Activity-3

MODULE-3

⇒ Mathematical Induction (PMI)

Q1) Prove by PMI that $1+2+3+\dots+n = \frac{n(n+1)}{2}$

Soln Let $P(n)$ be the statement that $1+2+3+\dots+n = \frac{n(n+1)}{2}$

Step (i) put $n=1$

$L.H.S = 1$

$R.H.S = \frac{1(1+1)}{2} = \frac{1 \times \cancel{2}}{\cancel{2}} = 1$

$\therefore L.H.S = R.H.S$

$\therefore P(1)$ is true

Step (ii) Assume that $P(n)$ is true for $n=K$

Let $P(K)$ is true

$1+2+3\dots+K \;\; \frac{K(K+1)}{2}$

Step (iii) Now, we have to prove that $P(n)$ is true for $n=(K+1)$

L.H.S of $P(K+1)$

$1+2+3+\dots+K+K+1 = \frac{(K+1)(K+2)}{2}$

Now,, $1+2+3\dots+K+K+1$

$= \frac{K(K+1)}{2} + (K+1)$

$= (K+1)\left[\frac{K}{2}+1\right]$

$= \frac{(K+1)(K+2)}{2}$ R.H.S

$\therefore P(n)$ is true for $n \in N$

$\therefore$ By $P(n)$ $P(n)$ is true $\forall n \in N$ // proved.

Q2) prove by PMI that $1^2+2^2+3^3+ \cdots +n^2 = \frac{n(n+1)(2n+1)}{6}$

Sol) let $P(n)$ be the Statement that $1^2+2^2+3^2+ \cdots +n^2 = \frac{n(n+1)(2n+1)}{6}$

Step (i) Inductive base

put $n=1$

$L.H.S = 1^2 = 1$

$R.H.S = \frac{1(1+1)(2\times1+1)}{6}$

$= \frac{1\times2\times3}{6}$

$= \frac{\cancel{6}}{\cancel{6}} = 1$

$\therefore$ L.H.S = R.H.S

$\therefore$ $P(1)$ is true

Step (ii) Inductive hypothesis

Assume that $P(n)$ is true for $n=k$

$1^2+2^2+3^2+ \cdots +k^2 = \frac{k(k+1)(2k+1)}{6}$

$\therefore$ $P(k)$ is true.

Step (iii) Inductive step

we have to prove that $P(n)$ is true for $n=(k+1)$

$1^2+2^2+3^2+ \cdots + (k+1)^2 = \frac{k+1\,(k+2)(2k+3)}{6}$

$L.H.S = 1^2+2^2+3^2 \cdots + k^2 + (k+1)^2$

$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$ [from step (ii)]

$= (k+1)\left[\frac{k(2k+1)}{6} + (k+1)\right]$

$= (k+1)\left[\frac{2k^2+k+6k+6}{6}\right]$

$= (k+1)\left[\frac{2k^2+7k+6}{6}\right]$

$= (k+1)\left[\frac{2k^2+4k+3k+6}{6}\right]$

$= (k+1)\left[\frac{(k+2)(2k+3)}{6}\right]$ = R.H.S

$\because$ L.H.S = R.H.S

$\therefore$ P(n) is true for $n \in N$ //

proved //

Q3) Prove by PMI that $\frac{1}{1.2} + \frac{1}{2.3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

Soln let P(n) be the statement that $\frac{1}{1.2} + \frac{1}{2.3} + \cdots + \frac{1}{n(n+1)} = \frac{1}{n+1}$

Step (i) inductive base

Put $n=1$

$L.H.S = \frac{1}{1.2} = \frac{1}{2}$

$R.H.S = \frac{1}{1+1} = \frac{1}{2}$

$\therefore$ L.H.S = R.H.S $\therefore$ P(1) is true,

Step (ii) inductive hypothesis

Assume that P(n) is true for $n = k$

$\frac{1}{1.2} + \frac{1}{2.3} + \cdots + \frac{1}{k(k+1)} = \frac{k}{k+1}$

$\therefore$ P(k) is true,

Step (iii) inductive test

we have to prove that P(n) is true for $n = (k+1)$

$1^3 + 2^3 + 3^3 + \cdots + (k+1)^3$

L.H.S $1^3 + 2^3 + 3^3 + \cdots + k^3 + (k+1)^2$

$= \left[\frac{k(k+1)}{2}\right]^2 + (k+1)^3$ (from step ii)

