

Indian Institute of Engineering Science and Technology, Shibpur
B.Tech. 1st Semester Mid Semester Examination, February 2021
Subject: Mathematics - I (MA 1101)

Time : 45 minutes

Full Marks : 30

Answer any FIVE questions. Only the first five questions answered by the candidate will be evaluated.

1. Let $u, v \in \mathbb{R}$ such that $0 < u < v$. Show that

$$\frac{v-u}{1+v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v-u}{1+u^2}$$

and hence deduce that, [4+2]

$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1} \left(\frac{4}{3} \right) < \frac{\pi}{4} + \frac{1}{6}.$$

2. Prove the following inequality by using Taylor's theorem, [6]

$$\log \left(\frac{1+x}{1-x} \right) > 2x \text{ if } 0 < x < 1.$$

3. Find the points on the parabola $y^2 = 8x$ at which the radius of curvature is $\frac{125}{16}$. [6]

4. Show that a necessary condition for convergence of an infinite series $\sum u_n$ is that $\lim_{n \rightarrow \infty} u_n = 0$. Show through an example that it is not sufficient condition of convergence. [4+2]

5. Evaluate the following integrals:

(a) $\int_0^{\pi/2} \sqrt{\tan \theta} \, d\theta$

(b) $\int_0^1 x^4 \left[\log\left(\frac{1}{x}\right) \right]^3 dx$ [3+3]

6. Solve the following differential equations:

(a) $\frac{d^2 y}{dx^2} + 4y = \cos x$

(b) $x^2 \frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + 6y = x$ [2+4]

7. Discuss the nature of the following series [6]

$$\sum_{n=1}^{\infty} \frac{4.7.\dots(3n+1)}{1.2.\dots n} x^n, \quad x > 0.$$