BLUE BOOK

Achiever

INTERNAL ASSESSMENT BOOK

Name: M. SHIVA KUMAR

Subject: MATHS   Class: MCA (AIML)

Sl.No.	PARTICULARS	Test Date	Page No	Marks Awarded	Signature of Staff Incharge
1	TEST - I				
2	TEST - II				
3	TEST - III				
4					
5					

Certificate

This is to certify that Smt. / Sri...has satisfactorily completed the course of Assignment prescribed by the...............................University for the semester ....Degree Course in the Year 20   - 20

MARKS	
MAX	OBTAINED

M. [signature]

Signature of the Student   Signature of H.O.D.   Signature of the Staff Member (Incharge of the Batch)

Activity - 2

Q. Let A be the set of all triangles in a plane and let R be a relation if it is reflexive symmetric and transitive Show that R is an equivalence relation in A.

Solu The relation satisfies the following properties

(i) Reflexivity

let Δ be an arbitrary triangle A. Then,

$\Delta \cong \Delta \Rightarrow (\Delta, \Delta) \in R$ for all values of Δ in A.

$\therefore$ R is Reflexive.

(ii) Symmetry

let $\Delta_1, \Delta_2 \in A$ such that $(\Delta_1, \Delta_2) \in R$ then

$(\Delta_1, \Delta_2) \in R \Rightarrow \Delta_1 \cong \Delta_2$

$\Rightarrow \Delta_2 \cong \Delta_1$

$\Rightarrow (\Delta_2, \Delta_1) \in R$

$\therefore$ R is Symmetric

(iii) Transitivity

let $\Delta_1, \Delta_2, \Delta_3 \in A$ Such that (Δ_1, Δ_2) and $(\Delta_2, \Delta_3) \in R$ then

$(\Delta_1, \Delta_2) \in R$ and $(\Delta_2, \Delta_3) \in R$,

$\Rightarrow \Delta_1 \cong \Delta_2 \quad (\Delta_2, \Delta_3) \in R.$

$\Rightarrow \Delta_1 \cong \Delta_3$

$\Rightarrow (\Delta_1, \Delta_3) \in R$

$\therefore$ R is transitive

Thus R is reflexive, Symmetric and transitive.

Hence, R is an equivalence relation.

Q2 Let A be the set of all line in $x=y$ plane and let R be a relation in A, defined by

$$R = \{(L_1, L_2) : L_1 \parallel L_2\}$$

Show that R is an equivalance relation in A $y = 3x+5$

Soln The given relation satisfies the following properties

(i) Reflexivity

Let be an arbitrary line in A, then

$L \parallel L \Rightarrow (L, L) \in R$ $L \in A$

Thus, R is reflexive

(ii) Symmetry

let $L_1, L_2 \in A$ such that $(L_1, L_2) \in R$, then

$(L_1, L_2) \in R \Rightarrow L_1 \parallel L_2$

$\Rightarrow L_2 \parallel L_1$

$\Rightarrow (L_2, L_1) \in R$

$\therefore$ R is symmetric

(iii) Transitivity

Let $L_1, L_2, L_3 \in A$ such that $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$

Then $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$

$\Rightarrow L_1 \parallel L_2$ and $(L_2, L_3) \cancel{\in R}$

$\Rightarrow L_1 \parallel L_3$

$\Rightarrow (L_1, L_3) \in R$

$\therefore$ R is transitive

Thus R is reflexive, symmetric and transitive

Here equivlence relation.

③ Let S be the set of all real number and let R be a relation S defined by $R = \{(a,b) : a \le b^2\}$

Show that R Satisfies None of reflexivity, symmetric and transitivity.

Sol: (i) Non reflexivity

clearly, $\frac{1}{2}$ is a reall number and $\frac{1}{2} \le (\frac{1}{2})^2$ is not true

$\therefore (\frac{1}{2}, \frac{1}{2}) \notin R$

Hence, R is not reflexive.