$= \frac{k^2(k+1)^2}{2} + (k+1)^3$

$= (k+1)^2\left[\frac{k^2}{2} + k+1\right]$

$= (k+1)^2 \left[\frac{k(k+1)}{2}\right]^2$ R.H.S

$\therefore$ P(n) is true for $\forall n \in N$ proved //

Q4) Prove by PMI that $1^3+2^3+3^3+\dots n^3 = \left[\frac{n(n+1)}{2}\right]^2$

Sol^n let $P(n)$ be the statement that $1^3+2^3+3^3+\dots n^3$
$= \left[\frac{n(n+1)}{2}\right]^2$

Step (i) inductive base

put $n=1$

$L.H.S = 1^3 = 1$

$R.H.S = \left[\frac{1(1+1)}{2}\right]^2 = 1$

$\because L.H.S. = R.H.S \quad \therefore P(1)$ is true

Step (ii) inductive hypothesis

Assume that $P(n)$ is true for $n=k$

$1^3+2^3+3^3+\dots k^3 \left[\frac{k(k+1)}{2}\right]^2$

$\therefore P(k)$ is true

Step (iii) inductive test

we have to prove that $P(n)$ is true for $n=(k+1)$

$1^3+2^3+3^3+\dots+(k+1)^3$

L.H.S

$1^3+2^3+3^3+\dots+k^3+(k+1)^3$

$= \left[\frac{k(k+1)}{2}\right]^2 + (k+1)^3$ (from step ii)

$= \frac{k^2(k+1)^2}{2} + (k+1)^3$

$= (k+1)^2 \left[\frac{k^2}{2}+k+1\right]$

$= (k+1)^2 \left[\frac{k^2+2k+2}{2}\right]$

$= (k+1)^2 \left[\frac{k(k+1)}{2}\right]^2$ R.H.S

$\therefore$ p(n) is true for $\forall n \in N$ proved//

$\Rightarrow$ Counting Principle

Q5) in a survey of 6 People, it was found that, 25 read News week Magazine 26 read time, 26 read fortune.

9 read both: News week & fortune,

11 read both Newweek & time

8 read both time & fortune

3 read all three magazine

a) Find the no of People who read at least one of the 3 Magazine.

Soln Given

$|N| = 25 \quad |N \cap F| = 9 \quad U = 60$

$|T| = 26 \quad |N \cap T| = 11 \quad |N \cap T \cap F| = 3$

$|F| = 26 \quad |T \cap F| = 8$

a) atleast one of the 3 magazine

N or T or F

$|N \cup T \cup F| = ?$

$|N \cup T \cup F| = |N| + |T| + |F| - |N \cap T| - |N \cap F| - |T \cap F| + |N \cap T \cap F|$

$= 25 + 26 + 26 - 9 - 11 - 8 + 3$

$= 52$//

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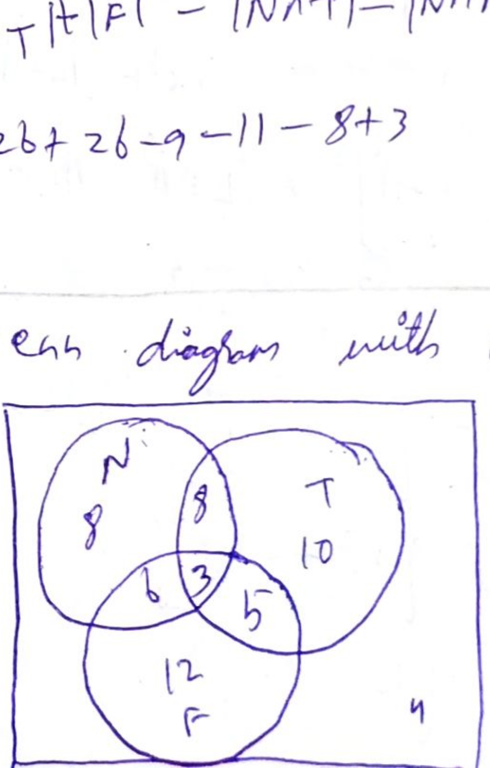
b) Fill in the venn diagram with correct value in eight regions.



