MATHEMATICS

1. Considering only the principal values of the inverse trigonometric functions, the value of
$\frac{3}{2}\cos^{-1}\sqrt{\frac{2}{2+\pi^2}} + \frac{1}{4}\sin^{-1}\frac{2\sqrt{2}\pi}{2+\pi^2} + \tan^{-1}\frac{\sqrt{2}}{\pi}$ is ____________

2. Let α be a positive real number. let f: $\mathbb{R} \to \mathbb{R}$ and g: $(\alpha, \infty) \to \mathbb{R}$ be the functions defined by f(x)=sin($\frac{\pi x}{12}$) and g(x)=$\frac{2log_e(\sqrt{x}-\sqrt{\alpha})}{log_e(e^{\sqrt{x}}-e^{\sqrt{\alpha}})}$
then the value of $\lim_{x\to\alpha} f(g(x))$ is ____________

3. In a study about a pandemic, data of 900 persons was collected. It was found that

   190 persons had symptom of fever,
   220 persons had symptom of cough,
   220 persons had symptom of breathing problem,
   330 persons had symptom of fever or cough or both,
   350 persons had symptom of cough or breathing problem or both,
   340 persons had symptom of fever or breathing problem or both,
   30 persons had all three symptoms

   If a person is chosen randomly from these 900 persons, then the probability that the person has at most one symptom is ____________

4. Let z be a complex number with non-zero imaginary part.
If $\frac{2+3z+4z^2}{2-3z+4z^2}$
is a real number, then the value of $|z|^2$ is ____________

5. Let $\overline{z}$ denote the complex conjugate of a complex number z and let $i = \sqrt{-1}$. In the set of complex numbers, the number of distinct roots of the equation
$\overline{z} - z^2 = i(\overline{z} + z^2)$

is ___________

6. Let $l_1, l_2, ..., l_{100}$ be consecutive terms of an arithmetic progression with common difference d_1, and let $w_1, w_2, ..., w_100$ be consecutive terms of another arithmetic progression with common difference d_2, where $d_1 d_2 =$ 10. For each i = 1, 2,...,100, let R_i be a rectangle with length L_i, width W_i and area A_i. If $A_{51} - A_{50} = 1000$, then the value of $A_{100} - A_{90}$ is ___________

7. The number of 4-digit integers in the closed interval [2022, 4482] formed by using the digits 0,2,3,4,6,7 is___________

8. Let ABC be the triangle with AB = 1, AC = 3 and $\angle BAC = \frac{\pi}{2}$. If a circle of radius $r > 0$ touches the sides AB, AC and also touches internally the circumcircle of the triangle ABC, then the value of r is___________

9. consider the equation
$\int_1^e \frac{(log_e x)^{\frac{1}{2}}}{x(a-(log_e x)^{\frac{3}{2}})^2} dx = 1$,a $\in (-\infty, 0) \cup (1, \infty)$
which of the following statements is/are TRUE?
(A) No a satisfies the above equation
(B) An integer a satisfies the above equation
(C) An irrational number a satisfies the above equation
(d) More than one a satisfy the above equation

10. Let $a_1, a_2, a_3, ...$ be an arithmetic progression with $a_1 = 7$ and common difference 8. Let T_1, T_2, T_3,... be such that $T_1 = 3$ and $T_{n+1} - t_n = a_n$ for $n \geq 1$. Then, which of the following is/are TRUE?
(A)$T_{20} = 1604$ (B)$\sum_{K=1}^{20} T_k = 10510$
(C)$T_{30} = 3454$ (D)$\sum_{K=1}^{30} T_k = 35610$

11. Let $P_1 and P_2$ be two planes given by
$P_1 : 10x + 15y + 12z - 60 = 0$
, $P_2 : -2x + 5y + 4z - 20 = 0$.
Which of the following straight lines can be an edge of some tetrahedron whose two faces lie on P_1 and P_2 ?
$(A) \frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{5}$
$(B) \frac{x-6}{-5} = \frac{y}{2} = \frac{z}{3}$
$(C) \frac{x}{-2} = \frac{y-4}{5} = \frac{z}{4}$

