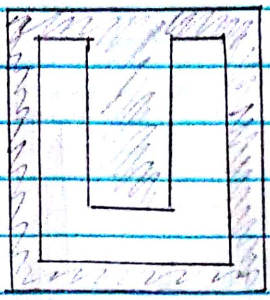
Homework 3

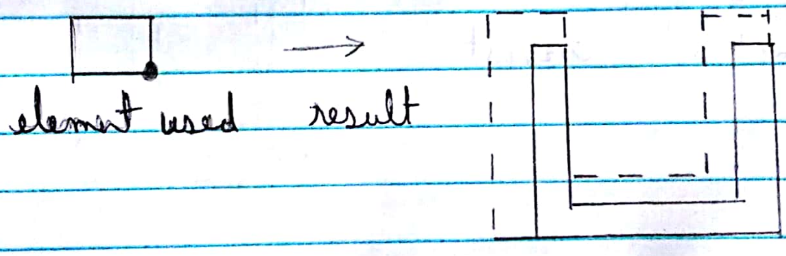
Problem 1



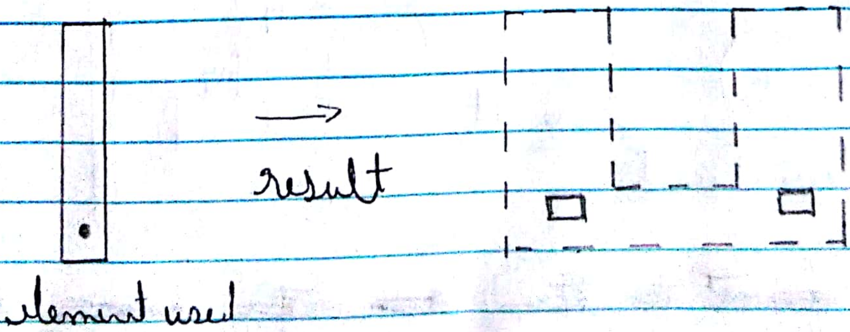
Sample Image provided

Eg Squibbled area is background

a) The first image can be obtained by using the below image, using erosion operation

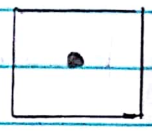


b) The given image can be transformed in the new image using the ~~the~~ below operator and performing an erosion ~~function~~ operation on it



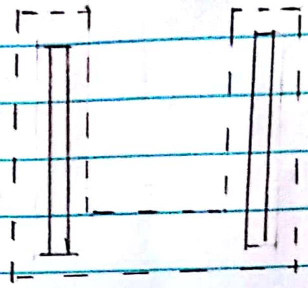
c) The resulting image required in this part takes 2 operations

1st erosion operation



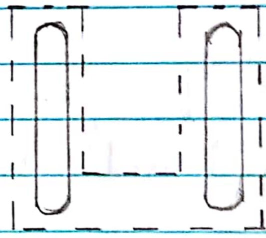
element used

$\longrightarrow$ result



The above result was then dilated using the following operator

element used $\longrightarrow$ result

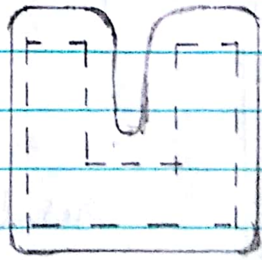


d) The required image can be obtained again by 1 operation

1st dilation operation

element used

$\longrightarrow$ result



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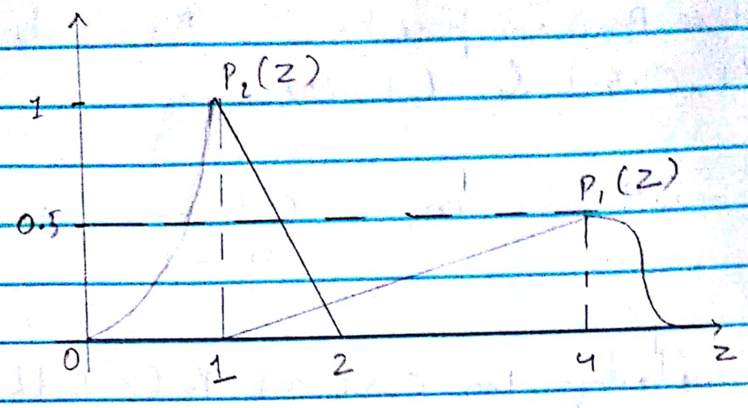
Problem 2



We are given that P_1 denotes the probability occurrence of $P_1(z)$ and P_2 denotes the probability occurrence of $P_2(z)$

As we can see in the probability density provided that majority of pixel lies between 1 to 4 we can obtain their occurrence by using the area of individual probability.

