Lab Assignment 4: Inverted Pendulum

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Teamwork Participation Pledge :: Team 1

I attest that I have made a fair and equitable contribution to this lab and submitted assignment.

My signature also indicates that I have followed the University of Michigan Honor Code, while working on this lab and assignment.

I accept my responsibility to look after all of the equipment assigned to me and my team, and that I have read and understood the X50 Lab Rules.

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1.

need figure to show coordinate frames!

a. Physical Model

We model the physical system as a pair of rigid links, the horizontal arm and the pendulum mounted to the end of the arm, neglecting the presence of the encoder cable, using a lumped parameter model.

b. Assumptions

We assume that all of our links are rigid and assume allowing us to neglect work done by the internal constraining forces of the system. This assumption very nearly holds because e are using stiff metals and bearings that are well fitted which should all but prevent internal losses due to collisions and deformations. We neglect air resistance because of the small cross sectional areas and low velocities combined with the difficult in constructing an appropriate model and determining its forces. We assume that the axis of rotation of the horizontal arm is fixed in space and perfectly level with the ground. This assumption overlaps with neglecting losses due to internal constraints and ensures that there will be no potential energy term associated with the arm and avoids the need to consider the motor stand, the table beneath it etc. We also neglect the presence of the encoder cable connected to the arm. We found that the cable has a large impact on our system however its influence would be very difficult to model because it is not fixed to the arm and because the forces it introduces are highly non linear.

c. Mathematical Modeling

Forwards Kinematics

Let ${}^0F$ represent the fixed world coordinate frame, with $\mathbf{{}^0\hat{z}}$ aligned with the motor shaft and with $\mathbf{{}^0\hat{x}}$ pointing along the initial direction of the horizontal arm when the system is initialized. Let the origin of ${}^0F$ be located to coincide with the motor axis and the axis of rotation of the pendulum. Let ${}^1F$, representing the local frame of the horizontal arm, be attached to the horizontal arm, with $\mathbf{{}^1\hat{z}}$ aligned with the motor shaft and $\mathbf{{}^1\hat{x}}$ aligned with the motor arm pointed towards the pendulum. Let ${}^2F$ be located such that its origin is along $\mathbf{{}^1\hat{x}}$ at a distance l_1, with $\mathbf{{}^2\hat{x}}$ pointing along the pendulum and $\mathbf{{}^2\hat{z}}$ along the $-\mathbf{{}^1\hat{x}}$ direction. Where l_1 is the distance from the axis of rotation of the motor to the center of mass of the pendulum along the $\mathbf{{}^1\hat{x}}$ direction. Let θ denote the angle between $\mathbf{{}^0\hat{x}}$ and $\mathbf{{}^1\hat{x}}$ and let α denote the angle between $\mathbf{{}^1\hat{z}}$ and $\mathbf{{}^2\hat{x}}$ measured clockwise when viewed from $\mathbf{{}^1\hat{x}}$. We assume that the motor and the horizontal arm are rigidly connected and can be treated as a single inertia for purposes of our energy calculations below.

Then:

$$ {}^0F = R_z(\theta)T_x(l_1)R_y(-\pi/2)R_z(\alpha)\left({}^2F\right) $$

Let l_{2c} be the distance along $\mathbf{{}^2x}$ from the origin of ${}^2F$ to the center of mass of the pendulum. Then the location of the center of mass of the pendulum in the world coordinate system $\mathbf{{}^0P_{2c}}$ is given as:

$$ \mathbf{{}^0P_{2c}} = R_z(\theta)T_x(l_1)R_y(-\pi/2)R_z(\alpha)\left(\mathbf{{}^2P_{2c}}\right) = R_z(\theta)T_x(l_1)R_y(-\pi/2)R_z(\alpha)\left[l_{2c}, 0, 0\right]^T $$

$$ \mathbf{{}^0P_{2c}} = \begin{bmatrix} -l_{2c}\sin(\theta)\sin(\alpha) + l_1\cos(\theta) \\ l_{2c}\cos(\theta)\sin(\alpha) + l_1\sin(\theta) \\ l_{2c}\cos(\alpha) \end{bmatrix} $$

