Lab Assignment 1

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Question 1

a

The circuit diagram can be seen in figure 1.

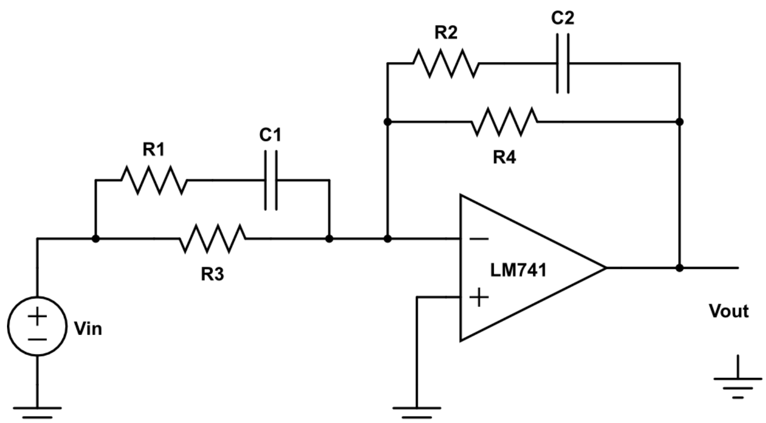

Figure 1: A lead-lag compensator circuit diagram for question 1.

b

If we consider the impedance of the group R_1, C_1 and R_3 as Z_1 and the impedance of the group R_2, C_2 and R_4 as Z_2, we can get the circuit diagram seen in figure 2.This circuit essentially operates as an inverting amplifier and hence the relationship between the output and input is given as:

$$\frac{V_{out}(s)}{V_{in}(s)} = -\frac{Z_2}{Z_1}$$

Where:

$$Z_1 = \frac{R_3 * (R_1 + (1/sC_1))}{R_3 + (R_1 + (1/sC_1))} = \frac{R_3(sR_1C_1 + 1)}{s(R_1C_1 + R_3C_1) + 1} \text{ , and}$$

$$Z_2 = \frac{R_4 * (R_2 + (1/sC_2))}{R_4 + (R_2 + (1/sC_2))} = \frac{R_4(sR_2C_2 + 1)}{s(R_2C_2 + R_4C_2) + 1}$$

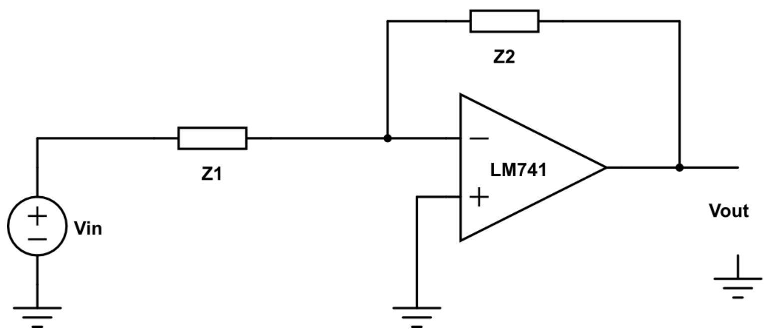

Figure 2: Simplified lead-lag compensator circuit diagram.

Thus the gain of the system can now be re-written as:

$$\frac{V_{out}(s)}{V_{in}(s)} = -\frac{R_4(sR_2C_2+1)}{(s(R_2C_2+R_4C_2)+1)}\frac{(s(R_1C_1+R_3C_1)+1)}{R_3(sR_1C_1+1)}$$

$$= -\frac{R_4}{R_3}\frac{(sR_2C_2+1)}{(s(R_2C_2+R_4C_2)+1)}\frac{(s(R_1C_1+R_3C_1)+1)}{(sR_1C_1+1)}$$

We see that the op-amp operates as an inverting amplifier. In order to match the required transfer function, we need to account for the additional $-\frac{R_4}{R_3}$ gain, which we can do by passing the output of the circuit through another inverting amplifier which has a gain of $-\frac{R_3}{R_4}$[1]. Comparing our final transfer function with what is asked in the question, we get:

$$\Big(-\frac{R_4}{R_3}\times -\frac{R_3}{R_4}\Big)\frac{(sR_2C_2+1)}{(s(R_2C_2+R_4C_2)+1)}\frac{(s(R_1C_1+R_3C_1)+1)}{(sR_1C_1+1)} = \frac{(1+0.1s)(1+5s)}{(1+0.01s)(1+10s)}$$

