HW 1

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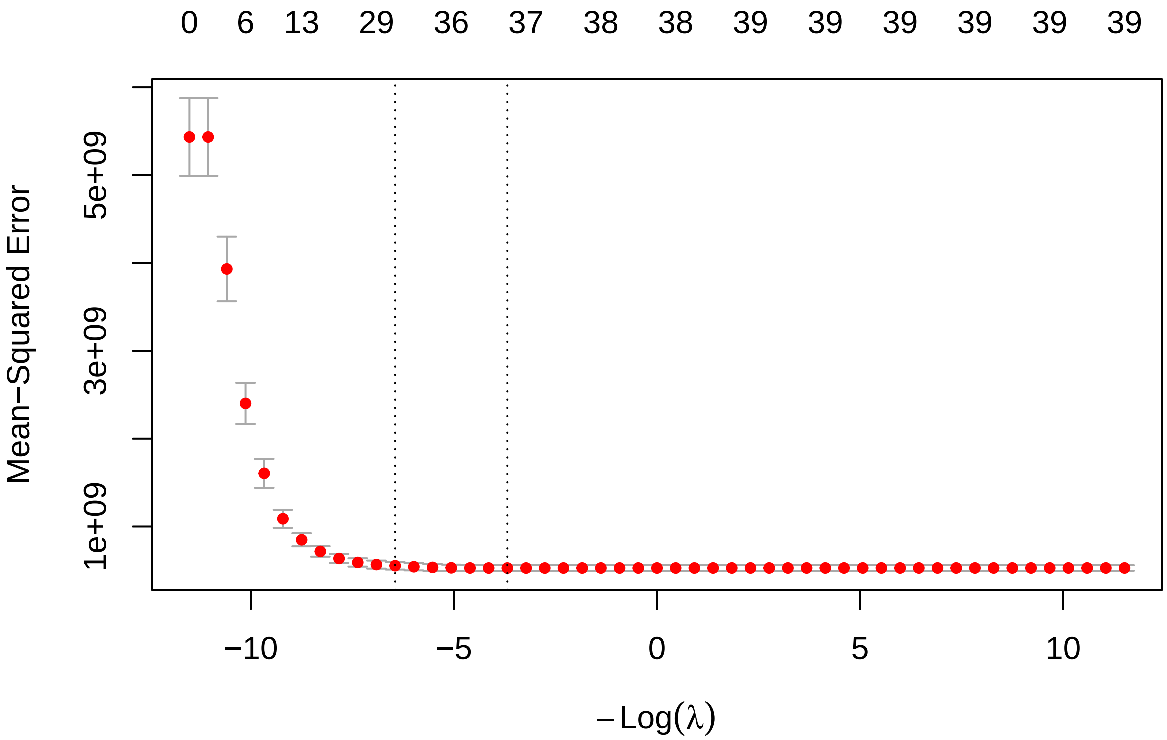
```
housing_train = read.csv("./data/housing_training.csv") |> janitor::clean_names()
housing_test = read.csv("./data/housing_test.csv") |> janitor::clean_names()

y = housing_train |> pull(sale_price)
x = model.matrix(sale_price ~., housing_train)[,-1]

x_test = model.matrix(sale_price ~ ., housing_test)[,-1]
y_test = housing_test$sale_price
```