From this position we can then determine the linear velocity of the center of mass of the pendulum in the world frame denoted $\mathbf{{}^0\dot{P}_{2c}}$. Let J denote the matrix of partial derivatives of $\mathbf{{}^0P_{2c}}$ with respect to θ and α. Then the $\mathbf{{}^0\dot{P}_{2c}}$ is given by.

$$
{}^{\mathbf{0}}\dot{\mathbf{P}}_{\mathbf{2c}} = J \begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} -l_{2c}sin(\alpha)\cos(\theta) - l_1 \sin(\theta) & -l_{2c}\sin(\theta)\cos(\alpha) \\ -l_{2c}sin(\alpha)\sin(\theta) + l_1\cos(\theta) & l_{2c}\cos(\theta)\cos(\alpha) \\ 0 & -l_{2c}\sin(\alpha) \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \end{bmatrix}
$$

We can also determine the angular velocity of the pendulum about its center of mass in the pendulum fixed frame ${}^2F$ by the following:

$$
\omega_{\mathbf{2}} = \dot{\theta}({}^{\mathbf{1}}\hat{\mathbf{z}}) + \dot{\alpha}({}^{\mathbf{2}}\hat{\mathbf{z}}) = \left(R_y(-\pi/2)R_z(\alpha)\right)^{-1}\left[0, 0, \dot{\theta}\right]^T + \dot{\alpha}({}^{\mathbf{2}}\hat{\mathbf{z}})
$$

$$
{}^{\mathbf{2}}\omega_{\mathbf{2}} = \begin{bmatrix} \dot{\theta}\cos(\alpha) \\ -\dot{\theta}\sin(\alpha) \\ \dot{\alpha} \end{bmatrix}
$$

Rather than concern our selves with the forwards kinematics of the horizontal arm, because we assume that its only motion is a rotation about ${}^{\mathbf{0}}\hat{\mathbf{z}}$ which allows us to simply apply the parallel axis theorem to obtain its moment of inertia

System Energy

Our system is composed of two rigid bodies, then the horizontal arm and the pendulum. Each has an energy denoted E_1 and E_2 respectively. Using the forwards kinematics derived above we can compute the kinetic and potential energies associated with each body as a function of θ, α, $\dot{\theta}$, and $\dot{\alpha}$.

$$
E_1 = T_1 + V_1 \qquad E_2 = T_2 + V_2
$$

By our assumption above that ${}^{\mathbf{1}}\hat{\mathbf{z}}$ is parallel to the direction of gravity, the term V_1 can be taken to be 0. Furthermore, rather than expand T_1 into linear and rotational components, we can simply use a single rotation term with a modified moment of inertia, I_{1z1}. Let l_{1c} be the distance along ${}^{\mathbf{1}}\hat{\mathbf{x}}$ from the origin to the center of mass of the rotor ${}^1P_{1c}$, and let m_1 be the mass of the horizontal arm. Note that we must also include the rotor inertia :

$$
E_1 = I_{1z1}\dot{\theta}^2 = \left(I_{1zzc} + I_{rotor} + m_1 l_{1c}^2\right)\dot{\theta}^2
$$

The expression for E_2 is much more complex, we will break the kinetic energy into a translational and a rotational component.

Because the pendulum is perpendicular to the horizontal arm which is perpendicular to the ground, the potential energy of the pendulum, V_2 only depends on alpha, the distance from the axis of rotation to the center of mass along ${}^{\mathbf{2}}\hat{\mathbf{x}}$, l_{2c}, the mass of the pendulum, m_2, and the acceleration due to gravity, g. Without loss of generality, we assume that zero potential energy occurs for $\alpha = 0$.

