Assignment 3 Qmm

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```
library(kableExtra)
```

```
## Warning in !is.null(rmarkdown::metadata$output) && rmarkdown::metadata$output
## %in% : 'length(x) = 3 > 1' in coercion to 'logical(1)'
```

```
Transportation_Model <-matrix(c("$20","$14","$25","$400",100,
 "$12","$15","$14","$300",125,
 "$10","$12","$15","$500",150,
 80,90,70,"-","-"),ncol = 5,byrow = TRUE)
colnames(Transportation_Model)<-c("Warehouse 1","Warehouse 2","Warehouse 3","Production Cost","P
roduction Capacity")
rownames(Transportation_Model)<-c("Plant A","Plant B","Plant C","Monthly Demand")
tab<-as.table(Transportation_Model)
```

```
tab%>%
 kable()%>%
 kable_classic()%>%
 column_spec(2,border_left = TRUE) %>%
 column_spec(6,border_left = TRUE)%>%
 row_spec(3,extra_css = "border-bottom:dotted;")
```

	Warehouse 1	Warehouse 2	Warehouse 3	Production Cost	Production Capacity
Plant A	$20	$14	$25	$400	100
Plant B	$12	$15	$14	$300	125
Plant C	$10	$12	$15	$500	150
Monthly Demand	80	90	70	•	•

```
#"Creating a visual representation that illustrates all the potential routes for supplying goods
from three different suppliers to four different demanders."

library(igraph)
```

```
##
## Attaching package: 'igraph'
```

```
## The following objects are masked from 'package:stats':
##
##     decompose, spectrum
```

```
## The following object is masked from 'package:base':
##
##     union
```

```
sources <- c("S1", "S2", "S3")
supply <- c(100, 125, 150)

destinations <- c("W1", "W2", "W3")
demand <- c(80, 90, 70)
```

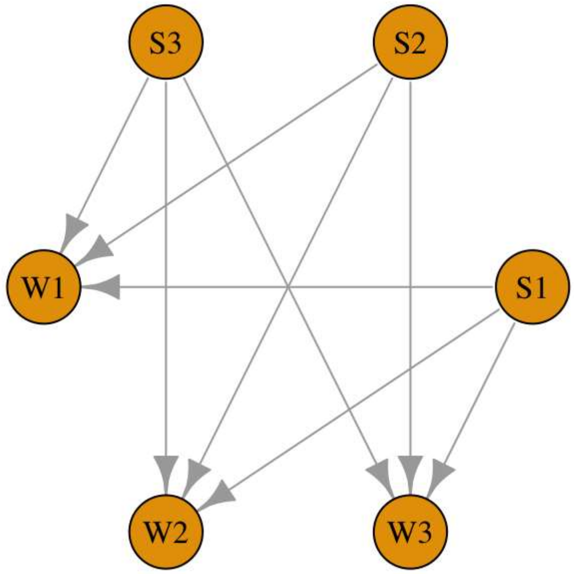
```
#create an empty graph with the total mnumber of vertices
vertices<-length(sources)+length(destinations)
g<-graph.empty(n=vertices,directed=TRUE)
```

```
#"Include nodes or points in the graph to represent the supply sources and demand destinations"
V(g)$name<-c(sources, destinations)
```

```
#"Establish connections between every supply source and every demand destination in the graph."
for(i in 1:length(sources)) {
  for(j in 1:length(destinations)) {
    weight<- min(supply[i], demand[j])
    if(weight>0)
      g<-add_edges(g, edges = c(sources[i],destinations[j]), weight=weight)
  }
}
```

```
#Define a layout with x-coordinates for sources and destinations
layout<-layout_in_circle(g, order = c(1,2,3,4,5,6))
```

```
#"Display the graph on a plot, showing the directed edges and utilizing the user-defined layout.
plot(g, layout = layout,vertex.label.color = "black",vertex.size = 30, edge.arrow.size=1.0)
```



1)-

Formulate and solve this transportation problem using R

“In this scenario, the overall supply amounts to 375 units, yet the total demand or warehouse capacity is limited to just 240 units. To reconcile this disparity, it’s necessary to introduce a fictitious demander or warehouse capacity. This essentially implies that there’s an available inventory that can be utilized by either all or one of the manufacturing plants.”

