Math for Data Science-Practice Problems

1. Minimize $f(x) = \frac{x^2 \cos(x) - x}{10}$ using gradient descent method.

2. Use gradient descent to approximate sin(x) with a 5-degree polynomial within the range -3 < x <3. Explore the result for different step sizes.

3. a) Using Taylor series find an approximate value of the function $f(x, y, z) = e^{xyz}$ at (1,1,0).

Q3(b) (i) Derive the two variable second order Taylor series approximation, below, to f(x,y) = $x^3 + y^3 - 6xy$ centred at (a,b) = (-5,3)

$$f(x,y) \approx Q(x,y) = f(a,b) + \left.\frac{\partial f}{\partial x}\right|_{(a,b)} (x-a) + \left.\frac{\partial f}{\partial y}\right|_{(a,b)} (y-b)$$

$$+ \frac{1}{2!}\left[\left.\frac{\partial^2 f}{\partial x^2}\right|_{(a,b)} (x-a)^2 + 2\left.\frac{\partial^2 f}{\partial x \partial y}\right|_{(a,b)} (x-a)(y-b) + \left.\frac{\partial^2 f}{\partial y^2}\right|_{(a,b)} (y-b)^2\right]$$

Q3(b)(ii) Calculate and state this approximate value at (x,y) = (4.5,-5)
Q3(b)(iii) Calculated and state the actual value of f(x,y) at (4.5,-5)
Q3(b)(iv) Calculated and state the error, Q(x,y) – f(x,y) at (4.5,-5)

4. Find the extremum of $f(x, y, z) = x^3 + y^3 + z^3 - 9xy - 9xz + 27x$ subject to $x^2 + y^2 + z^2 = 1$.

5. Find the minimum of the given function using Stochastic gradient descent method.

x	-11.5152	-9.8181	-5.4545	-1.3333	3.5151	6.1818	8.1212	10.0606
y	7.7352	-7.9215	2.5563	0.17515	-1.5019	3.1837	-2.5537	-9.1497

Or find the attached excel sheet for the entire data.

6. Find the minimum of f(x, y) = $3x^2+y^2$ with initial values x_0 = 1 and y_0 = 3 with learning rate $\alpha = 0.9$ using Ada Grad method.

7. Find the minimum of f(x, y) = $3x^2+y^2$ with initial values x_0 = 1 and y_0 = 3 with learning rate $\alpha = 0.1$, $\rho = 0.9$ using RMS prop method.

8. Find the minimum of f(x,y) = $3x^2+y^2$ with initial values x_0 = 1 and y_0 = 3 with learning rate $\alpha = 0.2$, $\beta 1 = 0.9$, $\beta 2 = 0.999$ using Adam method.