac{3.62}{2\pi} = 0.576 \text{ Hz}$$

$$c = f\lambda = 0.576 \times 2.99 = 1.72 \text{ m/s}$$

4. For transverse waves on a string, $v = \sqrt{F/\mu}$. The general form of the equation for waves traveling in the +x-direction is $y(x,t) = A\cos(kx - \omega t)$. For waves traveling in the −x-direction it is $y(x,t) = A\cos(kx + \omega t)$. Comparison to the general equation gives A = 8.50 mm, k =172 rad/m and ω = 2730 rad/s.

(a) $v = \omega / k = 2730/172 = 15.9$ m/s.
$t = L/v = 1.5 / 15.9 = 0.094$ s.

(b) $\lambda = 2\pi/k = 0.0365$ m. The number of wavelengths along the length of the string is $L/\lambda \sim 41$

(c) For a wave traveling in the opposite direction,
y(x,t) = (8.50 mm)cos([172 rad/m]x +[2730 rad/s]t)

5. We take the wave function to be $D(x,t) = A\sin(kx - \omega t)$. The wave speed is given by $v = \frac{\omega}{k} = \frac{\lambda}{f}$,

while the speed of particles on the cord is given by $\frac{\partial D}{\partial t}$.

$$\frac{\partial D}{\partial t} = -\omega A\cos(kx - \omega t) \quad \rightarrow \quad \left(\frac{\partial D}{\partial t}\right)_{\max} = \omega A$$

$$\omega A = v = \frac{\omega}{k} \quad \rightarrow \quad A = \frac{1}{k} = \frac{\lambda}{2\pi} = \frac{10.0\,\text{cm}}{2\pi} = 1.59\,\text{cm}$$

Answers

1. a) k = 0.157 rad/cm, ω = 50.3 rad/s, T = 0.125 s, v = 320 cm/s.
   b) y = 15.0 cos (0.157x – 50.3t)
2. a) ω = 31.4 rad/s, k = 1.57 rad/m, y = 0.012 sin (1.57x – 31.4t)
   b) 3.77 m/s, 118 m/s^2