Project

Robustness of Hidden Information in Digital Data

PD Dr.Andreas Jakoby

Discrete Wavelet Transform

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Overview

- Introduction
- Limitations of FFT
- FFT vs Wavelets
- Wavelets
- Types fo Wavelets
- Discrete Wavelet Transfrom

Introduction

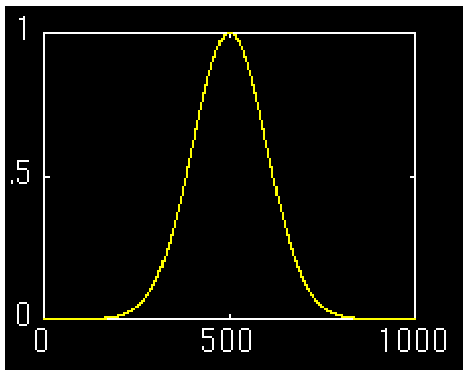
- Signal decomposition
- Fourier Transform
  - Frequency domain ☺
  - Temporal domain ☹

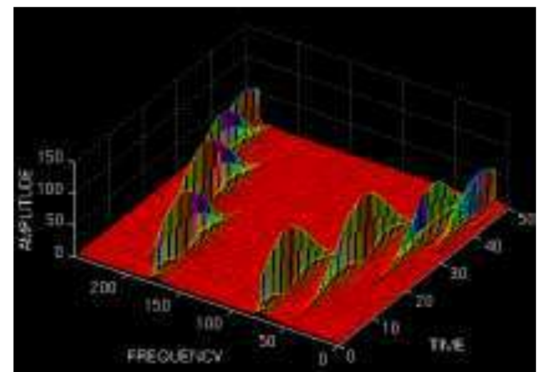
Time information?

STFT (Window size Limitation)

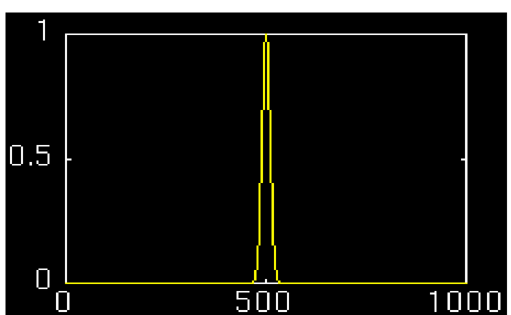
Time -Frequency localization depends on window size.

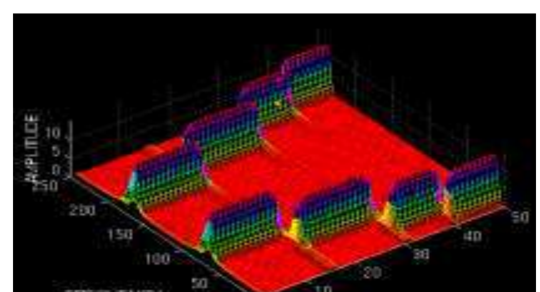
–Wide window good frequency localization, poor time localization





–Narrow window good time localization, poor frequency localization.