$V = 60$

$U - |N \cup T \cup F| = 60 - 52$

$= 8$

$|N \cap T \cap F| = 3$

$|N \cap F| = 9 - 3 = 6$

$|N \cap T| = 11 - 3 = 8$

$|T \cap F| = 8 - 3 = 5$

$|N| = 25 - 8 - 6 - 3 = 8$

$|T| = 28 - 8 - 3 - 5 = 10$

$|F| = 26 - 6 - 5 - 3 = 12$

c) Find the No of People who read exactly one magazine,

$N + T + F = 8 + 10 + 12$

$= 30$//.

Combinations

Q6) A farmer buys 3 cow, 2 pigs & 4 hens from a man who has 6 cow, 5 pigs & 8 hens How many choices does the farmer have,

Solu
$^6C_3$ = cows

$^3C_2$ = pigs

$^4C_8$ = hens

According to principle of counting

$$^6C_3 * {}^3C_2 \times {}^4C_8 = \frac{6!}{3!3!} \times \frac{5!}{2!3!} \times \frac{8!}{4! \times 4!}$$

$$= \frac{6 \times 5 \times 4}{3 \times 2} \times \frac{5 \times 4}{1 \times 2} \times \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2}$$

$$= 20 \times 10 \times 17 = 1400$$//

Q 7) 6 men, 5 women, To form the Committee of 5 members you Should have 2 women.

Soln Given
total = 11
Members = 5

According to Combination

$$^nC_r = \frac{n!}{r!(n-r)!}$$

$$^5C_2 \times C_3 = \frac{5!}{2!3!} \times \frac{6!}{3!3!}$$

$$= \frac{5 \times 4 \times \cancel{3 \times 2 \times 1}}{2 \times 1 \times \cancel{3 \times 2 \times 1}} \times \frac{6 \times 5 \times 4 \times \cancel{3 \times 2 \times 1}}{3 \times 2 \times 1 \times \cancel{3 \times 2 \times 1}}$$

$$= \frac{5 \times 4}{2 \times 1} \times \frac{6 \times 5 \times 4}{3 \times 2 \times 1}$$

$$= \frac{20}{2} \times \frac{120}{6}$$

$$= 10 \times 20$$

$$= 200 \;//$$

=> Inclusion - Exclusion principle

Q 8) A computer company must hire 20 programmers to handle System programming jobs and 30 programmers for applications programming of those hired, are expected to perform jobs of both types.

How many programmers must be hired?

Solu Given

$|A| = 20$

$|B| = 30$

$|A \cap B| = 5$

$|A \cup B| = ?$

$\therefore |A \cup B| = |A| + |B| - |A \cap B|$

$= 20 + 30 - 5$

$= 50 - 5$

$= 45$ //

Q Solve the recurrence relation $a_r - 7a_r + 10a_{r-2} = 0$ given that $a_0 = 0$, $a_1 = 3$

Solu Given recurrence relation is

$$a_r - 7a_{r-1} + 10a_{r-2} = 0 \rightarrow ①$$

and given that $a_0 = 0$, $a_1 = 3$

This is second order recurrence relation

The characteristic equation is

$$M^2 - 7M + 10 = 0$$

$$\Rightarrow (M-2)(M-5) = 0$$

$$\Rightarrow M = 2, 5$$

$\therefore$ The general solution is

$$a_r = C_1(2)^r + C_2(5)^r \rightarrow ②$$

Putting in eqn ② , $a_0 = 0$ i.e $a_r = 0$ and $a = 0$

$$C_1(2)^0 + C_2(5)^0 = 0$$

$$\Rightarrow C_1 + C_2 = 0 \rightarrow ③$$

Again putting in equation (2) $a_1 = 3$ i.e $a_r = 3$ and $r = 1$

$$c_1 (2)^1 + c_2 (5)^1 = 3$$

$$\Rightarrow 2c_1 + 5c_2 = 3 \rightarrow (4)$$

Solving eqn (3) & (4) we get

$$c_1 = -1 \text{ and } c_2 = 1$$

$\therefore$ The required general solution is put in eqn (2)

$$\boxed{a_r = 5^r - 2^r}$$

Proved //