Lab Assignment 1

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Question 1

Question 2

a

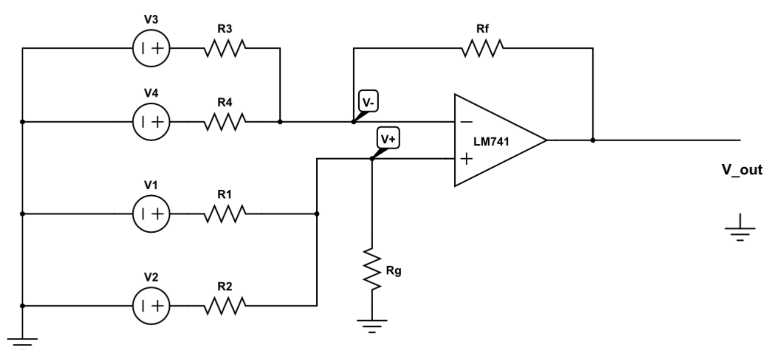

Figure 1: The required circuit diagram for question 2.

b

Circuit analysis:

At the non-inverting terminal, we effectively have a circuit represented by figure 2. We can use Kirchoff's voltage law along with the superposition theorem, we see:

Voltage at V^+ due to V_1, V_1^+, is given by:

$$V_1^+ = \frac{\frac{R_2 R_g}{R_2+R_g}}{R_1 + \frac{R_2 R_g}{R_2+R_g}} V_1$$

$$\therefore V_1^+ = \left(\frac{1}{\frac{R_1}{R_g} + \frac{R_1}{R_2} + 1} \right) V_1$$

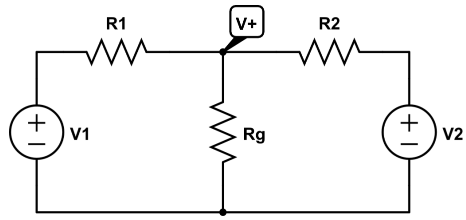

Figure 2: Equivalent circuit at the non-inverting terminal

Similarly, voltage at V^+ due to V_2, V_2^+, is given by:

$$V_2^+ = \frac{\frac{R_1 R_g}{R_1+R_g}}{R_2 + \frac{R_1 R_g}{R_1+R_g}} V_2$$

$$\therefore V_2^+ = \frac{1}{\frac{R_2}{R_g} + \frac{R_2}{R_1} + 1} V_2$$

Using the superposition theorem to recombine the values, we find that $V+$ is given by:

$$V^+ = V_1^+ + V_2^+$$

$$V^+ = \frac{1}{\frac{R_1}{R_g} + \frac{R_1}{R_2} + 1} V_1 + \frac{1}{\frac{R_2}{R_g} + \frac{R_2}{R_1} + 1} V_2$$

We can now move on to the inverting terminal of the op-amp. The corresponding equivalent circuit is shown in figure 3. Analyzing this circuit, we see that:

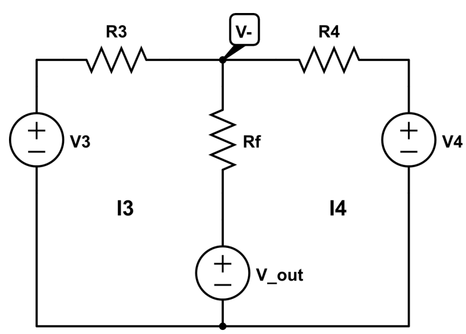

Figure 3: Equivalent circuit at the non-inverting terminal

$$I_3 R_3 = V_3 - V^-$$

$$I_4 R_4 = V_4 - V^-$$

$$V^- - V_{out} = (I_3 + I_4) R_f$$

Substituting from the above equations, we get:

$$V^- - V_{out} = \left(\frac{V_3 - V^-}{R_3} + \frac{V_4 - V^-}{R_4} \right) R_f$$

Reordering the terms, we get:

$$V_{out} = V^- - \Big(\frac{V_3 - V^-}{R_3} + \frac{V_4 - V^-}{R_4}\Big)\, R_f$$

$$\implies V_{out} = V^-\Big(\frac{R_f}{R_3} + \frac{R_f}{R_4} + 1\Big) - \Big(\frac{R_f}{R_3}\Big)V_3 - \Big(\frac{R_f}{R_4}\Big)V_4$$

We also know that for an ideal op-amp, $V^+ = V^-$, hence we get:

$$V_{out} = \Big(\frac{R_f}{R_3} + \frac{R_f}{R_4} + 1\Big)\Big(\frac{1}{\frac{R_1}{R_g} + \frac{R_1}{R_2} + 1}\Big)V_1 + \Big(\frac{R_f}{R_3} + \frac{R_f}{R_4} + 1\Big)\Big(\frac{1}{\frac{R_2}{R_g} + \frac{R_2}{R_1} + 1}\Big)V_2 - \Big(\frac{R_f}{R_3}\Big)V_3 - \Big(\frac{R_f}{R_4}\Big)V_4$$

This is equation is the mathematical representation of the problem statement which which required $V_{out} = a_1V_1 + a_2V_2 - a_3V_3 - a_4V_4$, i.e. :

$$a_1 = \Big(\frac{R_f}{R_3} + \frac{R_f}{R_4} + 1\Big)\Big(\frac{1}{\frac{R_1}{R_g} + \frac{R_1}{R_2} + 1}\Big)$$

$$a_2 = \Big(\frac{R_f}{R_3} + \frac{R_f}{R_4} + 1\Big)\Big(\frac{1}{\frac{R_2}{R_g} + \frac{R_2}{R_1} + 1}\Big)$$

$$a_3 = \Big(\frac{R_f}{R_3}\Big)$$

$$a_4 = \Big(\frac{R_f}{R_4}\Big)$$

c

[DO WE HAVE ANY MORE??]