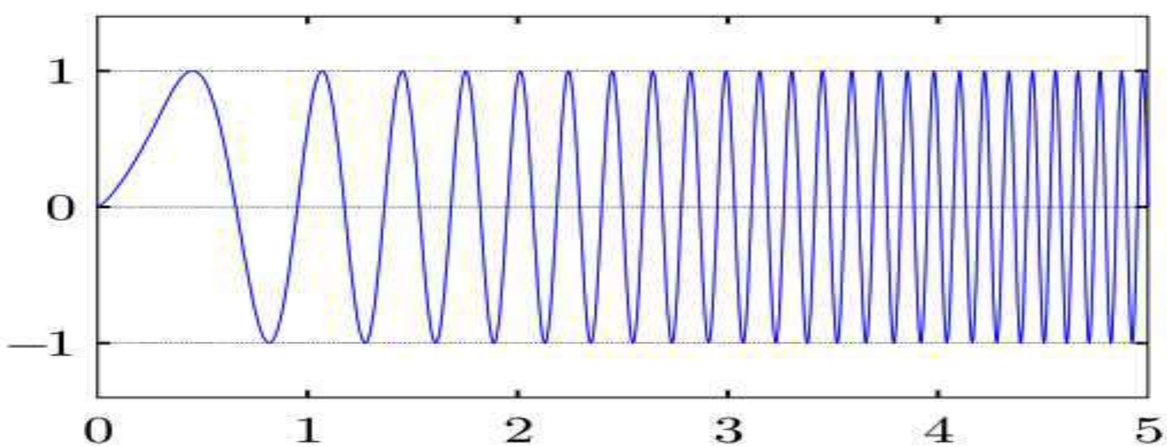




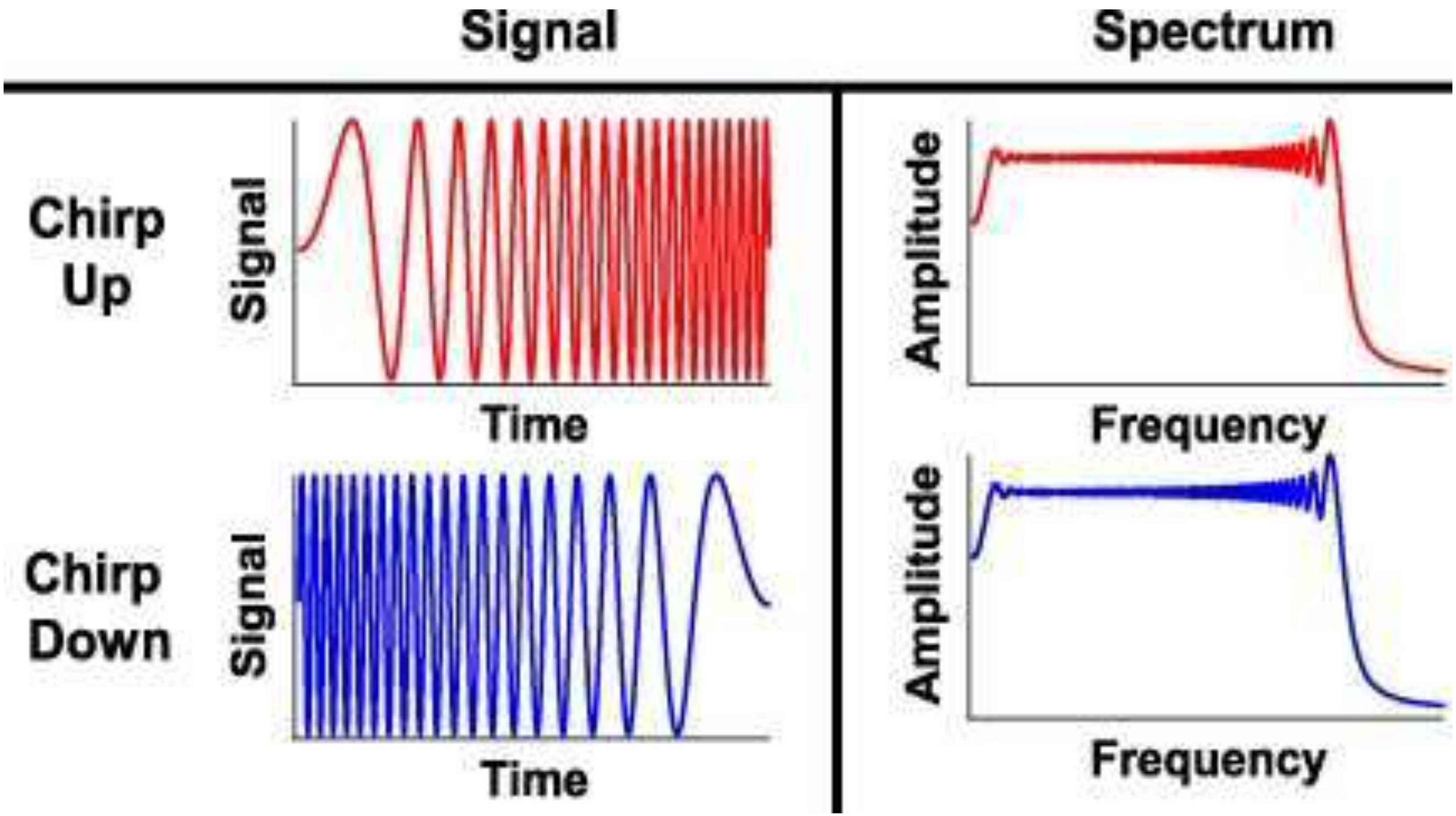
Limitations of Fourier Transform:

To show the limitations of Fourier Transform, we chose a well-known signal in SONAR and RADAR applications, called the Chirp.

A Chirp is a signal in which the frequency increases ('up-chirp') or decreases ('down-chirp').



Limitations of Fourier Transform:

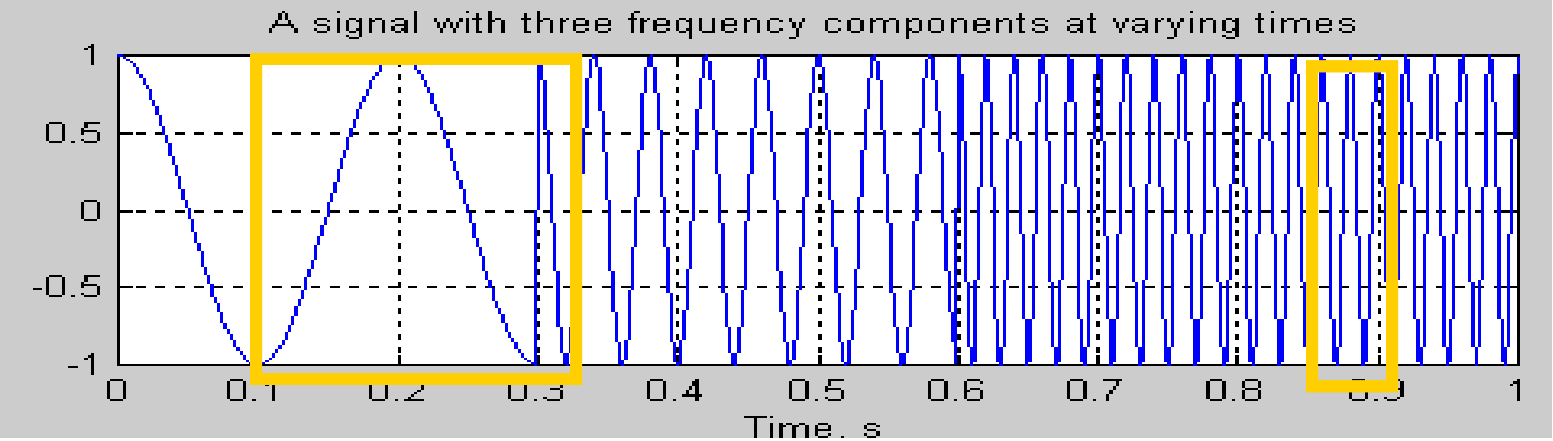


Wavelet Transform

Uses a variable length window, e.g.:

Narrower windows are more appropriate at high frequencies

Wider windows are more appropriate at low frequencies



What is wavelet?

A function that “waves” above and below the x-axis with the following properties:

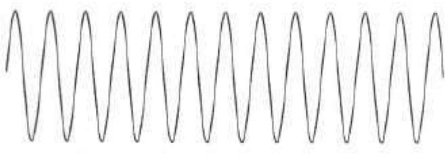
–Varying frequency

–Limited duration

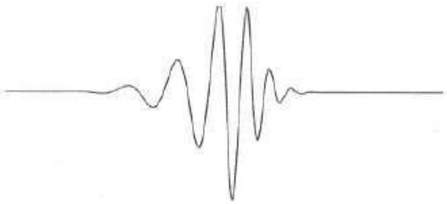
–Zero average value

- This is in contrast to sinusoids, used by FT, which have infinite duration and constant frequency.