So $\underset{\text{area}}{P_1(z)} = \frac{1}{2} \times (4-1) \times (0.5) = \frac{1}{2} \times 3 \times \frac{1}{2} = \frac{3}{4}$

Area of $P_2(z) = \frac{1}{2} \times (2-1) \times 1 = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

$\therefore P_1 = \frac{3/4}{3/4 + 1/2} = \frac{3/4}{3/4 + 2/4} = \frac{3}{5}$ (probability occurrence of P_1)

$P_2 = \frac{1/2}{3/4 + 1/2} = \frac{1/2}{3/4 + 2/4} = \frac{2}{5}$ (probability occurrence of P_2

Evaluating the equation of line of $P_1(2)$
we know points of $P_1(2)$ line are ~~(0.5, 4)~~ &
(4, 0.5) & (1, 0)

$\therefore$ Slope $m_1 = \frac{0.5 - 0}{4 - 1} = \frac{0.5}{3} = \frac{1}{6}$

Similarly in case of $P_2(2)$ the points are (1, 1)
(2, 0)

$\therefore$ Slope $m_2 = \frac{0 - 1}{2 - 1} = \frac{-1}{1} = -1$

So equation of line is:

$y = m_1 x + c \Rightarrow 0.5 = 4\left(\frac{1}{6}\right) + c$

$\Rightarrow \frac{1}{2} = \frac{2}{3} + c \Rightarrow c = \frac{1}{2} - \frac{2}{3} \Rightarrow c = \frac{3-4}{6} =$

$\therefore y = \frac{1}{6}x - \frac{1}{6}$ — $P_1(2)$

equation of second line

$y = m_2 x + c \Rightarrow 1 = (-1)1 + c$

$\Rightarrow 1 = -1 + c \Rightarrow c = 2$

$\therefore y = (-1)x + 2$

$\Rightarrow y = -x + 2$ — $P_2(2)$

Q4

we know $P_1 \, P_1(z) = P_2 \, P_2(z)$ for optimal thresho

$\Rightarrow P_1 \left(\frac{1}{6}x - \frac{1}{6} \right) = P_2 \left(-x + 2 \right)$

$\Rightarrow \frac{3}{5} \left(\frac{1}{6}x - \frac{1}{6} \right) = \frac{2}{5} \left(-x + 2 \right)$

$\Rightarrow \frac{3}{30} \left(x - 1 \right) = \frac{2}{5} \left(-x + 2 \right)$

$\Rightarrow \frac{1}{\cancel{10}\,2} (x - 1) = \frac{2}{\cancel{5}} (-x + 2)$

$\Rightarrow x - 1 = 4(-x + 2)$

$\Rightarrow x - 1 = -4x + 8$

$\Rightarrow x + 4x = 1 + 8$

$\Rightarrow 5x \cancel{-6x} = 9$

$\Rightarrow x = \frac{9}{5} = 1.8$

Hence the optimal threshold value is 1.8

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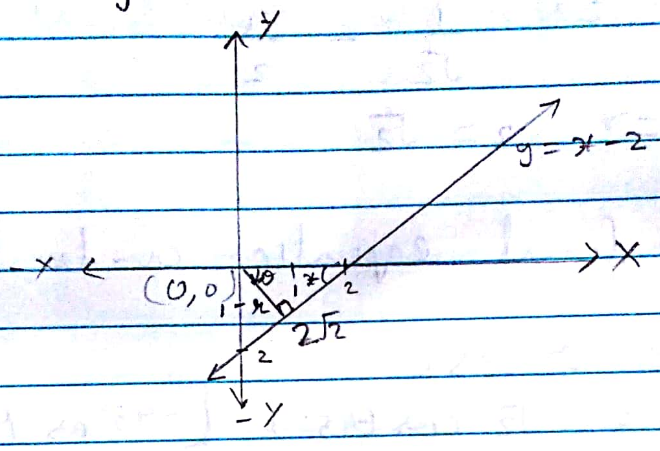
Problem 3

a) $y = x - 2$ (given)

