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Q) find CF

① $\frac{d^4y}{dx^4} - 2\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = \sin x$

$(D^4 - 2D^3 + D^2)y = \sin x$

AE is

$D^4 - 2D^3 + D^2 = 0$

$D^2(D^2 - 2D + 1) = 0$

$D^2 D(D-1)(D-1) = 0$

$D = 0, 0, 1, 1$

$y_c = e^{0x}(c_1 + c_2 x) + e^{1x}(c_3 + c_4 x)$

② $(D^4 - m^4)y = 0$

AE is

$D^4 - m^4 = 0$

let $D^2 = t$

$t^2 = m^4$

$t = \pm m^2$

$D^2 = \pm m^2$

$D^2 = m^2$	$D^2 = -m^2$
$D = \pm m$	$D = \pm mi$

$D = m, -m, mi, -mi$.

$\therefore$

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CF is

$y_c = e^{mx} + e^{-mx} + e^{0x}(c_3 \cos mx + c_4 \sin mx)$

$y_c = e^{mx} + e^{-mx} + c_3 \cos x + c_4 \sin x$

a) find PI of

① $(D^2 - 6D + 9)y = e^{3x}$

$PI = \frac{1}{f(D)} x$

$= \frac{1}{(D^2-6D+9)} e^{3x}$

if we put $D=3$ $D^2-6D+9=0$

$\therefore PI = \frac{x e^{3x}}{2D-6}$

$PI = \frac{x^2 e^{3x}}{2}$ (diffentiating)

② $(D^2+1)y = \cos x$

$PI = \frac{1}{D^2+1} \cdot \cos x$

if we put $D^2 = -1^2$

$\therefore -1^2+1=0$

$=$

$\frac{x}{2D} \cdot \cos x$ $\frac{1}{D}$ = integration

$PI = \frac{x}{2} \sin x$

③ $(D^2-1)y = x^3$

$$PI = \frac{1}{(D^2-1)} x^3$$

$$= \frac{-1}{1-D^2} \cdot x^3$$

$$= -[1-D^2]^{-1} x^3$$

$$= -[1+D^2 \ldots] x^3$$

$$= [-1-D^2] x^3$$

$$= -x^3 - 6x$$

$$= -(x^3+6x)$$

Q) Slove

$$\frac{d^2y}{dx^2} - 4y = (1+e^x)^2 + 3$$

$$D^2y - 4y = 1 + e^{2x} + 2e^x + 3$$

$$(D^2-4)y = e^{2x} + 2e^x + 4$$

AE

$$D^2 - 4 = 0$$

$$D^2 = 4$$

$$D = \pm 2$$

$$y_c = c_1 e^{2x} + c_2 e^{-2x}$$

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$PI = \frac{1}{f(D)} x$

$= \frac{1}{(D^2-4)} e^{2x} + \frac{1}{(D^2-4)} 2e^{x} + \frac{4}{(D^2-4)} e^{0x}$

$= \frac{x}{2D} e^{2x} \quad 2\frac{1}{(1-4)} e^{x} + \frac{4}{0-4}$

$\frac{x}{2}\int e^{2x}.dx - \frac{2}{3}e^{x} - 1$

$PI = \frac{x}{4} e^{2x} - \frac{2}{3} e^{x} - 1$

$\therefore y = CC + PI$

$= c_1 e^{2x} + c_2 e^{-2x} + \frac{x e^{2x}}{4} - \frac{2}{3}e^{x} - 1$

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Tutorial 3

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Q) $x^2 \frac{d^2y}{dx^2} - 2y = x^2 + \frac{1}{x}$

put $x = e^z$

$z = \log x$

$\therefore x^2 \frac{d^2y}{dx^2} = D(D-1)y$

$\therefore D(D-1)y - 2y = e^{2z} + e^{-z}$

$(D^2 - D - 2)y = e^{2z} + e^{-z}$

$\therefore$ AE $= D^2 - D - 2 = 0$

$(D+1)(D-2)$

$D = -1, \quad D = 2$

$\therefore CF = c_1 e^{-z} + c_2 e^{2z}$ --- (1)

$PI = \frac{1}{(D^2 - D - 2)} (e^{2z} + e^{-z})$

$\frac{1}{D^2 - D - 2} (e^{2z}) + \frac{1}{D^2 - D - 2} (e^{-z})$

$$\frac{z}{2D-1} e^{2z} + \frac{z}{2D-1} e^{-z}$$

put $D=2)$ and put $D=(-1)$

$$\frac{z\, e^{2z}}{4-1} + \frac{z}{-2-1} e^{-z}$$

$$\frac{z \cdot e^{2z}}{3} - \frac{z e^{-z}}{3} \quad \text{--- (2)}$$

from (1) & (2)

$$y = c_1 e^{-z} + c_2 e^{2z} + \frac{z}{3} e^{2z} - \frac{z}{3} e^{-z}$$

$$y = c_1 x^{-1} + c_2 x^2 + \frac{\log x}{3} \cdot x^2 - \frac{\log x}{3} \cdot x^{-1}$$

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Q2) $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\log(1+x)$

put $1+x = e^z$

$z = \log(1+x)$

$\frac{dz}{dx} = \frac{1}{1+x}$

$\therefore \frac{dy}{dz} = \frac{1}{(1+x)} \frac{dy}{dz}$

$(1+x)\frac{dy}{dx} = Dy$ where $D = \frac{d}{dz}$

similarly

$(1+x)^2 \frac{d^2y}{dx^2} = D(D-1)y$

$\therefore$ given eqn ① becomes.

$D(D-1)y + Dy + y = 4z$

$(D^2+1)y = 4z$

$\therefore$

$AE = D^2+1 = 0$

$D = \pm i$

$y_z = \cos z + \sin z$ --- (2)

Now,

$PI = \frac{1}{(D^2+1)} 4z$

$= 4 \frac{1}{(D^2+1)} z$

$= 4 (1+D^2)^{-1} z$

$= 4 (1-D^2+\ldots) z$

$= 4z$

$\therefore PI = 4z$ --- (3)

from (2) & (3)

$y = \cos z + \sin z + 4z$

$y = \cos \log(1+x) + \sin \log(1+x) + 4 \log(1+x)$

Q3) $\frac{dx}{dt} - wy = a\cos pt$

$\frac{dy}{dt} + wx = a\sin pt$

let $\frac{d}{dt} = D$

$\therefore$ $Dx - wy = a\cos pt$ -- ①

$Dy + wx = a\sin pt$ -- ②

eqn ① multiply D and eqn ② by w

$\therefore$ $D^2x \cancel{- Dwy} = D\, a\cos pt$

$\cancel{wDy} + w^2x = w\, a\sin pt$

$D^2x + w^2x = -ap\sin pt + aw\sin pt$

$(D^2+w^2)x = aw\sin pt - ap\sin pt$

AE

$D^2+w^2=0$

$D^2 = -w^2$

$D = \pm wi$

$y_x = c_1\cos wt + c_2\sin wt$ -- ③

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$$PI = \frac{1}{f(D)} x$$

$$= \frac{1}{(D^2+\omega^2)} (a\omega \sin pt - ap \sin pt)$$

$$PI = a\omega \frac{1}{D^2+\omega^2} (\sin pt) - ap \frac{1}{D^2+\omega^2} (\sin pt)$$

$$= \frac{a\omega}{-p^2+\omega^2} \sin pt - \frac{ap}{-p^2+\omega^2} \sin pt$$

$$= \frac{a \sin pt (\omega - p)}{\omega^2 - p^2}$$

$$PI = \frac{a \sin pt (\omega - p)}{\omega^2 - p^2} \quad \text{--} (4)$$

$$x = c_1 \cos \omega t + c_2 \sin \omega t + \frac{a \sin pt (\omega - p)}{\omega^2 - p^2} \quad \text{-} (5)$$

putting in eqn (1)

$$\frac{d}{dt} (c_1 \cos \omega t + c_2 \sin \omega t - \frac{a \sin pt (\omega - p)}{\omega^2 - p^2}) - a \cos pt = \omega y$$

$$-c_1 \omega \sin \omega t + c_2 \omega \cos \omega t + \frac{\cos pt (\omega - p) ap}{\omega^2 - p^2} - a \cos pt = y$$

