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**Micromechanics (AS5990)**

**Homework-2**

Due Date: 14 Mar 2023

**Consider a composite consisting of aligned fibres in a matrix. Assume now that the matrix is epoxy (bulk modulus $\kappa_0 = 2.83$ GPa, shear modulus $\mu_0 = 1.31$ GPa), and the inclusions are glass fibres (bulk modulus $\kappa_1 = 51.2$ GPa, shear modulus $\mu_1 = 35.2$ GPa). Assume that all the fibres are aligned in the $\underline{e}_3$ coordinate direction, and the fibres are distributed randomly in the transverse plane.**

1. **Use the Eshelby solution for a cylindrical inclusion in an isotropic matrix (c.f. the previous homework) to compute the effective stiffness tensor $\bar{\underset{\approx}{\mathbf{C}}}$ of the composite using the Mori-Tanaka method.**

   Let $f$ be the volume fraction of the fibres. Using the Mori-Tanaka method,

$$\bar{\underset{\approx}{\mathbf{C}}}^{MK} = \left[(1-f)\underset{\approx}{\mathbf{C}}_0 + f\underset{\approx}{\mathbf{C}}_1\underset{\approx}{\mathbf{A}}_1\right]\left[(1-f)\underset{\approx}{\mathbf{I}} + f\underset{\approx}{\mathbf{A}}_1\right]^{-1}$$

   where

$$\underset{\approx}{\mathbf{A}}_1 = \left[\underset{\approx}{\mathbf{I}} + \underset{\approx}{\mathbf{S}}\underset{\approx}{\mathbf{C}}_0^{-1}(\underset{\approx}{\mathbf{C}}_1 - \underset{\approx}{\mathbf{C}}_0)\right]^{-1}$$

   and the nonzero components of $\underset{\approx}{\mathbf{S}}$ are

$$S_{1111} = S_{2222} = \frac{5-4\nu_0}{8(1-\nu_0)}$$
$$S_{1122} = S_{2211} = \frac{-1+4\nu_0}{8(1-\nu_0)}$$
$$S_{1133} = S_{2233} = \frac{\nu_0}{2(1-\nu_0)}$$
$$S_{2323} = S_{3131} = 0.25$$
$$S_{1212} = \frac{3-4\nu_0}{8(1-\nu_0)}$$

   For a fixed volume fraction $f = 0.3$, The effective stiffness tensor using the Mori-

Tanaka method is

$$\bar{\underset{\approx}{\mathbf{C}}}^{MK} = \begin{bmatrix} 7.1483 & 2.9204 & 2.7203 & 0 & 0 & 0 \\ 2.9204 & 7.1483 & 2.7203 & 0 & 0 & 0 \\ 2.7203 & 2.7203 & 29.6328 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.3212 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.3212 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.1139 \end{bmatrix} \text{GPa} \tag{1}$$

2. **Show that the resulting stiffness tensor has transverse isotropic symmetry.**

   From equation(1)

$$\begin{aligned} \bar{\mathbf{C}}_{1111} &= \bar{\mathbf{C}}_{2222} \\ \bar{\mathbf{C}}_{1133} &= \bar{\mathbf{C}}_{2233} \\ \bar{\mathbf{C}}_{1313} &= \bar{\mathbf{C}}_{2323} \\ \bar{\mathbf{C}}_{1212} &= \frac{1}{2}\left(\bar{\mathbf{C}}_{1111} - \bar{\mathbf{C}}_{1122}\right) \\ \bar{\mathbf{C}}_{3333} &\neq 0. \end{aligned}$$

   Hence, the Mori-Tanaka stiffness tensor has transverse isotropic symmetry.

3. **Use the Eshelby solution for a cylindrical inclusion in a transversely isotropic matrix to compute the effective stiffness tensor $\bar{\underset{\approx}{\mathbf{C}}}$ of the composite using the self-consistent method.**

   The nonzero components of the Eshelby tensor for a cylindrical inclusion in a transversely isotropic matrix are

$$\begin{aligned} S_{1111} = S_{2222} &= \frac{5C_{11} + C_{12}}{8C_{11}} \\ S_{1122} = S_{2211} &= \frac{3C_{12} - C_{11}}{8C_{11}} \\ S_{1133} = S_{2233} &= \frac{C_{13}}{2C_{11}} \\ S_{2323} = S_{3131} &= 0.25 \\ S_{1212} &= \frac{3C_{11} - C_{12}}{8C_{11}} \end{aligned}$$

   where $\underset{\approx}{\mathbf{C}}$ is the stiffness tensor of the matrix.