```
library(kableExtra)
Transportation_Model<-matrix(c("$20","$14","$25","$0","$400",100,
 "$12","$15","$14","$0","$300",125,
 "$10","$12","$15","$0","$500",150,
 80,90,70,135,"-",375),ncol = 6,byrow = TRUE)
colnames(Transportation_Model)<-c("Warehouse 1","Warehouse 2","Warehouse 3","Dummy","Production Cost","Production Capacity")
rownames(Transportation_Model)<-c("Plant A","Plant B","Plant C","Monthly Demand")
tab<-as.table(Transportation_Model)
```

```
tab%>%
 kable()%>%
 kable_classic()%>%
 column_spec(2,border_left = TRUE) %>%
 column_spec(6,border_left = TRUE)%>%
 row_spec(3,extra_css = "border-bottom:dotted;")
```

	Warehouse 1	Warehouse 2	Warehouse 3	Dummy	Production Cost	Production Capacity
Plant A	$20	$14	$25	$0	$400	100
Plant B	$12	$15	$14	$0	$300	125
Plant C	$10	$12	$15	$0	$500	150
Monthly Demand	80	90	70	135	•	375

Now Demand = supply = 375 The total demand and total supply both amount to 375 units.

“The goal of the objective function is to reduce transportation costs to a minimum.”

$$\begin{aligned} Min\,TC = {} & 420X_{11} + 414X_{12} + 425X_{13} + 0X_{14} + \\ & 312X_{21} + 315X_{22} + 314X_{23} + 0X_{24} + \\ & 510X_{31} + 512X_{32} + 515X_{33} + 0X_{34} \end{aligned}$$

##Constraints

#Supply Constraints

For Plant A:

$$X_{11} + X_{12} + X_{13} + X_{14} \leq 100 \;\; (PLANT \;\; A)$$

For Plant B:

$$X_{11} + X_{12} + X_{13} + X_{14} \leq 125 \;\; (PLANT \;\; B)$$

For Plant C:

$$X_{11} + X_{12} + X_{13} + X_{14} \leq 150 \;\; (PLANT \;\; C)$$

#Demand Constraints

From Destination 1:

$$X_{11} + X_{21} + X_{31} \geq 80 \;\; (Destination \;\; 1)$$

From Destination 2:

$$X_{12} + X_{22} + X_{32} \geq 90 \;\; (Destination \;\; 2)$$

From Destination 3:

$$X_{13} + X_{23} + X_{33} \geq 70 \;\; (Destination \;\; 3)$$

Non-negativity of the decision variables:

$$X_{ij} \geq 0 \;\; where \;\; i = 1,2,3 \;\; and \;\; j = 1,2,3,4$$

“Now, we will solve the previously defined transportation model using the LP Solve package.”

```
library(lpSolve)
```

```
"As demand doesn't match supply, we'll introduce dummy variables to balance the problem. Now, w
e'll construct a matrix for the specified objective function."
```

```
## [1] "As demand doesn't match supply, we'll introduce dummy variables to balance the problem.
Now, we'll construct a matrix for the specified objective function."
```

```
tp_model <-matrix(c(420,414,425,0,
 312,315,314,0,
 510,512,515,0),ncol = 4,byrow = TRUE)
tp_model
```

```
##      [,1] [,2] [,3] [,4]
## [1,]  420  414  425    0
## [2,]  312  315  314    0
## [3,]  510  512  515    0
```

```
library("lpSolveAPI")
library("lpSolve")
library("tinytex")
```

```
#"Setting the signs and values for the rows in the problem."
row.signs <- rep("<=",3)
row.rhs <- c(100,125,150)
```

```
#Since it's supply function it cannot be greater than the specified units.
#Defining the column signs and column values
col.signs <- rep(">=",4)
col.rhs <- c(80,90,70,135)
```

```
#Since it's demand function it can be greater than the specified units.
#"Executing the lp.transport function."
lptrans1 <- lp.transport(tp_model,"min", row.signs,row.rhs,col.signs,col.rhs)
```

```
lptrans1$solution #"Obtaining the values of 12 decision variables."
```

```
##      [,1] [,2] [,3] [,4]
## [1,]   10   90    0    0
## [2,]   55    0   70    0
## [3,]   15    0    0  135
```

“Out of the 12 decision variables, we find that 6 of them have non-zero values. Analyzing these non-zero values reveals that Plant A allocates its entire supply of 100 to two different destinations, namely destination 1 and 2. Plant B distributes its output to destinations 1 and 3, without supplying destinations 2 and 4. Plant C’s production is exclusively channeled to the first warehouse.

On the demand side, Warehouse 1 is the sole recipient of supplies from all three suppliers. Demanders 2, 3, and 4, on the other hand, source their supplies from suppliers 3, 1, and 2, respectively.

The minimized cost, as determined by the objective function, represents the value achieved after optimizing the transportation process."

