**Indian Institute of Engineering Science and Technology, Shibpur**
B.Tech 2nd Semester Mid-Semester Examination, February 2024

**Subject: Physics (PH1201)**

**Full Marks: 30** ***Answer All Questions*** **Time: 2 Hours**

1. (a) Find constants $a$, $b$ and $c$ so that $\vec{A} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is a conservative field. Find the potential corresponding to the field $\vec{A}$.

(b) Find a unit vector perpendicular to the surface of the paraboloid of revolution $z = x^2 + y^2$ at the point (1, 2, 5).

(c) Find the work done in moving a particle in a force field $\vec{F} = 3xy\,\hat{\imath} - z\hat{\jmath} + 10x\,\hat{k}$ along the straight line from the origin to the point (1, 5, 8).

(d) Express the equation of the cone $z^2 = 3(x^2 + y^2)$ in spherical coordinates.

(e) Express the position vector of a point in space in cylindrical coordinate systems. Find the unit basis vectors in cylindrical coordinates $\{\hat{\rho}, \hat{\varphi}, \hat{z}\}$ in terms of $\{\hat{\imath}, \hat{\jmath}, \hat{k}\}$. [(1+2)+1+2+1+(1+2)]

2. (a) State Faraday's law and obtain its differential form. Use Maxwell's equations to show that an electric field vector can propagate in free space like a wave with the speed of light.

(b) An oscillating voltage $V(t) = V_0 \cos(\omega t)$ is applied across a square-shaped parallel plate capacitor having plate separation '$d$' and side length '$a$' ($d << a$). Find the expression of the displacement current density.

(c) In a charge-free and current-free region, an electric field associated with an electromagnetic wave propagating along positive $y$ direction is given by

$$\vec{E} = -\hat{x}\,E_0\; e^{i\left(\frac{\pi a}{3}y - \sqrt{3}\pi b t\right)}$$

Find the ratio of $a$ and $b$. Obtain the direction and magnitude of the associated magnetic field and Poynting vector. Assume $a$ and $b$ are constants. [(2+3)+1+(1+3)]

3. (a) Consider the one dimensional potential: $V(x) = 3x^2 + 2x^3$. Obtain the equilibrium points and check their stability. A particle of unit mass is slightly displaced from the stable equilibrium point. Obtain the frequency of oscillation.
(b) A wave group is given by $f(x,t) = A\,e^{-(x-vt)^2}$ Plot the wave group at $t=0$ and $t = 1/v$. Check if it satisfies the wave equation.

(c) The frequency of a wave in a dispersive medium is given by $\omega^2 = a^2 + c^2k^2$, where $a$ and $c$ are constants and $k$ is the wave number. Show that, $v_p v_g = c^2$, where $v_p$ and $v_g$ are the phase and group velocities respectively.

(d) Assume the general solution of the wave equation for a stretched string of tension $T$ and mass per unit length $\rho$ as,

$$y(x,t) = (A\cos kx + B\sin kx)\,(C\cos \omega t + D\sin \omega t),$$

where, $A$, $B$, $C$, $D$ are constants and $\omega = k\sqrt{T/\rho}$ has usual meaning. Write down the appropriate boundary conditions to find out the suitable solution for stretched string of length $L$ rigidly fixed at both ends. Find out the frequency of the $n^{\text{th}}$ mode of vibration.

[(2+1)+2+2+(2+1)]