Recitation 3

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Brief overview

- Review of important theorems (contraction, fixed point, convergence) - **5 min**
- Solving examples on rate of convergence (linear, superlinear etc.) - **25 min**
- Discussion about problems 5, 7 & 8 from HW 2 (newton's method with 2 same roots, secant method order of convergence) - **30 min**
- Interesting things about Newton's method - **15 min**
    - Can be generalized to linear systems
        - $x^{k+1} = x^k - (\nabla F(x^k))^{-1} F(x^k)$, or
        - $\nabla F(x^k) \Delta x^k = - F(x^k), \quad x^{k+1} = x^k + \Delta x^k$ (Newton's iteration)
    - Newton's method used in optimization looks different because it solves for $f'(x) = 0$ (to find optima), so don't get confused!
        - $x_{k+1} = x_k - f'(x)/f''(x)$
    - Newton's method is quadratic only near ξ, i.e. fixed point. Otherwise, it roughly halves the error. Read ref. 1 for more info.
    - Newton's method can also work on complex numbers! Read ref. 4 for more info, as well as to look at some pretty fractals.
    - Newton/secant methods converge slower when dealing with functions with multiplicity >= 2. Read ref. 5 for more info. This is one way to prevent that (i.e. converge faster than linear!):
        - $x_{k+1} = x_k - m.f(x)/f'(x)$, where m = multiplicity

Helpful resources

1. A casual (but interesting) introduction to Newton’s method
2. A good lecture on rates of convergence
3. A proof of Newton’s method’s quadratic convergence
4. Newton’s method, complex numbers and pretty fractals
5. Newton’s method on functions with multiple same roots
6. Secant method and why it’s order of convergence is the golden ratio