PHYSICS

1. A solenoid having 70 turns per cm current flowing in solenoid is 2 amp. Find magnetic field inside the solenoid.

(1) $860\ \pi \times 10^{-4}$ T (2) $560\ \pi \times 10^{-4}$ T
(3) $280\ \pi \times 10^{-4}$ T (4) $360\ \pi \times 10^{-4}$ T

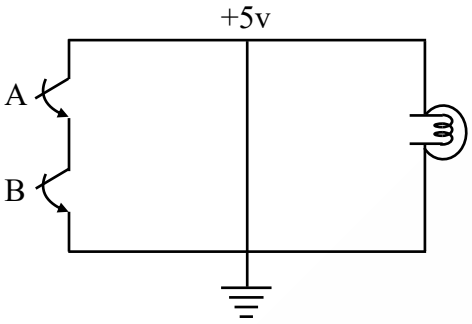
Ans. **(2)**

Sol. $B = \mu_0 \pi I$

$$B = 4\pi \times 10^{-7} \times \frac{70}{10^{-2}} \times 2$$

$$B = 560\pi \times 10^{-4}\ T$$

2.



(1) NAND (2) NOR (3) OR (4) AND

Ans. **(1)**

Sol.

A	B	Output
0	0	1
1	0	1
0	1	1
1	1	0

3. Separation between earth and sun is given by 1.5×10^6 km. Time period of another planet is 2.83 year. Find distance of another planet from sun?

(1) 3×10^6 km (2) 2×10^7 km (3) 3×10^7 km (4) 2×10^6 km

Ans. **(1)**

Sol. $T^2 \propto R^3$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$$

$$\left(\frac{1}{2.83}\right)^2 = \left(\frac{1.5\times10^6}{R_2}\right)^3$$

$R_2 = (1.5 \times 10^6)\ (2.83)^{2/3}$ km

$= (1.5 \times 10^6)\ (8)^{1/3}$

$= 3 \times 10^6$ km

4. Choose the correct options based on the column shown below.

1. TV signal				(P) 12 GHz
2. Satellite				(Q) 30 MHz
3. AM				(R) 88 MHz
4. FM				(S) 1 MHz
	1	2	3	4
(1)	P	Q	R	S
(2)	Q	P	S	R
(3)	S	Q	R	P
(4)	P	Q	S	R

Ans. **(2)**

5. If two vectors $\dot{P} = \hat{i} + 2m\hat{j} + m\hat{k}$ & $\dot{Q} = 4\hat{i} - 2\hat{j} + m\hat{k}$ are perpendicular to each other, then find value of m.

(1) m = 3 (2) m = 2 (3) m = 8 (4) m = 1

Ans. **(2)**

Sol. $\dot{P} \cdot \dot{Q} = 0$

$(\hat{i} + 2m\hat{j} + m\hat{k}).(4\hat{i} - 2\hat{j} + m\hat{k}) = 0$

$4 - 4m + m^2 = 0$

$m^2 - 2m - 2m + 4 = 0$

$m(m - 2) - 2(m - 2) = 0$

$m = 2$

6. A photon is emitted from n = 4 to n = 1 level in hydrogen atom the corresponding wavelength for this transfer will be [hc = 1240 nm eV].

(1) 88.2 nm (2) 121.7 nm (3) 102.5 nm (4) 97.3 nm

Ans. **(4)**

Sol. $\Delta E = \frac{hc}{\lambda}$

$l = \frac{hc}{\Delta E_{4-1}} = \frac{1240\text{nm eV}}{12.75\text{ eV}} = 97.3\text{ nm}$

7. When ${}_{Z}X^{240}$ nucleus goes for fission, energy released is 200 MeV. Total energy released when 120g of this sample is _____ 10^{25} MeV.

Ans. **6**

Sol. $n_A = \frac{120}{240} = \frac{1}{2}$

$E_{total} = \frac{1}{2} \times 6.02 \times 10^{23} \times 200 MeV = 6 \times 10^{25}$ MeV

8. In an electromagnetic wave electric field and magnetic field is given by

$E = E_0 \sin(kx - \omega t + \phi)$

$B = B_0 \sin(kx - \omega t + \phi)$

Find correct relation.

(1) $\frac{\omega}{k} = \frac{E_0}{B_0}$ (2) $\frac{k}{\omega} = \frac{E_0}{B_0}$

(3) $\frac{\omega}{k} = B_0$ (4) $\omega k = E_0 B_0$

Ans. **(1)**

Sol. $E_0 = B_0 C$

Speed of light $C = \frac{\omega}{k}$

$\frac{E_0}{B_0} = \frac{\omega}{k}$

9. If all the particles have same kinetic energy, The relation between the wavelengths of alpha particle, electron and proton is :

(1) $\lambda_\rho > \lambda_\alpha > \lambda_e$ (2) $\lambda_e > \lambda_\rho > \lambda_\alpha$

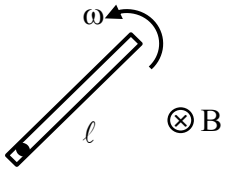
(3) $\lambda_\alpha > \lambda_e > \lambda_\rho$ (4) $\lambda_\alpha > \lambda_\rho > \lambda_e$

Ans. **(2)**

$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mk}}$

$\therefore \quad \mu_e < m_\rho < m_\alpha \quad \therefore \quad \lambda_e > \lambda_\rho > \lambda_\alpha$

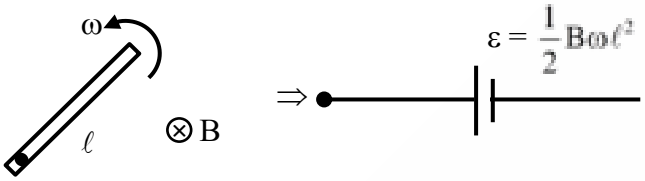
10. A rod of length ℓ is rotating in a uniform magnetic field as shown in figure. Then induced e.m.f across its ends is.



