Vehicle Routing Problem using Quantum Computing

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- This is where we want to make use of quantum properties of superposition so that we can explore this vast solution space quickly

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- backend tries to find the lowest energy state of this physical system
- which is equivalent to finding the minimum of our objective function

Hamiltonian in $Y_{i\alpha}$

- minimizing cost

$$H_A = A \cdot \sum_{v=1}^{V} \sum_{i=1}^{N_0} \sum_{j=1}^{N_0} c_{ij}^v \sum_{\alpha=1}^{N_0-1} y_{i\alpha}^v y_{j\alpha+1}^v + A \cdot \sum_{v=1}^{V} \sum_{i=1}^{N_0} c_{0i}^v \left(y_{i1}^v + \sum_{\alpha=2}^{N_0} \left(1 - \sum_{\substack{j=1 \\ j \neq i}}^{N_0} y_{j(\alpha-1)}^v \right) y_{i\alpha}^v \right)$$
$$+ A \cdot \sum_{v=1}^{V} \sum_{i=1}^{N_0} c_{i0}^v \left(y_{iN_0}^v + \sum_{\alpha=1}^{N_0-1} y_{i\alpha}^v \left(1 - \sum_{\substack{j=1 \\ j \neq i}}^{N_0} y_{j(\alpha+1)}^v \right) \right)$$

- constraints to prevent sub-tours

$$H_B = B \cdot \sum_{i=1}^{N_0} \left(1 - \left(\sum_{\alpha=1}^{N_0} \sum_{v=1}^{V} y_{i\alpha}^v \right) \right)^2$$

$$H_C = C \cdot \sum_{\alpha=1}^{N_0} \left(1 - \left(\sum_{i=1}^{N_0} \sum_{v=1}^{V} y_{i\alpha}^v \right) \right)^2$$

Hamiltonian in $Y_{i\alpha}$

- order delivery quantity

$$H_D = D \sum_{v=1}^{k} \left(\sum_{i=1}^{N_0} \sum_{\alpha=1}^{N_0} q_i y_{i\alpha}^v - Q_v \right)$$

- time constraints

$$H_E = E \cdot \sum_{v=1}^{V} \sum_{i=1}^{N_0} \sum_{j=1}^{N_0} t_{ij}^v \sum_{\alpha=1}^{N_0-1} y_{i\alpha}^v y_{j\alpha+1}^v + E \cdot \sum_{v=1}^{V} \sum_{i=1}^{N_0} t_{0i}^v \left(y_{i1}^v + \sum_{\alpha=2}^{N_0} \left(1 - \sum_{\substack{j=1 \\ j \neq i}}^{N_0} y_{j(\alpha-1)}^v \right) y_{i\alpha}^v \right)$$
$$+ E \cdot \sum_{v=1}^{V} \sum_{i=1}^{N_0} t_{i0}^v \left(y_{iN_0}^v + \sum_{\alpha=1}^{N_0-1} y_{i\alpha}^v \left(1 - \sum_{\substack{j=1 \\ j \neq i}}^{N_0} y_{j(\alpha+1)}^v \right) \right)$$

Comparison of time for classical and quantum

N	K	classical route: V1	V2	classical time	classical cost	CQM cost	BQM cost	vehicle routes-v1	v2	constraint satisfied	time
4	2	Vehicle_1 path: []	0 ->2 -> 4 -> 1 -> 3 -> 0	0.080123901	112.7471745	153.757	112.747 (D= 10000, E=100)	Vehicle_1 path: []	Vehicle 2 : 0 -> 2 -> 4 -> 3 -> 1 -> 0	yes	42.519 ms
5	2	Vehicle_1 path: []	0 -> 2 -> 4 -> 5 -> 3 -> 1 -> 0	0.1208	110.3649677	152.6	126.389 (D= 1000,E=100)	Vehicle_1 path: []	Vehicle 2 : 0 -> 2 -> 3 -> 1 -> 4 -> 5 -> 0	yes	73.678 ms
6	2	Vehicle_1 path: []	0 -> 2 -> 4 -> 5 -> 3 -> 1 -> 6 -> 0	0.248588133	107.23425	146.82	254.172	Vehicle_1 path: []	Vehicle_2 path: ['0 -> 1', '1 -> 3', '2 -> 6', '3 -> 5', '4 -> 2', '5 -> 4', '6 -> 0']	yes	31.822
7	2	0 -> 2 -> 4 -> 5 -> 0	0 -> 7 -> 3 -> 1 -> 6 -> 0	0.253687143	222.1871036	265.50	312.579	Vehicle_1 path: ['0 -> 3', '1 -> 5', '3 -> 1', '5 -> 0']	Vehicle_2 path: ['0 -> 2', '2 -> 4', '4 -> 7', '6 -> 0', '7 -> 6']	yes	15.927
8	2	0 -> 2 -> 8 -> 0	0 -> 7 -> 4 -> 5 -> 3 -> 1 -> 6 -> 0	0.369401932	225.1299349	269.27	-	Vehicle_1 path: ['0 -> 3', '1 -> 5', '3 -> 1', '5 -> 0']	Vehicle_2 path: ['0 -> 2', '2 -> 8', '4 -> 7', '6 -> 0', '7 -> 6', '8 -> 4']	yes	31.849
9	2	0 -> 2 -> 8 -> 0	0 -> 7 -> 9 -> 4 -> 5 -> 3 -> 1 -> 6 -> 0	0.54723405	228.7424815	264.382	-	Vehicle_1 path: ['0 -> 1', '1 -> 3', '3 -> 5', '5 -> 0']	Vehicle_2 path: ['0 -> 2', '2 -> 9', '4 -> 8', '6 -> 0', '7 -> 6', '8 -> 7', '9 -> 4']	yes	31.852

Quantum Computing: Advantages and Limitations

Quantum superposition enables us to perform multiple calculations at once as opposed to one at a time on classical computers

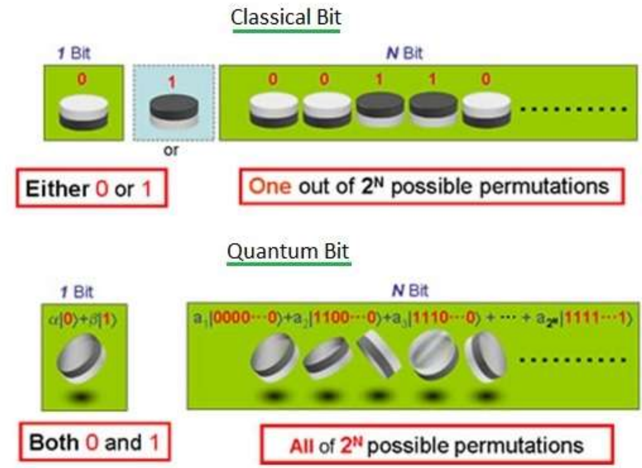

Figure: classical bit vs Qubit

IBM Roadmap for Quantum Computers

Since hardware is improving, it will not be long until quantum computers will demonstrate significant speed up. We have to be ready for the next big thing!