Assignment 2

Q1) Find the directional derivative of function $\phi = xy^2 + yz^3$ at $(1,-1,1)$ in direction of normal to surface at point $(1,2,2)$ $x^2+y^2+z^2$

→

$\phi = xy^2 + zy^3$

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k} \quad -② $$

$$\nabla\phi = y^2\hat{i} + (2xy + z^3)\hat{j} + 3yz^2\hat{k}$$

let normal to surface ψ

$\psi = x^2+y^2+z^2$

grad $\psi = \nabla\psi$

$= 2x\hat{i} + 2y\hat{j} + 2z\hat{k} \quad -①$

put 1, 2, 2 in eqn ①

$\bar{a} = 2\hat{i} + 4\hat{j} + 4\hat{k}$

$$\hat{a} = \frac{2\hat{i} + 4\hat{j} + 4\hat{k}}{6}$$

Now put 1, -1, 1 in eqn ②

$\nabla\phi(1,-1,1) = \hat{i} - \hat{j} - 3\hat{k}$

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$$D.d = \nabla\phi \cdot \hat{a}$$

$$= (\hat{i} - \hat{j} - 3\hat{k}) \cdot \frac{(2\hat{i} + 4\hat{j} + 4\hat{k})}{6}$$

$$\frac{2}{3}(\hat{i} - \hat{j} - 3\hat{k})(\hat{i} + 2\hat{j} + 2\hat{k})$$

$$= \frac{\cancel{2}}{3}(1 - 2 - 6) = \frac{-7}{3}$$

(b) find the direction derivative of the function $\phi = e^{2x-y-z}$ at $(1,1,1)$ in the direction of tangent to the curve

$$x = e^{-t},\ y = 2\sin t + 1,\ z = t - \cos t \quad \text{at } t = 0.$$

→ $\phi = e^{2x-y-z}$

$$\nabla\phi = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}$$

$$= (2.e^{2x-y-z})\hat{i} + (-e^{2x-y-z})\hat{j} + (-e^{2x-y-z})\hat{k}$$

$$= e^{2x-y-z}(2\hat{i} - \hat{j} - \hat{k})$$

$$\nabla\phi \text{ at } (1,1,1) = e^{2-1-1}(2\hat{i} - \hat{j} - \hat{k})$$

$$= 2\hat{i} - \hat{j} - \hat{k} \quad \text{—(1)}$$

Now,

$$\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}.$$

~~tangent~~ $\frac{\partial\bar{r}}{\partial t} = -\cancel{e^{-t}\hat{i} + (2\sin t + 1)}\hat{j}$

tangent

$$\frac{d\bar{r}}{dt} = -e^{-t}\hat{i} + (2\cos t \cancel{\ })\hat{j} + (1+\sin t)\hat{k}$$

tangent at $t=0$

$$\frac{d\bar{r}}{dt} = -\hat{i} + 2\hat{j} + \hat{k}$$

$$\bar{a} = -\hat{i} + 2\hat{j} + \hat{k}$$

$$\hat{a} = \frac{-\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{6}} \quad (2)$$

Now,

$$dd = \nabla\phi \cdot \hat{a}$$

$$(2\hat{i} - \hat{j} - \hat{k})\frac{(-\hat{i} + 2\hat{j} + \hat{k})}{\sqrt{6}}$$

$$\frac{-2-2-1}{\sqrt{6}} = \frac{-5}{\sqrt{6}}$$

Q2)

