11/2/19

Statistics.

UNIT-3.

$x_1, x_2, x_3 \cdots x_n$.

$$\boxed{\text{Mean} = \bar{x} = \frac{\Sigma x}{n}}$$

$$\boxed{\text{Varience} = V = \frac{1}{n} \Sigma (x - \bar{x})^2}$$

or

$$V = \left(\frac{1}{n} \Sigma x^2\right) - (\bar{x})^2.$$

Standard deviation.

$$\boxed{\sigma = \sqrt{V} = \sqrt{\frac{1}{n} \Sigma x^2 - (\bar{x})^2}}$$

$x_1, x_2 \cdots x_n$

$f_1 \; f_2 \cdots f_n$.

$N = \Sigma f$.

$$\boxed{\bar{x} = \frac{1}{N} \Sigma f x}$$

$$\boxed{V = \frac{1}{N} \Sigma f (x - \bar{x})^2}$$

or

$$N = \frac{1}{N} \Sigma f x^2 - (\bar{x})^2.$$

$$\boxed{\sigma = \sqrt{\frac{1}{N} \Sigma f x^2 - (\bar{x})^2}}$$

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$\boxed{\text{Coefficient of variance} = \frac{S.D}{\text{Mean}} \times 100}$ should be small for consistency

$d = x - A$

$\bar{x} = \bar{d} + A$

$V_x = V_d$

$u = \frac{x - A}{h}$

$\bar{x} = h\bar{u} + A$

$V_x = hV_u$

$\sigma_x = h\sigma_u$

* **Moments:**

Central moments OR moment about mean r^{th} moment about mean $\bar{x}$ is μ_r &

$$\boxed{\mu_r = \frac{1}{N} \Sigma f (x - \bar{x})^r}$$

$\mu_1 = 0$

$\mu_2 = \frac{1}{N} \Sigma f (x - \bar{x})^2$

$\mu_3 = \frac{1}{N} \Sigma f (x - \bar{x})^3$

$\mu_4 = \frac{1}{N} \Sigma f (x - \bar{x})^4$

* **General moments:**

r^{th} moment about value A is μ_r'

$$\mu_r' = \frac{1}{N} \Sigma f (x - A)^r$$

can never be 0 $\mu_1' = \frac{1}{N} \Sigma f (x - A)$

$\mu_2' = \frac{1}{N} \Sigma f (x - A)^2$

$\mu_3' = \frac{1}{N} \Sigma f (x - A)^3$

$\mu_4' = \frac{1}{N} \Sigma f (x - A)^4$

If $u = \frac{x - A}{h}$

$\mu_r' = h^r \frac{1}{N} \Sigma f u^r$

$\mu_1 = h \frac{1}{N} \Sigma f u$

$\mu_2 = h^2 \frac{1}{N} \Sigma f u^2$

$\mu_3' = h^3 \frac{1}{N} \Sigma f u^3$

$\mu_4' = h^4 \frac{1}{N} \Sigma f u^4$

Relation between central moments & general moments:-

If 1st four moments of a distribution about value 'A' are μ_1', μ_2', μ_3' and μ_4' then

$\mu_1 = 0$

$\mu_2 = \mu_2' - \mu_1'^2$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$$\boxed{A.M = \bar{x} = \mu_1' + A}$$

Variance $= \mu_2$

$S.D = \sigma = \sqrt{\mu_2}$

coefficient of skewness = β_1

$\boxed{\beta_1 = \frac{\mu_3^2}{\mu_2^3}}$

Small because negatively skewed data

$\boxed{\gamma = \sqrt{\beta_1}}$ Positive skewed data has large value.

coefficient of kurtosis.

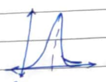
$\boxed{\beta_2 = \frac{\mu_4}{\mu_2^2}}$

~~Positive skewed data~~.

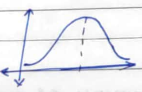
$\boxed{\gamma_2 = \beta_2 - 3}$

Median - The middle value in ascending over of data.

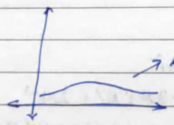
When $\beta_2 > 3$ it is known as Leptokurtic distribution.



When $\beta_2 = 3$ it is known as Normal or Mesokurtic Distribution (Mode & median nearly same)



When $\beta_2 < 3$ Plytikurtic Distribution.



The first four moments of a distribution about the value are 2, 20, 40 and 50. Calculate the first four moments about mean. Also find mean, standard Deviation & coefficient of skewness and kurtosis.

$A = 5$

$\mu_1' = 2$

$\mu_2' = 20$

$\mu_3' = 40$

$\mu_4' = 50$

$\mu_1 = 0$

$\mu_2 = \mu_2' - \mu_1'^2$

$= 20 - 4$

$= 16$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$= 40 - 3(20)(2) + 2(2)^3$

$= 40 - 120 + 16$

$= -80 + 16$

$= \cancel{-72}\ -64$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$= 50 - 4(40)(2) + 6(20)(8) - 3(2)^4$

$= 50 - 320 + 960 - 48$

$= 642$

$= 50 - 4(40)(2) + 6(20)(4) - 48$

$= 50 - 320 + \cancel{960}\ 480 - 48$

$= 162$

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$\mu_r = {}^rC_0 \mu_r' - {}^rC_1 \mu_{r-1}' \mu_1' + {}^rC_2 \mu_{r-2}' \mu_1'^2 + \dots (-1)^{r-1} (\mu_1')^r$

Mean $= \mu_1' + A$

$= 2+5$

$\bar{x} = 7$.

$\sigma = \sqrt{\mu_2} = \sqrt{16} = 4$.

$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(64)^3}{(16)^3} = 1$. Considerable skewness.

$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{162}{(16)^2} = 0.6328$

$\beta_2 < 3$

Platykurtic distribution.

Q.2] From the following data, find first four central moments on skewness and kurtosis.

x	2.0	2.5	3.0	3.5	4.0	4.5	5.0.
F	4	36	60	80	70	40	10.

x	f.	$u = \frac{x-3.5}{0.5}$	fu	fu^2	fu^3	fu^4.
2.0	4	−3	−12	36	−108	324
2.5	36	−2	−72	144	−288	576
3.0	60	−1.	−60	60	−60	60
3.5	90	0	~~70~~	~~70~~	~~70~~	~~70~~
4.0	70	1	70	70	70	70
4.5	40	2	80	160	320	640.
5.0	10.	3	30	90	270	810.
	N = 310.		Σfu = 36	Σfu^2 = 560	Σfu^3 = 204	Σfu^4 = 2480

F : FY : FY^2 : FY^3 : FY^4 CALC

$\mu_1' = h \cdot \frac{1}{N} \Sigma fu$.

$\mu_2' = h^2 \frac{1}{N} \Sigma fu^2$.

$\mu_3' = h^3 \frac{1}{N} \Sigma fu^3$

$\mu_4' = h^4 \frac{1}{N} \Sigma fu^4$.

$\therefore \mu_1' = 0.5 \frac{1}{310} (36) = 0.05$

$\mu_2' = (0.5)^2 \frac{1}{310} (560) = 0.45$

$\mu_3' = (0.5)^3 \frac{1}{310} (204) = 0.08$

$\mu_4' = (0.5)^4 \frac{1}{310} (2480) = 0.5$

Moments about mean.

