Indian Institute of Engineering Science and Technology, Shibpur
B.Tech.(All groups) 1st Semester Mid-Term Examination, February 2022
Subject: Mathematics-I(MA-1101)

Time : 45 minutes **Full Marks : 30**

Answer any FIVE(05) questions.

[**Only the first five answers will be considered for evaluation.**]

1. (a) If $y = x^{n-1} logx$, then prove with the help of Leibnitz's theorem that

$$y_n = \frac{(n-1)!}{x}.$$

(b) If $y = cos(msin^{-1}x)$, then prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0.$$

Hence find y_n for $x = 0$, when n is even.

[2+(3+1)=6]

2. (a) Show that Lagrange's mean value theorem is not applicable to the function

$$f(x) = \begin{cases} xsin(\frac{1}{x}) & for \;\; x \neq 0 \\ 0 & for \;\; x = 0 \end{cases}$$

in $[-1,\ 1]$.

(b) In the mean value theorem $f(x) = f(0) + hf^{'}(\theta h), 0 < \theta < 1$, show that the limiting value of θ as $h \to 0^+$ is $\frac{1}{2}$ when $f(x) = cosx$.

[3+3=6]

3. (a) Find the Cauchy's form of remainder after n terms in the expansion of $log(1+x)$ in power of x.

(b) Expand $sinx$ in infinite series in powers of x stating the validity of such expansion.

[2+(3+1)=6]

4. (a) Find the radius of curvature at origin for the curve

$$x^4 - y^4 + x^3 - y^3 + x^2 - y^2 + y = 0.$$

(b) Find the point on $y = 4x - x^2$, where the curvature is maximum.

[3+3=6]

5. (a) Test the convergence of the series

$$\frac{6}{1.3.5}+\frac{8}{3.5.7}+\frac{10}{5.7.9}+...\ to\ \ \infty$$

(b) Test the convergence of the series

$$1+\frac{x}{2}+\frac{x^2}{5}+\frac{x^3}{10}+...\ to\ \ \infty, (x>0).$$

[3+3=6]

6. (a) Discuss the convergence of the series

$$1+\frac{2^2}{3^2}x+\frac{2^2.4^2}{3^2.5^2}x^2+\frac{2^2.4^2.6^2}{3^2.5^2.7^2}x^3+...\ \ to\ \ \infty, (x\neq 1).$$

(b) Solve the differential equation $\dfrac{d^2y}{dx^2}-y=4xe^x.$

[3+3=6]

7. (a) Solve the differential equation

$$x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4y=2x^2.$$

(b) Apply the method of variation of parameters to solve the differential equation

$$\frac{d^2y}{dx^2}+4y=\tan 2x.$$

[3+3=6]