Qtext :

Represent the standard inner product in $\mathbb{R}^3$ with respect to the basis $(1,0,1),(0,1,1),(1,1,0)$. Also find the inner product between $(-2,1,3)$ and $(-4,5,9)$. [4 Marks]

Qtext :

(a) Let A and B be two $n \times n$ matrices. A Linear Algebra Professor asked whether rank of B and AB are the same if A has full rank. What should be the answer? Justify.
(b) If $A^2 = A$, $B^2 = B$ and $I - A - B$ is invertible then help the students to prove A and B are of same rank. [5 Marks]

Qtext :

A student in MTech datascience was revising Linear Algebra for his end semester exam. Then he came across a 3 by 3 matrix named $\mathbf{G}$ in a linear algebra textbook.

$$\mathbf{G} = \begin{bmatrix} 2 & \gamma & 0 \\ \gamma & 2 & \gamma \\ 0 & \gamma & 2 \end{bmatrix}$$

where γ is an unknown real number. His friend named A1 then asked him the following questions about this matrix :

(a) For which possible values of parameter γ, can we be sure that the matrix $\mathbf{G}$ will only have non-zero eigenvalues.

(b) If all the eigenvalues of $\mathbf{G}$ are known to be positive then what is the possible range of values that parameter γ can take.

Derive answer to the questions asked by A1. [4 Marks]

Qtext :

Consider a vector $\mathbf{r} \in \mathbb{R}^k$. Here $k > 102$. Consider a matrix $\mathbf{M}$ defined as follows : $\mathbf{M} = \mathbf{r}\mathbf{r}^\mathbf{T}$. Based on this definition answer the following :

(a) Howmanynon-zero eigenvalues does the matrix M have ?.Derive all the nonzero eigenvalues.

(b) Find the sum of absolute value of eigenvalues of this matrix i.e $\sum_{i=1}^{k} |\lambda_i|$ where λ_i is the i^{th} eigenvalues of $\mathbf{M}$.

(c) How many non-zero singular values does $\mathbf{M}^\mathbf{T}\mathbf{M}$ have. Also, derive the value of all non-zero singular value of $\mathbf{M}^\mathbf{T}\mathbf{M}$.

[4 Marks]

Qtext :

If $\{v_1, v_2, \ldots, v_n\}$ is an orthonormal set in V and if $w \in V$, then show that $u = w - \sum_{i=1}^{n} < w, v_i > v_i$ is orthogonal to each of $v_i (i = 1, 2, \ldots, n)$.

[3 Marks]

Qtext :

Let the vector $f = 2x_1^2 + 2x_1x_2 + 3x_2^2 - x_1 - x_2$. Calculate the gradient of f and find a point where it vanishes, if there exists such a point.
[3 Marks]

Qtext :

(a) Let $V = \{p(x)/p(x) = \sum_{i=0}^{5} c_i x^i, p(a) = 0\}$ be set of all polynomials of degree less than or equal to 5 with a as its root with usual scalar multiplication and polynomial addition. Is V a vector space? Justify. If yes, find a basis for V
(b) Let $V = \{A \in M_{2\times 2}(R)/A^{-1} \text{ exists}\}$ be set of all invertible real matrices of order 2 with usual scalar multiplication and matrix addition. Is V a vector space? Justify.
(c) If we replace matrix addition by matrix multiplication in the above

set, is V a vector space? Justify. [3 Marks]

Qtext :

Can the function $f(x) = 1 + 2x + 3x^2/2$ be the Taylor's series expansion to three terms for a function of the form $e^{\alpha x+\beta}$ around $x = 0$? If so, find α and β. Otherwise, explain why the given polynomial cannot be a Taylor series expansion of a function of the form $e^{\alpha x+\beta}$? [4 Marks]