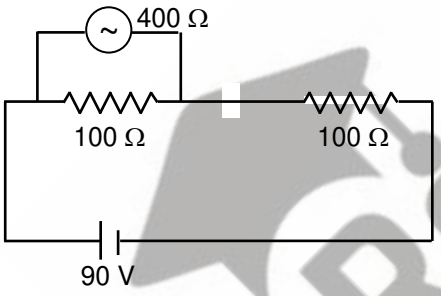
(1) $B\omega\ell^2$ (2) $\frac{B\omega\ell^2}{2}$ (3) $\frac{B\omega\ell^2}{4}$ (4) $\frac{B\omega\ell^2}{8}$

Ans. (2)

Sol.

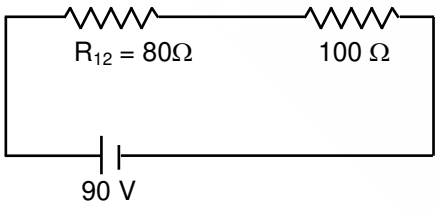


11. Find reading of voltmeter ?



Ans. 40

Sol.



$$\frac{1}{R_{12}} = \frac{1}{100} + \frac{1}{400} = \frac{5}{400}$$

$$R_{12} = 80$$

$$V_{12} = 90 \times \frac{800}{(80+100)} = \frac{90\times 80}{180} = 40V$$

12. When a parallel beam of white light incident on convex lens split into different colours the phenomenon is called.

(1) Spherical aberration (2) Chromatic aberration

(3) Polarization (4) Diffraction

Ans. (2)

13. If frequency can be represented as f = (radius)^a (density)^b (surface tension)^c. Find a, b, c?

(1) $a=\frac{3}{2},\ b=\frac{1}{2},\quad c=\frac{-1}{2}$ (2) $a=\frac{-3}{2},\ b=\frac{-1}{2},\quad c=\frac{1}{2}$

(3) $a=\frac{-3}{2},\ b=\frac{1}{2},\quad c=\frac{-1}{2}$ (4) $a=\frac{1}{2},\ b=\frac{3}{2},\quad c=\frac{-1}{2}$

Ans. **(2)**

Sol. $M^0L^0T^{-1} = L^a (ML^{-3})^b (MT^{-2})^c$

$M^0L^0T^{-1} = L^a M^bL^{-3b} M^cT^{-2c}$

Equivalent the power of MLT

$M \Rightarrow \quad 0 = b + c$

$L \Rightarrow \quad 0 = a - 3b$

$T \Rightarrow \quad -1 = -2c$

$a=\frac{-3}{2},\ b=\frac{-1}{2},\quad c=\frac{1}{2}$

14. A dielectric of 3.5 is inserted and the distance between the plates is doubled. Find new capacitance, if original capacitance was 7.5 pF?

Ans. **13.33**

Sol. $C' = \frac{K\varepsilon_0 A'}{d'} = \frac{7}{2} \times \frac{\varepsilon_0 A}{2d} = \frac{7}{4} \times \frac{15}{2} = \frac{105}{8}$ pF

15. Statement-I : If we move upward and downward from the surface of earth surface acceleration due to gravity decreases in both upward and downward direction.

Statement-II : Acceleration due to gravity changes by same amount when we go up to height h and depth d when h = d.

Choose the correct options based on above statements.

(1) Both statement-I and Statement-II are true.

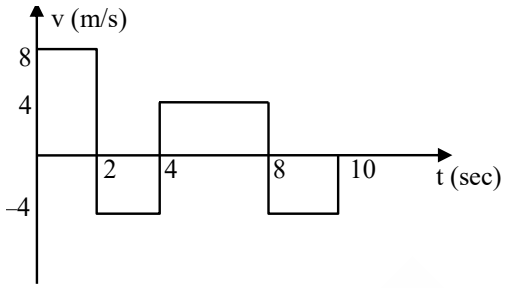
(2) Statement-I is true and Statement-II is false.

(3) Statement-I is false and Statement-II are true.

(4) Both statement-I and Statement-II are false.

Ans. **(2)**

16. A particle follows the above V - t graph, then the ratio of distance travelled and displacement of particle is given by :



(1) 3 : 1 (2) 1 : 3 (3) 2 : 3 (4) 3 : 2

Ans. **(1)**

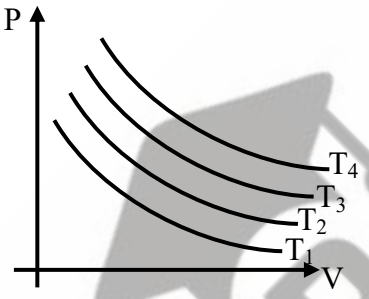
Sol. Distance = 16 + 8 + 16 + 8 = 48 m

Displacement = 16 + 16 – 8 – 8 = 16 m

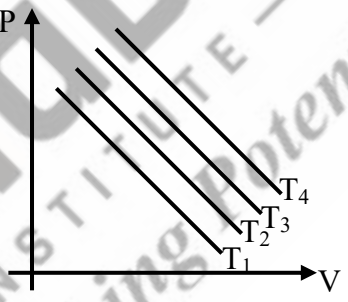
Ratio = $\frac{48}{16} = 3$

17. For an Isothermal expansion of an ideal gas in a closed container at different temperature P-V graph is given. Then choose the correct graph where $T_1 > T_2 > T_3 > T_4$.

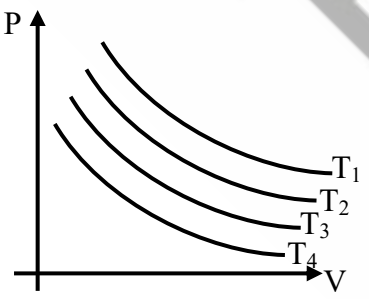
(1)



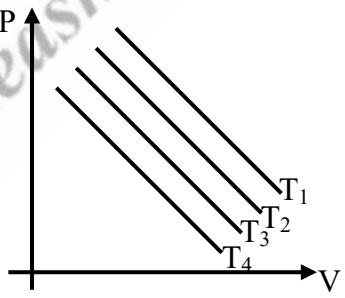
(2)



(3)



(4)



Ans. **(3)**

Sol. PV = C ; C = constant

If temperature will increase then C will increase.