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$$c_2 \cos\omega t - c_1 \sin\omega t + \frac{\cos pt \cdot ap}{\omega(\omega+p)} = y$$

$$\therefore y = c_2 \cos\omega t - c_1 \sin\omega t + \frac{ap}{\omega(\omega+p)} \cos pt$$

Tutorial 4

Q) find the fourier cosine transform of

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 < x < 2 \\ 0 & x > 2 \end{cases}$$

fourier cosine transform

$$f(x) = \frac{2}{\pi}\int_0^\infty f_c(\lambda) . \cos\lambda x . d\lambda$$

$$f_c(\lambda) = \int_0^\infty f(u) . \cos\lambda u . du$$

$$= \int_0^1 u\cos\lambda u . du + \int_1^2 2\cos\lambda u . du - \int_1^2 u\cos\lambda u . du$$

$$= \left[\frac{u\sin\lambda u}{\lambda}\right]_0^1 - \int_0^1 \frac{\sin\lambda u}{\lambda} + \left[\frac{2\sin\lambda u}{\lambda}\right]_1^2 - \left[\frac{u}{\lambda}\sin\lambda u\right]_1^2$$

$$+ \int_1^2 \frac{\sin\lambda u}{\lambda} du$$

$$= \frac{\sin\lambda u}{\lambda} + \left[\frac{\cos\lambda u}{\lambda^2}\right]_0^1 + \frac{2\sin 2\lambda}{\lambda} - \frac{2\sin\lambda}{\lambda}$$

$$- \frac{2\sin 2\lambda}{\lambda} + \frac{\sin\lambda}{\lambda} - \frac{\cos\lambda u}{\lambda^2}$$

$$- \frac{\sin\lambda}{\lambda}$$

$$= \frac{\sin\lambda}{\lambda} - \frac{\cos\lambda}{\lambda^2} - \frac{1}{\lambda^2}$$

$$- \frac{\cos 2\lambda}{\lambda^2} + \frac{\cos\lambda}{\lambda^2}$$

$$= \frac{2\cos\lambda - \cos 2\lambda - 1}{\lambda^2} \quad -①$$

$$\therefore \quad f(x) = \frac{2}{\pi}\int_0^\infty f_c(\lambda).\cos\lambda x.d\lambda$$

$$= \frac{2}{\pi}\int_0^\infty \left(\frac{2\cos\lambda - \cos 2\lambda - 1}{\lambda^2}\right) \cos\lambda x.d\lambda$$

Q2) find fouriers sine transform of e^{-3x} $x \geq 0$

$$f(x) = \frac{2}{\pi}\int_0^\infty f_s(\lambda).\cos\lambda x.d\lambda$$

$$f_s(\lambda) = \int_0^\infty e^{-3u}.\sin\lambda u.du$$

$$= \left[\frac{e^{-3u}}{(-3)^2+\lambda^2}\left[-3\sin\lambda u - \lambda\cos\lambda u\right]\right]_0^\infty$$

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$$\left[\frac{-e^{-3u}}{\lambda^2+9}\left(3\sin\lambda u+\lambda\cos\lambda u\right)\right]_0^\infty$$

$$\frac{1}{\lambda^2+9}\cdot\lambda$$

$$=\frac{\lambda}{\lambda^2+9}$$

Q3) find fourier Transform of $f(x)\begin{cases} x^2 & |x|\le 1 \\ 0 & |x|>1 \end{cases}$

$$f(x)\int_{-\infty}^{\infty} f(u).\,e^{-i\lambda u}.du$$

$$=\int_{-1}^{1} u^2 e^{-i\lambda u}.du$$

as $f(-x)=(-x^2)$
$=x^2$

$f(x)=x^2$

as $f(x)=f(-x)$
$\therefore$ give function is even for $-1<x<1$

$$\therefore f(\lambda)=\int_0^1 u^2\cos\lambda u.du$$

$$F(\lambda) = \left[\frac{u^2 \sin\lambda u}{\lambda} - 2u\left(\frac{-\cos\lambda u}{\lambda^2}\right) + 2\left(\frac{-\sin\lambda u}{\lambda^3}\right)\right]_0^1$$

$$F(\lambda) = \frac{\sin\lambda}{\lambda} + \frac{2\cos\lambda}{\lambda^2} - \frac{2\sin\lambda}{\lambda^3}$$

Q4) Find fourier sine Transform of $e^{-3x}\cos^2 hx$

$\therefore$ f(x) $\cosh x = \dfrac{e^{2x}+e^{-2x}}{2}$

$$\therefore f(x) = e^{-3x}\cos^2 hx = e^{-3x}\left(\frac{e^{2x}+e^{-2x}}{2}\right)$$

$$= \frac{e^{-x}}{2} + \frac{e^{-5x}}{2}$$

Now,

$$F_s(\lambda) = \int_0^\infty f(u)\cdot\sin\lambda u . du$$

$$\int_0^\infty \frac{e^{-u}}{2}\sin\lambda u . du + \int_0^\infty e^{-5x}\sin\lambda u . du$$

$$= \frac{1}{2}\left[\frac{e^{-u}}{(-1)^2+\lambda^2}(-\sin\lambda u - \lambda\cos\lambda u)\right]_0^\infty$$

$$+ \frac{1}{2}\left[\frac{e^{-5u}}{(-5)^2+\lambda^2}[-5\sin\lambda u - \lambda\cos\lambda u]\right]_0^\infty$$

$$= \frac{1}{2}\left[\frac{-e^{-u}}{\lambda^2+1}(\sin\lambda u + \lambda\cos\lambda u)\right]_0^\infty$$

$$+ \frac{1}{2}\left[\frac{-e^{-5u}}{\lambda^2+25}(5\sin\lambda u + \lambda\cos\lambda u)\right]_0^\infty$$

$$= \frac{1}{2}\left[\frac{1}{\lambda^2+1}\times\lambda + \frac{1}{2}\,\frac{\lambda}{\lambda^2+25}\right]$$

$$= \frac{\lambda}{2}\left[\frac{1}{\lambda^2+1} + \frac{1}{\lambda^2+25}\right]$$

∴ fourier sine transform

$$F_s(\lambda) = \frac{\lambda}{2}\left[\frac{1}{\lambda^2+1} + \frac{1}{\lambda^2+25}\right]$$

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Tutorials 5

Q1) In inductor of 0.5 Henries is connected in series with resister of 6 ohms, a capacitor of 0.02 farads, a generator having alternative voltage given by 24 sin 10t, t ≥ 0 and switch k.

1) Set up diffential equation for the instenteniour charge on the condensor

2) find charge and current at time t when charge on capacitor is zero when the switch k is closed at t=0

→ $L\frac{dI}{dt} + RI + \frac{Q}{C} = E_0 \sin t$

$\frac{d^2Q}{dt^2} + \frac{R}{L} I + \frac{Q}{LC} = \frac{E_0}{L} \sin t$

Here

$L = 0.5H$

$R = 6\Omega$

$C = 0.02F$

$\frac{d^2Q}{dt^2} + 12 \cdot \frac{dQ}{dt} + 100\,Q = 48 \sin 10t$

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$(D^2 Q + 12DQ + 100Q) = 48 \sin 10t$

$AE = D^2 + 12D + 100 = 0$

$$D = \frac{-12 \pm \sqrt{144 - 400}}{2}$$

$$= \frac{-12 \pm \sqrt{-256}}{2}$$

$D = -6 \pm 8i$

$$\therefore y_c = e^{-6t}[c_1 \cos 8t + c_2 \sin 8t] \quad \text{---} (1)$$

Now

$$PI = \frac{1}{D^2 + 12D + 100}(48 \sin 10t)$$

put $D^2 = -10^2$

$$= \frac{1}{\cancel{-100} + 12D + \cancel{100}}(48 \sin 10t)$$

$$= 4 \frac{1}{D} \sin 10t$$

$$= 4 \int \sin 10t$$

$$= \frac{-4}{10} \cos 10t$$

$$PI = \frac{-2}{5} \cos 10t \quad \text{----} (2)$$

Q2)