While building this circuit, we made the following assumptions:

- We are using an ideal op-amp, i.e. the input impedance is infinite and the output impedance is negligible. This implies that the op-amp does not load it's input circuit and loading effects are not seen on the op-amps output terminal.
- The voltage at the non-inverting terminal is equal to the voltage at the non-inverting terminal.
- The open loop op-amp gain is infinite.
- The op-amp will operate in an ideal manner when the output is not saturated above or below its supply limits.

d

We used the following components in our circuits:

Component Name	**Component Type**	**Component Specification**
R_f	Resistor	9.880 $K\Omega$
R_g	Resistor	9.860 $K\Omega$
R_1	Resistor	49.3 $K\Omega$
R_2	Resistor	24.65 $K\Omega$
R_3	Resistor	3.293 $K\Omega$
R_4	Resistor	2.470 $K\Omega$

e

Physical implementation.

f

John

g

John

h

The beating signal is achieved as a sinusoidal wave which has an amplitude that also varies sinusoidally. In effect, the output is $\sin(\omega_a t_a)\sin(\omega_b t_b)$. We can use trigonometric identities as follows to rearrange this sinusoidal product as a sum as follows:

$$\sin(\omega_a t)\sin(\omega_b t) = \frac{\cos(\omega_a t + \omega_b t) - \cos(\omega_a t + \omega_b t)}{2}$$

We use the product-to-sum identities to break up he product of the sins

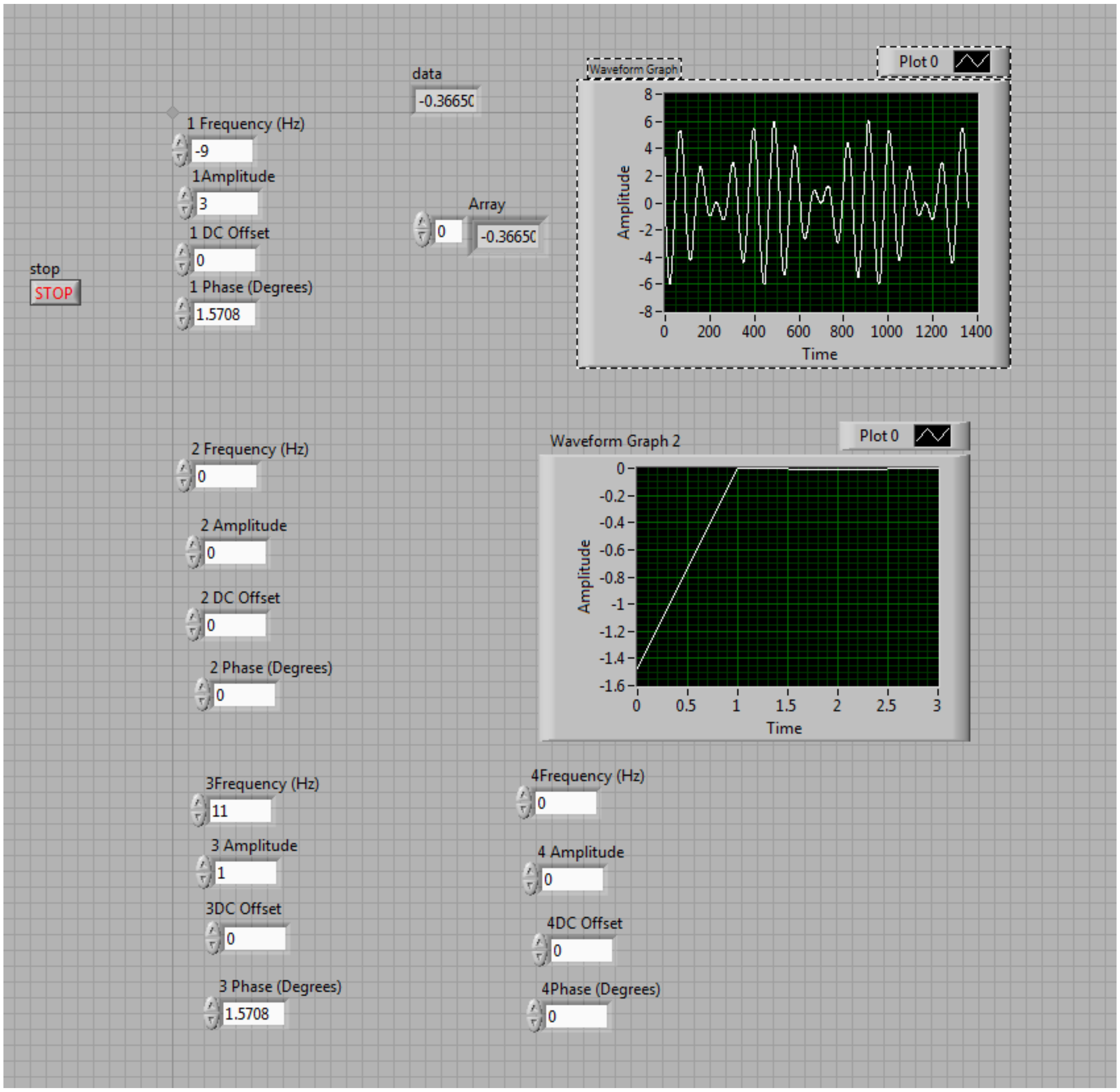

Figure 4: The LabView front-end of a system that produces $\sin(\omega_a t_a)\sin(\omega_b t_b)$.