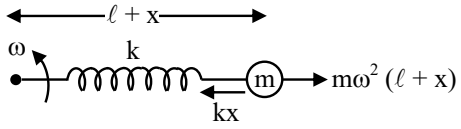
P = $\frac{C}{V}$ $\rightarrow$ rectangular hyperbola

18. A block of mass 200 gm is connected with a spring of spring constant 12.5 N/m. It is rotating in horizontal plane with angular speed 5 rad/sec. Find ratio of elongation in spring and natural length?

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) $\frac{1}{3}$ (4) $\frac{1}{2}$

Ans. (1)

Sol.



$kx = m\omega^2 (\ell + x)$

$(k - m\omega^2)x = m\omega^2 \ell$

$$\frac{x}{\ell} = \frac{m\omega^2}{k - m\omega^2} = \frac{0.2 \times 25}{\frac{25}{2} - 0.2 \times 25}$$

$$\frac{x}{\ell} = \frac{2}{3}$$

19. A wire is extended by 20% keeping its volume is constant. Find the percentage change in its resistance.

Ans. 44

Sol. $R = \frac{\rho\ell}{A} = \frac{\rho\ell}{V/\ell} = \frac{\rho\ell^2}{V} \propto \ell^2$

$\ell \rightarrow 1.2\ \ell$

$$\frac{\Delta R}{R} = \frac{1.44R - R}{R} \times 100\% = 44\%$$

20. S-1 $\rightarrow$ Steel is used in construction of a bridge and house.

S–2 $\rightarrow$ Modulus of elasticity of steel is high.

(1) S-1 & S-2 both are true

(2) S-1 is true & S-2 is false

(3) S-1 is false & S-2 is true

(4) S-1 & S-2 both are false

Ans. (1)

21. A lens of refractive index 1.5 and focal length 18 cm in air is submerged in water change in focal length of lens is ($\mu_w = \frac{4}{3}$)

Ans. 54

Sol. $\frac{1}{18} = (1.5-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ....(1)

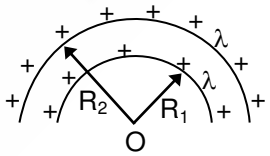
$\frac{1}{f} = \left(\frac{1.5}{\frac{4}{3}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ....(2)

$\frac{\text{Eq(1)}}{\text{Eq(2)}}$: $\frac{f}{18} = \frac{1.5-1}{\frac{9}{8}-1} = \frac{1/2}{1/8}$

$f = 18 \times 4 = 72$ cm

change in focal length = 72 – 18 = 54 cm

22. Two semicircular arcs of linear charge density λ are placed as shown in figure. Find the potential at the point O.



(1) $\frac{2\lambda}{\varepsilon_0}$ (2) $\frac{\lambda}{\varepsilon_0}$ (3) $\frac{\lambda}{2\varepsilon_0}$ (4) $\frac{3\lambda}{\varepsilon_0}$

Ans. (3)

Sol. $\frac{K[\lambda(\pi R_1)]}{R_1} + \frac{K\lambda(\pi R_2)}{R_2} = 2k\lambda\pi = \frac{\lambda}{2\varepsilon_0}$

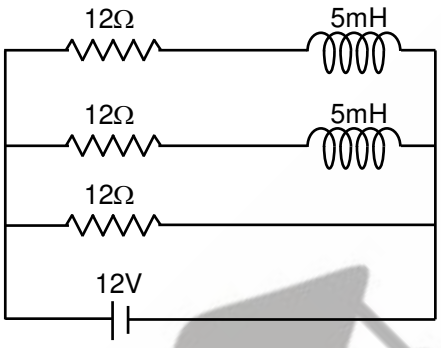
23. Ratio of molar heat capacity at constant pressure and at constant volume for monoatomic and diatomic gas is?

(1) 25 : 21 (2) 21 : 25 (3) 16 : 25 (4) 25 : 16

Ans. (1)

Sol. $\frac{\frac{5}{3}}{\frac{7}{5}} \Rightarrow \frac{5}{3}\times\frac{5}{7} = \frac{25}{21}$

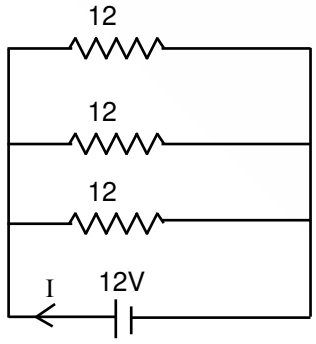
24.



Current through the battery after long time is:

Ans 3

Sol.



After long time

$R_{eq} = \frac{12}{3} = 4\Omega$

$$I = \frac{V}{R_{eq.}} = \frac{12}{4} = 3A$$

25. A solid cylinder of radius R and length L have moment of inertia I_1 and a second solid cylinder of radius $\frac{R}{2}$ and length $\frac{L}{2}$ cut from it have moment of inertia I_2. Find $\frac{I_1}{I_2}$.

(1) 64 (2) 32 (3) 128 (4) 256

Ans. (2)

Sol. $I_1 = M\left(\frac{R^2}{4} + \frac{L^2}{12}\right)$

$I_1 = \frac{M}{4}\left(R^2 + \frac{L^2}{3}\right)$

$M = \rho\pi R^2 L$

$M_2 = \rho\pi\frac{R^2}{8}L = \frac{M}{8}$

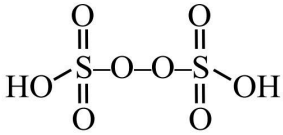
$I_2 = \frac{M}{8} \times \frac{1}{4}\left[\frac{R^2}{4} + \frac{L^2}{12}\right]$

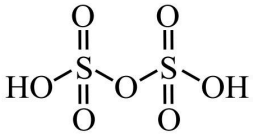
$= \frac{M}{128}\left(R^2 + \frac{L^2}{3}\right)$

CHEMISTRY

1. Sum of π-bonds in one molecule each of Peroxydisulphuric acid & Pyrosulphuric acid is:

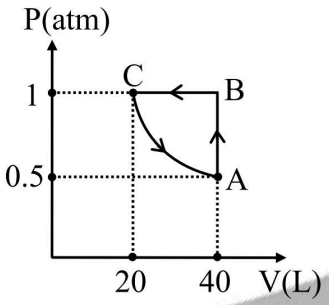
Ans. **8** **(Chemical Bonding)**

Sol. Peroxydisulphuric acid  π-bonds = 4

Pyrosulphuric acid  π-bonds = 4

Sum = 4 + 4 = 8

2.