$$
V_2 = (\cos(\alpha) - 1)\, l_{2c} m_2 g
$$

The translation kinetic energy of the pendulum T_{2T} is given by the following expression:

$$
T_{2T} = \frac{1}{2} m_2 \left({}^{\mathbf{0}}\dot{\mathbf{P}}_{\mathbf{2c}}\right)^2 = \frac{1}{2} m_2 \left({}^{\mathbf{0}}\dot{\mathbf{P}}_{\mathbf{2c}}\right)^T \left({}^{\mathbf{0}}\dot{\mathbf{P}}_{\mathbf{2c}}\right) = \frac{1}{2} m_2 \left[\dot{\theta}, \dot{\alpha}\right] J^T J \begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \end{bmatrix}
$$

$$
T_{2T} = \dot{\theta}^2\left(l_{2c}^2 \sin^2(\alpha) + l_1^2\right) + 2 l_1 l_{2c}\dot{\theta}\dot{\alpha}\cos(\alpha) + \left(\dot{\alpha} l_{2c}\right)^2
$$

The rotational kinetic energy of the pendulum is given by the following expression where I_2 is the 3 by 3 moment of inertia tensor of the pendulum.

$$
T_{2R} = \frac{1}{2}{}^{\mathbf{2}}\omega_{\mathbf{2}}{}^T I_2 {}^{\mathbf{2}}\omega_{\mathbf{2}} = \frac{1}{2}\left[\dot{\theta}^2\left(I_{2xx}\cos^2(\alpha) - 2I_{2xy}\sin(\alpha)\cos(\alpha) + I_{2yy}\sin^2(\alpha)\right) + 2I_{2xz}\dot{\theta}\dot{\alpha}\sin(\alpha) + I_{2zz}\dot{\alpha}^2\right]
$$

Note this is where the arbitrary sign flip is!!!

As discussed later I_{2xy} and I_{2yz} are very nearly 0 and have been neglected for the rest of the derivation.

$$T_{2R} = \frac{1}{2}\left[\dot{\theta}^2\left(I_{2xx}\cos^2(\alpha) + I_{2yy}\sin^2(\alpha)\right) + I_{2zz}\dot{\alpha}^2\right] - I_{2xz}\dot{\theta}\dot{\alpha}\cos(\alpha)$$

Lagrange's Equation and non-Linear Equations of Motion

The Lagrange method can be used to compute the equations of motion of complex dynamical systems using the total system energy derived above along with the generalized coordinates of the system q_i and generalized non-conservative forces on the system Q_{NCi}. These non-conservative forces are the damping forces exerted on both the pendulum and the horizontal arm determined by b_α and b_θ respectively, the motor torque exerted on the horizontal arm, τ_m, and the force of coulomb friction exerted on both the pendulum and the horizontal arm denoted $\tau_{C\alpha}$ and $\tau_{C\theta}$ respectively.

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} - \frac{\partial \mathcal{L}}{\partial q_i} = Q_{NCi}$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} - \frac{\partial \mathcal{L}}{\partial \theta} = -b_\theta\dot{\theta} + \tau_m + \tau_{C\theta}$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\alpha}} - \frac{\partial \mathcal{L}}{\partial \alpha} = -b_\alpha\dot{\alpha} + \tau_{C\alpha} \tag{1}$$

For our friction model here we must make a simplifying assumption to avoid having to estimate internal constraint forces. In a correct model to handle static friction correctly if the joint angle, $\dot{\theta}$ or $\dot{\alpha}$ is 0, then we must estimate the torque exerted on the joint attempting to overcome static friction. This torque can really be thought of as the constraint torque required to keep the angle fixed, but our Lagrange method sacrifices knowledge of these constraint forces to simplify the equation of motion. To avoid having to estimate this constraint torque we make a simplifying assumption. We only consider the external torques acting on each joint, rather than the internal forces at the joint. These external torques are known, simply the motor torque, τ_m for the horizontal arm and the torque exerted by gravity on the pendulum τ_g.

$$\tau_g = gm_2 l_{2c}\sin(\alpha)$$

With this assumption we can use our previous coulomb friction model with no change. Note that we again use two different frictional torques for the horizontal arm because our previous experiments with the motor discovered significant differences in friction in the forwards and reverse directions. The pendulum has been modeled with a single friction torque because we have no reason to believe that the direction of rotation changes friction.