(a) show that $\nabla^2\left[\nabla\left(\frac{\bar{r}}{r^2}\right)\right] = \frac{2}{r^4}$

by using property,

$$\nabla\left(\frac{\bar{r}}{r^2}\right)$$

$\bar{r}\cdot\bar{r} = |\bar{r}|^2 = r^2$

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$$\nabla\left(\frac{\bar{r}}{r^2}\right) = \nabla(\bar{r}\cdot r^{-2}) = \bar{r}\cdot\nabla r^{-2} + r^{-2}\nabla\bar{r}$$

$$= \bar{r}\cdot\frac{(-2r^{-3})\cdot\bar{r}}{r} + 3r^{-2}$$

$$= -2r^{-2} + 3r^{-2}$$

$$= r^{-2}$$

$$= \frac{1}{r^2}$$

LHS

$$\nabla^2\left(\frac{1}{r^2}\right)$$

$$\nabla(\nabla r^{-2}) = \nabla\left(\frac{-2r^{-3}\cdot\bar{r}}{r}\right) = \nabla(-2r^{-4}\cdot\bar{r})$$

$$= -2\left[\bar{r}\cdot\nabla r^{-4} + r^{-4}\nabla\bar{r}\right]$$

$$= -2\left(\bar{r}\cdot\frac{(-4r^{-5})\cdot\bar{r}}{r} + 3r^{-4}\right)$$

$$-2\left(-4r^{-6}\cdot r^2 + 3r^{-4}\right]$$

$$2\cdot\left(4r^{-4} - 3r^{-4}\right]$$

$$= \frac{2}{r^2}$$

$= RHS$

Hence LHS = RHS

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(b) show that

$$\nabla^4 . e^r = e^r + \frac{4}{r} . e^r$$

→ LHS

$$\nabla^3 (\nabla e^r)$$

$$\nabla^3 \left(\frac{e^r}{r} . \bar{r} \right)$$

using property,

$$\nabla . \phi . \bar{F} = \nabla . \phi . \bar{F} + \phi \nabla \bar{F}$$

put $\phi = \frac{e^r}{r}$ $\bar{F} = \bar{r}$

$$\therefore \nabla . \phi . \bar{F} = \nabla (e^r . r^{-1}) . \bar{r} + \frac{e^r}{r} . \nabla r$$

$$= \left[e^r . \nabla r^{-1} + r^{-1} \nabla e^r \right] \bar{r} + \frac{3e^r}{r}$$

$$= \left[-e^r . \frac{r^{-2}}{r} . \bar{r} + r^{-1} \frac{e^r}{r} . \bar{r} \right] . \bar{r} + \frac{3e^r}{r}$$

$$= e^r \left[r^{-3} . \bar{r} + r^{-2} . \bar{r} \right] \bar{r} + \frac{3e^r}{r}$$

$$= e^r \left[r^{-3} . r^2 + r^{-2} . r^2 \right] + \frac{3e^r}{r}$$

$$= \frac{-e^r}{r} + e^r + \frac{3e^r}{r}$$

$$= e^r + \frac{2e^r}{r}$$

$$\nabla^2\left[\nabla\left(e^r + \frac{2e^r}{r}\right)\right]$$

$$\nabla^2\left[\frac{e^r}{r}\cdot\bar{r} + 2\left(\frac{r.\nabla e^r - e^r.\nabla r}{r^2}\right)\right]$$

$$\nabla^2\left[\frac{\bar{r}.e^r}{r} + 2\left(\frac{r.\frac{e^r.\bar{r}}{r} - \frac{e^r.\bar{r}}{r}}{r^2}\right)\right]$$

$$\nabla^2\left[\frac{\bar{r}.e^r}{r} + 2\left(\frac{r.e^r.\bar{r} - e^r.\bar{r}}{r^3}\right)\right]$$

$$\nabla^2\left[\frac{\bar{r}.e^r}{r} + 2e^r.\bar{r}\left(\frac{r-1}{r^3}\right)\right]$$

$$\nabla^2\left[e^r.\bar{r}\left(\frac{1}{r} + \frac{2r-2}{r^3}\right)\right]$$

$$\nabla^2\left[e^r.\bar{r}\left(\frac{r^2+2r-2}{r^3}\right)\right]$$

$$\nabla^2\left[\nabla^2 e^r\right] = \nabla^4 e^r$$

but $\nabla^2 e^r = e^r\left(1+\frac{2}{r}\right)$

$$\therefore f(r) = e^r\left(1+\frac{2}{r}\right)$$

$$f'(r) = e^r\left(1+\frac{2}{r}\right) + e^r\left(\frac{-2}{r^2}\right)$$

$$e^{r}\left[1+\frac{2}{r}-\frac{2}{r^2}\right]$$

$$f''(r)=e^{r}\left[1+\frac{2}{r}-\frac{2}{r^2}\right]+e^{r}\left[-\frac{2}{r^2}+\frac{4}{r^3}\right]$$

