2.2 a)

RR bot has a total of 4 equilibrium points.

They are:

1) $$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

2) $$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} \pi \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

3) $$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} \pi \\ \pi \\ 0 \\ 0 \end{bmatrix}$$

4) $$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \pi \\ 0 \\ 0 \end{bmatrix}$$

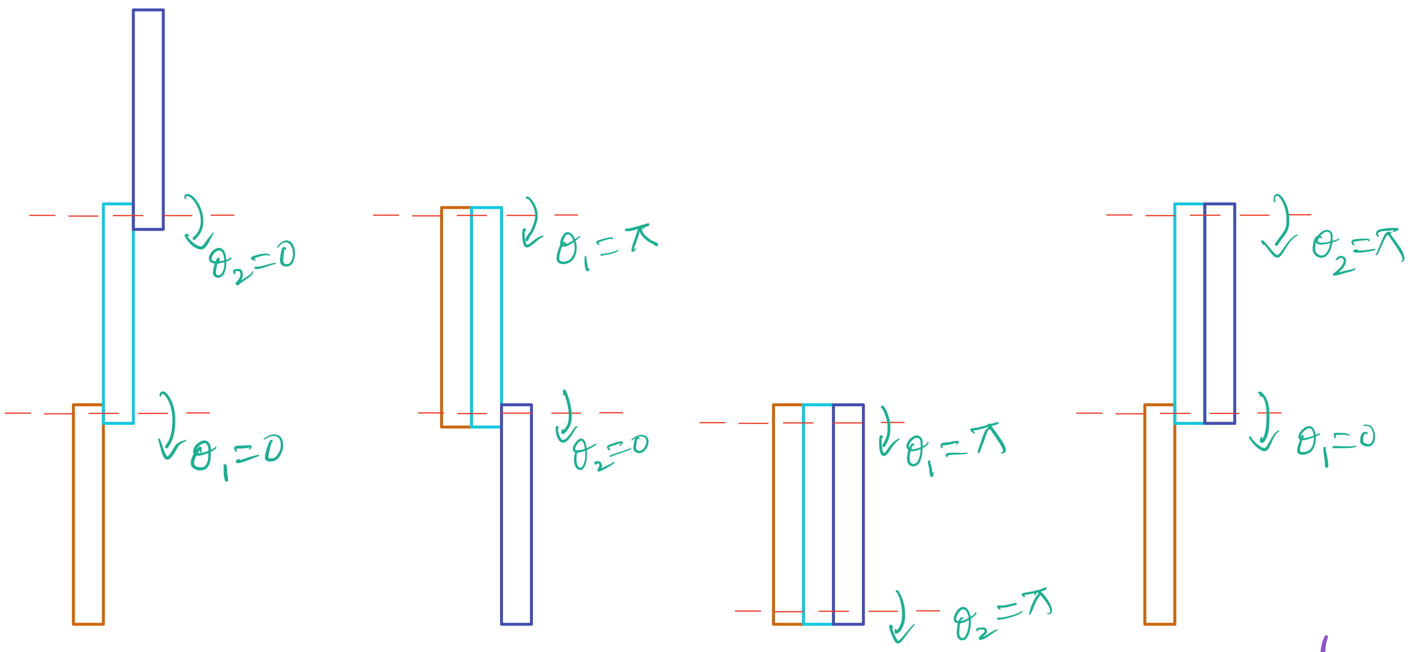
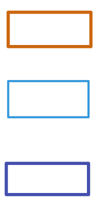


Figure 1: side views of RR bot drawn at each equilibrium point.



shows the base

shows the first link

shows the second link

Note: Matlab only returns three equilibrium points
$[\pi,\pi,0,0]$ is not returned by Matlab's solve function

2.2 b) Jacobian Linearization:

Note: General linearized state space matrix (A) and input matrix (B) are too long to write here. They are printed with Matlab script.

Jacobian linearization around each of the 4 equilibrium points:

i) At $[0,0,0,0]^T$ is

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 12.5769 & -11.9611 & 0 & 0 \\ -16.9227 & 46.1565 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1.7250 & -4.4345 \\ -4.4345 & 14.8902 \end{bmatrix}$$

ii) At $[\pi,0,0,0]^T$ is

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -12.5769 & 11.9611 & 0 & 0 \\ 16.9227 & -46.1565 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1.7250 & -4.4345 \\ -4.4345 & 14.8902 \end{bmatrix}$$

iii) At $[0, \pi, 0, 0]^T$ is

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 12.5769 & -11.9611 & 0 & 0 \\ -8.2310 & -22.2343 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1.7250 & 0.9845 \\ 0.9845 & 4.0522 \end{bmatrix}$$

iv) At $[\pi, \pi, 0, 0]$ is

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -12.5769 & 11.9611 & 0 & 0 \\ 8.2310 & 22.2343 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1.7250 & 0.9845 \\ 0.9845 & 4.0522 \end{bmatrix}$$

2.2 c)

