Indian Institute of Engineering Science and Technology, Shibpur

B. Tech. First Semester Final Examination, March 2021

Subject: Mathematics-I (MA1101)

Full Marks: 50 **Time: 1 hour 30 minutes**

Answer any FIVE (05) questions.

[Only the first five answers will be considered for evaluation]

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1. a) Using Maclaurin series, show that

$$\sin x > \ x - \frac{1}{6}x^3 \ \text{ if } \ 0 < x < \frac{\pi}{2}$$

b) Find the radius of curvature at any point of the cycloid

$$x = a(\theta + \sin\theta)$$
$$y = a(1 - \cos\theta)$$

c) Find the asymptotes of the curve

$$x^2y^2 - x^2y - xy^2 + x + y + 1 = 0$$

(3+4+3=10)

2. a) Test for the convergence (*any two*) of the following series:

i) $\frac{x}{y} + \frac{x(x+1)}{y(y+1)} + \frac{x(x+1)(x+2)}{y(y+1)(y+2)} + \dots \infty,$ where $x > 0,\ y > 0$

ii) $1 + \frac{2^P}{2!} + \frac{3^P}{3!} + \dots \infty,$ where $P > 0$

iii) $\sum_{n=1}^{\infty} \frac{n^3 + a}{2^n + a}$, where a is a constant.

b) Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{2^n}{n^2} (x-3)^n$

(3+3+4= 10)

3. a) If $u = f(x,y)$, where $x = r\, cos\theta$ and $y = r\sin\theta$ then prove that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$

b) If V= $cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then prove that

$$x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} + \frac{1}{2}\cot V = 0$$

(5+5=10)

4. a) If $f(x,y) = \begin{cases} xy\frac{x^2-y^2}{x^2+y^2} \ when \ x^2+y^2 \neq 0 \\ 0 \ when \ x^2+y^2 = 0 \end{cases}$ show that

$$f_{xy} \neq f_{yx} \quad at\ (0,0)$$

b) Examine the convergence of $\int_0^\infty \frac{x^2 dx}{4x^4 + 25}$

c) Show that $\Gamma(n+1) = n\Gamma(n)\ \ for\ \ n > 0$

(4+3+3=10)

5. a) Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}$

b) Prove that $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$,where $P_n(x)$ is the Legendre polynomial of order n.

(6+4=10)

6. a) Solve the differential equation

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$$

in series about the ordinary point x=0.

b) Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$, where $J_n(x)$ is the Bessel function of first kind of order n.

(6+4=10)

7. a) Evaluate $\iint_R |x+y| dxdy$ over the region $R = \{(x,y): -1 \le x \le 1\ and\ -1 \le y \le 1\}$.

b) Evaluate $\iint (x+y)^4 dxdy$, where the double integration is over the parallelogram in the xy-plane with vertices (1, 0), (3,1), (2,2) and (0,1).

(5+5=10)