$\therefore$
$$Q = e^{-6t}[c_1\cos 8t + c_2\sin 8t] - \frac{2}{5}\cos 10t$$

given that $t=0, Q=0$

$$0 = c_1 - \frac{2}{5}$$

$$\therefore c_1 = \frac{2}{5} \quad --- (3)$$

when $c=0, t=0, \frac{dQ}{dt}=0$

$$\therefore \frac{dQ}{dt} = e^{-6t}\left[\frac{-16}{5}\sin 8t + 8c_2\cos 8t\right] + \frac{20}{5}\sin 10t + (-6).e^{-6t}\left[\frac{2}{5}\cos 8t + c_2\sin 8t\right]$$

$$t=0 \qquad \frac{dQ}{dt}=0.$$

$$0 = [8c_2] + (-6)\left(\frac{2}{5}\right)$$

$$\boxed{c_2 = \frac{3}{10}}$$

$$Q = e^{-6t}\left[\frac{2}{5}\cos 8t + \frac{3}{10}\sin 8t\right] - \frac{2}{5}\cos 10t$$

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Q2) ~~An uncharged~~ ~~condensor~~

Now we find I

$$\frac{dQ}{dt} = I = e^{-6t}\left[-\frac{16}{5}\sin 8t + 8 \times \frac{3}{10}\cos 8t\right]$$

$$+ \frac{20}{5}\sin 10t + (-6)\, e^{-6t}\left[\frac{2}{5}\cos 8t + \frac{3}{10}\sin 8t\right]$$

$$I = e^{-6t}\left[-\frac{16}{5}\sin 8t + \frac{24}{10}\cos 8t\right] + 4\sin 10t$$

$$-6\, e^{-6t}\left[\frac{2}{5}\cos 8t + \frac{3}{10}\sin 8t\right]$$

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Q2) An uncharged condensor of capacity C charged by appling on emf of value $E \sin \frac{t}{\sqrt{LC}}$ through the leads of inductance L and neglegible resistance. charge Q on the plate of condensor satisfies the differential equation

$$\frac{d^2Q}{dt^2} + \frac{Q}{LC} = \frac{E}{L} \sin \frac{t}{\sqrt{LC}}$$

prove that charge at time t is given by

$$Q = \frac{EC}{2}\left[\sin \frac{t}{\sqrt{LC}} - \frac{t}{\sqrt{LC}} \cos \frac{t}{\sqrt{LC}}\right]$$

given,

$$\frac{d^2Q}{dt^2} + \frac{Q}{LC} = \frac{L}{C} \sin \frac{t}{\sqrt{LC}}$$

$$D^2Q + \frac{1}{LC} Q = \frac{L}{C} \sin \frac{t}{\sqrt{LC}}$$

AE is

$$D^2 + \frac{1}{LC} = 0$$

$$D^2 = \frac{-1}{LC}$$

$$D = \pm \frac{1}{\sqrt{LC}} i$$

$$\therefore y_c = c_1 \cos \frac{t}{\sqrt{LC}} + c_2 \sin \frac{t}{\sqrt{LC}} \quad \text{--- ①}$$

Now

$$PI = \frac{1}{f(D)} X$$

$$= \frac{1}{\left(D^2 + \frac{1}{LC}\right)} \cdot \frac{E}{L} \sin\left(\frac{t}{\sqrt{LC}}\right)$$

$$PI = \frac{t}{2D} \cdot \frac{E}{L} \sin\left(\frac{t}{\sqrt{LC}}\right) \cdot \sqrt{LC}$$

$$= \frac{L}{C} \cdot \frac{t}{2D} \sin\left(\frac{t}{\sqrt{LC}}\right)$$

$$= -\frac{E}{L} \cdot \frac{t}{2} \cos \frac{t}{\sqrt{LC}} \sqrt{LC}$$

$$\cancel{PI = -\frac{L\sqrt{L}}{2\sqrt{C}} t \cos \frac{t}{\sqrt{LC}}} \quad \text{--- ②}$$

$$\therefore Q = c_1 \cos \frac{t}{\sqrt{LC}} + c_2 \sin \frac{t}{\sqrt{LC}} - \frac{E}{2L} t \cos \frac{t}{\sqrt{LC}} \quad \text{--- ③}$$

at $Q = 0,\ t = 0$

$$\therefore 0 = c_1 \cos\left(\frac{0}{\sqrt{LC}}\right) + c_2 \sin\left(\frac{0}{\sqrt{LC}}\right) - \frac{E}{2L} \cdot 0 \cos\left(\frac{0}{\sqrt{LC}}\right)$$

$$0 = c_1 \times 1 + c_2 \times 0 - 0$$

$$\boxed{\therefore c_1 = 0}$$

$\frac{d\theta}{dt} = 0$ $\theta = 0,$ $t = 0$

differentiating eqn (3) w.r.t t we get,

$$\frac{d\theta}{dt} = \frac{d}{dt}\left(c_2 \sin\frac{t}{\sqrt{LC}} - \frac{E\sqrt{C}}{2\sqrt{L}} t \cos\frac{t}{\sqrt{LC}} \right)$$

$$= c_2 \cos\frac{t}{\sqrt{LC}} \times \frac{1}{\sqrt{LC}} - \frac{E\sqrt{C}}{2\sqrt{L}}\left[\cos\frac{t}{\sqrt{LC}} + t\left(-\sin\frac{t}{\sqrt{LC}} \cdot \frac{1}{\sqrt{LC}}\right)\right]$$

put $\frac{d\theta}{dt} = 0$, $t = 0$ we get,

$$0 = \frac{c_2}{\sqrt{LC}} \cos\left(\frac{0}{\sqrt{LC}}\right) - \frac{L\sqrt{L}}{2\sqrt{C}} \cos\frac{t}{\sqrt{LC}} - \frac{L\sqrt{L}}{2\sqrt{2}} \cdot t\left(-\sin\frac{t}{\sqrt{LC}}\right) \cdot \frac{1}{\sqrt{LC}}$$

$$0 = \frac{c_2}{\sqrt{LC}} \times 1 - \frac{L\sqrt{L}}{2\sqrt{C}} \times 1 - 0$$

$$0 = \frac{c_2}{\sqrt{LC}} - \frac{L\sqrt{L}}{2\sqrt{C}}$$

$$\frac{L\sqrt{L}}{2\cancel{\sqrt{C}}} = \frac{c_2}{\sqrt{L}\cdot\cancel{\sqrt{C}}}$$

$$c_2 = \frac{L\sqrt{L}\sqrt{L}}{2}$$

$$\boxed{c_2 = \frac{L^2}{2}}$$

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$$\therefore Q = \frac{L^2}{2}\sin\frac{t}{\sqrt{LC}} - \frac{L\sqrt{L}}{2\sqrt{C}} \cdot t\cos\left(\frac{t}{\sqrt{LC}}\right)$$

$$= \frac{L^2}{2}\sin\frac{t}{\sqrt{LC}} - \frac{L^2}{2\sqrt{LC}}\, t\cos\left(\frac{t}{\sqrt{LC}}\right)$$

$$= \frac{L^2}{2}\left[\sin\frac{t}{\sqrt{LC}} - \frac{1}{\sqrt{LC}} \cdot t \cdot \cos\left(\frac{t}{\sqrt{LC}}\right)\right]$$

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$$\frac{d^2 Q}{dt^2} + \frac{Q}{LC} = \frac{E}{L}\sin\left(\frac{t}{\sqrt{LC}}\right)$$

$$\left(D^2 + \frac{1}{LC}\right) Q = \frac{E}{L}\sin\left(\frac{t}{\sqrt{LC}}\right)$$