$$=e^{r}\left[1+\frac{2}{r}-\frac{4}{r^2}+\frac{4}{r^3}\right]$$

$$f''(r)+\frac{2}{r}f'(r)=$$

$$e^{r}\left[1+\frac{2}{r}-\frac{4}{r^2}+\frac{4}{r^3}\right]+\frac{2}{r}\left[1+\frac{2}{r}-\frac{2}{r^2}\right]e^{r}$$

$$e^{r}\left[1+\frac{2}{r}-\frac{4}{r^2}+\frac{4}{r^3}+\frac{2}{r}+\frac{4}{r^2}-\frac{4}{r^3}\right]$$

$$e^{r}\left[1+\frac{4}{r}\right]$$

$$e^{r}\left[1+\frac{4}{r}\right]$$

$$\therefore \nabla^2 e^{r}=e^{r}+\frac{4}{r}e^{r}$$

show that
vector field $\bar{F} = (2xz^3 + 6y)\hat{i} + (6x - 2yz)\hat{j} + (3x^2z^2 - y^2)\hat{k}$ is irrotational
hence find corresponding scaler field ϕ
such that $\bar{F} = \nabla\phi$

to prove the given vector field is irrotational

$$\nabla \times \bar{F} = 0 \quad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$\left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right)\hat{i} - \left(\frac{\partial F_3}{\partial x} - \frac{\partial F_1}{\partial z}\right)\hat{j} + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\hat{k}$$

$F_1 = 2xz^3 + 6y$
$F_2 = 6x - 2yz$
$F_3 = 3x^2z^2 - yz$

$$(-2y + 2y)\hat{i} - (6x^2z - 6xz^2)\hat{j} + (6 - 6)\hat{k}$$

$= 0.$

hence given vector fild is irrotational.

now

$\partial\phi = \bar{F} \cdot \bar{r}$

$\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\nabla\phi \cdot \bar{r}$

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$d\phi = (F_1\hat{i} + F_2\hat{j} + F_3\hat{k})(\hat{i}dx + \hat{j}dy + \hat{k}dz)$

$$((2xz^3 + 6y)\hat{i} + (6x - 2yz)\hat{j} + (3x^2z^2 - y^2)\hat{k})$$
$$(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$= 2xz^3dx + 6ydx + 6xdy - 2yzdy + 3x^2z^2dz - y^2dz$

$= \cancel{\ldots}$

$d\phi = d(x^2z^3) - d(y^2z) + d(z^3x^2)$

on integrating

$\phi = x^2z^3 - y^2z + z^3x^2$

Q3)

b) Show that vector field $\bar{F} = (y\sin z - \sin x)\hat{i} + (x\sin z + 2yz)\hat{j} + (xy\cos z + y^2)\hat{k}$ is irrotational Hence find corresponding scaler field.

Here $\bar{F} = F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$

$F_1 = y\sin z - \sin x$
$F_2 = x\sin z + 2yz$
$F_3 = xy\cos z + y^2$

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to show that given vetor field is irrotational