a - LASSO Model

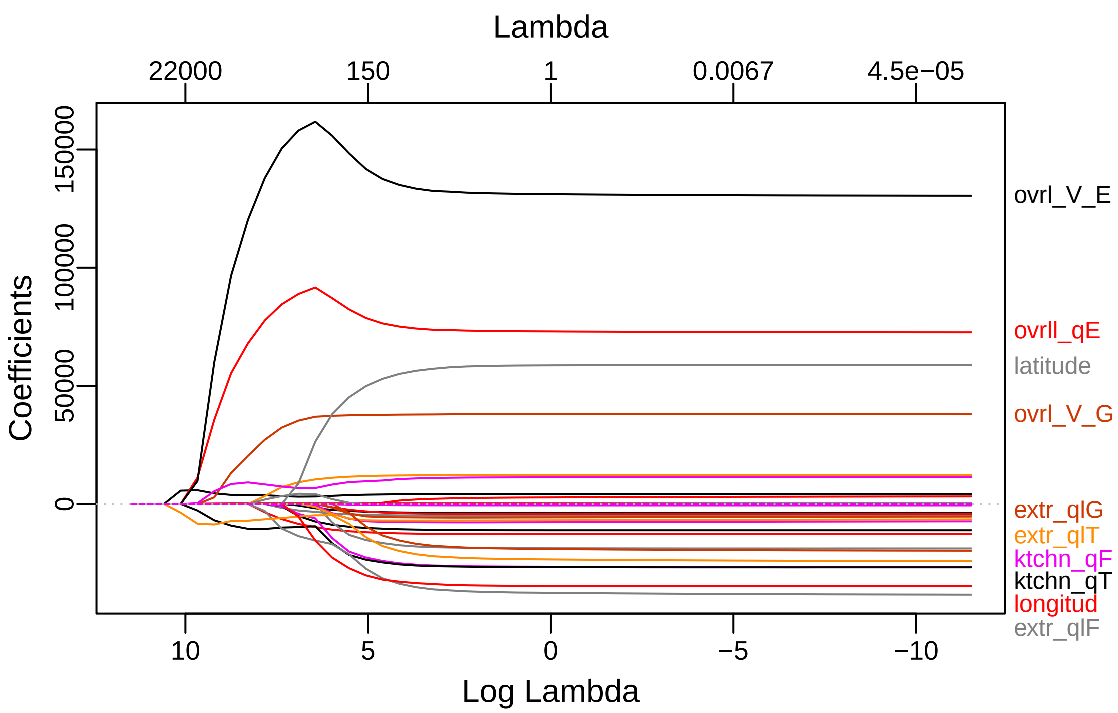
```
set.seed(1234)

lambda = 10^(seq(-5, 5, 0.2)) # lambda grid of lambda values for penalty tuning

# k-fold cross-validation for Lasso (alpha = 1) over the lambda values
lasso_cv = cv.glmnet(x, y,
                     alpha = 1,
                     lambda = lambda)

lambda_min = lasso_cv[["lambda.min"]] # minimum mean cross-validated error
lambda_1se = lasso_cv[["lambda.1se"]] # largest value of lambda such that error is within 1 standard er
CVM_min = lasso_cv |> broom::tidy() |> filter(lambda == lambda_min) |> pull(estimate)
CVM_1se = lasso_cv |> broom::tidy() |> filter(lambda == lambda_1se) |> pull(estimate)
```

The λ value with the smallest CVM (5.2673109×10^8) is 39.8107171.

```
plot(lasso_cv)
```




```r
plot_glmnet(lasso_cv$glmnet.fit)
```



Test Error with $\lambda = 39.81$:

```r
y_pred = predict(lasso_cv, newx = x_test, s = lambda_min, type = "response")
mspe_lasso <- mean((y_test - y_pred)^2)
```

The test error is 4.4304636×10^8.

```r
coef_1se = predict(lasso_cv, type = "coefficients", s = lambda_1se)
# Count non-zero coefficients to determine the number of predictors (excluding intercept)
num_predictors_1se = sum(coef_1se != 0) - 1
```

When using λ_{1SE}, there are 31 predictors.

b - Elastic Net Model

```r
set.seed(1234)

alpha = seq(0, 1, length = 21) # alpha grid ranging from 0 (Ridge) to 1 (Lasso)
lambda = 10^(seq(-5, 5, 0.2)) # lambda grid of lambda values for penalty tuning

# Iterate through each alpha to perform cross-validation and store results in a tibble
enet_cv_results = tibble(alpha = alpha) |>
  mutate(
    cv_fit = map(alpha, ~cv.glmnet(x, y, alpha = .x, lambda = lambda)), # runs glmnet for each alpha
    min_cvm = map_dbl(cv_fit, ~min(.x$cvm)) # finds min CVM for each model at each alpha
  )

# pull the alpha value with the lowest overall CVM
enet_alpha_min_cvm = enet_cv_results |>
  filter(min_cvm == min(min_cvm)) |>
  pull(alpha)

# pull the corresponding model with that optimal alpha
best_enet_cv_fit = enet_cv_results |>
  filter(alpha == enet_alpha_min_cvm) |>
  pull(cv_fit) |>
  pluck(1)

# pull 1SE lambda for optimal alpha
lambda_1se_enet = best_enet_cv_fit$lambda.1se

# make predictions using optimal alpha and 1SE lambda at that alpha
y_pred = predict(best_enet_cv_fit, newx = x_test, s = "lambda.1se")
mspe_elastic_net = mean((y_test - y_pred)^2)
```