Auxillary eqn is

$$D^2 + \frac{1}{LC} = 0$$

$$D^2 = \frac{-1}{LC}$$

$$D = \pm \frac{i}{\sqrt{LC}}$$

Roots are imaginary and distinct.

$$CF = e^0 \left[c_1 \cos\left(\frac{t}{\sqrt{LC}}\right) + c_2 \sin\left(\frac{t}{\sqrt{LC}}\right)\right]$$

Now

$$PI = \frac{1}{\left(D^2 + \frac{1}{LC}\right)} \left[\frac{E}{L}\sin\left(\frac{t}{\sqrt{LC}}\right)\right]$$

$$= \frac{tE}{L}\frac{1}{2D}\left[\sin\left(\frac{t}{\sqrt{LC}}\right)\right]$$

$$= \frac{tE}{2L}\left[-\cos\left(\frac{t}{\sqrt{LC}}\right)\right]\sqrt{LC}$$

Q) 1

$$Q = c_1 \cos\left(\frac{t}{\sqrt{LC}}\right) + c_2 \sin\left(\frac{t}{\sqrt{LC}}\right] - \frac{E}{2L} t\sqrt{LC} \cos\left(\frac{t}{\sqrt{LC}}\right]$$

at $t=0,\ Q=0$

$$\boxed{0=c_1}$$

$$\frac{dQ}{dt} = \frac{-1}{\sqrt{LC}} c_1 \sin\left(\frac{t}{\sqrt{LC}}\right) + \frac{1}{\sqrt{LC}} c_2 \cos\left(\frac{t}{\sqrt{LC}}\right)$$

$$- \frac{E}{2L}\sqrt{LC}\left[-\frac{1}{\sqrt{LC}} \sin\left(\frac{t}{\sqrt{LC}}\right) + \cos\left(\frac{t}{\sqrt{LC}}\right)\right.$$

at $t=0,\ \dot{Q}=0$.

$$0 = \frac{c_2}{\sqrt{LC}} - \frac{E}{2L}\sqrt{LC}$$

$$c_2 = \frac{E}{2L} LC$$

$$\therefore Q = \frac{ELC}{2L} \sin\left(\frac{t}{\sqrt{LC}}\right) - \frac{Et}{2L}\sqrt{LC} \cos\left(\frac{t}{\sqrt{LC}}\right)$$

$$\frac{EC}{2} \sin\left(\frac{t}{\sqrt{LC}}\right) - \frac{Et\sqrt{C}}{2\sqrt{L}} \cos\left(\frac{t}{\sqrt{LC}}\right)$$

$$Q = \frac{EC}{2}\left[\sin\left(\frac{t}{\sqrt{LC}}\right) - \frac{t}{\sqrt{LC}} \cos\left(\frac{t}{\sqrt{LC}}\right)\right]$$

Tutorial 6.

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Q) 1 find fourier sine transform of $\frac{e^{-x}}{x}$ $x>0$

Fourier sine transform

$$F_S(\lambda) = \int_0^\infty \frac{e^{-u}}{u} \sin\lambda u \, .du$$

$$I = \int_0^\infty \frac{e^{-u}}{u} \sin\lambda u.du$$

$$I' = \int_0^\infty \frac{\partial}{\partial\lambda} \left(\frac{e^{-u}}{u} \sin\lambda u.du\right)$$

$$= \int_0^\infty \frac{e^{-u}}{\cancel{u}} \cos\lambda u \, . \cancel{u}$$

$$= \int_0^\infty e^{-u}. \cos\lambda u \, . du$$

$$\left[\frac{e^{-u}}{1+\lambda^2}\left[-1\cos\lambda u + \lambda\sin\lambda u\right]\right]_0^\infty$$

$$0 - \left[\frac{1}{1+\lambda^2}\left[-1 \; +0\right]\right]$$

$$I' = \frac{1}{1+\lambda^2}$$

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$$\frac{d(I')}{d\lambda} = \frac{d}{d\lambda}\left(\frac{1}{1+\lambda^2}\right)$$

$$I(\lambda) = \tan^{-1}(\lambda) + A$$

put $\lambda = 0$

$$0 = \tan^{-1} 0 + A$$

$$A = 0$$

$\therefore$ $I(\lambda) = \tan^{-1}(\lambda)$

fourier ~~cosi~~ sine transform is $\tan^{-1}(\lambda)$

Q2) find fourier sine transform of e^{-x} $x \geq 0$

prove that

$$\int_0^\infty \frac{\lambda \sin m\lambda}{1+\lambda^2} \, d\lambda = \frac{\pi}{2} e^{-m}$$

$$F_s(\lambda) = \int_0^\infty f(u) . \sin \lambda u . du$$

$$= \int_0^\infty e^{-u} . \sin \lambda u . du$$

$$\cancel{\left[\frac{e^{-u}}{1+\lambda^2} \left[-1 \cos \lambda u - \lambda \sin \lambda u \right] \right]_0^\infty}$$

$$\cancel{- \left[\frac{e^{-u}}{1+\lambda^2} \left[\cos \lambda u + \lambda \sin \lambda u \right] \right]_0^\infty}$$

$$\cancel{- \left[0 - \left[\frac{\lambda (1)}{1+\lambda^2} \right] \right]}$$

$$\frac{\lambda}{1+\lambda^2}$$

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$$Fs(\lambda) \int_0^\infty e^{-u} \sin\lambda u \, du$$

$$Fs(\lambda) = \frac{\lambda}{1+\lambda^2}$$

Now, fourier sine integral

$$f(x) = \frac{2}{\pi} \int_0^\infty fs(\lambda) \sin\lambda x \, d\lambda$$

$$= \frac{2}{\pi} \int_0^\infty \frac{\lambda}{1+\lambda^2} \sin\lambda x \, d\lambda$$

put $x = m$

$$f(x) = e^{-m}$$

$$\cancel{f(x)} = \frac{2}{\pi} \int_0^\infty \frac{\lambda}{1+\lambda^2} \sin m\lambda \, d\lambda$$

when $x = 0$, $f(x) = 1$

$$e^{-m} = \frac{2}{\pi} \int_0^\infty \frac{\lambda}{1+\lambda^2} \sin m\lambda \, d\lambda$$

$$\frac{2e^{-m}}{\pi} = \int_0^\infty \frac{\lambda}{1+\lambda^2} \sin m\lambda \, d\lambda$$

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Q3) Slove integral equation ①

$$\int_0^\infty f(x)\sin\lambda x . dx = \begin{cases} 1-\lambda & 0 \le \lambda \le 1 \\ 0 & \lambda \ge 1 \end{cases}$$

LHS is fourier sine transform

$$F_s(\lambda) = \frac{2}{\pi}\int_0^\infty f(u)\sin\lambda u\, du$$

$$= \frac{2}{\pi}\int_0^1 (1-\lambda)\sin\lambda x . d\lambda + \frac{2}{\pi}\int_1^\infty 0$$

Corresponding fourier integral

$$f(x) = \frac{2}{\pi}\int_0^\infty F_s(\lambda)\ \sin\lambda x . d\lambda$$

$$\text{as } F_s(\lambda) = \int_0^\infty f(u).\sin\lambda u\ .du$$

$$\therefore f(x) = \frac{2}{\pi}\int_0^1 (1-\lambda)\sin\lambda x . d\lambda + \frac{2}{\pi}\int_1^\infty 0$$

$$= \frac{2}{\pi}\left[(1-\lambda)\left(\frac{-\cos\lambda x}{x}\right) - (-1)\left(\frac{-\sin\lambda x}{x}\right)\right]_0^1$$

$$F(x) = \frac{2}{\pi}\left[\frac{-\sin x}{x^2} + 1\right]$$

$$\frac{2}{\pi}\left[\frac{x^2 - \sin x}{x^2}\right]$$

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Tutorial 7

find z transform of

① $f(k) = \left(\frac{1}{3}\right)^{|k|}$

$Z\{f(k)\} = F(z) = \sum_{-\infty}^{\infty} f(k).z^{-k}$

from $-\infty$ to -1 $\quad |k| = -k$

0 to ∞ $\quad |k| = k$

$\sum_{-\infty}^{-1} \left(\frac{1}{3}\right)^{-k} z^{-k} + \sum_{0}^{\infty} \left(\frac{1}{3}\right)^{k} z^{-k}$