The self-consistent solution is

$$\bar{\underset{\approx}{\mathbf{C}}}^{SC} = \underset{\approx}{C}_0 + f(\underset{\approx}{C}_1 - \underset{\approx}{C}_0)\bar{\underset{\approx}{A}}_1$$

where

$$\bar{\underset{\approx}{A}}_1 = \left[\underset{\approx}{I} + \bar{\underset{\approx}{S}}\bar{\underset{\approx}{C}}_0^{-1}(\underset{\approx}{C}_1 - \bar{\underset{\approx}{C}}_0)\right]^{-1}$$

By taking the Mori-Tanaka stiffness tensor as the initial guess, for a volume fraction $f$=0.3 the estimated effective stiffness tensor using the self-consistent method is

$$\bar{\underset{\approx}{\mathbf{C}}}^{MK} = \begin{bmatrix} 7.6664 & 3.0745 & 2.8655 & 0 & 0 & 0 \\ 3.0745 & 7.6664 & 2.8655 & 0 & 0 & 0 \\ 2.8655 & 2.8655 & 29.6956 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5.7882 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5.7882 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.4347 \end{bmatrix} \text{GPa}$$

4. **Compute and plot Young's moduli of the composite along the fiber direction and transverse to the fiber direction as a function of the fiber volume fraction, $f$. Include the Mori-Tanaka and self-consistent estimate as well as the Voigt and Reuss bounds**

The Young's modulus in the fiber direction is

$$E_{cl} = E_1 \times f + E_0 \times (1 - f)$$

The young's modulus transverse to the fibre direction is

$$\frac{1}{E_{ct}} = \frac{f}{E_1} + \frac{1-f}{E_0}.$$

Figure 1 shows the estimation of Young's modulus in fiber direction for various methods. All of them coincide with each other except Reuss bound. Since the only possibility for the Young's modulus to be maximum is in fiber direction, It makes sense that both the Mori-Tanaka and self-consistent estimates agree with the Voigt bound.

Figure 2 shows the estimation of Young's modulus transverse to the fiber direction for various methods. $E_{ct}$ coincides with Reuss bound, while self-consistent and Mori-Tanaka estimates reside inside the Reuss and Voigt bounds which is the typical behaviour. And the difference between the Mori-Tanaka and self-consistent estimates is high for higher volume fractions.