1 mole of ideal gas undergoes above cyclic process.

Value of work done (in J) is : ($\ell n2 = 0.7$)

Ans. **608** **(Thermodynamics)**

Sol. $W = W_{AB} + W_{BC} + W_{CA}$

$= 0 - 1(20 - 40) + \left[-20\ell n\left(\frac{40}{20}\right)\right]$

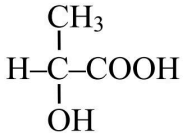
$= 20 - 20\ \ell n2$

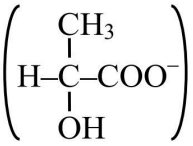
$= 20\ (1 - 0.7)$

$= 6$ L-atm

$= 6 \times 101.3$

$= 607.8\ J \approx 608\ J$

3.  (Lactic acid) has $K_a = 10^{-5}$

pH of a solution containing 0.005M anionic form of above acid  is :

(Nearest integer)

Ans. **8** **(Ionic Equilibrium)**

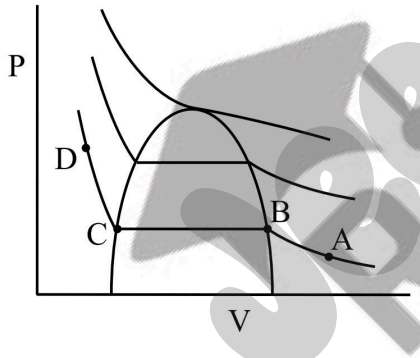
Sol. Salt of WA & SB

$$pH = \frac{1}{2}(pK_w + pK_a + \log C)$$

$$= \frac{1}{2}(14 + 5 - 3 + \log 5)$$

$$= 8.35 \approx 8$$

4. Which of the following statements are correct for given Andrew isotherm of CO_2



(i) Formation of liquid starts at point C.

(ii) From point B to C amount of liquid decreases.

(iii) Formation of liquid starts from point B.

(iv) At points B & C, both liquid & vapour coexist.

(1) i, ii (2) ii, iii (3) iii, iv (4) i, iv

Ans. **(3)** **(Real gas)**

Sol. (i) Formation of liquid ends at point C.

(ii) From B to C, amount of liquid increases.

5. Which of the following are concentration terms.

Mole, Mass%, Molality, Molarity, Mole fraction, ppm.

Ans. **5** **(Mole Concept)**

Sol. All other than mole.

6. Unipositive ion of an atom containing 55 protons contains how many s electrons?

Ans. **10** **(Atomic Structure)**

Sol. $_{55}Cs^{+}$: $1s^2 2s^2 2p^6 3s^2 3p^6\ 4s^2 3d^{10} 4p^6 5s^2 4d^{10} 5p^6$

Number of s-electrons = 2 + 2 + 2 + 2 + 2 = 10

7. $[Co(NH_3)_6]^{3+}$ is _____ hybridised and _____.

(1) d^2sp^3, Diamagnetic (2) d^2sp^3, Paramagnetic

(3) sp^3d^2, Diamagnetic (4) sp^3d^2, Paramagnetic

Ans. **1** **(Coordination Compounds)**

Sol. $Co^{3+}(3d^6)$ + SFL(CN = 6)

$\Rightarrow t_{2g}^{222}e_g^{00} \Rightarrow d^2sp^3$ and Diamagnetic

8. The metal which is extracted by oxidation and subsequent reduction from its ore is:

(1) Au (2) Cu (3) Fe (4) Al

Ans. **(1)** **(Metallurgy)**

Sol. $Au \xrightarrow[\substack{+O_2\\(\text{Oxidation})}]{NaCN} [Au(CN)_2]^- \xrightarrow[(\text{Reduction})]{Zn} Au\downarrow$

9. How many statement/statements is/are correct for physisorption?

(i) physisorption is highly specific in nature.

(ii) physisorption is monolayer in nature.

(iii) physisorption has zero activation energy

(iv) physisorption decreases with increasing temperature.

(v) physisorption has high $\Delta H_{Adsorption}$

Ans. **2 (iii, iv)** **(Surface Chemistry)**

Sol. (i) physisorption is less specific in nature.
(ii) physisorption is multimolecular layer
(iii) physisorption has low $\Delta H_{Adsorption}$

10. An ideal solution containing $X_A = 0.7$ has VP = 350 torr

Another ideal solution containing $X_B = 0.2$ has VP = 410 torr

P_A^o = ? (nearest integer)

Ans. **314** **(Solution & Colligative Properties)**

Sol. $0.7\,P_A^o + 0.3\,P_B^o = 350$

& $0.2\,P_A^o + 0.8\,P_B^o = 410$

$\therefore\ P_A^o = 314$ torr

11. H_2O_2 behave like reducing agent in which of the following reactions :

(1) $Fe^{+2} + H_2O_2 \longrightarrow Fe^{+3} + H_2O$

(2) $H_2S + H_2O_2 \longrightarrow SO_4^{2-} + H_2O$

(3) $HOCl + H_2O_2 \longrightarrow Cl^- + 2H_2O + O_2$

(4) $Mn^{+2} + H_2O_2 \longrightarrow MnO_2 + H_2O$

Ans. **(3)** **(p-Block (15-16 family))**

Sol. H_2O_2 reduces HOCl to Cl^- and itself gets oxidised to O_2.