$$\tau_{C\alpha} = \begin{cases} -\tau_g & \text{if } \dot{\alpha} = 0 \text{ and } |\tau_g| < \tau_{F\alpha} \\ -sgn(\tau_g)\cdot\tau_{FA} & \text{if } \dot{\alpha} = 0 \text{ and } |\tau_{motor}| > \tau_{F\alpha} \\ -sgn(\dot{\alpha})\cdot\tau_{F\alpha} & \text{if } \dot{\alpha} \neq 0 \end{cases}$$

$$\tau_{C\theta} = \begin{cases} -\tau_m & \text{if } \dot{\theta} = 0 \text{ and } 0 \le \tau_{motor} < \tau_{F\theta F} \\ -\tau_m & \text{if } \dot{\theta} = 0 \text{ and } -\tau_{F\theta R} \le \tau_m < 0 \\ -sgn(\tau_m)\cdot\tau_{F\theta F} & \text{if } \dot{\theta} = 0 \text{ and } \tau_m > \tau_{F\theta F} \\ -sgn(\tau_m)\cdot\tau_{F\theta R} & \text{if } \dot{\theta} = 0 \text{ and } \tau_m \le -\tau_{F\theta R} \ge \\ -sgn(\dot{\theta})\cdot\tau_{F\theta F} & \text{if } \dot{\theta} > 0 \\ -sgn(\dot{\theta})\cdot\tau_{F\theta R} & \text{if } \dot{\theta} < 0 \end{cases}$$

The right hand side of the equation contains partial derivatives of the Lagrangian, $\mathcal{L}$, which is given below as the difference of the total kinetic energy of the system, T, and the total potential energy of the system V. Using the expressions derived above for the energies of each component of the system we can construct an expression for $\mathcal{L}$ and take partial derivatives

$$\mathcal{L} = T - V = T_1 + T_2 - (V_1 + V_2) = T_1 + T_{2R} + T_{2T} - V_2$$

$$\mathcal{L} = \frac{1}{2}\dot{\theta}^2\left(I_{1z1} + C_2\sin^2(\alpha) + I_{2xx}\cos^2(\alpha) + m_2 l_1^2\right) + \frac{1}{2}C_1\dot{\alpha}^2 + C_3\dot{\theta}\dot{\alpha}\cos(\alpha) - C_4\cos(\alpha) + C_4$$

$$C_1 = I_{2zz} + m_2 l_{2c}^2 \qquad C2 = I_{2yy} + m_2 l_{2c}^2 \qquad C_3 = m_2 l_1 l_{2c} - I_{2xz} \qquad C_4 = l_{2c} m_2 g$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \ddot{\theta}\left(I_{1z1} + C_2\sin^2(\alpha) I_{2xx}\cos^2(\alpha) + m_2 l_1^2\right) + 2\left(C_2 - I_{2xx}\right)\dot{\theta}\dot{\alpha}\sin(\alpha)\cos(\alpha) + C_3\ddot{\alpha}\cos(\alpha) - C_3\dot{\alpha}^2\sin(\alpha)$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = 0$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\alpha}} = C_1\ddot{\alpha} + C_3\ddot{\theta}\cos(\alpha) - C_3\dot{\theta}\dot{\alpha}\sin(\alpha)$$

$$\frac{\partial \mathcal{L}}{\partial \alpha} = \dot{\theta}^2\left(C_2\sin(\alpha)\cos(\alpha) - I_{2xx}\cos(\alpha)\sin(\alpha)\right) - C_3\dot{\theta}\dot{\alpha}sin(\alpha) + C_4\sin(\alpha)$$

fix centering on this equation and probably the next one

$$\begin{aligned}\ddot{\theta}\left(I_{1z1} + C_2\sin^2(\alpha) + I_{2xx}\cos^2(\alpha) + m_2 l_1^2\right) + 2(C_2 - I_{2xx})\dot{\theta}\dot{\alpha}\sin(\alpha)\cos(\alpha) \\ + C_3\ddot{\alpha}\cos(\alpha) - C_3\dot{\alpha}^2\sin(\alpha) = -b_\theta\dot{\theta} + \tau_m + \tau_{C\theta}\end{aligned} \tag{2}$$

$$C_1\ddot{\alpha} + C_3\,\ddot{\theta}\cos(\alpha) - (C_2 - I_{2xx})\dot{\theta}^2\sin(\alpha)\cos(\alpha) - C_4\sin(\alpha) = -b_\alpha\dot{\alpha} + \tau_{C\alpha} \tag{3}$$

The pair of non-linear differential equations can be used to numerically model the behavior of the system, but they are not particularly useful for actual controller design. Both state space controls and our traditional techniques require the system to be linearized around a particular operating point. This allows us to generate a transfer function to approximate the behavior of the system and lets us construct root locii and bode plots to quantify system behavior.

d.

put some pictures of the non-linear model make make references to matlab code

e.

