27/4/18

Indian Institute of Engineering Science and Technology, Shibpur

Dual Degree (B.Tech-M.Tech) 2nd Semester (All Engineering Branches) Examination, April 2018

Subject: Mathematics-II (MA-201)

Time : 3 hours Full Marks : 70

(Use separate answer script for each half)

First Half

Answer any THREE questions

(Two marks are reserved for general proficiency)

1. (a) Prove that the necessary and sufficient condition for a non-empty subset W of a vector space $V(F)$ to be a subspace of V is

$$a,\ b \in F \text{ and } \alpha,\ \beta \in W \Rightarrow a\alpha + b\beta \in W.$$

[5]

(b) Show that the set W of the elements of the vector space $V_3(R)$ of the form $(x+2y,\ y,\ -x+3y)$, where $x,\ y \in R$ is a subspace of $V_3(R)$.

[3]

(c) Show that the vectors $(1,\ 2,\ 1)$, $(2,\ 1,\ 0)$, $(1,\ -1,\ 2)$ form a basis of R^3.

[3]

2. (a) Show that the mapping $T : V_3(R) \to V_2(R)$ defined as

$$T(a_1,\ a_2,\ a_3) = (3a_1 - 2a_2 + a_3,\ a_1 - 3a_2 - 2a_3)$$

is a linear transformation from $V_3(R)$ into $V_2(R)$.

[5]

(b) If A is an orthogonal matrix of order n, then show that (i) A is non-singular, (ii) $A^{-1} = A^T$.

[4]

(c) If A and B are orthogonal matrices of same order, then show that AB is also orthogonal.

[2]

3. (a) Reduce the following matrix to its Echelon form and hence find its rank:

$$P = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}.$$

[5]

P.T.O.

(b) Diagonalise the following matrix by finding the diagonalising matrix:

$$A= \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}.$$

[6]

4. (a) Determine the conditions under which the system of equations:

$$\begin{aligned} x + y + z &= 1 \\ x + 2y - z &= b \\ 5x + 7y + az &= b^2 \end{aligned}$$

admits of (i) only one solution, (ii) no solution. (iii) many solutions.

[6]

(b) Determine the eigen values and any one eigen vector of the following matrix:

$$A= \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}.$$

[5]

5. (a) Determine the Fourier series of the function:

$$f(x) = \begin{cases} 0, & -5 < x < 0 \\ 3, & 0 < x < 5 \end{cases}$$

and $f(x)$ is a periodic function of period 10. How should $f(x)$ be defined at $x = -5$, $x = 0$ and $x = 5$, so that its Fourier series converges to $f(x)$ for $-5 \leq x \leq 5$?

[6]

(b) Find a series of sines of multiples of x which will represent $f(x) = x$ in $0 \leq x < \pi$.

[5]

P.T.O.

SECOND HALF

Answer any THREE questions

(Two marks are reserved for general proficiency)

6.a) Find the velocities of the particles at the points (1, 2, 3) and (1, 0, 4) of a rigid body which is spinning with angular velocity 5 radians/ sec about the axis in the direction of $(-2\vec{i} + \vec{j} + 2\vec{k})$ passing through a fixed point (2, –1, –3).

b) Show that the vector field given by $\{(y + \sin z)\vec{i} + x\vec{j} + (x\cos z)\vec{k}\}$ is conservative. Find the scalar potential.

c) If $\vec{A}$ is differentiable vector function and ϕ is a differentiable scalar function of position (x, y, z), then prove that curl $(\phi\vec{A}) = (\text{grad}\,\phi) \times \vec{A} + \phi\,\text{curl}\,\vec{A}$. **3+ (1+3)+4**

7.a) If $\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20zx^2\vec{k}$, evaluate $\int_C \vec{F}.d\vec{r}$, from (0, 0, 0) to (1, 1, 1) along the curve C having parametric equations $x = t$, $y = t^2$, $z = t^3$.

b) If $\vec{A} = (4xy - 3x^2z^2)\vec{i} + 2x^2\vec{j} - 2x^3z\vec{k}$, prove that $\int_C \vec{A}.d\vec{r}$ is independent of the curve joining two given points.

c) State Stokes' theorem.
Applying Stokes' theorem to the vector function $\vec{F} = \varphi\vec{\nabla}\varphi$, show that $\int_\Gamma \varphi\vec{\nabla}\varphi \,.\, d\vec{r} = 0$, where Γ is a closed curve. **3+3+(1+4)**

8.a) If the real component of an analytic function be given by $log_e(x^2 + y^2)^{1/2}$, prove that the function is $log_e z + ic$, where $z = x + iy$, $i = \sqrt{(-1)}$ and c is a constant of integration.

b) Evaluate $\frac{1}{2\pi i}\oint_C \frac{\cos\pi z}{z^2 - 1}dz$, where C is a rectangle with vertices at $2 \pm i, -2 \pm i$. **6+5**

9.a) Show that

$$\oint_C \frac{dz}{(z-a)^n} = \begin{cases} 2\pi i, & if\ n = 1, \\ 0, & if\ n = 2,3,4, \dots \end{cases}$$

where C is a simple closed curve bounding a region having z = a as interior point. What are the values of the integral if n = 0, -1, -2, -3, … … ?

b) Expand $f(z) = \frac{z}{(z-1)(2-z)}$ in a Laurent series which is valid for $0 < |z - 2| < 1$.

c) Name the singularity of the function $f(z) = \frac{ze^z}{(z-a)^3}$ and find the residue of f(z) at that point. **(4+1)+3+3**

10.a) State and prove the residue theorem.

b) Use the method of contour integration to evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta}d\theta$. **5+6**