ASSIGNMENT – 2

MA 202

Name : SHIVAM TAYAL

Roll No : 19074016

Dept. : CSE (IDD)

1.) Urn contains :

4 white balls

6 black balls

4 white
6 black

No. of balls to be drawn = 5

Let 'W' represents a white ball is drawn and 'B' represents a black ball is drawn

The probability distribution of no. of white balls drawn without replacement is as follows :

$$P(0\ white) = \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} \times \frac{2}{6}$$

$$= \frac{1}{42}$$

$$P(1W, 4B) = \frac{4}{10} \times \frac{6}{9} \times \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} \times 5$$

$$= \frac{5}{21}$$

$$P(2W, 3B) = \frac{4}{10} \times \frac{3}{9} \times \frac{6}{8} \times \frac{5}{7} \times \frac{4}{6} \times 10 = \frac{10}{21}$$

$$P(3W, 2B) = \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8} \times \frac{6}{7} \times \frac{5}{6} \times 10 = \frac{5}{21}$$

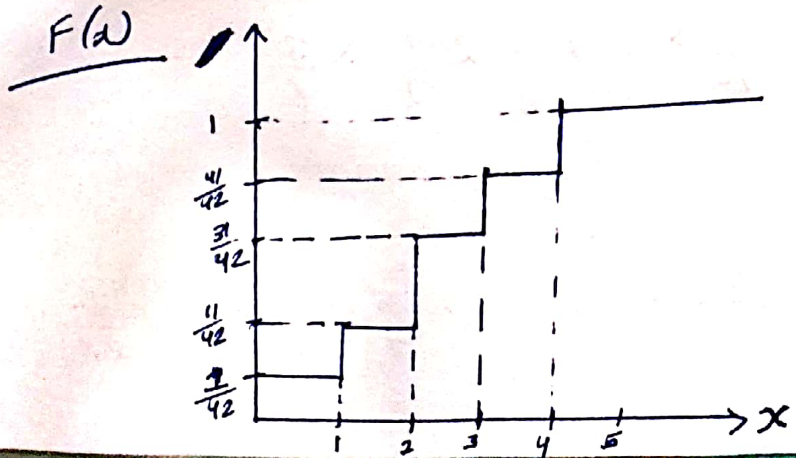
$$P(4W, 1B) = \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8} \times \frac{1}{7} \times \frac{6}{6} \times 5 = \frac{1}{42}$$

$$\frac{1}{42} + \frac{5}{21} + \frac{10}{21} + \frac{5}{21} + \frac{1}{42} = 1$$

So, we can see that sum of all probabilities is equal to 1, i.e. $\sum_{x=0}^{4} p(x) = 1$

Now, probability distribution fn. of x will be :

$$F(x) = \begin{cases} 0 & , -\infty < x < 0 \\ \frac{1}{42} & , 0 \le x < 1 \\ \frac{11}{42} & , 1 \le x < 2 \\ \frac{31}{42} & , 2 \le x < 3 \\ \frac{41}{42} & , 3 \le x < 4 \\ 1 & , 4 \le x < \infty \end{cases}$$



2.) Tossing a fair coin until head appears.
Let x be random variable representing the no. of toss required. So, x can be 1, 2, 3, ...
Let us take no. of turn required to be 'y'.
So, $P(x=y)$ means $(y-1)$ failure and success on y^{th} turn.

$P(x=y) = (1-p)^{y-1} p$ (p = probability of success)

P(getting a head) = $\frac{1}{2}$

$\therefore P(x=y) = \left(1-\frac{1}{2}\right)^{y-1} \times \frac{1}{2} = \left(\frac{1}{2}\right)^{y} = \frac{1}{2^{y}}$

$P(x=1) = \frac{1}{2}$, $P(x=2) = \frac{1}{4}$, $P(x=3) = \frac{1}{8}$

Probability fn. $P(x=y) = \frac{1}{2^{y}}$

$$\sum_{x=1}^{\infty} P(X=x) = \sum_{y=1}^{\infty} \frac{1}{2^{y}} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$$

$$= \frac{1}{2}\left[\frac{1}{1-\frac{1}{2}}\right] = \boxed{1}$$

3.) A random variable x has the following discrete distribution :

X	-3	-2	-1	0	1	2	3	4
$P(X=x)$	$2k^2$	k	$2k$	$3k$	$2k$	k	$7k^2$	k^2

We know that $\Sigma P(X=x) = 1$

$$\sum_{x=-3}^{4} P(X=x) = 2k^2 + k + 2k + 3k + 2k + k + 7k^2 + k^2 = 1$$

$$\Rightarrow 10k^2 + 9k = 1$$

$$\Rightarrow 10k^2 + 9k - 1 = 0$$

$$(10k-1)(k+1) = 0$$

It gives, $\Rightarrow k = \frac{1}{10}, -1$

But $k \neq -1$, since $P(x) \geq 0$

Therefore, the given distribution becomes :

X	-3	-2	-1	0	1	2	3	4
$P(X=x)$	$\frac{2}{100}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{1}{10}$	$\frac{7}{100}$	$\frac{1}{100}$

(i) $y = x^2$

So, range of y is $0, 1, 4, 9, 16$

let $y = g(x) = x^2$

For, $x_1 \neq x_2 \Rightarrow g(x_2) = g(x_1)$ $\{$ for $x = (1,-1)(2,-2)(3,-3)\}$

So, probability discrete functions y will not be same as that of X

$$P(y=9) = P(x=-3) + P(x=3)$$
$$= \frac{2}{100} + \frac{7}{100} = \frac{9}{100}$$

$$P(y=4) = P(x=-2) + P(x=2)$$
$$= \frac{1}{10} + \frac{1}{10} = \frac{2}{10}$$

$$P(y=1) = P(x=-1) + P(x=1)$$
$$= \frac{2}{10} + \frac{2}{10} = \frac{4}{10}$$

$$P(y=0) = P(x=0) = \frac{3}{10}$$

$$P(y=16) = P(x=4) = \frac{1}{100}$$

Probability distribution of $y = x^2$

y	0	1	4	9	16
$P(y=y)$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{2}{10}$	$\frac{9}{100}$	$\frac{1}{100}$

(ii) $y = |x-1| + |x+1| = g(x)$

Range of $y = 2, 4, 6, 8$

for $x_1 \neq x_2 \Rightarrow g(x_1) = g(x_2)$ {for $x_1 = (-1, 0, 1), (2, -2), (3, -3)$}

Probability fn. y will not be same as X.