Sinusoid



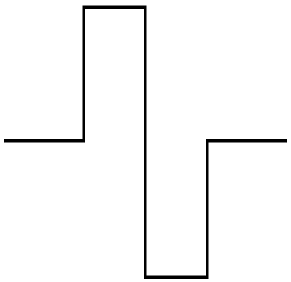
Wavelet



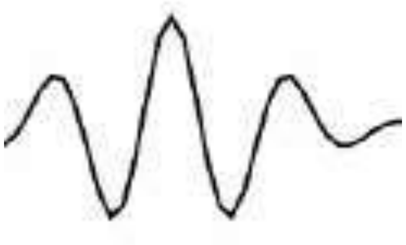
Types of Wavelets

There are many different wavelets, for example:

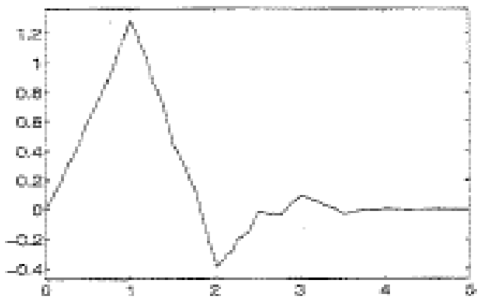
Haar



Morlet



Daubechies



Basis Functions Using Wavelets

Like sin( ) and cos( ) functions in the Fourier Transform, wavelets can define a set of basis functions ψk(t):

$$f(t) = \sum_k a_k \psi_k(t)$$

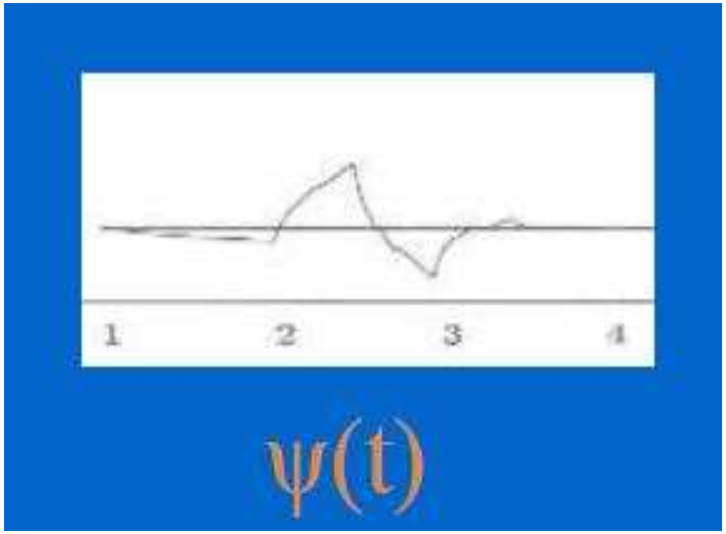
Span of ψk(t): vector space S containing all functions f(t) that can be represented by ψk(t).

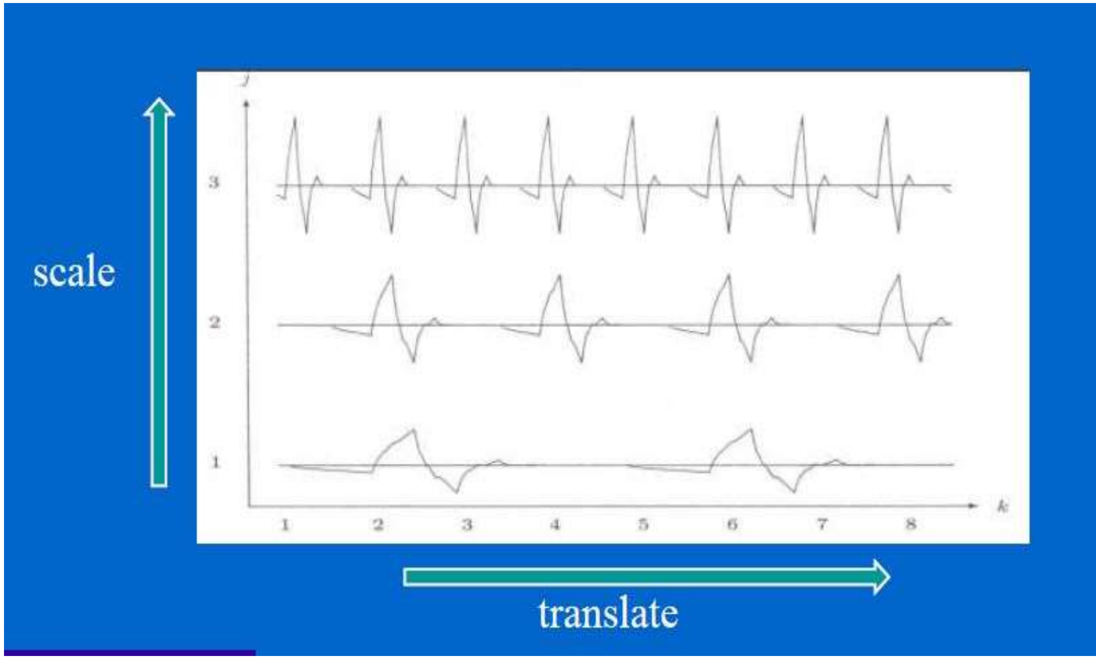
Basis Construction –"Mother" Wavelet

The basis can be constructed by applying translations and scaling's(stretch/compress) on the "mother" wavelet ψ(t):

$$\psi(s,\tau,t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right)$$

Example





Discrete wavelet transforms

Discrete wavelet transform (DWT), which transforms a discrete time signal to a discrete wavelet representation.

It converts an input series x0, x1, ..xm, into one high-pass wavelet coefficient series and one low-pass wavelet coefficient series (of length n/2 each) given by:

$$H_i = \sum_{m=0}^{k-1} x_{2i-m} \cdot s_m(z) \qquad (1)$$

$$L_i = \sum_{m=0}^{k-1} x_{2i-m} \cdot t_m(z) \qquad (2)$$

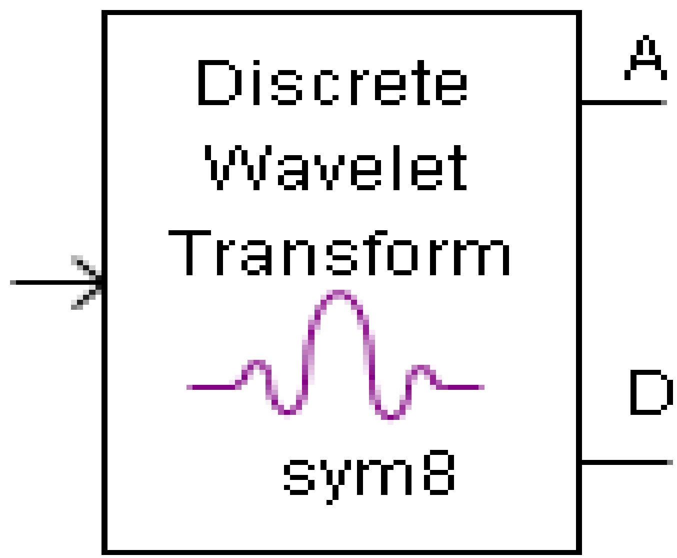
Discrete wavelet transforms (cont..d)

where $s_m(Z)$ and $t_m(Z)$ are called *wavelet filters*, K is the length of the filter, and i=0, ..., [n/2]-1.

