Indian Institute of Engineering Science and Technology, Shibpur
B.Tech. (All groups) 2nd Semester Mid Semester Examination, February 2024
Subject: Mathematics - II (MA-1201)

Time : 2 hours Full Marks : 30

Answer any FIVE questions.

[$5 \times 6 = 30$]

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1. (a) Let $\{\alpha, \beta, \gamma\}$ be a basis of the vector space $\mathbb{R}^3$. Then show that for any nonzero real number c, $\{\alpha + c\beta, \beta, \gamma\}$ is also a basis of $\mathbb{R}^3$. [2]

   (b) Let $\mathbb{R}$ be the field of real numbers. Justify whether the following sets form a subspace over $\mathbb{R}$ or not: [1+1]

   $$(i)\ \ S_1 = \{(x, y, z) \in \mathbb{R}^3 : x, y \text{ and } z \text{ are rational numbers}\},$$

   $$(ii)\ \ S_2 = \{(x, y, z) \in \mathbb{R}^3 : 2x + y + 5z = 1,\ \ x + y + z = 0\}.$$

   (c) Prove or disprove: If W_1 and W_2 are two subspaces of the vector space $\mathbb{R}^3$, then $W_1 \cup W_2$ is also a subspace of $\mathbb{R}^3$. [2]

2. (a) Find the basis and dimension of the following subspace W of $\mathbb{R}^3$ [1+1]

   $$W = \{(x, y, z) \in \mathbb{R}^3 : x + 2y + z = 0,\ 2x + y + 3z = 0\}.$$

   (b) Determine the rank of the matrix AA^T, where A be non zero column matrix of order $n \times 1$. [1]

   (c) Reduce the matrix $A = \begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ in its normal form. [3]

3. (a) Investigate for what values of λ and μ, the following linear equations

   $$x + 2y + 3z = 6,\ \ x + 3y + 5z = 9,\ \ 2x + 5y + \lambda z = \mu$$

   have (i) no solution (ii) unique solution (iii) infinite number of solutions. Find the solutions in cases (ii) and (iii). [3+3]

4. (a) Prove or disprove: The following system of linear equations

   $$2x + 3y + z = 0,\ \ x + 2y - 3z = 0,\ \ 4x - y - 2z = 0$$

   has a non-zero solutions. [2]

   (b) Prove that $r^n \vec{r}$ is an irrotational vector for any value of n but is solenoidal only if $n + 3 = 0$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$. [2+2]

PTO.

5. (a) Find the shortest distance between the lines $\vec{r} = \vec{\alpha} + t\vec{\beta}$ and $\vec{r} = \vec{\gamma} + s\vec{\delta}$ in terms of k, where $\vec{\alpha} = (1, 2, 3)$, $\vec{\beta} = (2, 3, 4)$, $\vec{\gamma} = (k, 3, 4)$ and $\vec{\delta} = (3, 4, 5)$. Also determine the value of k, if the lines are coplanar. [2+1]

(b) Find the moment of a force represented by $\overrightarrow{PQ}$ acting at the point P about an axis through the point $A(3, 1, 0)$ in the direction of the vector $(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$, where the position vectors of P and Q are $(\hat{\imath} + 3\hat{\jmath} + 2\hat{k})$ and $(3\hat{\imath} + 4\hat{\jmath} + 3\hat{k})$ respectively. [3]

6. (a) For the scalar point functions $u = x + y + z$, $v = x^2 + y^2 + z^2$ and $w = xy + yz + zx$, show that $\vec{\nabla} u$, $\vec{\nabla} v$, and $\vec{\nabla} w$ are coplanar vectors. [2]

(b) Prove that $\vec{F} = (2xy + z^3)\hat{\imath} + x^2\hat{\jmath} + 3xz^2\hat{k}$ is a conservative force field. Calculate the corresponding scalar potential of $\vec{F}$ and hence determine the work done by the force in moving an object from $(1, -2, 1)$ to $(3, 1, 4)$. [1+2+1]

7. (a) If $\vec{F} = (5x^2 + 6y)\hat{\imath} - (3x + 2y^2)\hat{\jmath} + 2xz^2\hat{k}$, then evaluate the line integral $\int_C \vec{F} . d\vec{r}$ from $(0, 0, 0)$ to $(1, 1, 1)$ along the path C given by the straight lines from $(0, 0, 0)$ to $(1, 0, 0)$, then to $(1, 1, 0)$ and then to $(1, 1, 1)$. [3]

(b) Evaluate $\int\int_S \vec{F} . \hat{n}\, dS$, where $\vec{F} = 18z\hat{\imath} - 12\hat{\jmath} + 3y\hat{k}$, S is the surface of the plane $2x + 3y + 6z = 12$ in the first octant and $\hat{n}$ is the unit outer normal vector of the surface S. [3]