$\mu_1 = 0$. $= 0$

$\mu_2 = \mu_2' - \mu_1'^2$ $= 0.45 - (0.05)^2 = 0.44$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$ $= 0.012$.

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^3 - 3\mu_1'^4$ $= \cancel{1}\ 0.489$

$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(0.012)^2}{(0.44)^3} = \frac{0.014}{0.085} = \cancel{0.164}\ 1.69 \times 10^{-3}$

Not considerable.

$\beta_2 = \frac{\mu_4}{\mu_2^2} = 2.525$

x is not the mean, it is the mid value.

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HW

Q.3) From the following data find first four central moments on skewness and kurtosis

class	0-10	10-20	20-30	30-40	40-50
Freq	6	26	47	15	6

Class	x	f	$u = \frac{x-25}{10}$	fu	fu^2	fu^3	fu^4
0-10	5	6	-2	-12			
10-20	15	26	-1	-26			
20-30	25	47	0	0			
30-40	35	15	1	15			
40-50	45	6	2	12			
		$\Sigma f = N = 100$					

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* Correlation and Regression.

Karl Pearson's coefficient of correlation.

x	x_1	x_2	$\cdots$	x_n
y	y_1	y_2		y_n

x	y	x^2	y^2	xy
x_1	y_1	x_1^2	y_1^2	x_1y_1
x_2	y_2	x_2^2	y_2^2	x_2y_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_n	y_n	x_n^2	y_n^2	x_ny_n
Σx	Σy	Σx^2	Σy^2	Σxy

n = no. of data points Karl Pearson's coefficient of correlation

$$\bar{x} = \frac{1}{n}\Sigma x, \quad \bar{y} = \frac{1}{n}\Sigma y$$

$$\sigma x = \sqrt{\frac{1}{n}\Sigma x^2 - (\bar{x})^2}$$

$$\sigma y = \sqrt{\frac{1}{n}\Sigma y^2 - (\bar{y})^2}$$

$$\text{covariance }(x,y) = \frac{1}{n}\Sigma xy - \bar{x}\bar{y}$$

$$\text{cov}(x,y) = \frac{1}{n}\Sigma(x-\bar{x})(y-\bar{y})$$

coefficient of correlation $= r$.

$$\boxed{r = \frac{\text{cov}(x,y)}{\sigma x \, \sigma y}}$$ or [illegible]

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Regression line of y on x

$$\boxed{y-\bar{y} = b_{yx}(x-\bar{x}) \quad \text{Here } b_{yx} = \frac{r\sigma_y}{\sigma_y}}$$

Regression line of x on y

$$\boxed{x-\bar{x} = b_{xy}(y-\bar{y}) \quad b_{xy} = r \cdot \frac{\sigma_x}{\sigma_y}}$$

1) Calculate the coefficient of correlation for the following table.

x	65	66	67	67	68	69	70	72
y	55	58	72	55	66	71	70	50

x : y : x2 : y2 : xy CALC

x	y	x^2	y^2	xy
65	55	4225	3025	3575
66	58	4356	3364	3828
67	72	4489	5184	4824
67	55	4489	3025	3685
68	66	4624	4356	4488
69	71	~~4900~~ 4761	5041	4899
70	70	4900	4900	4900
72	50	5184	2500	3600
$\Sigma = 544$	$\Sigma =$ 497	$\Sigma =$ 37028	$\Sigma =$ 30295	$\Sigma =$ 33799

$n = 8$.

$\bar{x} = 1/n\, \Sigma x = 1/8\,(544) = 68$

$\bar{y} = 1/n\, \Sigma y = 1/8 \times 497 = 62.125$.

$\sigma_x = \sqrt{(1/n\, \Sigma x^2) - (\bar{x})^2}$.

$= \sqrt{\cancel{(68)^2 -} (1/8 \times 37028) - (68)^2}$.

$= \sqrt{4628.5 - 4624}$

$= \sqrt{4.5} = 2.121$.

$\sigma_y = \sqrt{(1/8 \times 30295) - (\bar{y})^2}$

$= \sqrt{3924.37 - 3859.515}$

$= \sqrt{64.86}$

$= 8.05$.

$cov(x,y) = \frac{1}{n} \Sigma xy - \bar{x}\bar{y}$

$= \frac{1}{8}(33799) - 68 \times 62.125$

$= 4224.87 - 4224.5$

$= 0.37$.

$r = \frac{cov(x,y)}{\sigma_x \sigma_y} = \frac{0.37}{2.12 \times 8.05} = 0.0219$.

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2) Obtain regression lines for the following data.

x	6	2	10	4	8
y	9	11	5	8	7

x	y	x^2	y^2	xy
6	9	36	81	54
2	11	4	121	22
10	5	100	25	50
4	8	16	64	32
8	7	64	49	56
30	40	220	340	214

$n=5$.

$\bar{x} = \frac{1}{5} \Sigma x = \frac{1}{5} \times 30 = 6$.

$\bar{y} = \frac{1}{5} \Sigma y = \frac{1}{5} \times 40 = 8$.

$\sigma x = \sqrt{1/5\,(220) - 36}$.

$= \sqrt{44-36}. = \sqrt{8}$

$= 2.82$.

$\sigma y = \sqrt{(1/5 \times 340) - 64}$

$= \sqrt{68-64} = \sqrt{4} = 2$.

$cov(x,y) = -5.2$

$r = -0.918$.

$byx = r \cdot \frac{\sigma y}{\sigma x}$.

$= -0.918 \times \frac{2}{2.82}$

$byx = -0.65$.

$bxy = r \cdot \frac{\sigma x}{\sigma y}$

$= -0.918 \times \frac{2.82}{2}$.

$= -1.29$.

$y-8 = -0.65(x-6)$

$y-8 = -0.65x + 3.9$.

$y + 0.65x = 8 + 3.9$.

$y + 0.65x = 11.9$.

$\boxed{y = -0.65x + 11.9}$. y on x

$x-6 = -1.29.(y-8)$.

$x-6 = -1.29y + 10.32$.

$\boxed{x = -1.29y + 16.32}$ x on y.

3) Obtain regression lines for the following data.

x	2	3	5	9	10	12	15
y	2	5	8	10	12	15	16

Estimate y when $x=6$. (y on x)

x	2	3	5	7	9	10	12	15
y	2	5	8	10	12	14	15	16

x	y	x^2	y^2	xy
2	2	4	4	4
3	5	9	25	15
5	8	25	64	40
7	10	49	100.	70.
9	12	81	144.	108
10	14	100	196	140.
12	15	144	225	180
15	16	225	256.	240.
63	82	637	1014.	797.