$\frac{z}{3-z} + \frac{3z}{3z-1}$

ROC

$\left|\frac{z}{3}\right| < 1 \; ; \; \left|\frac{1}{3z}\right| < 1$

② $\frac{5^k}{k!}, \; k \geq 0$

$Z\{f(k)\} = \int_{-\infty}^{\infty} f(k) \cdot z^{-k}$

$= \int_{0}^{\infty} \frac{5^k}{k!} z^{-k}$

$e^{5/k}$

(3) $2^k \cos(3k+2)$ $\qquad k \geq 0$

$f(k) = 2^k \cos(3k+2)$

~~$z\{f(x)\} = \int_0^\infty f(k) \cdot e^{-k}$~~

$=$

$z\{f(k)\} = \sum_{-\infty}^{\infty} f(k) \cdot z^{-k}$

$= \sum_{-\infty}^{\infty} 2^k \cos(3k+2)$

$= 2^k \cos 3k \cdot \cos 2 - 2^k \sin 3k \cdot \sin 2$

$= \cos 2 \dfrac{z(z-\cos 3)}{z^2-2z\cos 3-1}$

$= \dfrac{z\sin 3 \sin 2 + z^2\cos 2 - z\cos 2\cos 3}{z^2-2z\cos 3-1}$

ROC $|z|>1$

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(6) $(k+1)\left(\frac{1}{3}\right)^k \quad k \geq 0$

we know that.

$$(1+k)\cdot a^k = \frac{z^2}{(z-a)^2} \quad |z|>|a|$$

$$z[f(k)] = \frac{z^2}{\left(z-\frac{1}{3}\right)^2}$$

$$z[f(k)] = \frac{9z^2}{(3z-1)^2} \quad |z|>|1/3|$$

) $\cos \frac{\pi k}{2} \quad k \geq 0$

we know that,

$$\cos\alpha k = \frac{z^2 - 2z\cos\alpha}{z^2-2z\cos\alpha+1} \quad k \geq 0$$

$$z[f(k)] = \frac{z^2 - z\cos(\pi/2)}{z^2 - 2z\cos(\pi/2)+1}$$

$$= \frac{z^2}{z^2+1}$$

ROC $|z|>1$

we know

$$\cos\alpha k = \frac{z^2 - z\cos\alpha}{z^2 - 2z\cos\alpha + 1}$$

$$z\{f(k)\} = \frac{z^2 - z\cos\pi}{z^2 - 2z\cos(\pi) + 1}$$

$$= \frac{z^2 + z}{z^2 + 2z + 1}$$

$$= \frac{z(z+1)}{(z+1)^2}$$

$$z\{f(k)\} = \frac{z}{z+1}$$

(7) $k\,5^k,\ k \geq 0$

$$z\{f(k)\} = -z\frac{d}{dz}\{F(z)\}$$

$$F(z) = \frac{z}{z-5} \qquad |z| > 5$$

$$\therefore z\{f(k)\} = -z\frac{d}{dz}\left(\frac{z}{z-5}\right)$$

$$= -z\left[\frac{(z-5)(1) - z(1)}{(z-5)^2}\right]$$

$$= \frac{-z[\cancel{z}-5-\cancel{z}]}{(z-5)^2}$$

$$\frac{5z}{(z-5)^2} \qquad |z|>5$$

$2^k \sinh 3k \qquad k \geq 0$

$$Z\{a^n \sinh n\theta\} = \frac{a z \sinh\theta}{z^2 - 2a z\cosh\theta + a^2}$$

$$= \frac{2z \sinh 3}{z^2 - 4z\cosh 3 + 4}$$

$u(k) \qquad k \geq 0$

$f(k) = 1$

$$Z\{u(k)\} = \frac{z}{z-1} \qquad |z|>1$$

$nc_k \qquad 0 \leq k \leq n$

$$Z\{nc_k\} = (1+z^{-1})^n \qquad |z|>0$$

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Tutorial 8

(Q) the scores obtained by two batsman A and B in 10 matches are given below Determine which batsman is more consistent.

A	30	44	66	62	60	34	80	46	20	38
B	34	46	70	38	55	48	60	34	45	30

x	y	x^2	y^2
30	34	900	1156
44	46	1936	2116
66	70	4356	4900
62	38	3844	1444
60	55	3600	3025
34	48	1156	2304
80	60	6400	3600
46	34	2116	1156
20	45	400	2025
38	30	1444	900

$\Sigma x = 480$ $\Sigma y = 460$ $\Sigma x^2 = 26152$ $\Sigma y^2 = 22626$

$$SD_x = \sqrt{\left(\frac{1}{n}\Sigma x^2\right) - (\bar{x})^2}$$

$$= \sqrt{\frac{1}{10}(26152) - (48)^2}$$

$$= \sqrt{2615.2 - (48)^2}$$

$$= \sqrt{311.2}$$

$$= 17.64$$

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for
the coefficient of variance $= \frac{SD}{mean} \times 100$

$= \frac{17.64}{48} \times 100$

$= 36.75$ -①

$SD_y = \sqrt{\frac{1}{n}(\Sigma y)^2 - (\bar{y})^2}$

$= \sqrt{\frac{1}{10}(22626) - (46)^2}$

$= \sqrt{2262.6 - 2116}$

$= \sqrt{146.6}$

$= 12.107$

∴ for B
Coefficient of variance $= \frac{SD}{mean} \times 100$

$= \frac{12.107}{46} \times 100$

$= 26.32$ -②

from ① & ② B is more consistant than A

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Q2> find the coefficient of skewness and kurtosis, mean and standard deviation of following data.

$n=100$, $\Sigma fd = 50$ & $\Sigma fd^2 = 1970$ $\Sigma d^3 f = 2958$
$\Sigma fd^4 = 86{,}752$
where $d = x - 48$

$B_1 = \frac{\mu_3^2}{\mu_2^3}$ $B_2 = \frac{\mu_4}{\mu_2^2}$

$\mu_1' = \frac{1}{N} \Sigma fd = \frac{1}{100} \times 50 = 0.5$

$\mu_2' = \frac{1}{N} \Sigma fd^2 = \frac{1}{100} \times 1970 = 19.7$

$\mu_3' = \frac{1}{N} \Sigma fd^3 = \frac{1}{100} \times 2958 = 29.58$

$\mu_4' = \frac{1}{N} \Sigma fd^4 = \frac{1}{100} \times 86752 = 867.52$

$\therefore \mu_1 = 0$

$\mu_2 = \mu_2' - \mu_1'^2$

$= 19.7 - (0.5)^2$

$\mu_2 = 19.45$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$

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$= 29.48 - 29.55 + 0.25$

$\mu_3 = 0.18$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1)^4$

$= (867.52) - 4(29.48)(0.5) + 6(19.7)(0.5)^2 - 3(0.5)^4$

$= 876.52 - 58.96 + 14.775 - 0.1575$

$= 837.54$

$\therefore$ coefficient of skewness $\beta_1 = \dfrac{\mu_3^2}{\mu_2^3}$

$= \dfrac{(0.18)^2}{(19.45)^3}$

$= 4.403 \times 10^{-6}$

coefficient of skewness $\beta_2 = \dfrac{\mu_4}{(\mu_2)^2}$

$= 2.214$

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Q3) find the co-relation coefficient from following data:

$N = 10 \quad \Sigma X = 300 \quad \Sigma y = 250 \quad \Sigma(X-30)^2 = 154$

$\Sigma(Y-25)^2 = 162 \quad \Sigma(x-30)(y-25) = 144$

→ $SD\,x = \sigma_x = \sqrt{\frac{1}{N}\Sigma (X-\bar{X})^2}$

$= \sqrt{\frac{1}{10} \times 154}$

$= 3.92$

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~~$r = cov(x,y) = \sigma y =$~~

$(SD)_y = \sigma_y = \sqrt{\frac{1}{N}(y-\bar{y})^2}$

$\sigma_y = \sqrt{\frac{1}{10}(162)}$

$\sigma_y = 4.02$

$cov(x,y) = \frac{1}{n}\Sigma(x-\bar{x})(y-\bar{y})$

$= \frac{1}{10} \times 144$

$= 14.4$

$r = \frac{cov(x,y)}{\sigma_x \sigma_y} = \frac{14.4}{3.92 \times 4.02}$

$r = 0.9138$

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Tutorial 9:

a) Slove the following by Jacobi method

$13x+5y-3z+u=18$

$2x+12y+z-4u=13$

$x-4y+10z+u=29$

$2x+y-3z+9u=31$

the given equation in standard form.

