MATHEMATICS

1. considering only the principal values of the inverse trigonometric functions, the value of $\frac{3}{2}\cos^{-1}\sqrt{\frac{2}{2+\pi^2}}+\frac{1}{4}\sin^{-1}\frac{2\sqrt{2}\pi}{2+\pi^2}+\tan^{-1}\frac{\sqrt{2}}{\pi} is$___________

2. Let α be a positive real number. let f: $\mathbb{R}\to\mathbb{R}$ and g: $(\alpha,\infty)\to\mathbb{R}$ be the functions defined by f(x)=sin($\frac{\pi x}{12}$) and g(x)=$\frac{2\log_e(\sqrt{x}-\sqrt{\alpha})}{\log_e(e^{\sqrt{x}}-e^{\sqrt{\alpha}})}$ then the value of $\lim_{x\to\alpha} f(g(x))$ is ___________

3. In a study about a pandemic, data of 900 persons was collected. It was found that

   - 190 persons had symptom of fever,
   - 220 persons had symptom of cough,
   - 220 persons had symptom of breathing problem,
   - 330 persons had symptom of fever or cough or both,
   - 350 persons had symptom of cough or breathing problem or both,
   - 340 persons had symptom of fever or breathing problem or both,
   - 30 persons had all three symptoms

   If a person is chosen randomly from these 900 persons, then the probability that the person has at most one symptom is ___________

4. Let z be a complex number with non-zero imaginary part. If $\frac{2+3z+4z^2}{2-3z+4z^2}$ is a real number, then the value of $|z|^2$ is ___________

5. Let $\overline{z}$ denote the complex conjugate of a complex number z and let $i=\sqrt{-1}$. In the set of complex numbers, the number of distinct roots of the equation
   $\overline{z}-z^2=i(\overline{z}+z^2)$ is ___________

6. Let $l_1,l_2,...,l_{100}$ be consecutive terms of an arithmetic progression with common difference d_1, and let $w_1,w_2,...,w_100$ be consecutive terms of another arithmetic progression with common difference d_2, where $d_1d_2=10$. For each i = 1, 2,...,100, let R_i be a rectangle with length L_i, width W_i and area A_i. If $A_{51}-A_{50}=1000$, then the value of $A_{100}-A_{90}$ is ___________

7. the number of 4-digit integers in the closed interval [2022, 4482] formed by using the digits $0, 2, 3, 4, 6, 7$ is

8. Let ABC be the triangle with AB $= 1$, AC $= 3$ and $\angle BAC = \frac{\pi}{2}$. If a circle of radius $r > 0$ touches the sides AB, AC and also touches internally the circumcircle of the triangle ABC, then the value of r is___________

9. consider the equation
$\int_1^e \frac{(\log_e x)^{\frac{1}{2}}}{x(a-(\log_e x)^{\frac{3}{2}})^2} dx = 1$,a $\in (-\infty, 0) \cup (1, \infty)$
which of the following statements is/are TRUE?

   A. No a satisfies the above equation
   B. An integer a satisfies the above equation
   C. An irrational number a satisfies the above equation
   D. More than one a satisfy the above equation

10. Let $a_1, a_2, a_3, ...$ be an arithmetic progression with $a_1 = 7$ and common difference 8. Let T_1, T_2, T_3,... be such that $T_1 = 3$ and $T_{n+1} - t_n = a_n$ for $n \geq 1$. Then, which of the following is/are TRUE?

   A. $T_{20} = 1604$
   B. $\sum_{K=1}^{20} T_k = 10510$
   C. $T_{30} = 3454$
   D. $\sum_{K=1}^{30} T_k = 357610$

11. Let $P_1 and P_2$ be two planes given by

   - $P_1 : 10x + 15y + 12z - 60 = 0,$
   - $P_2 : -2x + 5y + 4z - 20 = 0.$

   Which of the following straight lines can be an edge of some tetrahedron whose two faces lie on P_1 and P_2

   A. $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{5}$
   B. $\frac{x-6}{-5} = \frac{y}{2} = \frac{z}{3}$
   C. $\frac{x}{-2} = \frac{y-4}{5} = \frac{z}{4}$
   D. $\frac{x}{1} = \frac{y-4}{-2} = \frac{z}{3}$

12. Let S be the reflection of a point Q with respect to the plane given by
$\uparrow r = -(t + p)\hat{i} + t\hat{j} + (1 + p)\hat{k}$
where t, p are real parameters and $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along the three positive coordinate axes. If the position vectors of Q and S are $10\hat{i} + 15\hat{j} + 20\hat{k}$and $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ respectively, then which of the following is/are TRUE ?