$n = 8$.

$$\bar{x} = \frac{1}{8}\Sigma x = \frac{1}{8} \times 63 = 7.87.$$

$$\bar{y} = \frac{1}{8}\Sigma y = \frac{1}{8} \times 82 = 10.25$$

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* If two regression lines are given, to find min values of x and y ($\bar{x}$ and $\bar{y}$) solve two regression lines simultaneously, then root value of x is $\bar{x}$ and y is $\bar{y}$

* If two regression ~~tow~~ equations are given and to find correlation coefficient between x & y
  from regression line of y on x.
  coeff of x on RHS = b_{yx}.
  from regression line of x on y
  coeff of y on RHS = b_{xy}.

$$\boxed{r = \sqrt{b_{yx} \cdot b_{xy}}}$$

* If b_{yx} is +ve then r is +ve.
* If b_{yx} is -ve then r is -ve

[both b_{xy} & b_{yx} can either be positive], then b_{xy} or negative

20/2/19 The regression equations $8x - 10y + 66 = 0$ & $40x - 18y = 214$.
The value of variance of x is 9 find
1) The mean values of x & y
2) coefficient of correlation between x & y
3) standard deviation of y

A. Given regression lines
$$8x - 10y = -66.$$
$$40x - 18y = 214.$$
$V_x = 9 \Rightarrow \sigma_x = 3$

i) Mean values of x & y are
$x = +13$
$y = +17.$

ii) correlation coeff.
$$r = \sqrt{b_{yx} \cdot b_{xy}}$$

∴ Regression line of x on y is
$$40x - 18y = 214$$

∴ When coeff of x is bigger in magnitude than then that is regression line is x on y. (+ or - doesn't matter)

$$x = \frac{18}{40} y + \frac{214}{40}.$$

$$\boxed{b_{xy} = \frac{18}{40}.}$$

Regression line of y on x is

$$8x - 10y = -66.$$
$$10y = 8x + 66.$$
$$y = \frac{8}{10}x + \frac{66}{10}.$$

$$\boxed{b_{yx} = \frac{8}{10}.}$$

$$r = \sqrt{\frac{8}{10} \cdot \frac{18}{40}} = \sqrt{\frac{144}{400}} = \sqrt{0.36}. = 0.6$$

iii) $\frac{8}{10} = (0.6) \frac{\sigma_y}{3}.$

$$\sigma_y = \frac{8}{10} \times \frac{3}{0.6} = \frac{8 \times 3}{6 \cdot 2} = 4.$$

$$\boxed{\sigma_y = 4.}$$

Mean values also satisfy regression equation.

2) If the two lines of regression are $9x+y-\lambda=0$ & $4x+y=\mu$ and mean values of x & y ~~are~~ are 2 and -3 respective find values of μ and λ also find coefficient of correlation between x & y.

A. $\bar{x}=2$ & $\bar{y}=-3$

$\bar{x}, \bar{y}$ satisfy regression equation.

$\Rightarrow 9(2)+(-3)-\lambda=0$

$18-13=\lambda$

$\boxed{\lambda=15}$.

$\Rightarrow 4(2)+(-3)=\mu$

$\boxed{5=\mu}$

$\therefore$ Regression equations are

$9x-y-15=0$ $\quad$ $9x+y=15$

$4x+y=5$ $\quad$ $4x+y=5$

$9x-y=15$ is regression line of x on y

$4x+y=5$ is regression line of y on x

$9x=y+15$

$x=-\frac{1}{9}y+\frac{15}{9}$

$\boxed{b_{xy}=-\frac{1}{9}}$

$4x+y=5$

$y=-4x+5$

$\boxed{b_{yx}=-4}$

coefficient of determination $= r^2$

Probable error $-$ Ans $\times \sqrt{1-r^2}$

$r=\sqrt{b_{yx}\cdot b_{xy}}$

$= r=\sqrt{-4\times-\frac{1}{9}} = \sqrt{\frac{4}{9}} = -0.667$. $= \frac{\cancel{2}}{3} = -0.667$

As regression coefficients are negative $\therefore$ correlation coefficient is also negative

PROBABILITY.

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$.

or

$P(A\cup B)=1-P(A'\cap B')$.

$P(A\cap B)=P(A)\cdot P(B)$ $\quad$ when A, B are independent.

Binomial distribution.

n = no. of trials

P = probability of success in one trial

$q=1-p$ probability of failure.

Probability of r success $= P(X=r)$

$$\boxed{P(X=r)={}^{n}C_{r}\,p^{r}\cdot q^{n-r}}$$

Mean $= np$

Variance $= npq$

Probability of atleast r

$P(x \geq r) = P(r) + P(r+1) \cdots + P(n)$

$P(x \geq r) = 1 - P(x < r)$

$= 1 - [P(0) + P(1) + \cdots + P(r-1)]$

Probability of atmost r success

$P(x \leq r) = 1 - P(x > r)$

Q] Probability that a male aged 60 will live upto 70 years is 0.65. Now, 10 men ~~are~~ of age 60 are selected randomly, what is the probability that 6 or more will live upto the age of 70

A. $n = 10$

$p = 0.65$

$q = 0.35$

$P(x \geq 6) = P(x=6) + P(x=7) + P(x=8) + P(x=9) + P(x=10)$

$P(x=r) = {}^nC_r\, p^r q^{n-r}$

$= {}^{10}C_6 (0.65)^6 q^4 = {}^{10}C_6 (0.65)^6 (0.35)^4$

$= 210 \times 0.075 \times 0.015$

$= 0.236$

${}^{10}C_7 (0.65)^7 (0.35)^3$

$= 0.252$

${}^{10}C_8 (0.65)^8 (0.35)^2$

$= 0.175$

Expected freq (or Theoretical freq)

$= P \times N$

$=$ prob $\times$ sample size

${}^{10}C_9 (0.65)^9 (0.35)$

$= 0.072$

${}^{10}C_{10} (0.65)^{10} \cancel{(0.35)^0} (1)$

$= 0.013$

$\therefore P(x = r) = 0.748$

27/2/19 Hypergeometric distribution:

Negative Binomial distribution.

Sample size N is divided into two categories

N

Type I — Type II

M — Non-M.

$= {}^MC$ n objects are selected.

Probability of n, r are of type I.

$$= \boxed{\frac{{}^MC_r \cdot {}^{Non\text{-}M}C_{n-r}}{{}^NC_n}}$$

1) Among the ~~200~~ 200 employees 160 are union members and others are non-union members. If 4 employees are chosen at random, what is the probability that atleast one employee is a union members:

$$\frac{{}^{160}C_0\, {}^{40}C_4}{{}^{200}C_4} + \frac{{}^{160}C_1\, {}^{40}C_3}{{}^{200}C_4} + \frac{{}^{160}C_2\, {}^{40}C_2}{{}^{20}C_4} + \frac{{}^{160}C_3\, {}^{40}C_1}{{}^{20}C_4}$$

or

$1-P(x<1)$

$= 1-P(x=0)$

$1-\frac{{}^{160}C_0 \, {}^{40}C_4}{{}^{200}C_4}$

$= 1-1.4\times10^{-3}$

$= ~~0.0014~~ \; 0.99$

Poission Distribution:

$\lambda = $ mean.

Probability of r successes =

$$\boxed{P(x=r) = \frac{e^{-\lambda}\lambda^r}{r!}}$$

Mean = Variance   when n is large apply poission.

1) The average number of misprints per page of a book are 1.5 Assuming the distribution to be poission, find the number of pages containing more than 1 misprint if the book contains 900 pages.

A. Mean $\lambda = 1.5$

By poission probability

$P(x=r) = \frac{e^{-\lambda}\lambda^r}{r!} = \frac{e^{-1.5}(1.5)^r}{r!}$

$P(x>1) = 1-P(x\leq 1)$.