In practice, such transformation will be applied recursively on the low-pass series until the desired number of iterations is reached.

DWT

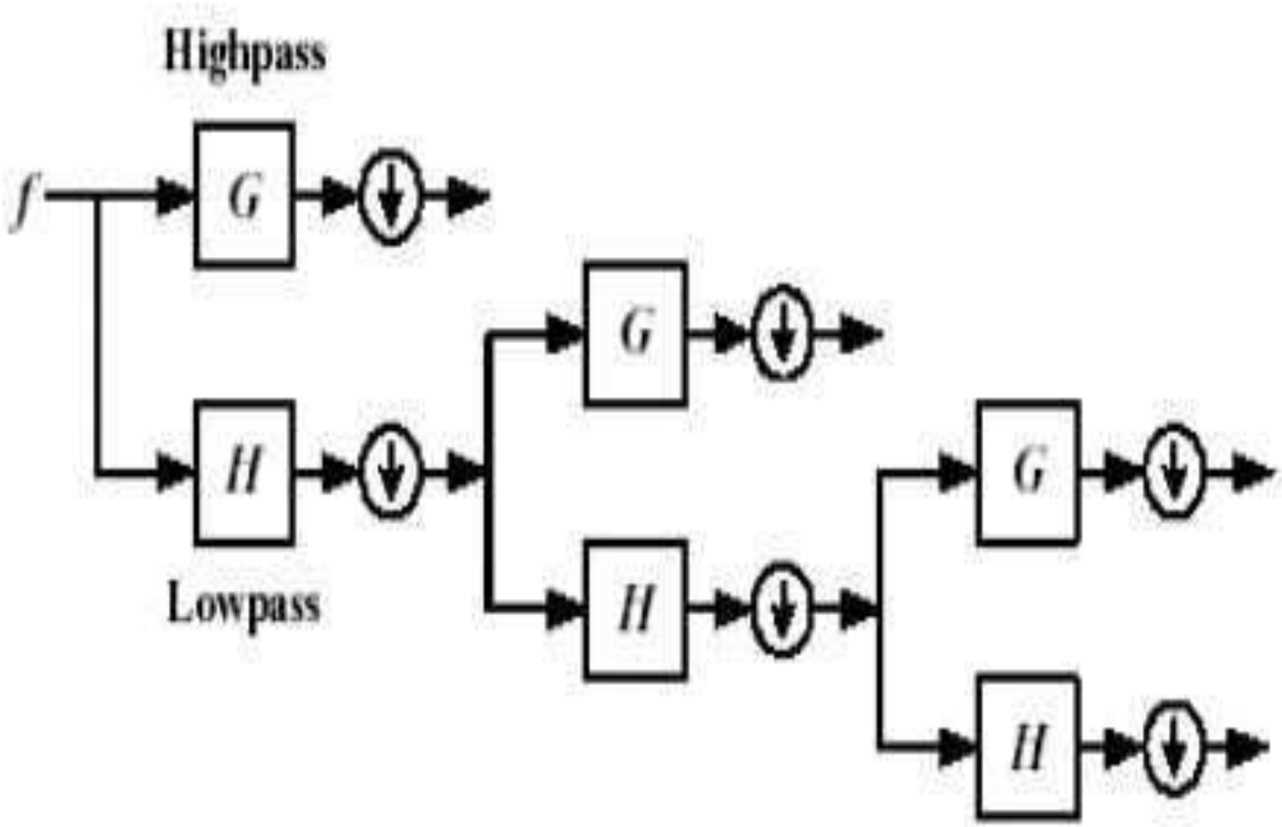
Input signal



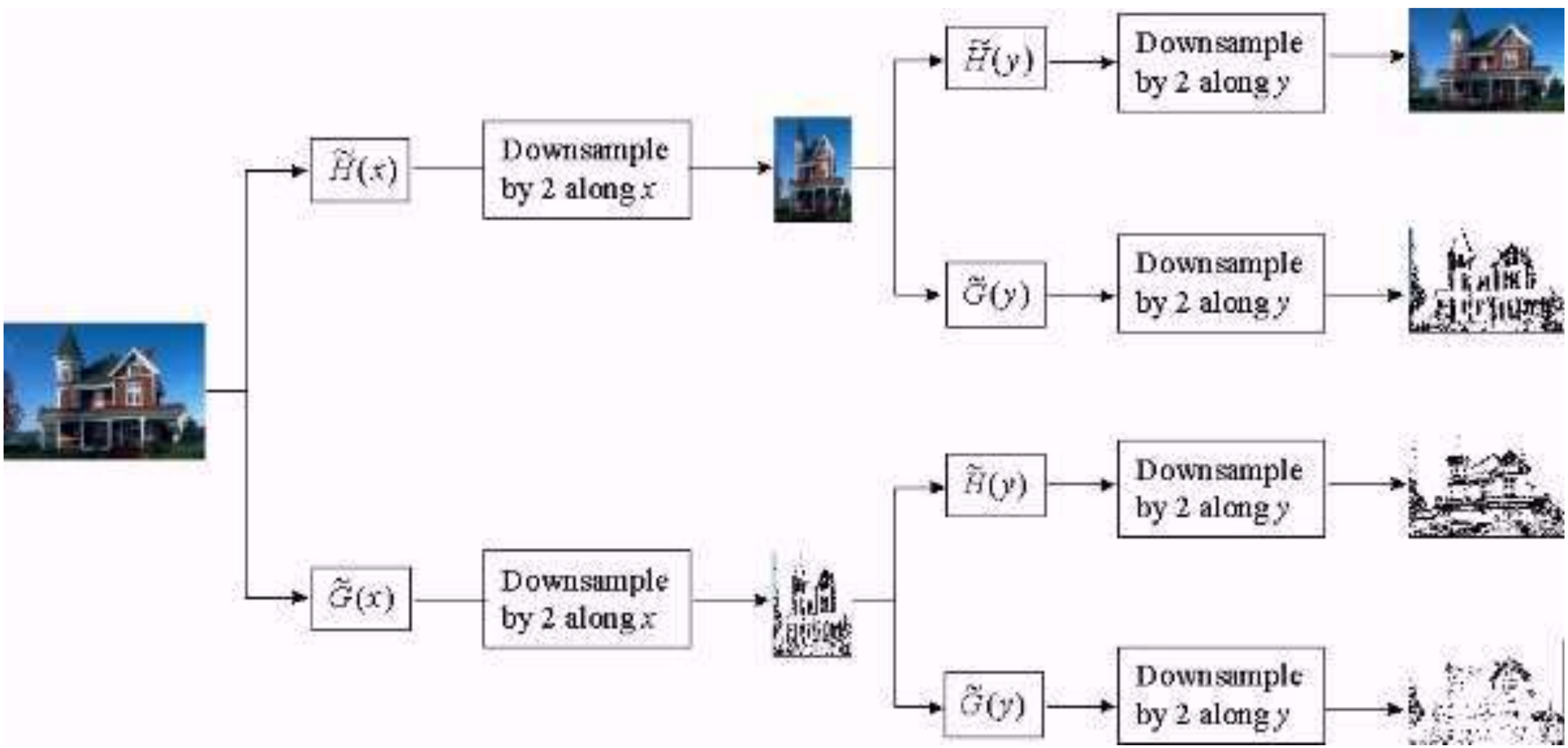
low frequency content of the input signal component.