Linearized Equations of Motion

We will linearize the equations of motion about $\theta = 0$, $\alpha = 0$. By taylor expansion about this point and dropping higher order terms we obtain:

$$\sin(\alpha) \approx 0 \qquad \cos(\alpha) \approx \alpha \qquad \sin(\theta) \approx 0 \qquad \cos\theta \approx \theta$$

We note that the coulomb friction terms from the previous non-linear equations must be dropped completely as there is no reasonable linearization of the effects of friction. We also note that we expect $\dot{\theta}$ and $\dot{\alpha}$ to be relatively small, which implies:

$$\dot{\theta}\dot{\alpha} \approx 0$$

Applying these to equations (2) and (3) yields (4) and (5) respectively.

$$\ddot{\theta}\left(I_{1z1} + I_{2xx} + m_2 l_1^2\right) + \ddot{\alpha}C_3 = -b_\theta\dot{\theta} + \tau_m \tag{4}$$

$$\ddot{\alpha}C_1 + \ddot{\theta}C_3 - \alpha C_4 = -b_\alpha \dot{\alpha} \tag{5}$$

Using equations (4) and (5) together we can generate two independent equations for $\ddot{\alpha}$ and $\ddot{\theta}$.

$$C_5 = (I_{1z1} + I_{2xx} + m_2 l_1^2)$$

$$\ddot{\theta} = \frac{C_3 b_\alpha \dot{\alpha} - C_3 C_4 \alpha - C_1 b_\theta \dot{\theta} + C_1 \tau_m}{C_5 C_1 - C_3^2} \tag{6}$$

$$\ddot{\alpha} = \frac{-C_5 b_\alpha \dot{\alpha} + C_3 b_\theta \dot{\theta} + \alpha C_4 C_5 - C_3 \tau_m}{C_5 C_1 - C_3^2} \tag{7}$$

State Space Model

need some basic state space explanation here clearly state A, B, C, and D matricies need to look back at formulation for their meaning

$$Q = [q_1, q_2, q_3, q_4]^T = [q_1, \dot{q}_1, q_3, \dot{q}_3]^T = [\theta, \dot{\theta}, \alpha, \dot{\alpha}]^T$$
$$\dot{Q} = [\dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{q}_4]^T = [\dot{q}_1, \ddot{q}_1, \dot{q}_3, \ddot{q}_3]^T = [\dot{\theta}, \ddot{\theta}, \dot{\alpha}, \ddot{\alpha}]^T$$

$$A_{ij} = \frac{\partial \dot{q}_j}{\partial q_i}$$

need to format this matrix to look better

$$A = \begin{bmatrix} \frac{\partial\dot{\theta}}{\partial\theta} & \frac{\partial\dot{\theta}}{\partial\dot{\theta}} & \frac{\partial\dot{\theta}}{\partial\alpha} & \frac{\partial\dot{\theta}}{\partial\dot{\alpha}} \\ \frac{\partial\ddot{\theta}}{\partial\theta} & \frac{\partial\ddot{\theta}}{\partial\dot{\theta}} & \frac{\partial\ddot{\theta}}{\partial\alpha} & \frac{\partial\ddot{\theta}}{\partial\dot{\alpha}} \\ \frac{\partial\dot{\alpha}}{\partial\theta} & \frac{\partial\dot{\alpha}}{\partial\dot{\theta}} & \frac{\partial\dot{\alpha}}{\partial\alpha} & \frac{\partial\dot{\alpha}}{\partial\dot{\alpha}} \\ \frac{\partial\ddot{\alpha}}{\partial\theta} & \frac{\partial\ddot{\alpha}}{\partial\dot{\theta}} & \frac{\partial\ddot{\alpha}}{\partial\alpha} & \frac{\partial\ddot{\alpha}}{\partial\dot{\alpha}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{\partial\ddot{\theta}}{\partial\dot{\theta}} & \frac{\partial\ddot{\theta}}{\partial\alpha} & \frac{\partial\ddot{\theta}}{\partial\dot{\alpha}} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{\partial\ddot{\alpha}}{\partial\dot{\theta}} & \frac{\partial\ddot{\alpha}}{\partial\alpha} & \frac{\partial\ddot{\alpha}}{\partial\dot{\alpha}} \end{bmatrix}$$

