Indian Institute of Engineering Science and Technology, Shibpur

B.Tech (Group V to VIII) 1st Semester, End Semester Examination, February, 2023

Mechanics (AM 1101)

Full marks: 50 Time: 3 Hours

All notations have their usual meanings

Marks for each question are shown in the right

(i) Take $g = 9.81\ m/s^2$.

(ii) Answer **question no 1 (compulsory)** and any **Two (02)** questions from **Group A**, and any **Five (05)** questions from **Group B**

(iv) **Answer both Group A and Group B in SINGLE ANSWER SCRIPT**

[Group A: Statics]

[Answer **question no 1 (compulsory)** and any **Two (02)** questions from the rest of **Group A**)

1. Determine the maximum and minimum moments of inertia with respect to centroidal axes through C for the composite of the two rectangular areas shown in **Fig.1.** Find the angle β measured from the x axis to the axis of maximum moment of inertia. (8)

2. Determine the distance, $\overline{H}$ from the bottom of the base plate to the centroid of the built-up structural section shown in **Fig. 2.** (6)

3. The special box wrench with head B swiveled at C to the handle A will accommodate a range of sizes of hexagonal bolt heads. For the nominal size shown in **Fig. 3** where the center O of the bolt and the pin C are in line with the handle, compute the magnitude of the force supported by the pin at C if P = 100 N. Assume the surface of the bolt head to be smooth. (6)

4. Determine the force in the member BF as shown in **Fig. 4.** (6)

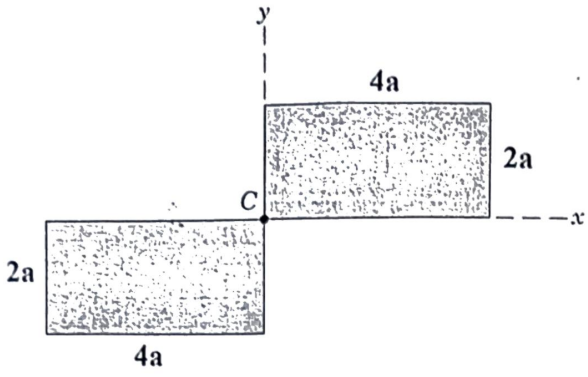
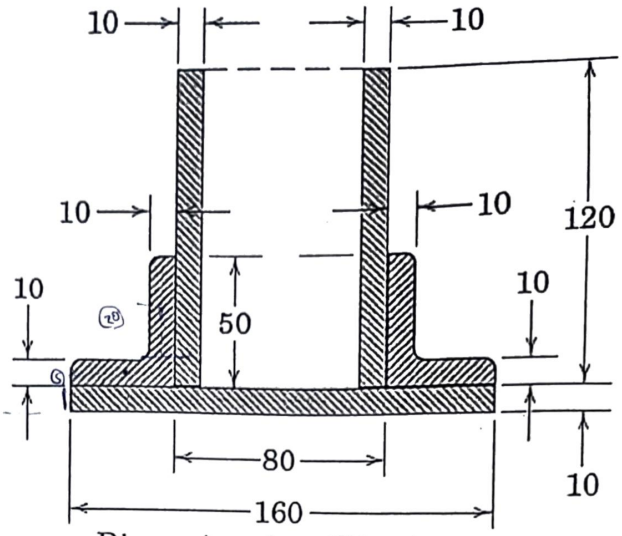


