

Cross Entropy loss

$$L = \text{Cross E}(A, P)$$

A = Ground Truth
distribution

P = Predicted distribution

$$A \in \mathbb{R}^{m \times n}$$

m = batch size

$$P \in \mathbb{R}^{m \times n}$$

n = Input dim.

$$L \in \mathbb{R}^m$$

Forward Propagation

$$L = \text{Cross E}(A, P)$$

$$L_i = - \sum_{j=1}^n A_{ij} \log P_{ij}$$

$$1 \leq i \leq m$$

Derivative

$$L_i = - \sum_{j=1}^n A_{ij} \log P_{ij} \quad \text{--- (1)}$$

$$\frac{\partial L}{\partial P} \in \mathbb{R}^{m \times n}$$

Here P is
predicted
distribution

by (1)

$$\left(\frac{\partial L}{\partial P} \right)_{ij} = \frac{\partial L_i}{\partial P_{ij}} = \frac{-A_{ij}}{P_{ij}} \quad \begin{matrix} 1 \leq i \leq m \\ 1 \leq j \leq n \end{matrix}$$

\Rightarrow often for batch size one,

$$A = [0, \dots, 1, \dots, 0]_{1 \times m}$$

$$P = [P_1, \dots, P_k, \dots, P_m]_{1 \times m}$$

In this case

$$\left(\frac{\partial L}{\partial P} \right)_i = \frac{\partial L_i}{\partial P_i} = \begin{cases} \frac{-1}{P_i} & , i=k \\ 0 & , \text{else} \end{cases}$$

And Same, we can repeat for each element in batch.