We know the equation of a line in (r, θ) plane changes to $r = x\cos\theta + y\sin\theta$



in the given equation

$y = x - 2$

when $x = 0$

$\Rightarrow y = -2$

when $y = 0$

$\Rightarrow x = 2$

Distance between $(0,2)$ & $(2,0)$

$$\Rightarrow \quad d = \sqrt{(2-0)^2 + (0-2)^2}$$

$$\Rightarrow \quad d = 2\sqrt{2}$$

Now we know $\sin x = \frac{P}{h}$

$$\Rightarrow \sin x = \frac{2}{2\sqrt{2}} \Rightarrow \sin x = \frac{1}{\sqrt{2}}$$

$$\therefore \quad x = 45^\circ$$

we know sum of angle of a triangl is 180

$$\therefore \quad 90 + x + \theta = 180$$

$\Rightarrow 90 + 45 + \theta = 180 \Rightarrow \theta = 45^\circ$ (anti clock-wise direction)

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calculating 'r'

we know $\cos\theta = \frac{b}{h}$

$\Rightarrow \cos(45) = \frac{\cancel{b}\, r}{2}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{r}{2}$

$\Rightarrow r = \sqrt{2}$

So the final equation can be formed as under

$x = r\cos\theta$

$\Rightarrow x = \sqrt{2}\cos(-45^\circ)$ [-45° as θ is in anticlockwise direction]

$\Rightarrow x = \sqrt{2} \times \frac{1}{\sqrt{2}} = 1$

$\therefore x = 1$

$y = r\sin\theta$

$\Rightarrow y = \sqrt{2}\sin(-45^\circ)$ as θ is in anticlockwise direction

$\Rightarrow y = -\sqrt{2}\sin(45^\circ)$

$\Rightarrow y = -\sqrt{2} \times \frac{1}{\sqrt{2}} = -1$

So final equation is

$r = \cos\theta - \sin\theta$

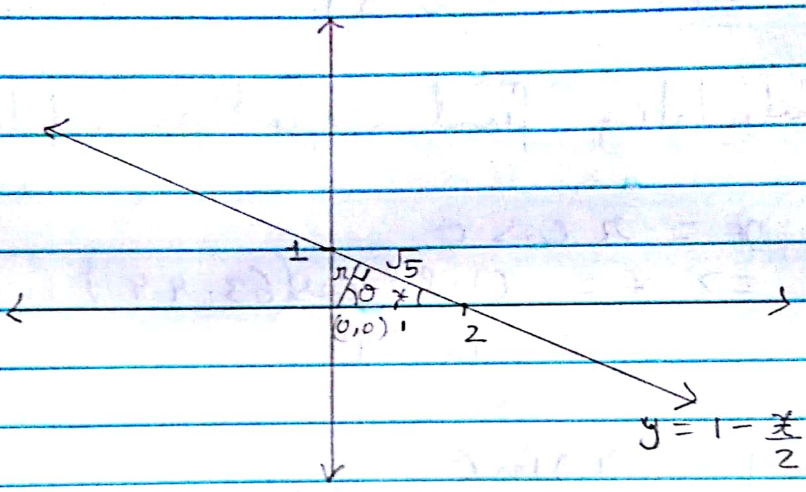
another line $y = 1 - \frac{x}{2}$ given

putting $y = 0$

$\Rightarrow x = 2$

putting $x = 0 \Rightarrow y = 1$

So the line is represented as under



Distance b/w (0,1) & (2,0)

$d = \sqrt{(2-0)^2 + (0-1)^2}$

$\Rightarrow \sqrt{4+1} = \sqrt{5}$

we know $\sin x = \frac{P}{h}$

$\Rightarrow \sin x = \frac{1}{\sqrt{5}}$

$\Rightarrow x = 26.56^\circ$

$\therefore \theta + 90 + x = 180$

$\Rightarrow \theta + 90 + 26.56 = 180^\circ$

$\Rightarrow \theta = 63.44^\circ$

Calculating r

$$\cos\theta = \frac{b}{4r}$$

$$\Rightarrow \cos\theta = \frac{r}{2}$$

$$\Rightarrow \cos(63.44°) = \frac{r}{2}$$

$$\Rightarrow r = 0.89$$

Calculating final equation of line

$$x = r\cos\theta$$
$$\Rightarrow x = 0.89\cos(63.44)$$
$$= 0.399$$

$$y = r\sin\theta$$
$$\Rightarrow y = 0.89(\sin(63.44))$$
$$= 0.799$$

$$\therefore r = 0.399\cos\theta + 0.799\sin\theta$$

Problem 3

b) To show the equation $x\cos\theta + y\sin\theta = P$ represents a curve / sinusoide for each image point (x, y) in (P, θ) hough space.

