

Calculating $Q(n, k)$:

We will try to develop a recursion relation to calculate $Q(n, k)$.

$Q(n, k)$ = no. of partitions of 'n' such that the maximum number in each partition is 'k'.

Now, no. of partitions such that max. no. is at most $(k-1)$ i.e. less than 'k' = $Q(n, k-1)$

i.e., the no. of partitions such that the max. no. in each partition is not 'k' = $Q(n, k-1)$

no. of partitions such that the max. no. is only 'k' in each partition = $Q(n-k, k)$ (because all partitions of $(n-k)$ will have only 'k' as the maximum no.)

Hence, by Inclusion exclusion principle,

$$Q(n, k) = Q(n, k-1) + Q(n-k, k) \quad \forall k > 1, n \geq k$$

Calculating $R(n, k)$:

no. of partitions of 'n' such that each partition has less than k numbers = $R(n, k-1)$

no. of partitions of 'n' such that each partition has only 'k' elements = $R(n-k, k)$ (\because each number of a partition can be at least 1)

\therefore By IEP:

$$R(n, k) = R(n, k-1) + R(n-k, k) \quad \forall k > 1, n \geq k.$$

Hence, as the characteristic equation of $R(n, k)$ and $Q(n, k)$ are the same, hence $Q(n, k) = R(n, k) \quad \forall n, k.$