the high frequency components, of the input signal at various levels (up to level 6)

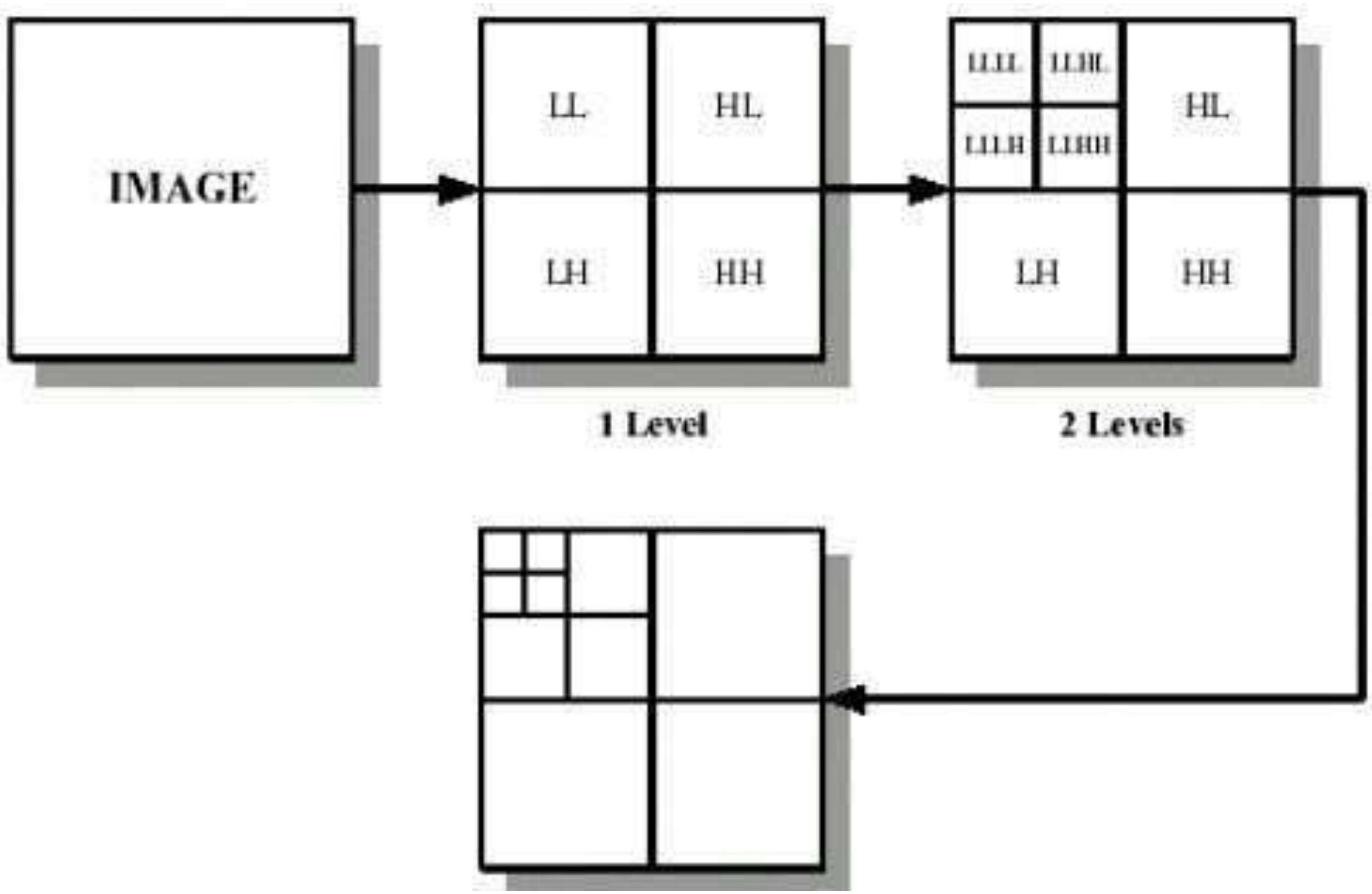
2-D DWT for Image



2-D DWT for Image



2-D DWT for Image



Compression algorithms using DWT

Embedded zero-tree (EZW)

- Use DWT for the decomposition of an image at each level.
- Scans wavelet coefficients subband by subband in a zigzag manner.

Set partitioning in hierarchical trees (SPHIT)

- Highly refined version of EZW.

Perform better at higher compression ratio for a wide variety of images than EZW.

Compression algorithms using DWT (cont..)

Zero-tree entropy (ZTE)

- Quantized wavelet coefficients into wavelet trees to reduce the number of bits required to represent those trees.
- Quantization is explicit instead of implicit, make it possible to adjust the quantization according to where the transform coefficient lies and what it represents in the frame.
- Coefficient scanning, tree growing, and coding are done in one pass.
- Coefficient scanning is a depth first traversal of each tree

Performance

- Peak Signal to Noise ratio used to be a measure of image quality.