$x=\frac{1}{13}[18-u+3z-5y]$

$y=\frac{1}{12}[13+4u-z-2x]$

$z=\frac{1}{10}[29-u+4y-x]$

$u=\frac{1}{9}[31+3z-y-2x]$

initially $x=0, y=0, z=0, u=0$

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$\therefore x = \frac{1}{13} \times (18 - 0 + 0 - 0) = 1.3846$

$y = \frac{(13 + 0 + 0 + 0)}{12} = 1.0833$

$z = \frac{29(0 + 0 + 0)}{10} = 2.9000$

$u = \frac{31(0 + 0 + 0)}{9} = 3.4444$

2nd iteration: put $x = 1.3846$, $y = 1.0833$, $z = 2.9000$, $u = 3.4444$

$x_1 = \frac{1}{13}[18 - 3.4444 + 3 \times 2.9 - 5 \times 1.0833] = 1.3722$

$y_1 = \frac{1}{12}[13 + 4 \times 3.4444 - 2.9 - 2 \times 1.3846] = 1.7589$

$z_1 = \frac{1}{10}[29 - 3.444 + 4 \times 1.0833 - 1.3846] = 2.8605$

$u_1 = \frac{1}{9}[31 + 3 \times 2.900 - 1.0833 - 2 \times 1.3846] = 3.9831$

3rd iteration

Put

$x = 1.3722$, $y = 1.7589$, $z = 2.8505$, $u = 3.9831$

$$x_2 = \frac{1}{13}(18 - 3.9831 + 3 \times 2.8505 - 5 \times 1.7589)$$

$$= 1.0595$$

$$y_2 = \frac{1}{12}(13 + 6 \times 3.9831 - 2.8505 - 2 \times 1.3722)$$

$$= 1.9998$$

$$z_2 = \frac{1}{10}(29 - 3.9831 + 4 \times 1.7589 - 1.3722)$$

$$= 3.0680$$

$$u_2 = \frac{1}{9}(31 + 3 \times 2.8505 - 1.7589 - 2 \times 1.3722)$$

$$= 3.8942$$

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Q2) slove by Guass seidal method:

$28x + 3y - z = 32$
$x + 3y + 10z = 24$
$2x + 17y + 4z = 35$

arranging equation.

$28x + 3y - z = 32$ — ①
$2x + 17y + 4z = 35$ — ②
$x + 3y + 10z = 24$ — ③

~~in 1st equation put $y=0, z=0$.~~

$x = \frac{1}{28}[32 + z - 3y]$ — ①

$y = \frac{1}{17}[35 - 4z - 2x]$ — ②

$z = \frac{1}{10}[24 - 3y - x]$ — ③

put $y=0, z=0$ in eqⁿ ①.

$x_1 = \frac{32}{28} = 1.1429$

put $x = 1.1429$ and $z = 0$ in eqn ②

$$y = \frac{1}{17}[35 - 4\times 0 - 2\times 1.1429]$$

$y_1 = 1.9244$

Put $x = 1.1429$, $y = 1.9244$ in eqn ③

$$z = \frac{1}{10}[24 - 3\times 1.9244 - 1.1429]$$

$z_1 = \cancel{1.9089}\ 1.7084$

$x = 1.1429$, $y = 1.9244$, $z = \cancel{1.9089}\ 1.7084$

in eqn ① Put $y = 1.9244$, $z = 1.9089$

$$x_2 = \frac{1}{28}[32 + 1.9089 - 3\times 1.9244]$$

$\boxed{x_2 = 1.0048}$

put $x = 1.0048$, $z = 1.9089$ in eqn ②.

$$y_2 = \frac{1}{17}[35 - 4\times 1.9089 - 2\times 1.0048]$$

$\boxed{y_2 = 1.4915}$

1st iteration

$$x_1 = \frac{1}{28}[32 + 1.7089 - 3 \times 1.9244]$$

$$\boxed{x_1 = 0.9977}$$

$$y_1 = \frac{1}{17}[35 - 4 \times 1.7089 - 2 \times 0.9977]$$

$$\boxed{y_1 = 1.5394}$$

$$z_1 = \frac{1}{10}[24 - 3 \times 1.5394 - 0.9977]$$

$$\boxed{z_1 = 1.8390}$$

2nd iteration.

$$x_2 = \frac{1}{28}[32 + 1.8390 - 3 \times 1.5395]$$

$$\boxed{x_2 = 1.0436}$$

$$y_2 = \frac{1}{17}[35 - 4 \times 1.8390 - 2 \times 1.0436]$$

$$\boxed{y_2 = 1.5033}$$

$$z_2 = \frac{1}{10}[24 - 3 \times 1.5033 - 1.0436]$$

$$\boxed{z_2 = 1.8947}$$

Tutorial 9

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Q) using eulers method approximate $y(0.6)$ given $\frac{dy}{dx} = \frac{y-x}{y+x}$ $y(0)=1$

in step of 0.2

→ $f(x) = \frac{y-x}{y+x}$

	x	y
$y_0 = 1$	$x_0 = 0$	y_0
$x_0 = 0$	$x_1 = x_0 + h = 0.2$	y_1
$h = 2$	$x_2 = x_1 + h = 0.4$	y_2
	$x_3 = x_2 + h = 0.6$	y_3

using eulers method.

$y_{n+1} = y_n + h f(x_n, y_n)$

$y_1 = y_0 + h f(x_0, y_0)$

$y_1 = 1 + 0.2\ f(0,1)$

$1 + 0.2 \times 1$

$1 + 0.2$

$\boxed{y_1 = 1.20}$

2^{nd} iteration

$x_1 = x_0 + h = 0.2$

$y_2 = y_1 + h f(x_1, y_1)$

$y_2 = 1.2 + 0.2\ f(0.2, 1.2)$

$= 1.2 + 0.2 \left(\frac{1.2 - 0.2}{1.2 + 0.2}\right)$

$\boxed{y_2 = 1.3429}$

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3rd iteration $x_2 = x_1 + h = 0.4$

$y_3 = y_2 + h\, f(x_2, y_2)$

$y_3 = 1.3429 + 0.2\ f(0.4, 1.3429)$

$= 1.3429 + 0.2 \times 0.5410$

$= 1.3429 + 0.1082$

$y_3 = 1.4511$

Q2) given $f(x) = 2 + \sqrt{xy}$ and $y = 1$ at $x = 1$ find approximate vallu of $y(1.2)$ using modified eulers method perform 3 el'iteration each.

$f(x) = 2 + \sqrt{xy}$

$y_0 = 1$

$x_0 = 1$

assume $h = 0.2$

using eulers modified method.

$$y_{n+1}^{m+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1}^{m})\right]$$

$f(x_0, y_0) = 2 + \sqrt{1.1}$

$= 2+1$

$f(x_0, y_0) = 3$

$x_1 = 0.2 + 1 = 1.2$

by eulers method

$y_1 = y_0 + h f(x_0, y_0)$

$y_1 = 1 + 0.2 \times 3$

$= 1 + 0.6$

$y_1 = 1.6$

$f(x_1, y_1^0) =$

$2 + \sqrt{1.6 \times 1.2}$

3.3856

let $y_1 = y_1^0 = 1.6$

$\therefore$ using eules modified method

$y_1^{(1)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(0)})$

$y_1^{(1)} = 1 + \frac{0.2}{2} [3 + 3.3856]$

$y_1^{(1)} = 1.6386$

2^{nd} modification.