$\nabla \times \bar{F} = 0$

$$\therefore \nabla \times \bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$= \left(\frac{\partial}{\partial y} F_3 - \frac{\partial}{\partial z} F_2\right)\hat{i} - \left(\frac{\partial}{\partial x} F_3 - \frac{\partial}{\partial z} F_1\right)\hat{j} + \left(\frac{\partial}{\partial x} F_2 - \frac{\partial}{\partial y} F_1\right)\hat{k}$$

$$= (x\cos z + 2y - x\cos z - 2y)\,\hat{i} - (y\cos z - y\cos z)\,\hat{j}$$

$$+ (\sin z - \sin z)\,\hat{k}$$

$= 0$

$\therefore \nabla \times \bar{F} = 0$

Now,

$$d\phi = \bar{F}\, d\bar{r}$$

$$= (F_1\hat{i} + F_2\hat{j} + F_3\hat{k}) \cdot (dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$$

$$= F_1\,dx + F_2\,dy + F_3\,dz$$

$$= \cancel{\cos x}\; y\sin z\,dx - \sin x\,dx + 2yz\cdot dy + x\sin z\,dy + xy\cos z\,dz + y^2 dz$$

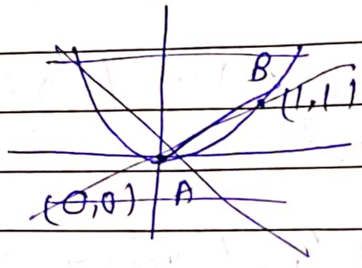
$$d\phi = d(y^2 z) + d(\cos x) + d(xy\sin z)$$

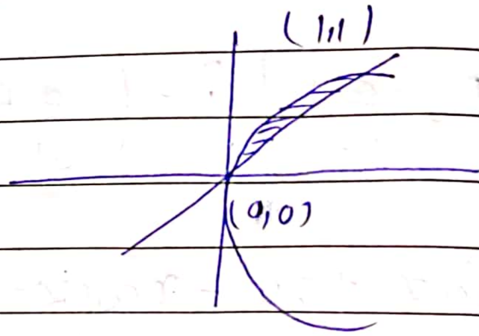
$\cos x + y^2 z + xy\sin z$

Q9) evaluate $\int_C F\cdot d\gamma$ for $F = (2x+y^2)\hat{i} + (3y-4x)\hat{j}$

along the following path:

① the parabolic arc $y^2 = x$ joing (0,0) and (1,1)





$$\gamma = x\hat{i} + y\hat{j} + z\hat{k}$$
$$d\gamma = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$\oint_C F\cdot d\gamma = \oint_C \left((2x+y^2)\hat{i} + (3y-4x)\hat{j}\right)(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$\oint F\cdot d\gamma = \oint_C (2x+y^2)\,dx + (3y-4x)\,dy \quad -①$$

Now, equation of line

$$\frac{x-0}{1-0} = \frac{y-0}{1-0}$$
$$y = x$$
$$dx = dy$$

Now put $x = y^2$

$dx = 2dy$ in eq^n ①.

$\int_c \bar{F}\cdot d\bar{r} = \int (2y^2+y^2)(2y\,dy) + (3y-4y^2)\,dy$

$= \int (4y^3 - 4y^2 + 3y)\,dy$

as the parabola varies from $y=0, y=1$.

$\oint_c \bar{F}\cdot d\bar{r} = \int_0^1 (6y^3 - 4y^2 + 3y)\,dy$

$= \frac{3}{2} - \frac{4}{3} + \frac{3}{2}$

$= \frac{5}{3}$

Q5) Evaluate $\int F\cdot dr$ for

$F = (2xy + 3z^2)\hat{i} + (x^2 + 4yz)\hat{j} + (2y^2 + 6xz)\hat{k}$

where c is the curve joining the points $(0,0,0)$ and $(1,1,1)$.

$\rightarrow \quad d\bar{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$x = t \qquad y = t^2 \qquad z = t^3$

$dx = dt \qquad dy = 2t\cdot dt \qquad dz = 3t^2 dt$

$\oint \bar{F}\cdot\bar{r} = \int F_1\cdot dx + F_2\,dy + F_3\,dz$

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$\int (2xy + 3z^2)\,dx + (x^2 + 4yz)\,dy + (2y^2 + 6xz)\,dz$