12. $AB_3(g)$ dissociates into gaseous products with following data:

$t_{1/2}$	4 sec.	2 sec.	1 sec.	0.5 sec.
P_o (AB_3)	50 torr	100 torr	200 torr	400 torr

Order of reaction is

Ans. **2** **(Chemical Kinetics)**

Sol. $t_{1/2} \propto \frac{1}{P_o} \Rightarrow$ II order

13. Number of unpaired electron in highest occupied molecular orbital of following species is :

	N_2	$N_2^{\oplus}$	O_2	$O_2^{\oplus}$
(1)	0	1	2	1
(2)	1	0	1	2
(3)	2	2	0	2
(4)	1	1	1	0

Ans. **(1)** **(Chemical Bonding)**

Sol. $N_2 \rightarrow \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \underset{\text{HOMO}}{\left[\pi 2p_x^2 = \pi 2p_y^2\right] \sigma 2p_z^2}$

$N_2^{\oplus} \rightarrow \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \underset{\text{HOMO}}{\left[\pi 2p_x^2 = \pi 2p_y^2\right] \sigma 2p_z^1}$

$O_2 \rightarrow \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \left[\pi 2p_x^2 = \pi 2p_y^2\right] \underbrace{\left[\pi^* 2p_x^1 = \pi^* 2p_y^1\right]}_{\text{HOMO}}$

$O_2^{\oplus} \rightarrow \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \left[\pi 2p_x^2 = \pi 2p_y^2\right] \underbrace{\left[\pi^* 2p_x^1 = \pi^* 2p_y^0\right]}_{\text{HOMO}}$

14. Which is good oxidising agent ?

(i) Sm^{+2} (ii) Ce^{+2} (iii) Ce^{+4} (iv) Tb^{+4}

(1) Sm^{+2} only (2) Ce^{4+}, Tb^{4+} (3) Ce^{+4} only (4) Ce^{2+} only

Ans. **(2)** **(f-Block)**

Sol. Ce^{4+} & Tb^{4+} are good oxidising agents (both get reduced to +3).

15. $K_2Cr_2O_7$ paper acidified with dil. H_2SO_4 turns green when exposed to :

(1) SO_2 (2) SO_3 (3) CO_2 (4) H_2S

Ans. **(1)** **(d-Block)**

Sol. $SO_2 \xrightarrow[H^+]{K_2Cr_2O_7} \underset{\text{(green)}}{Cr^{3+}} + SO_4^{2-}$

16. α-particle, proton & electron have same kinetic energy. Select correct order of their de-Broglie wavelength.

(1) $\lambda_e > \lambda_p > \lambda_\alpha$ (2) $\lambda_\alpha > \lambda_e > \lambda_p$ (3) $\lambda_p = \lambda_\alpha = \lambda_e$ (4) $\lambda_p > \lambda_e > \lambda_\alpha$

Ans. **(1)** **(Atomic Structure)**

Sol. $\lambda = \dfrac{h}{m \cdot v} = \dfrac{h}{\sqrt{2 \cdot m \cdot K.E.}}$

as K.E. is same $\Rightarrow \lambda \propto \dfrac{1}{\sqrt{m}}$

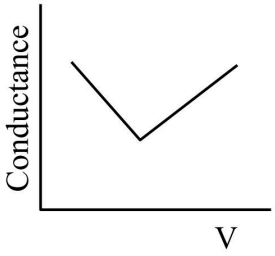
mass of electron = 9.1×10^{-31} kg

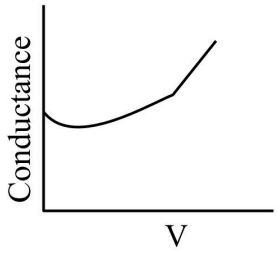
mass of proton = 1.67×10^{-27} kg

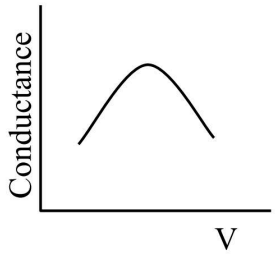
mass of α-particle = 6.68×10^{-27} kg

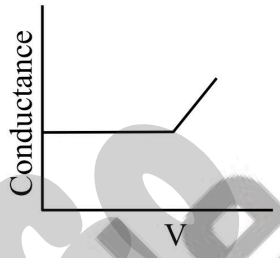
$\Rightarrow \lambda_e > \lambda_p > \lambda_\alpha$

17. Which of the following is correct graph for conductometric titration between benzoic acid & NaOH ?

(1) 

(2) 

(3) 

(4) 

Ans. **(2)** **(Electrochemistry)**

18. S_1 : Be^{+2} has higher SRP than other alkaline earth metals.

S_2 : Be^{+2} has higher hydration energy and greater Δ_aH (atomisation enthalpy) than other alkaline earth metals.

(1) Both S_1 & S_2 are true

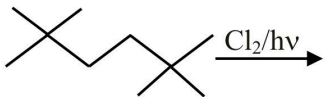
(2) S_1 is true ; S_2 is false

(3) S_1 is false ; S_2 is true

(4) Both S_1 & S_2 are false

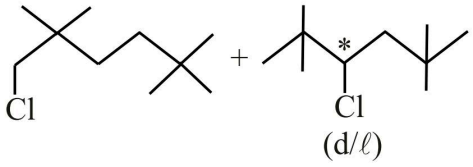
Ans. **(1)** **(s-Block)**

Sol. Be has least –ve SRP value because of high Δ_aH (atomisation enthalpy), inspite of maximum hydration energy.

19. 

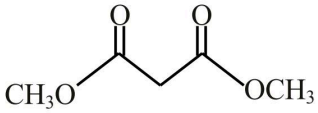
All possible monochloro products **[Hydrocarbons]**

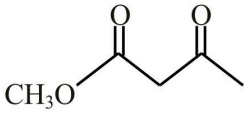
Ans. **(3)**

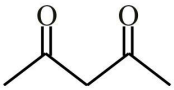
Sol. 