Fig. 1



Dimensions in millimeters

Fig. 2

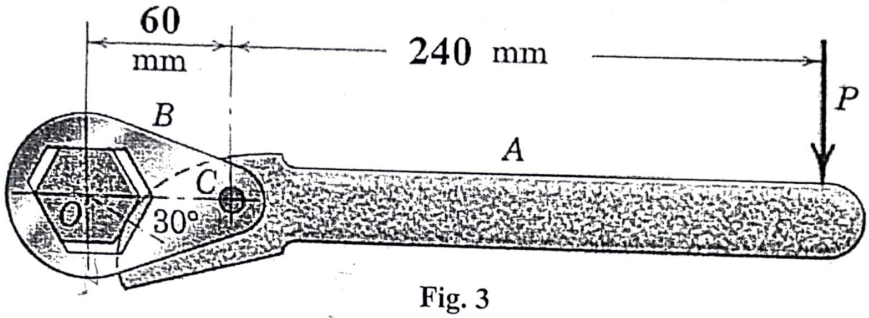


Fig. 3

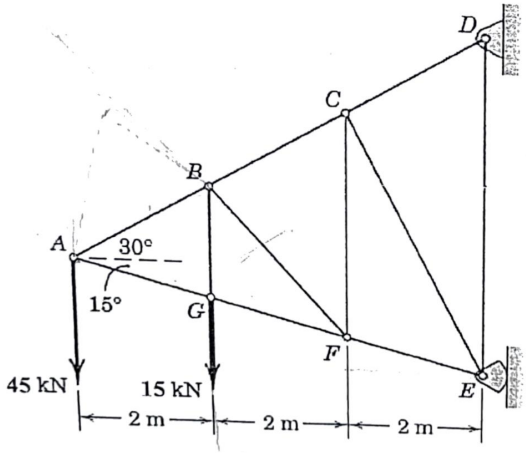


Fig. 4

[Group B: Dynamics]
(Answer any **Five (05)** questions from **Group B**)

5. The motion of the pin A in the fixed circular slot is controlled by the guide B, which is being elevated by its lead screw with a constant upward velocity $v_0 = 3$ m/sec for an interval of its motion shown in **Fig. 5**. Calculate both the normal and tangential components of acceleration of pin A as it passes the position for which $\theta = 40^0$. (6)

6. Neglect the diameter of the small pulley attached to body A and determine the magnitude of the total velocity of B in terms of the velocity v_A that body A has to the right as shown in **Fig. 6**. Assume that the cable between B and the pulley remains vertical and solve for a given value of x. (6)

7. The small object is placed on the inner surface of the conical dish at the radius shown in **Fig. 7**. If the coefficient of the static friction between the object and the conical surface is 0.35, for which range of angular velocities ω about the vertical axis will the block remain on the dish without slipping? Assume that the speed changes are made slowly so that any acceleration may be neglected. (6)

8. The force P, which is applied to the 10-kg block, initially at rest as shown in **Fig. 8**, varies linearly with time as indicated. If the coefficients of static and kinetic friction between the block and the horizontal surface are 0.60 and 0.40, respectively, determine the velocity of the block when $t = 3.5$ sec. (6)

9. A steel ball of mass m strikes a steel plate of mass m with a velocity $v_0 = 20$ m/s at an angle of 60^0 with the horizontal as shown in **Fig. 9**. The coefficient of restitution between the ball and the plate is 0.75. Compute the final velocities of both the masses immediately after the impact considering the plate is initially stationary. (6)

10. Cylinder B is connected with the block A as shown in **Fig. 10**. Neglecting all friction and mass of the pulleys, determine the accelerations of bodies A and B when release from rest. (6)

11. The nest of two springs is used to bring the 0.7 kg plunger A to a stop from a speed of 7 m/s and reserve its direction of motion as shown in **Fig. 11**. The inner spring increases the deceleration, and the adjustment of its position is used to control the exact point at which the reversal takes place. If this point is to correspond to a maximum deflection $\delta = 250$ mm for outer spring, specify the adjustment of the inner spring by determining the distance s. The outer spring has stiffness of 400 N/m and the inner one has stiffness of 200 N/m. (6)

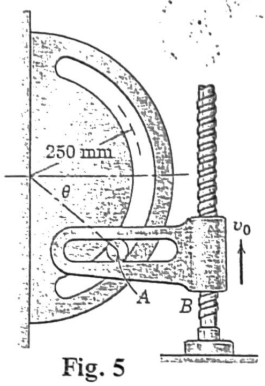


Fig. 5

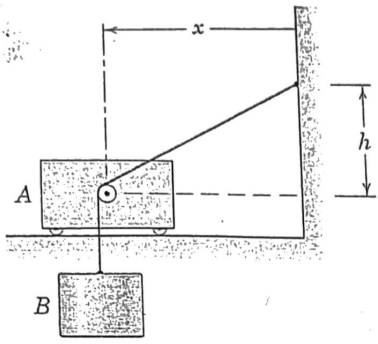


Fig. 6

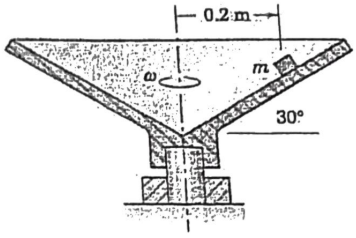


Fig. 7

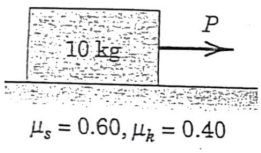
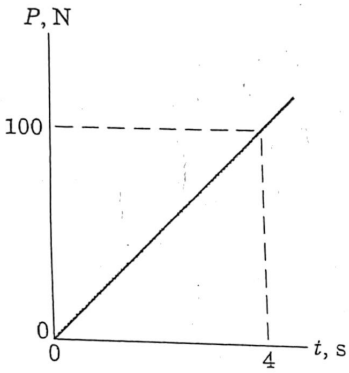