$= 1-P[P(x=0)+P(x=1)]$

$= 1-\left[\frac{e^{-1.5}(1.5)^0}{0!}+\frac{e^{-1.5}(1.5)^1}{1!}\right]$.

$= 1-\left[e^{-1.5}+1.5e^{-1.5}\right]$

$= 1-\left[e^{-1.5}(2.5)\right] = 0.443$

Expected no. of pages

$N\times P(x>1)$

$= 900\times 0.443$

$= 398.7$.

$= \approx 399$ pages.

2) Fit a poission distribution to the following data.

x	0	1	2	3	4	5
f	150	154	60	35	10	1

$P(x=r) = \frac{e^{-\lambda}\lambda^r}{r!}$

x	f	fx		Theoretical freq / Expected freq NxP (410 x P(x=r)) $\cdot$ 146.3
0	150	0	$P(x=0) = 0.35$	
1	154	154	$P(x=1) = 0.36$	150.76
2	60	120	$P(x=2) = 0.189$	77.63
3	35	105	$P(x=3) = 0.06$	26.657
4	10	40	$P(x=4) = 0.016$	6.86.
5	1	5	$P(x=5) = 0.0034$	1.414.
	$\Sigma=410$	$\Sigma=424$		

$\text{Mean} = \lambda = \frac{\Sigma fx}{\Sigma f} = \frac{424}{410} = 1.03$

Normal distribution.

$$\boxed{y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$ Continuous distribution

μ = Mean

σ = Standard deviation.

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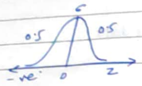
To find probability

$z = \frac{x-\mu}{\sigma}$.

Area under normal curve is 1



Mean, median modes is same
symmetric curve

$\int_{-\infty}^{\infty} y \cdot dx = 1$

put $x = x_1$ in z.

$z_1 = \frac{x_1 - \mu}{\sigma}$

Area under z_1 = $A(z_1)$ given Area under normal curve from 0 to $|z_1|$

$\underline{P(x \leq x_1)}$

Case 1] z_1 — +ve

$P(x < x_1) = 0.5 + A(z_1)$

Case 2] z_1 — -ve.

$P(x < x_1) = 0.5 - A(z_1)$

$\underline{P(x > x_1)}$

case 1] z_1 — +ve

$0.5 - A(z_1)$

case 2] z_1 — -ve

$0.5 + A(z_1)$

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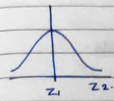
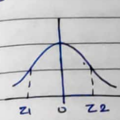
case 1] z_1 and z_2 are of the same sign [when mean and standard deviation are given - normal distribution]



$P(x_1 < x < x_2) = A(z_2) - A(z_1)$

case 2] z_1 and z_2 are of opposite signs



$P(x_1 < x < x_2) = A(z_1) + A(z_2)$

Q: In a certain examination test, 10,000 students appeared. Average marks obtained were 50%. with 50 5% marks are normally distributed. Find number of students expected to get more than 60% marks (Given $z = 2, A = 0.4772$)

A. $z = \frac{x-\mu}{\sigma}$

$= \quad z = \frac{x-50}{5} \quad P(x > 60) = ?$ at $x = 60$ $z_1 = \frac{60-50}{5} = \frac{10}{5} = 2$

at $z = 2, A(2) = 0.4772$

$P(x > 60) = 0.5 - A(2) = 0.5 - 0.4772$

$= 0.0228.$

Expected number of students = $10k \times 0.0228 = 228$ students.

$\therefore$ 228 students got more than 60% marks.

UNIT-4

Computer oriented Mathematics
Numerical methods

Solutions of simultaneous equations

★ **Jacobi method**

To solve,

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$ — ①

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$ — ②

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$ — ③

1) Rearrange equations so that diagonal elements are bigger in magnitude.

2) Rewrite equations as from ①

$$x_1 = \frac{1}{a_{11}}[b_1 - a_{12}x_2 + a_{13}x_3]$$

$$x_2 = \frac{1}{a_{22}}[b_2 - a_{21}x_1 - a_{23}x_3] \text{ from ②}$$

$$x_3 = \frac{1}{a_{33}}[b_3 - a_{31}x_1 - a_{32}x_2] \text{ from ③}$$

4) If k^{th} approximation/iteration to roots is x_1^k, x_2^k, x_3^k, then by Jacobi method $k+1^{th}$ iteration is

$$x_1^{k+1} = \frac{1}{a_{11}}[b_1 - a_{12}x_2^k - a_{13}x_3^k]$$

$$x_2^{k+1} = \frac{1}{a_{22}}[b_2 - a_{21}x_1^k - a_{23}x_3^k]$$

$$x_3^{k+1} = \frac{1}{a_{33}}[b_3 - a_{31}x_1^k - a_{32}x_3^k]$$

Let initial iteration be x_1^0, x_2^0, x_3^0

1st iteration = $x_1^1 = \frac{1}{a_{11}}[b_1 - a_{12}x^0 - a_{13}x_3^0]$

$$x_2^1 = \frac{1}{a_{22}}[b_2 - a_{21}x^0 - a_{23}x^0]$$

$$x_3^1 = \frac{1}{a_{33}}[b_3 - a_{31}x^0 - a_{32}x_2^0]$$

Q. Use Jacobi method to solve

$3x + 20y - z = -18$, $20x + y - 2z = 17$ and $2x - 3y + 2z = 25$

A. Rearranging equations.

$$\left.\begin{aligned} 20x + y - 2z &= 17 \\ 3x + 20y - z &= -18 \\ 2x - 3y + 2z &= 25 \end{aligned}\right\} \text{Arranging for diagonal elements}$$

$x = \frac{1}{20}[17 - y + 2z]$ $\quad y = \frac{1}{20}[-18 - 3x + z]$ $\quad z = \frac{1}{2}[25 - 2x - 3y]$

If k^{th} approximation to x, y, z is x^k, y^k, z^k then by Jacobi method $k+1$th iteration is

$x^{k+1} = \frac{1}{20}[17 - y^k + 2z^k]$ $\quad y^{k+1} = \frac{1}{20}[-18 - 3x^k + z^k]$ $\quad z^{k+1} = \frac{1}{2}[25 - 2x^k - 3y^k]$

Let initial approximation be $x^0 = 0, y^0 = 0, z^0 = 0$, 1st iteration is

$$x^1 = \frac{1}{20}[17 - 0 + 0] = \frac{17}{20} = 0.85$$

$$y^1 = \frac{1}{20}[-18] = -0.9$$

$$z^1 = \frac{1}{2}[25] = \frac{25}{2} = 12.5$$

2nd iteration

$$x^2 = \frac{1}{20}[17 - 0 + 0] + 2(12.5)) = 2.145$$

$$y^2 = \frac{1}{20}[-18 - 3(0.85) + 12.5] = -0.4025$$

$$z^2 = \frac{1}{2}[25 - 2(0.85) + 3(-0.9)] = 1.03$$

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★ Gauss Seidal method.

To solve

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$

1) Rearranging eqⁿ so that diagonal elements are bigger in magnitude.