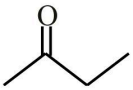
(d/ℓ)

20. Which of the following is most easily deprotonated ?

(a) 

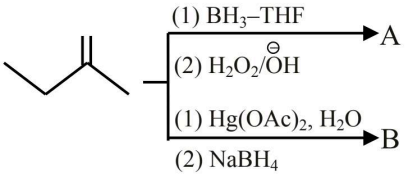
(b) 

(c) 

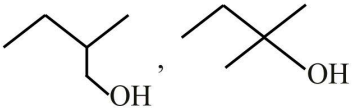
(d) 

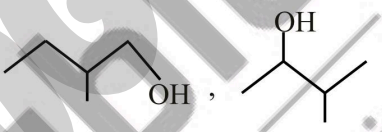
(1) a (2) b (3) c (4) d

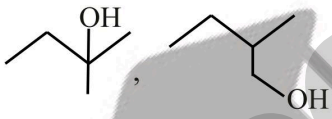
Ans. **(3)** **[GOC-2]**

21. 

A & B are respectively

(1) 

(2) 

(3) 

(4) 

Ans. **(1)** **[Hydrocarbons]**

22. Average human being requires nearly ___ times more air than the food

(1) 12–15 (2) 100 (3) 40-50 (4) 75

Ans. **(1)** **[Environmental]**

23. **Statement-I :** Aniline and other aryl amines are usually colourless

Statement-II : Aniline and other arylamines get coloured on storage due to atmospheric oxidation

(1) Both Statement-I and Statement-II are correct.

(2) Both Statement-I and Statement-II are incorrect.

(3) Statement-I is correct but Statement-II is incorrect.

(4) Statement-I is incorrect but Statement-II is correct.

Ans. **(1)** **[Aromatic compound]**

Sol. Both are correct

24. **Assertion (A) :** Benzene is more stable than hypothetical cyclohexatriene

Reason (R): The delocalised π-electrons cloud is attracted more strongly by the nuclei of the carbon atoms than the electron cloud localised between two carbon atoms.

(1) Both **(A)** and **(R)** are true but **(R)** is not the true explanation of **(A)**

(2) **(A)** is false but **(R)** is true.

(3) **(A)** is true but **(R)** is false

(4) Both **(A)** and **(R)** are true and **(R)** is the true explanation of **(A)**

Ans. **(4)** **[Hydrocarbon]**

25. Match the column

(P) Antifertility drugs	(A) Norethindrone
(Q) Anti histamines	(B) Seldane
(R) Tranquilizers	(C) Meprobamate
(S) Antibiotics	(D) Penicillin

(1) P → (A), Q → (B), R → (C), S → (D)

(2) P → (A), Q → (C), R → (B), S → (D)

(3) P → (D), Q → (C), R → (B), S → (A)

(4) P → (A), Q → (D), R → (B), S → (C)

Ans. **(1)** **[Chemistry in every day life]**

26. How many tripeptides can be formed from the amino acid valine and proline?

Ans. **8** **[Biomolecules]**

MATHEMATICS

SECTION-A

1. Find number of numbers greater then 7000 which can be formed by using the digits 3, 5, 6, 7 and 8. Repetition of digits is not allowed.

(1) 68 (2) 168 (3) 120 (4) 172

Ans. **(2)**

Sol. Number of digit number

7				$4 \times 3 \times 2 = 24$

8				$4 \times 3 \times 2 = 24$

Number of 5 digit number

					$5! = 120$

$\therefore$ Total number of numbers = 24 + 24 + 120 = 168

2. $\int\limits_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}\,dx$ is equal to-

(1) 2π (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{3}$ (4) π

Ans. **(1)**

Sol. $\int\limits_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{24}{\sqrt{\frac{9}{4}-x^2}} = 24.\sin^{-1}\frac{2x}{3}\Big|_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} = 24\left(\sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}\frac{1}{\sqrt{2}}\right) = 2\pi$

3. If system of equation $x + 2y = 6$, $x - 3y + 72z = 0$, $x + y + \lambda z = \mu + 9$ has infinite solution then ordered pair (λ, μ) is

(1) $\left(\frac{72}{5}, \frac{-21}{5}\right)$ (2) $\left(\frac{21}{5}, \frac{-72}{5}\right)$ (3) $\left(\frac{-21}{5}, \frac{72}{5}\right)$ (4) $\left(\frac{-21}{5}, \frac{-72}{5}\right)$

Ans. **(1)**

Sol. $\begin{vmatrix} 1 & 2 & 0 \\ 1 & -3 & 72 \\ 1 & 1 & \lambda \end{vmatrix} = 0 \qquad \Rightarrow \lambda = \frac{72}{5}$

$$\Delta_x = \begin{vmatrix} 6 & 2 & 0 \\ 0 & -3 & 72 \\ \mu+9 & 1 & \lambda \end{vmatrix} = 0$$

solving $\mu = -\frac{21}{5}$

4. Consider a 3 × 3 matrix P such that $|\text{adj (adj (adj P))}| = (12)^4$, then find $|P^{-1} \text{ . adj } P|$

(1) $2\sqrt{3}$ (2) $\sqrt{3}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\frac{1}{\sqrt{3}}$

Ans. (1)

Sol. $|P|^{2^3} = 12^4 \Rightarrow |P|^8 = 12^4 \Rightarrow |P| = 12^{\frac{1}{2}} = 2\sqrt{3}$

$|P^{-1} \text{ adj } P| = |P^{-1}|\,|\text{adj } P| = \frac{1}{|P|} \times |P|^2 = |P| = 2\sqrt{3}$

5. Let $f(x) = \frac{2^{2x}}{2^{2x}+2}$, then $\sum_{r=1}^{2022} f\left(\frac{r}{2023}\right)$ is

(1) 1010 (2) $\frac{2023}{2}$ (3) 1011 (4) $\frac{2021}{2}$

Ans. **(3)**

Sol. $f(x) = \frac{4^x}{4^x+2} \Rightarrow f(x) + f(1-x) = 1$

$$\therefore \sum_{r=1}^{2022} f\left(\frac{r}{2023}\right) = \left[f\left(\frac{1}{2023}\right)+f\left(\frac{2022}{2023}\right)\right] + \left[f\left(\frac{2}{2023}\right)+f\left(\frac{2021}{2023}\right)\right] + ...$$

$$.....+ \left[f\left(\frac{1011}{2023}\right)+f\left(\frac{1012}{2023}\right)\right] = 1011$$