(ii) Non Symmetry

consider the real number $\frac{1}{2}$ and 1

clearly, $\frac{1}{2} \le 1^2 \Rightarrow (\frac{1}{2}, 1) \in R$

But, $1 \le (\frac{1}{2})$ is not true and so $(1, \frac{1}{2}) \notin R$

Thus, $(\frac{1}{2}, 1) \in R$ but $(1, \frac{1}{2}) \notin R$

Hence, R is not symmetric

(iii) Non Transitivity

Consider the real number 2 -2 and 1 clearly, $2 \le (-2)^2$ &

$-2 < (1)^2$ but $2 \le 1^2$ is not true, thus $(2, -2) \in R$ and $(-2, 1) \in R$

but $(2, 1) \notin R$

Hence, R is not transitive

$\Rightarrow$ Equivalence class and Partitions

Q4) which of these collection of subset are partitions of

$S = \{-3, -2, -1, 0, 1, 2, 3\}$

(a) $\{-3, -1, 1, 3\}$, $\{-2, 0, 2\}$

(b) $\{-3, -2, -1, 0\}$, $\{0, 1, 2, 3\}$

c) $\{-3,3\}, \{-2,2\} \{-1,1\}, \{0\}$

d) $\{-3,-2,2,3\}, \{-1,1\}$

Soln

a) $S_1 = \{-3,-1,1,3\}$ and $S_2 = \{-2,0,2\}$

$S_1 \cap S_2 = \phi$ and $S_1 \cup S_2 = \{-3,-1,1,3) -2,0,2\} = S$

Also $= S_1 \neq \phi$ and S_2

By this defination of Partition, the given collection of subset is a Partition

$\{-3,-1,1,3\} \cap \{-2,0,2\} = \phi$ (Yes)

b) $S_3 = \{-3,-2,-1,0\}$ and $S_4 = \{0,1,2,3\}$

$S_3 \cap S_4 \neq \phi$

Therefore, the given collection of subset is not a Partition

c) $S_5 = \{-3,-2\}, S_6 = \{-2,2\}, S_7 = \{-1,1\}, S_8 = \{0\}$

$S_5 \cap S_6 \cap S_7 \cap S_8 = \phi$ (Yes)

$S_5 \cup S_6 \cup S_7 \cup S_8 = \{-3,3,-2,2,-1,1,0\} = S$

Also, $S_5 \neq 0$, $S_6 \neq \phi$, $S_7 \neq \phi$, $S_8 \neq \phi$

By the definition of Partition the given collection of subset is a Partition.

d) $S_9 = \{-3,-2,2,3\}$ and $S_{10} = \{-1,1\}$

$S_9 \cap S_{10} = \phi$ and $S_9 \cup S_{10} = \{-3,-2,2,3,-1,1 \neq S$

$\therefore$ the given collection of subset is not a Partition

⑤ Show that the relation R on the set of all bit strings such that sRt iff s and t contains the same numbes of 1s is an equivalence relation.

Soln

A = Set of all bit String

$R = \{(s,t) \mid s$ and t have the same numbes of $1s\}$

(i) Reflexivity: $\forall s \in A\ (s,s) \in R$

$(s,s) \in R$ means s & s have same numbers of 1s

$\therefore$ R is Reflexive.

(ii) Symmetry: $\forall s, t \in A\ [(s,t) \in R \rightarrow (t,s) \in R]$

$(s,t) \in R$ means s & t have same numbers of 1s

$(t,s) \in R$ means t and s have same numbers of 1s

$\therefore$ R is Symmetric

(iii) Transitivity $\forall s, t, u \in A\ [(s,t) \in R \wedge (t,u) \in R) \rightarrow (s,v) \in R$

if $(s,t) \in R \wedge (t,v) \in R$ then $(s,v) \in R$ because.

s and t have the same numbes of 1s t and v have the same numbes of 1s: then it is obivous that s and v have the same numbers of 1s

$\therefore$ R is transitive

$\Rightarrow$ Function.

Q6) Show that the function $f: N \to N$, defined by

$$f(x) = \begin{cases} x+1, & \text{if } x \text{ is odd} \\ x-1, & \text{if } x \text{ is even} \end{cases}$$

is one-one and onto

Soln: Suppose $f(x_1) = f(x_2)$

Case 1: when x_1 is odd and x_2 is even

In this case, $f(x_1) = f(x_1) \Rightarrow x_1 + 1 = x_2 - 1$

$\Rightarrow x_2 - x_1 = 2$

This is a controduction, since the difference between an odd integer and an even integer can never be 2.

$\therefore$ in this case, $f(x_1) \neq (x_2)$

Similarly, when x_1 is even and x_2 is odd, then $f(x_1) \neq (x_2)$

Case 2: when x_1 and x_2 are both odd.