2) $x_1 = \frac{1}{a_{11}}[b_1 - a_{12}x_2 - a_{13}x_3]$

$x_2 = \frac{1}{a_{22}}[b_2 - a_{21}x_1 - a_{23}x_3]$

$x_3 = \frac{1}{a_{33}}[b_3 - a_{31}x_1 - a_{32}x_2]$

3) If x_1^k, x_2^k, x_3^k is the k^{th} approximate roots of x_1, x_2, x_3 then ~~By~~ seidal method $k+1^{th}$ iteration is

$x_1^{k+1} = \frac{1}{a_{11}}[b_1 - a_{12}x_2^k - a_{13}x_3^k]$

$x_2^{k+1} = \frac{1}{a_{22}}[b_2 - a_{21}x_1^{k+1} - a_{23}x_3^k]$

$x_3^{k+1} = \frac{1}{a_{33}}[b_3 - a_{31}x_1^{k+1} - a_{32}x_2^{k+1}]$

4) Let initial iteration be x_1^0, x_2^0, x_3^0

then first iteration is –

$x_1 = 1/a_{11}[b_1 - a_{12}x^0 - a_{13}x_3^0]$

$x_2 = 1/a_{22}[b_2 - a_{21}x_1^1 - a_{23}x_3^0]$

$x_3 = 1/a_{33}[b_3 - a_{31}x_1^1 - a_{32}x_2^1]$

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Q] Find roots of the following equation correct to three decimal places by Gauss seidal method.

$2x - 3y + 20z = 25$
$20x + y - 2z = 17$
$3x + 20y - z = -18$

$20x + y - 2z = 17$. — ①
$3x + 20y - z = -18$. — ②
$2x - 3y + 20z = 25$. — ③

Arrangeing for diagonal elements.

$\cancel{x = 1/2[25 + 3y - 20z]}$
$\cancel{y = [17 - 20x + 2z]}$
$\cancel{z = -[-18 - 20y - 3x]} \quad \therefore \cancel{z = [18 + 20 + 3x]}$

$x = 1/20 \cdot [17 - y + 2z]$
$y = 1/20[-18 - 3x + z]$
$z = 1/20[25 - 2x + 3y]$

If k^{th} approximation to x, y, z is x^k, y^k, z^k then by Gauss seidal method, $k+1$ th iteration is.

$\cancel{x^{k+1} = 1/2[25 + 3y^k + 20z^k]}$
$\cancel{y^{k+1} = [17 - 20x^{k+1} + 2z^k]}$
$\cancel{z^{k+1} = [18}$

$x^{k+1} = 1/20[17 - y^k + 2z^k]$
$y^{k+1} = 1/20[-18 - 3x^{k+1} + z^k]$
$z^{k+1} = 1/20[25 - 2x^{k+1} + 3y^{k+1}]$

Let initial approximation be $x^0 = 0$, $y^0 = 0$, $z^0 = 0$.

1^{st} is.

$x^1 = 1/20[17 - y^0 + 2z^0] = 17/20 = 0.85$
$y^1 = 1/20[-18 - 3x^1 + z^0] = 1/20[-18 - 3(0.85)] = -1.027$
$z^1 = 1/20[25 - 2x^1 + 3y^1] = 1.010$.

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k	x^k	y^k	z^k
0	0	0	0
1	0.85	−1.027	1.01
2	1.00235	−0.999 ~~+0.001~~	0.9998
3	0.999	−0.99986	1.000041
4	0.99997	−0.99999	1.0000045

$x \approx 1$ or $x = 1$

$y \approx -1$ or $y = -1$

$z \approx 1$ or $z = 1$.

HW

$27x + 6y - z = 85$

$x + y + 54z = 110$

$6x + 15y + 2z = 72$

* **Euler's Method.**

To solve

$\frac{dy}{dx} = f(x,y)$

step length $= h$

x_0	$y_0 = y(x_0)$
$x_1 = x_0 + h$	$y_1 = y(x_1)$
$x_2 = x_1 + h$	$y_2 = y(x_2)$
$x_3 = x_2 + h$	$y_3 = y(x_3)$
⋮	⋮
$x_n = x_{n-1} + h$	$y_n = y(x_n)$

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By Euler's method

$y_1 = y_0 + hf(x_0 y_0)$

find $f(x_1, y_1)$

$y_2 = y_1 + hf(x_1, y_1)$

$$\boxed{y_{n+1} = y_n + hf(x_n, y_n)}$$

Q] Find Euler's method, find approximate value of y when $x = 0.6$ of $\frac{dy}{dx} = 1 - 2xy$ given that at $x=0$, $y=0$ take $h = 0.2$

A. $x_0 = 0$. $y_0 = 0$.

$h = 0.2$

$\frac{dy}{dx} = 1 - 2xy$

$f(x,y) = 1 - 2xy$

~~$y(0.6)$~~ $= y_0 = y(x_0) = y(0)$

$y_1 = y(x_1) = y(0.2+0) = y(0.2)$

$y_2 = y(x_2) = y(0.4)$

$y_3 = y(x_3) = y(0.6)$

$y(0.6) = ?$

$x_1 = x_0 + h = 0 + 0.2 = 0.2$

$y_1 = y(x_1) = y(0.2)$

By Euler's formula

$y_1 = y_0 + h f(x_0 y_0)$

$f(x_0 y_0) = 1 - 2x_0 y_0$

$= 1 - 2(0)(0)$

$= 1$

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$y_1 = 0 + (0.2)(1)$

$y_1 = 0.2$

$\therefore y(0.2) = 0.2$

$f(x_1, y_1) = 1 - 2x_1y_1$

$= 1 - 2\cdot(0.2)(0.2)$

$= 0.92$

$x_2 = x_1 + h = 0.2 + 0.2 = 0.4$

$y(x_2) = y(0.4) = y_2$

$= 0.2 + 0.2(0.92)$

$y_2 = 0.384$

$f(x_2 y_2)$

$\cancel{x_3} = \cancel{x_2 + h} = 1 - 2(0.4)(0.384)$

$= 0.6928$

$x_3 = x_2 + h = 0.4 + 0.2 = 0.6$

$y_3 = y(x_3) = y(0.6)$

By Euler's method

$y_3 = y_2 + hf(x_2 y_2)$

$= 0.384 + 0.2(0.6928)$

$y_3 = 0.52256$

$\therefore y(0.6) = 0.52256$

HW

Using Euler's method.

Given that $\frac{dy}{dx} = \frac{y-x}{y+x}$ with boundary conditions

$y = 1$ when $x = 0$ find approximate value of or find $y(0.1)$ by ~~of~~ Euler's method.

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$x_0 = 0 \quad y_0 = 1$

$y_1 = y_0 + h(f x_0 y_0)$

$= 1 + 0.1\left(\frac{1+0}{1+0}\right)$

$= 1.1$

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Modified Euler's method.

To solve

$\frac{dy}{dx} = f(x, y)$

$y(x_0) = y_0$, h = step length.

$x_1 = x_0 + h$

By Euler's method

$y_1 = y_0 + hf(x_0 y_0)$

Let $y_1 = y_1^0$.

find $f(x_1, y_1^0)$

By modified Euler's method.

1st modification.

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0 y_0) + f(x_1 y_1^0)\right]$$

find $(x_1 y_1^{(1)})$

2nd modification.