$f(x_1, y_1^{(1)}) = 2 + \sqrt{1.6386 \times 1.2}$

$= 3.4022$

$y_1^{(2)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(2)})]$

$= 1 + 0.1 [3 + 3.4022]$

$y_1^{(2)} = 1.6402$

3^{rd} modification

$$f(x_1, y_1^{(2)}) = 2 + \sqrt{1.6402 \times 1.2}$$

$$= 3.4030$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)}) \right]$$

$$= 1 + 0.1 \left[3 + 3.4030 \right]$$

$$y_1^{(3)} = 1.6403$$

$\therefore$

x_0	y
$x_0 = 1$	$y_0 = 1$
$x_1 = x_0 + h = 1.2$	$y_1 = 1.6403$

Q3) apply Rengy Kutta 4th order method to approximate y(0.2) given that

$$\frac{dy}{dx} = 1 + \frac{2xy}{1+x^2} \qquad y(0) = 0$$

→ $x_0 = 0$

$y_0 = 0$

assume $h = 0.2$

$$f(x) = \frac{1+2xy}{1+x^2}$$

according to 4th order Runge Kutta metha

$$y(x+h) = y(x) + K$$

$$K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

here $x_0 = 0$

$x_1 = x_0 + h$

$x_1 = 1.2$

$y_1(1.2) = y(0) + K$ --- (1).

$K_1 = h\, f(x_0, y_0)$

$= 0.2\, f(0,0)$

$= 0.2 \times 1$

$\boxed{K_1 = 0.2}$

$k_2 = h\, f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= 0.2\, f(0.1, 0 + 0.1)$

$= 0.2\, f(0.1, 0.1)$

$\boxed{k_2 = 0.20396}$

$k_3 = h\, f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= 0.2\, f\left(0.1, 0 + \frac{0.20396}{2}\right)$

$\boxed{k_3 = 0.204038}$

$k_4 = h\, f(x_0 + h, y_0 + k_3)$

$h\, f(0.2, 0.204038)$

$= 0.2\, f(0.2, 0.204038)$

$\boxed{k_4 = 0.21569}$

$\therefore k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0.2 + 2 \times 0.20396 + 2 \times 0.204038 + 0.21569]$

$\boxed{k = 0.205281}$

$\therefore y_1 = y_0 + k$

$= 0 + 0.205281$

$\boxed{y_1 = 0.205281}$

$\boxed{y(0.2) = 0.205281}$

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Tutorial 11

Q)

Show that $u = y^3 - 3x^2y$ is harmonic function find harmonic consecutive 'v' determine the anylytic function $f(z) = u + iv$ in terms z.

→ $u = y^3 - 3x^2y$

$\frac{\partial u}{\partial x} = -6xy$

$\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x}$

$\partial v = -6xy\, \partial y$

$\int \partial v = \int -6xy . \partial y + f(x)$

$v = \frac{-6xy^2}{2} + f(x)$

$v = -3xy^2 + f(x)$ — ①

$\frac{\partial u}{\partial y} = 3y^2 - 3x^2$ —② & $\frac{\partial v}{\partial x} = -3y^2 + f'(x)$ ③

by CR equation

$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

$3y^2 - 3x^2 = +\cancel{3y^2} - f(x)$

$f'(x) = 3x^2$

4x1

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$\int f'(x) = \int 3x^2$

$f(x) = x^3$ -- (2).

$v = -3xy^2 + f(x)$

$v = -3xy^2 + x^3$ -- (3)

$u = y^3 - 3x^2y$

$u_x = -6xy$

$u_{xx} = -6y$ - (1)

Now

$u_y = 3y^2 - 3x^2$

$u_{yy} = 6y$ - (2)

Now

$u_{xx} + u_{yy} = 6y - 6y = 0$

∴ by Laplas transfoom

$u_{xx} + u_{yy} = 0$

∴ given function is harmonic.

$u = y^3 - 3x^2 y$

$\frac{\partial u}{\partial x} = -6xy$

$\frac{\partial u}{\partial y} = 3y^2 - 3x^2$

Now,

$$\partial v = \frac{\partial v}{\partial x} . \partial x + \frac{\partial v}{\partial y} . \partial y$$

$$= \left(\frac{-\partial u}{\partial y}\right) \partial x + \left(\frac{\partial u}{\partial x}\right) \partial y$$

$\cancel{\partial v = (-3y^2 3x^2) \partial x + (-6xy) \partial y}$

$\cancel{\partial v = -3y^2 \partial x - 6xy \partial y}$

$\cancel{\int \partial v = -\int 3y^2 . \partial x - \int 6xy . \partial y}$

$= -3y^2$

$\cancel{\partial v = -\int (3y^2 - 3x^2) \partial x + \int -6xy . \partial y}$

$\cancel{dv = -[3y^2 - 3x^2] dx + [-6xy] dy}$

exact.

$\cancel{dv = [-3y^2 + 3x^2] dx + -6xy . dy}$

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$$u = y^3 - 3x^2y$$

$$\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x}$$

$$\partial v = \int (-6xy)\,\partial y$$

$$v = -3xy^2 + f(x) \quad -① $$

Now By CR equation

$$\frac{\partial u}{\partial y} = 3y^2 - 3x^2$$

&

$$\frac{\partial v}{\partial x} = -3y^2 + f'(x)$$

$$\therefore \frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x}$$

$$\therefore 3y^2 - 3x^2 = 3y^2 - f'(x)$$

$$f'(x) = 3x^2$$

$$f(x) = x^3 + c$$

$$\therefore v = x^3 - 3xy^2 + c$$

Now

Put $x = z$, $y = 0$

$$F(z) = u + iv$$

$$F(z) = y^3 - 3x^2 + (x^3 - 3xy^2 + c)\,i$$

$$F(z) = z^3 i + ci$$

Q2) if $f(z) = u+iv$ is an analytic function find $f(z)$ if $u+v = e^{-x}(\cos y - \sin y)$

$$u+v = e^{-x}(\cos y - \sin y)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = -e^{-x}(\cos y - \sin y) \quad -①$$

differentiating w.r.t y

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} = e^{-x}(-\sin y - \cos y) \quad -②$$

using CR equation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} = -e^{-x}(\sin y + \cos y) \quad -③$$

① + ③

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$$2\frac{\partial u}{\partial x} = -e^{-x}\cos y + e^{-x}\sin y - e^{-x}\sin y - e^{-x}\cos y$$

$$2\frac{\partial u}{\partial x} = -2e^{-x}\cos y$$

$$\frac{\partial u}{\partial x} = -e^{-x}\cos y$$

eqⁿ ① - ③

$$2\frac{\partial v}{\partial x} = -e^{-x}\cos y + e^{-x}\sin y + e^{-x}\sin y + e^{-x}\cos y$$