Now put $x = t$

$y = t^2$

$z = t^3$

$\int (2t^3 + 3t^6)\,dt + (t^2 + 4t^5)(2t \cdot dt) +$

$(2t^4 + 6t^4)(3t^2)\,dt$

$\int (2t^3 + 3t^6 + 2t^3 + 8t^6 + 24t^6)\,dt$

$\cancel{\int 3t^6 + 4t^3\,dt}$ $\int 35t^6 + 2t^3\,dt$

for given point $0,0,0,\ 1,1,1$

$\frac{x-0}{1-0} = \frac{y-0}{1-0} = \frac{z-0}{1-0} = t$

$x = t \qquad y = t \qquad z = t$

as x varies from 0 to 1

t varies from 0 to 1

$\int_0^1 (35t^6 + 2t^3)\,dt$

$\left[\frac{35}{7}t^7 + \frac{2}{4}t^4\right]_0^1$

$5 + 1$

$\boxed{6}$

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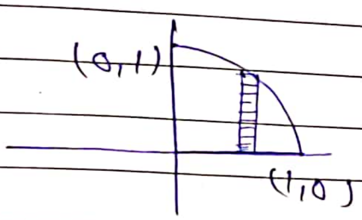
Q8) using greens theorem

evaluate $\int_C F\cdot dr$ for $F = x\hat{i} + y^2\hat{j}$ over the first quadrant of the circle $x^2+y^2=1$



according to greens theorem

$$\int_C u\,dx + v\,dy = \iint_A \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dx\,\partial y$$

here

$F = u\hat{i} + v\hat{j} = x\hat{i} + y^2\hat{j}$

$dr = dx\hat{i} + dy\hat{j}$

$\therefore \int F\cdot dr = \int u\,dx + v\,dy$

here x varies from 0 to 1

y varies from 0 to $1-x^2$

$$\int_0^1\int_0^{1-x^2} \left(\frac{\partial y^2}{\partial x} - \frac{\partial x}{\partial y}\right) dx\,dy$$

$$\int_0^1\int_0^{1-x^2} 0 \cdot dx\cdot dy$$

0

Q8) If $F(z) = u+iv$ is analytic function then prove that,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |F(z)|^4 = 16 |F(z)|^2 |F'(z)|^2$$

→ $g(z) = u+iv$

$|g(z)| = |u+iv|$

$= \sqrt{u^2+v^2}$

$|g(z)|^2 = u^2+v^2$

$|g(z)|^4 = (u^2+v^2)^2 = g$

$\frac{\partial g}{\partial x} = \frac{\partial}{\partial x}(u^2+v^2)^2 = \cancel{2(uu_x + 2vv_x)}$

$\frac{\partial}{\partial x}(u^4+v^4+2u^2v^2)$

$4uu_x + 4vv_x + 2(2uu_x + 2vv_x)$

$\frac{\partial^2 g}{\partial x^2} = 4(u_x)^2 + 4u\,u_{xx} + 4(v_x)^2 v_{xx} +$

$4(u_x)^2 + 4uu_{xx} + 4(v_x)^2 + 4vv_x$

$= 8 \quad + 8(u_x)^2 + 8(v_x)^2 +$

$|f(z)|^2 = f(z)\,\bar{f}(\bar{z})$

$\therefore (|f(z)|^2)^2 = |f(z)|^2 (\bar{f}(\bar{z}))^2$

and

$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4\frac{\partial^2}{\partial z \partial \bar{z}}$

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LHS $4 \frac{\partial^2}{\partial z \partial \bar{z}} (f(z))^2 (\bar{f}(\bar{z}))^2 = 4 \frac{\partial}{\partial z} \frac{\partial}{\partial \bar{z}} (f(z))^2 (\bar{f}(\bar{z}))^2$

$= 4 \frac{\partial}{\partial z} (f(z))^2 \cdot \frac{\partial}{\partial \bar{z}} (\bar{f}(\bar{z}))^2$

$= 8 \frac{\partial}{\partial z} \cdot \frac{\partial}{\partial \bar{z}} f'(z) + 2 \bar{f}(z) \bar{f}'(\bar{z})$

$= 16 f(z) f(z) \cdot \bar{f}(\bar{z}) f'(z) \bar{f}'(\bar{z})$

$= 16 |f(z)|^2 |f'(z)|^2$

Q9) Show that $u = 3x^2 - 3y^2 + 2y$ is harmonic and hence find the function v such that $f(z) = u + iv$ is analytic and express $f(z)$ in terms of z.

