Q. Find roots of the following equations correct to three decimal places using Gauss Seidal method.

$$2x - 3y + 20z = 25$$
$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$

Soln:

Rearrange eqn so that diagonal elements are bigger in magnitude.

$$20x + y - 2z = 17 \quad \text{——} \; (1)$$
$$3x + 20y - z = -18 \quad \text{——} \; (2)$$
$$2x - 3y + 20z = 25 \quad \text{——} \; (3)$$

(1) $\Longrightarrow$

$$x = \frac{1}{20}\left[17 - y + 2z\right]$$

(2) $\Longrightarrow$

$$y = \frac{1}{20}\left[-18 - 3x + z\right]$$

(3) $\Longrightarrow$

$$z = \frac{1}{20}\left[25 \cancel{+3y} - 2x + 3y\right]$$

If x^k, y^k, z^k is k^{th} app. then By Seidal method.

$$x^{k+1} = \frac{1}{20}\left[17 - y^k + 2z^k\right]$$

$$y^{k+1} = \frac{1}{20}\left[-18 - 3x^{(k+1)} + z^{(k)}\right]$$

$$z^{k+1} = \frac{1}{20}\left[25 - 2x^{k+1} + 3y^{k+1}\right]$$

Page No.:

Date: / /

$$x^1 = \frac{1}{20}\left[17 - 0 + 0\right]$$

$$= 0.85$$

$$y^1 = \frac{1}{20}\left[-18 - 3(0.85) + 0\right]$$

$$= -1.027$$

$$z^1 = \frac{1}{20}\left[25 - 2(0.85) + 3(-1.027)\right]$$

$$= 1.01$$

$$x^2 = \frac{1}{20}\left[17 - (-1.027) + 2(1.01)\right]$$

$$= 1.0023$$

$$y^2 = \frac{1}{20}\left[-18 - 3(1.0023) + 3(1.01)\right]$$

$$= -0.9998$$

$$z^2 = \frac{1}{20}\left[25 - 2(1.0023) + 3(-0.9998)\right]$$

$$= 0.9998$$

	k	x^k	y^k	z^k
	0	0	0	0
$x \approx 1$	1	0.85	-1.027	1.01
$y \approx -1$	2	1.00235	-0.9998	0.9998
$z \approx 1$	3	0.9998	-0.99986	1.000041
Final Ans.	4.	0.999997	-0.99999	1.0000045

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Page No.:

Date: / /

Euler's Method -

To solve

$$\frac{dy}{dx} = f(x, y)$$

with $y(x_0) = y_0$

and step length: h

$\Rightarrow$

$x_1 = x_0 + h$

find $f(x_0, y_0)$

By Euler's method.

$y_1 = y_0 + h f(x_0, y_0)$

find (x_1, y_1)

$y_2 = y_1 + h f(x_1, y_1)$

$$\boxed{y_{i+1} = y_i + h f(x_i, y_i)}$$

Step length: h

x_0	$y_0 = y(x_0)$
$x_1 = x_0 + h$	$y_1 = y(x_1)$
$x_2 = x_1 + h$	$y_2 = y(x_2)$
$x_i = x_{i-1} + h$	$y_i = y(x_i)$

Q. Using Euler's method, find approximate value of y when $x = 0.6$ of

$$\frac{dy}{dx} = 1 - 2xy$$ given that

at $x = 0$, $y = 0$

take $h = 0.2$.

Soln: Given,

$x_0 = 0$, $y_0 = 0$

& $h = 0.2$

$$\frac{dy}{dx} = 1 - 2xy$$

$f(x, y) = 1 - 2xy$

$y(0.6) = ?$

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Page No.:

Date: / /

$x_1 = x_0 + h = 0 + 0.2$

$= 0.2$

$y_1 = y(x_1) = y(0.2)$

By Euler's formula,

$y_1 = y_0 + h f(x_0 y_0)$

$h = 0.2, \quad x_0 = 0, \quad y_0 = 0$

$f(x_0 y_0) = 1 - 2x_0 y_0$

$= 1 - 2(0)(0)$

$= 1$

$y_1 = 0 + (0.2)(1)$

$y_1 = 0.2$

$y(0.2) = 0.2$

$f(x_1, y_1) = 1 - 2x_1 y_1$

$= 1 - 2(0.2)(0.2)$

$= 0.92$

$y(x_2) = y(x_1 + h)$

$= y(0.4)$

$= y_2$

By Euler's Method.