To investigate stability, it is sufficient to check the eigen values of linearized state matrix around each equilibrium point.

i) At $[0,0,0,0]^T$ the eigen values are

$$e = \begin{bmatrix} 7.1676 \\ 2.7129 \\ -7.1676 \\ -2.7129 \end{bmatrix}$$

There are positive eigen values for the linearized state matrix around this equilibrium. Hence, the system is not stable around this equilibrium point

ii) At $[\pi,0,0,0]^T$ eigen values are

$$e = \begin{bmatrix} 0 + 7.1676i \\ 0 - 7.1676i \\ 0 + 2.7129i \\ 0 - 2.7129i \end{bmatrix}$$

since none of the eigen values have positive real parts, the system is stable around this equilibrium point. Since real parts are zero, it is Marginally stable

iii) At $[0, \pi, 0, 0]'$, the eigen values are

$$e = \begin{bmatrix} -3.8995 \\ 3.8995 \\ 4.9864i \\ -4.9864i \end{bmatrix}$$

Two of the eigen values have real values > 0

$\therefore$ The system is not stable around this equi-librium point

iv) At $[\pi, \pi, 0, 0]^T$, the eigen values are

$$e = \begin{bmatrix} 4.9864 \\ 3.8995i \\ -3.8995i \\ -4.9864 \end{bmatrix}$$

Two of the Eigen values have the real part > 0

$\therefore$ The system is not stable around this equilibri-um point.

2.2 d) Controllability test around 'upward' configuration

upward configuration is when both the joints have zero rotation.

Equilibrium point at this configuration is

$$Eq = [0, 0, 0, 0]^T$$

State matrix around this equilibrium is

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 12.5769 & -11.9611 & 0 & 0 \\ -16.9227 & 46.1565 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1.725 & -4.4345 \\ -4.4345 & 14.8902 \end{bmatrix}$$

$$C = [B \quad AB \quad A^2B \quad \dots \quad A^{n-1}B]$$

rank (C) = 4

C is full rank

∴ The system is controllable around this configuration

2.2 e) State-feed back control

$$u = -Kx$$

Eigen values are placed at

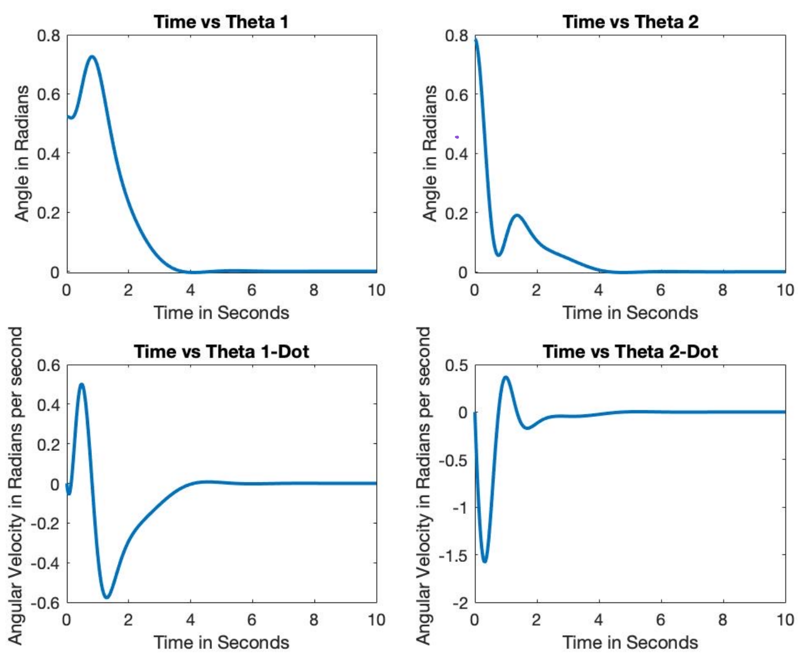
$$e = \begin{bmatrix} -1+2i \\ -1-2i \\ -2 \\ -1.5 \end{bmatrix}$$

Computed gain values for these eigen values are

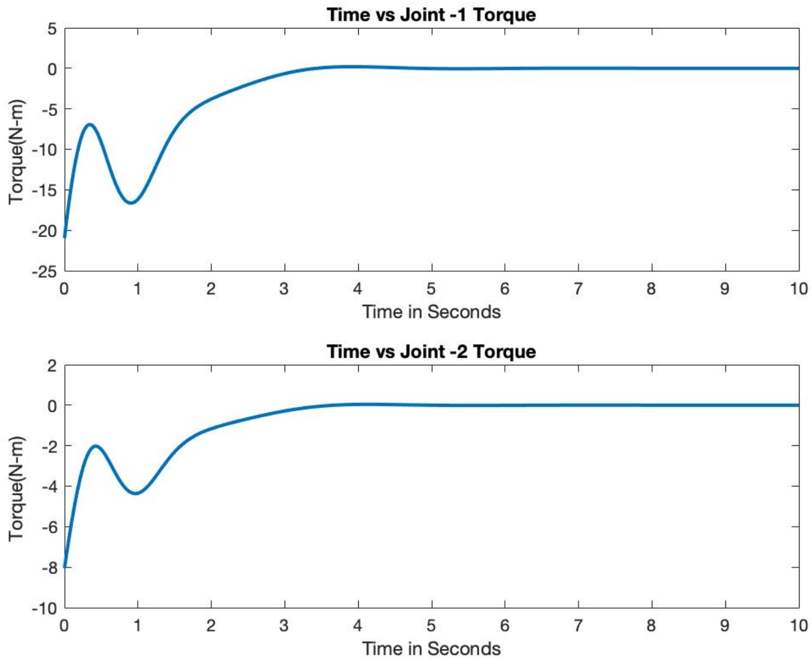
$$K = \begin{bmatrix} 19.6705 & 13.5741 & 4.5331 & 7.2672 \\ 4.4649 & 7.2775 & 1.2178 & 2.3712 \end{bmatrix}$$