$(D)\frac{x}{1} = \frac{y-4}{-2} = \frac{z}{3}$

12. Let S be the reflection of a point Q with respect to the plane given by
$\uparrow r = -(t+p)\hat{i} + t\hat{j} + (1+p)\hat{k}$
where t, p are real parameters and $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along the three positive coordinate axes. If the position vectors of Q and S are $10\hat{i} + 15\hat{j} + 20\hat{k}$and $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ respectively, then which of the following is/are TRUE ?
(A) $3(\alpha + \beta) = -101$
(B) $3(\beta + \gamma) = -71$
(C) $3(\gamma + \alpha) = -86$
(D) $3(\alpha + \beta + \gamma) = -121$

13. Consider the parabola $y^2 = 4x$. Let S be the focus of the parabola. A pair of tangents drawn to the parabola from the point p = (-2, 1) meet the parabola at P_1 and P_2. Let Q_1 and Q_2 be points on the lines $SP_1 and SP_2$ respectively such that PQ_1 is perpendicular to SP_1 and PQ_2 is perpendicular to SP_2. Then, which of the following is/are TRUE ?
(A) $SQ_1 = 2$
(B) $Q_1Q_2 = \frac{3\sqrt{10}}{5}$
(C) $PQ_1 = 3$
(D) $SQ_2 = 1$

14. Let $\mid M \mid$ denote the determinant of a square matrix M. Let g:$[0, \frac{\pi}{2}] \to \mathbb{R}$ be the function defined by
$g(\theta) = \sqrt{f(\theta) - 1} + \sqrt{f(\frac{\pi}{2} - \theta) - 1}$ where
$$f(\theta) = \frac{1}{2}\begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix} + \begin{vmatrix} \sin\pi & \cos\left(\theta + \frac{\pi}{4}\right) & \tan\left(\theta - \frac{\pi}{4}\right) \\ \sin\left(\theta - \frac{\pi}{4}\right) & -\cos\frac{\pi}{2} & \log_e\left(\frac{4}{\pi}\right) \\ \cot\left(\theta + \frac{\pi}{4}\right) & \log_e\left(\frac{\pi}{4}\right) & \tan\pi \end{vmatrix}$$
Let p(x) be a quadratic polynomial whose roots are the maximum and minimum values of the function $g(\theta) and p(2) = 2 - \sqrt{2}$. Then, which of the following is/are TRUE ?
(A) $p(\frac{3+\sqrt{2}}{4}) < 0$
(B) $p(\frac{1+3\sqrt{2}}{4}) > 0$
(C) $p(\frac{5\sqrt{2}-1}{4}) > 0$
(D) $p(\frac{5-\sqrt{2}}{4}) < 0$

15. consider the following lists:

List-I	List-II
$(I) x \in [-\frac{2\pi}{3}, \frac{2\pi}{3}] : \cos x + \sin x = 1$	(P) has two elements

$(II) x \in [-\frac{5\pi}{18}, \frac{5\pi}{18}] : \sqrt{3}\tan 3x = 1$ (Q) has three elements
$(III) x \in [-\frac{6\pi}{5}, \frac{6\pi}{5}] : 2\cos(2x) = \sqrt{3}$ (R) has four elements
$(IV) x \in [-\frac{7\pi}{4}, \frac{7\pi}{4}] : \sin x - \cos x = 1$ (s) has five elements
(V) (T) has six elements

The correct option is :

(A) $(I) \to (P); (II) \to (S); (III) \to (P); (IV) \to (S)$
(B) $(I) \to (P); (II) \to (P); (III) \to (T); (IV) \to (R)$
(C) $(I) \to (Q); (II) \to (P); (III) \to (T); (IV) \to (S)$
(D) $(I) \to (Q); (II) \to (S); (III) \to (P); (IV) \to (R)$

16. Two players, $P_1 and P_2$, play a game against each other. In every round of the game, each player rolls a fair die once, where the six faces of the die have six distinct numbers. Let x and y denote the readings on the die rolled by $P_1 and P_2$, respectively. If $x > y$, then P_1 scores 5 points and P_2 2 scores 0 point. If $x = y$, then each player scores 2 points. If $x < y$, then P_1 scores 0 point and P_2 scores 5 points. Let X_i and Y_i be the total scores of $P_1 and P_2$, respectively, after playing the i^{th} round.