6. If $\frac{dy}{dx} = \frac{3y^2-x^2}{3xy}$, $y(1) = 1$, find $6y^2(e)$

(1) e^2 (2) $\frac{e^2}{2}$ (3) $\frac{e^2}{3}$ (4) $3e^2$

Ans. (3)

Sol. $y = mx \Rightarrow \frac{dy}{dx} = m + x\frac{dm}{dx}$

$$m + x\frac{dm}{dx} = \frac{3m^2x^2-x^2}{3mx^2} = \frac{3m^2-1}{3m}$$

$$x\frac{dm}{dx} = \frac{3m^2-1-3m^2}{3m}$$

$$3m\,dm = -\frac{dx}{x}$$

$$3\frac{m^2}{2} = -\ell n\,x + c$$

$$\frac{3}{2}\frac{y^2}{x^2} = -\ell n\, x + c$$

Given $x = 1, y = 1$

$$\Rightarrow \quad c = \frac{3}{2}$$

$$\frac{3}{2}\frac{y^2}{x^2} = -\ell n\, x + \frac{3}{2}$$

At $x = e$, $\frac{3}{2}\frac{y^2}{e^2} = -1 + \frac{3}{2} = \frac{1}{2}$

$3y^2 = e^2$

$y^2(e) = \frac{e^2}{3}$

$\therefore \quad 6y^2(e) = 2e^2$

7. If $\frac{1^3+2^3+3^3+.....+n^3}{1.3+2.5+3.7.....n\text{ terms}} = \frac{9}{5}$ then the value of n is-

(1) 5 (2) 8 (3) 9 (4) 10

Ans. **(1)**

Sol. $$\frac{\left(\frac{n(n+1)}{2}\right)^2}{2.\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}} = \frac{9}{5}$$

$$\Rightarrow \frac{\frac{n(n+1)}{4}}{\frac{2n+1}{3}+\frac{1}{2}} = \frac{9}{5}$$

$$\Rightarrow \frac{\frac{3}{2}n(n+1)}{4n+2+3} = \frac{9}{5}$$

$$\Rightarrow \frac{15}{2}(n^2 + n) = 9(4n + 5)$$

$5n^2 + 5n = 24n + 30$

$5n^2 - 19n - 30 = 0$

$5n^2 - 25n + 6n - 30 = 0$

$(5n + 6)(n - 5) = 0 \Rightarrow n = 5$

8. $\left(\frac{1+\cos\frac{2\pi}{9}+i\sin\frac{2\pi}{9}}{1+\cos\frac{2\pi}{9}-i\sin\frac{2\pi}{9}}\right)^3$ is equal to

(1) $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$ (2) $-\frac{1}{2}-\frac{\sqrt{3}}{2}i$ (3) $\frac{1}{2}-\frac{\sqrt{3}}{2}i$ (4) $\frac{1}{2}+\frac{\sqrt{3}}{2}i$

Ans. **(1)**

Sol. $\left(\frac{2\cos^2\frac{\pi}{9}+2i\cos\frac{\pi}{9}\cdot\sin\frac{\pi}{9}}{2\cos^2\frac{\pi}{9}-2i\cos\frac{\pi}{9}\cdot\sin\frac{\pi}{9}}\right)^3 = e^{i\frac{2\pi}{3}} = -\frac{1}{2}+i\frac{\sqrt{3}}{2}$

9. If $f(x) = x^3 + x^2 f'(1) + xf''(2) - f'''(3)$. Then the relation between $f'(1)$, $f''(2)$, $f'''(3)$

(1) $f(0) = f'(1) + 3f''(2) + f'''(3)$ (2) $f(0) = 2f'(1) + 3f''(2) - f'''(3)$

(3) $f(0) = 2f'(1) - f''(2) + f'''(3)$ (4) $f(0) = 3f'(1) - f''(2) - 3f'''(3)$

Ans. **(3)**

Sol. $f'(x) = 3x^2 + 2xf'(1) + f''(2) \Rightarrow f'(1) + f''(2) + 3 = 0$

$f''(x) = 6x + 2f'(1) \Rightarrow 2f'(1) - f''(2) + 12 = 0$

$f'''(x) = 6$

$\therefore f'(1) = -5$

$f''(2) = 2$

$f'''(3) = 6$

$f(0) = -6$

10. $\sim(p \wedge (p \rightarrow \sim q))$ is equivalent to-

(1) $p \rightarrow q$ (2) $p \wedge q$ (3) $p \vee q$ (4) $p \leftrightarrow q$

Ans. **(1)**

Sol. $\sim p \vee (\sim(p \rightarrow \sim q))$

$\sim p \vee (p \wedge q) = p \rightarrow q$

11. The sum of coefficients of first 3 terms in the expansion of $\left(x-\frac{3}{x^2}\right)^n$ is 376. Find the coefficient of x^4.

(1) 695 (2) 410 (3) 405 (4) 395

Ans. (3)

Sol. ${}^nC_0 - {}^nC_1(3) + {}^nC_2(9) = 376$

$1 - 3n + \frac{9n(n-1)}{2} = 376$

$2 - 6n + 9n^2 - 9n = 752$

$9n^2 - 15n - 750 = 0$

$3n^2 - 5n - 250 = 0$

$\Rightarrow n = 10$

$T_{r+1} = {}^{10}C_r(x)^{10-r}\left(\frac{-3}{x^2}\right)^r$

$T_3 = 405$

12. If $\lim\limits_{x\to a}$ [x – 5] – [2x + 2] = 0, (where [ ] denotes greatest integer function) then 'a' belongs to

(1) $\left(-\frac{15}{2}, -\frac{13}{2}\right)$ (2) $\left[-\frac{15}{2}, -\frac{13}{2}\right)$ (3) $\left(-\frac{15}{2}, -\frac{13}{2}\right]$ (4) $\left[-\frac{15}{2}, -\frac{13}{2}\right]$

Ans. **(1)**

Sol. f(x) is continuous $\forall x \in R - \left\{n + \frac{1}{2}\right\}$, $n \in I$