$y_2 = y_1 + h f(x_1 y_1)$

$= 0.2 + 0.2 [0.92]$

$y_2 = 0.384$

$f(x_2, y_2) = 1 - 2x_2 y_2$

$= 1 - 2(0.4 \times 0.384)$

$= 0.6928$

$y(x_3) = y(0.6)$

$= y_3$

$\therefore$ By Euler's Method,

$y_3 = y_2 + hf(x_2 y_2)$

$= 0.384 + 0.2\,(0.6928)$

$y_3 = \cancel{1.7696}\ 0.52256$

$$\boxed{y(0.6) = 0.52256}$$

Q. Using Euler's method, Find.

Soln: Given,

$$\frac{dy}{dx} = \frac{y-x}{y+x}$$

with boundary conditions,

$y = 1$ when $x = 0$

find $y(0.1)$ by Euler's method.

Consider $h = 0.1$.

$x_0 = 0$, $y_0 = 1$

$x_1 = x_0 + h = 0 + 0.1$

$= 0.1$

$y_1 = y(x_1) = y(0.1)$

Page No.:

Date: / /

By Eulers method

$$y_1 = y_0 + h f(x_0 y_0)$$

$h = 0.1$, $x_0 = 0$, $y_0 = 1$

$$f(x_0 y_0) = 1 - 2x_0 y_0$$
$$= 1 - 2(0)(1)$$
$$= 1$$

$$y_1 = 1 + 1(0.1)$$

$$\boxed{y_1 = 1.1}$$

Modified Eulers method :-

To solve.

$$\frac{dy}{dx} = f(x, y)$$

$y(x_0) = y_0$

$h =$ step length

Find $f(x_0 y_0)$

$x_1 = x_0 + h$

By Euler's method

$$y_1 = y_0 + hf(x_0 y_0)$$

Let $y_1 < y_1^{0}$

Find $f(x_1 y_1^{0})$

By Modified Euler's method 1st modification.

$(x_1, y_1^{(0)})]$

$.9091]$

Date: / /

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

find $f(x_1, y_1^{(1)})$

2nd modified value of y_1

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

3rd itration

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$

$$y_1 \approx y_1^{(k)}$$

$$x_2 = x_1 + h$$

$$y_2 = y_1 + h f(x_1, y_1)$$

Let,

$$y_2^{0} = y_2$$

find $(x_2\ y_2)$

By Euler's modified method

$$y_2^{1} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^{0})\right]$$

2nd modified value,

$$y_2^{(2)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^{(1)})\right]$$

Page No.:

Date: / /

Q. Given that

$$\frac{dy}{dx} = \frac{y-x}{y+x} \quad \text{with} \quad y(0)=1 \quad \text{and} \quad y(0.1)$$

By Euler's modified method by performing two itration each with $h = 0.05$

Sol^n:

$x_0 = 0 \quad , \quad y_0 = 1$

$h = 0.05$

$x_1 = x_0 + h$

$= 0.05$

$$f(x_0 y_0) = \frac{y_0 - x_0}{y_0 + x_0}$$

$$= \frac{1-0}{1+0}$$

$= 1$

By Eulers method,

$y_1 = y_0 + h\, f(x_0 y_0)$

$= 1 + (0.05)(1)$

$y_1 = 1.05$

$y_1^0 = 1.05$

$$f(x_1 y_1) = \frac{1.05 - 0.05}{1.05 + 0.05}$$

$= 0.9091$

By Modified Euler's method.