- The PSNR between two images having 8 bits per pixel or sample interms of decibels(dBs)is given by:

$$PSNR=10\log10\left(\frac{255^2}{MSE}\right)$$

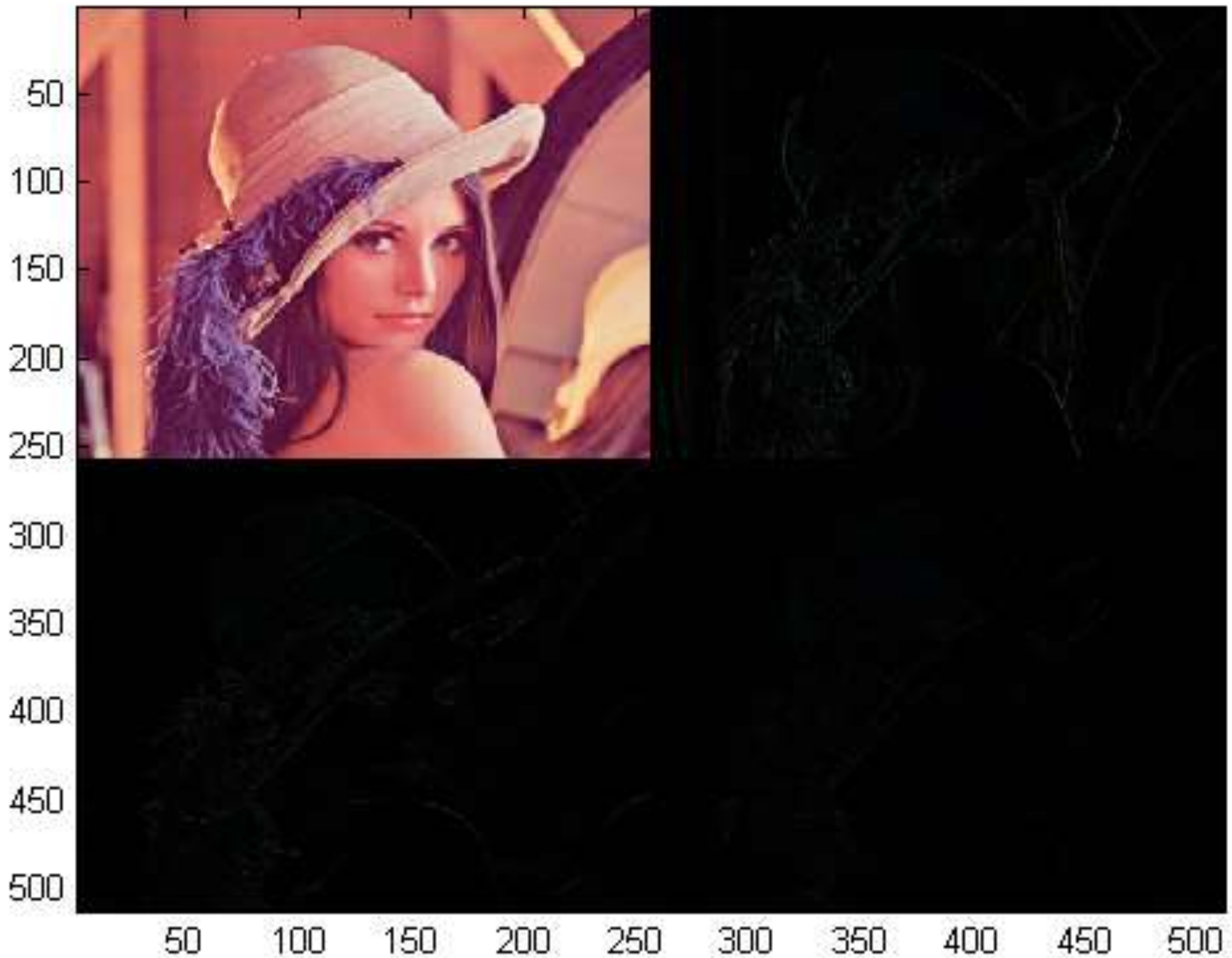
mean square error (MSE)

- Generally when PSNR is 40 dB or greater, then the original and the reconstructed images are virtually indistinguishable by human observers

Lenna Image

Source: http://sipi.usc.edu/database/

Lenna DWT

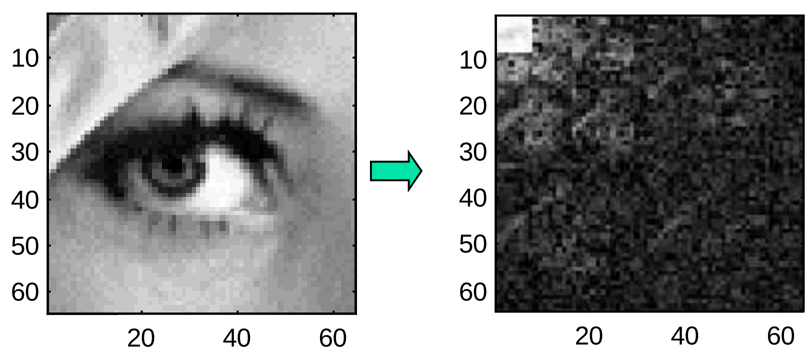


50
100
150
200
250
300
350
400
450
500
50 100 150 200 250 300 350 400 450 500

DWT for Image Compression

- Block Diagram

→ 2D Discrete Wavelet Transform → Quantization → Entropy Coding →



2D discrete wavelet transform (1D DWT applied alternatively to vertical and horizontal direction line by line) converts images into “sub-bands” Upper left is the DC coefficient Lower right are higher frequency sub-bands.