$P(Y=2) = P(X=-1) + P(X=0) + P(X=1)$

$= \frac{2}{10} + \frac{3}{10} + \frac{2}{10} = \frac{7}{10}$

$P(Y=4) = P(X=-2) + P(X=2)$

$= \frac{1}{10} + \frac{1}{10} = \frac{2}{10}$

$P(Y=6) = P(X=-3) + P(X=3)$

$= \frac{2}{100} + \frac{7}{100} = \frac{9}{100}$

$P(Y=8) = P(X=4) = \frac{1}{100}$

$\therefore$ Probability distribution fn. of $Y = |X-1| + |X+1|$

y	2	4	6	8
$P(Y=y)$	$\frac{7}{10}$	$\frac{2}{10}$	$\frac{9}{100}$	$\frac{1}{100}$

4.) Given that,

$$F_x(x) = \begin{cases} 0 & , -\infty < x < 0; \\ 1/5 & , 0 \leq x < 1; \\ 3/5 & , 1 \leq x < 3; \\ 1 & , 3 \leq x < \infty; \end{cases}$$

we know that cumulative distribution fn.

$$F(x) = P(x \leq X)$$

Range of random variable x is $0, 1, 3$

$$P(x=0) = P(0 \leq x < 1) - P(-\infty < x < 0)$$

$$= \frac{1}{5} - 0 = \frac{1}{5}$$

$$P(x=1) = P(1 \leq x < 3) - P(0 \leq x < 1)$$

$$= \frac{3}{5} - \frac{1}{5} = \frac{2}{5}$$

$$P(x=3) = P(3 \leq x < \infty) - P(1 \leq x < 3)$$

$$= 1 - \frac{3}{5} = \frac{2}{5}$$

$$P_x(x) = \begin{cases} 1/5 & , x = 0 \\ 2/5 & , x = 1 \\ 2/5 & , x = 3 \\ 0 & , \text{otherwise} \end{cases}$$

5.) Let x be the random variable representing no. of screws defective in the lot, then $x = 0, 1, 2, 3, 4, 5, 6$

Probability of a screw to be defective $= \frac{1}{50} = p$ (success)

Probability of a screw to be non-defective $= \frac{49}{50} = q$

Clearly, x is a binomial random variable with parameters,

$$n = 6, \quad p = \frac{1}{50}, \quad q = \frac{49}{50}$$

We know that,

$$P(X = x) = {}^nC_x (p)^x q^{n-x}$$

(i) 2 defective screws in the lot :

$x = 2$

$$P(X = 2) = {}^6C_2 \left(\frac{1}{50}\right)^2 \left(\frac{49}{50}\right)^4 = \frac{15 \times (49)^4}{(50)^6}$$

(ii) 0 defective screws in the lot :

$x = 0$

$$P(X = 0) = {}^6C_0 \left(\frac{1}{50}\right)^0 \left(\frac{49}{50}\right)^6 = \left(\frac{49}{50}\right)^6$$

(iii) At most two defective screws :

$$P(x \leq 2) = P(x=0) + P(x=1) + P(x=2)$$

$$= \left(\frac{49}{50}\right)^6 + \frac{6 \times 1}{50} \times \left(\frac{49}{50}\right)^5 + \frac{15 \times (49)^4}{(50)^6}$$

$$= \frac{(49)^4}{(50)^6} \left[49^2 + 6 \times 49 + 15\right]$$

$$= \frac{2710 \times (49)^4}{(50)^6}$$

6.) Let x be a random variable denoting number of defective pieces, $x = 0, 1, 2, \ldots$

Sample = 30

Probability of product to be defective $= \frac{1}{10} = 'p'$

Probability of product to be non-defective $= \frac{9}{10} = 'q'$

x is a binomial random variable with parameters,

$n = 30,\ p = \frac{1}{10},\ q = \frac{9}{10}$

(i) Probability of 2 defective products is :

$P(X=x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$

$P(X=2) = {}^{30}C_{2} \left(\frac{1}{10}\right)^{2} \times \left(\frac{9}{10}\right)^{28}$

$P(X=2) = \frac{30!}{2!\,28!} \times \frac{9^{28}}{10^{30}} = \frac{15 \times 29 \times 9^{28}}{10^{30}} = \frac{435 \times 9^{28}}{10^{30}}$

$P(X=2) = \underline{0.2276}$

(ii) Using Poisson distribution $P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

Now, $\lambda = np = 30 \times \frac{1}{10} = 3$

$P(X=2) = \frac{e^{-3}(3)^{2}}{2!} = \frac{9}{2} \times e^{-3} = 4.5 \times 0.0498$

$P(X=2) = \underline{0.2241}$

Error $= 0.2276 - 0.2241 = \underline{0.0035}$

Error in % $= \frac{0.0035}{0.2276} \times 100 = \underline{1.537\%}$