$\therefore$ $\lim\limits_{x\to a} f(x) = f(a)$

Hence [a – 5] – [2a + 2] = 0

$\Rightarrow$ [a] – [2a] = 7

$a \in I$ $a = -7$

$a \notin I$ $a = I + f$

–I – [2f] = 7

Case-I : $f \in \left(0, \frac{1}{2}\right)$ Case-II : $f \in \left(\frac{1}{2}, 1\right)$

–I = 7 I = –8

I = –7

a ∈ (–7.5, –6.5)

At $a = n + \frac{1}{2}$, $n \in I$ $\Rightarrow a \in (-7.5, -7)$

LHL ≠ RHL

$\therefore$ a ∈ (–7.5, –6.5)

SECTION-B

13. Let $a_1, a_2, ..., a_6$ are in Arithmetic Progression where $a_1 + a_3 = 10$. If mean of $a_1, a_2, ..., a_6$ is $\frac{19}{2}$, then find the value of $8\sigma^2$ (where σ^2 denotes the variance of given numbers)

Ans. 210

Sol. $a_1, a_2, ..., a_6$

mean = $\frac{19}{2}$

variance = σ^2

$a_1 + a_3 = 10$

$8\sigma^2 = ?$

$$\frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6}{6} = \frac{19}{2}$$

$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 57$

$a_2 + a_4 + a_5 + a_6 = 47$

$\sigma^2 = \frac{1}{6}\sum x_i^2 - \left(\frac{19}{2}\right)^2$

$a_1 + d + a_1 + 3d + a_1 + 4d + a_1 + 5d = 47$

$4a_1 + 13d = 47$

$a_1 + a_1 + 2d = 10$

$a_1 + d = 5$

$4a_1 + 13(5 - a_1) = 47$

$a_1 = 2, d = 3$

2, 5, 8, 11, 14, 17

$\sigma^2 = \frac{1}{6}(4 + 25 + 64 + 121 + 196 + 289) - \left(\frac{19}{2}\right)^2$

$= \frac{1}{6} \times 699 - \frac{361}{4} = \frac{699}{6} - \frac{361}{4}$

$\therefore 8\sigma^2 = 210$

2. If urn 1 contain 7 red & 3 green balls, urn2 contain 3 red and 2 green balls, urn 3 contain λ red & 2 green balls. One urn is selected at random & one ball is drawn. If probability of getting red ball is 0.6 then find value of λ.

Ans. **(2)**

Sol. $\frac{1}{3}\left[\frac{7}{10} + \frac{3}{5} + \frac{\lambda}{\lambda+2}\right] = 0.6 \Rightarrow \quad \cdot 7 + \cdot 6 + \frac{\lambda}{\lambda+2} = 1.8 \quad \Rightarrow \frac{\lambda}{\lambda+2} = \cdot 5 = \frac{1}{2} \Rightarrow 2\lambda = \lambda + 2$

$\lambda = 2$

3. Relation R on the set P = {a, b , c, d} is given by R = {(a, b), (b, c), (b, d)}. Find the minimum number of ordered pairs to be added in R so that it is an equivalence relation.

Ans. 13

Sol. R = {(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (b, c), (c, b), (b, d), (d, b), (a, c), (c, a), (c, d), (d, c), (a, d), (d, a)}

minimum no. of ordered pairs = 13

4. Consider a matrix of order 5 × 5 which can be formed using numbers 0 or 1. How many such matrices can be formed in which sum of elements in each column & each row is 1.

Ans. 120

Sol.
$$\begin{bmatrix} - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \end{bmatrix}$$

I row has 5 options to place '1'

II row has 4 options

III row has 3 options

IV row has 2 options

V row has 1 options

so total matrix = 5 × 4 × 3 × 2 × 1 = 120

5. Consider a function f(x) such that $f(x + y) = f(x) \cdot f(y)$ & $f(1) = 3$. If $\sum_{k=1}^{n} f(k) = 3279$. Find 'n'.

Ans. **7**

Sol. Put $x = y = 1$, $f(2) = 3^2$

Put $x = 2, y = 1$, $f(3) = 3^3$

and so on

$\Rightarrow f(x) = 3^x ; x \in N$

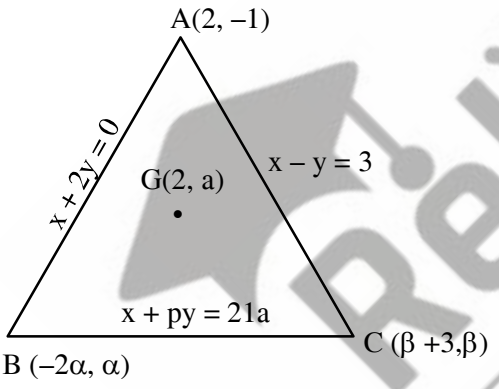
$\sum_{r=1}^{n} f(r) = 3 + 3^2 + ... + 3^n = 3279$

$\Rightarrow n = 7$

6. Consider a triangle formed by lines AC : $x - y = 3$, AB : $x + 2y = 0$ & BC : $x + py = 21a$. If centroid is $(2, a)$, find $\ell(BC)^2$.

Ans. 17

Sol. $\frac{-2\alpha + 2 + \beta + 3}{3} = 2 \Rightarrow \beta = 1 + 2\alpha$ so C $(2\alpha + 4, 1 + 2\alpha)$



$\frac{\alpha - 1 + \beta}{3} = a \Rightarrow \alpha + \beta = 3a + 1 \Rightarrow \alpha + 2\alpha + 1 = 3\alpha + 1 = 3a + 1 \Rightarrow \alpha = a$, $\beta = 1 + 2a$

B & C lies on $x + py = 21a$

$\Rightarrow$ $-2\alpha + p\alpha = 21\alpha$ & $2\alpha + 4 + p(1 + 2\alpha) = 21a$

also $-2a + pa = 21a$ $2a + 4 + p + 2pa = 21a$

$pa = 23a$ $2a + 4 + p + 46a = 21a$

$\Rightarrow$ $a = 0$ or $p = 23$ (rejected) $p + 4 = -27a$

$p = -4$

so B(0, 0), C(4, 1)

$BC = \sqrt{16+1} = \sqrt{17}$

so $(BC)^2 = 17$