A. $3(\alpha+\beta) = -101$

B. $3(\beta+\gamma) = -71$

C. $3(\gamma+\alpha) = -86$

D. $3(\alpha+\beta+\gamma) = -121$

13. Consider the parabola $y^2 = 4x$. Let S be the focus of the parabola. A pair of tangents drawn to the parabola from the point p = (-2, 1) meet the parabola at P_1 and P_2. Let Q_1 and Q_2 be points on the lines $SP_1 and SP_2$ respectively such that PQ_1 is perpendicular to SP_1 and PQ_2 is perpendicular to SP_2. Then, which of the following is/are TRUE ?

    A. $SQ_1 = 2$

    B. $Q_1Q_2 = \frac{3\sqrt{10}}{5}$

    C. $PQ_1 = 3$

    D. $SQ_2 = 1$

14. Let $\left|M\right|$ denote the determinant of a square matrix M. Let g:$[0, \frac{\pi}{2}] \to \mathbb{R}$ be the function defined by
$g(\theta) = \sqrt{f(\theta)-1} + \sqrt{f(\frac{\pi}{2}-\theta)-1}$ where
$f(\theta) = \frac{1}{2}\begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix} + \begin{vmatrix} \sin\pi & \cos\left(\theta+\frac{\pi}{4}\right) & \tan\left(\theta-\frac{\pi}{4}\right) \\ \sin\left(\theta-\frac{\pi}{4}\right) & -\cos\frac{\pi}{2} & \log_e\left(\frac{4}{\pi}\right) \\ \cot\left(\theta+\frac{\pi}{4}\right) & \log_e\left(\frac{\pi}{4}\right) & \tan\pi \end{vmatrix}$
Let p(x) be a quadratic polynomial whose roots are the maximum and minimum values of the function $g(\theta) and p(2) = 2-\sqrt{2}$. Then, which of the following is/are TRUE ?

    A. $p(\frac{3+\sqrt{2}}{4}) < 0$

    B. $p(\frac{1+3\sqrt{2}}{4}) > 0$

    C. $p(\frac{5\sqrt{2}-1}{4}) > 0$

    D. $p(\frac{5-\sqrt{2}}{4}) < 0$

15. consider the following lists:

List-I	List-II
$(I) x \in [-\frac{2\pi}{3}, \frac{2\pi}{3}] : \cos x + \sin x = 1$	(P) has two elements
$(II) x \in [-\frac{5\pi}{18}, \frac{5\pi}{18}] : \sqrt{3}\tan 3x = 1$	(Q) has three elements
$(III) x \in [-\frac{6\pi}{5}, \frac{6\pi}{5}] : 2\cos(2x) = \sqrt{3}$	(R) has four elements
$(IV) x \in [-\frac{7\pi}{4}, \frac{7\pi}{4}] : \sin x - \cos x = 1$	(s) has five elements
(V)	(T) has six elements $\left|T\right|$he correct option is :

    A. $(I) \to (P); (II) \to (S); (III) \to (P); (IV) \to (S)$

B. $(I) \rightarrow (P); (II) \rightarrow (P); (III) \rightarrow (T); (IV) \rightarrow (R)$

C. $(I) \rightarrow (Q); (II) \rightarrow (P); (III) \rightarrow (T); (IV) \rightarrow (S)$

D. $(I) \rightarrow (Q); (II) \rightarrow (S); (III) \rightarrow (P); (IV) \rightarrow (R)$

16. Two players, $P_1 and P_2$, play a game against each other. In every round of the game, each player rolls a fair die once, where the six faces of the die have six distinct numbers. Let x and y denote the readings on the die rolled by $P_1 and P_2$, respectively. If $x > y$, then P_1 scores 5 points and P_2 2 scores 0 point. If $x = y$, then each player scores 2 points. If $x < y$, then P_1 scores 0 point and P_2 scores 5 points. Let X_i and Y_i be the total scores of $P_1 and P_2$, respectively, after playing the i^{th} round.