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0 y_0) + f(x_1 y_1^{(1)})\right]$$

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3^{rd} modification

$$y^{3} = y_0 + \frac{h}{2}[f(x_0 y_0) + f(x_1 y_1^{(2)})]$$

$y_1 \approx y_1^{(3)}$, find $f(x_1, y_1)$

$x_2 = x_1 + h$

$y_2 = y_1 + h f(x_1 y_1)$

Let $y_2^{0} = y_2$

find $(x_2 y_2)$

By Eulers modified method

$$y_2' = y_1 + \frac{h}{2}[f(x_1 y_1) + f(x_2 y_2^{0})]$$

2^{nd} modified value

$$y_2^{(2)} = y_1 + \frac{h}{2}[f(x_1 y_1) + f(x_2 y_2^{(1)})]$$

Q] Given that $\frac{dy}{dx} = \frac{y-x}{y+x}$ with $y(0) = 1$ find $y(0.1)$ by Euler's modified method by performing 2 iteration each with $h = 0.05$

$y_0 = x_0 + h$.

$\cancel{x_1} = 0 + 0.05 = \cancel{0.05}\ 0.05 = x_1$

$y_0 = 1$

$$f(x_0 y_0) = \frac{y_0 - x_0}{y_0 + x_0} = \frac{1-0}{1+0} = 1$$

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By Euler's method.

$$y_1 = y_0 + h f(x_0 y_0)$$
$$= 1 + (0.05)(1).$$

$y_1 = 1.05$

$$y_1^{0} = 1.05\ f(x_1 (y_1^{0}) = \frac{1.05 - 0.05}{1.05 + 0.05} = \frac{1}{1.1} = 0.909$$

1^{st} modification.

$$y_1' = y_0 + \frac{h}{2}[f(x_0 y_0) + f(x_1 y_1^{0})]$$

$$= 1 + \frac{0.05}{2}[1 + 0.909]$$

$y_1' = 1.0477$.

$$y_1' \neq f(x_1 y^{(1)}) = \frac{1.0477 - 0.05}{1.0477 + 0.05} = 0.9089$$

2^{nd} modification.

$$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0 y_0) + f(x_1 y_1^{(1)})]$$

$$= 1 + \frac{0.05}{2}[1 + 0.9089]$$

$= 1.0477.$

$\therefore y_1 = 1.0477.$

$$f(x_1 y_1) = \frac{y_1 - x_1}{y_1 + x_1} = \frac{1.0477 - 0.05}{1.0477 + 0.05} = 0.9089$$

$x_2 = x_1 + h = 0.05 + 0.05 = 0.1$

By Euler's method

$y_2 = y_1 + h \cdot f(x_1 y_1)$

$= 1.0477 + 0.05\,(0.9089)$

$y_2 = 1.0931.$

let $y_2 = y_2^{0}$

$\therefore y_2^{0} = 1.0931$

$\therefore f(x_2 y_2^{0}) = \cancel{0.1} \; \dfrac{1.0931 - 0.1}{1.0931 + 0.1}.$

$= \cancel{0.90757} \; 0.83237$

By modified Euler's method.

<u>1st modification</u>:

$y_2^{1} = y_1 + \dfrac{h}{2}\left[f(x_1 y_1) + f(x_2 y_2^{0})\right]$

$= 1.0477 + \dfrac{0.05}{2}\left[0.9089 + 0.8323\right]$

$y_2^{1} = 1.09123.$

$f(x_2 y_2^{(1)}) = \dfrac{1.09123 - 0.1}{1.09123 + 0.1} = 0.83210.$

<u>2nd modification</u>:

$y_2^{(2)} = y_1 + \dfrac{h}{2}\left[f(x_1 y_1) + f(x_2 y_2^{(1)})\right]$

$= 1.0477 + \dfrac{0.05}{2}\left[0.9089 + 0.83210\right]$

$= 1.09122$

$\therefore y_2 = 1.09122$

Q) Using Euler's method (modified) obtain a solution of the equation.

$\dfrac{dy}{dx} = x + |\sqrt{y}|.$ or $x + \sqrt{y} = f(xy)$

with initial conditions

$y = 1$ at $x = 0$

for the range $0 \le x \le 0.6$. in steps of 0.2

$f(x_0 y_0) = 0 + \sqrt{1} = 0 + 1 = 1$

$y_1 = y_0 + h f(x_0 y_0)$

$= 1 + 0.2\,(1)$

$y_1 = \cancel{0}\; 1.2$

taking $y_1 = y_1^{0}$

$\therefore y_1^{0} = 1.2 \quad f(x_1 y_1^{0}) = \cancel{\dfrac{1.2 - 0.2}{1.2 + 0.2}} = \cancel{\dfrac{1}{1.4}} = \cancel{0.71428}.$ $0.2 + \sqrt{1.2} = 1.2954.$

<u>1st modification</u>:

$y_1^{1} = y_0 + \dfrac{h}{2}\left[f(x_0 y_0) + f(x_1 y_1^{0})\right]$

$= 1 + \dfrac{0.2}{2}\left[1 + 1.29544\right]$

$= \cancel{1+}\; 1.22954 \quad ; f(x_1 y_1^{(1)}) = 0.2 + \sqrt{1.22954}$

$= 1.30884.$

<u>2nd modification</u>:

$y_1^{2} = y_0 + \dfrac{h}{2}\left[f(x_0 y_0) + f(x_1 y_1^{1})\right].$

$= 1 + \dfrac{0.2}{2}\left[1 + 1.30884\right]$

$= 1.33088$

$\log_e \rightarrow \times \log_{10}$
$(\ln)$ $(\times \log)$

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Q] solve the following by Euler's modified method.

$$\frac{dy}{dx} = \log(x+y)$$

$y(1) = 2$

$x_0 = 1$ $y_0 = 2$ at $x = 1.2$ & 1.4 with $h = 0.2$

$f(x,y) = \ln(x+y)$.

$f(x_0 y_0) = \ln(1+2) = \ln(3)$
$= 1.0986$.

$x_1 = x_0 + h$.
$= 1 + 0.2 = 1.2$

$y_1 = y_0 + h f(x_0 y_0)$
$= 2 + 0.2(1.0986)$.
$= 2.21972$

$\therefore y_1 = 2.21972$

$y_1^0 = 2.21972$

$\therefore f(x_1 y_1^0) = \ln(1.2 + 2.21972)$
$= 1.22955$

1st modification.

$y_1' = y_0 + \frac{h}{2}[f(x_0 y_0) + f(x_1 y_1^0)]$

$= 2.\cancel{21972} + \frac{0.2}{2}[1.0986 + 1.22955]$

$= \cancel{2.4525}$. 2.23281

$\therefore f(x_1 y_1^{(1)}) = \ln(1.2 + 2.23281)$
$= 1.233379$

2nd modification.

$y_1^2 = y_0 + \frac{h}{2}[f(x_0 y_0) + f(x_1 y_1')]$

$= 2 + 0.1[1.0986 + 1.233379]$
$= 2.23319$.

$\therefore y_1 = y_1^2 = 2.23319$

$f(x_1 y_1) = \ln(1.2 + 2.23319)$
$= 1.23348$.