Consider a point $(1, 1)$.
let this be in an image

So for $\theta = 45°$
$P = 1\cos 45° + 1\sin 45°$

$P = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

for $\theta = 0°$

$P = 1\cos 0 + 1\sin 0$
$= 1 \times 1 + 1 \times 0$
$= 1$

for $\theta = 90°$
$P = 1\cos 90° + 1\sin 90°$
$= 1 \times 0 + 1 \times 1$
$= 1$

for $\theta = -45°$
$P = 1\cos(-45°) + 1\sin(-45°)$
$= 1 \times \frac{1}{\sqrt{2}} + 1 \times \frac{-1}{\sqrt{2}} = 0$

for $\theta = -90°$

$P = 1\cos(-90°) + 1\sin(-90°)$

$\cancel{P = 1 \times 0 + 1 - (-1)}$

$P = 1 \times 0 + 1(-1) = -1$

Let us consider a new point $(2, 0)$

Then for $\theta = 45°$

$P = 2\cos 45 + 0 = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}$

for $\theta = 0°$

$P = 2\cos 0° + 0 = 2 \times 1 = 2$

for $\theta = 90°$

$P = 2\cos 90° + 0 = 2 \times 0 + 0 = 0$

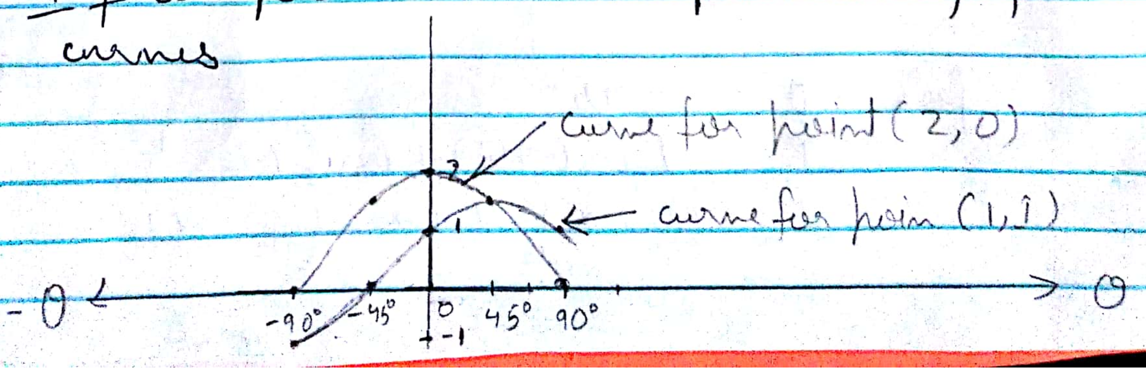
for $\theta = -45°$

$P = 2\cos(-45°) = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}$

for $\theta = -90°$

$P = 2\cos(-90°) + 0 = 2 \times 0 = 0$

If we plot the above points we get following curves



As we can see from the figure that point (1,1) & (2,0) both gave us curve or a sinoside in (P, θ) we can conclude that every point (x, y) can give us a sinoside in hough space ~~(B)~~, (P, θ)

Amplitude & phase relation

The amplitude of a point (x, y) is going to increase with higher values of x & y as $\cos\theta$ and $\sin\theta$ can give us maximum values in -1 to 1 and x is a ~~multiple~~ multiplied with $\cos\theta$ similar is the case for y, Hence the maximum value that can be achived depends on x & y itself and hence (x, y) is directly proportional to amplitude, in other words aplitude will rise with increase in x & y values.

A phase is basically a point on the waveform so the phase value is itself the value x & y and their multiplication with sin & cos angles. A phase value can only be ~~d~~ determined by $x\cos\theta + y\sin\theta$, which provide us P for some value of θ.

No there is no change in period (or frequency) of sinusoid with the image point (x, y) as the

frequency is the number of ~~st~~ cycles in one time period but in ~~our~~ though space the cycle can be passed in one time frame is always one ~~hence~~ as it is dependent on the angle.
Hence the frequency does not change with the image point.