Assignment No.1

Q1] Solve the following Differential Equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^{3x} + \sin 2x$

Ans: $y = c_1 e^x + c_2 e^{2x} + e^{3x}\left(\frac{x}{2} - \frac{3}{4}\right) + \frac{1}{20}(3\cos 2x - \sin 2x)$

Q2] Solve the following differential equations by method of variation of parameters:

1] $(D^2 - 6D + 9)\, y = \frac{e^{3x}}{x^2}$ $Ans: y = (c_1 + c_2 x)\, e^{3x} - e^{3x}(1 + \log x)$

2] $(D^2 + 9)\, y = \frac{1}{1 + \sin 3x}$ $Ans: y = (c_1 \cos 3x + c_2 \sin 3x) + \frac{1}{9}(-1 + \sin 3x - 3x\cos 3x + \sin 3x.\log(1 + \sin 3x))$

3] $\frac{d^2y}{dx^2} + 4y = \tan 2x$ $Ans: y = c_1 \cos 2x + c_2 \sin 2x - \frac{1}{4}\cos 2x \log(\sec 2x + \tan 2x)$

Q3] Solve the following differential equation:

$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2 \log x$ $Ans: y = x^2(c_1 \cos \log x + c_2 \sin \log x) + x^2 \log x$

Q4] Solve the following differential equations:

1] $(x+2)^2\frac{d^2y}{dx^2} + 3(x+2)\frac{dy}{dx} + y = 4\sin[\log(x+2)]$

ANS: $y = [c_1 + c_2 \log(x+2)](x+2)^{-1} - 2\cos[\log(x+2)]$

2] $(2x+1)^2\frac{d^2y}{dx^2} - 6(2x+1)\frac{dy}{dx} + 16y = 8(2x+1)^2$

ANS: $y = [c_1 + c_2 \log(2x+1)](2x+1)^2 - (2x+1)^2[\log(2x+1)]^2$

Q5] Solve simultaneously $2\frac{dx}{dt} - x + 3y = \sin t$; $2\frac{dy}{dt} + 3x - y = \cos t$

Q6] An e.m.f. Esinpt is applied at t=0 to a circuit containing a condenser C and inductance L in series.

The current I satisfies the equation $L\frac{dI}{dt} + \frac{1}{C}\int I\, dt = E \sin pt$, $where$ $I = -\frac{dQ}{dt}$ If $p^2 = \frac{1}{LC}$ and

initially current I and charge Q are zero then show that the current in the circuit at time t is given by

$\frac{E}{2L} t \sin pt$

7] Find the Fourier Sine Transform of

$$f(x) = \begin{cases} 1 - x^2 & |x| \le 1 \\ 0 & |x| \succ 1 \end{cases}$$

8] Find the Fourier Cosine Transform of $f(x) = \begin{cases} x^2 & 0 \le x \le a \\ 0 & x \succ a \end{cases}$ and find Fourier cosine integral representation.

9] Using Fourier integral representation, show that $\int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda = \begin{cases} \frac{\pi}{2} & 0 \le x \prec 1 \\ \frac{\pi}{4} & x = 1 \\ 0 & x \succ 1 \end{cases}$

10] Find the Fourier Sine Transform of $e^{-2x} \sinh x$.

11] Using inverse sine Transform, find f(x) ,if $F_s[\lambda] = e^{-\lambda}$, $\lambda \ge 0$.

12] Find Z transform of the following

1] $f(k) = \sin(4k + 3)$; $k \ge 0$ 2]. $f(k) = 3^k \cos(2k - 5)$; $k \ge 0$

3] $f(k) = e^{-2k} \sinh 3k$; $k \ge 0$ 4] $f(k) = ke^{3k} \sin(4k + 5)$; $k \ge 0$

5] $f(k) = k3^k$; $k \ge 0$ 6] $f(k) = \frac{a^k}{k}$; $k \ge 1$

13]Find inverse Z-transform of

1. $F(z) = \frac{1}{\left(z - \frac{1}{4}\right)\left(z - \frac{1}{5}\right)}$, $|z| < \frac{1}{5}$
2. $F(z) = \frac{1}{(z-4)(z-5)}$, $4 < |z| < 5$
3. $F(z) = \frac{1}{(z-a)^2}$, $|z| < |a|$
4. $F(z) = \frac{1}{(z-2)(z-3)}$, $|z| > 3$
5. $F(z) = \frac{1}{(z-5)^3}$, $|z| > 5$

6. 6. $F(z) = \frac{z^2}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{3}\right)}, \ |z| > \frac{1}{2}$
7. $F(z) = \frac{3z^2 + 2z}{z^2 - 3z + 2}, \ 1 < |z| < 2$

14] Solve the following difference equations

1. $f(k+2) + 3f(k+1) + 2f(k) = 0 ; \ k \geq 0, \ f(0) = 0, \ f(1) = 1.$
2. $f(k+1) - f(k) = 1; f(0) = 0, \ k \geq 0.$