→ $u = 3x^2 - 3y^2 + 2y$

$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x} (3x^2 - 3y^2 + 2y)$

$\frac{\partial u}{\partial x} = 6x$

$\frac{\partial^2 u}{\partial x^2} = 6$ — (1)

and

$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y} (3x^2 - 3y^2 + 2y)$

$= -6\cancel{x}y + 2$

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$$\frac{\partial^2 u}{\partial y^2} = -6 \quad -(2)$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -6+6 = 0$$

hence by laplas equation given function is harmonic.

Now,

$$u = 3x^2 - 3y^2 + 2y$$

$$\frac{\partial u}{\partial x} = 6x \quad -(1) \qquad , \frac{\partial u}{\partial y} = -6y + 2 \quad -(2)$$

by CR equation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\therefore \frac{\partial v}{\partial y} = 6x$$

$$\partial v = 6x \, \partial y$$

integrating w.r.t y

$$v = 6xy + f(x)$$

$$\frac{\partial v}{\partial x} = 6y + f'(x) \quad -(1)$$

by CReqn.

$$-\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$

$$= \cancel{6y} - 2 = \cancel{6y} + f'(x)$$

$$=$$

$$f'(x) = 2$$

$f(x) = -2x + C$

$\therefore V = 6xy - 2x + c$

$f(z) = u + iv$

$= (3x^2 - 3y^2 + 2y) \quad + i(6xy - 2x)$

by milne thomson's theorem

put $x = z$

$y = 0$

$\boxed{f(z) = 3z^2 - i(2z) + c}$

Q10) find analytic function $f(z) = u + iv$ if

$u - v = (x-y)(x^2 + 4xy + y^2)$

$u - v = x^3 + 4x^2y + y^2x - x^2y - 4xy^2 - y^3$ ①

differting eqn ① partially w.r.t x

$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = 3x^2 + 8xy + y^2 - 2xy - 4y^2$ ②

differting eqn ② partially w.r.t y

$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial y} = 4x^2 + 2xy - x^2 - 8xy - 3y^2$ ③

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$$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} = 3x^2 - 3y^2 + 6xy \quad (1)$$

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial y} = 3x^2 - 6xy - 3y^2 \quad (2)$$

adding (1) + (2).

$$\cancel{\frac{\partial u}{\partial x}} - \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} - \cancel{\frac{\partial v}{\partial y}} = 6x^2 - 6y^2$$

by CR eqn

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\frac{\partial v}{\partial x} = -3x^2 + 3y^2$$

eqn (1) - (2)

$$2\frac{du}{dx} = -x^2 + 6xy + y^2 + x^2 + 6xy - y^2$$

$$2\frac{du}{dx} = 6xy$$

$$F'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

$f'(z) = 6xy + i(3y^2 - 3x^2)$

using milne thomson theorem,
put $x = z$, $y = 0$

$f'(z) = 0 + i(0 - 3z^2)$

$f'(z) = -i3z^2$

$f(z) = -6iz + c$

Q11) z plane pts $(1, i, -1)$
w plane pts $(i, 0, -1)$

$$\frac{(w-w_2)(w_1-w_3)}{(w-w_3)(w_1-w_2)} = \frac{(z-z_2)(z_1-z_3)}{(z-z_3)(z_1-z_2)}$$