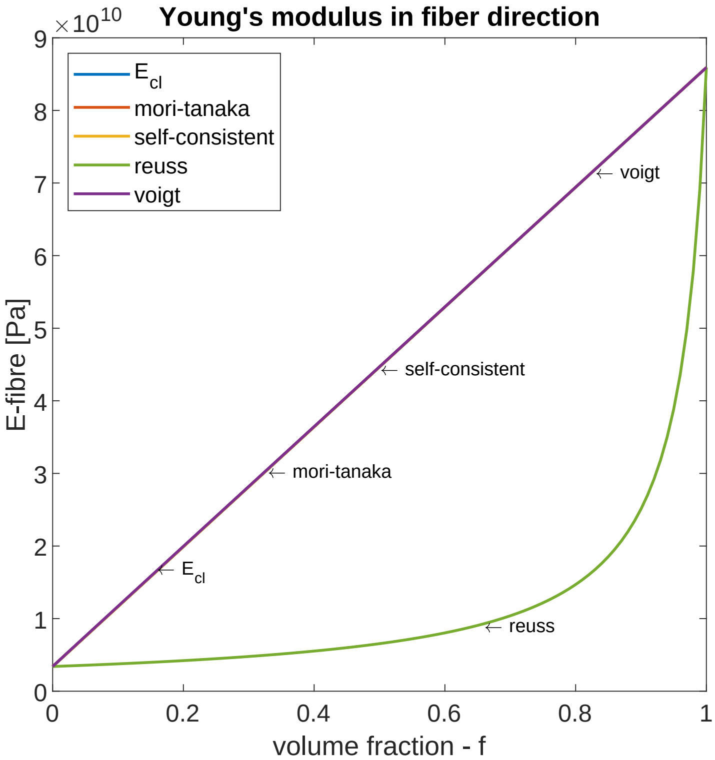


Figure 1: Young's modulus in fiber direction

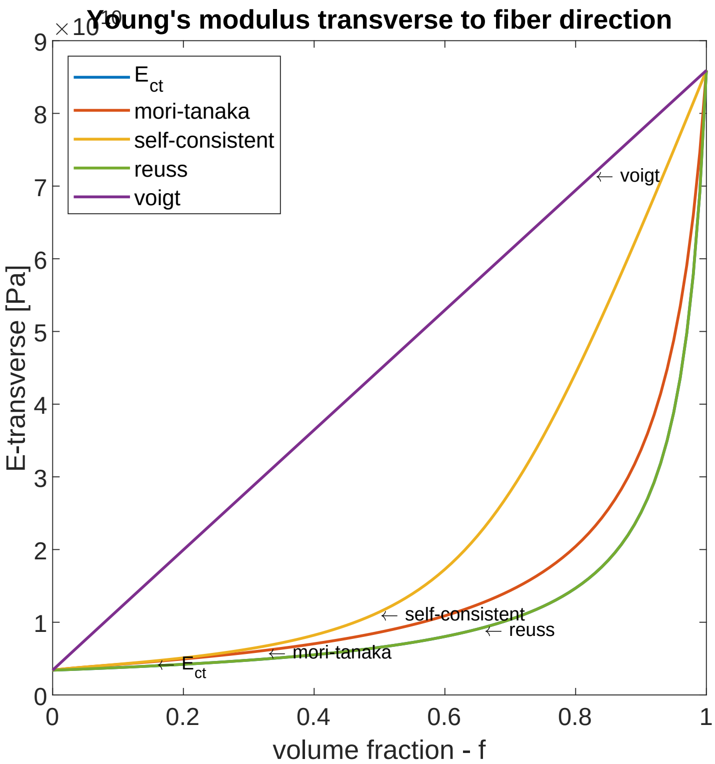


Figure 2: Young's modulus transverse to the fiber direction

5. **Compute and plot the shear moduli in the plane of symmetry and any plane normal to the plane of symmetry as a function of $f$. Include the Mori-Tanaka and self-consistent estimate as well as the Voigt and Reuss bounds.**

   The in-plane shear modulus is

$$\frac{1}{\mu_{12}} = \frac{f}{\mu_1} + \frac{1-f}{\mu_0}$$

   The out-of-plane shear modulus is

$$\mu_{23} = \mu_{31} = \mu_1 \times f + \mu_0 \times (1-f)$$

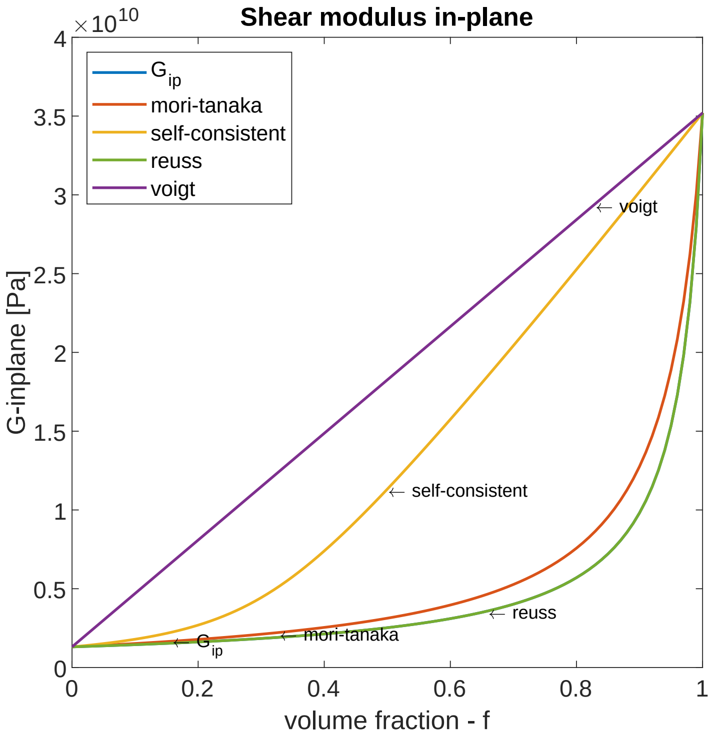


Figure 3: Shear modulus in plane of symmetry

   Figure 3 shows the estimation of shear modulus in the plane of symmetry for various methods. Figure 4 shows the estimation of shear modulus in the plane perpendicular to the plane of symmetry for various methods. In both cases, The behaviour is similar to particular composites and the in-plane shear modulus is a little less than the out of plane shear modulus for both Mori-Tananka and self-consistent estimates.