1st modification,

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0 y_0) + f(x_1 y_1^{(0)})\right]$$

$$= 1 + \frac{0.05}{2}\left[1 + 0.9091\right]$$

Date: / /

$$y_1^{(1)} = 1.0477$$

$$f(x_1, y_1^{(1)}) = \frac{1.0477 - 0.05}{1.0477 + 0.05}$$

$$= 0.9089$$

2nd modification

$$y_2^{(2)} = y_0 + \frac{h}{2}\left[f(x_0 y_0) + f(x_1, y_1^{(1)})\right]$$

$$= 1 + \frac{0.05}{2}\left[1 + 0.9089\right]$$

$$y_2^{(2)} = 1.0477$$

$$y_1 = 1.0477.$$

$$f(x_1, y_1) = \frac{y_1 - x_1}{y_1 + x_1}$$

$$= \frac{1.0477 - 0.05}{1.0477 + 0.05}$$

$$= 0.9089$$

$$x_2 = x_1 + h$$
$$= 0.05 + 0.05$$
$$= 1$$

By Eulers method,

$$y_2 = y_1 + h f(x_1, y_1)$$
$$= 1.0477 + 0.05(0.9089)$$

$$y_2^{\circ} = y_2 = 1.093145$$

$$f(x_2 y_2^0) = \frac{y_2^0 - x_2}{y_2^0 + x_2}$$

$$= \frac{1.093145 - 0.1}{1.09314 + 0.1}$$

$$= 0.8323$$

By Modified Euler's method

$$y_2^{(1)} = y_1 + \frac{h}{2}\left[f(x_1 y_1) + f(x_2 y_2^0)\right]$$

$$= 1.0477 + \frac{0.05}{2}\left[0.9089 + 0.8323\right]$$

$$y_2^{(1)} = 1.09123$$

$$f(x_2 y_2^{(1)}) = \frac{1.09123 - 0.1}{1.09123 + 0.1}$$

$$= 0.8323$$

2nd itration

$$y_2^{(2)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2 y_2^{(1)})\right]$$

$$= 1.0471 + \frac{0.05}{2}\left[0.9089 + 0.8323\right]$$

$$= 1.0912$$

$$y_2 = y_2^{(2)} = 1.0912$$

$$y(x_2) = y(0.1) = 1.0912$$

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Q. Using Euler modified method. Obtain a solution of the eqn.

$$\frac{dy}{dx} = x + |\sqrt{y}|$$

with initial conditions $y = 1$, at $x = 0$. For the value $0 \le x \le 0.6$ in steps of 0.2.

Q. Solve the following by Euler's modified method.

$$\frac{dy}{dx} = \log(x+y)$$

$$y(1) = 2$$

at $x = 1.2$ and 1.4 with $h = 0.2$

Soln: $f(x,y) = \ln(x+y)$

$x_0 = 1$, $y_0 = 2$

$f(x_0 y_0) = \ln(1+2) = \ln(3)$

$f(x_0 y_0) = 1.0986$

$x_1 = x_0 + h = 1 + 0.2 = 1.2$

By Euler's method

$y_1 = y_0 + hf(x_0, y_0)$

$y_1 = 2 + 0.2[1.0986]$

$y_1 = 2.2197 = y_1^{(0)}$

$f(x_0 y_1^0) = \ln(x_1 + y_1) = \ln(1.2 + 2.21972)$

$f(x_0 y_1^0) = 1.2295$

By Modified Euler's method

1st itration

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0 y_0) + f(x_1 y_1^{(0)})\right]$$

$$= 2 + \frac{0.2}{2}[1.0986 + 1.2295]$$

$$y_1^{(1)} = 2.23281$$

$f(x_1, y_1^{(1)}) = \ln(1.2 + 2.23281)$

$f(x_1, y_1^{(1)}) = \cancel{2.23}\ 1.23256$

2nd itration.