$x_2 = x_1 + h$.
$= 1.2 + 0.2 = 1.4$.

By Euler's method

$y_2 = y_1 + h f(x_1 y_1)$
$= \cancel{2.21972} + 0.2(1.23348) + 2.23319$

$y_2 = \cancel{2.466416}$ 2.47988

$\therefore y_2^0 = 2.47988$.

$\therefore f(x_2, y_2^0) = \ln(1.4 + 2.47988)$
$= 1.35580$.

1st modification.

$y_2' = y_1 + \frac{h}{2}[f(x_1 y_1) + f(x_2 y_2^0)]$

$= 2.23319 + 0.1[1.23348 + 1.35580]$

$y_2' = 2.49211$

$f(x_2, y_2') = \ln(1.4 + 2.49211)$
$= 1.35895$

2nd modification:

$y_2^2 = y_1 + \frac{h}{2}[f(x_1 y_1) + f(x_2 y_2^1)]$

$= 2.23319 + 0.1[1.23348 + 1.35895]$
$= 2.49243$.

$\therefore y_2^2 = y_2 = 2.49248$.

★ Runge Kutta Fourth order Method.

To solve
$\frac{dy}{dx} = f(x, y)$; $y(x_0) = y_0$ with step h.

$k_1 = hf(x_0 y_0)$
$k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$.

$k_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2})$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$\therefore \boxed{k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]}$.

$y_1 = y_0 + k$.

To find $y(x_2) = y_2$

$k_1 = hf(x_1 y_1)$
$k_2 = hf(\frac{x_1 + h}{2}, y_1 + \frac{k_1}{2})$

$k_3 = hf(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2})$.

$k_4 = hf(x_1 + h, y_1 + k_3)$

$\therefore k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4)$

$y_2 = y_1 + k$.

Q] Apply Runge Kutta fourth order method to find approximate value ~~for~~ of y for $x = 0.2$ in steps of 0.1 if $\frac{dy}{dx} = x + y^2$ given that $y = 1$ when $x = 0$.

A. $f(x \cdot y) = x + y^2$
$x_0 = 0$; $y_0 = 1$ $h = 0.1$
$x_1 = x_0 + h = 0 + 0.1$

$k_1 = hf(x_0 y_0)$
$= 0.1 f(0\ 1)$
$= 0.1[0 + (1)^2]$
$k_1 = 0.1$

$k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{h}{2})$.

$= 0.1 \times f(0 + \frac{0.1}{2}, 1 + \frac{0.1}{2})$

$= 0.1 f(0.05, 1.05)$
$= 0.1 \times [(0.05) + (1.05)^2]$
$= 0.11525$

$k_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2})$

$= 0.1 \times f(0 + \frac{0.1}{2}, \frac{1 + 0.11525}{2})$

$= 0.1 \times f(0.05, 1.057625)$

$= 0.1 \times [0.05 + (1.057625)^2]$

$= 0.11685$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= 0.1 \times f(0 + 0.1, 1 + 0.11685)$

$= 0.1 \times f(0.1, 1.11685)$

$= 0.1 \times [0.1 + (1.11685)]$

$= 0.13473$

$\therefore k = \frac{1}{6}[0.1 + 2(0.11525) + 2(0.11685) + 0.13473]$

$= 0.116488$

$y_1 = y_0 + k$ $\quad x_1 = x_0 + h$

$y_1 = 1 + 0.116488$ $\quad = 0 + 0.1 = 0.1$

$= 1.116488$

To find $y(x_2) = y_2$

$k_1 = hf(x_1, y_1)$

$= 0.1 \times f(0.1, 1.1)$

$= 0.1 \times [(0.1) + (1.116488)^2]$

$= 0.134654 = k_1$

$k_2 = hf(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2})$

$= 0.1 \times f(0.1 + \frac{0.1}{2}, 1.116488 + \frac{0.13465}{2})$

$= 0.1 \times f(0.15, 1.183813)$

$= 0.1 \times [(0.15) + (1.183813)^2]$

$= 0.155141 = k_2$

$k_3 = hf(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2})$

$= 0.1 \times f(0.1 + \frac{0.1}{2}, 1.116488 + \frac{0.155141}{2})$

~~$0.1 \times f(0.15,$~~

$= 0.157577 = k_3$

$k_4 = h \cdot f(x_1 + h, y_1 + k_3)$

$= 0.1 \times f(0.1 + 0.1, 1.116488 + 0.157577)$

$= 0.1 \times f(0.2, 1.274065)$

$= 0.1 \times [0.2 + (1.274065)^2]$

$= 0.18349$

$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0.134654 + 2 \times 0.155141 + 2 \times 0.157577 + 0.28349]$

$= \cancel{0.174244}$ 0.157070

HW) Solve (RK) $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2, 0.4$ & $h = 0.2$

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$y_2 = y_1 + k$

$y_2 = 1.116488 + 0.157070$

$= 1.273558$

$x_2 = x_1 + h$

$= 0.1 + 0.1$

$= 0.2$

$k_1 = hf(x_2 y_2)$

$= 0.1 \times f(0.2, 1.273558)$

$= 0.1 \times [0.2 + (1.273558)^2]$

$= 0.18219$

$k_2 = 0.21122.$

$k_3 = 0.21521224.$

$k_4 = 0.251644$

$k = 0.5919.$

2) Using Runge Kutta fourth order method calculate $y(0.2)$

given $\frac{dy}{dx} - \frac{2xy}{1+x^2} = 1$ $\quad y(0) = 0.$ $\quad [x_0 = 0, y_0 = 0]$

$h = 0.2$ (as not mentioned)

$\frac{dy}{dx} = 1 + \frac{2xy}{1+x^2}$

$f(xy) = 1 + \frac{2xy}{1+x^2}$

$x_0 = 0 \quad y_0 = 0$

$k_1 = h \cdot f(x_0 y_0)$

$= 0.2 \times \cancel{f}\left[1 + \frac{2(0)(0)}{1+(0)}\right].$

$= 0.2 \times 1 = 0.2$

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$k_2 = h \cdot f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right).$

$= 0.2 \times f\left(0 + \frac{0.2}{2}, 0 + \frac{0.2}{2}\right).$

$= 0.2 \times f(0.1, 0.1)$

$= 0.2\left[1 + \frac{2(0.1)(0.1)}{1 + (0.1)^2}\right] \quad = 0.2 \times \left[1 + \frac{0.02}{1.01}\right]$

$= \cancel{0.2 \times f} \quad k_2 = 0.20396.$

$k_3 = h \cdot f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right).$

$= 0.2 \times f\left(0.1, \frac{0.20396}{2}\right) = 0.2 \times f(0.1, 0.10198)$

$= 0.2 \times \left[1 + \frac{2 \times 0.1 \times 0.10198}{1 + (0.1)^2}\right].$

$= \cancel{0.2 \times [1 +} \quad 0.20403.$

$k_4 = h \cdot f(x_0 + h, y_0 + k_3)$

$= 0.2 \times f(0.2, 0.20403)$

$= 0.2\left[1 + \frac{2 \times 0.2 \times 0.20403}{1 + (0.2)^2}\right] = 0.2\left[1 + \frac{0.081612}{1.04}\right]$

$= 0.21569$

$k = \frac{1}{6}[0.2 + 2 \times 0.20396 + 2 \times 0.20403 + 0.21569]$

$= k = 0.20527.$

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★ Solution of Algebraic and ~~For~~ Transcendental Equations. ★

1) Newton Raphson method.

