20/11/19

**INDIAN INSTITUTE OF ENGINEERING SCIENCE AND TECHNOLOGY, SHIBPUR**

B.Arch. 1st Semester Final Examination, 2019

Subject : Mathematics-IA (MA-1102)

Full Marks : 50 Time : 3 hours

Use Separate answer script for each half

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Answer any **SIX** questions taking **THREE** from each half.

One mark is reserved for general proficiency in each half.

# First Half

1. a) State Lagrange's Mean Value theorem. Give its geometrical interpretation.

   b) In Mean value theorem determine the value of c lying between a and b if $f(x) = x(x-1)(x-2)$ and $a = 0,\ b = \frac{1}{2}$. [(1+3)+4]

2. a) Find the radius of curvature at any point p(r, $\theta$) on the curve $r = a(1-\cos\theta)$ and show that it varies as $\sqrt{r}$.

   b) Prove that $\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1} b - \sin^{-1} a < \frac{b-a}{\sqrt{1-b^2}}$, for 0<a<b<1. [4+4]

3. a) State Maclaurin's theorem. Using Maclaurin's series show that $\sin x > x - \frac{x^3}{6}$ if $0 < x < \frac{\pi}{2}$.

   b) Verify Cauchy's Mean value theorem for the function $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{\sqrt{x}}$ for $x \in [1,2]$. [4+4]

4. a) If $y = tan^{-1}x$, prove that

$$(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$$

   b) Test the convergence of the series $\frac{1+2}{2^3} + \frac{1+2+3}{3^3} + \frac{1+2+3+4}{4^3} + \cdots$ [4+4]

5. Show that the series $\sum \frac{n^2-1}{n^2+1} x^n$ , $x > 0$ is convergent if $0 < x < 1$ and divergent if $x > 1$. Also, using Raabe's test, prove that the series is divergent at $x = 1$. [8]

## Second Half

6. a) If z be a function of x and y and $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$.

b) If $x = r\ \cos\theta$, $y = r\sin\theta$, find the value of $\frac{\partial(r,\theta)}{\partial(x,y)}$. [4+4=8]

7. a) State and prove Euler's theorem.

b) If $u = cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$. [(1+4)+3]

8. a) State the necessary condition for the extremum of the function f(x, y) at the point (a, b).

b) Examine maximum or minimum of the function $f(x,y) = 2x^2 - xy + 2y^2 - 20x$.

c) Show that $f(x,y) = \frac{x^2}{y} + \frac{2y^2}{x}$ is a homogeneous function of degree one. [3+4+1]

9. a) Using Lagrange's method of multipliers find the maximum and minimum values of the function $x^2 + y^2$ subject to $3x + 2y = 6$.

b) Using Taylor's theorem expand $x^2y - 3y - 2$ in powers of $(x - 1)$. [5+3]

10. a) Change the order of the integration $\int_{-a}^{a}\int_{0}^{\sqrt{a^2-y^2}} f(x,y)\, dx\, dy$.
Determine the value of the integration when $f(x,y) = 1$.

b) Find the area of the region bounded by the curves $x = y^2$ and $y = x$. [(2+2)+4]