The selected tuning parameters are $\alpha = 0.45$ and $\lambda_{1SE} = 1584.89$. The model with these parameters has a test error of 4.2029165×10^8.

Applying the 1SE rule is a bit more complicated with elastic net, because it has two parameters, α and λ. The 1SE rule, when used in the Lasso model, is easy to implement because a larger value for λ means fewer non-zero coefficients and a more parsimonious model. In elastic net models, this will work when α is fixed at a single value.

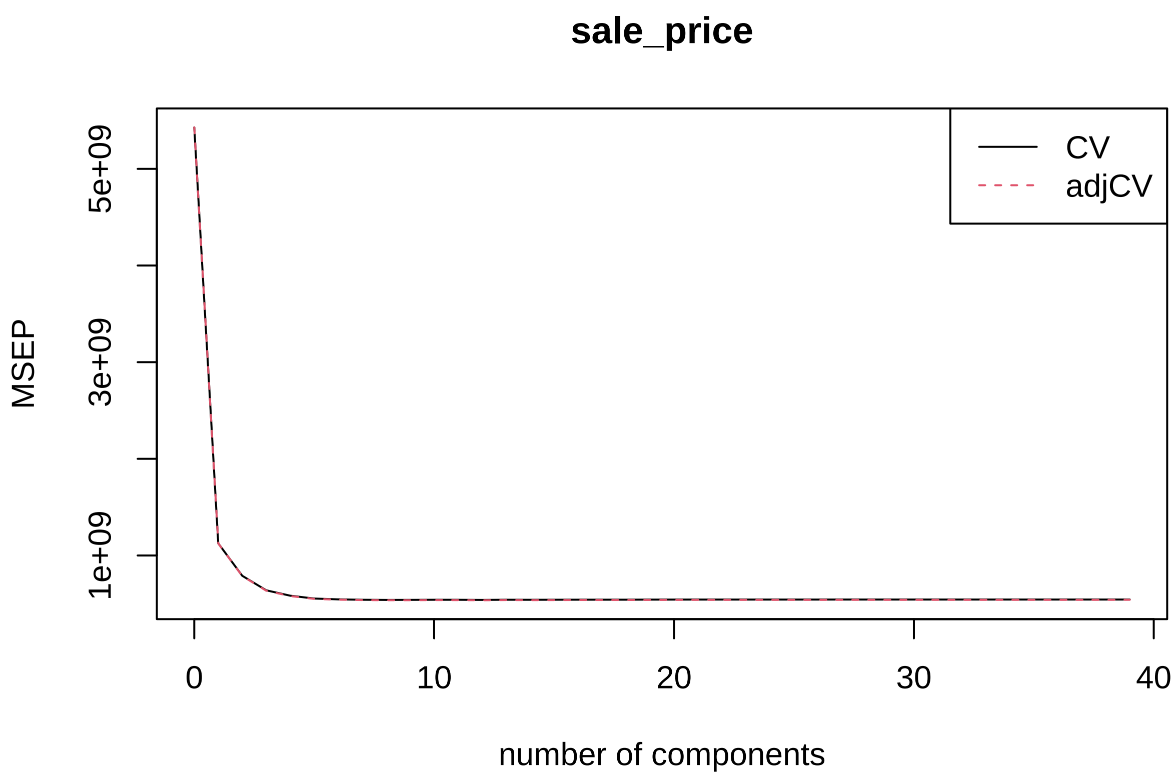
c - Partial Least Squares (PLS) Model

```r
set.seed(1234)

pls_mod <- plsr(sale_price ~ .,
                data = housing_train,
                scale = TRUE, # similar scaling importance as PCR
```

```r
                validation = "CV")
summary(pls_mod)
```

```
## Data:    X dimension: 1440 39
##  Y dimension: 1440 1
## Fit method: kernelpls
## Number of components considered: 39
##
## VALIDATION: RMSEP
## Cross-validated using 10 random segments.
##        (Intercept)  1 comps  2 comps  3 comps  4 comps  5 comps  6 comps
## CV           73685    33553    28106    25289    24162    23546    23362
## adjCV        73685    33537    28060    25207    24086    23471    23295
##        7 comps  8 comps  9 comps  10 comps  11 comps  12 comps  13 comps
## CV       23277    23238    23250     23272     23269     23240     23282
## adjCV    23210    23173    23182     23200     23196     23170     23207
##        14 comps  15 comps  16 comps  17 comps  18 comps  19 comps  20 comps
## CV        23266     23279     23295     23290     23294     23312     23315
## adjCV     23193     23205     23219     23215     23219     23235     23238
##        21 comps  22 comps  23 comps  24 comps  25 comps  26 comps  27 comps
## CV        23323     23322     23322     23322     23323     23324     23326
## adjCV     23245     23245     23244     23244     23245     23246     23248
##        28 comps  29 comps  30 comps  31 comps  32 comps  33 comps  34 comps
## CV        23326     23326     23326     23327     23327     23327     23327
## adjCV     23248     23248     23248     23248     23248     23248     23248
##        35 comps  36 comps  37 comps  38 comps  39 comps
## CV        23327     23327     23327     23327     23326
## adjCV     23248     23248     23248     23248     23266
##
## TRAINING: % variance explained
##             1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps
## X             20.02    25.93    29.67    33.59    37.01    40.03    42.49
## sale_price    79.73    86.35    89.36    90.37    90.87    90.99    91.06
##             8 comps  9 comps  10 comps  11 comps  12 comps  13 comps  14 comps
## X             45.53    47.97     50.15     52.01     53.69     55.35     56.86
## sale_price    91.08    91.10     91.13     91.15     91.15     91.16     91.16
##             15 comps  16 comps  17 comps  18 comps  19 comps  20 comps
## X              58.64     60.01     62.18     63.87     65.26     67.10
## sale_price     91.16     91.16     91.16     91.16     91.16     91.16
##             21 comps  22 comps  23 comps  24 comps  25 comps  26 comps
## X              68.44     70.12     71.72     73.35     75.20     77.27
## sale_price     91.16     91.16     91.16     91.16     91.16     91.16
##             27 comps  28 comps  29 comps  30 comps  31 comps  32 comps
## X              78.97     80.10     81.83     83.55     84.39     86.34
## sale_price     91.16     91.16     91.16     91.16     91.16     91.16
##             33 comps  34 comps  35 comps  36 comps  37 comps  38 comps
## X              88.63     90.79     92.79     95.45     97.49    100.00
## sale_price     91.16     91.16     91.16     91.16     91.16     91.16
##             39 comps
## X             100.24
## sale_price     91.14
```

```r
# plot cross-validated MSEP for PLS
validationplot(pls_mod, val.type = "MSEP", legendpos = "topright")
```




```
# determine the optimal number of components
cv_mse <- RMSEP(pls_mod)
ncomp_cv <- which.min(cv_mse$val[1,,]) - 1
ncomp_cv
```

```
## 8 comps
##       8
```

```
# calculate test MSE
predy2_pls <- predict(pls_mod, newdata = housing_test,
                      ncomp = ncomp_cv)

mspe_pls = mean((y_test - predy2_pls)^2)
```

There are 8 components in the partial least squares model, with an MSPE of 4.4021794×10^8.

d - Comparing Models

```
summary = tibble(
  model = c("LASSO", "Elastic Net", "PLS"),
  mspe = c(mspe_lasso, mspe_elastic_net, mspe_pls)
)

knitr::kable(summary)
```

model	mspe
LASSO	443046363
Elastic Net	420291645
PLS	440217938

Comparing the mean squared predicted error across the three models, the elastic net model is the best model for making predictions. This means that the model likely benefits from the balance of LASSO and Ridge (λ, α) penalties, as compared to the LASSO model, which only has the λ penalty.