$$\frac{\partial v}{\partial x} = e^{-x}\sin y$$

$f(z) = u + iv$

$$= \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

$$= e^{-x}\cos y + i\, e^{-x}\sin y$$

usining milie Theomson

$x = z$

$y = 0$

$f'(z) = -e^{-z}$

$f(z) = e^{-z} + c.$

Tutorial 12

Q 1) evaluate
$$\int_C \frac{2z^2+z+5}{\left(z-\frac{3}{2}\right)^2} dz$$

where C is ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$

$$\int \frac{f(z)\,dz}{(z-a)^{n+1}} = \frac{2\pi i}{n!} f^n(a)$$

here $n=1$, $a=3/2$

$$f(3/2) = \left[\frac{d}{dz}[2z^2+z+5]\right]_{z=3/2}$$

$$= [4z+1]_{z=3/2}$$

$$= 6+1$$

$$f(3/2) = 7$$

$$\therefore \int \frac{2z^2+z+5}{(z-3/2)^2} dz = \frac{2\pi i}{1!} \times 7$$

$$= 14\pi i$$

Q2) evaluate $\int \frac{2z^2+zz+1}{(z+1)^3(z-3)}$ where c is circle

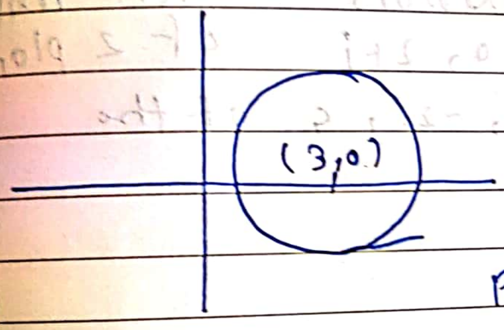
$|z-3|=2$

given circle

$|z-3|=2$

$|x+iy-3|=2$

$(x-3)^2+y^2=4$



Pole $z=-1$
is outside the circle

formula

$$\int_c f(z)dz = 2\pi i \sum \text{Res } f(z)$$

residue at $z=3$

$$\lim_{z\to3} (z-3) \frac{(2z^2+2z+1)}{(z+1)^3 (z-3)}$$

$$= \frac{2\times9+6+1}{4^3}$$

$$= \frac{25}{64}$$

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$$\int_C f(z)\,dz = 2\pi i \times \sum \text{Residue } f(z)$$

$$= 2\pi i \times \frac{25}{64}$$

$$\int_C f(z)\,dz = \frac{25\pi i}{32}$$

Q3) find bilinear transformation which maps the point $-i, 0, 2+i$ of z plane onto ~~po~~ points $0, -2i, 4$ of the w-plane.

→ $w = \dfrac{a+bz}{c+dz}$

if $z = -i$ $\quad w = 0$

$0 = a + b(-i)$

$\therefore a = ib$

$$b = \frac{a}{i}$$

if $z = 0$

$w = -2i$

$$w = \frac{a+bz}{c+dz}$$

$\therefore -2i = \frac{a}{c}$

$a = -2ic$

$c = \frac{a}{-2i}$

if $z = 2+i$ $\quad w = 4$

$w = \frac{a+bz}{c+dz}$

$\cancel{w = z+i \quad + w} = 4$

$4 = \frac{a+b(2+i)}{c+d(2+i)}$

$4c + 4d(2+i) = a + b(2+i)$

we know

$c = \frac{a}{-2i}$ & $b = \frac{a}{i}$

$\therefore 4\left(\frac{-a}{2i}\right) + 4d(2+i) = a + \frac{a}{i}(2+i)$

$\therefore -\frac{2a}{i} + (8+4i)d = a + \frac{2a}{i} + a$

$= 2a + \frac{2a}{i} + \frac{2a}{i}$

$(8+4i)d = \frac{2a+4a}{i}$

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$$d = \frac{2ai + 4a}{i(8+4i)}$$

$$= \frac{2a(i+2)}{4(2i-1)}$$

$$= \frac{a(i+2)}{2(2i-1)}$$

$$\therefore \omega = \frac{a+bz}{c+dz}$$

$$\frac{a + \frac{a}{i}(z)}{\frac{a}{-2i} + \frac{a}{2}\left(\frac{i+2}{2i-1}\right)z}$$

$$\omega = \frac{\frac{2(i+2)}{i}}{\left(\frac{-i}{i} + \left(\frac{2+i}{2i-1}\right)z\right)}$$

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Tutorial 13

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Show that $u = x^3 - 3xy^2 + 3x^2 - 3y^2$ is harmonic. find the harmonic conjugate, v of u such that

$f(z) = u + iv$ analytic determine $f(z)$ interms of z.

→ To show the function is harmonic

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2 + 6x$$

$$\frac{\partial^2 u}{\partial x^2} = 6x + 6 \quad -①$$

$$\frac{\partial u}{\partial y} = -6xy - 6y$$

$$\frac{\partial^2 u}{\partial y^2} = -6x - 6 \quad -②.$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -6x + 6x + 6 - 6 = 0$$

∴ given function is harmonic.

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Now

$u = x^3 - 3xy^2 + 3x^2 - 3y^2$

$$\left.\begin{aligned} \frac{\partial u}{\partial x} &= \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} &= -\frac{\partial v}{\partial x} \end{aligned}\right\} \text{by CR equation}$$

Now,

$\frac{\partial u}{\partial x} = 3x^2 - 3y^2 + 6x$

$\therefore \frac{\partial v}{\partial y} = 3x^2 - 3y^2 + 6x$ by CR equation

$v = \int 3x^2 - 3y^2 + 6x + f(x)$

$v = 3x^2y - y^3 + 6xy + f(x)$

$\frac{\partial v}{\partial x} = 6xy + 6y + f'(x)$

by CR equation

$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

$\cancel{-6xy} - 6y - f'(x) = \cancel{-6xy} - 6y$

$f'(x) = 0$

x = constant

$f(x)$ = constant.

let $f(x) = c$.

$$v = 3x^2y - y^3 + 6xy + c$$

$\therefore$

$$f(z) = u + iv$$

$$(x^3 - 3xy^2 + 3x^2 - 3y^2) + i(3x^2y - y^3 + 6xy + c)$$

by milnethonson,

put $x = z$

$y = 0$

$$f(z) = (z^3 + 3z^2) + i(c)$$

$$f(z) = z^3 + 3z^2 + ic$$

alternative.

$$v = \frac{\partial v}{\partial x} \cdot dx + \frac{\partial v}{\partial y} \cdot dy$$

by CR equation

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

$$\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x}$$

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$$dv = \frac{-\partial u}{\partial y} \cdot dx + \frac{\partial u}{\partial x} \cdot dy$$

$$dv = -\left(-6xy - 6y\right) dx + 3x^2 - 3y^2 + \ldots$$

by using exact.

put values

$$\frac{\partial u}{\partial y} = -6xy - 6y$$

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2 + 6x$$

using

$$v = \int_{y=\text{const}} (+6xy + 6y)\, dx + \int_{x=\text{const}} (\cancel{3x^2} - 3y^2 + \cancel{6x})\, dy$$

vs

$$v = \frac{36x^2}{2} y + 6xy + \cancel{3x^2y} - \frac{3y^3}{3} + \cancel{6xy} + c$$

$$\boxed{v = 3x^2y + 6xy - y^3 + c}$$

$$f(z) = u + iv$$

If $f(z)$ is analytic function then show that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^n = n^2 |f(z)|^{n-2} |f'(z)|^2$$

LHS

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^n$$

$$\left(4\frac{d}{dz}\frac{d}{d\bar{z}}\right)|f(z)|^n$$

$$\left(4\frac{d}{dz}\frac{d}{d\bar{z}}\right)\left[|f(z)|^2\right]^{n/2}$$

$$\left(4\frac{d}{dz}\frac{d}{d\bar{z}}\right)\left[f(z) \cdot \bar{f}(\bar{z})\right]^{n/2}$$

$$\left(4\frac{d}{dz}\frac{d}{d\bar{z}}\right)\left[f(z) \cdot \bar{f}(\bar{z})\right]^{n/2}$$

$$4\frac{d}{dz}[f(z)]^{n/2} \cdot \frac{d}{d\bar{z}}[\bar{f}(\bar{z})]^{n/2}$$

$$= 4\left\{ \frac{n}{2} f'(z) \left[f(z)\right]^{n/2-1} \cdot \frac{n}{2}\left[f(\bar{z})\right]^{n/2-1} \bar{f}'(\bar{z}) \right\}$$

$$n^2 \left\{ f(z) \cdot \bar{f}(\bar{z}) \right\}^{n/2-1} |f'(z)|^2$$

$$n^2 \, |f(z)|^{n-2} \, |f'(z)|^2$$

RHS

10/10

22/4/19