In this case $f(x_1) = f(x_2) \Rightarrow x_1 + 1 = x_2 - 1$

$\Rightarrow x_1 = x_2$

$\therefore f$ is one-one

Case 3: when x_1 and x_2 are both even

in this case, $f(x_1) = f(x_2) \Rightarrow x_1 - 1 = x_2 - 1$

$\Rightarrow x_1 = x_2$

$\therefore f$ is one-one

In order to show that f is onto, let $y \in N$ (the codomain)

Case 1: when y is odd

in this case $(y+1)$ is even

$\therefore f(y+1) = (y+1) - 1 = y$

Case 2: when y is even

in this case, $(y-1)$ is even

$\therefore f(y-1) = y-1+1 = y$

Thus, each $\in N$ (co-domain of f) has its pre-image in dom(f)

$\therefore f$ is onto

Hence, f is one-one onto.

Q7) Show that $f: N \to N$. defined by

$$f(x) \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{f } n \text{ is even} \end{cases}$$

is a many-one onto function.

Soln we have

$$f(1) = \frac{(1+1)}{2} = \frac{2}{2} = 1 \text{ and } f(2) = \frac{2}{2} = 1$$

Thus, $f(1) = f(2)$ while $1 \neq 2$

$\therefore f$ is many-one

In order to show that f is onto, consider on arbitrary element $n \in N$

if n is odd then $2n$ is even and $f(2n) = \frac{2n}{2} = n$

Thus, for each $n \in N$ (whether even or odd) there exists its pre-image in N.

$\therefore f$ is onto

Hence, f is many-one onto.

Q8) Let $A = R - \{3\}$ and $B = R - \{1\}$

Let $f : A \to B : f(x) = \frac{x-2}{x-3}$ for all values of $x \in A$.

Show that f is one-one and onto

Soln

f is one-one since

$$f(x_1) = f(x_2) \Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$$\Rightarrow (x_1 - x_2)(x_2 - 3) = (x_1 - 3)(x_2 - 2)$$

$$\Rightarrow x_1 x_2 - 3x_1 - 2x_2 + 6 = x_1 x_2 - 2x_1 - 3x_2 + 6$$

$$\Rightarrow x_1 = x_2$$

Let $y \in B$ such that $y = \frac{x-2}{x-3}$

then, $(x-3)y = (x-2) \Rightarrow x = \frac{(3y-2)}{(y-1)}$

Clearly, x is defined when $y \neq 1$

Also, $x = 3$ will give us $1 = 0$, which is false.

$\therefore x \neq 3$

And $f(x) = \frac{\left(\frac{3y-1}{y-1} - 2\right)}{\left(\frac{3y-2}{y-1} - 3\right)} = y$

Thus, for each $y \in B$, there exists $x \in A$ such that $f(x) = y$.

$\therefore f$ is onto

Hence f is onto one-one

Q9) Let A and B be two non-empty sets. Show that the function $f = (A \times B) \to (B \times A) : f(a, b) = (b, a)$ is a bijective function.

Solu f is one-one since

$f(a_1, b_1) = f(a_2, b_2) \Rightarrow (b_1, a_1) = (b_2, a_2)$
$\Rightarrow (a_1 = a_2)$ and $b_1 = b_2$
$\Rightarrow (a_1, b_1) = (a_2, b_2)$

in order to show that f is onto, let (b, a) be an arbitrary element of $(B \times A)$

Then $(b, a) \in (B \times A)$
$\Rightarrow b \in B$ and $a \in A$
$\Rightarrow (a, b) \in (A \times B)$

Thus, for each $(b, a) \in (B \times A)$, there exists $(a, b) \in A \times B$ such that
$f(a, b) = (b, a)$
$\therefore f$ is onto
thus f is one-one onto and hence bijective.

Q10) consider function $f: X \to Y$ and define a relation R in X by $R = \{(a, b) : f(a) = f(b)\}$ that R is an equivalence relation,

Solu (i) reflexivity

Let $a \in X$ then
$f(a) = f(a) \Rightarrow (a, a) \in R$
R is reflexive

(ii) symmetry

let $(a, b) \in R$ then
$(a, b) \in R \Rightarrow f(a) = f(b) \Rightarrow f(b) = f(a)$
$\Rightarrow (b, a) \in R$

(iii) Transitivity

let $(a, b) \in R$ and $(b, c) \in R$ then
$(a, b) \in R$, $(b, c) \in R$
$\Rightarrow f(a) = f(b)$ & $f(b) = f(c)$ $\Rightarrow (a, c) \in R$
R is transitive//