$$\frac{\partial\ddot{\theta}}{\partial\ddot{\theta}} = -\frac{C_1 b_\theta}{C_5 C_1 - C_3^2} \qquad \frac{\partial\ddot{\theta}}{\partial\alpha} = -\frac{C_3 C_4}{C_5 C_1 - C_3^2} \qquad \frac{\partial\ddot{\theta}}{\partial\dot{\alpha}} = \frac{C_3 b_\alpha}{C_5 C_1 - C_3^2}$$
$$\frac{\partial\ddot{\alpha}}{\partial\dot{\theta}} = \frac{C_3 b_\theta}{C_5 C_1 - C_3^2} \qquad \frac{\partial\ddot{\alpha}}{\partial\alpha} = \frac{C_4 C_5}{C_5 C_1 - C_3^2} \qquad \frac{\partial\ddot{\alpha}}{\partial\dot{\alpha}} = -\frac{C_5 b_\alpha}{C_5 C_1 - C_3^2}$$

$$B = \begin{bmatrix} \frac{\partial\dot{\theta}}{\partial\tau_m} \\ \frac{\partial\ddot{\theta}}{\partial\tau_m} \\ \frac{\partial\dot{\alpha}}{\partial\tau_m} \\ \frac{\partial\ddot{\alpha}}{\partial\tau_m} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{C_1}{C_5 C_1 - C_3^2} \\ 0 \\ -\frac{C_3}{C_5 C_1 - C_3^2} \end{bmatrix}$$

System Transfer Functions

To generate the transfer functions it is more straight forwards to proceed from equations (4) and (5) than from (6) and (7). Applying the Laplace transform (4) and (5) yields and performing arithmetic operation to isolate $\theta(s)$ and $\alpha(s)$ yields the following transfer functions from motor torque, $\tau(s)$ to $\theta(s)$ and $\alpha(s)$ respectively.

$$\frac{\theta(s)}{\tau_m(s)} = \frac{s^2 C_1 + s b_\alpha - C_4}{(s^2 C_1 + s b_\alpha - C_4)(s^2 C_5 + s b_\theta) - s^4 C_3^2} \tag{8}$$

$$\frac{\alpha(s)}{\tau_m(s)} = \frac{-s^2 C_3}{(s^2 C_1 + s b_\alpha - C_4)(s^2 C_5 + s b_\theta) - s^4 C_3^2} \tag{9}$$

If we were to neglect the effects of damping, then (8) and (9) would reduce to the following:

$$\frac{\theta(s)}{\tau_m(s)} \approx \frac{s^2 C_1 - C_4}{s^2 \left[s^2 \left(C_5 C_1 - C_3^3\right) - C_4 C_5\right]}$$

$$\frac{\alpha(s)}{\tau_m(s)} \approx \frac{-s^2 C_3}{s^2 \left[s^2 \left(C_5 C_1 - C_3^3\right) - C_4 C_5\right]}$$

In the simplified transfer function for $\alpha(s)$ above it appears as if we can cancel the but this not the case as we see when we examine equation (9).

2. Parameter Identification

3. Comparing Linear and Non-Linear Models

4. State Space Controls in Simulink

5. Swing Up Controller in Simulink

6. LabView Implementation

a.

b.

c.

d.

7. Effects on LabView Loop Rate on Stability

8. Classical Controller

9. Another Swing Up Controller

10. Parasitic Effects