LIST-I	**LIST-II**
(I) Probability of $(X_2 \geq Y_2)$ is	$(P) = \frac{3}{8}$
(II) Probability of $(X_2 > Y_2)$ is	$(Q) = \frac{11}{16}$
(III) Probability of $(X_3 = Y_3)$ is	$(R) = \frac{5}{16}$
(IV) Probability of $(X_3 > Y_3)$ is	$(S) = \frac{355}{864}$
(V)	$(T)\frac{77}{432}$

the correct option is:

A. $(I) \rightarrow (Q); (II) \rightarrow (R); (III) \rightarrow (T); (IV) \rightarrow (S)$

B. $(I) \rightarrow (Q); (II) \rightarrow (R); (III) \rightarrow (T); (IV) \rightarrow (S)$

C. $(I) \rightarrow (P); (II) \rightarrow (R); (III) \rightarrow (Q); (IV) \rightarrow (S)$

D. $(I) \rightarrow (P); (II) \rightarrow (R); (III) \rightarrow (Q); (IV) \rightarrow (T)$

17. Let p,q amd r be nonzero real numbers that are the $10^{th}, 100^{th}, and 1000^{th}$ terms of a harmonic progression, respectively. Consider the following system of linear equations:

$$x + y + z = 1$$

$$10x + 100y + 1000z = 0$$

$$qrx + pry + pqz = 0$$

LIST-I	**LIST-II**
(I) If $\frac{q}{r} = 10$, then the system of linear equations has	(P) $x = 0, y = \frac{10}{9}, z = -\frac{1}{9}$ as a solution
(II) If $\frac{p}{r} \neq 100$, then the system of linear equations has	(Q) $x = \frac{10}{9}, y = -\frac{1}{9}, z = 0$ as a solution
(III) If $\frac{p}{q} \neq 10$, then the system of linear equations has	(R) infinitely many solutions
(IV) If $\frac{p}{q} = 10$, then the system of linear equations has	(S) no solution

A. $(I) \rightarrow (T); (II) \rightarrow (R); (III) \rightarrow (S); (IV) \rightarrow (T)$

B. $(I) \rightarrow (Q); (II) \rightarrow (S); (III) \rightarrow (S); (IV) \rightarrow (R)$

C. $(I) \rightarrow (Q); (II) \rightarrow (R); (III) \rightarrow (P); (IV) \rightarrow (R)$

D. $(I) \rightarrow (T); (II) \rightarrow (S); (III) \rightarrow (P); (IV) \rightarrow (T)$

18. consider the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ let $H(\alpha, 0), 0 < \alpha < 2$,be a point. A straight line drawn through H parallel to the y-axis crosses the ellipse and its auxiliary circle at points E and F respectively, in the first quadrant. The tangent to the ellipse at the point E intersects the positive x-axis at a point G. Suppose the straight line joining F and the origin makes an angle ϕ with the positive x-axis

LIST-I	**LIST-II**
(I) if $\varphi = \frac{3}{4}$, then the area of the triangle FGH is	$(p)\frac{(\sqrt{3}-1)^4}{8}$
(II) if $\varphi = \frac{\pi}{3}$, then the area of the triangle FGH is	(Q)1
(III) if $\varphi = \frac{\pi}{6}$, then the area of the triangle FGH is	(R)$\frac{3}{4}$
(IV) if $\varphi = \frac{\pi}{12}$, then the area of the triangle FGH is	(S)$\frac{1}{2\sqrt{3}}$
(V)	$(T)\frac{3\sqrt{3}}{2}$

A. $(I) \rightarrow (R); (II) \rightarrow (S); (III) \rightarrow (Q); (IV) \rightarrow (P)$

B. $(I) \rightarrow (R); (II) \rightarrow (T); (III) \rightarrow (S); (IV) \rightarrow (P)$

C. $(I) \rightarrow (Q); (II) \rightarrow (T); (III) \rightarrow (S); (IV) \rightarrow (P)$

D. $(I) \rightarrow (Q); (II) \rightarrow (S); (III) \rightarrow (Q); (IV) \rightarrow (P)$