DWT for Image Compression

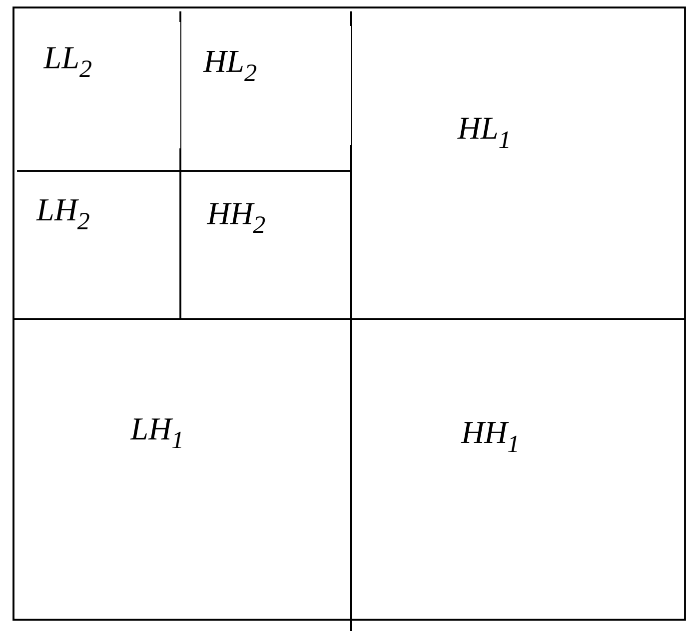
- Image Decomposition
  - Scale 1

LL_1	HL_1
LH_1	HH_1

    - 4 subbands: LL_1, HL_1, LH_1, HH_1
    - Each coeff. $\leftrightarrow$ a 2*2 area in the original image
    - Low frequencies: $0 < |\omega| < \pi/2$
    - High frequencies: $\pi/2 < \omega < \pi$

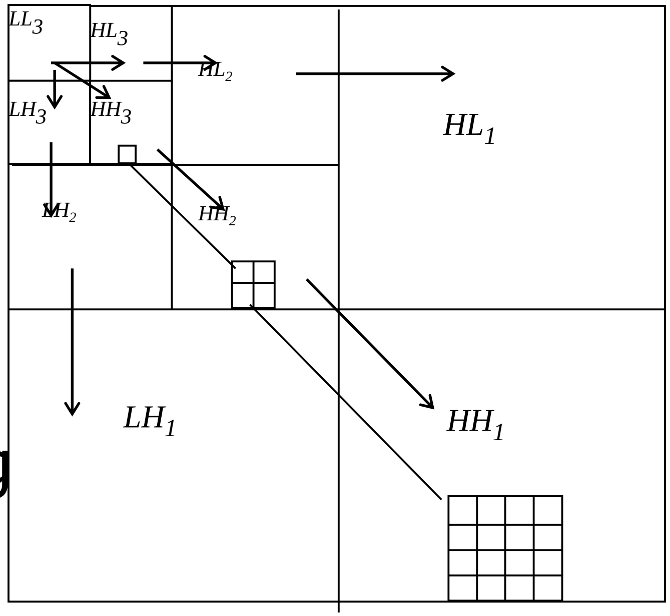
DWT for Image Compression

- Image Decomposition
  - Scale 2 LL_2, HL_2, LH_2, HH_2
  - 4 subbands:
  - Each coeff. $\leftrightarrow$ a 2*2 area in scale 1 image
  - Low Frequency: $0 < |\omega| < \pi/4$
  - High frequencies: $\pi/4 < \omega < \pi/2$



DWT for Image Compression

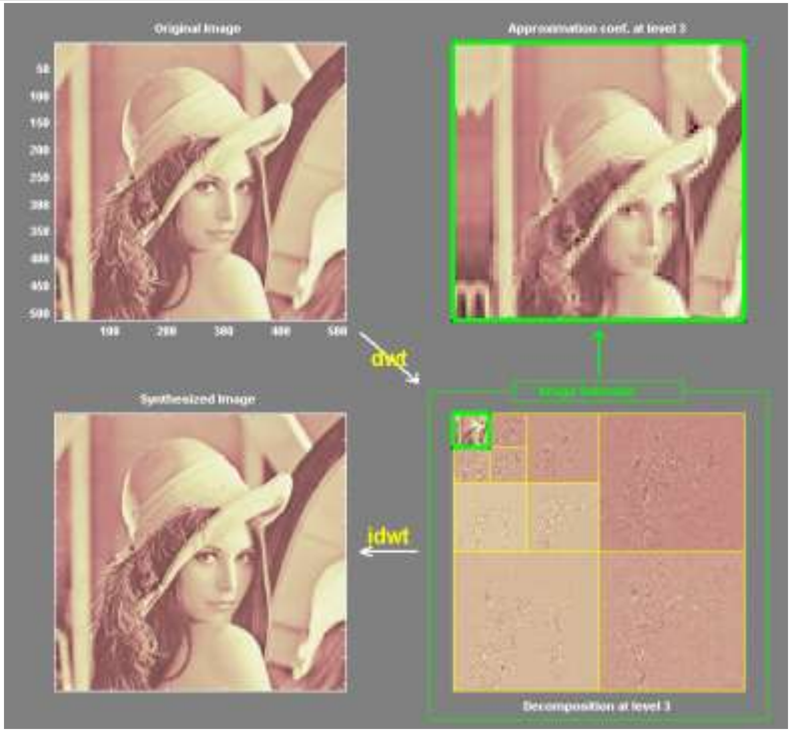
- Image Decomposition
  - Parent
  - Children
  - Descendants: corresponding coeff. at finer scales
  - Ancestors: corresponding coeff. at coarser scales

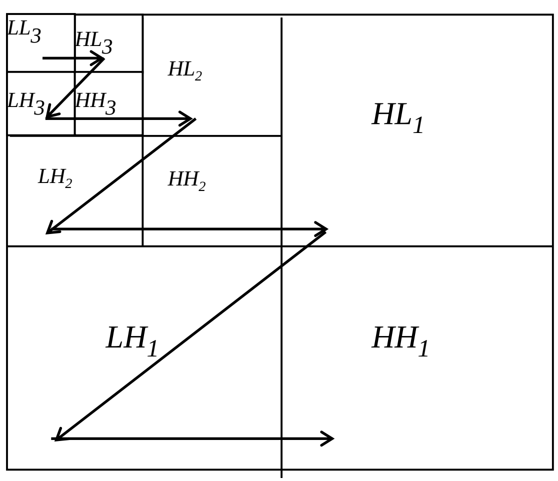


DWT for Image Compression

Image Decomposition

- Feature 1:
  - Energy distribution similar to other TC: Concentrated in low frequencies
- Feature 2:
  - Spatial self-similarity across subbands