Comparing the LHS and the RHS, we see:

$$R_1C_1 = 0.01$$

$$R_2C_2 = 5$$

$$R_1C_1 + R_3C_1 = 0.01 + R_3C_1 = 0.1$$

$$\implies R_3C_1 = 0.09$$

$$R_2C_2 + R_4C_2 = 5 + R_4C_2 = 10$$

$$\implies R_4C_2 = 5$$

$$\therefore R_4 = R_2$$

C

While building this circuit, we made the following assumptions:

- We are using an ideal op-amp, i.e. the input impedance is infinite and the output impedance is negligible. This implies that the op-amp does not load it's input circuit and loading effects are not seen on the op-amps output terminal.
- The voltage at the non-inverting terminal is equal to the voltage at the non-inverting terminal.
- The open loop op-amp gain is infinite.

[1] For practical purposes this second op-amp is implemented in LabView as a negative gain.

- The op-amp will operate in an ideal manner when the output is not saturated above or below its supply limits.
- We used potentiometers to tune the resistors to their required value but none of the components used were ideal components. But for the most part we assumed the values of the resistors were accurate and that capacitors had negligible internal resistance.

d

We can determine the ratings of the components by arbitrarily setting C_1 and C_2. We set the capacitances because the values available to us is limited whereas we have significantly more flexibility with choosing resistances. By fixing C_1 and C_2 we find:

$$R_1 = \frac{0.01}{C_1}$$

$$R_2 = \frac{5}{C_2}$$

$$R_3 = \frac{0.09}{C_1} \text{ , and}$$

$$R_4 = R_2$$

We additionally see that the gain of the inverting op-amp lead-lag circuit is proportional to $\frac{R_4}{R_3}$. We want to limit this so that the output does not get saturated wile trying to amplify the signal by a large amount. As we need to use an input amplitude of up to 2V in part (i), for a input supply of +/- 15V, we would like to limit the op-amp output to +/- 10V. Hence we want the gain, $\frac{R_4}{R_3}, \approx 5$.

$$\frac{R_4}{R_3} \approx 5 \implies \frac{(5/C_2)}{(0.09/C_1)} \approx 5$$

$$\therefore \frac{C_1}{C_2} = 0.09$$

For this circuit we chose $C_1 =$ BLALALALAL, hence ratings of the other components became:

Component Name	**Component Type**	**Component Specification**
C_1	Electrolytic Capacitor	μF
C_2	Electrolytic Capacitor	μF
R_1	Resistor	$K\Omega$
R_2	Resistor	$K\Omega$
R_3	Resistor	$K\Omega$
R_4	Resistor	$K\Omega$

e

Physical implementation.

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i

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Question 2

a

The circuit diagram can be seen in figure 3.

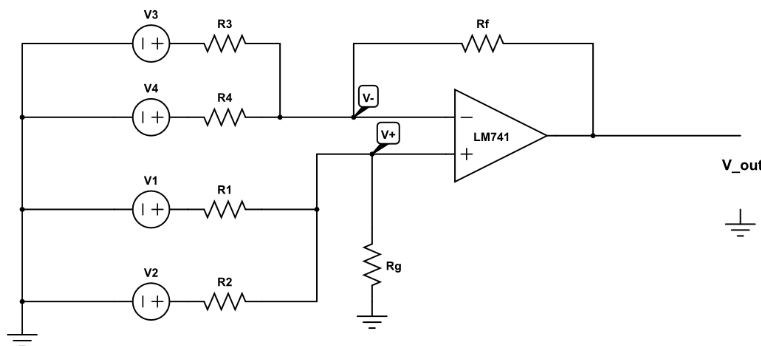

Figure 3: The required circuit diagram for question 2.

b

Circuit analysis:

At the non-inverting terminal, we effectively have a circuit represented by figure 4. We can use Kirchoff's voltage law along with the superposition theorem, we see:

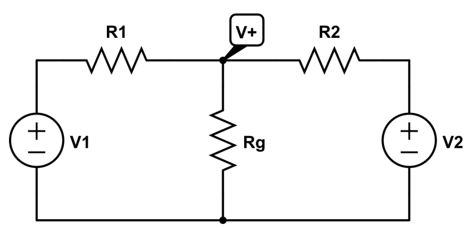

Figure 4: Equivalent circuit at the non-inverting terminal

Voltage at V^+ due to V_1, V_1^+, is given by:

$$V_1^+ = \frac{\frac{R_2 R_g}{R_2+R_g}}{R_1 + \frac{R_2 R_g}{R_2+R_g}} V_1$$

$$\therefore V_1^+ = \Big(\frac{1}{\frac{R_1}{R_g} + \frac{R_1}{R_2} + 1}\Big)\ V_1$$

Similarly, voltage at V^+ due to V_2, V_2^+, is given by:

$$V_2^+ = \frac{\frac{R_1 R_g}{R_1+R_g}}{R_2 + \frac{R_1 R_g}{R_1+R_g}}\ V_2$$

$$\therefore V_2^+ = \frac{1}{\frac{R_2}{R_g} + \frac{R_2}{R_1} + 1}\ V_2$$

Using the superposition theorem to recombine the values, we find that $V+$ is given by:

$$V^+ = V_1^+ + V_2^+$$

$$V^+ = \frac{1}{\frac{R_1}{R_g} + \frac{R_1}{R_2} + 1}\ V_1 + \frac{1}{\frac{R_2}{R_g} + \frac{R_2}{R_1} + 1}\ V_2$$

We can now move on to the inverting terminal of the op-amp. The corresponding equivalent circuit is shown in figure 5. Analyzing this circuit, we see that:

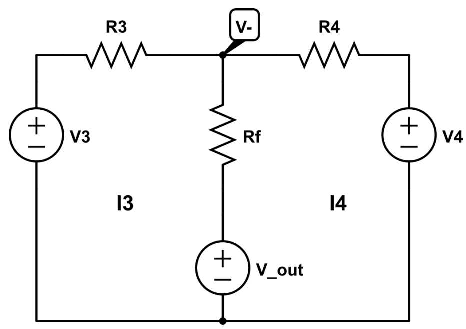

Figure 5: Equivalent circuit at the non-inverting terminal

$$I_3 R_3 = V_3 - V^-$$

$$I_4 R_4 = V_4 - V^-$$

$$V^- - V_{out} = (I_3 + I_4) R_f$$

Substituting from the above equations, we get:

$$V^- - V_{out} = \Big(\frac{V_3 - V^-}{R_3} + \frac{V_4 - V^-}{R_4}\Big)\ R_f$$

Reordering the terms, we get:

$$V_{out} = V^- - \Big(\frac{V_3 - V^-}{R_3} + \frac{V_4 - V^-}{R_4}\Big)\ R_f$$

$$\implies V_{out} = V^-\Big(\frac{R_f}{R_3} + \frac{R_f}{R_4} + 1\Big) - \Big(\frac{R_f}{R_3}\Big)V_3 - \Big(\frac{R_f}{R_4}\Big)V_4$$

We also know that for an ideal op-amp, $V^+ = V^-$, hence we get:

$$V_{out} = \Big(\frac{R_f}{R_3} + \frac{R_f}{R_4} + 1\Big)\Big(\frac{1}{\frac{R_1}{R_g} + \frac{R_1}{R_2} + 1}\Big)V_1 + \Big(\frac{R_f}{R_3} + \frac{R_f}{R_4} + 1\Big)\Big(\frac{1}{\frac{R_2}{R_g} + \frac{R_2}{R_1} + 1}\Big)V_2 - \Big(\frac{R_f}{R_3}\Big)V_3 - \Big(\frac{R_f}{R_4}\Big)V_4$$

This is equation is the mathematical representation of the problem statement which which required $V_{out} = a_1V_1 + a_2V_2 - a_3V_3 - a_4V_4$, i.e. :

$$a_1 = \Big(\frac{R_f}{R_3} + \frac{R_f}{R_4} + 1\Big)\Big(\frac{1}{\frac{R_1}{R_g} + \frac{R_1}{R_2} + 1}\Big) = 1$$

$$a_2 = \Big(\frac{R_f}{R_3} + \frac{R_f}{R_4} + 1\Big)\Big(\frac{1}{\frac{R_2}{R_g} + \frac{R_2}{R_1} + 1}\Big) = 2$$

$$a_3 = \Big(\frac{R_f}{R_3}\Big) = 3$$

$$a_4 = \Big(\frac{R_f}{R_4}\Big) = 4$$

c

[DO WE HAVE ANY MORE??]
While building this circuit, we made the following assumptions:

- We are using an ideal op-amp, i.e. the input impedance is infinite and the output impedance is negligible. This implies that the op-amp does not load it's input circuit and loading effects are not seen on the op-amps output terminal.
- The voltage at the non-inverting terminal is equal to the voltage at the non-inverting terminal.
- The open loop op-amp gain is infinite.
- The op-amp will operate in an ideal manner when the output is not saturated above or below its supply limits.
- We used potentiometers to tune the resistors to their required value but none of the components used were ideal components. But for the most part we assumed the values of the resistors were accurate and that capacitors had negligible internal resistance.

d

We started by choosing R_f, R_g and R_g to arbitrary resistances such that they were large enough to make sure that other resistances in the circuit were not small. By this we mean that the values of the other resistances were in the order of Kilo-Ohms. From section **b**, we can rewrite gains a_1 and a_2 in terms of a_3 and a_4 as follows:

$$a_1 = \Big(a_3 + a_4 + 1\Big)\Big(\frac{1}{\frac{R_1}{R_g} + \frac{R_1}{R_2} + 1}\Big) = 1$$

$$= (8)\Big(\frac{1}{\frac{R_1}{R_g} + \frac{R_1}{R_2} + 1}\Big) = 1$$

We can rewrite this as:

$$8R_2R_g = R_1R_2 + R_1R_g$$

$$\implies R_g = \frac{R_1R_2}{7R_2 - R_1}$$

Similarly for a_2, we see:

$$a_2 = \Big(a_3 + a_4 + 1\Big)\Big(\frac{1}{\frac{R_2}{R_g} + \frac{R_2}{R_1} + 1}\Big) = 2$$

$$= (8)\Big(\frac{1}{\frac{R_2}{R_g} + \frac{R_2}{R_1} + 1}\Big) = 2$$

We can re-arrange and re-write the above equations as:

$$R_g = \frac{R_1 R_2}{3R_1 - R_2}$$

Equating the two equations which equal R_g, we get:

$$\frac{R_1 R_2}{7R_2 - R_1} = \frac{R_1 R_2}{3R_1 - R_2}$$

Thus, we get:

$$R_2 = \frac{R_1}{2}$$

Thus we arbitrarily chose $R_f = R_g = 10\ K\Omega$ and $R_1 = 50\ K\Omega$ We ended up using the following components in our circuit:

Component Name	Component Type	Component Specification
R_f	Resistor	9.880 $K\Omega$
R_g	Resistor	9.860 $K\Omega$
R_1	Resistor	49.3 $K\Omega$
R_2	Resistor	24.65 $K\Omega$
R_3	Resistor	3.293 $K\Omega$
R_4	Resistor	2.470 $K\Omega$

e

Physical implementation.

f

John

g

John

h

The beating signal is achieved as a sinusoidal wave which has an amplitude that also varies sinusoidally. In effect, the output is $\sin(\omega_a t)\sin(\omega_b t)$. We can use trigonometric identities to rearrange this sinusoidal product as a sum as follows:

$$\sin(\omega_a t)\sin(\omega_b t) = \frac{\cos(\omega_a t + \omega_b t) - \cos(\omega_a t + \omega_b t)}{2}$$

We use the product-to-sum identity to break up the product of the sines into a sum of cosines. We can again rewrite the cosines in terms of sines by taking into account the phase difference.

$$\frac{\cos(\omega_a t + \omega_b t) - \cos(\omega_a t + \omega_b t)}{2} = \frac{\sin((\omega_a t + \omega_b t) + \pi/2) - \sin((\omega_a t + \omega_b t) + \pi/2)}{2}$$

This way, we can achieve a beating signal as a sum of sinusoids. For the purposes of this question, we chose $\omega_a = 1$ and $\omega_b = 10$ The values for each of the output terminals is as follows:

The required LabView screen shot can be seen in figure 6

	V1	V2	V3	V4
Amplitude (Volts)	3	0	1	0
Frequency (Hz)	-9	0	11	0
Offset (Volts)	0	0	0	0
Phase (Radians)	$\frac{\pi}{2}$	0	$\frac{\pi}{2}$	0

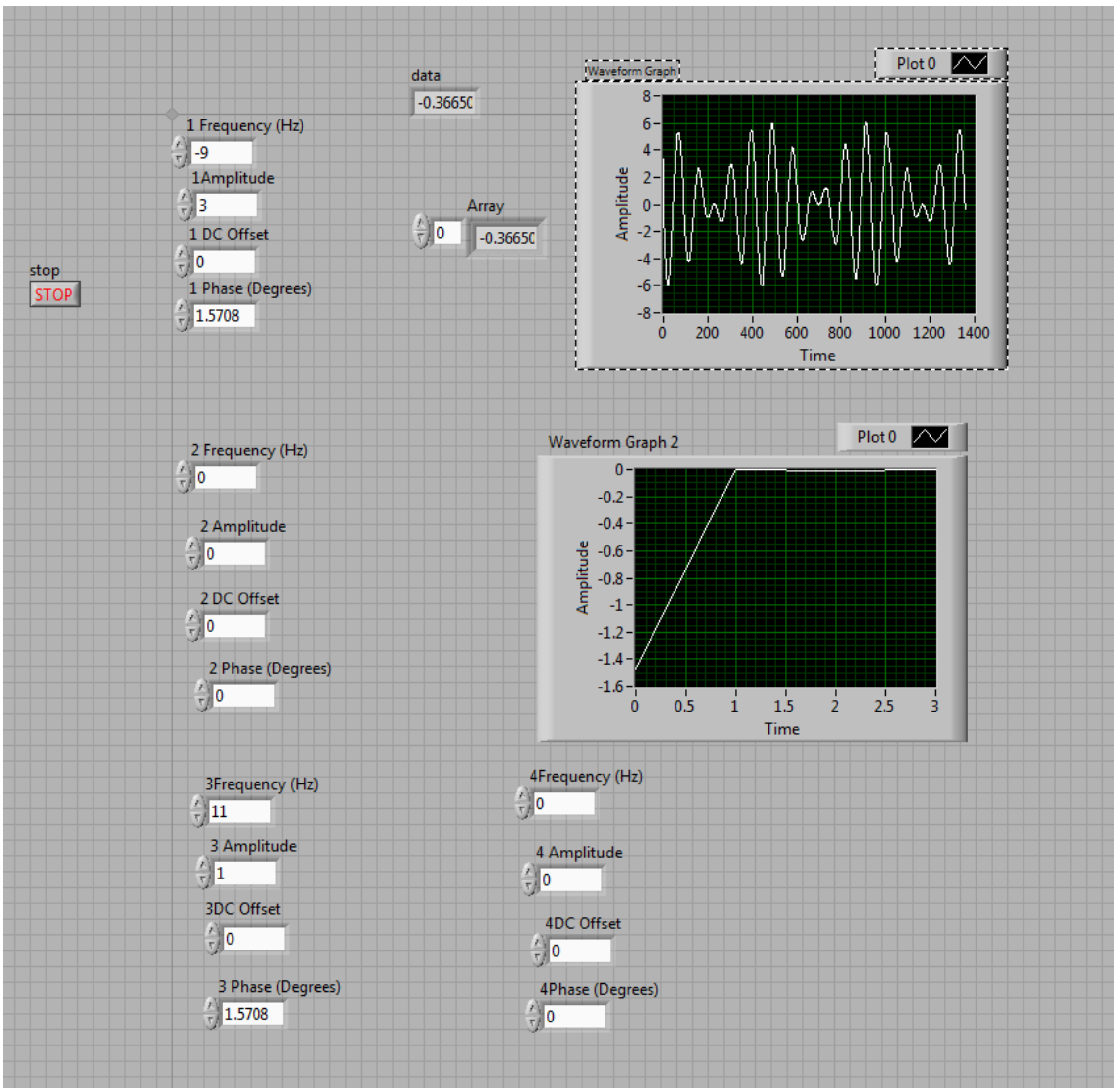

Figure 6: The LabView front-end of a system that produces $\sin(\omega_a t_a)\sin(\omega_b t_b)$.