$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0 y_0) + f(x_1 y_1^{(1)})]$

$= 2 + \frac{0.2}{2}[1.0986 + 1.23256]$

$y_1 = y_1^{(2)} = 2.2331$

$y(1.2) = 2.2331$

$f(x_1, y_1) = \ln(1.2 + 2.2331)$

$= 1.23346$

$x_2 = x_1 + h = 1.2 + 0.2 = 1.4$

By Euler's method,

$y_2 = y_1 + h f(x_1 y_1)$

$y_2^0 = 2.2331 + (0.2)[1.23346]$

$= 2.4797$

$f(x_2 y_2^0) = \ln(1.4 + 2.4797)$

$= 1.35575$

By Modified Euler's

1st modification,

$y_2^{(1)} = y_1 + \frac{h}{2}[(f(x_1 y_1) + f(x_2 y_2^0)]$

$= 2.2331 + \frac{0.2}{2}[1.23319 + 1.35575]$

$y_2^{(1)} = 2.4921$

* Runge-kutta Fourth order Method -

T solve,

$$\frac{dy}{dx} = f(x, y);$$

$$y(x_0) = y_0$$

with Step h :

$$k_1 = hf(x_0 y_0)$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

$$k = \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

$$\boxed{y_1 = y_0 + k}$$

To find $y(x_1) = y_2$

$$k_1 = hf(x_1, y_1)$$

$$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_1 + h, y_1 + k_3)$$

$$k = \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

$$\boxed{y_2 = y_1 + k.}$$

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Q. Apply Runge-kutta fourth order method to find approximate value of y for $x = 0.2$ if in steps of 0.1 if

$$\frac{dy}{dx} = x + y^2$$

given that $y = 1$ when $x = 0$.

Soln: $f(x, y) = x + y^2$

$x_0 = 0, \quad y_0 = 1$

$h = 0.1$

$x_1 = x_0 + h = 0 + 0.1 = 0.1$

$k_1 = hf(x_0 y_0)$

$= 0.1 f(0, 1)$

$= 0.1 (0 + (1)^2)$

$\boxed{k_1 = 0.1}$

$k_2 = hf(x_0 + h/2, y_0 + k/2)$

$k_2 = 0.1 f\left(0 + \frac{0.1}{2}, 1 + \frac{0.1}{2}\right)$

$= 0.1 f(0.05, 1.05)$

$= (0.1)[0.05 + (1.05)^2]$

$\boxed{k_2 = 0.11525}$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= 0.1 f\left(0 + \frac{0.1}{2}, 1 + \frac{0.11525}{2}\right)$

$= 0.1 f(0.05 + 1.057625)$

$\boxed{k_3 = 0.11685}$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= 0.1 f(0 + 0.1, 1 + 0.11685)$

$k_4 = 0.1 [0.1 + (1.11685)^2] = 0.13473$

Page No.:
Date: / /

$$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= \frac{1}{6}[0.1 + 2(0.11525) + 2(0.11685) + 0.13473]$$

$$\boxed{k = 0.1165}$$

$y_1 = y_0 + k$

$y_1 = 1 + 0.1165$

$y(0.1) = y_1 = 1.1165$

$x_2 = x_1 + h = 0.1 + 0.1 = 0.2$

$k_1 = h f(x_1, y_1)$

$= 0.1 f(0.1, 1.1165)$

$= 0.1 [0.1 + (1.1165)^2]$

$$\boxed{k_1 = 0.1347}$$

$$k_2 = h f\left(x_1 + \frac{h}{2}, y_1 + \frac{k}{2}\right)$$

$$= 0.1 f\left(0.1 + \frac{0.1}{2}, 1.1165 + \frac{0.1347}{2}\right)$$

$$\boxed{k_2 = 0.1551}$$

$$k_3 = h f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$= 0.1 f\left(0.1 + \frac{0.1}{2}, 1.1165 + \frac{0.1551}{2}\right)$$