LIST-I	**LIST-II**
(I) Probability of $(X_2 \geq Y_2)$ is	$(P) = \frac{3}{8}$
(II) Probability of $(X_2 > Y_2)$ is	$(Q) = \frac{11}{16}$
(III) Probability of $(X_3 = Y_3)$ is	$(R) = \frac{5}{16}$
(IV) Probability of $(X_3 > Y_3)$ is	$(S) = \frac{355}{864}$
(V)	$(T) \frac{77}{432}$

the correct option is:
(A) $(I) \to (Q); (II) \to (R); (III) \to (T); (IV) \to (S)$
(B) $(I) \to (Q); (II) \to (R); (III) \to (T); (IV) \to (S)$
(C) $(I) \to (P); (II) \to (R); (III) \to (Q); (IV) \to (S)$
(D) $(I) \to (P); (II) \to (R); (III) \to (Q); (IV) \to (T)$

17. Let p,q amd r be nonzero real numbers that are the $10^{th}, 100^{th}, and 1000^{th}$ terms of a harmonic progression, respectively. Consider the following system of linear equations:

$$x + y + z = 1$$

$$10x + 100y + 1000z = 0$$

$$qrx + pry + pqz = 0$$

LIST-I	**LIST-II**
(I) If $\frac{q}{r} = 10$, then the system of linear equations has	(P) $x = 0, y = \frac{10}{9}, z = -\frac{1}{9}$ as a solution
(II) If $\frac{p}{r} \neq 100$, then the system of linear equations has	(Q) $x = \frac{10}{9}, y = -\frac{1}{9}, z = 0$ as a solution
(III) If $\frac{p}{q} \neq 10$, then the system of linear equations has	(R) infinitely many solutions
(IV) If $\frac{p}{q} = 10$, then the system of linear equations has	(S) no solution

(A) $(I) \rightarrow (T); (II) \rightarrow (R); (III) \rightarrow (S); (IV) \rightarrow (T)$
(B) $(I) \rightarrow (Q); (II) \rightarrow (S); (III) \rightarrow (S); (IV) \rightarrow (R)$
(C) $(I) \rightarrow (Q); (II) \rightarrow (R); (III) \rightarrow (P); (IV) \rightarrow (R)$
(D) $(I) \rightarrow (T); (II) \rightarrow (S); (III) \rightarrow (P); (IV) \rightarrow (T)$

18. consider the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ let $H(\alpha, 0), 0 < \alpha < 2$,be a point. A straight line drawn through H parallel to the y-axis crosses the ellipse and its auxiliary circle at points E and F respectively, in the first quadrant. The tangent to the ellipse at the point E intersects the positive x-axis at a point G. Suppose the straight line joining F and the origin makes an angle ϕ with the positive x-axis

LIST-I	**LIST-II**
(I) if $\varphi = \frac{3}{4}$, then the area of the triangle FGH is	$(p)\frac{(\sqrt{3}-1)^4}{8}$
(II) if $\varphi = \frac{\pi}{3}$, then the area of the triangle FGH is	(Q)1
(III) if $\varphi = \frac{\pi}{6}$, then the area of the triangle FGH is	(R)$\frac{3}{4}$
(IV) if $\varphi = \frac{\pi}{12}$, then the area of the triangle FGH is	(S)$\frac{1}{2\sqrt{3}}$
(V)	$(T)\frac{3\sqrt{3}}{2}$

(A) $(I) \rightarrow (R); (II) \rightarrow (S); (III) \rightarrow (Q); (IV) \rightarrow (P)$
(B) $(I) \rightarrow (R); (II) \rightarrow (T); (III) \rightarrow (S); (IV) \rightarrow (P)$
(C) $(I) \rightarrow (Q); (II) \rightarrow (T); (III) \rightarrow (S); (IV) \rightarrow (P)$
(D) $(I) \rightarrow (Q); (II) \rightarrow (S); (III) \rightarrow (Q); (IV) \rightarrow (P)$