Fig. 8



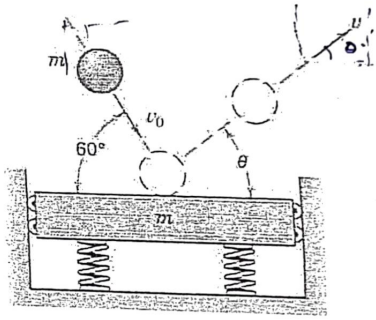


Fig. 9

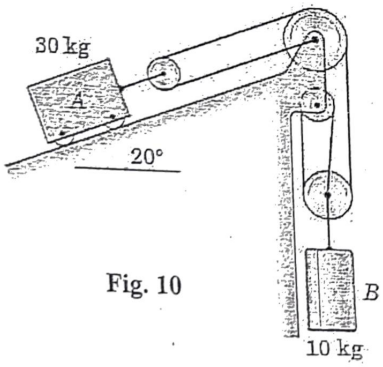


Fig. 10

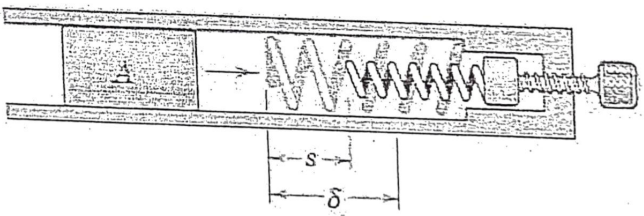


Fig. 11

Indian Institute of Engineering Science and Technology, Shibpur

B.Tech 1st Semester Final Term Examination, February 2023

Subject: MATHEMATICS-I (MA1101)

F.M. - 50 Time: 3 hours

Answer any **Five** questions

1. (a) Given $y = f(x) = (1+2x)^{-\frac{1}{2}}$
   (i) Prove that $(1+2x)y_{n+1} + (2n+1)y_n = 0$ where $y_n = \frac{d^n y}{dx^n}$.
   (ii) Expand $f(x)$ by Maclaurin's Theorem with remainder after n terms. Write remainder in Lagranges form.

   (b) For the curve $x^3 + y^3 - 3axy = 0$, prove that the radius of curvature at the point $(\frac{3}{2}a, \frac{3}{2}a)$ is numerically equal to $\frac{3a}{16}\sqrt{2}$.

   (c) Find all the asymptotes of the curve $x^3 + x^2y - xy^2 - y^3 + x^2 - y^2 = 2$ and show that two of which are parallel. (4+3+3)

2. (a) Given

$$f(x,y) = \begin{cases} xy\dfrac{x^2 - y^2}{x^2 + y^2} & : (x,y) \neq (0,0) \\ 0 & : (x,y) = (0,0) \end{cases}$$

   Show that $\frac{\partial^2 f}{\partial x \partial y} \neq \frac{\partial^2 f}{\partial y \partial x}$ at the origin.

   (b) If $u(x,y) = x\phi(\frac{y}{x}) + \psi(\frac{y}{x})$ then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$ assuming $u_{xy} = u_{yx}$.

   (c) Let $y = F(x,t)$ where $u = x+ct$, $v = x-ct$, c is a constant. Show that $\frac{\partial^2 y}{\partial u \partial v} = 0$ reduce to $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$. (4+3+3)

3. (a) Show that the expansion of $f(x,y) = sin(xy)$ in powers of $(x-1)$ and $(y-\frac{\pi}{2})$ upto and including second degree terms is $1 - \frac{1}{8}(x-1)^2\pi^2 - \frac{1}{2}\pi(x-1)(y-\frac{\pi}{2}) - \frac{1}{2}(y-\frac{\pi}{2})^2$.

   (b) Find the maximum value of $f(x,y) = sinx + siny + sin(x+y)$ where $0 \leq x \leq \pi$, $0 \leq y \leq \pi$.