$$\boxed{k_3 = 0.1576}$$

$k_4 = h f(x_1 + h, y_1 + k_3)$

$= 0.1 f(0.1 + 0.1, 1.1165 + 0.1576)$

$$\boxed{k_4 = 0.1823}$$

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Page No.:
Date: / /

$$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= \frac{1}{6}[0.1347 + 2(0.1551) + 2(0.1576) + 0.1823]$$

$$k = 0.15705$$

$$y_2 = y_1 + k$$

$$y_2 = 1.1165 + 0.15705$$

$$\boxed{y_2 = 1.27355}$$

Q. Using Runge kutta fourth order method calculate $y(0.2)$ given

$$\frac{dy}{dx} = \frac{2xy}{1+x^2} = 1$$

$$y(0) = 0$$

Sol^n.,

$$\frac{dy}{dx} = 1 + \frac{2xy}{1+x^2}$$

$$f(x,y) = 1 + \frac{2xy}{1+x^2}$$

$$x_0 = 0, \quad y_0 = 0$$

$$h = 0.2$$

$$k_1 = hf(x_0 y_0)$$

$$= 0.2 f(0,0)$$

$$= 0.2\left[1 + \frac{2(0)(0)}{1+0}\right]$$

$$= 0.2[1+0]$$

$$\boxed{k_1 = 0.2}$$

Page No.:
Date: / /

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$= 0.2 f\left(0 + \frac{0.2}{2}, 0 + \frac{0.2}{2}\right)$$

$$= 0.2 f(0.1, 0.1)$$

$$= 0.2\left[1 + \frac{2(0.1)(0.1)}{1+(0.1)^2}\right]$$

$k_2 =$

H.W.

Q. Using Runge kutta fourth order method solve

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$$

with $y(0) = 1$ at $x = 0.2, 0.4$

* Solution of Algebric & Transendental eqn:-

To solve.

$f(x) = 0$

* Newton Raphison Method:-

To solve,

$f(x) = 0$

Let $x = x_0$ be initial root

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

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Q. Using Newton Raphson method. Find approximate root of $3x = \cos x + 1$ correct to three decimal places.

Soln: $f(x) = 3x - \cos x - 1$

$f'(x) = 3 + \sin x$

By Newton Raphson method.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Let, $x_0 = 0$ be

$f(x_0) = f(0) = 0 - \cos 0 - 1$

$f(0) = -2$

$f'(x_0) = f'(0) = 3 + \sin 0$

$= 3$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

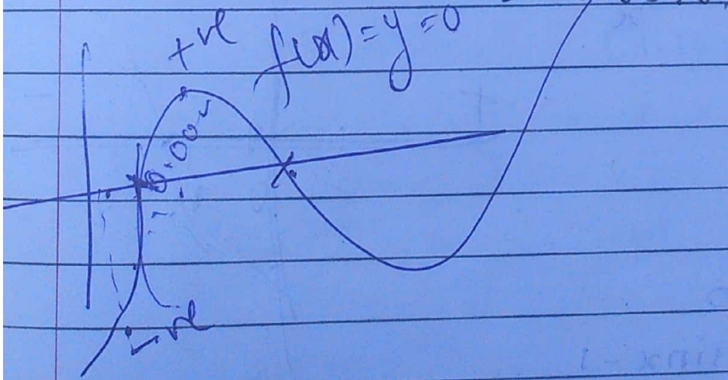
$$= 0 - \frac{-2}{3}$$

$$x_1 = \frac{2}{3} = 0.667$$

~~E~~ $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$$= 0.667 - \frac{0.2153}{3.6186}$$

$$= 0.6076$$



Page No.:

Date: / /

* Bisection Method :-

- To find root of $f(x) = 0$
- Find x_0 & x_1 such that, $f(x_0)$ & $f(x_1)$ has opposite signs.
- Let $a = x_0$ & $b = x_1$

$f(a) = -ve$; $f(b) = +ve$.