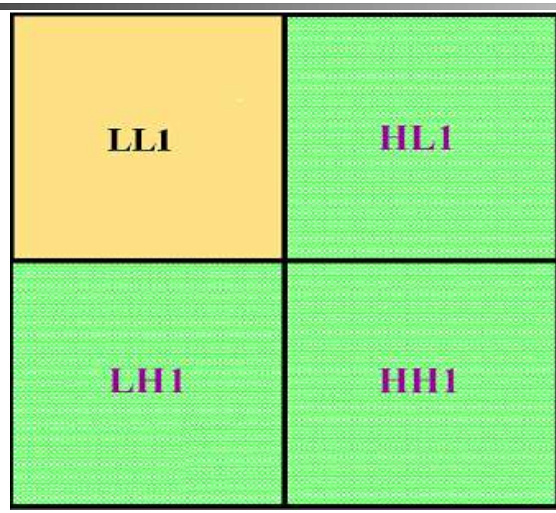




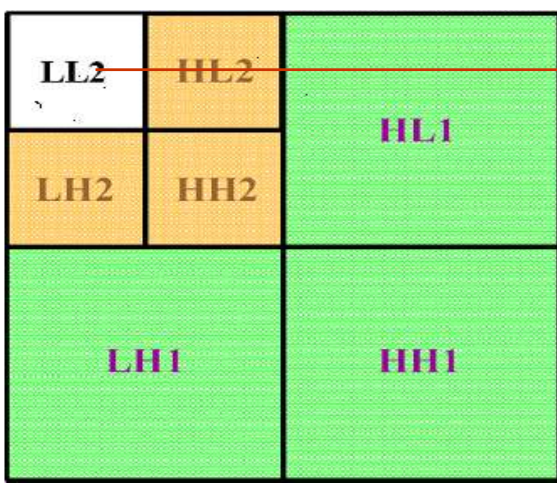
The scanning order of the subbands for encoding the significance map.

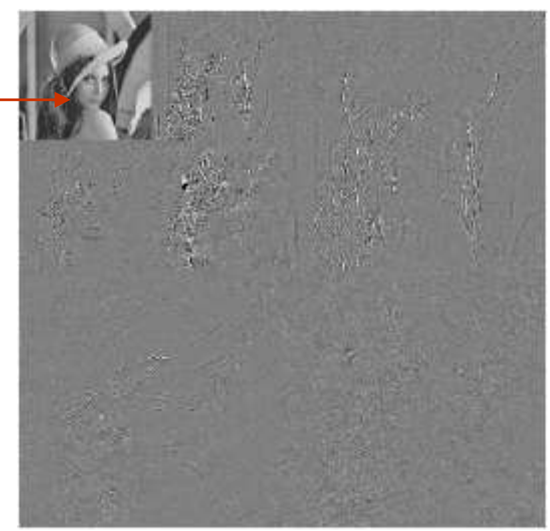
Resolution and Resolution-Increments

1-level DWT

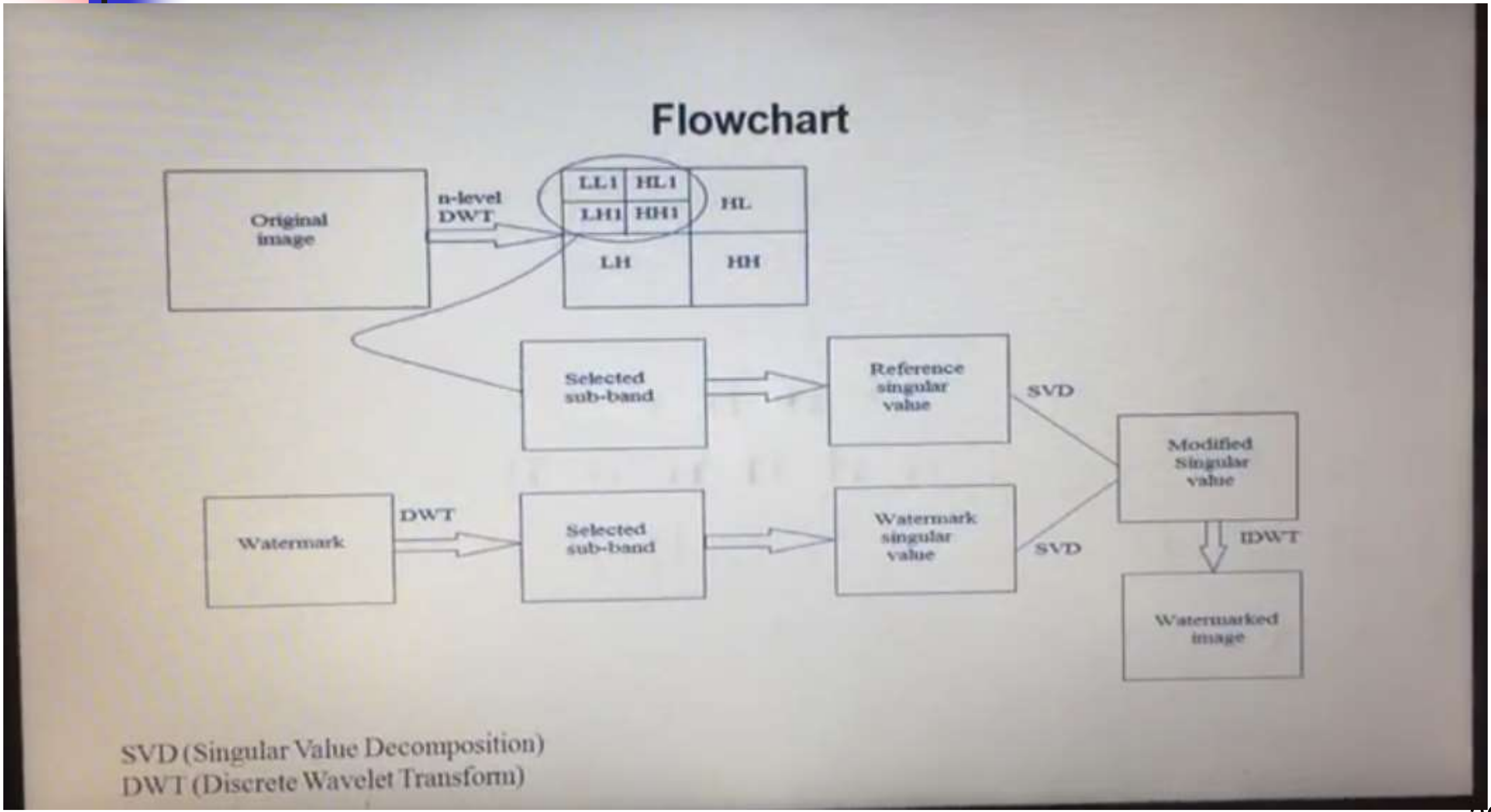


2-level DWT





DWT Watermarking



Thank You!

Any question ??