```
lptrans1$objval #"Obtaining the minimized cost, which corresponds to the value of the objective function."
```

```
## [1] 88250
```

2)-Dual of the primal transportation model

"Consider the dual variables linked to the two categories of constraints."

$$U_i \ = \ supply\ constraints\ and\ V_i \ = \ demand\ constraints$$

"In the dual problem, the objective function represents the value added (VA) for the suppliers. The positive portion signifies the gains achieved through delivering products to the demanders, while the negative part accounts for the cost of production. The gap between these components signifies the value added, which can be thought of as profit. Consequently, the objective in the dual problem is to maximize this value added (VA)."

The objective function of the dual is

$$Max\ Z = 80V_1 + 90V_2 + 70V_3 - 100U_1 - 125U_2 - 150U_3$$

Constraints:

$$V_j - U_i \geq C_{ij}$$

As production cost has been given to us so above constraint will be rewritten as:

The source 1 transports goods to four possible destinations. That is

$$V_1 - U_1 \geq C_{11}$$

$$V_1 - 400 \geq 20$$

$$V_2 - U_1 \geq C_{12}$$

$$V_2 - 400 \geq 14$$

$$V_3 - U_1 \geq c_{13}$$

$$V_3 - 400 \geq 25$$

$$V_4 - U_1 \geq c_{14}$$

$$V_4 - 400 \geq 0$$

The source 2 transports goods to four possible destinations.

$$V_1 - U_2 \geq C_{21}$$

$$V_1 - 300 \geq 12$$

$$V_2 - U_2 \geq C_{22}$$

$$V_2 - 300 \geq 15$$

$$V_3 - U_2 \geq C_{23}$$

$$V_3 - 300 \geq 14$$

$$V_4 - U_2 \geq C_{24}$$

$$V_4 - 300 \geq 0$$

The source 3 transports goods to four possible destinations.

$$V_1 - U_3 \geq C_{31}$$

$$V_1 - 500 \geq 10$$

$$V_2 - U_3 \geq C_{32}$$

$$V_2 - 500 \geq 12$$

$$V_3 - U_3 \geq c_{33}$$

$$V_3 - 500 \geq 15$$

$$V_4 - U_3 \geq C_{34}$$

$$V_4 - 500 \geq 0$$

$$All\ V_j \geq 0\ for\ j = 1,2,3,4\ and\ U_i \geq 0\ for\ i = 1,2,3$$

Economic Interpretation of the Dual

$1.MR = MC$

Our dual constraint is $V_j - U_i \geq C_{ij}$. This means $V_j \geq U_i + C_{ij}$. In more detail,$V_3 \geq 400 + 25$. The left side is the per unit income got by selling one unit of the item. This is what we call MR (marginal revenue) in economics.The right side is the per unit cost of making and shipping great. This is called MC (minimal expense). Provider 1 continues to expand creation and delivery to the objections 1 and 2 as long as $V_3 \geq 400 + 25$, that is as long as $MR \geq MC$. On the inverse, provider 1 lessens creation and transportation if$V_3 \leq 400 + 25$, that is, $MR \leq MC$. These both are dynamic circumstances where either creation increments or diminishes. When $V_3 = 400 + 25$, that is,$MR = MC$, the maker neither increments creation nor diminishes it. This is the very thing we call balance for benefit expansion. Accordingly, transportation cost minimization issue is comparable to benefit augmentation in the double and which winds up with $MR = MC$.

2. Hiring or not hiring shipping company for shipping goods If $V_j - U_i \geq C_{ij}$, the provider straightforwardly supplies merchandise from the source to the objective. Be that as it may, assuming the provider finds some other delivery organization who can ship products from the source to the objective fulfilling $V_j - U_i \leq C_{ij}$,then the provider employs the delivery organization as opposed to straightforwardly include in moving products. On the off chance that the maker finds a delivery organization who will transport merchandise fulfilling the constraints $\leq$ rather than $\geq$,then, at that point, the maker recruits the transportation organization.

In this way, if $V_j - U_i \geq C_{ij}$, maker (provider) and the transporter will be something similar. In any case, in the event that $V_j - U_i \leq C_{ij}$, producer(supplier) simply creates merchandise and recruits one more delivery organization for the transportation of products.