Indian Institute of Engineering Science and Technology, Shibpur

B.Tech. 2nd Semester Mid term Examination, June 2021

Subject: Mathematics - II (MA 1201)

Time: 45 minutes | Full Marks: 30

Answer any FIVE questions. Only the first five questions answered will be evaluated.

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1. Define vector space over the field of Real numbers. Show that the intersection of two subspaces of a vector space is a subspace. Give an example to show that the union of two subspaces may not be a subspace. [2+3+1]

2. Define subspace of a vector space over the field R of Real numbers. Prove that a subset S of a vector space V is a subspace if
   i. $\alpha + \beta \in S$ for all α, β in S.
   ii. $c \cdot \alpha \in S$ for c in R and α in S. [1+5]

3. Define linear transformation between two sets with an example. Show that the transformation T defined by $T\{f(x)\} = \frac{d}{dx}\{f(x)\}$ is a linear transformation. Prove that the Eigen values of an orthogonal matrix are of unit modulus. [1+1+4]

4. Define rank of a matrix. Find the rank of the following matrix by reducing it to normal form

$$\begin{pmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{pmatrix}$$ [1+5]

5. Given the system of equations :

$$x_1 + 4x_2 + 2x_3 = 1$$
$$2x_1 + 7x_2 + 5x_3 = 2m$$
$$4x_1 + nx_2 + 10x_3 = 2m + 1\,,$$

   find for what values of m and n the system has (i) a unique solution (ii) no solution (iii) many solutions. [6]

6. Define shortest distance between two skew lines. Find the shortest distance between two skew lines $\boldsymbol{r} = \boldsymbol{r_1} + t\boldsymbol{a}$ and $\boldsymbol{r} = \boldsymbol{r_2} + t\mathbf{b}$, where t is a scalar and $\boldsymbol{r_1}$, $\boldsymbol{a}$, $\boldsymbol{r_2}$, $\mathbf{b}$ are vectors with co-ordinates $(1, -2, 3)$, $(2, 1, 1)$, $(-2, 2, -1)$ and $(-3, 1, 2)$ respectively. Find the moment about the point $B\,(3, -1, 3)$ of a force $\boldsymbol{F}\,(4, 2, 1)$ passing through the point $A\,(5, 2, 4)$. [1+3+2]

7. Define curl of a vector point function and give its physical interpretation. Find div $\boldsymbol{F}$ and $curl\ \boldsymbol{F}$, where $\boldsymbol{F} = grad(x^3 + y^3 + z^3 - 3xyz)$. [1+2+3]

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