To solve $f(x)=0$

let $x = x_0$ be initial root.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

Q. using Newton Raphason method, find approximate root of $3x = \cos x + 1$ correct to three decimal places.

A. $f(x) = 3x - \cos x - 1$

$f'(x) = 3 + \sin x$.

By Newton Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

let $x_0 = 0$ be

$f(x_0) = f(0)$

$= \cancel{9n}\ 0 - \cos 0 - 1$.

$= 0 - 1 - 1 = -2$

$f'(0) = 3 + \sin(0)$

$= 3$.

$$\therefore x_1 = 0 - \left[\cancel{\frac{3(1) - \cos(1) - 1}{3 + \sin 1}}\right], \frac{f(x_0)}{f'(x_0)}$$

$$= 0 - \left(\frac{-2}{3}\right) = \frac{2}{3} = 0.667$$

Degree

$$f(x_1) = \frac{3(1) - \cos(1) - 1}{3 + \sin(1)}$$

$= 0.33145$

$f'(x) = 3 + \sin 1$

$= \cancel{3f}\cdot 3.0174.$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.667 - \frac{0.33145}{3.0174}$$

$= 0.667 - 0.1098$

$= 0.5572$

Radian

$$f(x) = \frac{3(1) - \cos(1) - 1}{3 + \sin 1}$$

$= 0.37998$

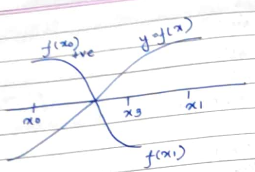
$f'(x) = 3 + \sin 1$

$= 3.8414$.

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.667 - \frac{0.37998}{3.8414}$$

$= 0.56808$.



* Bisection method – To find root of equation $f(x)=0$

1) find x_0 and x_1 such that $f(x_0)$ and $f(x_1)$ has opposite signs

let $a = x_0$ and $b = x_1$

$f(a) = -ve \quad f(b) = +ve$

By bisection method

$$x_2 = \frac{a+b}{2}$$

If $f(x_2)$ and $f(x_1)$ has opposite signs then root lies between x_2 and x_1 $\quad a = x_2,\ b = x_1$

$$x_3 = \frac{a+b}{2}$$

else if

$f(x_0)$ & $f(x_2)$ has opposite signs then root lies between x_0 and x_2

let $a = x_0$ and $b = x_2$

$$x_3 = \frac{a+b}{2}$$

Q. Using bisection method, find app root of

$$\sin x = \frac{1}{x}$$

A $\quad \sin x = \frac{1}{x} \quad \therefore x\sin x = 1$

$x\sin x - 1 = 0$

$f(x) = x\sin x - 1$

$f(0) = 0\sin 0 - 1$

$f(0) = -1$

$f(1) = 1\sin 1 - 1$

$= -0.1585$

$f(2) = 2\sin 2 - 1$

$= 0.8185$

$f(1)$ and $f(2)$ are of opposite signs $\therefore$ root lies between 1 and 2

$a = 1$ and $b = 2$

By bisection method.

$$x_2 = \frac{a+b}{2} = \frac{1+2}{2} = 1.5$$

$f(x_2) = f(1.5)$

$f(1.5) = 1.5\sin(1.5) - 1$

$f(1.5) = 0.4962$ +ve and $f(1) = -ve$

$\therefore$ root lies between 1 and 1.5

$a = 1 \quad b = 1.5$

By bisection method

$$x_3 = \frac{a+b}{2} = \frac{1+1.5}{2} = 1.25$$

$f(1.25) = 1.25\sin(1.25) - 1$

$= 0.186$ +ve

$f(1) = -ve$

$\therefore$ root lies between 1 and 1.25

$\therefore a = 1 \quad b = 1.25$

$$x_4 = \frac{a+b}{2} = \frac{1.25+1}{2} = 1.125$$

$f(x_4) = 1.125\sin(1.125) - 1$

$f(1.125) = 0.0154$ +ve

root lies between 1 and 1.125

$a=1 \quad b=1.125$

$x_5 = \frac{1+1.125}{2} = 1.0625$

$f(1.0625) = 1.0625 \sin(1.0625) - 1$

$f(1.0625) = -0.0752$

$f(1.0625) = -ve$ & $f(1.125)$ +ve

$\therefore$ root lies between 1.0625 and 1.125

* Regula Falsi method to find approximate root of $f(x)=0$

Find x_0 and x_1 such that $f(x_0)$ and $f(x_1)$ are of opposite sign

By Regula Falsi method

$$\boxed{x_2 = x_0 - \frac{x_1 x_0}{f(x_1)-f(x_0)} \cdot f(x_0)}$$

$f(x_2)$ and $f(x_1)$ are of opposite signs
root lies between x_1 and x_2, replace

$x_0 = x_1$ and $x_1 = x_2$

else

$f(x_2)$ and $f(x_0)$ are of opposite sign then root lies between x_0 and x_2.

6/3/19 Find the real root of the equation $x^3-2x+5=0$ by Regula Falsi Method.

A $f(x) = x^3-2x+5$

$f(0) = 5$

$f(1) = 1-2+5 = 4$. $\qquad f(-1) = -1+2+5 = 6$

$f(2) = 8-4+5 = 9$. $\qquad f(-2) = -8+4+5 = 1$.

$f(3) = 27-6+5 = 26$. $\qquad f(-3) = -27+6+5 = -16$.

$f(-2)$ & $f(-3)$ are of opposite signs.

$\therefore$ root lies between -2 & -3

$x_0 = -3$ and $x_1 = -2$

$f(x_0) = f(-3) < 0 \quad f(x_1) = f(-2) > 0$.

$-16 < 0 \qquad 1 > 0$.

By Regula Falsi method.

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1)-f(x_0)} \cdot f(x_0)$$

$$= -3 - \frac{(-2)-(-3)}{1+16} \cdot (-16)$$

$$= -3 - \frac{(-2+3)\cdot(-16)}{17}$$

$$= -3 - \frac{(-16)}{17} = -3 + \frac{16}{17}.$$

$$= -\frac{45}{17}. = -2.0588.$$

$f(x_2) = f(-2.0588)$

$= (-2.0588)^3 - 2(-2.0588) + 5$

$= 0.3907 > 0$.

$\therefore f(-3)$ & $f(-2.0588)$ has opposite sign

$\therefore$ root lies between -2.0588 & -3

Let $x_0 = -3$ and $x_1 = -2.0588$

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} \cdot f(x_0).$$

$$x_2 = -3 - \frac{(-2.0588+3)}{0.3907+16} \cdot (-16).$$

$$= -3 - \frac{(0.9412)(-16)}{0.3907+16}.$$

$$= -3 + \frac{15.0592}{16.3907}.$$

$$= -3 + 0.918764$$

$$= -2.08123$$

$$f(x_2) = f(-2.08123)$$

$$= (-2.08123)^3 - 2(-2.08123) + 5$$

$$= 0 \cancel{.14757}.\ 0.14757.$$

6/3/19

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