2.2 f) Plotting state Trajectories, and control inputs

Trajectories:

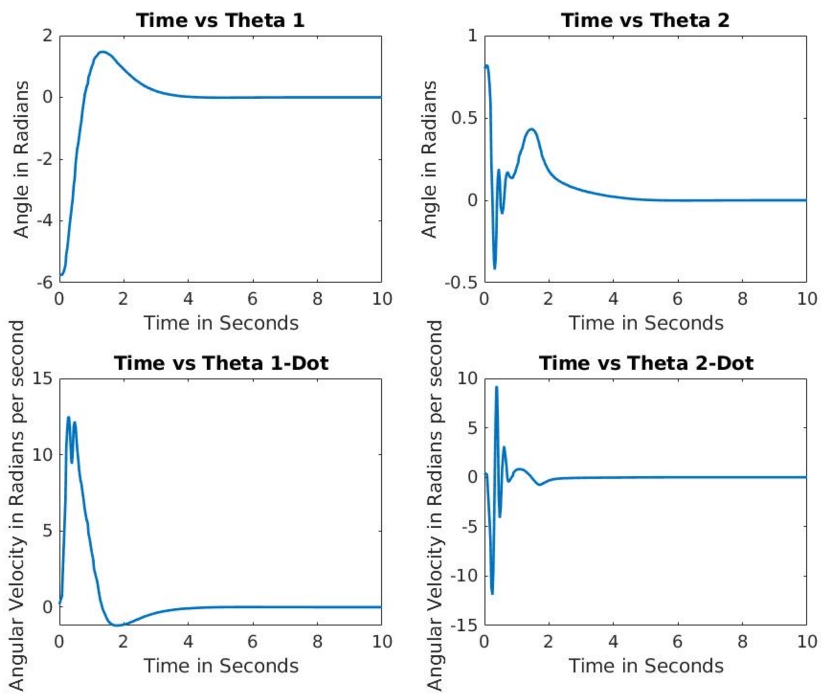


Control inputs:

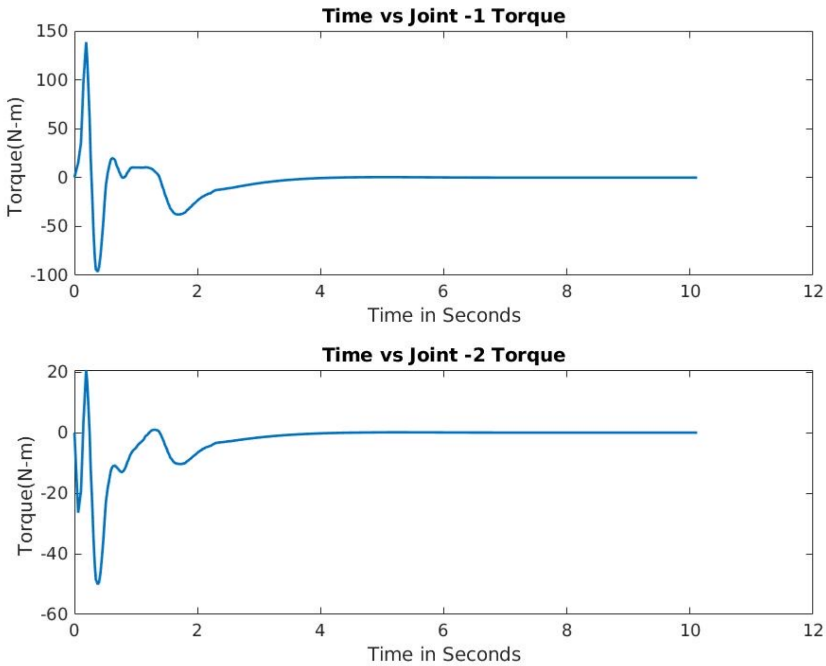


2.2 g)

Output Trajectories:



output of control inputs:



These trajectories are not matching with those plotted in the previous answer. This is because we are dealing with a physical object that has extra properties such as friction that are not considered while modelling the problem.