$w_1 = i$

$w_2 = 0$

$$\frac{(w-0)(i-(-1))}{(w-(-1))(i-0)} = \frac{(z-i)(i-(-1))}{(z-(-1))(1-i)}$$

$$\frac{w}{w+1} \times \frac{i+1}{i} = \frac{z}{1-i} \times \frac{i}{1+i}$$

$$\frac{\cancel{w+1}}{w+1} \quad \frac{w+1-1}{w+1} = 1 - \frac{1}{w+1}$$

$$\frac{w+1-1}{w+1} = \frac{z-i}{z+i} \times \frac{2i}{1+1}$$

$$1 - \frac{1}{w+1} = \frac{(z-i)}{z+1} \times i$$

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$$\frac{1}{\omega+1} = 1 - \frac{(z-i)\cdot i}{z+1}$$

$$\frac{1}{\omega+1} = \frac{z+1-(iz+1)}{z+1}$$

$$\frac{1}{\omega+1} = \frac{z\cdot i}{z+1}$$

$$= \frac{z+1}{z(1-i)}$$

$$\omega = \frac{z+1}{z(1-i)} - 1$$

$$= \frac{z+1-z(1-i)}{z(1-i)}$$

$$= \frac{zi+1}{z(1-i)}$$

$$= \frac{iz+1}{z(1-i)} \times \frac{(1+i)}{(1+i)}$$

$$\frac{z(i-1) - i(i-1)}{2z}$$

$$\omega = \frac{(z-i)(i-1)}{2z}$$

Q(2) $w = \dfrac{2z-1}{2z+1}$

$(2z+1)w = 2z-1$

$2wz + w = 2z-1$

$2z(w-1) = -1-w$

$z = \dfrac{w+1}{2(1-w)}$

Put $z = x+iy$, $w = u+iv$

$x+iy = \dfrac{u+iv+1}{2(1-(u+iv))}$

$= \dfrac{1(1+u)+iv}{2[(1-u)-iv]} \times \dfrac{(1-u)+iv}{(1-u)+iv}$

$= \dfrac{1}{2} \dfrac{((1-u^2)+iv(1+u)+i(1-u)v-iv^2)}{v^2+(1-u)^2}$

$= \dfrac{1}{2}\left[\dfrac{1-u^2-v^2+i(v+uv+v-uv)}{v^2+(1-u)^2}\right]$

$= \dfrac{1}{2}\left[\dfrac{1-u^2-v^2+i(2v)}{v^2+(1-u)^2}\right]$

$x+iy = \dfrac{1}{2}\dfrac{1-u^2-v^2}{u^2+(1-u)^2} + i\dfrac{2v}{2(v^2+(1-u)^2)}$

by camparison,

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$x = \dfrac{1-u^2-v^2}{2(v^2+(1-u)^2}$

$y = \dfrac{v}{v^2+(1-u)^2}$

map the st. $2y = x$

$$\frac{2v}{v^2+(1-u)^2} = \frac{1-u^2-v^2}{2(u^2+(1-u)^2}$$

$$4v = 1-u^2-v^2$$

$$u^2+v^2+4v-1=0$$

Q13) $\displaystyle\int \frac{4z^2+2dz}{z^2-1}$ $c =$ circle $|z-1|^2 = \frac{1}{2}$

$\displaystyle\int f(z) = 2\pi i \sum \frac{f(z)}{\text{residue}}.$

$$\oint_c \frac{4z^2+2}{(z-1)(z+1)}$$

$z = -1$
$z = 1$

$z = -1$ lies outside the circle
$z = 1$ is inside the circle.

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$z=-1$ is not inside the circle.

$z=1$ is inside the circle.

$$\lim_{z \to -1} \frac{d}{dz}\left[\frac{2z^2+2z+1}{z-3}\right]$$

$$\lim_{z \to 1} \frac{\left((z-3)(4z+2) - (2z^2+4z+1)(1)\right)}{(z-3)^2}$$

$$\frac{(-1-3)(-4+2)-(2-4+1)}{(-4)^2}$$

$$\frac{8-1}{16}$$

$$= \frac{7}{16}.$$

$$\therefore \oint_C f(z).dz = 2\pi i \times \frac{7}{16}$$

$$= \frac{7\pi i}{8}.$$