6. **Plot the Poisson's ratio of the composite for loading along the fiber direction as a function of $f$. Include the Mori-Tanaka and self-consistent estimate as well as the Voigt and Reuss bounds**

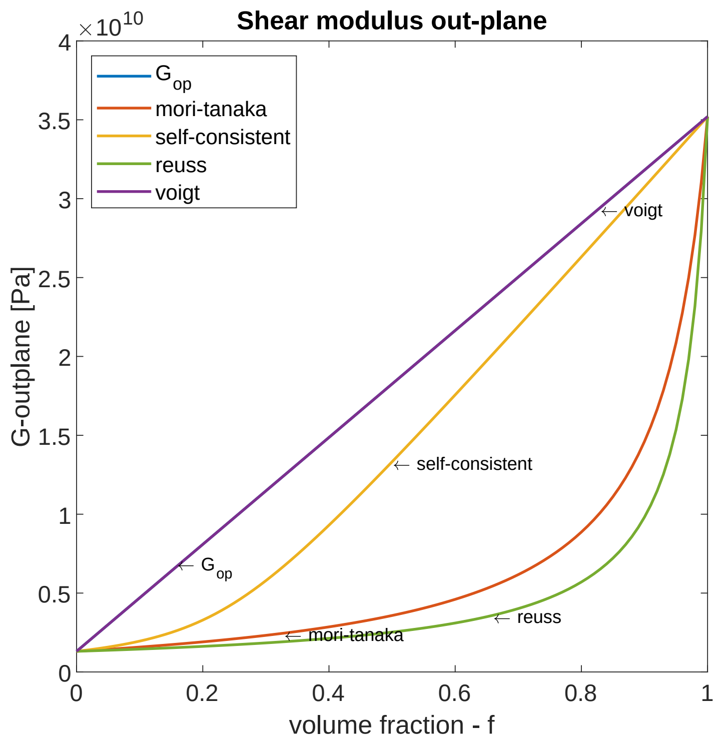


Figure 4: Shear modulus in plane of symmetry

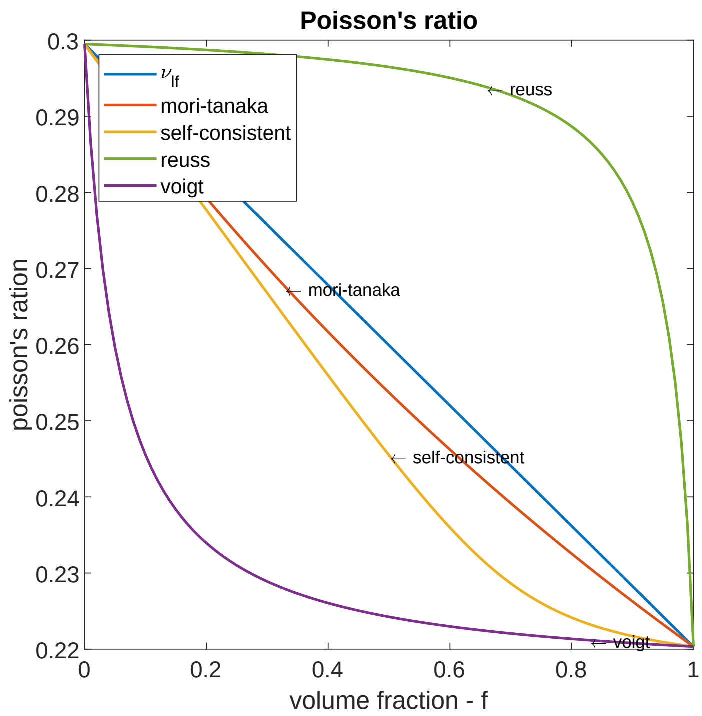


Figure 5: Poisson's ratio

The Poisson's ratio of the composite for loading along the fiber direction as a function of $f$ is

$$\nu_{12} = \nu_1 \times f + \nu_0 \times (1 - f)$$

Figure 5 shows the estimation of Poisson's ratio for various methods. Similar to particular composites the bounds are flipped. the self-consistent estimate, Mori-Tanaka estimate and $\nu_{12}$ are close to each other.