By Bisection method

$$x_2 = \frac{a+b}{2}$$

If $f(x_2)$ & $f(x_1)$ has opp. sign then root lies between x_2 & x_1

$a = x_2$; $b = x_1$

$$x_3 = \frac{a+b}{2}$$

Else if $f(x_0)$ & $f(x_2)$ has opp. sign then root lies betn x_0 & x_2.

Let $a = x_0$ & $b = x_2$

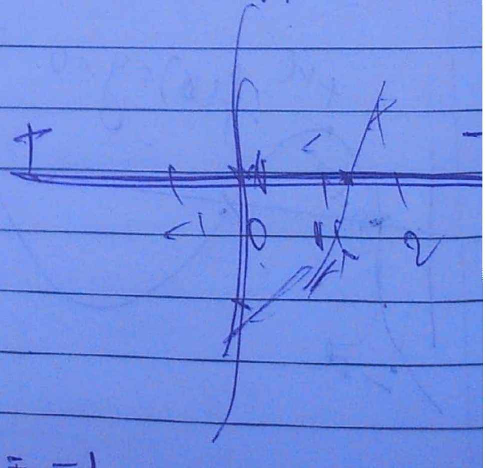
$$x_3 = \frac{a+b}{2}$$

* Examples -

(1) Using Bisection method. Find approximate root of $\sin x = \frac{1}{x}$

Soln.

$$\sin x = \frac{1}{x}$$



$\Rightarrow x\sin x = 1$

$x\sin x - 1 = 0$

$f(x) = x\sin x - 1$

$f(0) = 0\sin 0 - 1 = -1$

$f(1) = 1\sin 1 - 1$

$x \rightarrow$ 1.125 1.25 1.5 2

$f(x) \rightarrow$ − + + +

Page No.:

Date:

$f(1) = -0.1585$

$f(2) = 2\sin(2) - 1$; $f(2) = 0.8185$

$\therefore$ $f(1)$ & $f(2)$ are of opp. sign

Root lies betn 1 & 2

$a = 1$, $b = 2$

By Bisection method

$$x_2 = \frac{a+b}{2} = \frac{1+2}{2} = \underline{1.5}$$

$f(x_2) = f(1.5)$

$f(1.5) = 1.5\sin(1.5) - 1$

$f(1.5) = 0.4962$ (+ve)

& $f(1)$ is '−ve'

$\therefore$ Root lies betn 1 & 1.5

$a = 1$, $b = 1.5$

By Bisection method

$$x_3 = \frac{a+b}{2} = \frac{1+1.5}{2} = \underline{1.25}$$

$f(1.25) = 1.25\sin(1.25) - 1$

$= 0.186$ (+ve)

$f(1)$ is (−ve)

Root lies betn 1 and 1.25

$\therefore$ By Bisection method

$$x_4 = \frac{1+1.25}{2} = \underline{\underline{1.125}}$$

$f(x_4) = (1.125)\sin(1.125) - 1$

$= 0.0154$ (+ve)

Root lies betn 1 & 1.125

$$x_5 = \frac{1+1.125}{2} = \underline{\underline{1.0625}}$$

$f(1.0625) = (1.0625)\sin(1.0625) - 1$

$= -0.0712$ (−ve)

$f(1.125)$ is +ve

Page No.:
Date: / /

Root lies betn 1.0625 & 1.125

$$x_6 = \frac{a+b}{2} = \frac{1.0625 + 1.125}{2} = 1.093$$

* Regula False Method :-

To find approximate root of $f(x) = 0$
find x_0 & x_1 such that $f(x_0)$ & $f(x_1)$ are of opposite sign.

By Regula Method

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0)$$

If $f(x_2)$ & $f(x_1)$ are opposite sign.

Root lies betn x_1 & x_2

$x_0 = x_1$ & $x_1 = x_2$

Else,

$f(x_2)$ & $f(x_0)$ are of opp. root lies between x_0 & x_2

Q1. Find the real roots of the eqn.

$$x^3 - 2x + 5 = 0$$

Soln :

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