   (c) If $\psi(x) = (x-a)^m(x-b)^n$ where m,n are positive integers then show that $\zeta \in (a,b)$ in Rolle's Theorem divides the line segment $a \leq x \leq b$ in the ratio $m:n$. (4+3+3)

4. (a) Solve : $(1+x)^2D^2y+(1+x)Dy-y=2sin(log(1+x))$ where $D\equiv\frac{d}{dx}$, $D^2\equiv\frac{d^2}{dx^2}$.

(b) Using Variation of parameter method solve $\frac{d^2y}{dx^2}-y=\frac{2e^{-x}}{1+e^{-x}}$.

(c) Derive **Rodrigue's formula** $P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$. (4+3+3)

5. (a) (i) Test for convergence of the infinite series $1+\frac{1}{2.3}+\frac{1.3}{2.4.5}+\frac{1.3.5}{2.4.6.7}+\ldots\ldots$

(ii) Find the radius of convergence of the power series $\sum_{n=1}^{\infty}\frac{n!}{n^n}x^n$.

(b) Using **Gamma function** evaluate : $\int_0^\infty x^4 4^{-x}dx$.

(c) Express the integral $\int_0^{2a}x^2\sqrt{2ax-x^2}dx$ in terms of **Beta function** and find the value. ((2+2)+3+3)

6. (a) Change the order of integration $\int_0^1\int_{x^2}^{2-x}xydydx$ and then evaluate.

(b) Show that $\int_0^1\int_0^x\sqrt{x^2+y^2}dxdy=\frac{1}{3}[\frac{1}{\sqrt{2}}+\frac{1}{2}sinh^{-1}1]$ by using the transformation $x=u,\ y=uv$.

(c) Evaluate $\iint_{\mathbb{R}}\frac{dxdy}{\sqrt{x^2+y^2}}$ where $\mathbb{R}=\{|x|\leq 1,\ |y|\leq 1\}$. (4+3+3)

7. (a) Solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^xsinx$.

(b) Prove that $J_{\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}sinx$ where $J_n(x)$ is the Bessel function of First kind and of order n.

(c) Show that $\int_1^5\frac{dx}{\sqrt{x^4-1}}$ converges but $\int_0^3\frac{dx}{(3-x)\sqrt{x^2+1}}$ diverges. (4+3+3)

END

Indian Institute of Engineering Science and Technology, Shibpur
B.Tech. 1st Semester Final Examinations, February, 2023

Introduction to Computing (CS - 1101)
(Group V, VI, VII, VIII)

Time: 3 hours **Full Marks: 50**

Answer any five questions (not exceeding five).
All parts of the same question must be answered together.
For the programming problems use C language.

1. Derive the truth table for Full Adder and obtain the expressions for sum and carry. Implement the Full Adder using Half Adders and external gates.

[(4 + 3) + 4 = 10]

2. (a) Perform the arithmetic operations $(+16)_{10} + (-12)_{10}$ and $(-16)_{10} - (-12)_{10}$ in binary using the 2's complement representation.
(b) Consider a decimal number $(125)_{10}$ and represent it in 2's complement for using 8-bits.

[(4 + 4) + 2 = 10]

3. (a) What do you understand by call by value and call by address? Explain them with suitable examples.
(b) Write a recursive function to generate first n terms of the following series, where n is the input given by the user.
Series: 0, 1, 1, 2, 3, 5, 8, 13, 21, . . .

[5 + 5 = 10]

4. (a) What are the benefits of functions in programming?
(b) Write a program to reverse a string and print it. The string should be input by the user.

[4 + 6 = 10]

5. Write a function `int maximum(int array[], int n)` that takes an integer array and its length as arguments, and returns the maximum element present in the array. For example, the maximum of [5, 9, 20, 21, 8] should return as 21. Write the complete program to test the function.

[10]

6. (a) Compare structure and union in C language.
(b) Write a function `int strl(char strarr[])` to calculate the length of a string without using any standard library function and also write the complete program to test the function.

[4 + 6 = 10]

7. Define a structure to represent a complex number. Write a function to add two such complex numbers passed as parameters. Also write a function to display the result in (X + iY) or (X - iY) (whichever seems appropriate) format. Write the complete program to test the function.

[2 + 2 + 2 + 4 = 10]

8. Write a program to create a file (say `myfile.txt`) and store n number of records of students with the fields as RollNo, Name and Marks in that file, where n is the input given by the user.

[10]